CSE-351 _ HW2
Abdurrahman BULUT

Q1) $H(s) = (2s+3)/(s^2+5s+6)$

a) if $f(t) = e^{-3t}u(t)$, zero-state response = ?

$$F(s) = \frac{1}{s+3}$$

$$Y(s) = H(s).F(s) = \frac{1}{s+3} \cdot \frac{2s+3}{s^2+5s+6} = \frac{2s+3}{(s+3)^2.(s+2)} = \frac{k}{s+2} + \frac{a_0}{(s+3)^2} + \frac{a_1}{s+3}$$

$$k = \frac{2s+3}{(s+3)^2}\Big|_{s=-2} = \frac{-1}{1} = -1$$

$$a_0 = \frac{2s+3}{s+2}\Big|_{s=-3} = \frac{-3}{-1} = 3$$

$$Y(s) = \frac{-1}{s+2} + \frac{3}{(s+3)^2} + \frac{a_1}{s+3} = \frac{2s+3}{(s+2).(s+3)^2}$$

To compute a_1, Multiply both sides by s and let $s \to \infty$,

$$-1 + 0 + a_1 = 0$$

$$a_1 = 1$$

$$Y(s) = \frac{-1}{s+2} + \frac{3}{(s+3)^2} + \frac{1}{s+3}$$

$$y(t) = (-e^{-2t} + 3te^{-3t} + e^{-3t})u(t) \Rightarrow [-e^{-2t} + (1+3t)e^{-3t})u(t)$$

b) the differential equation relating output $y(t)$ to the input $f(t)$.

$$Y(s) = F(s).H(s)$$

$$Y(s) = F(s). \frac{2s+3}{s^2+5s+6}$$

$$(s^2+5s+6).Y(s) = (2s+3).F(s)$$

$$(D^2+5D+6).y(t) = (2D+3)f(t)$$

$$\Rightarrow \boxed{\frac{d^2y}{dt^2} + \frac{5dy}{dt} + 6y(t) = 2\frac{df}{dt} + 3f(t)}$$

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c) inverse laplace transform of $(s+2) / s.(s+1)^2$

$$H(s) = \frac{s+2}{s(s+1)} = \frac{k}{s} + \frac{a_0}{(s+1)^2} + \frac{a_1}{s+1}$$

cover-up

$$k = \frac{s+2}{s.(s+1)^2}\Big|_{s=0} = \frac{2}{1} = 2$$

$$a_0 = \frac{s+2}{s.(s+1)^2}\Big|_{s=-1} = \frac{1}{-1} = -1$$

$$H(s) = \frac{2}{s} - \frac{1}{(s+1)^2} + \frac{a_1}{s+1}$$

To compute a_1, multiply both sides by s and let $s \to \infty$

$$2 + 0 + a_1 = 0$$

$$a_1 = -2$$

$$H(s) = \frac{2}{s} - \frac{2}{s+1} - \frac{1}{(s+1)^2}$$

$$H(s) = [2 - 2e^{-t} - te^{-t}].u(t)$$

$$\boxed{h(t) = [2 - (2+t).e^{-t}].u(t)}$$

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(Q2)

Equation in delay form

$$2y[k]-3y[k-1]+y[k-2]=4f[k]-3f[k-1]$$

$$y[k] \Leftrightarrow Y[z] \quad , \quad y[k-1] \Leftrightarrow \frac{1}{z}Y[z] \quad , \quad y[k-2] \Leftrightarrow \frac{1}{z^2}Y[z]+1$$

$$2Y[z]-\frac{3}{z}Y[z]+\frac{1}{z^2}+1=\frac{4z}{z-0{,}25}-\frac{3}{z-0{,}25}$$

$$\left(2-\frac{3}{z}+\frac{1}{z^2}\right)Y[z]=-1+\frac{4z-3}{z-0{,}25}$$

$$\frac{z^2}{z^2} \Big/ \left(2-\frac{3}{z}+\frac{1}{z^2}\right)Y[z]=\frac{3z-2{,}75}{z-0{,}25}$$

$$(2z^2-3z+1).\frac{Y[z]}{z}=\frac{z.(3z-2{,}75)}{z-0{,}25}$$

$$\frac{Y[z]}{z}=\frac{z.(3z-2{,}75)}{(2z^2-3z+1)(z-0{,}25)}=\frac{z.(3z-2{,}75)}{(2z-1).(z-1).(z-0{,}25)}$$

$$\frac{Y[z]}{z}=\frac{z.(3z-2{,}75)}{2.(z-0{,}5).(z-1).(z-0{,}25)}$$ Cover-up Method

$$\frac{Y[z]}{z}=\frac{5/2}{z-0{,}5}+\frac{1/3}{z-1}-\frac{4/3}{z-0{,}25}$$

$$\boxed{y[k]=\left[\frac{5}{2}(2)^{-k}+\frac{1}{3}-\frac{4}{3}(4)^{-k}\right]u[k]}$$

(Q3) Find the inverse z-transform of $z(-5z+22)/(z+1).(z-2)^2$

$$\frac{H(z)}{z} = \frac{-5z+22}{(z+1).(z-2)^2}$$

$$= \frac{k}{z+1} + \frac{a_0}{(z-2)^2} + \frac{a_1}{z-2}$$

$$\frac{-5z+22}{\cancel{(z+1)}(z-2)^2}\Big|_{k=-1} = \frac{27}{9} \Rightarrow k = 3$$

$$\frac{-5z+22}{(z+1).(z-2)^2}\Big|_{a=2} = \frac{12}{3} = 4 = a_0$$

$$\frac{H(z)}{z} = \frac{3}{z+1} + \frac{4}{(z-2)^2} + \frac{a_1}{z-2}$$

Multiply both sides by z and let $z \to \infty$

$$3 + 0 + a_1 = 0$$

$$a_1 = -3$$

$$H(z) = 3\frac{z}{z+1} + 4\frac{z}{(z-2)^2} - 3\frac{z}{z-2}$$

$$\boxed{h[k] = [3.(-1)^k - 3.(2)^k + 2k(2)^k]\, u(k)}$$

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