CONFIDENTIAL

**UNIVERSITI TEKNOLOGI MALAYSIA**

**FINAL EXAMINATION**

**SEMESTER II 2022/2023**

| | |
|---|---|
| **SUBJECT CODE** | **: SCSI/SECI 1143/2143** |
| **SUBJECT NAME** | **: PROBABILITY & STATISTICAL DATA ANALYSIS** |
| **YEAR/COURSE** | **:** |
| **TIME** | **:** |
| **DATE** | **:** |
| **VENUE** | **:** |

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**INSTRUCTIONS TO THE STUDENTS:**

Answer all questions in the answer booklet.

Do all calculations in 3 decimal points.

| | |
|---|---|
| **NAME** | |
| **MATRIC NO.** | |
| **SECTION** | |
| **LECTURER** | |

(This question paper consist of 13 pages including this pages)

**QUESTION 1** **[10 MARKS]**

a) A recent study investigated the usage of ChatGPT towards the students. ChatGPT is a tool that provide language learning, translation and many more. Researcher observed that a representative sample of students from Faculty of Computing found that 158 out of 367 students are active using the ChatGPT.

i) Compute a point estimate of the true proportion of all students who active using the ChatGPT.

(1 mark)

ii) Construct and interpret a 95% confidence interval for the true proportion of all students who actively using ChatGPT services.

(2 marks)

b) In a study of coffee lover among staff in UTM, 30 randomly selected staffs were asked how much time they drink coffee in a day. The time is assumed normally distributed and the sample standard deviation was 6 hours. At $\alpha = 0.01$, is there enough evidence to reject the claim that the population variance is at 35 hours.

i) State the null and alternative hypothesis. (1 mark)

ii) Calculate the test statistic value. (2 marks)

iii) Find the critical value. (1 mark)

iv) Draw and shade the critical region (rejection area) and what can you conclude from the test. (2 marks)

**QUESTION 2** **[20 MARKS]**

a) A research study was conducted to examine the differences between older and younger adults on perceived life satisfaction. A pilot study was conducted to examine this hypothesis. 13 older adults (over the age of 70) and 16 younger adults (between 20 and 30) were give a life satisfaction test. The summary of the score results for the test is given in Table 1.

**Table 1**: Data on perceived life satisfaction

| Older adults | Younger adults |
|---|---|
| $s_1$=1.21 | $s_2$=0.63 |

Test at the $\alpha = 0.05$ level of significance whether there is a different in the variability between the two groups?

(10 marks)

b) The iron contents of fruits before and after applying farm yard manure were observed as in Table 2.

**Table 2**: The iron contents of fruits before and after applying farm yard manure

| Fruit No | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Before Applying | 11.6 | 18.7 | 15.9 | 20.6 | 10.1 | 17.4 | 7.2 | 12.2 | 11.7 |
| After Applying | 10.0 | 21.6 | 13.9 | 22.8 | 11.5 | 16.2 | 8.1 | 10.8 | 9.6 |

Is there any significant difference between the mean iron contents in the fruits before and after the farm yarn manure at $\alpha = 0.05$ level of significance?

(10 marks)

**QUESTION 3** **[20 MARKS]**

a) A staff member of an emergency medical service wishes to determine whether the number of accidents is equally distributed during a week. A week was selected at random, and the following data (Table 3) were obtained. Is there evidence to reject the hypothesis that the number of accident is equally distributed throughout the week, at 95% confidence level.

**Table 3**: Distribution number of accident during a week

| Day | Mon. | Tues. | Wed. | Thurs. | Fri. | Sat. | Sun. |
|---|---|---|---|---|---|---|---|
| No. of accident | 28 | 21 | 15 | 14 | 32 | 23 | 19 |

(8 marks)

b) A study is being conducted to determine whether there is relationship between jogging and blood pressure. A random sample of 210 subjects is selected, and the raw data is shown in Table 4.

**Table 4**: Data from 210 respondents

| Jogging Status | Blood pressure level | Count |
|---|---|---|
| Joggers | Low | 34 |
| Non joggers | Low | 15 |
| Joggers | Moderate | 57 |
| Non joggers | Moderate | 63 |
| Joggers | High | 21 |
| Non joggers | High | 20 |

Based on the data,

i) Construct the two ways contingency table based on data in Table 4.

(2 marks)

ii) At $\alpha = 0.05$ level of significance, test the claim that jogging and blood pressure are not related.

(10 marks)

**QUESTION 4** **[15 MARKS]**

A study of the amount of rainfall and the quantity of air pollution removed produced the following data (Table 5). Assume that $x$ and $y$ are random variables with a bivariate normal distribution.

**Table 5**: Rainfall and air pollution data

| Daily rainfall, $x$ (0.01 cm) | Particulate removed, $y$ ($\frac{\mu g}{m^3}$) |
|---|---|
| 4.3 | 126 |
| 4.5 | 121 |
| 5.9 | 116 |
| 5.6 | 118 |
| 6.1 | 114 |
| 5.2 | 118 |
| 3.8 | 132 |
| 2.1 | 141 |
| 7.5 | 108 |

a) Draw (sketch) the scatter plot.

(2 marks)

b) Given that $\sum x^2 = 244.26, \sum y^2 = 133786$, calculate *r*.

(10 marks)

c) Does the value of *r* value found in 4(b) consistent with the pattern of the scatter plot in 4(a)?

(1 mark)

d) Is the *r* value found in 4(b) suggest a strong linear relationship? If yes, do the changes in *x* and *y* will go in the same direction?

(2 marks)

**QUESTION 5** **[15 MARKS]**

Accu-Copiers, Inc., sells and services the Accu-500 copying machine. As part of its standard service contract, the company agrees to perform routine service on this copier. To obtain information about the time it takes to perform routine service, Accu-Copiers has collected data for 11 service calls. The data are given in Table 6. Whilst, the R output from fitting a least squares regression line to the data is shown in Figure 1.

**Table 6**: The service time data

| Service Call | Number of copiers serviced, $x$ | Number of minutes required, $y$ |
|---|---|---|
| 1 | 4 | 109 |
| 2 | 2 | 58 |
| 3 | 5 | 138 |
| 4 | 7 | 189 |
| 5 | 1 | 37 |
| 6 | 3 | 82 |
| 7 | 4 | 103 |
| 8 | 5 | 134 |
| 9 | 2 | 68 |
| 10 | 4 | 112 |
| 11 | 6 | 154 |

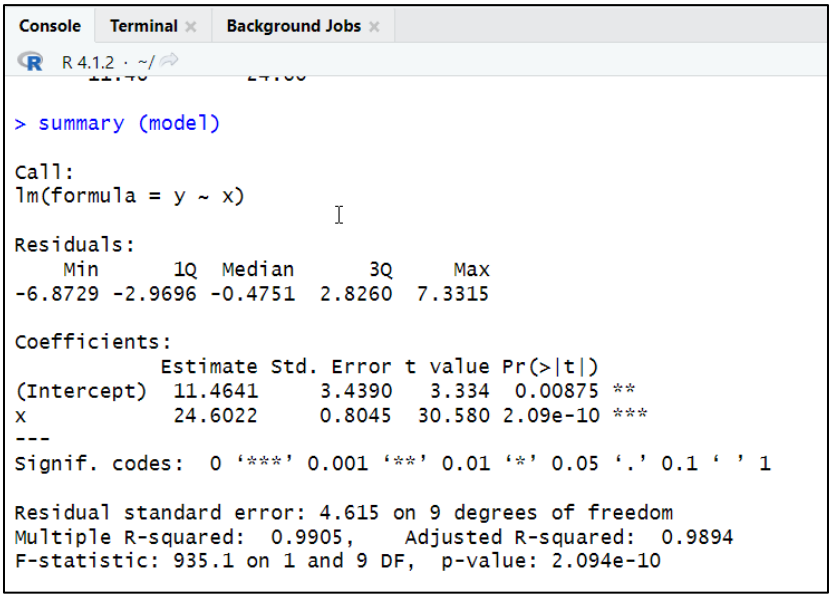


**Figure 1**: R Output

a) Find the least squares point estimates $b_0$ and $b_1$ on the R output (Figure 1) and write their value.

(2 marks)

b) Interpret $b_0$ and $b_1$. Does the interpretation of $b_0$ make practical sense?

(3 marks)

c) Write the equation of the least squares line.

(2 marks)

d) Use the least square line to compute a point estimate of the mean time to service four copiers.

(2 marks)

e) How many percent of the variation in the number of minutes required to service is explained by the variation in the number of copiers serviced?

(1 mark)

f) Conduct the *t*-test to check whether there is a linear relationship between $x$ and $y$. Use $\alpha = 0.05$

(5 marks)

**QUESTION 6** **[20 MARKS]**

A scientist wanted to see if mice is sensitive to colour. He set up an experiment using mice and different door colours to see if there is any colour that the mice would be attracted to. Three sets of five mice were randomly selected to be placed in a standard maze but with different colour doors: red, green, and black. The objective of the experiment is to identify the time for each mouse to run in the maze and reach the door. He recorded the response time (in minutes), to which the time required for each mouse to complete the maze (Table 7).

**Table 7**: Response time recorded for each mouse to complete the maze.

| Door Colour | Response time (minutes) | | | | |
|---|---|---|---|---|---|
| | M1 | M2 | M3 | M4 | M5 |
| Red | 9 | 11 | 10 | 9 | 15 |
| Green | 20 | 21 | 23 | 17 | 30 |
| Black | 6 | 5 | 8 | 14 | 7 |

a) Find the mean, $\bar{x}$, and standard deviation, $s$, for each door colour.

(6 marks)

b) Calculate the test statistics, $F$.

(10 marks)

c) At the 0.01 level of significance, test if the data provide sufficient evidence to conclude that mice are sensitive to different colour.

(4 marks)

**List of Formula:**

(a) Point Estimate and CI

$$\hat{p} \pm (z\ critical\ value)\sqrt{\frac{\hat{p}(1-\hat{p}}{n}}$$

(b) Hypothesis Testing for one sample

$$Test\ Statistic, z_0 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$Test\ Statistic, t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$Test\ Statistic, \chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

(c) Hypothesis Testing for two samples

$$Test\ Statistic, z_0 = \frac{\bar{x}_1 - \bar{x}_2 - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$Test\ Statistic, t_0 = \frac{\bar{x}_1 - \bar{x}_2 - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}; s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$$

$$Test\ Statistic, T_0 = \frac{\bar{x}_1 - \bar{x}_2 - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}; v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(s_1^2/n_1\right)^2}{n_1 - 1} + \frac{\left(s_2^2/n_2\right)^2}{n_2 - 1}}$$

$$Test\ Statistic, F_0 = \frac{s_1^2}{s_2^2}$$

(d) Chi-Square Test

$$Test\ Statistic, \chi^2 = \sum_{i}^{n} \frac{(o_i - e_i)^2}{e_i}$$

$$Degree\ of\ Freedom\ for\ two-way\ contigency\ table = (r-1)(c-1)$$

(e) Correlation

$$Pearson\ Correlation\ Coefficient, r = \frac{\sum xy - \frac{(\sum x \sum y)}{n}}{\sqrt{\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)\left(\sum y^2 - \frac{(\sum y)^2}{n}\right)}}$$

$$Spearman\ Correlation\ Coefficient, r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)}\ where\ d_i = y_i - x_i$$

$$Test\ Statistic, t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}}$$

(f) Regression

$$b_1 = \frac{\sum xy - \frac{(\sum x \sum y)}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

(g) Analysis of variance (ANOVA)

$$Test\ Statistic, F = \frac{ns^{\frac{2}{x}}}{s_p^2}$$

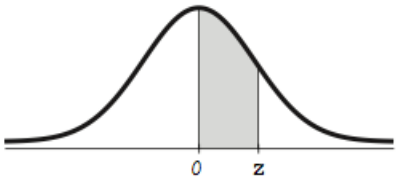


**TABLE A-2** Standard Normal (z) Distribution

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| 0.1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| 0.2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| 0.3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| 0.4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| 0.5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| 0.6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| 0.7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| 0.8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| 0.9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 * | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 * | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |
| 3.10 and higher | .4999 | | | | | | | | | |

*NOTE:* For values of z above 3.09, use 0.4999 for the area.

*Use these common values that result from interpolation:

| z score | Area |
|---|---|
| 1.645 | 0.4500 |
| 2.575 | 0.4950 |

From Frederick C. Mosteller and Robert E. K. Rourke, *Sturdy Statistics*, 1973, Addison-Wesley Publishing Co., Reading, MA. Reprinted with permission of Frederick Mosteller.

**STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.**

| Z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **-3.9** | .00005 | .00005 | .00004 | .00004 | .00004 | .00004 | .00004 | .00004 | .00003 | .00003 |
| **-3.8** | .00007 | .00007 | .00007 | .00006 | .00006 | .00006 | .00006 | .00005 | .00005 | .00005 |
| **-3.7** | .00011 | .00010 | .00010 | .00010 | .00009 | .00009 | .00008 | .00008 | .00008 | .00008 |
| **-3.6** | .00016 | .00015 | .00015 | .00014 | .00014 | .00013 | .00013 | .00012 | .00012 | .00011 |
| **-3.5** | .00023 | .00022 | .00022 | .00021 | .00020 | .00019 | .00019 | .00018 | .00017 | .00017 |
| **-3.4** | .00034 | .00032 | .00031 | .00030 | .00029 | .00028 | .00027 | .00026 | .00025 | .00024 |
| **-3.3** | .00048 | .00047 | .00045 | .00043 | .00042 | .00040 | .00039 | .00038 | .00036 | .00035 |
| **-3.2** | .00069 | .00066 | .00064 | .00062 | .00060 | .00058 | .00056 | .00054 | .00052 | .00050 |
| **-3.1** | .00097 | .00094 | .00090 | .00087 | .00084 | .00082 | .00079 | .00076 | .00074 | .00071 |
| **-3.0** | .00135 | .00131 | .00126 | .00122 | .00118 | .00114 | .00111 | .00107 | .00104 | .00100 |
| **-2.9** | .00187 | .00181 | .00175 | .00169 | .00164 | .00159 | .00154 | .00149 | .00144 | .00139 |
| **-2.8** | .00256 | .00248 | .00240 | .00233 | .00226 | .00219 | .00212 | .00205 | .00199 | .00193 |
| **-2.7** | .00347 | .00336 | .00326 | .00317 | .00307 | .00298 | .00289 | .00280 | .00272 | .00264 |
| **-2.6** | .00466 | .00453 | .00440 | .00427 | .00415 | .00402 | .00391 | .00379 | .00368 | .00357 |
| **-2.5** | .00621 | .00604 | .00587 | .00570 | .00554 | .00539 | .00523 | .00508 | .00494 | .00480 |
| **-2.4** | .00820 | .00798 | .00776 | .00755 | .00734 | .00714 | .00695 | .00676 | .00657 | .00639 |
| **-2.3** | .01072 | .01044 | .01017 | .00990 | .00964 | .00939 | .00914 | .00889 | .00866 | .00842 |
| **-2.2** | .01390 | .01355 | .01321 | .01287 | .01255 | .01222 | .01191 | .01160 | .01130 | .01101 |
| **-2.1** | .01786 | .01743 | .01700 | .01659 | .01618 | .01578 | .01539 | .01500 | .01463 | .01426 |
| **-2.0** | .02275 | .02222 | .02169 | .02118 | .02068 | .02018 | .01970 | .01923 | .01876 | .01831 |
| **-1.9** | .02872 | .02807 | .02743 | .02680 | .02619 | .02559 | .02500 | .02442 | .02385 | .02330 |
| **-1.8** | .03593 | .03515 | .03438 | .03362 | .03288 | .03216 | .03144 | .03074 | .03005 | .02938 |
| **-1.7** | .04457 | .04363 | .04272 | .04182 | .04093 | .04006 | .03920 | .03836 | .03754 | .03673 |
| **-1.6** | .05480 | .05370 | .05262 | .05155 | .05050 | .04947 | .04846 | .04746 | .04648 | .04551 |
| **-1.5** | .06681 | .06552 | .06426 | .06301 | .06178 | .06057 | .05938 | .05821 | .05705 | .05592 |
| **-1.4** | .08076 | .07927 | .07780 | .07636 | .07493 | .07353 | .07215 | .07078 | .06944 | .06811 |
| **-1.3** | .09680 | .09510 | .09342 | .09176 | .09012 | .08851 | .08691 | .08534 | .08379 | .08226 |
| **-1.2** | .11507 | .11314 | .11123 | .10935 | .10749 | .10565 | .10383 | .10204 | .10027 | .09853 |
| **-1.1** | .13567 | .13350 | .13136 | .12924 | .12714 | .12507 | .12302 | .12100 | .11900 | .11702 |
| **-1.0** | .15866 | .15625 | .15386 | .15151 | .14917 | .14686 | .14457 | .14231 | .14007 | .13786 |
| **-0.9** | .18406 | .18141 | .17879 | .17619 | .17361 | .17106 | .16853 | .16602 | .16354 | .16109 |
| **-0.8** | .21186 | .20897 | .20611 | .20327 | .20045 | .19766 | .19489 | .19215 | .18943 | .18673 |
| **-0.7** | .24196 | .23885 | .23576 | .23270 | .22965 | .22663 | .22363 | .22065 | .21770 | .21476 |
| **-0.6** | .27425 | .27093 | .26763 | .26435 | .26109 | .25785 | .25463 | .25143 | .24825 | .24510 |
| **-0.5** | .30854 | .30503 | .30153 | .29806 | .29460 | .29116 | .28774 | .28434 | .28096 | .27760 |
| **-0.4** | .34458 | .34090 | .33724 | .33360 | .32997 | .32636 | .32276 | .31918 | .31561 | .31207 |
| **-0.3** | .38209 | .37828 | .37448 | .37070 | .36693 | .36317 | .35942 | .35569 | .35197 | .34827 |
| **-0.2** | .42074 | .41683 | .41294 | .40905 | .40517 | .40129 | .39743 | .39358 | .38974 | .38591 |
| **-0.1** | .46017 | .45620 | .45224 | .44828 | .44433 | .44038 | .43644 | .43251 | .42858 | .42465 |
| **-0.0** | .50000 | .49601 | .49202 | .48803 | .48405 | .48006 | .47608 | .47210 | .46812 | .46414 |

**STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.**

| Z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.0** | .50000 | .50399 | .50798 | .51197 | .51595 | .51994 | .52392 | .52790 | .53188 | .53586 |
| **0.1** | .53983 | .54380 | .54776 | .55172 | .55567 | .55962 | .56356 | .56749 | .57142 | .57535 |
| **0.2** | .57926 | .58317 | .58706 | .59095 | .59483 | .59871 | .60257 | .60642 | .61026 | .61409 |
| **0.3** | .61791 | .62172 | .62552 | .62930 | .63307 | .63683 | .64058 | .64431 | .64803 | .65173 |
| **0.4** | .65542 | .65910 | .66276 | .66640 | .67003 | .67364 | .67724 | .68082 | .68439 | .68793 |
| **0.5** | .69146 | .69497 | .69847 | .70194 | .70540 | .70884 | .71226 | .71566 | .71904 | .72240 |
| **0.6** | .72575 | .72907 | .73237 | .73565 | .73891 | .74215 | .74537 | .74857 | .75175 | .75490 |
| **0.7** | .75804 | .76115 | .76424 | .76730 | .77035 | .77337 | .77637 | .77935 | .78230 | .78524 |
| **0.8** | .78814 | .79103 | .79389 | .79673 | .79955 | .80234 | .80511 | .80785 | .81057 | .81327 |
| **0.9** | .81594 | .81859 | .82121 | .82381 | .82639 | .82894 | .83147 | .83398 | .83646 | .83891 |
| **1.0** | .84134 | .84375 | .84614 | .84849 | .85083 | .85314 | .85543 | .85769 | .85993 | .86214 |
| **1.1** | .86433 | .86650 | .86864 | .87076 | .87286 | .87493 | .87698 | .87900 | .88100 | .88298 |
| **1.2** | .88493 | .88686 | .88877 | .89065 | .89251 | .89435 | .89617 | .89796 | .89973 | .90147 |
| **1.3** | .90320 | .90490 | .90658 | .90824 | .90988 | .91149 | .91309 | .91466 | .91621 | .91774 |
| **1.4** | .91924 | .92073 | .92220 | .92364 | .92507 | .92647 | .92785 | .92922 | .93056 | .93189 |
| **1.5** | .93319 | .93448 | .93574 | .93699 | .93822 | .93943 | .94062 | .94179 | .94295 | .94408 |
| **1.6** | .94520 | .94630 | .94738 | .94845 | .94950 | .95053 | .95154 | .95254 | .95352 | .95449 |
| **1.7** | .95543 | .95637 | .95728 | .95818 | .95907 | .95994 | .96080 | .96164 | .96246 | .96327 |
| **1.8** | .96407 | .96485 | .96562 | .96638 | .96712 | .96784 | .96856 | .96926 | .96995 | .97062 |
| **1.9** | .97128 | .97193 | .97257 | .97320 | .97381 | .97441 | .97500 | .97558 | .97615 | .97670 |
| **2.0** | .97725 | .97778 | .97831 | .97882 | .97932 | .97982 | .98030 | .98077 | .98124 | .98169 |
| **2.1** | .98214 | .98257 | .98300 | .98341 | .98382 | .98422 | .98461 | .98500 | .98537 | .98574 |
| **2.2** | .98610 | .98645 | .98679 | .98713 | .98745 | .98778 | .98809 | .98840 | .98870 | .98899 |
| **2.3** | .98928 | .98956 | .98983 | .99010 | .99036 | .99061 | .99086 | .99111 | .99134 | .99158 |
| **2.4** | .99180 | .99202 | .99224 | .99245 | .99266 | .99286 | .99305 | .99324 | .99343 | .99361 |
| **2.5** | .99379 | .99396 | .99413 | .99430 | .99446 | .99461 | .99477 | .99492 | .99506 | .99520 |
| **2.6** | .99534 | .99547 | .99560 | .99573 | .99585 | .99598 | .99609 | .99621 | .99632 | .99643 |
| **2.7** | .99653 | .99664 | .99674 | .99683 | .99693 | .99702 | .99711 | .99720 | .99728 | .99736 |
| **2.8** | .99744 | .99752 | .99760 | .99767 | .99774 | .99781 | .99788 | .99795 | .99801 | .99807 |
| **2.9** | .99813 | .99819 | .99825 | .99831 | .99836 | .99841 | .99846 | .99851 | .99856 | .99861 |
| **3.0** | .99865 | .99869 | .99874 | .99878 | .99882 | .99886 | .99889 | .99893 | .99896 | .99900 |
| **3.1** | .99903 | .99906 | .99910 | .99913 | .99916 | .99918 | .99921 | .99924 | .99926 | .99929 |
| **3.2** | .99931 | .99934 | .99936 | .99938 | .99940 | .99942 | .99944 | .99946 | .99948 | .99950 |
| **3.3** | .99952 | .99953 | .99955 | .99957 | .99958 | .99960 | .99961 | .99962 | .99964 | .99965 |
| **3.4** | .99966 | .99968 | .99969 | .99970 | .99971 | .99972 | .99973 | .99974 | .99975 | .99976 |
| **3.5** | .99977 | .99978 | .99978 | .99979 | .99980 | .99981 | .99981 | .99982 | .99983 | .99983 |
| **3.6** | .99984 | .99985 | .99985 | .99986 | .99986 | .99987 | .99987 | .99988 | .99988 | .99989 |
| **3.7** | .99989 | .99990 | .99990 | .99990 | .99991 | .99991 | .99992 | .99992 | .99992 | .99992 |
| **3.8** | .99993 | .99993 | .99993 | .99994 | .99994 | .99994 | .99994 | .99995 | .99995 | .99995 |
| **3.9** | .99995 | .99995 | .99996 | .99996 | .99996 | .99996 | .99996 | .99996 | .99997 | .99997 |

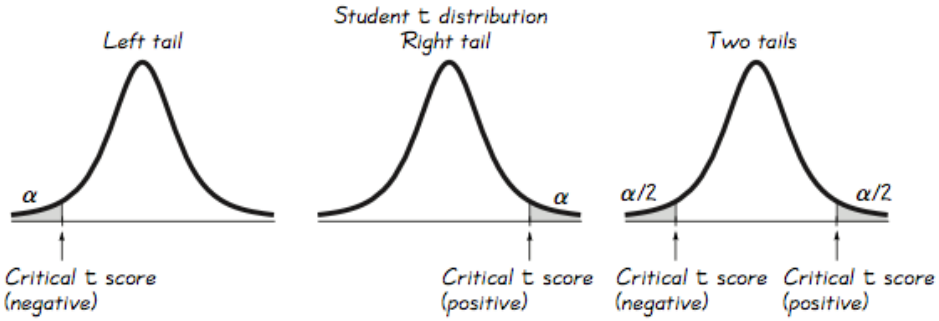


**TABLE A-3** t Distribution

| | α | | | | | |
|---|---|---|---|---|---|---|
| Degrees of Freedom | .005 (one tail) .01 (two tails) | .01 (one tail) .02 (two tails) | .025 (one tail) .05 (two tails) | .05 (one tail) .10 (two tails) | .10 (one tail) .20 (two tails) | .25 (one tail) .50 (two tails) |
| 1 | 63.657 | 31.821 | 12.706 | 6.314 | 3.078 | 1.000 |
| 2 | 9.925 | 6.965 | 4.303 | 2.920 | 1.886 | .816 |
| 3 | 5.841 | 4.541 | 3.182 | 2.353 | 1.638 | .765 |
| 4 | 4.604 | 3.747 | 2.776 | 2.132 | 1.533 | .741 |
| 5 | 4.032 | 3.365 | 2.571 | 2.015 | 1.476 | .727 |
| 6 | 3.707 | 3.143 | 2.447 | 1.943 | 1.440 | .718 |
| 7 | 3.500 | 2.998 | 2.365 | 1.895 | 1.415 | .711 |
| 8 | 3.355 | 2.896 | 2.306 | 1.860 | 1.397 | .706 |
| 9 | 3.250 | 2.821 | 2.262 | 1.833 | 1.383 | .703 |
| 10 | 3.169 | 2.764 | 2.228 | 1.812 | 1.372 | .700 |
| 11 | 3.106 | 2.718 | 2.201 | 1.796 | 1.363 | .697 |
| 12 | 3.054 | 2.681 | 2.179 | 1.782 | 1.356 | .696 |
| 13 | 3.012 | 2.650 | 2.160 | 1.771 | 1.350 | .694 |
| 14 | 2.977 | 2.625 | 2.145 | 1.761 | 1.345 | .692 |
| 15 | 2.947 | 2.602 | 2.132 | 1.753 | 1.341 | .691 |
| 16 | 2.921 | 2.584 | 2.120 | 1.746 | 1.337 | .690 |
| 17 | 2.898 | 2.567 | 2.110 | 1.740 | 1.333 | .689 |
| 18 | 2.878 | 2.552 | 2.101 | 1.734 | 1.330 | .688 |
| 19 | 2.861 | 2.540 | 2.093 | 1.729 | 1.328 | .688 |
| 20 | 2.845 | 2.528 | 2.086 | 1.725 | 1.325 | .687 |
| 21 | 2.831 | 2.518 | 2.080 | 1.721 | 1.323 | .686 |
| 22 | 2.819 | 2.508 | 2.074 | 1.717 | 1.321 | .686 |
| 23 | 2.807 | 2.500 | 2.069 | 1.714 | 1.320 | .685 |
| 24 | 2.797 | 2.492 | 2.064 | 1.711 | 1.318 | .685 |
| 25 | 2.787 | 2.485 | 2.060 | 1.708 | 1.316 | .684 |
| 26 | 2.779 | 2.479 | 2.056 | 1.706 | 1.315 | .684 |
| 27 | 2.771 | 2.473 | 2.052 | 1.703 | 1.314 | .684 |
| 28 | 2.763 | 2.467 | 2.048 | 1.701 | 1.313 | .683 |
| 29 | 2.756 | 2.462 | 2.045 | 1.699 | 1.311 | .683 |
| Large ($z$) | 2.575 | 2.326 | 1.960 | 1.645 | 1.282 | .675 |

**TABLE A-4** Chi-Square ($\chi^2$) Distribution

| Degrees of Freedom | Area to the Right of the Critical Value | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.995 | 0.99 | 0.975 | 0.95 | 0.90 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| 1 | — | — | 0.001 | 0.004 | 0.016 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.071 | 12.833 | 15.086 | 16.750 |
| 6 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 |
| 7 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 | 18.549 | 21.026 | 23.337 | 26.217 | 28.299 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 7.042 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 |
| 15 | 4.601 | 5.229 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 |
| 16 | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 |
| 17 | 5.697 | 6.408 | 7.564 | 8.672 | 10.085 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 |
| 19 | 6.844 | 7.633 | 8.907 | 10.117 | 11.651 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 |
| 20 | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 |
| 21 | 8.034 | 8.897 | 10.283 | 11.591 | 13.240 | 29.615 | 32.671 | 35.479 | 38.932 | 41.401 |
| 22 | 8.643 | 9.542 | 10.982 | 12.338 | 14.042 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 |
| 23 | 9.260 | 10.196 | 11.689 | 13.091 | 14.848 | 32.007 | 35.172 | 38.076 | 41.638 | 44.181 |
| 24 | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 | 33.196 | 36.415 | 39.364 | 42.980 | 45.559 |
| 25 | 10.520 | 11.524 | 13.120 | 14.611 | 16.473 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 |
| 26 | 11.160 | 12.198 | 13.844 | 15.379 | 17.292 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 |
| 27 | 11.808 | 12.879 | 14.573 | 16.151 | 18.114 | 36.741 | 40.113 | 43.194 | 46.963 | 49.645 |
| 28 | 12.461 | 13.565 | 15.308 | 16.928 | 18.939 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 |
| 29 | 13.121 | 14.257 | 16.047 | 17.708 | 19.768 | 39.087 | 42.557 | 45.722 | 49.588 | 52.336 |
| 30 | 13.787 | 14.954 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |
| 40 | 20.707 | 22.164 | 24.433 | 26.509 | 29.051 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 |
| 50 | 27.991 | 29.707 | 32.357 | 34.764 | 37.689 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 |
| 60 | 35.534 | 37.485 | 40.482 | 43.188 | 46.459 | 74.397 | 79.082 | 83.298 | 88.379 | 91.952 |
| 70 | 43.275 | 45.442 | 48.758 | 51.739 | 55.329 | 85.527 | 90.531 | 95.023 | 100.425 | 104.215 |
| 80 | 51.172 | 53.540 | 57.153 | 60.391 | 64.278 | 96.578 | 101.879 | 106.629 | 112.329 | 116.321 |
| 90 | 59.196 | 61.754 | 65.647 | 69.126 | 73.291 | 107.565 | 113.145 | 118.136 | 124.116 | 128.299 |
| 100 | 67.328 | 70.065 | 74.222 | 77.929 | 82.358 | 118.498 | 124.342 | 129.561 | 135.807 | 140.169 |

**TABLE E**

*F* critical values (continued)

| Degrees of freedom in the denominator | | Degrees of freedom in the numerator | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | *p* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 8 | .100 | 3.46 | 3.11 | 2.92 | 2.81 | 2.73 | 2.67 | 2.62 | 2.59 | 2.56 |
| | .050 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 |
| | .025 | 7.57 | 6.06 | 5.42 | 5.05 | 4.82 | 4.65 | 4.53 | 4.43 | 4.36 |
| | .010 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 |
| | .001 | 25.41 | 18.49 | 15.83 | 14.39 | 13.48 | 12.86 | 12.40 | 12.05 | 11.77 |
| 9 | .100 | 3.36 | 3.01 | 2.81 | 2.69 | 2.61 | 2.55 | 2.51 | 2.47 | 2.44 |
| | .050 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 |
| | .025 | 7.21 | 5.71 | 5.08 | 4.72 | 4.48 | 4.32 | 4.20 | 4.10 | 4.03 |
| | .010 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 |
| | .001 | 22.86 | 16.39 | 13.90 | 12.56 | 11.71 | 11.13 | 10.70 | 10.37 | 10.11 |
| 10 | .100 | 3.29 | 2.92 | 2.73 | 2.61 | 2.52 | 2.46 | 2.41 | 2.38 | 2.35 |
| | .050 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 |
| | .025 | 6.94 | 5.46 | 4.83 | 4.47 | 4.24 | 4.07 | 3.95 | 3.85 | 3.78 |
| | .010 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 |
| | .001 | 21.04 | 14.91 | 12.55 | 11.28 | 10.48 | 9.93 | 9.52 | 9.20 | 8.96 |
| 11 | .100 | 3.23 | 2.86 | 2.66 | 2.54 | 2.45 | 2.39 | 2.34 | 2.30 | 2.27 |
| | .050 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 |
| | .025 | 6.72 | 5.26 | 4.63 | 4.28 | 4.04 | 3.88 | 3.76 | 3.66 | 3.59 |
| | .010 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 |
| | .001 | 19.69 | 13.81 | 11.56 | 10.35 | 9.58 | 9.05 | 8.66 | 8.35 | 8.12 |
| 12 | .100 | 3.18 | 2.81 | 2.61 | 2.48 | 2.39 | 2.33 | 2.28 | 2.24 | 2.21 |
| | .050 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 |
| | .025 | 6.55 | 5.10 | 4.47 | 4.12 | 3.89 | 3.73 | 3.61 | 3.51 | 3.44 |
| | .010 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 |
| | .001 | 18.64 | 12.97 | 10.80 | 9.63 | 8.89 | 8.38 | 8.00 | 7.71 | 7.48 |
| 13 | .100 | 3.14 | 2.76 | 2.56 | 2.43 | 2.35 | 2.28 | 2.23 | 2.20 | 2.16 |
| | .050 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 |
| | .025 | 6.41 | 4.97 | 4.35 | 4.00 | 3.77 | 3.60 | 3.48 | 3.39 | 3.31 |
| | .010 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 |
| | .001 | 17.82 | 12.31 | 10.21 | 9.07 | 8.35 | 7.86 | 7.49 | 7.21 | 6.98 |
| 14 | .100 | 3.10 | 2.73 | 2.52 | 2.39 | 2.31 | 2.24 | 2.19 | 2.15 | 2.12 |
| | .050 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 |
| | .025 | 6.30 | 4.86 | 4.24 | 3.89 | 3.66 | 3.50 | 3.38 | 3.29 | 3.21 |
| | .010 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 |
| | .001 | 17.14 | 11.78 | 9.73 | 8.62 | 7.92 | 7.44 | 7.08 | 6.80 | 6.58 |
| 15 | .100 | 3.07 | 2.70 | 2.49 | 2.36 | 2.27 | 2.21 | 2.16 | 2.12 | 2.09 |
| | .050 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 |
| | .025 | 6.20 | 4.77 | 4.15 | 3.80 | 3.58 | 3.41 | 3.29 | 3.20 | 3.12 |
| | .010 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 |
| | .001 | 16.59 | 11.34 | 9.34 | 8.25 | 7.57 | 7.09 | 6.74 | 6.47 | 6.26 |
| 16 | .100 | 3.05 | 2.67 | 2.46 | 2.33 | 2.24 | 2.18 | 2.13 | 2.09 | 2.06 |
| | .050 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 |
| | .025 | 6.12 | 4.69 | 4.08 | 3.73 | 3.50 | 3.34 | 3.22 | 3.12 | 3.05 |
| | .010 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 |
| | .001 | 16.12 | 10.97 | 9.01 | 7.94 | 7.27 | 6.80 | 6.46 | 6.19 | 5.98 |
| 17 | .100 | 3.03 | 2.64 | 2.44 | 2.31 | 2.22 | 2.15 | 2.10 | 2.06 | 2.03 |
| | .050 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 |
| | .025 | 6.04 | 4.62 | 4.01 | 3.66 | 3.44 | 3.28 | 3.16 | 3.06 | 2.98 |
| | .010 | 8.40 | 6.11 | 5.19 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 |
| | .001 | 15.72 | 10.66 | 8.73 | 7.68 | 7.02 | 6.56 | 6.22 | 5.96 | 5.75 |





**TABLE E**

*F* critical values (continued)

| Degrees of freedom in the numerator | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 12 | 15 | 20 | 25 | 30 | 40 | 50 | 60 | 120 | 1000 |
| 2.54 | 2.50 | 2.46 | 2.42 | 2.40 | 2.38 | 2.36 | 2.35 | 2.34 | 2.32 | 2.30 |
| 3.35 | 3.28 | 3.22 | 3.15 | 3.11 | 3.08 | 3.04 | 3.02 | 3.01 | 2.97 | 2.93 |
| 4.30 | 4.20 | 4.10 | 4.00 | 3.94 | 3.89 | 3.84 | 3.81 | 3.78 | 3.73 | 3.68 |
| 5.81 | 5.67 | 5.52 | 5.36 | 5.26 | 5.20 | 5.12 | 5.07 | 5.03 | 4.95 | 4.87 |
| 11.54 | 11.19 | 10.84 | 10.48 | 10.26 | 10.11 | 9.92 | 9.80 | 9.73 | 9.53 | 9.36 |
| 2.42 | 2.38 | 2.34 | 2.30 | 2.27 | 2.25 | 2.23 | 2.22 | 2.21 | 2.18 | 2.16 |
| 3.14 | 3.07 | 3.01 | 2.94 | 2.89 | 2.86 | 2.83 | 2.80 | 2.79 | 2.75 | 2.71 |
| 3.96 | 3.87 | 3.77 | 3.67 | 3.60 | 3.56 | 3.51 | 3.47 | 3.45 | 3.39 | 3.34 |
| 5.26 | 5.11 | 4.96 | 4.81 | 4.71 | 4.65 | 4.57 | 4.52 | 4.48 | 4.40 | 4.32 |
| 9.89 | 9.57 | 9.24 | 8.90 | 8.69 | 8.55 | 8.37 | 8.26 | 8.19 | 8.00 | 7.84 |
| 2.32 | 2.28 | 2.24 | 2.20 | 2.17 | 2.16 | 2.13 | 2.12 | 2.11 | 2.08 | 2.06 |
| 2.98 | 2.91 | 2.85 | 2.77 | 2.73 | 2.70 | 2.66 | 2.64 | 2.62 | 2.58 | 2.54 |
| 3.72 | 3.62 | 3.52 | 3.42 | 3.35 | 3.31 | 3.26 | 3.22 | 3.20 | 3.14 | 3.09 |
| 4.85 | 4.71 | 4.56 | 4.41 | 4.31 | 4.25 | 4.17 | 4.12 | 4.08 | 4.00 | 3.92 |
| 8.75 | 8.45 | 8.13 | 7.80 | 7.60 | 7.47 | 7.30 | 7.19 | 7.12 | 6.94 | 6.78 |
| 2.25 | 2.21 | 2.17 | 2.12 | 2.10 | 2.08 | 2.05 | 2.04 | 2.03 | 2.00 | 1.98 |
| 2.85 | 2.79 | 2.72 | 2.65 | 2.60 | 2.57 | 2.53 | 2.51 | 2.49 | 2.45 | 2.41 |
| 3.53 | 3.43 | 3.33 | 3.23 | 3.16 | 3.12 | 3.06 | 3.03 | 3.00 | 2.94 | 2.89 |
| 4.54 | 4.40 | 4.25 | 4.10 | 4.01 | 3.94 | 3.86 | 3.81 | 3.78 | 3.69 | 3.61 |
| 7.92 | 7.63 | 7.32 | 7.01 | 6.81 | 6.68 | 6.52 | 6.42 | 6.35 | 6.18 | 6.02 |
| 2.19 | 2.15 | 2.10 | 2.06 | 2.03 | 2.01 | 1.99 | 1.97 | 1.96 | 1.93 | 1.91 |
| 2.75 | 2.69 | 2.62 | 2.54 | 2.50 | 2.47 | 2.43 | 2.40 | 2.38 | 2.34 | 2.30 |
| 3.37 | 3.28 | 3.18 | 3.07 | 3.01 | 2.96 | 2.91 | 2.87 | 2.85 | 2.79 | 2.73 |
| 4.30 | 4.16 | 4.01 | 3.86 | 3.76 | 3.70 | 3.62 | 3.57 | 3.54 | 3.45 | 3.37 |
| 7.29 | 7.00 | 6.71 | 6.40 | 6.22 | 6.09 | 5.93 | 5.83 | 5.76 | 5.59 | 5.44 |
| 2.14 | 2.10 | 2.05 | 2.01 | 1.98 | 1.96 | 1.93 | 1.92 | 1.90 | 1.88 | 1.85 |
| 2.67 | 2.60 | 2.53 | 2.46 | 2.41 | 2.38 | 2.34 | 2.31 | 2.30 | 2.25 | 2.21 |
| 3.25 | 3.15 | 3.05 | 2.95 | 2.88 | 2.84 | 2.78 | 2.74 | 2.72 | 2.66 | 2.60 |
| 4.10 | 3.96 | 3.82 | 3.66 | 3.57 | 3.51 | 3.43 | 3.38 | 3.34 | 3.25 | 3.18 |
| 6.80 | 6.52 | 6.23 | 5.93 | 5.75 | 5.63 | 5.47 | 5.37 | 5.30 | 5.14 | 4.99 |
| 2.10 | 2.05 | 2.01 | 1.96 | 1.93 | 1.91 | 1.89 | 1.87 | 1.86 | 1.83 | 1.80 |
| 2.60 | 2.53 | 2.46 | 2.39 | 2.34 | 2.31 | 2.27 | 2.24 | 2.22 | 2.18 | 2.14 |
| 3.15 | 3.05 | 2.95 | 2.84 | 2.78 | 2.73 | 2.67 | 2.64 | 2.61 | 2.55 | 2.50 |
| 3.94 | 3.80 | 3.66 | 3.51 | 3.41 | 3.35 | 3.27 | 3.22 | 3.18 | 3.09 | 3.02 |
| 6.40 | 6.13 | 5.85 | 5.56 | 5.38 | 5.25 | 5.10 | 5.00 | 4.94 | 4.77 | 4.62 |
| 2.06 | 2.02 | 1.97 | 1.92 | 1.89 | 1.87 | 1.85 | 1.83 | 1.82 | 1.79 | 1.76 |
| 2.54 | 2.48 | 2.40 | 2.33 | 2.28 | 2.25 | 2.20 | 2.18 | 2.16 | 2.11 | 2.07 |
| 3.06 | 2.96 | 2.86 | 2.76 | 2.69 | 2.64 | 2.59 | 2.55 | 2.52 | 2.46 | 2.40 |
| 3.80 | 3.67 | 3.52 | 3.37 | 3.28 | 3.21 | 3.13 | 3.08 | 3.05 | 2.96 | 2.88 |
| 6.08 | 5.81 | 5.54 | 5.25 | 5.07 | 4.95 | 4.80 | 4.70 | 4.64 | 4.47 | 4.33 |
| 2.03 | 1.99 | 1.94 | 1.89 | 1.86 | 1.84 | 1.81 | 1.79 | 1.78 | 1.75 | 1.72 |
| 2.49 | 2.42 | 2.35 | 2.28 | 2.23 | 2.19 | 2.15 | 2.12 | 2.11 | 2.06 | 2.02 |
| 2.99 | 2.89 | 2.79 | 2.68 | 2.61 | 2.57 | 2.51 | 2.47 | 2.45 | 2.38 | 2.32 |
| 3.69 | 3.55 | 3.41 | 3.26 | 3.16 | 3.10 | 3.02 | 2.97 | 2.93 | 2.84 | 2.76 |
| 5.81 | 5.55 | 5.27 | 4.99 | 4.82 | 4.70 | 4.54 | 4.45 | 4.39 | 4.23 | 4.08 |
| 2.00 | 1.96 | 1.91 | 1.86 | 1.83 | 1.81 | 1.78 | 1.76 | 1.75 | 1.72 | 1.69 |
| 2.45 | 2.38 | 2.31 | 2.23 | 2.18 | 2.15 | 2.10 | 2.08 | 2.06 | 2.01 | 1.97 |
| 2.92 | 2.82 | 2.72 | 2.62 | 2.55 | 2.50 | 2.44 | 2.41 | 2.38 | 2.32 | 2.26 |
| 3.59 | 3.46 | 3.31 | 3.16 | 3.07 | 3.00 | 2.92 | 2.87 | 2.83 | 2.75 | 2.66 |
| 5.58 | 5.32 | 5.05 | 4.78 | 4.60 | 4.48 | 4.33 | 4.24 | 4.18 | 4.02 | 3.87 |

*(Continued)*