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MATH 118

Report for Final Presentation

Poisson Distribution and the Poisson Process

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Chapter 1
Introduction to Poisson Distribution and the Poisson Process

Like many statistical tools and probability metrics, the Poisson Distribution was originally applied to the world of gambling. The French mathematician Simeon-Denis Poisson developed his function in 1830 to describe the number of times a gambler would win a rarely won game of chance in a large number of tries. The distribution published together with his probability theory in his book.

Chapter 2
2.1 Definition of Poisson Experiment

Experiments yielding numerical values of a random variable X, the number of outcomes occurring during a given time interval or in a specified region, are called Poisson experiments. The given time interval may be of any length, such as a minute, a day, a week, a month, or even a year, also the specified region could take many forms. For instance, it could be a length, an area, a volume, a period of time, etc. The 'X' is called a Poisson random variable.

2.2 Definition of Poisson Process

A Poisson Process is a model for a series of discrete event where the *average time* between events is known, but the exact timing of events is random. It is usually used in scenarios where we are counting the occurrences of certain events that appear to happen at a certain rate. For example, suppose that from historical data, we know that earthquakes occur in a certain area with a rate of 2 per month. Other than this information, the timings of earthquakes seem to be completely random. Second example may be the number of car accidents at a site or in an area. A Poisson experiment is derived from the Poisson process.

A Poisson Process meets the following criteria:

- Events are independent of each other. The occurrence of one event does not affect the probability another event will occur.
- The average rate (events per time period) is constant.
- Two events cannot occur at the same time.

2.3 Definition of Poisson Distribution

In statistics, a Poisson distribution is a probability distribution that can be used to show how many times an event is likely to occur within a specified period of time. Poisson distributions are often used to understand independent events that occur at a constant rate within a given interval of time. An example, a certain fast-food restaurant gets an average of 3 visitors to the drive-through per minute. This is just an average, however. The actual amount can vary. A Poisson distribution can be used to analyze the probability of various events regarding how many customers go through the drive-through. Difference between Poisson distribution and Poisson process is that Poisson process is one where the time between successive occurrences of an event. If the time is divided into equal intervals, then the number of occurrences of an event in each time interval follows a Poisson distribution.

Chapter 3

The Poisson Distribution

The Formula for the Poisson Distribution is:

$$f(k;\lambda t) = \frac{(\lambda t)^k e^{-\lambda t}}{k!} \qquad \textbf{or} \qquad f(x;\mu) = \frac{\mu^x\ \mathrm{e}^{-\mu}}{x!},$$

Those two formulas are exactly same but some resources use one of them. In the second formula, μ=λ.t is used.

- *e* is Euler's number (*e* = 2.71828...)
- f(*x*; μ) is The **Poisson probability** that exactly *x* successes occur in a Poisson experiment, when the mean number of successes is μ.
- *x, k* is the number of occurrences.

  x, k=0, 1, 2, ….
- *x*! is the factorial of *x.*
- λ is equal to the expected values of *x* when that is also equal to its variance.
- To specify the number of occurrences, x or k can be used.

The formula below is more understandable.

$$P(k \text{ events in time period}) = e^{-\frac{\text{events}}{\text{time}} * \text{time period}} * \frac{\left(\frac{\text{events}}{\text{time}} * \text{time period}\right)^k}{k!}$$

Example:

The average number of homes sold by a company is 2 homes per day. What is the probability that exactly 3 homes will be sold tomorrow?

Solution:

μ = 2; since 2 homes are sold per day, on average.

x = 3; since we want to find the likelihood that 3 homes will be sold tomorrow.

e = 2.71828

$f(x;\mu) = \frac{\mu^x \, e^{-\mu}}{x!}$, $= (2.71828^{-2})\,(2^3) / 3! = (0.13534)\,(8) / 6 = 0.180$

So, the probability of selling 3 homes tomorrow = 0.180

Example:

A complex software system averages 7 errors per 5,000 lines of code. What is the probability of exactly 2 errors in 5,000 lines of randomly selected lines of code?

Solution:

$$f(x;\mu) = \frac{\mu^x \ e^{-\mu}}{x!}, \quad = (2.71828^{-7})\ (7^2)\ /\ 2! \ = (0.000922)\ (49)\ /\ 2$$

$$= 0.022$$

Chapter 4
Cumulative Poisson Distribution

A cumulative poisson distribution is used to calculate the probability of getting at least x successes in a poisson experiment.

$$F(x,\lambda) = \sum_{k=0}^{x} \frac{e^{-\lambda}\lambda^x}{k!}$$

- *e* is Euler's number (*e* = 2.71828...)
- *k* is the number of occurrences.
  k=0, 1, 2, ….
- *k*! is the factorial of *k.*
- λ is equal to the expected values of *k* during the given interval.

Here, x is the Poisson random variable which refers to the number of success. Our formula is sum of Poisson’s from k is equal to 0 to x.

Chapter 5
Examples

Example:

A complex software system averages 7 errors per 5,000 lines of code. What is the probability of 2 or less than errors in 5,000 lines of randomly selected lines of code?

Solution:

- $\lambda = 7$; since there are 7 error on average.
- x = 0, 1 or 2
- e = 2.71828

$F(x, \lambda) = \sum_{k=0}^{x} \frac{e^{-\lambda} \lambda^{x}}{k!}$ => $P(x \leq 2, 7) = P(0; 7) + P(1; 7) + P(2; 7)$

$P(x \leq 2, 7) = ((e^{-7}) (7^0) / 0!) + ((e^{-7}) (7^1) / 1!) + ((e^{-7}) (7^2) / 2!)$

= 0.000912 + 0.0064 + 0.0223

= 0.0296

Example:

At a junction, there are an average of 6 accidents in 4 months. According to this,

a) What is the probability of 8 accidents at this junction in the next 4 months?

Solution:

- μ = 6; since there are 6 accidents, on average.
- x = 8; since we want to find the probability of 8 accidents.
- e = 2.71828

$$f(x;\mu) = \frac{\mu^x \, e^{-\mu}}{x!},$$

= $(2.71828^{-6})\ (6^{8})$ / 8! = (0.00248) (1,679,616) / 40,320 = 0.1033

b) What is the probability of 5 accidents at this junction in the next 6 months?

Solution:

$$f(x;\mu) = \frac{\mu^x \, e^{-\mu}}{x!},$$

- New μ = 9; If there is an average of 6 accidents in 4 months, it will be 9 in 6 months.

- x = 5; since we want to find probability of 5
- e = 2.71828

= $(2.71828^{-9})\ (9^{5})$ / 5! = (0.0001234) (59,049) / 120 = 0.061

c) What is the probability of 2 or less than accidents at this intersection in the next year?

Solution:

$$F(x, \lambda) = \sum_{k=0}^{x} \frac{e^{-\lambda}\lambda^{x}}{k!}$$

- New λ = 18; If there is an average of 6 accidents in 4 months, it will be 18 in a year.
- x = 0, 1 or 2
- e = 2.71828

=> $P(x \leq 2, 18) = P(0; 18) + P(1; 18) + P(2; 18)$

$P(x \leq 2, 7) = ((e^{-18}) (18^{0}) / 0!) + ((e^{-18}) (18^{1}) / 1!) + ((e^{-18}) (18^{2}) / 2!)$

$= 1.53*10^{-8} + 2.74*10^{-7} + 2.48*10^{-6} = 2.76*10^{-6}$

Conclusion

The Poisson Distribution can be a helpful statistical tool we can use to evaluate and improve business operations.

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