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CHAPTER 2

Euclidean Vector Space

Introduction to Vector

- **Definition**: Vector is a quantity having direction as well as magnitude, especially as determining the position of one point in space relative to another.
- A vector can be represented geometrically by a directed line segment that start at a point **A** (initial point) and end at point **B** (terminal point).

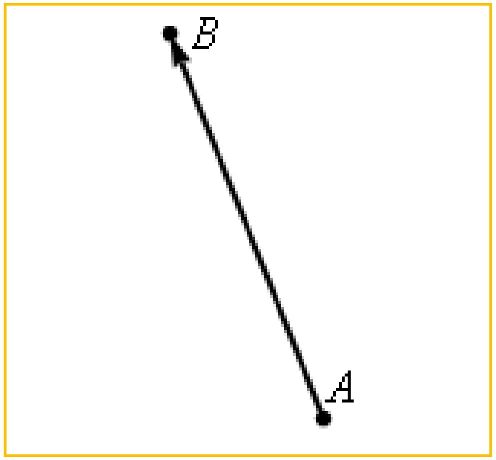


Figure 1: Example of a vector in 2-space

Introduction to Vector

- Vectors are typically denoted with a **b**oldface lower case letter. For instance we could represent the vector by **v, w, a** or **b.**

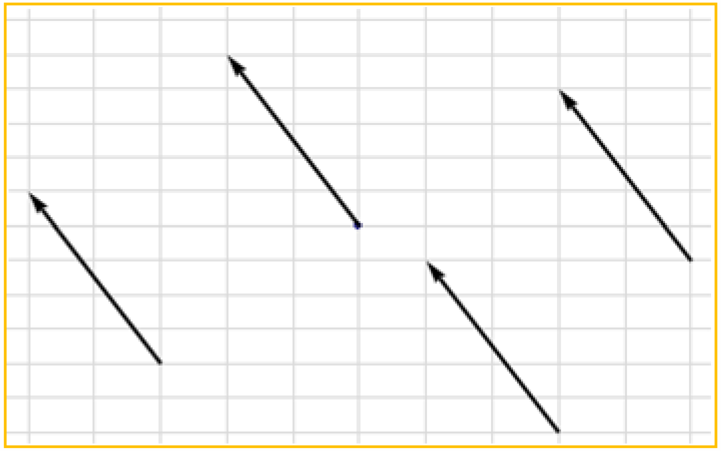
- Also when we've explicitly given the initial and terminal points we will often represent the vector as,

$$\mathbf{v} = \overrightarrow{AB}$$

Introduction to Vector

- Two vectors with the same magnitude but different directions **are different** vectors and likewise two vectors with the same direction but different magnitude **are different.**
- Vectors with the same direction and same magnitude are called **equivalent** and even though they may have different initial and terminal points we think of them as equal and so if **v** and **u** are two equivalent vectors we will write

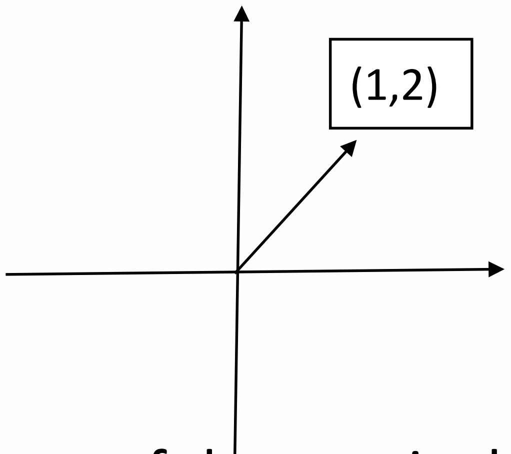
$$\mathbf{v} = \mathbf{u}$$



Equivalence vectors

Introduction to Vector

- We use coordinate system for easily visualize the vector.
- **Example:** (Fig. 2) In 2-space, suppose that **v** is vector whose the initial point at the origin (0,0) and the terminal point at (1,2).



We call the coordinate of the terminal point as components of **v** and write, **v** = (1,2)

Definition

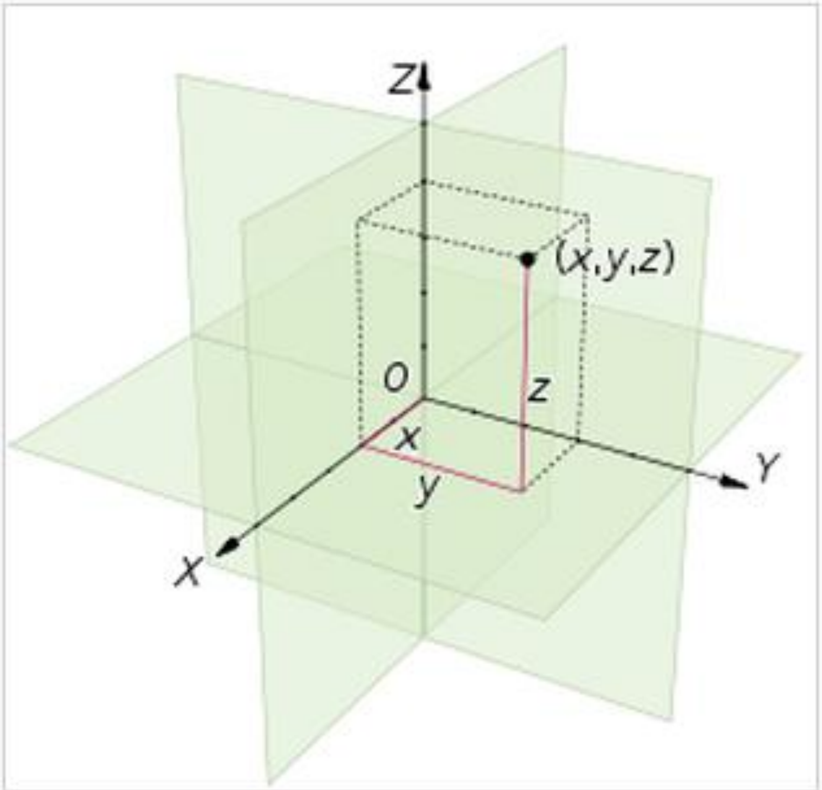
A sequence of n real numbers $(a_1, a_2, . . . , a_n)$ is called an ordered n-tuple, where n is a positive integer. The set of all ordered n-tuples is called n-space, denoted $\mathbf{R}^n$.

Example:

$\mathbf{R}^2$ represent a vector $\mathbf{v}$ with 2-coordinates;

$$\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Definition



Every point in three-dimensional Euclidean space is determined by three coordinates.

Vector - R^n

Example:

R^2:

Row vector: $v = \begin{bmatrix} 1 & 2 \end{bmatrix}$; Column vector: $v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

R^3:

Row vector: $v = \begin{bmatrix} 1 & 3 & 4 \end{bmatrix}$; Column vector: $v = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$

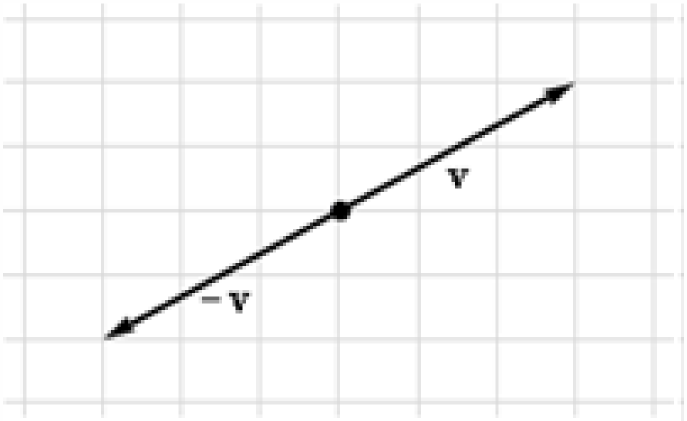
R^4:

Row vector: $v = \begin{bmatrix} 3 & 4 & 5 & \pi \end{bmatrix}$; Column vector: $v = \begin{bmatrix} 1 \\ 3 \\ 4 \\ \pi \end{bmatrix}$

Basic Arithmetic in Vector

- The first is the **zero vector**. The zero vector, denoted by **0**, is a vector with no length.

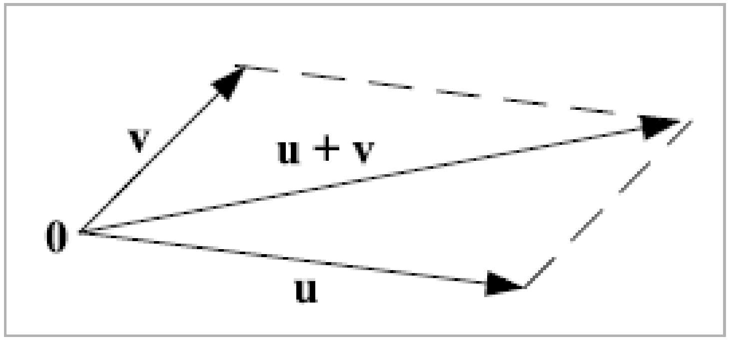
- If **v** is a vector then the negative of the vector, denoted by **–v**, is defined to be the vector with the same length as **v** but has the opposite direction .



negative of a vector

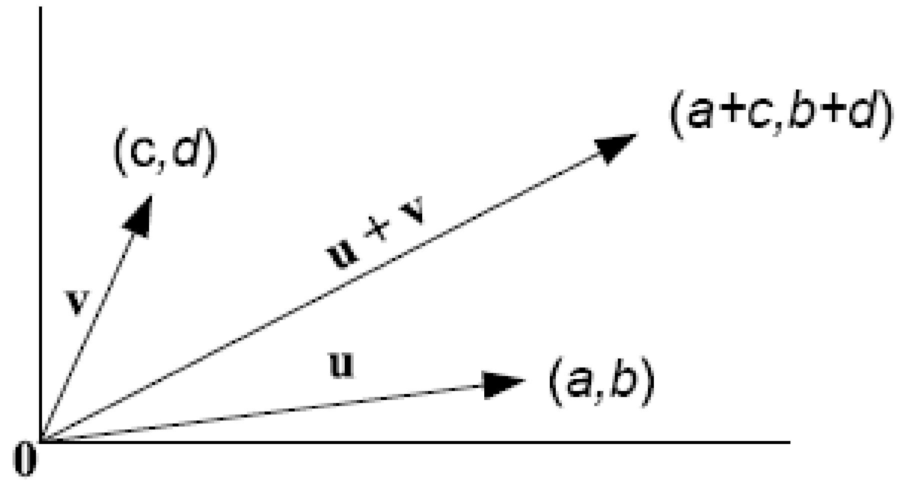
Basic Arithmetic in Vector

- **Addition**
  - The resultant **u+v** of 2 vectors u and v is obtained by the parallelogram law.
  - u+v is the diagonal of the parallelogram formed by u and v.

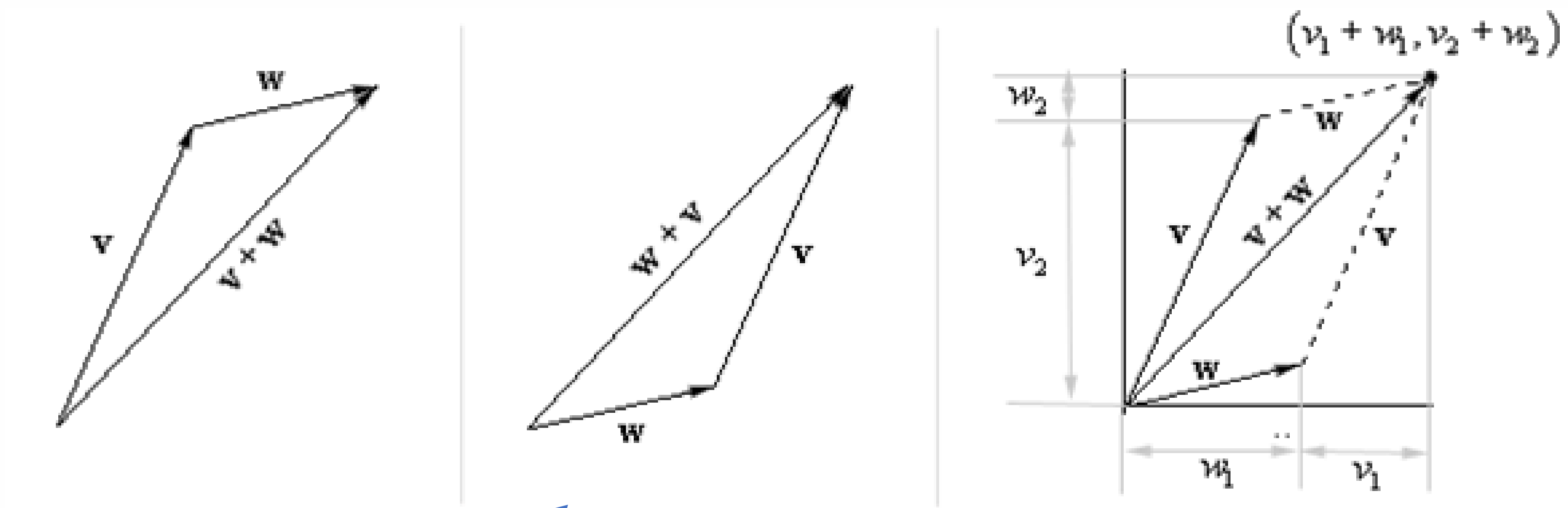


Basic Arithmetic in Vector

- If (a,b) and (c,d) are the endpoint of the vectors u and v, then (a+c, b+d) will be the endpoint of u+v.



Basic Arithmetic in Vector



Triangle rule for vector addition

Parallelogram rule for vector addition

Let **v** = (v_1,v_2) and **w** =(w_1, w_2) .
Then the sum of **v** + **w** can be demonstrated as the figures above.

Basic Arithmetic in Vector

- Let **u** and **v** be vectors in $\boldsymbol{R}^n$:

$$\mathbf{u} = \left(u_1, u_2, \ldots.., u_n\right); \ \mathbf{v} = \left(v_1, v_2, \ldots., v_n\right)$$

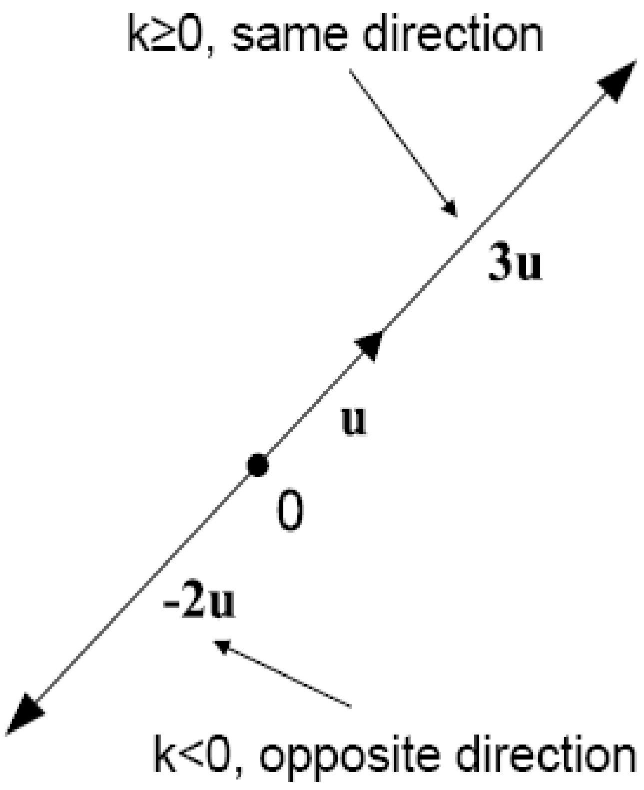
The sum (addition) of **u** and **v** is:

$$u + v = (u_1 + v_1, u_2 + v_2 + \cdots, u_n + v_n), \text{ or}$$

$$u + v = \begin{bmatrix} u_1 \\ u_2 \\ . \\ . \\ u_n \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ . \\ . \\ v_n \end{bmatrix} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ . \\ . \\ u_n + v_n \end{bmatrix}$$

Basic Arithmetic in Vector

- **Scalar multiplication**
  - The product **ku** of a real number k by a vector u is obtained by multiplying the magnitude of u by k.



Basic Arithmetic in Vector

The scalar product (multiplication) of c and $\boldsymbol{u}$ is:

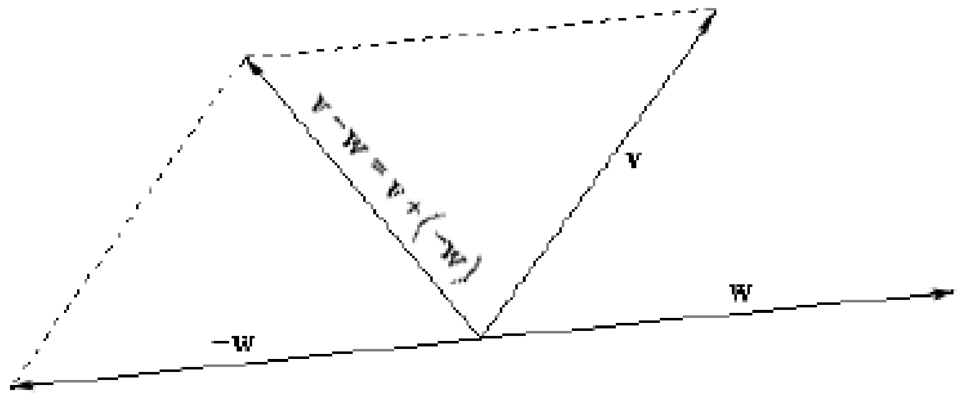
$$c\boldsymbol{u} = c[u_1 \quad u_2 \cdots \quad u_n] = [cu_1 \quad cu_2 \cdots \quad cu_n]$$

or,

$$c\boldsymbol{u} = c\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} = \begin{bmatrix} cu_1 \\ cu_2 \\ \vdots \\ cu_n \end{bmatrix}$$

Basic Arithmetic in Vector

- Suppose that we have two vectors **v** and **w** then the **difference** of **w** from **v**, denoted by **v-w** is defined to be,



$$\mathbf{v} - \mathbf{w} = \mathbf{v} + (-\mathbf{w})$$

Basic Arithmetic in Vector

Example

Let, $\boldsymbol{u} = (1, 4, 5, -3)$ and $\boldsymbol{v} = (8, 1, -2, -1)$

Find:

1) $\boldsymbol{u} + \boldsymbol{v}$
2) $2\boldsymbol{u}$
3) $2\boldsymbol{u} - 3\boldsymbol{v}$

Properties of Vector R^n

Let **u**, **v** and **w** be vectors in $\boldsymbol{R}^n$, and let c and k be scalar. The following algebraic properties hold.

1. Commutative property: **u** + **v** = **v** + **u**
2. Associative property: (**u** + **v**) + **w** = **u** + (**v** + **w**)
3. Additive identity: The vector **0** satisfies **0** + **u** = **u** + **0** = **u**
4. Additive inverse: for every vector **u**, the vector –**u** satisfies

   u - **u** = **u** + (- **u**) = **0**

5. 1**u** = **u**
6. ck(**u**) = c(k**u**) = k(c**u**)
7. (c + k) **u** = c**u** + k**u**
8. c(**u** + **v**) = c**u** + c**v**

Properties of Vector R^n

Example

Let, $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$; $\mathbf{w} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$;

Verify that the associative property holds for these three vectors. Also verify that for any scalars c and k, $c(k\mathbf{u}) = (ck)\mathbf{u}$

Properties of Vector R^n

Example - Solution

Associative property: **(u + v) + w = u + (v + w)**

$(\mathbf{u}+\mathbf{v})+\mathbf{w}:$

$$\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}+\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right)+\begin{bmatrix} 4 \\ -3 \end{bmatrix}=\begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

$\mathbf{u}+(\mathbf{v}+\mathbf{w}):$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}+\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}+\begin{bmatrix} 4 \\ -3 \end{bmatrix}\right)+=\begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

Properties of Vector R^n

Example - Solution

scalar: $c(k\mathbf{u}) = (ck)\mathbf{u}$

$c(k\mathbf{u})$:

$$c\left(k\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = c\left(\begin{bmatrix} k \\ -k \end{bmatrix}\right) = \begin{bmatrix} ck \\ -ck \end{bmatrix}$$

$ck(\mathbf{u})$:

$$ck\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} ck \\ -ck \end{bmatrix}$$

Euclidean Norm

- The **norm** (length) of the vector **v** is defined by:

$$\|\mathrm{v}\| = \sqrt{v_1^2 + v_2^2 + .. + v_n^2}$$

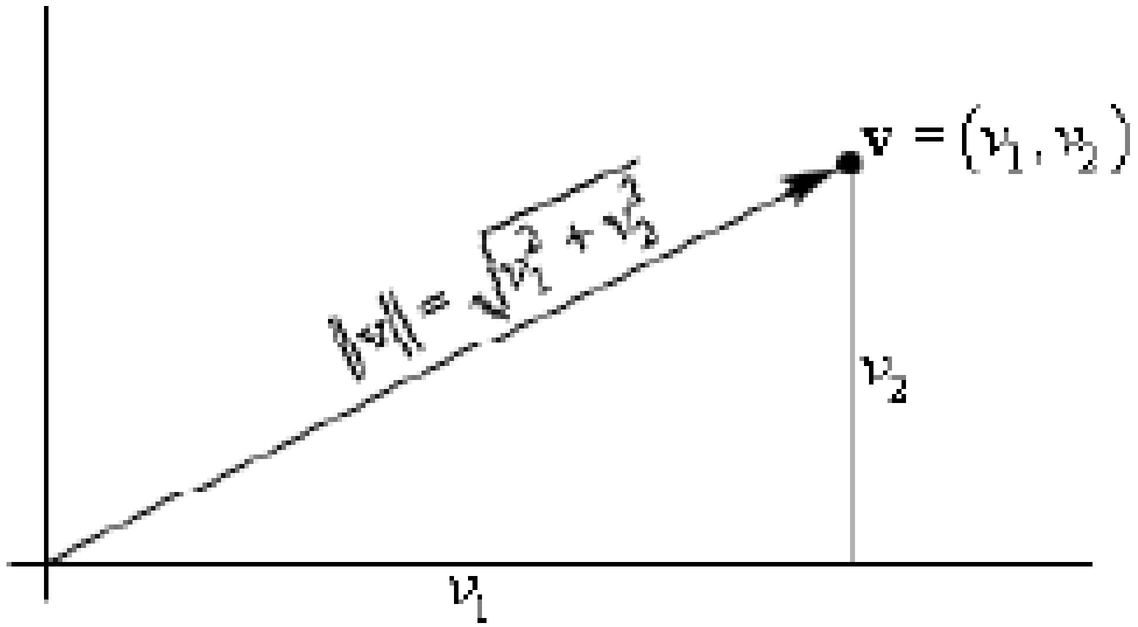
- If **v** is a vector in $\boldsymbol{R}^2$ then,

$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$$

and if **v** is in $\boldsymbol{R}^3$ then,

$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

Euclidean Norm



Euclidean Distance

- Let, **u** and **v** are vectors;

$$\mathbf{u} = (u_1, u_2, \ldots, u_n) \qquad \mathbf{v} = (v_1, v_2, \ldots, v_n)$$

- The **distance** between **u** and **v**, in $\boldsymbol{R}^n$ is known as Euclidean Distance and can be computed using the following formula:

$$d(\mathbf{u}, \mathbf{v}) = \|\mathbf{u} - \mathbf{v}\| = \sqrt{(u_1 - v_1)^2 + (u_2 - v_2)^2 + \ldots + (u_n - v_n)^2}$$

Euclidean Norm

Example

Let, **u** = (1, -2, 4,1) and **v** = (3, 1, -5, 0). Find the distance and norm of these two vectors.

Solution:

Distance:

$$d(u,v) = \sqrt{(1-3)^2 + (-2-1)^2 + (4-(-5))^2 + (1-0)^2}$$
$$= \sqrt{4+9+81+1} = \sqrt{95}$$

Norm/length:

$$\|v\| = \sqrt{3^2 + 1^1 + (-5)^2 + 0^2} = \sqrt{9+1+25+0}$$
$$= \sqrt{35}$$

Dot Product in $\boldsymbol{R}^n$

(i) Angle - known

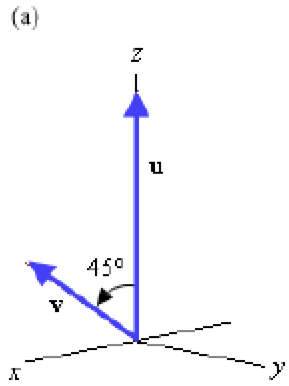
If **u** and **v** are two vectors in 2-space ($\boldsymbol{R}^2$) or 3-space ($\boldsymbol{R}^3$), and θ is the angle between them, then the dot product, denoted by $\mathbf{u} \cdot \mathbf{v}$ is defined as,

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\|\,\|\mathbf{v}\| cos\theta$$

Dot Product in R^n

Example

Find the dot product for the following pairs of vectors: **u** =(0, 0, 3) and **v** = (2, 0, 2) and θ = 45°



$$\|\mathbf{u}\| = \sqrt{0+0+9} = 3 \qquad \|\mathbf{v}\| = \sqrt{4+0+4} = \sqrt{8} = 2\sqrt{2}$$

$$\mathbf{u}\cdot\mathbf{v} = (3)\left(2\sqrt{2}\right)\cos(45) = 6\sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = 6$$

Dot Product in R^n

(ii) Angle – unknown

Suppose that $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$ are two vectors in 3-space then,

$$\mathbf{u} \bullet \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

Similarly, if $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$ are two vectors in 2-space then

$$\mathbf{u} \bullet \mathbf{v} = u_1 v_1 + u_2 v_2$$

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Cross Product in R^n

- **Cross product** of vectors is different with the **dot products** because the first one produced **new vectors** while the second one gives **a scalar** as it result.
- It only applicable to vector in 3-space.
- For vector **u**= (u_1,u_2,u_3) and **v**= (v_1,v_2,v_3) in $\boldsymbol{R}^3$ the cross product in defined by:

(a) $\mathbf{u} \times \mathbf{v} = (u_2v_3 - u_3v_2,\ u_3v_1 - u_1v_3,\ u_1v_2 - u_2v_1)$ Vector notation

or,

(b) $$\mathbf{u}\times\mathbf{v} = \left(\begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}, -\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}, \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \right)$$ determinant notation

Cross Product in R^n

(c) In 3x3 determinant:

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

Note: Refer page 68 (module) for examples (2.7 & 2.8).

Exercise #1

Given $\mathbf{u}$ = (5, -2) , $\mathbf{v}$ = (0, 7) and $\mathbf{w}$ = (4, 10) are vectors. Compute the following:

(a) $\mathbf{u} \bullet \mathbf{u}$

(b) $\|\mathbf{u}\|^2$

(c) $\mathbf{u} \bullet \mathbf{w}$

(d) $(-2\mathbf{u}) \bullet \mathbf{v}$

(e) $\mathbf{u} \bullet (-2\mathbf{v}) \quad =$

Exercise #2

i) Simplified the vector expression below using vector addition and scalar multiplication: 5(**v**-2**u**) -3(**v**-4**w**) + 3(**u** - **v** + **w**).

ii) Suppose **u** = (1,3,-2,7) and **v** = (0,7,2,2) are vectors in R^4. Find the Euclidean distance between **u** and **v** in R^4.

iii) Given that **u** is a vector of length 2, **v** is a vector of length 3 and the angle between these vectors is 45°. Find the value of **u** • **v**

iv) Find the cross products of vector **a** = (2 ,– 1 , 3) and **b** = (-1 , 2 ,4).

Linear Combination - Set of Vectors

Given a set of *n* vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$, which have a same length of *m*, a linear combination of these vectors is defined as the vector:

$$\mathrm{v} = c_1 \mathrm{v}_1 + c_2 \mathrm{v}_2 + \ldots.. + c_n \mathrm{v}_n .$$

Here $c_1, c_2, \ldots., c_n$ is a scalar.

Linear Combination - Set of Vectors

Example

Consider a set of vectors,

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \ v_2 = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$$

Can we express the following vectors

$$u = \begin{bmatrix} 9 \\ 2 \\ 7 \end{bmatrix}, \ w = \begin{bmatrix} 4 \\ -1 \\ 8 \end{bmatrix}$$

as linear combinations of $\mathbf{v}_1$ and $\mathbf{v}_2$?

Linear Combination - Set of Vectors

Example - Solution

For the first vector $\mathbf{u}$ to be a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$, we must have

$$\begin{pmatrix} 9 \\ 2 \\ 7 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix} \Rightarrow \begin{bmatrix} 1 & 6 \\ 2 & 4 \\ -1 & 2 \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \\ 7 \end{pmatrix}$$

which is a linear system. The augmented matrix is

$$\tilde{\mathbf{A}} = \left[\begin{array}{cc|c} 1 & 6 & 9 \\ 2 & 4 & 2 \\ -1 & 2 & 7 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 6 & 9 \\ 0 & -8 & -16 \\ 0 & 8 & 16 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 6 & 9 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

Thus,

$$\begin{pmatrix} 9 \\ 2 \\ 7 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix} = -3 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + 2 \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix}$$

From the last matrix, $c_1 = -3$ and $c_2 = 2$

Linear Combination - Set of Vectors

Example - Solution

For the second vector $\mathbf{w}$ to be a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$, we must have

$$\begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix} \Rightarrow \begin{bmatrix} 1 & 6 \\ 2 & 4 \\ -1 & 2 \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$$

The last matrix shows that the system is inconsistent

which is a linear system. The augmented matrix is

$$\tilde{\mathbf{A}} = \left[\begin{array}{cc|c} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 8 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & -8 & -9 \\ 0 & 8 & 12 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & -8 & -9 \\ 0 & 0 & 3 \end{array}\right]$$

Thus, $\mathbf{w}$ cannot be expressed as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

Linear Combination - Set of Vectors

Example

Show that the matrix, $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ is a linear combination of the matrices

$$M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } M_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Linear Combination - Set of Vectors

Example - Solution

Firstly we must find the values of c_1,c_2 and c_3 such that

$$c_1\mathbf{M}_1 + c_2\mathbf{M}_2 + c_3\mathbf{M}_3 = \mathbf{A}$$

That is,

$$c_1\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + c_2\begin{bmatrix}0 & 1\\1 & 1\end{bmatrix} + c_3\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} = \begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$$

Perform the scalar multiplication, we get

$$\begin{bmatrix}c_1 & 0\\0 & c_1\end{bmatrix} + \begin{bmatrix}0 & c_2\\c_2 & c_2\end{bmatrix} + \begin{bmatrix}c_3 & c_3\\c_3 & c_3\end{bmatrix} = \begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$$

Linear Combination - Set of Vectors

Example - Solution

Then, perform the addition, we get

$$\begin{bmatrix} c_1 + c_3 & c_2 + c_3 \\ c_2 + c_3 & c_1 + c_2 + c_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{aligned} c_1 + \quad + c_3 &= 1 \\ + c_2 + c_3 &= 1 \\ c_1 \quad + c_2 + c_3 &= 0 \end{aligned}$$

Solve these equations, we obtain $c_1 = c_2 = -1$ and $c_3 = 2$.
Thus, matrix A is a linear combination of matrices $\mathbf{M_1}$, $\mathbf{M_2}$ and $\mathbf{M}_3$

Exercise #3

Determine whether the vector

$$v = \begin{bmatrix} -1 \\ 1 \\ 10 \end{bmatrix}$$

is a linear combination of the following vectors.

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \; ; \quad v_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix} \quad \text{and} \quad v_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}$$

Exercise #4

Let $v_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} -2 \\ 4 \\ -6 \end{bmatrix}, v_3 = \begin{bmatrix} -1 \\ 5 \\ -6 \end{bmatrix}$ and $v_4 = \begin{bmatrix} 3 \\ -5 \\ 8 \end{bmatrix}$.

Is $w = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$ a linear combination of vectors v_1 and v_3? Justify your answer.

Linear Independent / Dependent

Let

$$\mathbf{v}_1 = \begin{pmatrix} v_{11} \\ v_{21} \\ \vdots \\ v_{m1} \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} v_{12} \\ v_{22} \\ \vdots \\ v_{m2} \end{pmatrix}, \quad \cdots, \quad \mathbf{v}_n = \begin{pmatrix} v_{1n} \\ v_{2n} \\ \vdots \\ v_{mn} \end{pmatrix}$$

Then

$$c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n = \begin{pmatrix} v_{11} \\ v_{21} \\ \vdots \\ v_{m1} \end{pmatrix} c_1 + \begin{pmatrix} v_{12} \\ v_{22} \\ \vdots \\ v_{m2} \end{pmatrix} c_2 + \cdots + \begin{pmatrix} v_{1n} \\ v_{2n} \\ \vdots \\ v_{mn} \end{pmatrix} c_n = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix} = 0$$

which is an $m \times n$ homogeneous system and it has a trivial solution. If this trivial solution is the only solution, then the given vectors are linearly independent. If there are non-trivial solutions, then the vectors are linearly dependent.

Linear Independent / Dependent

Example

Determine whether the vectors,

$$v_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix},\ v_2 = \begin{bmatrix} 5 \\ 6 \\ -1 \end{bmatrix},\ v_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

are linearly independent or not.

#Solution – Refer page 72 (module)

Linear Independent / Dependent

Example

Vectors $\boldsymbol{i} = (1,0,0), \boldsymbol{j} = (0,1,0), \boldsymbol{k} = (0,0,1)$ in R^3

$$c_1\boldsymbol{i} + c_2\boldsymbol{j} + c_3\boldsymbol{k} = 0$$
$$c_1 = (1,0,0) + c_2 = (0,1,0) + c_3 = (0,0,1)$$

This implies that $c_1 = 0$, $c_2 = 0$ and $c_3 = 0$

Thus, set $S = \{\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}\}$ is linearly independent

Linear Independent / Dependent

Example

Given a vector,

$\mathbf{v_1} = (1, -2, 3, -4)$,

$\mathbf{v_2} = (-1, 3, 4, 2)$ and

$\mathbf{v}_3 = (1, 1, -2, -2)$.

Determine whether the above vectors are linearly independent or not.

#Solution – Refer page 73 (module)

Exercise #5

Determine whether the vectors

$$\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix},\ \boldsymbol{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix} \text{and } \boldsymbol{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}$$

are linearly independent or linearly dependent.

Exercise #6

Determine whether the vectors

$$\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \ \boldsymbol{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}, \ \boldsymbol{v}_3 = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} \text{and } \boldsymbol{v}_4 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

are linearly independent or linearly dependent.

Exercise #7

Let $v_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} -2 \\ 4 \\ -6 \end{bmatrix}, v_3 = \begin{bmatrix} -1 \\ 5 \\ -6 \end{bmatrix}$ and $v_4 = \begin{bmatrix} 3 \\ -5 \\ 8 \end{bmatrix}$.

Is v_1, v_2, v_3 and v_4 are linearly independent? Justify your answer.