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Vulnerability Analysis of Railway Networks in Case of Multi-Link Blockage

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Abstract

In this paper, we propose a methodology to analyze the most critical links of a railway network based on flow interdiction. Our strategy for network interdiction is to maximize network disruption by removing the links with the greatest impact to the system. For this purpose, we first introduce our primary model to determine vulnerable links based on routing costs, which are based on the minimum cost model. Next, we propose a heuristic approach to solve this model with partial enumeration of network components to assess the most vulnerable parts. Since an important factor in system vulnerability is flow, we introduce the time-space network flow model as the second model to simulate train flow in the network. After interdicting critical links in the railway network, the trains are scheduled in the residual network with considerations of various factors including customer demand, track and station capacities, and time planning horizon. The paper includes a computational instance which has been analyzed by the proposed models under various disruption scenarios, and the results are compared with full enumeration of network components using a network scan method. The accuracy of obtained results indicates the effectiveness of the proposed method in addition to fast computational time compared to the enumeration method.

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Keywords: Railway network, Vulnerability, Scheduling, Combinatorial optimization

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1. Introduction

An important indicator of the economic growth of every society is its transportation infrastructures. Among various transport modes, railways have a vital role in the national economy by moving great amount of passengers and goods across the country. In Iran, railroads annually shipped over 35 million tons of goods and 25 million passengers along the 10,000 km integrated network which play a key role in the economy which contribute to 8.5% Gross Domestic Product (GDP) of the country (RAI, 2014). In light of the above, it is evident that disruption in the network operation resulting from temporary track blockage would not only have an unfavourable impact on the customer service but also an adverse effect on the economy. Examples of such events include natural disasters like earthquake, flood, or extreme weather conditions, and less frequently occurring events like terrorist attack or strike which cause the stations and tracks to be closed. Such disruptions often necessitate rescheduling and/or rerouting of trains and invariably result in longer trip times and unexpected delays. Therefore, railway authorities may be eager to know the most vulnerable nodes/links of their network in order to establish protection priorities to improve those parts of the networks.

This paper focuses on the model building methodology to analyze the most critical links of a railway network based on the concept of interdiction. Our strategy for network interdiction is to maximize network disruption by removing the links with the greatest impact to the system. For this purpose, we first introduce our primary model to determine vulnerable links based on routing cost. Next, we propose a heuristic approach to solve this model with partial enumeration of network components to assess the most vulnerable parts. Since an important factor in system vulnerability is flow, the time-space flow network model is introduced to model real train flow in the network.

The paper is structured as follows. In the next section, related studies on the transportation vulnerability are described and an overview of the most recent accomplishments on this topic is provided. Section 3 introduces our models of vulnerability assessment in two subsections. Section 4 applies the proposed methodology to a numerical case. The final section, section 5, summarizes the main results and conclusion, and also provides potential directions for future research.

2. Literature review

The vulnerability of transportation networks has been a well-researched topic in recent years. The initial engagements in this domain were introduced by interdiction models. They identify critical nodes and links by modeling a game between an adversary and an operator of the network, who routes flow through the network after the adversary makes his attack. Fulkerson and Harding (1977) were among the first to study how to interdict arcs in a network to maximally increase the length of the shortest path. Wollmer (1964) provided a model for interdicting a maximum-flow network; they are followed by others (Israeli and Wood, 2002). Researchers have also considered other objectives such as minimizing the maximum reliability path (Pan and Morton, 2008) and minimizing the maximum profit (Lim and Smith, 2007). Further, Church et al. (2004) presented models for interdicting a set of facilities. However, early optimization attempts had a limited success and could not incorporate many real-life characteristics of rail operations.

The concept of vulnerability of transportation network was first introduced in the literature by Berdica (2002). She defines vulnerability as susceptibility to disruptions that could cause considerable reductions in network service or the ability to use a particular network link or route at a given time. Jenelius et al. (2006) argued that road network vulnerability is composed of the probability and consequences (represented by an increased generalized travel cost) of single or multiple link failures.

Until now, most work in transport network vulnerability analysis has focused on the road network and personal vehicle travel with consideration network structure and travel patterns of the users (Jenelius, 2009, Scott et al., 2006, Taylor, 2012). Among these studies, railway transportation is the mode which has received less attention compared to other modes. A review by Wang et al. (2014) revealed only 8 present of research efforts focused on vulnerability of railway systems. Besides, the research methodology and vulnerability characteristics of different transportation modes varied significantly.

In some recent studies, Burdett and Kozan (2014) proposed an approach to prepare a robust timetable that is capable of detecting the critical operations and their impacts on others in case of any delay. Gedik et al. (2014)

presented a two-stage mixed integer programming interdiction model to identify the impacts of vulnerable infrastructure elements in a rail transportation network. Their model captures the movement of unit trains and identifies the critical nodes in the network whose unavailability causes the largest destruction in terms of total transportation and delay costs. However, they give a vague outline of integrating k-shortest path method into the main model to find alternative routes after disruption. Moreover, they consider the constraint of balancing train flow only at terminal stations rather than every intermediate station. In another research, Khaled et al. (2015) proposed heuristics to identify critical rail infrastructure based on the increased cost (delay) incurred when the element is disrupted. They considered the capacities at yards and links and the congestion factor that may occur during catastrophes. Zhang et al. (2015) also developed a methodology based on the gravity model to consider infrastructure interdependencies. Some of the above studies selected vulnerable parts of a network by defining some indicators while the other ones developed models that require explicit enumeration of network components. Although the above efforts have promising results for modelling railway vulnerability, but addressing the problem of railway vulnerability demands more comprehensive model building effort, considering real-life rail operation characteristics like simultaneous re-routing and re-scheduling of trains in case of disruption.

The main contribution of our work is its analytical aspect of mathematical programming model which attempts to incorporate real-life situations. That is, the model presented here aims to facilitate determining critical links of a railway network with consideration of supply and demand interaction under different disruption scenarios. A disruption scenario is the failure, or partial or full closure, of one link or a combination of different links (Dehghani et al., 2014). The proposed models also have the capability to consider capacity of each component of network in a time and space horizon. Also from the computational viewpoint, our proposed methodology unlike previous ones do not need full enumeration of all links to find most vulnerable ones.

3. Vulnerability modeling of railway transportation system

One way to study the vulnerability of a network is to identify the critical nodes and links in the network. The ability to account for link attributes is a prerequisite to assess the potential damage to the network. As mentioned previously, links connect the nodes to form a network that can be used to send flows (passenger and freight) between each origin to each destination. We invite reader to Assad (1980), Ahuja et al. (2005), and Nemani and Ahuja (2011) for a variety of modeling techniques that have been developed for the rail transportation systems.

Consider a railway network with a set of stations and tracks. We model this physical network by a graph, its stations by nodes and its tracks by links. Let $G = (N, E)$ be an undirected graph with node set N and link set E. The link $ij \in E$ has a travel cost C_{ij}. The trains travel from their origin station set $O(r \in R)$ to destination station set $D(r \in R)$ to satisfy demands for goods, where R is the set of routes between each OD pair.

In this section, we describe the development of vulnerability analysis in two stages. In first stage described in section 3.1, we introduce our primary model which determines the most vital (or vulnerable) links based on routing costs. In the second stage described in section 3.2, we develop a scheduling model to re-optimize train movements on the residual network with available nodes and links. The notations shown in Table 1 are adopted for the mathematical formulation of vulnerability analysis on a railway network.

Table 1. Notations

Sets		Parameters	
N	set of nodes (or stations)	Δ	length of time period
E	set of links (or tracks)	τ_{ij}	travel time of link ij ∈ A in multiples of Δ
R	set of routes between all OD pairs	TC_{ij}	track capacity of link ij ∈ A
O(r)	origin station of route r	nt_i	loading capacity at station i ∈ N
D(r)	destination station of route r	h_i	demand at destination station i ∈ D(r)
A(r)	set of links on route r	g_r	cost of moving a train on route r ∈ R
I(r)	set of nodes in route r ∈ R	q	waiting cost per unit time period at each station

T	set of time periods	C_{ij}	travel cost of link ij ∈ E
		Variables	
$F^t_{r,i}$	number of trains departing from i ∈ O(r) on route r ∈ R at time period t ∈ T	x^r_{ij}	indicator variable, 1 if link ij ∈ E is on route r ∈ R; 0 otherwise.
$W^t_{r,i}$	number of trains waiting at i ∈ D(r) on route r ∈ R at time period t ∈ T	y_{ij}	interdiction variable, 1 if link ij ∈ E is disrupted; 0 otherwise.

3.1. Primary Vulnerability Model

The primary model in our vulnerability assessment framework is based on the minimum cost flow problem with considerations of multiple origins and destinations (ODs). We formulate this problem as a bi-level integer linear programming model by introducing variables x^r_{ij} to represent whether link ij is located on route$r \in R$ and y_{ij} as an interdiction variable which is 1 if link ij is blocked and 0 otherwise. The number of interdicted links is specified by parameter n with according to a disruption scenario. The disruption scenario determines the type of failure in the network, which it might be a single-link or multiple-link disruption.

The lower level in the primary model minimizes the routing costs in the network with available links and is specified by objective function (2c) and constraint sets (2d)-(2g). However, the upper level maximizes the routing costs by blocking the most critical links and includes objective function (2a) and constraint set (2b). In other words, while the lower level minimizes the flow cost in the residual network, the upper level seeks the critical link(s) with the most flow cost when interdicted. Accordingly, the primary model is proposed as follows:

$$\max Z(y) \tag{2a}$$

$$\sum_i \sum_j y_{ij} = 2n \qquad \forall i, j \in N \tag{2b}$$

$$Z(y) = \min \sum_{r \in R} \sum_i \sum_j c_{ij} x^r_{ij} \tag{2c}$$

$$\sum_j x^r_{ij} - \sum_j x^r_{ji} = \begin{cases} 0 & if \quad \forall i \notin \{O(r), D(r)\} \\ 1 & if \quad \forall i \in O(r) \\ -1 & if \quad \forall i \in D(r) \end{cases} \tag{2d}$$

$$x_{ij} \le 1 - y_{ij} \qquad \forall i, j \in N \tag{2e}$$

$$y_{ij} = y_{ji} \tag{2f}$$

$$x^r_{ij}\, ,\, y_{ij} \in \{0,1\} \tag{2g}$$

Objective function (1a) maximizes the routing cost. Constraint set (2b) sets the number of interdicted links according to the disruption scenario. Since links are undirected in our problem, there is no difference between link ij and linkji. Objective function (2c) minimizes the routing cost of the residual network. Constraint set (2d) ensures the flow conservation at all nodes. Constraint set (2e) guarantees that no train travels along link ij if it is disrupted. The symmetry of network flow at links is met through constraint set (2f). Finally, the domain of variables is determined by constraint set (2g).

A common approach to solve such max-min, bi-level problem is to enumerate all possible combinations of network components by comparing the total cost before and after the disruption (interdiction), namely performing a network scan. However, a practical issue when analysing combinations of failures is that the number of failure combinations to evaluate increases exponentially as the number of components of the network increases and as the number of simultaneous component failures of interest increases. To overcome this difficulty, our proposed approach is to reduce the bi-level problem to a single level maximization problem. Since the lower level has always an integral optimal solution, we reformulate this model to achieve a single level model. For this purpose, first we fix y variables and the lower level problem becomes a linear programming problem, which possesses the totally uni-modularity property to ensure that every basic feasible solution is naturally integer. Using the dual of inner problem, it can be converted to a single-level problem. The following represents the dual variables of inner problem:

$$Z(y) = \min \sum_{r\in R}\sum_{i}\sum_{j} c_{ij}x_{ij}^{r} \qquad \left[duals\right]$$

$$\sum_{j} x_{ij}^{r} - \sum_{j} x_{ji}^{r} = \begin{cases} 0 & if \quad \forall i \notin \{O(r), D(r)\} \\ 1 & if \quad \forall i \in O(r) \\ -1 & if \quad \forall i \in D(r) \end{cases} \qquad \left[\alpha_i^r\right]$$

$$x_{ij} \le 1 - y_{ij} \qquad \forall i, j \in N \qquad \left[u_{ij}^r\right]$$

The resulting single-level problem becomes:

$$\max \quad W(y) = \sum_{r}(\alpha_O^r - \alpha_D^r) + \sum_{r}\sum_{i}\sum_{j} u_{ij}^r(1 - y_{ij}) \tag{3a}$$

$$\begin{aligned} &\alpha_i^r - \alpha_j^r + u_{ij}^r \le c_{ij} \qquad \forall ij \in E \\ &\alpha_j^r - \alpha_i^r + u_{ij}^r \le c_{ji} \qquad \forall ij \in E \end{aligned} \tag{3b}$$

$$\alpha_i^r : urs \tag{3c}$$

$$u_{ij}^r \le 0 \tag{3d}$$

Constraint sets (2b),(2c) and (2g)

It should be noted that the objective function contains non-linear terms $u_{ij}^r y_{ij}$ which are a product of a binary variable and a continuous variable. We can linearize them by substituting variable $\bar{u}_{ij}^r = u_{ij} y_{ij}$ and applying constraint set (4a)-(4d).

$$\bar{u}_{ij} - My_{ij} \le 0 \tag{4a}$$

$$-u_{ij}^r + \bar{u}_{ij}^r \le 0 \tag{4b}$$

$$u_{ij}^r - \bar{u}_{ij}^r + My_{ij} \le M \tag{4c}$$

$$\bar{u}_{ij}^r \ge 0 \tag{4d}$$

Where M is a big number. Finally, our primary model of vulnerability would be:

$$\max \quad W(y) = \sum_{r}(\alpha_O^r - \alpha_D^r) + \sum_{r}\sum_{i}\sum_{j}(u_{ij}^r - \bar{u}_{ij}^r) \tag{5a}$$

Constraint sets (2b-2c),(2g) ,(3b- 3d) and (4a-4d)

The resulted model is a single-level mixed-integer problem (MIP) which does not require network scan procedure and therefore could be solved easily. When this model is optimized, the value of y vector becomes known. If the variable y_{ij} takes 1, the link ij would be the most critical link at the railway network according to disruption scenario. The marginal value associated with constraint set 3b specifies the routes of residual network for OD pairs. The next section is devoted to describe train scheduling model in the degraded network.

3.2. Scheduling model

In this section, we model the problem of train scheduling in the residual network. After finding critical links through primary vulnerability model, those links are removed from the network and trains are scheduled in the residual network. For this purpose, after surveying various scheduling models, we adapted time- space flow network introduced by Lawley (2008). This model keeps track of train movement at intermediate stations by considering travel time, dwell time and capacity of each link and station. However, we customized the original model slightly to deal with specific aspects of scheduling at a degraded network.

Time–space models are commonly used to model train scheduling and interested readers are referred to Sherali and Tuncbilek (1997), and Sherali and Suharko (1998) for more details.

Our model allows the user to specify both the time period and the planning horizon such that each station on route $r \in R$ is replicated for time periods over the planning horizon. The trains are scheduled to time periods so they may have to wait from period t to $t+1$ at station i until the track becomes available or to arrive to the next subsequent station j at period $t + \tau_{ij}$.The model schedules trains from origin stations to destination ones to maximize the demand satisfied by departing the most number of departing trains and minimize the total waiting times during a given planning horizon.

We now present the mathematical formulation of the time-space network flow model. The constraints ensure flow conservation at nodes and guarantee that track and station capacities are not exceeded. It is reminded that routes in time-space model are predefined, meaning that the output of routing model described in section 3.1 is the input of scheduling model.

$$C = \min \Big(\sum_{i|i\in O(r)} \sum_{t\in T} F^t_{r,i} g_r + \sum_{i|i\in I(r)} \sum_{t\in T} q\Delta W^t_{r,i} \Big) \tag{6a}$$

$$s.t. \quad W^{t-1}_{r,j} + F^{t-\tau_{ij}}_{r,i} = W^t_{r,j} + F^t_{r,j} \qquad \forall r\in R, t\in T \tag{6b}$$

$$W^{t-1}_{r,i} = W^t_{r,i} + F^t_{r,i} \qquad \forall i \in O(r), r\in R, t\in T \tag{6c}$$

$$\sum_{t\in T} F^t_{r,i} \le nt_i \qquad \forall i\in O(r), r\in R \tag{6d}$$

$$\sum_{t\in T} F^t_{r,i} = 0 \qquad \forall i\in D(r), r\in R \tag{6e}$$

$$\sum_{r|ij\in A(r)} \sum_{t=t'-\tau_{ij}}^{t'} F^t_{r,i} \le TC_{ij} \qquad \forall t\in T, ij\in A(r) \tag{6f}$$

$$\sum_{r|i\in D(r)} W^t_{r,i} \ge h_i \qquad t = |T|, \forall i\in N \tag{6g}$$

$$F^t_{r,i}\ , W^t_{r,i} \ge 0\ , \mathrm{int} \tag{6h}$$

The objective function (6a) minimizes the cost of departing trains and waiting time at stations. The first term in the objective function represents the traveling cost of loaded trains departing from an origin station during the planning horizon and the second deals with the time that trains spend in stations. Constraint set (6b) balances flow at each station of route r and time period t. The total number of waiting and departing trains at each station and time period t, $W^{\mathrm{t}}_{\mathrm{r,j}}$ and $F^t_{r,j}$ respectively, equals the sum of waiting trains at time period $t-1$ and the number of trains arrives at that station at time period t. Constraint set (6c) balances flow at the origin station of route r. Constraint set (6d) ensures the number of trains leaving every origin station is no greater than the loading capacity of that station.

Constraint set (6e) guarantees no train leaves destination stations. Constraint set (6f) limits the number of traveling trains on each track segment to its capacity. Demands at destination stations are satisfied by constraint set (6g). Finally, constraint set (6h) restricts all variables to be non-negative integers.

The above IP formulation schedules train movement in pre-specified routes. Time-space network flow model can be applied following the primary vulnerability analysis described in Section 3.1 to optimize train movements in the degraded network after removing critical links. Figure 1 shows the concept of vulnerability analysis of the proposed models in summary.

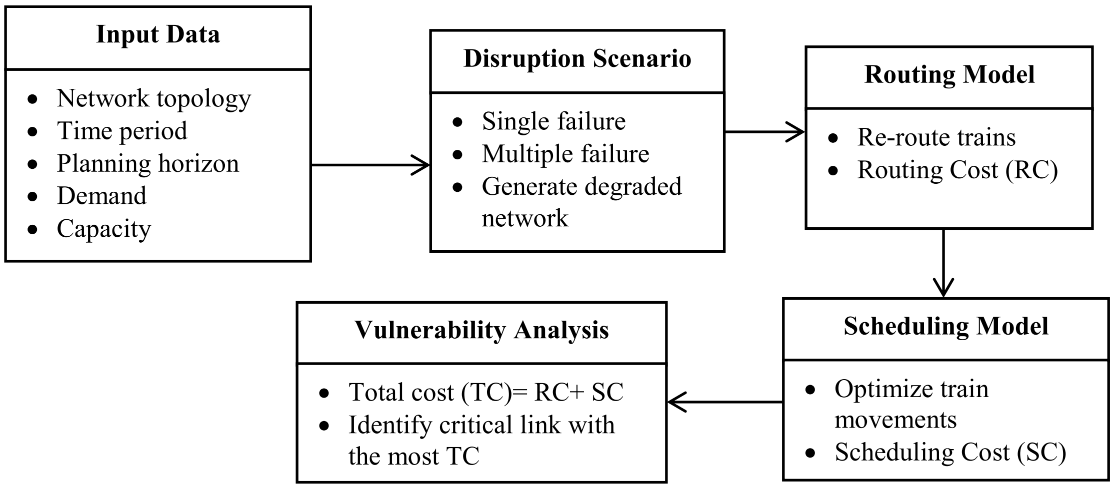

Fig.1. Concept of vulnerability analysis with proposed models

4. Numerical experiment

The proposed models of vulnerability are implemented in GAMS 24.1.2 language running on a PC with Intel Core i5 2.60 GHz CPU and 8.00 GB RAM. In this section, we present the result of a numerical experiment to show the capability of the proposed model in finding vulnerable links in the railway network and computational effectiveness of the heuristic method in comparison with complete enumeration, namely network scan.

Consider a hypothetical railway network with 9 nodes, 19 links, and 3 OD pairs as shown in Figure 2. The label on each link in Figure 3 is the length of links connecting two adjacent nodes. In our practical application, we experimented with 12h planning horizon and time interval of 1 hour. As the planning period increases and time interval length decreases, the number of variables and constraints increases exponentially and the model becomes more difficult to solve in a reasonable amount of time. Therefore, we should keep them as practically possible as limited to reach optimal solution in a reasonable computational time. Moreover, we consider a same average speed for unit train in order to specify travel time on each link. We also consider three types of scenarios according to the number of disrupted links. Table 1 shows input parameters to the model.

Table 1: Input parameters of railway network

Parameter	Value
O-D Pairs (r)	r1: L-B r2: D-B r3: P-W
Demand at destination station (unit train)	B: 15 W: 10
Loading capacity of origin station (unit train)	L, D, P: 20
Track capacity (unit train)	10
Holding capacity of each station (unit train)	10
Train dispatching cost ($)	1
Train waiting cost ($/h)	1
Train travel cost ($/mile)	1

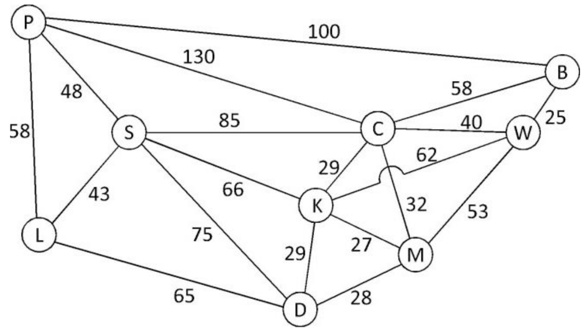

Fig. 2. Test rail network

We examine the criticality of network's component using three different failure scenarios. In the first scenario, single blockage of link is evaluated while the second and third types of scenarios consider combinations of two and three link disruptions, respectively. Results of the analysis for this small instance are presented in Table 2. In addition, the results of vulnerability evaluation of the presented instance with network scan method are incorporated in Table 2. In this approach, the criticality of each element is evaluated by comparing the total cost of routing and scheduling (objective function 7c) after disruption to (removal of) elements one by one. However, a practical issue during analysing combinations of failures in this method is that the number of failure combinations to evaluate increases exponentially as the number of components of the network increases and as the number of simultaneous component failures of interest increases. As the Table 2 shows, the result of this method fully matches that of proposed models.

Table 2: Results of vulnerability assessment for test network

Disruption Scenario	Number of Link Failures	Proposed Model		Network Scan	
		Vulnerable Links	Total Cost ($)	Vulnerable Links	Total Cost ($)
First	1	P-B	522	P-B	522
Second	2	B-W, B-C	611	B-W, B-C	611
Third	3	P-L, B-W, B-C	644	P-L, B-W, B-C	644

To show the result of train scheduling after disruption, we illustrate the corresponding time-space network flow model of the third scenario in Fig. 3. Moreover, Fig. 4 shows the re-routing of trains between three OD pairs in this network for this scenario. To satisfy demand at B station, 10 and 5 unit trains depart L and D stations at t_0, respectively. Similarly, 10 trains depart *P* station to arrive *W* at t_6. As the Figure 4 shows, the optimal routes from origin stations *D* and *L* to destination point *B* have the common link *PB*. As the link capacity is restricted to 10 trains in this example, the trains on origin-destination *D-B* have to wait at *P* and *S* stations until the link *PB* is released.

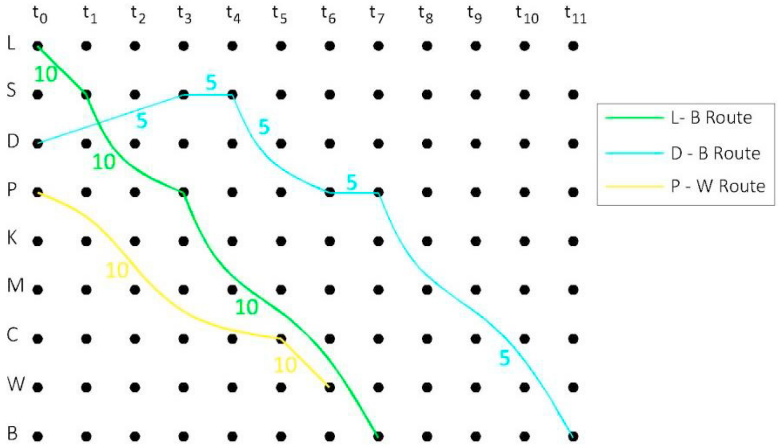

Fig.3. Time-space network for 3rd disruption scenario

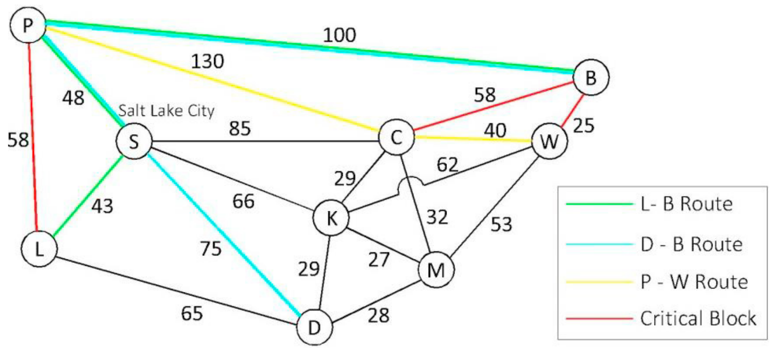

Fig.4. Trains re-routing at disrupted network under 3rd disruption scenario

The computational time of the integrated model is also compared with network scan method in Table 3. It easily depicts the superiority of the proposed models with regard to computational time.

Table 3: Comparison of solution time between proposed method and network scan

Disruption Scenario	Number of Link Failures	Computation Time (sec.)	
		Proposed model	Network Scan
First	1	0.2	2
Second	2	0.2	24
Third	3	0.3	144

5. Conclusions

The study on the vulnerability of railway networks has been considered much less extensively than that on the vulnerability of road and highway systems. Moreover, when a disruption occurs in the railway network, it is necessary to reschedule the timetable according to available links and stations. In this study, we address the evaluation of railway link vulnerability to disruption. In the section 3.1, we proposed the primary vulnerability model which was based on minimum flow cost. In fact, this primary model identifies those critical links with the maximum total travelling cost in case of removal. In addition, this model re-route trains to destinations in the degraded network. Although this model could be solved by explicit enumeration of all links, but the number of failure combinations increases exponentially as the number of railway links increases. To cope with this issue, we made another contribution by converting this bi-level model to a single level mixed integer linear programming (MIP) which can be solved by standard methods.

Moreover, the customer demand and system capacity are major concerns in the railway system. With introducing time-space network flow model, we considered these subjects in a scheduling model. The output of primary vulnerability model is used as input for time-space network flow model. Another contribution of this study is that this scheduling model is used along with routing model. The numerical instance illustrates the application of model in real situation in addition of its superiority to network scan method.

In summary, we tried to confront the vulnerability of railway infrastructure from an analytical viewpoint in this research effort. Nevertheless, the model can be improved by defining disruption probabilities and simulating disruption events (other than full closure) as direction for future research. Considering these issues can provide more realistic results.

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