Network Science

Lecture-1 30·09·24

Textbook- Network Science (Barabasi) Chapter 0 - Personal introduction

* degree, degree distribution

classification by degree distribution Paul Erdos?

* Network behind student movement

Introduction

* If we can explain the props of a system by the component's prop then this is not a complex system.

* A complex system does something beyond the union of the component.

* Universality of network characteristics
  - does not depend on the source of the network
  - properties independent of domain
  - Analysis is domain independent

- Solution can be extended to specific domain

Chars of Network Science

- interdisciplinary
- Empirical, data driven
- Quantitative and mathematical

Computational:

—o—

Denset Subgraph ⟶ each a suspected group

Management:

7 October 2024 Lecture - 2

Graph Theory for Network Science:

* Some mismatch in notation between graph theory and net. science
* Koninsberga
* Euler's proof is the birth of network/graph theory
* degree of all vertex has to be even

* This is because if I enter a city through a path, then I have to take another path to leave.

* Euler only proved necessity.

* sufficiency is proved much later

* Eulerian graph

Network and Graph:

nodes, vertices N

links, edges L

network, graph (N, L)

* refer to real system as network
* " " underlying math as graph.

A common Language

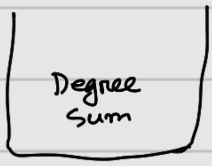
Degree: k_A (undir) $\qquad$ k_B^{in} k_B^{out} (Dir.)

First Theorem of Graph Theory.

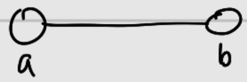
Degree Sum Formula: $\sum_{v \in V} \deg(v) = 2m$; $m \to$ # edges

Applicable to simple and multigraph

proof:



bag



each edge contributes 2 in the bag.

multigraph $\to$ multiedge OR loop (self) present

Degree Sequence: List of vertex degrees. Non increasing order

exa: 5, 4, 3, 3, 3, 2

Degree Set: distinct degrees in degree sequence

exa: {5, 4, 3, 2}

Degree seq: Necessary and sufficient condn

sum ⟷ even

* # odd degree is even.

To construct a graph from seq:

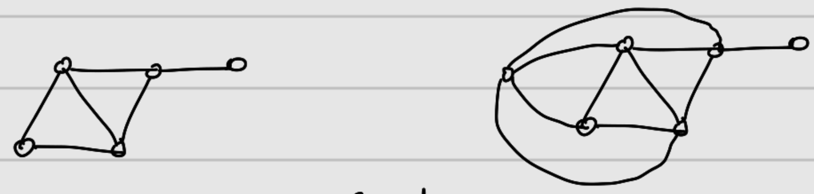
pair up the odd degrees. add edge between pairs, Then add self loops.

* Graphic Sequence: Degree Sequence of simple graph

* 2,2,0 not a graphic sequence

* Is given a sequence Graphic Sequence? ← This type of problems are called realizability problem

hakemi - 1962 hekel - ...



3, 3, 3, 2, 1 $\xleftrightarrow[\text{only if}]{\text{If and}}$ 4, 4, 4, 4, 3, 1

graphic seq. graphic seq.

$\xrightarrow{\hspace{2em}}$ Straight forward

$\xleftarrow{\hspace{2em}}$ Not straight forward

Degree Distribution: $p(k)$: probability that a randomly chosen node has degree k.

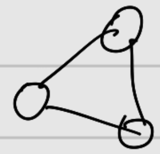
Subgraphs: $G=(V,E)$ $G'=(V',E')$

if $V' \subseteq V$, $E' \subseteq E$ then G' is a subgraph of G.

Vertex induced subgraph: particular subset of vertex and all the edges between them.

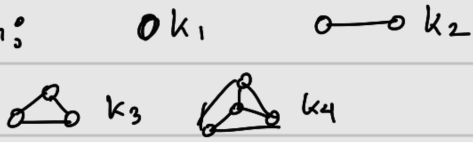
Edge induced subgraph: Similar

Singleton: 

Triad: 

Dyad: 

Clique: a subgraph which is a complete graph

Complete Graph: 

Cluster: Subgraph which is not a clique but very dense.

Egocentric Network: Degree-1 network
" -1.5 "
. . .

* a particular vertex, and their neighbors forming a subgraph → Degree-1

* add edges between the neighbors to the graph → Degree 1.5 network.

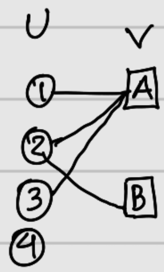
* Clustering Coefficient : edges between neighbors / total possible edge between them.

* Neighbor's neighbors ⟶ Degree-2 ??

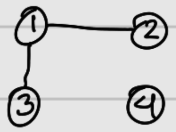
Digraph ⟶ directed graph

Graph Representation :

Bipartite Networks :



Projection U



Pathology :

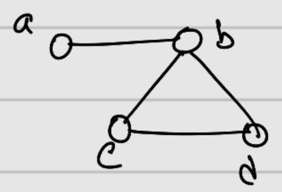
Walk : $W = v_0, e_1, \ldots, v_{f-1}, e_f, v_f$ Can visit any edge/vertex multiple times

Trail : edge cannot be repeated

Path : Vertex cannot be repeated (except end vertices) → end vertices same then cycle.

↓

edges cannot be repeated



$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & & & & \\ b & & & & \\ c & & & & \\ d & & & & \end{array} \quad \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{matrix} a & b & c & d \end{matrix} \\ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \end{array} = A$$

$$A^2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

$A^2 \longrightarrow$ entries indicate how many walks of length 2

<u>Connected Graph:</u>

<u>Connected Components:</u> maximal connected subgraph.

<u>Circuit:</u> closed trail

<u>Eulerian trail:</u> trail that has every edge exactly once.

<u>Eulerion Circuit:</u>

Constructive proof for sufficiency

Diameter
Radius
Eccentricity
Center

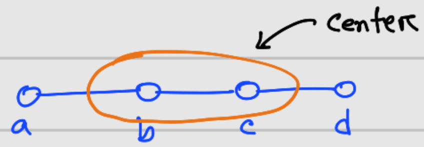
Diameter: (of a graph) max among all pairwise shortest distance

Radius: (of a graph)

Eccentricity: (of a vertex) max of distance of all other vertex from it

(of a graph)
diameter: max of eccentricity
radius: min of "

Center: (of a graph) vertex induced subgraphs by the nodes with minimum eccentricity



For any tree, center is either a single vertex or an edge

Finding Tree center algo: recursively delete leaf until left with a vertex or edge

Centrality: is a measure of determining which nodes are most influential.

Depending on application, various ways to define centrality.

Degree Centality: centrality = degree

Closeness Centrality: avg (shortest path length to all other nodes)

Center = node with minimum

Betweenness Centrality: Can be measured for both vertex and edge.

Take all pairwise shortest path. count how many times a node / edge appears in the shortest paths.

If between two nodes multiple path are shortest, then include all.

Eigenvector centrality:

importance depends on neighbor's importance.

— x — centrality — x —

Average path length/distance: ^ . . . . .

Hamiltonian Path: a path in G that includes every nodes in G.

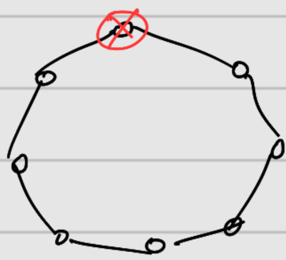
If the path is cycle, then it's called Hamiltonian Cycle and the graph is called Hamiltonian Graph.

necessary condition: ① Connected ② Cycle presents . . . . -.

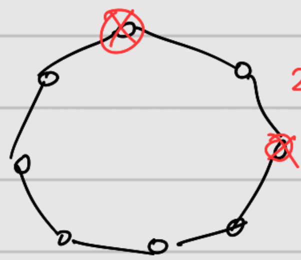
another 〃 〃 :

For each subset S, the number of connected components in G-S is at most G.

This is true for any cycle.

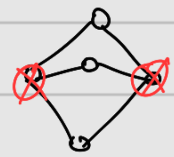


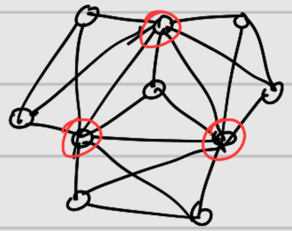
1



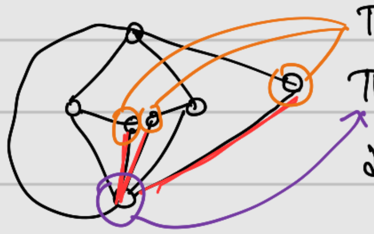
2

graph b in slide not Hamiltonian





d not Ham Expl:



To include these in a cycle This node is required and it's degree will be 3. but in a cycle all vertices' degree is 2.

Ore's Theorem ⟶ sufficient condition

Connected Graph (undirected): connected vs adjacent

Component: maximal connected subgraph

Strongly and weakly connected (Directed): weakly → connected after removing direction

strongly → " with directed edge.

Connectivity (K): min #vertices to make the graph disconnected or a single vertex graph K_1.

K_n → Complete graph of n vertices.

K_4 → longest K_n that can be drawn without edge crossing.

k-Connected: connectivity $\geq k$

Separating set / Vertex cut: set of vertex to remove to make it disconnecte

Edge Connectivity: min #edge to make it disconnected (κ'

k-edge- Connected: . . . .

$\delta(G) \longrightarrow$ minimum degree

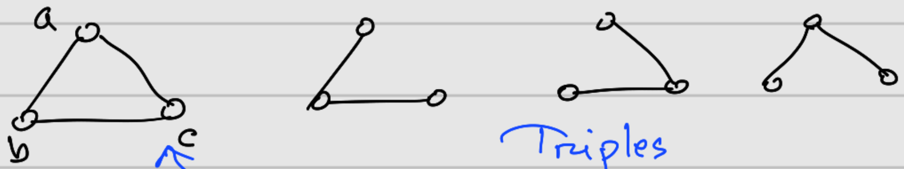
$$\kappa(G) \leq \kappa'(G) \leq \delta(G)$$

Block - Cutvertex Tree: . . . .

(of a node)
Clustering Coefficient: $C_i = \dfrac{e_i}{\frac{k_i(k_i-1)}{2}}$??

possible edges among neighbors

Global clustering coefficient:



Triples

of

Triple is very important in Social Science.

This is called forbidden structure.

Complex Network Conference

04 November, 2024 L-5

Random Network Model

Real world net এর mathematical model create করতে চাই।

Fractal Geometry Erdos number

Erdos - Renyl

G(N,p) model বেশি study করা হচ্ছে

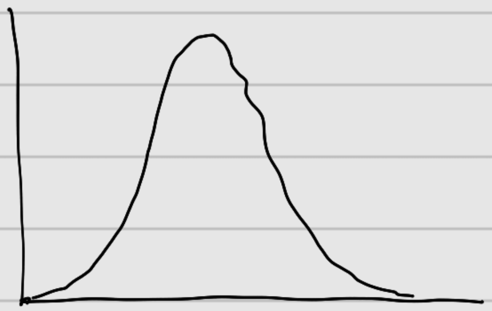
$\hookrightarrow$ #links is variable

p(L) : The prob to have exactly L links

$$p(L) = \binom{\binom{N}{2}}{L} p^L (1-p)^{\frac{N(N-1)}{2} - L}$$ Binomial Distribution

Degree Distribution

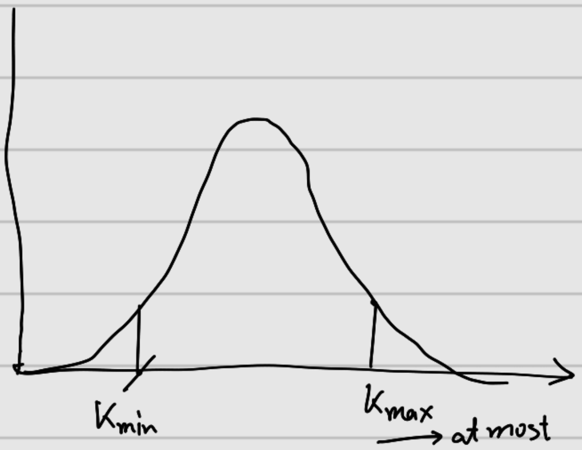
$p(k) \longrightarrow$ prob. that a randomly chosen node has degree k.



$$p(k) = \binom{N-1}{k} p^k (1-p)^{N-1-k}$$

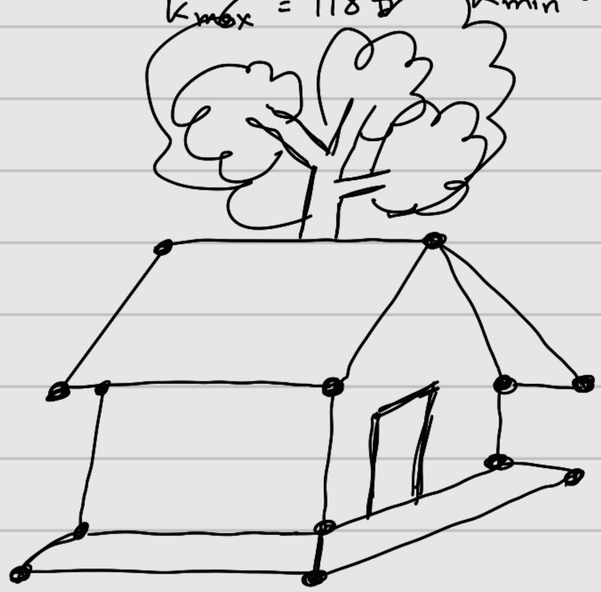
poisson distr শুধু degree এর উপর depend করছে

Real Networks are not Poisson



$\langle k \rangle = 1000 \quad N = 10^9$

$k_{max} = 1185 \quad k_{min} = 816$; not realistic



The evolution of a random Network

Rafi vai (boss of random net|K)

[case closed]

hehe

Shuaib Vai Zindabad