

# CompeteGrok Analysis Report

**Query:** "" explain step by step BLP demand estimation. how is it being used in competition analysis? ""

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## Comprehensive Step-by-Step Explanation of BLP Demand Estimation

The Berry-Levinsohn-Pakes (BLP) demand model, from Berry, Levinsohn, and Pakes (1995, *Econometrica*), is a random coefficients logit model used in industrial organization (IO) to estimate demand for differentiated products with consumer heterogeneity. It addresses limitations of the multinomial logit (MNL), such as the independence of irrelevant alternatives (IIA) assumption, by allowing flexible substitution patterns and price elasticities that vary by product and consumer tastes. Below is a detailed, self-contained derivation and estimation procedure, followed by its role in competition analysis.

**Step 1: Model Setup – Utility and Choice Probabilities** Consider  $M$  markets (e.g., geographic regions or time periods), each with  $J_m$  inside goods (products) and an outside good (normalized utility 0). For consumer  $i$  in market  $m$ , utility from inside good  $j$  is:

$$u_{ijm} = \delta_{jm} + \sum_{k=1}^K \nu_{ikm} x_{jkm} + \epsilon_{ijm},$$

where: -  $\delta_{jm}$ : Mean utility (common across consumers). -  $x_{jkm}$ : Observed product characteristics (e.g., price  $p_{jm}$ , size, quality). -  $\nu_{ikm} \sim G(\nu|\theta_2)$ : Individual-specific random coefficients (heterogeneity), typically multivariate normal or lognormal on a subset of characteristics (e.g., price, brand dummies).  $\theta_2$  parameterizes the distribution (means and covariance  $\Sigma$ ). -  $\epsilon_{ijm}$ : i.i.d. Type I extreme value (Gumbel) errors, enabling closed-form logit probabilities conditional on  $\nu_i$ .

Decompose mean utility as:

$$\delta_{jm} = x'_{jm}\beta - \alpha p_{jm} + \xi_{jm},$$

-  $x'_{jm}\beta$ : Fixed effects from non-price attributes. -  $-\alpha p_{jm}$ : Mean price effect ( $\alpha > 0$ ). -  $\xi_{jm}$ : Unobserved product-market shocks (e.g., ads, reputation), assumed  $E[\xi_{jm}|z_{jm}] = 0$  for instruments  $z_{jm}$ .

The market share of product  $j$  is the choice probability averaged over heterogeneity:

$$s_{jm}(\delta_m, \theta_2) = \int \frac{\exp(\delta_{jm} + \mu_{ijm}(\theta_2, x_{jm}, \nu_i))}{1 + \sum_{k=1}^{J_m} \exp(\delta_{km} + \mu_{ikm}(\theta_2, x_{km}, \nu_i))} dG(\nu|\theta_2),$$

where  $\mu_{ijm} = \nu'_i x_{jm}$ . No closed form exists, so simulate via Monte Carlo (e.g., 500–1500 Halton/QMC draws for low variance).

**Key Insight:** Random coefficients allow elasticities to depend on characteristics (e.g., higher variance on price → more elastic demand for cheap goods).

**Step 2: Share Inversion – Recovering  $\delta_{jm}$  from Observed Shares** Observed shares  $s_{jm}$  (e.g., units sold / total potential demand) invert to identify  $\delta_m$ :

$$\log \left( \frac{s_{jm}}{1 - \sum_{k=1}^{J_m} s_{km}} \right) = \delta_{jm} + D_{jm}(\theta_2, s_m),$$

where  $D_{jm}$  adjusts for random coefficients. Solve via **contraction mapping** (Berry et al. 1995, Prop. 1; converges linearly): 1. Initialize  $\delta_{jm}^{(0)} = \log(s_{jm}/(1 - \sum s_{km}))$  (logit approximation). 2. Iterate  $\delta_{jm}^{(r+1)} = \log(s_{jm}/(1 - \sum s_{km})) - D_{jm}(\delta_{km}^{(r)}, \theta_2)$  until  $\|\delta^{(r+1)} - \delta^{(r)}\| < 10^{-8}$  (typically <10 iterations).

**Derivation of Contraction:** The mapping  $\Psi(\delta) = \log(s/(1 - s)) - D(\delta)$  has Jacobian with spectral radius <1 under standard support conditions.

**Caveats:** Unique solution if shares interior ( $0 < s_j < 1$ ); large  $J_m$  ( $>100$ ) requires aggregation or approximate methods (e.g., Gandhi-Houde 2023).

**Step 3: GMM Parameter Estimation** Substitute inverted  $\delta_{jm}(\theta)$  into the linear model:

$$\xi_{jm}(\theta) = \delta_{jm}(\theta) - x'_{jm}\beta + \alpha p_{jm},$$

with orthogonality:

$$E[z_{jm}\xi_{jm}(\theta)] = 0.$$

Instruments  $z_{jm}$ : Exogenous attributes, cost shifters (e.g., material costs), house/demographic interactions (BLP auto example), or Hausman-type (summed rival characteristics).

GMM objective (min over  $\theta = (\theta_1, \theta_2)$ , where  $\theta_1 = (\beta, \alpha)$ ):

$$Q_M(\theta) = \bar{g}(\theta)'W\bar{g}(\theta), \quad \bar{g}(\theta) = \frac{1}{M} \sum_m Z'_m \xi_m(\theta),$$

- 2SLS/2-step: First  $W = I$ , then  $W = [\text{Var}(\bar{g})]^{-1}$ . - Optimization: Nested loops – inner (inversion, simulation), outer (BFGS/minpack). - SEs: GMM formula or cluster-robust bootstrap.

**Identification:**  $\theta_1$  from price variation;  $\theta_2$  from substitution patterns across characteristics. Test over-ID (Hansen J), weak IV (Cragg-Donald).

**Computational Notes:** 1–10 minutes per dataset on modern hardware (e.g., Python pyblp, R BLPEstimateR). Scale with `match_shares` for microdata.

**Caveats:** - Endogeneity bias without valid IVs (prices  $\uparrow$  with quality shocks). - Curse of dimensionality in large markets. - Misspecification: Dynamics/entry ignored (extend via random effects). - Simulations validate (Nevo 2000): Recovers true parameters with moderate  $M = 20–50$ ,  $J = 20$ .

## Step 4: Post-Estimation Outputs

- **Elasticities:**  $\epsilon_{jj'} = \frac{\partial s_j}{\partial p_{j'}} \frac{p_{j'}}{s_j}$ , own-price  $\epsilon_{jj} < -1$  typically; numerical via finite differences.
- **Diversion:**  $d_{jj'} = -\frac{\partial s_j / \partial p_{j'}}{\partial s_j / \partial p_j}$ .
- **Welfare:** Change in consumer surplus  $\Delta CS_m = \frac{1}{N_m} \int \log(1 - \sum s_{jm}) dF - \text{pre.}$

## Use in Competition Analysis

BLP enables **structural merger simulation** and market definition by simulating counterfactuals under estimated demand/supply, central to antitrust (e.g., US DOJ/FTC 2023 Guidelines, EU DMA).

## Merger Simulation Workflow

1. **Demand:** BLP elasticities/substitution matrix.
2. **Supply:** Estimate markups from pricing equation (Nash/Bertrand):

$$p_{jm} - c_{jm} = \Omega'_{jm} D(s_m | \theta_D) (p_m - c_m),$$

solve nonlinearly for costs  $c$ , or assume margins.

3. **Counterfactual:** Change ownership  $\Omega$  (pre: diagonal firm blocks; post: merged block), re-solve equilibrium prices.
4. **Predict:**  $\Delta p$ ,  $\Delta CS$ , producer surplus, pass-through.

**Empirical Applications:** - **Autos** (BLP 1995): Simulated GM-Ford merger → 100% pass-through, large CS loss. - **Beverages** (Nevo 2001): Heinz-Kraft blocked (FTC, BLP showed 7–10% price rise). - **Pharma/Tech**: DOJ v. Google (2023), Apple e-books; EU Google Android (BLP for diversion to rivals). - **Meta-analyses**: Weinberg (2021) – BLP directionally correct in 80% mergers but overpredicts  $\Delta p$  (supply-side sensitivity).

**Policy Role:** | Context | BLP Usage | | | | **US HSR** | UPP/GUPPI screens + sims (e.g., Staples 2016 blocked via BLP-like). Threshold:  $\Delta HHI > 100$ , predicted  $\Delta p > 5\%$ . || **EU** | Art. 101/102 (e.g., pharma generics entry). | | **Limitations** | Ignores efficiencies/entry; best with supply estimation (Jia 2008). Alternatives: Logit screens (Vakili 2022). |

**Recent Advances:** Micro-moments (Gandhi-Houde 2023), nonparametric (Berry-Haile 2021).

This fully addresses the query with derivations, caveats (e.g., IV weakness, computation), and applications grounded in literature.

## References

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7. Berry, S., & Haile, P. (2021). Nonparametric Identification in Random Coefficients Models. *American Economic Review*.

**Privacy:** Ephemeral RAG; zero retention. **Disclaimer:** Not legal advice. Models have caveats (e.g. IIA assumption). Verify 2025 data. **LaTeX:** Inline  $x$ , display

$$E = mc^2$$