FINAL EXAM SESSION 2019 2020 1

QUESTION 1 **15 MARKS**

a) Suppose A, B, and C are mutually exclusive events in a sample space S, $A \cup B \cup C = S$, and A and B have probabilities 0.2 and 0.6, respectively.

i. What is $P(A \cup B)$? (1 mark)

ii. What is P(C) (1 mark)

b) Suppose A and B are events in sample space S with probabilities 0.5 and 0.8, respectively. Suppose also that $P(A \cap B) = 0.7$. What is $P(A \cup B)$? (1 mark)

c) Suppose a sample space S consists of three outcomes: 0, 1, and 2. Let $A=\{0\}$, $B=\{1\}$, and $C=\{2\}$, and suppose P(A)=0.4, and P(B)=0.3. Find each of the following:

i. P(C) (1 mark)

ii. $P(A \cup C)$ (1 mark)

iii. $P(A' \cap B')$ (1 mark)

d) An urn contains 25 red balls and 15 blue balls. Two are chosen at random, one after the other, without replacement.

i. What is the probability that both balls are red? (1 mark)

ii. What is the probability that the second ball is red but the first ball is not? (1 mark)

iii. What is the probability that the second ball is red? (2 marks)

e) A student taking a multiple-choice exam does not know the answer to two questions. All have five choices for the answer. For one of the two questions, the student can eliminate two answer choices as incorrect but has no idea about the other answer choices. For the other question, the student has no clue about the answer at all. Assume that whether the student chooses the

correct answer on one of the questions does not affect whether the student chooses the correct answer on the other question.

i. What is the probability that the student will answer both question correctly? (1 mark)
ii. What is the probability that the student will answer exactly one of the questions correctly? (2 marks)
iii. What is the probability that the student will answer neither question correctly? (2 marks)

QUESTION 2 **[7 MARKS]**

When LHDN receives tax form, it puts them through a computer to flag forms that need to be invstigated further. The computer looks for errors in the forms, for example addition mistakes or incorrect deduction amounts. Suppose the computer correctly flags 80% of all returns that have errors, and it incorrectly flags 5% of error-free returns. Further, suppose that 15% of all tax returns have errors.

A tax return is flagged by the computer.

(hint : Let E = the return has error, Let F= return is flagged by computer)

i) What is the probability of the tax return has an error?
ii) What is the probability that the computer correctly flagged the tax return that have errors?
iii) What is the probability that the computer incorrecty flagged the tax return that have error free?
iv) What is the probability that it actually contains errors, given that the computer flagged it $P(E|F)$?
v) Based on the above results, Is it true the computer usage has improve LHDN performance in detecting thetax return that have errors? Justify your answer.

QUESTION 3 **[10 MARKS]**

a) Model the following situations as graphs.

Aida and Bertrand are friends. Aida is also friends with Chong and David. Bertrand, Chong and Emilin are all friends of each other.

b) Given the graph H in Figure 1

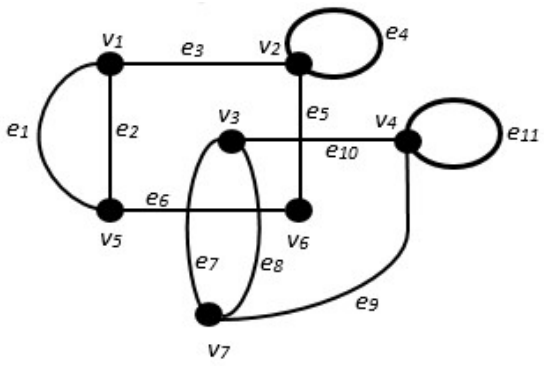


Figure 1: Graph H

(i) Construct the incidence matrix for graph H

(ii) List the degree of each of the vertices

(iii)Explain whether the graph is connected or disconnected?

c) An art museum arrange its current exhibit in the seven rooms(Room1,Room2, Room3, Room4,Room5,Room6 and Room 7). Each room is connected with other room by a door (a,b,c,d,e,f,g,h,i and j)and the floor plan for the museum is shown in Figure 2.

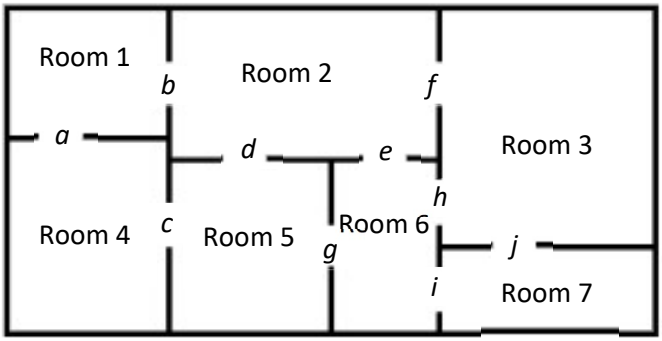
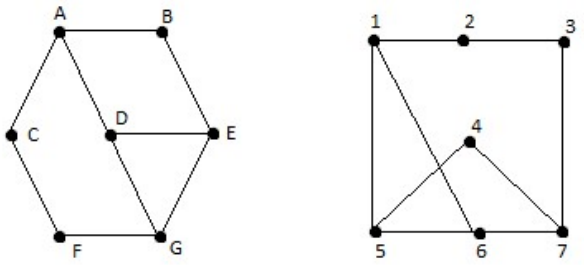


Figure 2: Floor plan for art museum

(i) Ramli wants to give a tour at the museum. Is there is a way to tour the exhibit by passing through each door exactly once? Explain. Determine which rooms must they begin and end the tour? Construct the trail of the tour.

(ii) Is it possible to tour the museum by visiting each room exactly once (not necessarily using every door)? If possible, state the theorem and construct the tour that begin and end at Room 1.

d) Determine whether the pair of graphs ($G_1$and G_2)in Figure 3 are isomorphic. If there are, give a vertex function that defines the isomorphic and their adjacency matrices. If they are not, give isomorphic invariant that they are not share.



G_1 G_2

Figure 3: Graph G_1 and G_2

QUESTION 4 **[15 MARKS]**

The following network in Figure 4 shows the time, in minutes, of train journeys between seven stations.

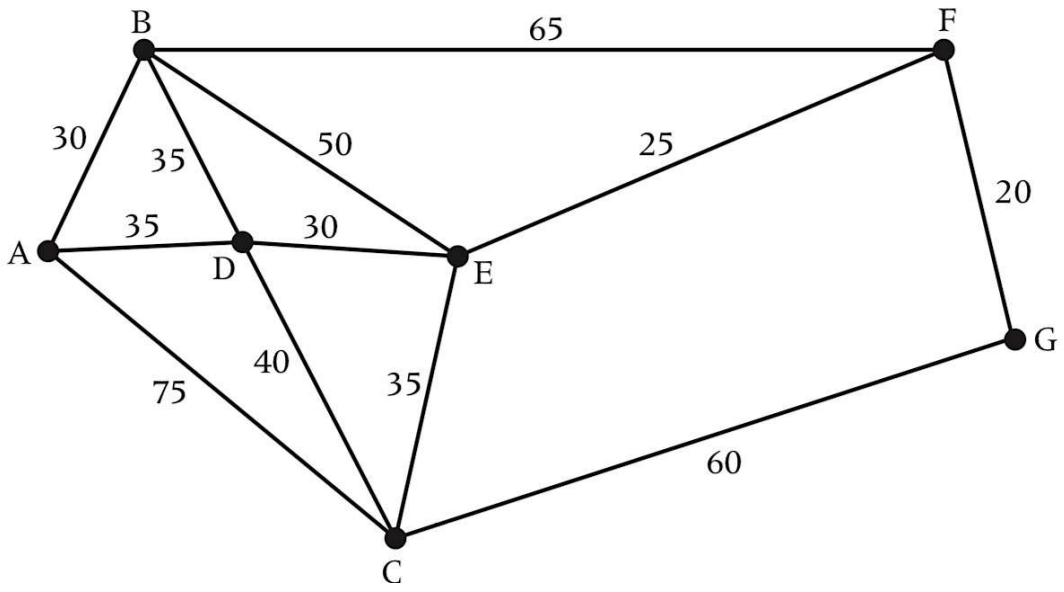


Figure 4

(a) Given that there is no time delay in passing through a station, use Dijkstra's algorithm to find the shortest time to travel from A to G. State the stations passed from A to G.

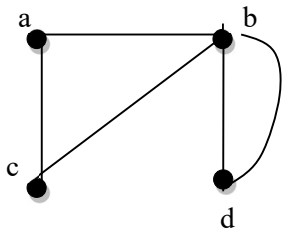
(12 marks)

(b) Find the shortest time to travel from A to G if in reality each time the train passes through a station, excluding A and G, an extra 10 minutes is added to the journey time.

(3 marks)

QUESTION 5 **[20 MARKS]**

a) Remove unnecessary edges in the following figure to make as a tree. (2 marks)



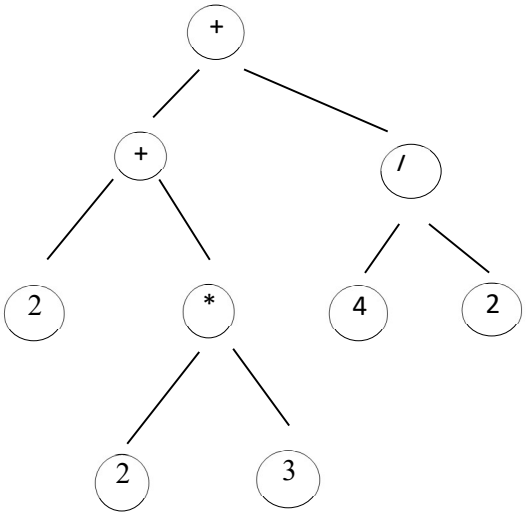
b) The organiser for chess tournament only schedule 45 games in order to determine the champion. Use a rooted tree model to determine the number of people that are allowed to register as player in this tournament. (3 marks)

c) Fibonacci sequence is given as follows,

1, 1, 2, 3, 5, …

Draw the five seven (T_5) rooted Fibonacci sequence given that T_1 andT_2 are both rooted tree consisting a single vertex and for $n=3, 4$.., the rooted T_n is constructed from a root with T_{n-1} as its left subtree and T_{n-2} as its right subtree. Draw the first seven rooted Fibonacci trees. What is the height of this tree? (5 marks)

d) Evaluate the expression obtained from in order traversal of the following tree.



e) Consider a set of 5 towns. The construction of a road between towns i and j is a_{ij} as in matrix below. Find the minimum cost road network connecting the towns with each other.

(6 marks)

$$\begin{pmatrix} 0 & 3 & 5 & 11 & 9 \\ 3 & 0 & 3 & 9 & 8 \\ 5 & 3 & 0 & +\infty & 10 \\ 11 & 9 & +\infty & 0 & 7 \\ 9 & 8 & 10 & 7 & 0 \end{pmatrix}$$

Draw the spanning tree obtain from (e) (2 marks)

QUESTION 6 **[23 MARKS]**

a) Construct a state transition diagram of a DFA that accepts all string over {**b, c, d**} that start with a letter **d** and end with a letter **b**. Examples of accepted strings are **db**, **ddb**, **dccb** etc.

(5 marks)

b) Construct a state transition diagram of a DFA, M with the input set {**m, t, u**} such that M accepts only the string '**utm**'. (4 marks)

c) Consider the following deterministic finite automata in Figure 5. Indicate whether the following string is accepeted or rejected.

(6 marks)

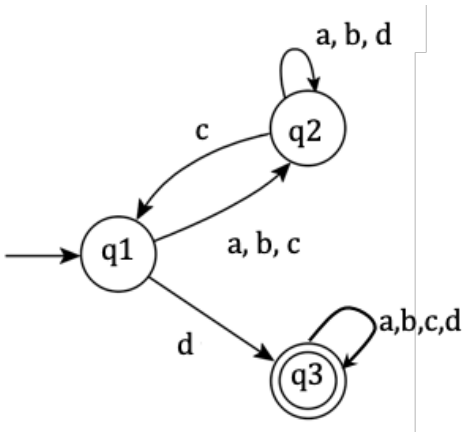


Figure 5

i. aaabccd

ii. abcdab

iii. bccabd

d) Suppose that a language, *L*, is a *C* programing language style comment such that $L = \{ w \mid w$ is a C-style comment$\}$with input alphabet, $\Sigma=\{a, *, /\}$. Examples of accepted and rejected strings are shown in Table 1:

Accepted Strings	Rejected Strings
/*a*/	/**
/**/	/**/a/*aa*/
/***/	aaa/**/
/*aaa*aaa*/	/*/

Construct a deterministic finite automaton (DFA) that accepts all strings in a C programming language style comment.

(9 marks)

QUESTION 7 **[10 MARKS]**

Figure 6 shows the design of a traffic light and barrier control at a crossing of a railway and a road. The railway consists of one track and is monitored by two sensors Left Sensor and Right Sensor which are located 1 kilometer away from the crossing.

Few assumptions for this design are:

- A train may come from either direction, and only one train can enter the sensor zone.
- A train may be long enough to cover one sensor or all two sensors at the same time.
- Each sensor (Left Sensor and Right Sensor) must send two signals to the controller. The first signal is to indicate that the train is entering the sensor zone while the second signal is to indicate that the train is leaving the sensor zone.

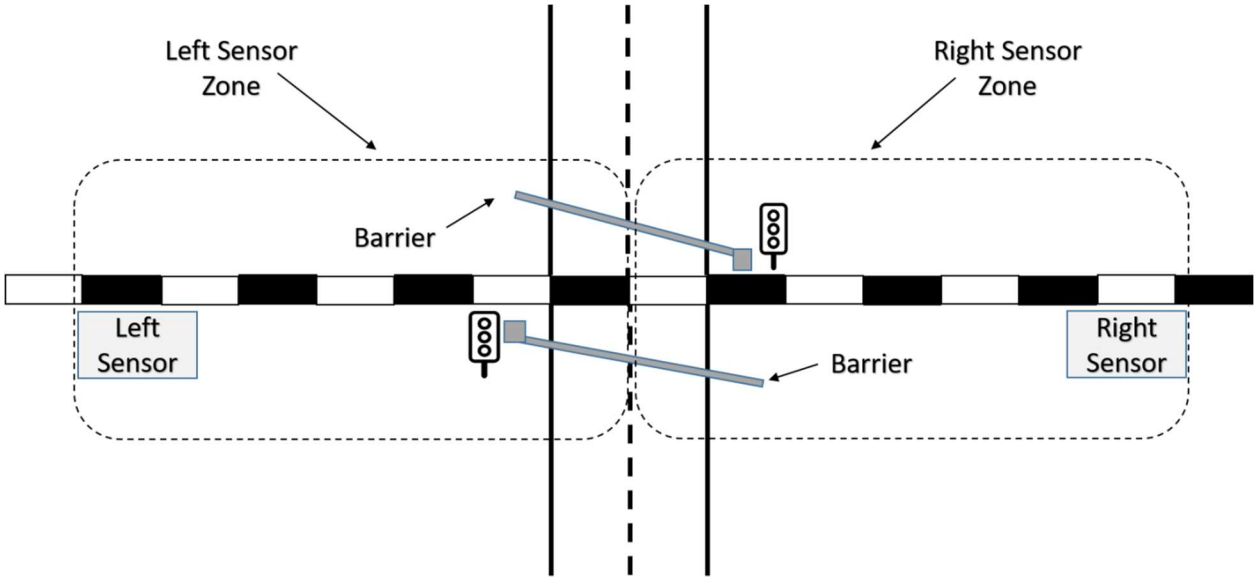


Figure 6

Let us consider a train is coming from the left side of the crossing. Construct a transition diagram for finite state machine to model the control system that switching the traffic lights and moving the barriers according to these following rules:

- Both traffic lights are in the state of showing green light with barriers are raised at the top position when there is no signal from any sensors
- Upon receiving the first signal from the Left Sensor indicating that a train is entering it sensor zone, the lights will be changed and the barriers will be lowered down. Both traffic lights will be in the state of flashing yellow light with lowering barriers.
- When the barriers are at the bottom position, the lights will be changed. Both traffic lights will be in the state of showing red light with the barriers at the bottom position.
- The Left Sensor will send the second signal when the train is leaving it sensor zone, while the Right Sensor will send it first signal indicating the train is entering it sensor zone. In this situation, both traffic lights and barriers remain in current state.
- Only when the Right Sensor sends it second signal which indicates that the train is leaving it sensor zone, the barriers will start raising up. In this condition, both traffic lights will remain in the state of showing red lights but the barriers now are raising to the top position.
- After the barriers reach the top position, the lights will be changed. Both traffic lights are in the state of showing green light with barriers have been raised at the top position.