SULIT

FACULTY OF COMPUTING
UTM Johor Bahru

UNIVERSITI TEKNOLOGI MALAYSIA
FINAL EXAMINATION SEMESTER 2 2022/2023

SUBJECT CODE	**:**	**SECI 1113**
SUBJECT NAME	**:**	**COMPUTATIONAL MATHEMATICS**
SECTION	**:**	**ALL SECTIONS**
TIME	**:**	
DATE/DAY	**:**	
VENUES	**:**	

INSTRUCTIONS:

PART 1:	**5 STRUCTURED QUESTIONS**	**(100 MARKS)**
TOTAL		**(100 MARKS)**

(Please Write Your Lecturer Name and Section In Your Answer Booklet)

Name	
I/C No.	
Year / Course	
Section	
Lecturer Name	

This question paper consists of **SEVEN (7)** printed pages including this page.

QUESTION 1 **20 MARKS**

To solve the nonlinear equation $f(x) = 0$,

i) Find the solution for $\sqrt[3]{12} - \frac{1}{3}x = x^2$using Bisection Method if x is located from 1 to 1.5. (10 Marks)

ii) Find the solution for $f(x) = x^2 - 5$ by using:

a) Secant Method if the initial approximations are 2 and 2.2.

b) Newton's Method if the initial approximation is 2.

Based on the solutions of Secant Method and Newton's Method, which method is more accurate. (10 Marks)

Note:

Do all CALCULATIONS in FOUR decimal places.

Approximates the solutions into TWO decimal places of ACCURACY.

QUESTION 2 **15 MARKS**

a) Consider the matrix **B**, =

$$\boldsymbol{B} = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 6 & 4 \\ 3 & 1 & 9 \end{bmatrix}$$

Use the Gerschgorin's Circle Theorem to determine region of all the eigenvalues of **B**. State the biggest circle. (4 Marks)

b) If the dominant eigenvalue of **C** is = 4. Use the **Shifted Power Method** to find the smallest eigenvalues of **C**. Let v = (0, 1, 0) and iterate until $\varepsilon < 0.005$. Do calculation in 3 decimal points. (11 Marks)

$$\boldsymbol{C} = \begin{bmatrix} 0 & 0 & -3 \\ -1 & -1 & -1 \\ 2 & -6 & 4 \end{bmatrix}$$

QUESTION 3 **30 MARKS**

a) A cylinder is filled with water to a height of 73 centimeters. The water is drained through a hole in the bottom of the cylinder and measurements are taken at two-second intervals. Table 1 below shows the height of the water level in the cylinder at different times.

Table 1

Time (seconds)	0.0	2.0	4.0	6.0	8.0
Water level (cm)	73.5	63.9	55.5	47.5	39.9

Estimate the water level at time 6.5 second using below methods. Do calculations in 3 decimal points.

i) Appropriate Newton method. (10 marks)

ii) Langrage interpolation (6 marks)

iii) If the true value of water level at time 6.5 second is 45.001. Find the absolute error obtain in (i) and (ii) (4 marks)

b) Number of man-hours the corresponding productivity (in units) are furnished below (Table 2). Determine the appropriate linear polynomial expression, $p(x) = a_0 + a_1 x$ applying the method of least squares method.

Table 2

Man-hour	3	5	7	9	11
Productivity (in units hundred)	9	10	11	13	18

Then by using the polynomial expression, do estimation for p(7.5). (10 marks)

QUESTION 4 **15 MARKS**

a) The position of an object at any time t is given by $s(t) = t^4 - 3t^3 + 40t^2 - 5$.

i. Use forward three-point difference with h=1 to estimate the velocity of the object at t=6. (4 marks)

ii. Use forward five-point difference with h=2 to estimate the velocity of the object at t=6. (6 marks)

b) Given the following data as in Table 3.

Table 3

x	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$f(x)$	0.549	0.542	0.525	0.496	0.456	0.412	0.375

i. Use central three-point difference to estimate $f''(0.5)$. (2 marks)

ii. Use central five-point difference to estimate $f''(0.5)$. (3 marks)

QUESTION 5 **20 MARKS**

a) Approximate the integral $\int_1^4 \frac{1}{x^2+x}\,dx$ using the following method with h = 0.5:

i) Trapezoidal rule (4 marks)

ii) Simpson 1/3 (3 marks)

iii) Simpson 3/8 (3 marks)

Do calculations in 4 decimal points.

b) Use Romberg integration to approximate the integral $\int_0^4 x^2 e^{-x}\,dx$.
Use $\varepsilon = 0.05$ and do all calculations in 4 decimal points.

(10 marks)

List of Formulas

Non Linear Equations

Bisection Method: $x_c = \dfrac{a+b}{2}$

Secant Method: $x_{i+2} = \dfrac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$, where $i = 0,1,2..n$

Newton's Method: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ where $n = 0,1,2,..m$

Eigenvalues

$$B_i = |\lambda - a_{ii}| \le r_i \text{ where } r_i = \sum_{\substack{j=1 \\ i \ne j}}^{n} |a_{ij}|$$

$$\mathbf{v}^{(k+1)} = \frac{1}{m_{k+1}} A\mathbf{v}^{(k)}, \; k = 0,1,2....$$

$$\sum_{i=1}^{3} \lambda_i = \sum_{i=1}^{3} a_{ii}$$

Interpolation

Newton Forward Difference Formula:

$$p_n(x) = y_k + r\Delta y_k + \frac{r(r-1)}{2!}\Delta^2 y_k + ... + \frac{r(r-1)...(r-n+1)}{n!}\Delta^n y_k$$

with $r = (x - x_k)/h$.

Newton Backward Difference Formula:

$$p_n(x) = y_k + r\nabla y_k + \frac{r(r+1)}{2!}\nabla^2 y_k + ... + \frac{r(r+1)...(r+n-1)}{n!}\nabla^n y_k$$

with $r = (x - x_k)/h$.

Lagrange Formula:

$$p_n(x) = L_0(x)y_0 + L_1(x)y_1 + ...L_n(x)y_n = \sum_{i=0}^{n} L_i(x)y_i$$

with

$$L_i(x) = \frac{(x-x_0)...(x-x_{i-1})(x-x_{i+1})...(x-x_n)}{(x_i-x_0)...(x_i-x_{i-1})(x_i-x_{i+1})...(x_i-x_n)} = \prod_{\substack{j=0 \\ j \ne i}}^{n} \frac{(x-x_j)}{(x_i-x_j)}$$

Least Square:

$$s_j = \sum_{k=0}^{n} x_k^j, j = 0,1,2,\ldots,2m$$

$$v_l = \sum_{k=0}^{n} x_k^l f_k, l = 0,1,2,\ldots,m$$

$$\begin{bmatrix} s_0 & s_1 & \ldots & s_m \\ s_1 & s_2 & \ldots & s_{m+1} \\ \ldots & \ldots & \ldots & \ldots \\ s_m & s_{m+1} & \ldots & s_{2m} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \ldots \\ a_m \end{bmatrix} = \begin{bmatrix} v_0 \\ v_1 \\ \ldots \\ v_m \end{bmatrix}$$

<u>Numerical Differentiation</u>

Two-Point Forward Difference

$$f'(x_i) \approx \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i} = \frac{f(x_i + h) - f(x_i)}{h}$$

Two-Point Backward Difference

$$f'(x_i) \approx \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} = \frac{f(x_i) - f(x_i - h)}{h}$$

Three-Point Forward Difference

$$f'(x_i) \approx \frac{1}{2h}[-3f(x_i) + 4f(x_i + h) - f(x_i + 2h)]$$

Three-Point Backward Difference

$$f'(x_i) \approx \frac{1}{2h}[f(x_i - 2h) - 4f(x_i - h) + 3f(x_i)]$$

Three-Point Central Difference

$$f'(x_i) \approx \frac{f(x_i + h) - f(x_i - h)}{2h}$$

$$f''(x_i) \approx \frac{f(x_i - h) - 2f(x_i) + f(x_i + h)}{h^2}$$

Five-Point Forward Difference

$$f'(x_i) \approx \frac{1}{12h}[-25f(x_i) + 48f(x_i + h) - 36f(x_i + 2h) + 16f(x_i + 3h) - 3f(x_i + 4h)]$$

Five-Point Central Difference

$$f'(x_i) \approx \frac{1}{12h}[f(x_i - 2h) - 8f(x_i - h) + 8f(x_i + h) - f(x_i + 2h)]$$

$$f''(x_i) \approx \frac{1}{12h^2}[-f(x_i - 2h) + 16f(x_i - h) - 30f(x_i) + 16f(x_i + h) - f(x_i + 2h)]$$

<u>Numerical Integration</u>

Trapezoidal Rule

$$\int_a^b f(x) \; dx = \frac{h}{2}\left(f_0 + f_N + 2\sum_{i=1}^{N-1} f_i\right)$$

Simpson

$$\int_a^b f(x) \; dx = \frac{h}{3}\left[(f_0 + f_N) + 4\sum_{i=1}^{N/2} f_{2i-1} + 2\sum_{i=1}^{N/2-1} f_{2i}\right]$$

$$\int_a^b f(x) \; dx = \frac{3h}{8}\left[(f_0 + f_N) + 3\sum_{i=1}^{N/3} (f_{3i-2} + f_{3i-1}) + 2\sum_{i=1}^{N/3-1} f_{3i}\right]$$

Romberg Integration

$$h_i = \frac{1}{2} h_{i-1}$$

$$R_{1,1} = \frac{h_1}{2}(f_0 + f_1)$$

$$R_{i,1} = \frac{1}{2}\left[R_{i-1,1} + h_{i-1}\sum_{k=1}^{2^{i-2}} f_{2k-1}\right] \text{for } i = 1,2,3....$$

$$R_{i,j} = \frac{4^{j-1} R_{i,j-1} - R_{i-1,j-1}}{4^{j-1} - 1} \text{ for } i = 2,3...N \text{ and } j = 2,3,...i$$