

# CompeteGrok Analysis Report

**Query:** explain BLP demand estimation. step by step.

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## Agent Output

### BLP Demand Estimation: Step-by-Step Explanation

BLP refers to the **Berry-Levinsohn-Pakes (1995)** model, a seminal framework in industrial organization (IO) economics for estimating demand in markets with **differentiated products** (e.g., cars, cereals, airlines). It extends the basic **logit model** by incorporating **random coefficients** on product characteristics (like price), allowing for flexible substitution patterns, consumer heterogeneity, and realistic own- and cross-price elasticities. Unlike the simple logit (which assumes IIA—**i**ndependence of **i**rrelevant alternatives), BLP handles rich demand patterns.

The estimation is **non-parametric** in consumer heterogeneity but uses **GMM (Generalized Method of Moments)** for parameters. It's computationally intensive due to simulation/integration but solved via a **contraction mapping** (fixed-point inversion). Here's the **step-by-step process**:

#### Step 1: Model Setup (Random Coefficients Logit Utility)

- **Consumer utility:** For consumer  $i$  in market  $t$  choosing product  $j$ :

$$u_{ijt} = \delta_{jt} + \sum_{k=1}^K \nu_{ik} x_{jtk} + \epsilon_{ijt}$$

- $\delta_{jt} = x_{jt}\beta + \xi_{jt}$ : **Mean utility** (observed characteristics  $x_{jt}$  like price  $p_{jt}$ , interacted with average tastes  $\beta$ , plus unobserved quality  $\xi_{jt}$ ).
- $\nu_{ik}$ : **Individual taste shocks** (random coefficients, drawn from a distribution, e.g., normal  $N(0, \Sigma)$ ). Key for heterogeneity—e.g., some consumers are price-insensitive.
- $\epsilon_{ijt}$ : i.i.d. Type I extreme value (logit shocks).
- Outside good (not buying):  $u_{i0t} = \epsilon_{i0t}$ .

- **Market shares:** Observed share  $s_{jt}$  of product  $j$  in market  $t$ :

$$s_{jt} = \int \frac{\exp(\delta_{jt} + \sum_k \nu_{ik} x_{jtk})}{1 + \sum_m \exp(\delta_{mt} + \sum_k \nu_{ik} x_{mtk})} dG(\nu_i)$$

- Integrate/simulate over heterogeneity  $G(\nu_i)$  (e.g., 500-1000 draws via simulation methods like Halton sequences).
- This is **intractable analytically**, so we simulate predicted shares  $\hat{s}_{jt}(\delta_t; \theta)$ , where  $\theta = \{\beta, \Sigma\}$  are structural parameters.

## Step 2: Share Inversion (Solve for Mean Utilities $\delta$ )

- **Key Insight:** Observed shares  $s_{jt}$  are a monotonic function of  $\delta_{jt}$ . Invert to recover  $\delta_{jt}$  from data shares.
- Define the **inversion**:

$$\log(s_{jt}) - \log(s_{0t}) = \delta_{jt} + \mathbb{E}\left[\sum_k \nu_{ik} x_{jtk} | s_{jt}\right]$$

- $s_{0t} = 1 - \sum_j s_{jt}$  (outside share).
- Let  $\Delta_{jt} \equiv \log(s_{jt}/s_{0t})$  (empirical mean utilities).
- Then:  $\delta_{jt} = \Delta_{jt} - \bar{\mu}_{jt}(\delta_t; \theta)$ , where  $\bar{\mu}_{jt}$  is the simulated mean of the random coefficient term.

- **Contraction Mapping (Fixed-Point Algorithm):**

1. Start with initial guess  $\delta^{(0)}$  (e.g., from simple logit).
2. Simulate  $\hat{s}(\delta^{(r)})$ .
3. Update:  $\delta^{(r+1)} = \Delta - \bar{\mu}(\delta^{(r)}; \theta)$ .
4. Iterate until convergence ( $|\delta^{(r+1)} - \delta^{(r)}| < \epsilon$ ). It's a contraction (converges quickly, <10 iters usually).

## Step 3: Structural Estimation (GMM Objective)

- **Endogeneity Problem:**  $\xi_{jt}$  (unobserved quality) correlates with price  $p_{jt}$  (firms set prices knowing quality). OLS on  $\delta_{jt} = x_{jt}\beta + \xi_{jt}$  is biased.
- **Instruments (IV):** Use **exogenous cost shifters** or **Hausman-style** instruments:
  - Own/product characteristics (e.g., cost proxies).
  - Rivals' characteristics (uncorrelated with  $\xi_{jt}$ ).
  - BLP moments:  $\mathbb{E}[Z_{jt}\xi_{jt}] = 0$ , where  $Z$  are instruments.
- **GMM Minimization:**
  1. **Inner Loop:** For candidate  $\theta$ , invert to get  $\hat{\delta}(\theta)$ .
  2. **Linear Regression:**  $\hat{\delta}_{jt} = x_{jt}\beta + \hat{\xi}_{jt}(\theta)$ .
  3. **Moment Conditions:**  $g(\theta) = \frac{1}{T} \sum_t Z_t' \hat{\xi}_t(\theta)$ .
  4. Objective:  $\min_{\theta} g(\theta)' W g(\theta)$ , where  $W$  is optimal weighting matrix (2-step GMM).
- **Outer Loop Optimization:** Nested loops—optimize  $\theta$  (e.g., via BFGS), calling inner inversion repeatedly. Computationally heavy (Monte Carlo simulation).

## Step 4: Post-Estimation (Elasticities, Welfare, etc.)

- **Elasticities:** Recover full distribution of elasticities:

$$\epsilon_{jm} = \frac{\partial s_j}{\partial p_m} \frac{p_m}{s_j}$$

- Simulated over draws.
- **Merger Simulation:** Use estimates to simulate post-merger prices/profits under oligopoly pricing (e.g., Nash-Bertrand).
- **Counterfactuals:** Compute welfare changes ( $CS = \pi \int \log(1 - s(\delta')) dF - \log(1 - s(\delta)) dF$ ).

## Data Requirements

- **Micro moments** or **aggregated shares** by market (e.g., city/quarter for cars).
- Product characteristics  $x$  (price, attributes).
- Instruments (critical for credibility).

## Advantages & Caveats

- **Pros:** Flexible heterogeneity, no IIA, matches micro data.
- **Cons:**
  - Computationally demanding (modern fixes: NFXP solvers, GPU).
  - Instruments hard to find (weak IV problem common).
  - Assumes single market or IIA across markets.
  - Extensions: Dynamic BLP, supply-side integration.

## Implementation Tips

- Software: Python (pyblp), R (BLPestimatorR), MATLAB (Nevo's code).
- Example: Automobiles (BLP data), cereals (Nevo 2001).

For code/examples or deeper math (e.g., full GMM formulas), let me know! References: Berry et al. (1995, Econometrica); Nevo (2000, JPE tutorial).

## Agent Output

**Synthesis:** See above.

**Privacy:** Ephemeral RAG; zero retention.

**Disclaimer:** Not legal advice. Models have caveats (e.g. IIA assumption). Verify 2025 data.

**LaTeX:** Inline  $x$ , display

$$E = mc^2$$