CSE 351 - HW1
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1.a $dy/dt + 2y(t) = f^2(t)$

Solution =>

$f(t) \longrightarrow \boxed{dy/dt + 2y(t) = f^2(t)} \longrightarrow y(t)$

& Applying superposition condition

$f_1(t) \rightarrow \boxed{h(t)} \rightarrow y_1(t) \quad \Rightarrow \quad \frac{dy_1}{dt} + 2y_1(t) = f_1^2(t) \quad / k_1$

$f_2(t) \rightarrow \boxed{h(t)} \rightarrow y_2(t) \quad \Rightarrow \quad \frac{dy_2}{dt} + 2y_2(t) = f_2^2(t) \quad / k_2$

$$\Rightarrow \frac{d}{dt}\Big[\underbrace{k_1y_1 + k_2y_2}_{\text{New output } y'(t)}\Big] + 2\Big[\underbrace{k_1y_1(t) + k_2y_2(t)}_{y'(t)}\Big] = \underbrace{k_1.f_1^2(t) + k_2.f_2^2(t)}_{f'(t)}$$

system

$k_1f_1(t) + k_2f_2(t) \rightarrow \boxed{} \rightarrow k_1y_1(t) + k_2y_2(t)$

$$k_1f_1^2(t) + k_2f_2^2(t) \neq [k_1f_1(t) + k_2f_2(t)]^2$$

when the input is $k_1f_1(t) + k_2f_2(t)$, then system output is not $k_1y_1(t) + k_2y_2(t)$

then system is non linear. ✓

1.b $dy/dt + 3ty(t) = t^2f(t)$

$f(t) \longrightarrow \boxed{\frac{dy(t)}{dt} + 3ty(t) = t^2f(t)} \longrightarrow y(t)$

& Applying superposition condition

$f_1(t) \rightarrow \boxed{h(t)} \rightarrow y_1(t) \quad \Rightarrow \quad \frac{dy_1}{dt} + 3ty_1(t) = t^2f_1(t) \quad / k_1$

$f_2(t) \rightarrow \boxed{h(t)} \rightarrow y_2(t) \quad \Rightarrow \quad \frac{dy_2}{dt} + 3ty_2(t) = t^2f_2(t) \quad / k_2$

$$\Rightarrow \frac{d}{dt}\Big[\underbrace{k_1y_1(t) + k_2y_2(t)}_{y'(t)}\Big] + 3t\Big[\underbrace{k_1y_1(t) + k_2y_2(t)}_{y'(t)}\Big] = t^2\big(\underbrace{k_1f_1(t) + k_2f_2(t)}_{f'(t)}\big)$$

when the input is $f'(t) = k_1f_1(t) + k_2f_2(t)$, then the output is $y'(t) = k_1y_1(t) + k_2y_2(t)$

So, it's a Linear system. ✓

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2.a) $\underbrace{(D^2+5D+6)}_{Q(D)} \cdot y(t) = \underbrace{(D+1)}_{P(D)} f(t)$, $y_0(0) = 2$, $\frac{d\, y_0(0)}{dt} = -1$, $y_0(t) = ?$ zero-input response

=> Solution:

$\rightarrow Q(\lambda) = Q(D)\big|_{D=\lambda}$

$= D^2+5D+6\big|_{D=\lambda}$

$Q(\lambda) = \lambda^2+5\lambda+6$ => characteristic polynomial

$= (\lambda+3).(\lambda+2)$ => $\lambda_1 = -3$, $\lambda_2 = -2$ // Characteristic roots

The characteristic modes are: e^{-3t} , e^{-2t}

The general solution:

$$y_0(t) = c_1 e^{-3t} + c_2 e^{-2t}$$

To find the unique solution, we find c_1 and c_2

$\dot{y}(0) = -1 \rightarrow y_0'(t) = -3c_1e^{-3t} - 2c_2e^{-2t}$

$\dot{y}_0(0) = -3c_1 - 2c_2 = -1$

$y_0(0) = 2 \rightarrow c_1 + c_2 = 2$

$c_1 = -3$

$c_2 = 5$

$\Rightarrow y_0(t) = c_1 e^{-3t} + c_2 e^{-2t}$

$= \boxed{-3e^{-3t} + 5e^{-2t}}$ → zero-input response ✓

%100

2.b) $h(t) = e^{-t}u(t)$, $f(t) = u(t)$ Find the zero state response of system $y(t)$?

$f(t) = u(t) \rightarrow \boxed{h(t) = e^{-t}u(t)} \rightarrow y(t) = ?$

$y(t) = \int_{-\infty}^{+\infty} f(\tau) \cdot h(t-\tau)\, d\tau$

$f(\tau) = f(t)\big|_{t=\tau} = u(\tau)$

$h(t-\tau) = h(t)\big|_{t=t-\tau} = e^{+\tau-t} \cdot u(\tau)$

$y(t) = \int_0^t e^{+\tau-t} \cdot d\tau = +e^{\tau-t}\Big|_0^t = 1 - e^{-t}$ result, $t \geq 0$

$y(t) = 0$, $t < 0$, then

$\boxed{y(t) = (1-e^{-t})u(t)}$ → zero-state response ✓

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3.a

$$y[k+1] + 2y[k] = \delta[k] \quad , \quad h[k] = ?$$

Solution;

$$E\,y[k] + 2y[k] = \delta[k]$$

$$\underbrace{(E+2)}_{Q(E)} \cdot y[k] = \delta[k]$$

characteristic polynomial

$$Q(\gamma) = Q(E)\big|_{E=\gamma} = \gamma + 2 = 0$$

$$\gamma_1 = -2$$

/ Non-repeated roots

characteristic root

$$y_0[k] = c_1\,\gamma_1^{\,k} = c_1 \cdot (-2)^k$$

$$h[k] = \frac{b_0}{a_0} \cdot \delta[k] + y_0[k] \cdot u[k]$$

$$a_0 = 2 \quad , \quad b_0 = 1$$

$$h[k] = \frac{1}{2} \cdot \delta[k] + c \cdot (-2)^k \cdot u[k]$$

- Iteration solution:

$$E\,y[k] + 2y[k] = f[k]$$

$$E\,h[k] + 2h[k] = \delta[k]$$

$$h[k+1] + 2h[k] = \delta[k]$$

for $k = -1$,

$$h[-1] = \delta[-1] = 0$$

$$h[0] = 0$$

$$h[0] = \frac{1}{2}\,\delta[0] + c \cdot (-2)^0$$

$$0 = 1/2 + c$$

$$c = -\frac{1}{2}$$

$$\boxed{h[k] = \frac{1}{2}\,\delta[k] - \frac{1}{2} \cdot (-2)^k \cdot u[k]}$$

3.6

$h(k) = (-2)^k u(k)$, $f(k) = e^{-k} u(k)$, what is the zero-state response of LTID system

$$y(k) = f(k) * h(k) = ?$$

Table 3.1 : Convolution Sums : #4 in textbook

$f_1(k) = \gamma_1^k u(k)$, $f_2(k) = \gamma_2^k u(k)$, $\gamma_1 \neq \gamma_2$

$$f_1(k) * f_2(k) = \left[\frac{\gamma_1^{k+1} - \gamma_2^{k+1}}{\gamma_1 - \gamma_2} \right] u(k)$$

$$y(k) = f(k) * h(k) = e^{-k} u(k) * (-2)^k u(k)$$

$$= \left[\frac{(e^{-1})^{k+1} - (-2)^{k+1}}{e^{-1} + 2} \right] u(k)$$