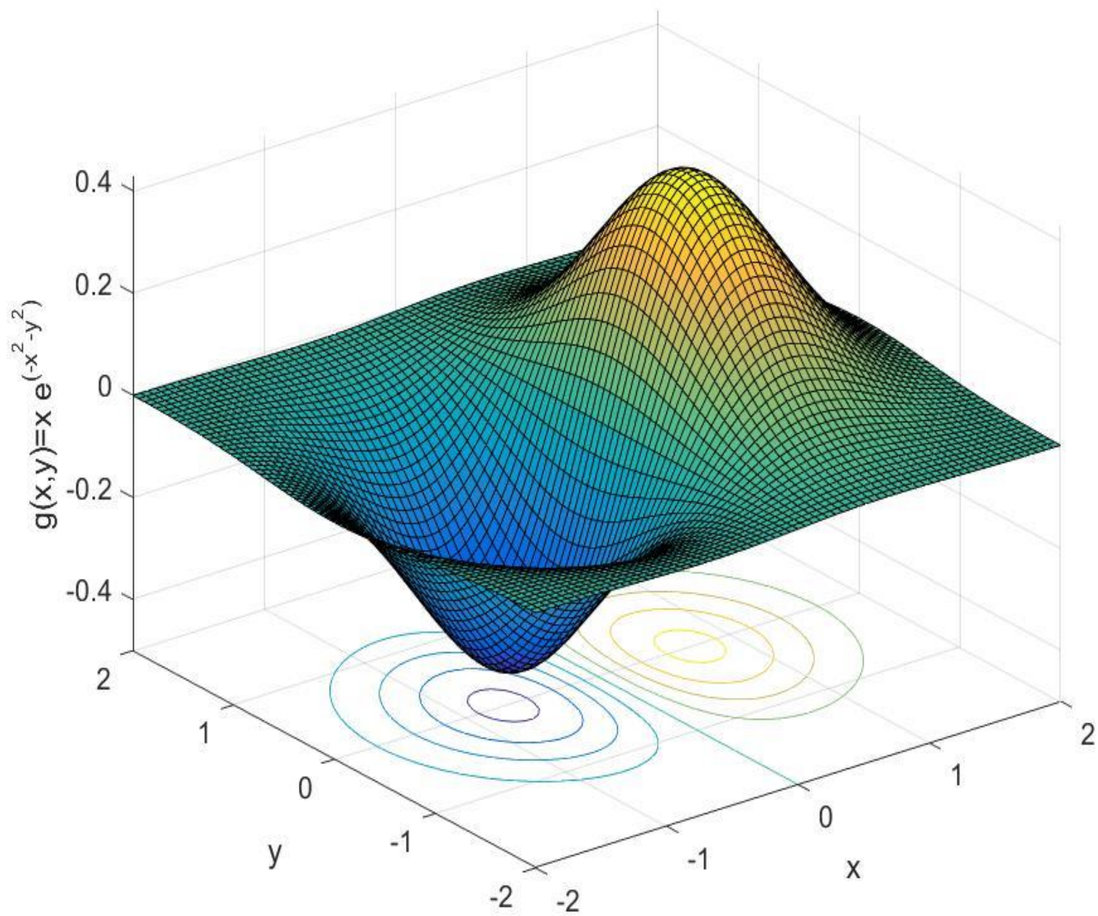
UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING
First Year Program – Core 8 and TrackOne

FIRST YEAR PROGRAM
ENGINEERING PROBLEM SOLVING LABS

MAT188: Laboratory #9
Three-Dimensional Plots



THREE-DIMENSIONAL PLOTS

In this lab, you will learn how to create three-dimensional plots to visualize data and use them to solve engineering problems. Three-dimensional plots are a useful tool for understanding sophisticated problems, allowing you to take experimental data and create a complex visual representation. They are more powerful than 2D plots and illustrate a mathematical model from a more informative perspective, which is often difficult to do by hand.

Learning Outcomes

By the end of this lab students will…

1) Know how to create a three-dimensional plot in MATLAB using both numeric and symbolic computation techniques, and
2) Further develop the ability to use matrix indexing to better understand how a function behaves.

Preparation *(Required to do* ***before*** *you come to the laboratory session)*

1. Read through this lab document.
2. Watch the posted ***Lab #9 Video Introduction, Part 1 and Part 2.***
3. Come up with three single variable functions that you would like to plot. You could make these up or perhaps you could find a function that is used to represent a physical phenomenon (e.g., Newton's law of gravitation, Einstein's (really Lorentz's) time-dilation formula, the normal distribution, etc.):

   $y_1(x) =$ ______________________________

   $y_2(x) =$ ______________________________

Related Reference Materials *(Not required, but may be a helpful resource)*

http://www.mathworks.com/help/matlab/examples/creating-3-d-plots.html
http://www.mathworks.com/help/matlab/visualize/representing-a-matrix-as-a-surface.html

Review – Labs 1-8

In previous labs, you have learned about how to work with matrices in MATLAB, and specifically how to:

1) *Define matrices* using semi-colons to separate the rows:

$A = \begin{bmatrix} -2 & 0 & 1 \\ 4 & -8 & 3 \end{bmatrix}$ can be defined using: `>> A = [-2 0 1;4 -8 3];`

$B = \begin{bmatrix} 0 & 8 \\ -1 & 5 \\ 0 & 4 \end{bmatrix}$ can be defined using: `>> B = [0 8;-1 5;0 4];`

2) *Doing basic matrix calculations and operations* including:
   Scaling (`3*A` or `A+5`), Addition (`A+3*A`), subtraction (5*`A-A`), matrix multiplication (`A*B`), element-by-element multiplication (`C.*C`), and transposes (`A'`).
3) *Index certain parts of a matrix* using the colon operator (:), such as:

```
>> A(2,1)
ans =
        4
>> A(2,2:3)
ans =
   -8   3
```

```
>> B(:,1)
ans =
                  0
                 -1
                  0
```

4) *Assign values to specific parts of a matrix* through commands such as:

```
>> A(2,:) = x.*exp(-2*x);
```

MATLAB Skills and Knowledge: Review and Maximum and Minimum Values

Defining Vectors and Matrices to Numerically and Graphically Represent Functions

To begin, complete the following exercise:

For the two functions that you identified above in your preparation, $y_1(x)$ and $y_2(x)$ write a short script that will:

(a) Create a single 2 x *n* matrix, `Y`, that contains the values of these two functions over a set domain for the independent variable (with *n* data points over this domain),

(b) Plot these two functions in a single figure over this domain through the use of different line types. This plot should be properly labeled and titled, and have a legend.

(c) Use the MATLAB commands `max` and `min` to find the maximum and minimum values of these two functions, $y_1(x)$ and $y_2(x)$ within the domain that you have chosen. ***Hint***: Review the online documentation for these functions at `max` and `min`, and the transpose operator, `transpose` may be useful to you. How is `max(Y)` different from `max(transpose(Y))`?

(d) Now use proper indexing of the matrix to plot these functions over half the domain.

MATLAB Skills and Knowledge: Three-Dimensional Plots

MATLAB can create many different kinds of 3D plots, but we will focus on the basic plots in this lab.

First, let's create a surface plot of the two-variable function $f(x, y) = z = y^2 - x^2$ over the domain $-3 \leq x, y \leq 3$.

We will start by defining the domains of the input variables (x and y) over the same values, but with different step sizes (the reason for this will be obvious soon):

```
>> domain_x = [-3:1:3]
>> domain_y = [-3:0.5:3]
```

In earlier labs, when you created 2D plots with one input variable, the input was a one-dimensional array (one vector). Now that we are creating a 3D plot with two input variables, the inputs must be two-dimensional arrays (i.e., two matrices).

The vectors `domain_x` and `domain_y` specify the domain of function f. Since this is a two-variable function, it applies to a set of (x, y) points in which x and y belong to `domain_x` and `domain_y` respectively.

The `meshgrid` function will allow you easily create a set of all pairs of x and y values, i.e., (x, y) points on the grid defined by $-3 \leq x, y \leq 3$. Notice how the function returns two matrices as outputs, which are assigned to the two variables `X` and `Y`.

```
>> [X,Y] = meshgrid(domain_x, domain_y)
```

What are the sizes of `X` *and* `Y`*?*
How do these compare to the sizes of `domain_x` *and* `domain_y`*?*
Look at the elements of `X` *and* `Y` *compared to* `domain_x` *and* `domain_y`*. Can you explain the differences?*

Now create the matrix `Z` with the value the function $f(x, y) = z$ over this domain. Notice how the dot operators are used for element-by-element operations to ensure that the function z is evaluated at each point within the (x, y) set.

```
>> Z = Y.^2-X.^2
```

Note: ***We have to use the matrices*** `X` and `Y` created through `meshgrid`, ***not the vectors*** `domain_x` and `domain_y`.

Observe how the different *rows* of `X, Y, Z` correspond to different y values, while the different *columns* correspond to different x values.

This means that as you move across the columns within a row, y is constant while x changes. As you move down the rows within a column, x is constant while y changes.

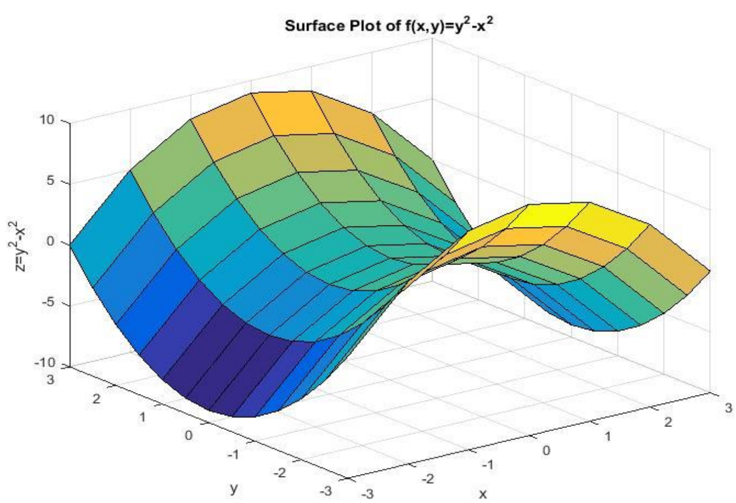
Now that we defined the function's values at these discrete points in the grid, we can plot it as a 3D figure. The two most common MATLAB functions to create 3D surface plots are `surf` and `mesh`, which have the same input syntax. As described in the MATLAB [Primer document] (Page 1-25):

> "`surf` displays both the connecting lines and the faces of the surface in color. `mesh` produces wireframe surfaces that color only the lines connecting the defining points."

Again note: For these plot commands, ***we have to use the matrices*** `X` and `Y` created through `meshgrid`, ***not the vectors*** `domain_x` and `domain_y`.

Let's use the `surf` (surface) plot first:

```
>> surf(X, Y, Z);
>> xlabel('x');ylabel('y');zlabel('z');
```



Experiment with the "Rotate 3D" button, ,in the toolbar to view the figure from different angles.

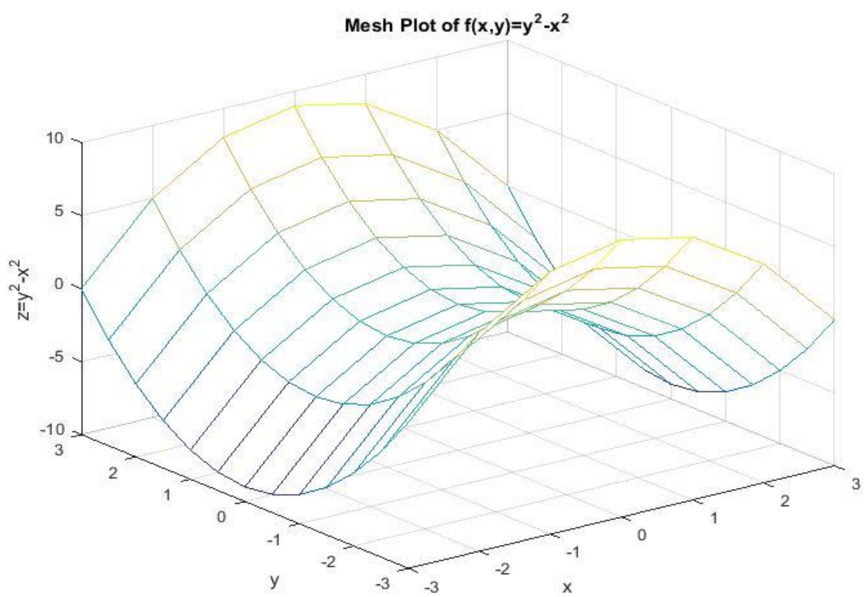
Observe how many "pieces", or segments there are with respect to the *x* and *y* domains.
How does this relate to the sizes of `domain_x` *and* `domain_y`*?*

There are many ways to change how the plot is displayed, and we will explore some of them. ***Try each of the following commands*** and use `help` or search MATLAB documentation to find out what each does.

`shading flat`
`shading interp`
`shading faceted` (the default)

Now let's try the `mesh` plot.

```
>> mesh(X, Y, Z);
```



Variations and Symbolic Plots

There are many other types of commands for creating 3D plots in MATLAB. These include:

Numerical methods: `surfc(X,Y,Z)`, `meshc(X,Y,Z)`, and `meshz(X,Y,Z)`

Symbolic methods:
`fsurf(f,[xmin xmax ymin ymax])`,
`fmesh(f,[xmin xmax ymin ymax])`, and
`fplot3(xt,yt,zt)`(particularly useful for plotting parametric curves in 3D)

Quickly edit your earlier commands to try out some of these so you can see the differences.

<u>Exercises</u>: Do the following exercises. Pick ONE to submit your work for this lab (see below):

1) **Function Plotting (challenging):** *Evaluate the behavior of* $g(x,y) = xe^{-x^2-y^2}$ *with a 3D plot and 2D plots using proper indexing*

   Consider the function $g(x,y) = xe^{-x^2-y^2}$

   a) Create a 3D `surfc` plot of this function for $-2 \leq x, y \leq 2$.
   b) Create the 2D plots of following functions and save them in one figure.
      i. $g\,(x\,,-2)$
      ii. $g\,(x,0)$
      iii. $g\,(-1,y)$

      You will have to first identify how to properly index the matrix `Z` to create this figure. For example, to do the first plot you should use the command:

      ```
      plot(domain_x,Z(1,:),'b-');
      ```

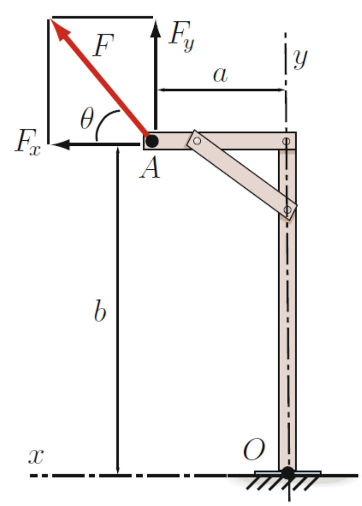
      Since the first row of `Z` corresponds to the constant value of $y = -2$ with x changing.

      Observe how the ROWS of Z correspond to different y values, and the COLUMNS of Z correspond to different x values. This means that as you move across the columns for a fixed row, your y is constant and your x changes. This is somewhat unintuitive.

   c) Use the `min` and `max` commands to find the maximum and minimum values of $g(x,y)$within this domain, and the points (x,y), where these occur.

   How do the 2D plots relate to the main 3D plot of f (x, y)?
   Are the maximum and minimum values what you would expect?

2) **Moment Calculation (easier):** *Consider the situation shown in the figure to the right. Plot the variation of the moment about the base point O caused by the force F as a function of length a and angle θ. It is known that $F = 500$ N, $b = 5$ m and $a \leq 10$ m.*



$$M_O(a,\theta) = 500(a\sin\theta - 5\cos\theta)$$

   a) Use a 3D plot to visualize how this moment at O changes with respect to a and θ.
   b) Generate three 2D plots that represent the following cases
      i. $M_O\,(a\,,0^{\circ})$
      ii. $M_O\,(5\text{ m},\theta)$
      iii. $M_O\,(10\text{ m},\theta)$
   c) Use the `min` and `max` commands to find the maximum and minimum values of $M_O(a,\theta)$, and the points (a,θ), where these occur.
   d) BONUS (Not required): Is it possible for $M_O = 0$ for any values of a and θ? If so, identify all such cases?

This lab is an individual submission.

For this you will submit your responses for ONE of the above exercises (i.e., either the **Function Plotting** OR the **Moment Calculation**). Please also include:

(a) Your 3D plot (make sure your figure axes are properly labeled),
(b) Your 2D plots (make sure your figure axes are properly labeled),
(c) A summary of the values and locations of the maximum and minimum values,
(d) A brief description, in your own words, of what the function "meshgrid" does, and
(e) A brief description about why your plots make sense.