

## UNIVERSITI TEKNOLOGI MALAYSIA FINAL EXAMINATION SEMESTER II 2019/2020

SUBJECT C	CODE	:	SCSI/SECI 1113

SUBJECT NAME : COMPUTATIONAL MATHEMATICS

SECTION :

TIME :

DATE/DAY :

VENUES :

Name	
Student Id:	
Section	
Lecturer Name	

This questions paper consists of SIX (6) printed pages excluding this page.

Question 1 [9 Marks]

a) Given

$$e^{x} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \frac{x^{4}}{4!} + \frac{x^{5}}{5!}$$

i) Estimate  $e^{0.5}$  using the above series.

(3marks)

- ii) Find the absolute error for the estimate value in (i) if the exact value for  $e^{0.5}$ =1.6489 (1 mark)
- b) Table 1 shows the petrol station which were cited for irregular dispensation by the Department of Agriculture and Energy. From the given data, determine which petrol station cheated their customer most? Justify your answer. (5 marks)

Table 1

Station	Actual Petrol dispensed	Petrol reading at pump	
A	9.90	10.00	
В	19.90	20.00	
С	29.80	30.00	
D	29.95	30.00	

Question 2 [20 Marks]

a) Given

$$f(x) = x^3 - 7x^2 + 14x - 6$$

i) Show that there is a root  $\alpha$  in interval [0,1]

(1 mark)

- ii) Find the minimum number of iterations needed by the bisection method to approximate the root,  $\alpha$  of f(x) = 0 on [0,1] with accuracy of 2 decimal points. (3 marks)
- iii) Find the root ( $\alpha$ ) of  $f(x) = x^3 7x^2 + 14x 6$  on [0,1] using the bisection method with accuracy of 2 decimal points. (6 marks)

b) Given

$$f(x) = \cos x - x^3$$

- i) Write the iterative formula for f(x) using Newton Method. (2 marks)
- ii) Determine which initial starting point  $x_0$  ( $x_0 = 0$  and  $x_0 = 0.5$ ) is appropriate to be used for finding root using Newton method. Give your justification. (3 marks)
- iii)Use the formula in (i) and the initial value chosen in (ii) to find a positive real root of f(x). Use  $\varepsilon = 0.0005$  (5 marks)

Question 3 [26 Marks]

a) Matrix A is given as follows

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

- i) Derive polynomial characteristics of matrix A using determinant approach. (4 marks)
- ii) Find all eigenvalues of matrix A using polynomial characteristics obtained in (i)

  (4 marks)
- iii) Find the approximate dominant eigenvalue and the associated eigenvector of matrix A using power method. Use  $v_0 = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix}$  and  $\varepsilon = 0.05$ . Calculate the error obtained in the estimated eigenvalue. (8 marks)
- b) Given matrix B,

$$B = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$

The dominant eigenvalue is 9. Find the smallest eigenvalue, the eigenvector associated with the smallest eigenvalue and the intermediate eigenvector of matrix B. Use

$$v_0 = \begin{bmatrix} 0.5\\1\\1 \end{bmatrix}$$
 and  $\varepsilon = 0.05$ .

(10 marks)

Question 4 [22 Marks]

a) The viscosity  $\mu$  of a fluid depends on the temperature T of the fluid according to a relationship represented by the data in Table 2.

 Table 2

 T (°C)
 5
 20
 30
 50
 55

  $\mu$  (N-sec/m²)
 0.0800
 0.0150
 0.0090
 0.0060
 0.0055

Use Lagrange interpolating polynomials to find an estimate for the viscosity at T=40. Do calculations in 4 decimal points. (10 marks)

b) Table 3 lists the population of Malaysia from 1960 to 2000.

 Table 3

 Year
 1960
 1970
 1980
 1990
 2000

 Population (in thousands)
 8,157
 10,804
 13,798
 18,029
 23,194

- i) Find the linear relationship between year and population. (8 marks)
- ii) Estimate the population of Malaysia in the year 1965 and 1995. (2 marks)
- iii)The population of Malaysia in 1965 and 1995 was 9,527,000 and 20,488,000 respectively. Calculate the relative error for the estimated values obtained in (ii).

(2 marks)

Question 5 [10 marks]

a) The vibration of cantilever structural beam is represented by the function  $s(t) = 5 \times \sin(0.8t)$  which is taken at the free end of the beam. The first derivative of the function would give the velocity value. By taking a time interval of 0.5s, determine the velocity at time, t = 3s using 3 points forward difference. (6 marks)

b) Values for  $f(x) = xe^x$  are given in Table 4. Use central five-point formulas to approximate f''(2.0). (4 marks)

Table 4

x	1.8	1.9	2.0	2.1	2.2
f(x)	10.889365	12.703199	14.778112	17.148957	19.855030

Question 6 [13 marks]

- Suppose we know that water is flowing into a pool at a rate  $f(t)=x\sqrt{x^3+1}$  liters per hours. Use Simpson's 1/3 Rule to estimate the total amount of water flows into the pool in the first two hours,  $\int_{0}^{2} f(t) dt$  with h=0.5 to 3 decimal places. (5 marks)
- b) Use the Romberg integration to find the approximation for the shaded area under the curve with equation  $f(x) = 4x^3 16x$  to 2 decimal places as shown in Figure 1 until  $\left|R_{i,j} R_{i,j-1}\right| < 0.05$  (8 marks)

