Introduction to Data Science

Notes

LaTeX Document by abd, more @ github.com/abdxdev/notes-maker
Report errors: abdulrahman.abd.dev@gmail.com | +92 312 4996133
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1 Week 0

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2 Week 1 and Week 2

> **WARNING**
> These formulas are written to be remembered after studying the slides

2.1 Mean

$$\text{Mean} = \frac{\sum X_i}{N}$$

Where:

- X_i = each value
- N = number of values

2.2 Median

Sort the data in ascending order.

2.2.1 If N is odd

$$\text{Median} = \text{Value at position } \left(\frac{N+1}{2}\right)$$

2.2.2 If N is even

$$\text{Median} = \frac{\text{Value at position } \left(\frac{N}{2}\right) + \text{Value at position } \left(\frac{N}{2}+1\right)}{2}$$

2.3 Mode

1. Identify the value(s) that occur most frequently.
2. If no value repeats, the dataset is said to have no mode.

2.4 Range

$$\text{Range} = \text{Max} - \text{Min}$$

2.5 Variance

- X_i = each value
- $\bar{X}$ = mean of the values
- N = number of values

2.5.1 Population Variance

$$\sigma^2 = \frac{\text{sum of squared deviations from the mean}}{N} = \frac{\sum(X_i - \bar{X})^2}{N}$$

2.5.2 Sample Variance

$$s^2 = \frac{\text{sum of squared deviations from the mean}}{N-1} = \frac{\sum(X_i - \bar{X})^2}{N-1}$$

2.6 Standard Deviation

TIP

Remember: S for Standard Deviation, S for Squareroot. So the standard deviation is the squareroot of the variance and not the other way around. (I always mess this up)

2.6.1 Population Standard Deviation

$$\sigma = \sqrt{\sigma^2}$$

2.6.2 Sample Standard Deviation

$$s = \sqrt{s^2}$$

2.7 Covariance

- X_i = each value of variable X
- Y_i = each value of variable Y
- $\bar{X}$ = mean of variable X
- $\bar{Y}$ = mean of variable Y
- N = number of values

2.7.1 Population Covariance

$$\mathrm{Cov}(X,Y) = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{N}$$

2.7.2 Sample Covariance

$$\mathrm{Cov}(X,Y) = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{N-1}$$

2.8 Correlation

- X_i = each value of variable X
- Y_i = each value of variable Y
- $\bar{X}$ = mean of variable X
- $\bar{Y}$ = mean of variable Y
- N = number of values

2.8.1 Population Correlation Coefficient

$$r = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

Where:

- $\sigma_X = \sqrt{\frac{\sum(X_i - \bar{X})^2}{N}}$
- $\sigma_Y = \sqrt{\frac{\sum(Y_i - \bar{Y})^2}{N}}$

2.8.2 Sample Correlation Coefficient

$$r = \frac{\text{Cov}(X, Y)}{s_X s_Y}$$

Where:

- $s_X = \sqrt{\frac{\sum(X_i - \bar{X})^2}{N-1}}$
- $s_Y = \sqrt{\frac{\sum(Y_i - \bar{Y})^2}{N-1}}$

3 Week 3

WARNING

These formulas are written to be remembered after studying the slides

3.1 Probability

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Where:

- $P(A)$ = Probability of event A

3.1.1 Addition Rule

Mutually Exclusive Events

$$P(A \cup B) = P(A) + P(B)$$

Non-Mutually Exclusive Events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

3.1.2 Multiplication Rule

Independent Events

$$P(A \cap B) = P(A) \times P(B)$$

Dependent Events

$$P(A \cap B) = P(A) \times P(B|A)$$

Where:

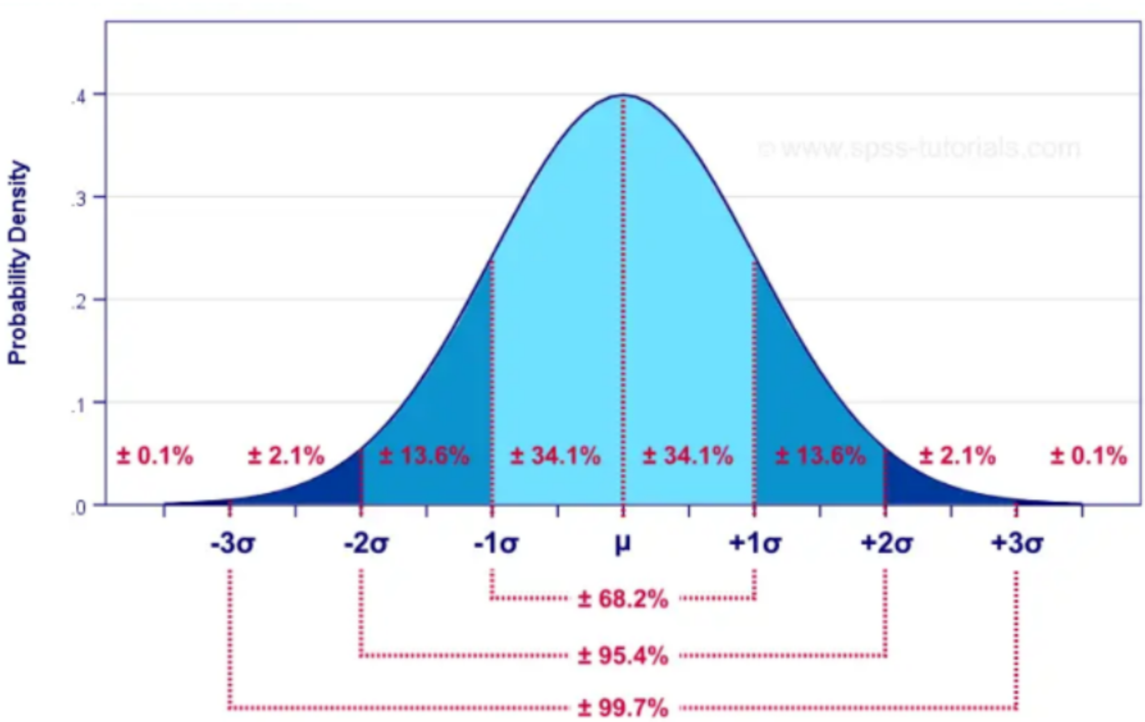
- $P(B|A)$ is the conditional probability of B given A.
- $P(B|A) = \frac{P(A \cap B)}{P(A)}$

NOTE

- A deck of cards has 52 cards.
- 4 suits: Hearts, Diamonds, Clubs, Spades.
- Each suit has 13 cards: Ace, 2-10, Jack, Queen, King.
- Probability of drawing an Ace: $P(Ace) = \frac{4}{52} = \frac{1}{13}$

3.2 Empirical Rule (68-95-99.7 Rule)

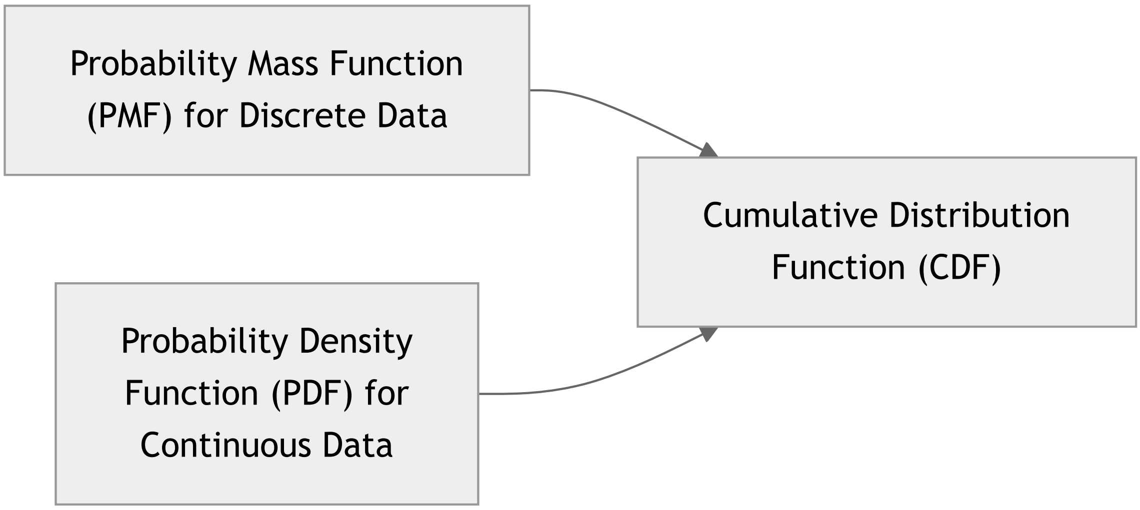
- About 68% of data falls within 1 standard deviation of the mean ($\mu \pm \sigma$).
- About 95% of data falls within 2 standard deviations of the mean ($\mu \pm 2\sigma$).
- About 99.7% of data falls within 3 standard deviations of the mean ($\mu \pm 3\sigma$).



3.3 Probability Distribution Functions

TIP

- This topic is confusing watch the video
- Probability Distribution Functions (PMF, PDF, CDF) - YouTube



> **NOTE**
> - PDFs means Probability Distribution Functions
> - PDF means Probability Density Function which is a type of Probability Distribution Function

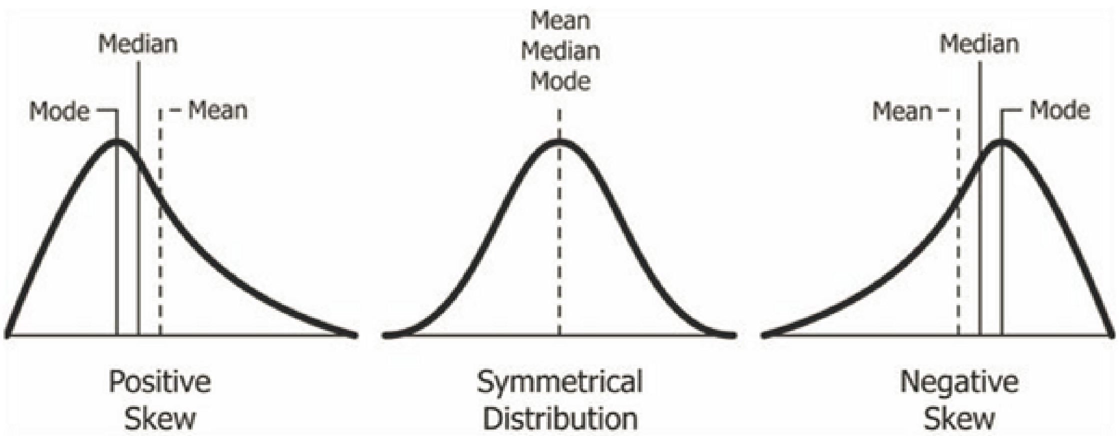
3.4 PDF of Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Where:

- μ = Mean (average)
- σ^2 = Variance
- σ = Standard deviation (data spread)
- π = Pi ≈ 3.14159 (`SHIFT` + `x10` on your calculator)
- e = Euler's number ≈ 2.71828 (`ALPHA` + `x10` on your calculator)

3.5 Skewness



- **Positive Skewness**: Tail on the right side (mean > median > mode)
- **Zero Skewness**: Symmetrical distribution (mean = median = mode)
- **Negative Skewness**: Tail on the left side (mean < median < mode)

3.6 PDF of Z - Distribution (standard normal distribution)

- Mean (μ) = 0
- Standard Deviation (σ) = 1

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

Where:

- z = Z-score
- π = Pi ≈ 3.14159 (SHIFT + x10 on your calculator)
- e = Euler's number ≈ 2.71828 (ALPHA + x10 on your calculator)

3.7 Z - Score

$$Z = \frac{X - \mu}{\sigma}$$

Where:

- X = value from the dataset
- μ = mean of the dataset
- σ = standard deviation of the dataset

NOTE

- Finding a value from **formula**: $f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$ for a given Z-score **IS NOT SAME AS** Finding a value from **Z-table** for a given Z-score
- Formula $f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$ gives PDF value for that Z-score.
- Z-table gives CDF from the left up to that Z-score.

4 Week 4

4.1 Types of Hypothesis Tests

One sample test
Test hypothesis about single population parameter.

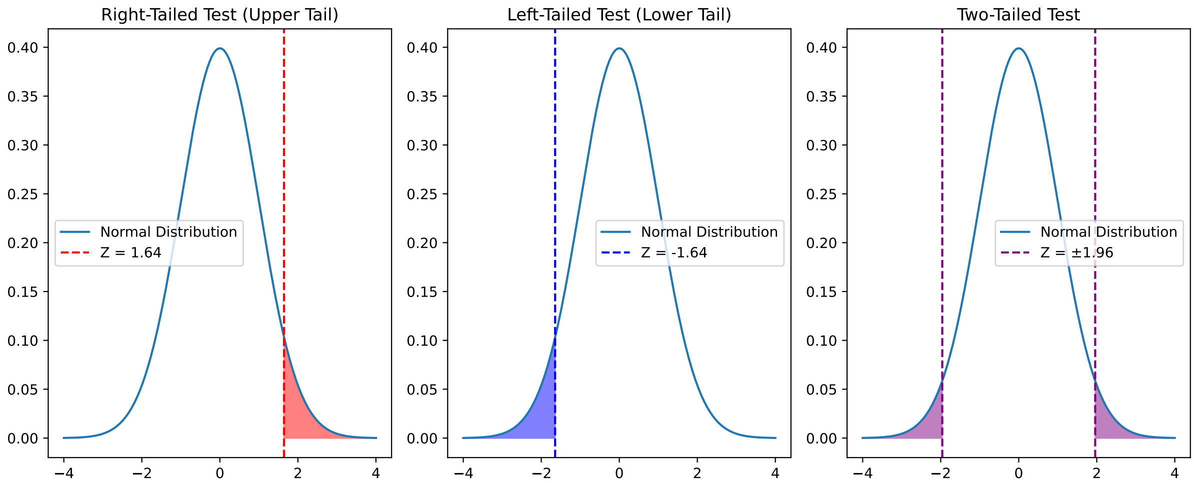
Two sample test
Compare 2 groups or samples.

Paired sample test
Compare two related samples.

Tests for Proportions
Test hypotheses about population proportions.

Chi-Square Tests
Test for independence or goodness-of-fit.

4.2 Tailed Tests for making Decisions



4.3 Hypothesis Testing

1. Choose Test Type (Right-Tailed, Left-Tailed, Two-Tailed)

TIP

Identify the keywords in the problem statement to determine the test type.

Test Type	Keywords
Right-tailed	greater than, more than, increase
Left-tailed	less than, decrease, lower
Two-tailed	correct, equal to, different from, not equal to, changed

2. Choose Z-critical (for $\alpha = 0.05$)

Test Type	Z-critical
Right-tailed	1.645 (for $\alpha = 0.05$)
Left-tailed	-1.645 (for $\alpha = 0.05$)
Two-tailed	± 1.96 (for $\alpha = 0.05$ and $\alpha/2 = 0.025$)

3. **State the Hypotheses**

- Null Hypothesis (H_0): Opposite of what you want to prove.
- Alternative Hypothesis (H_1): What you want to prove.

Test Type	Null Hypothesis (H_0)	Alternative Hypothesis (H_1)
Right-tailed	$\mu \leq \mu_0$	$\mu > \mu_0$
Left-tailed	$\mu \geq \mu_0$	$\mu < \mu_0$
Two-tailed	$\mu = \mu_0$	$\mu \neq \mu_0$

Where μ_0 is the population mean in given data.

4. **Z-test Statistic**

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{N}}$$

Where:

- $\bar{x}$ = Sample Mean

- μ_0 = Population Mean (under null hypothesis)
- σ = Population Standard Deviation
- N = Sample Size

5. **Hypothesis Testing**

- Z-Critical Value Method

Test Type	Critical Value Method
Right-tailed	Reject H_0 if Z-score > Z-critical
Left-tailed	Reject H_0 if Z-score < -Z-critical
Two-tailed	Reject H_0 if absolute Z-score $\geq$ Z-critical

- P-value Method

Test Type	P-value Calculation Formula
Right-tailed	P-value = $1 -$ table value of Z-score
Left-tailed	P-value = table value of Z-score
Two-tailed	P-value = $2 \times (1 -$ table value of absolute Z-score)

  - Reject H_0 if P-value $< \alpha$
  - Fail to reject H_0 if P-value $\geq \alpha$

6. **Conclusion**

- If you rejected H_0, conclude that there is enough evidence to support H_1. *(Most of the time this is what you want to prove)*
- If you failed to reject H_0, conclude that there is not enough evidence to support H_1.

4.4 Examples

4.4.1 Example 1

- The mean lifetime E[X] of the light bulbs produced by Lighting Systems Corporation is 1570 hours with a standard deviation of 120 hours.
- The president of the company claims that a new production process has led to an increase in the mean lifetimes of the light bulbs.
- If a worker tested 100 light bulbs made from the new production process and found that their mean lifetime is 1600 hours, test the hypothesis that E[X] is greater than 1570 hours using a level of significance 0.05.

Given

- Population Mean (μ_0) = 1570
- Sample Mean ($\bar{x}$) = 1600
- Population Standard Deviation (σ) = 120
- Sample Size (N) = 100
- Test Type = Right-Tailed Test
- Significance Level (α) = 0.05 (z-critical = 1.645)

Steps

1. **State the Hypotheses**

- The mean lifetime has not increased $H_0 : \mu \leq 1570$
- The mean lifetime has increased $H_1 : \mu > 1570$

2. **Calculate Z-test Statistic**

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{N}} = \frac{1600 - 1570}{120/\sqrt{100}} = \frac{30}{12} = 2.5$$

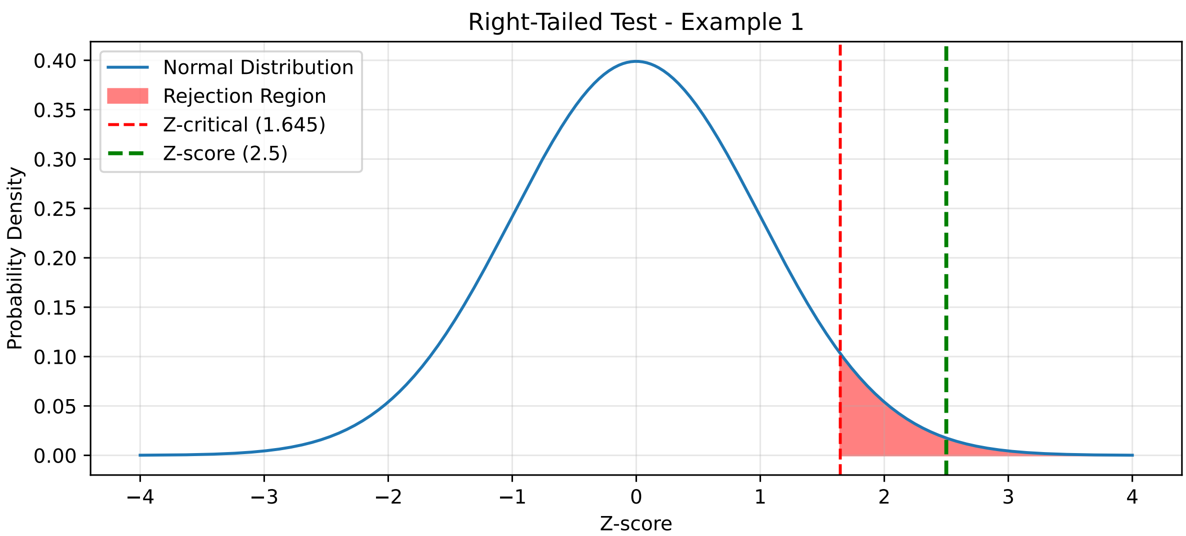
3. **Hypothesis Testing**

- **Z-critical Value Method**
  - Since this is a right-tailed test, Z-critical = 1.645
  - Since Z-score (2.5) > Z-critical (1.645), we reject H_0.

- **P-value Method**
  - P-value = 1 - table value of Z-score (2.5) = 1 - 0.9938 = 0.0062
  - Since P-value (0.0062) < Significance Level ($\alpha = 0.05$), we reject H_0.

4. **Conclusion**

The results confirm that the new production process significantly increased the mean lifetime of the light bulbs.



4.4.2 Example 2

The average weight of an iron bar population is 90kg. Supervisor believes that the average weight might be lower. Random samples of 36 iron bars are measured, and the average weight is 82kg and a standard deviation of 18kg. With a 95% confidence level, is there enough evidence to suggest the average weight is lower?

Given

- Population Mean (μ_0) = 90
- Sample Mean ($\bar{x}$) = 82
- Population Standard Deviation (σ) = 18
- Sample Size (N) = 36
- Test Type = Left-Tailed Test
- Significance Level (α) = 0.05 (z-critical = -1.645)

Steps

1. **State the Hypotheses**

   - The average weight is not lower $H_0 : \mu \geq 90$
   - The average weight is lower $H_1 : \mu < 90$

2. **Calculate Z-test Statistic**

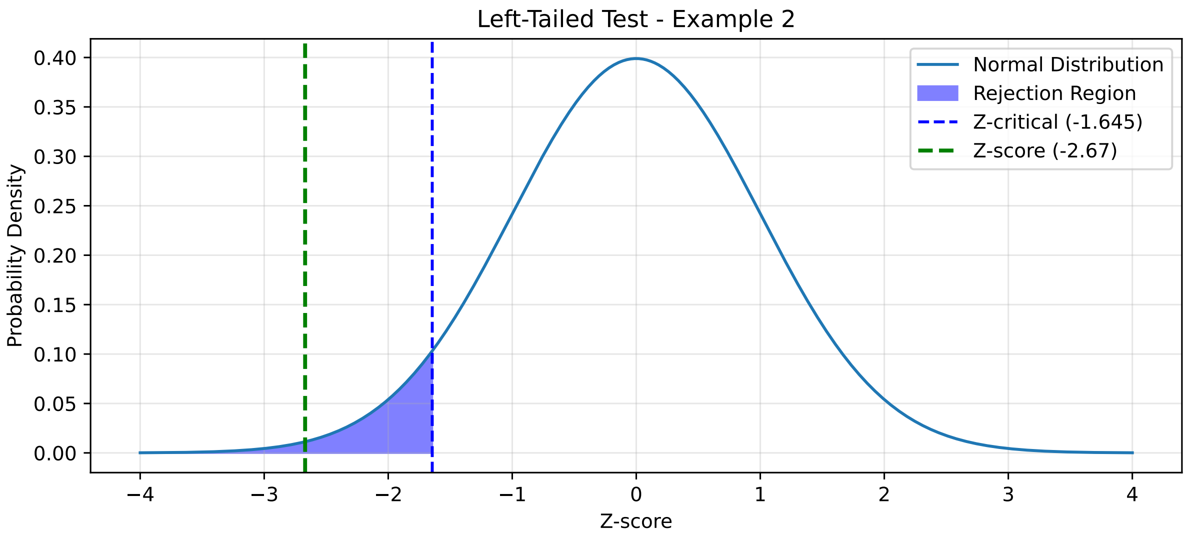
$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{N}} = \frac{82 - 90}{18/\sqrt{36}} = \frac{-8}{3} = -2.67$$

3. **Hypothesis Testing**

   - **Z-critical Value Method**
     - Since this is a left-tailed test, Z-critical = -1.645
     - Since Z-score (-2.67) < Z-critical (-1.645), we reject H_0.
   - **P-value Method**
     - P-value = table value of Z-score (-2.67) = 0.00466
     - Since P-value (0.00466) < Significance Level ($\alpha = 0.05$), we reject H_0.

4. **Conclusion**

   The results confirm that the average weight of the iron bars is significantly lower than 90kg.



4.4.3 Example 3

A machine produces bolts with a mean diameter of 10mm. The quality control team wants to check if the machine is still producing bolts with the correct diameter (not too large or too small). They take a random sample of 64 bolts and find the mean diameter is 10.3mm with a standard deviation of 1.2mm. At a significance level of 0.05, test whether the machine needs adjustment.

Given

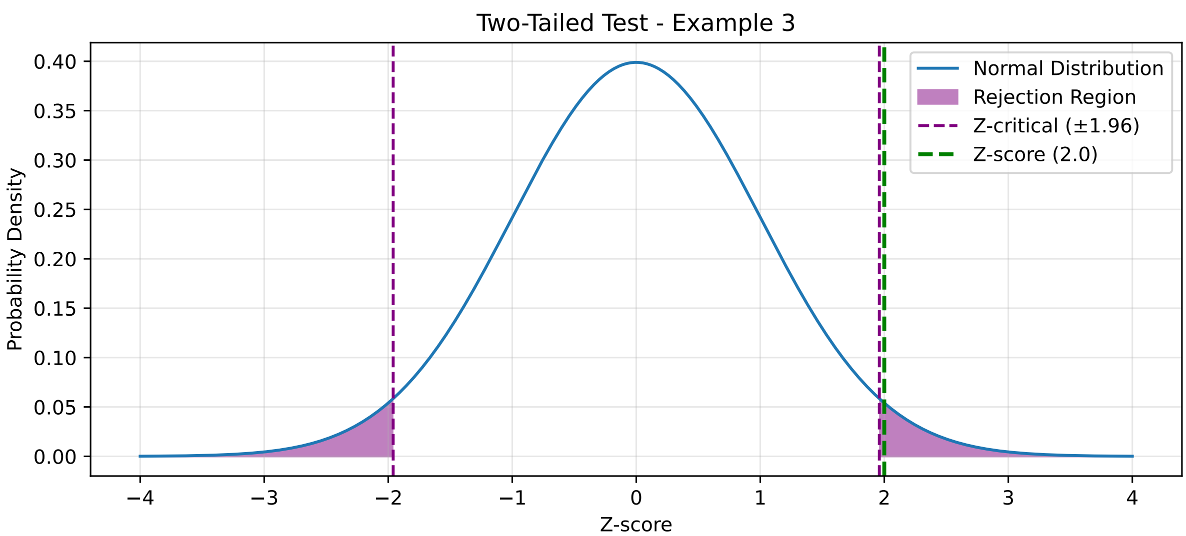
- Population Mean (μ_0) = 10
- Sample Mean ($\bar{x}$) = 10.3
- Population Standard Deviation (σ) = 1.2
- Sample Size (N) = 64
- Test Type = Two-Tailed Test
- Significance Level (α) = 0.05 (z-critical = ±1.96)

Steps

1. **State the Hypotheses**
   - The machine is working correctly $H_0 : \mu = 10$
   - The machine is not working correctly $H_1 : \mu \neq 10$
2. **Calculate Z-test Statistic**

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{N}} = \frac{10.3 - 10}{1.2/\sqrt{64}} = \frac{0.3}{0.15} = 2.0$$

3. **Hypothesis Testing**
   - **Z-critical Value Method**
     - Since this is a two-tailed test, Z-critical = ±1.96
     - Since |Z-score| (2.0) > Z-critical (1.96), we reject H_0.
   - **P-value Method**
     - P-value = 2 × (1 - table value of |Z-score|) = 2 × (1 - 0.9772) = 2 × 0.0228 = 0.0456
     - Since P-value (0.0456) < Significance Level ($\alpha = 0.05$), we reject H_0.
4. **Conclusion**
   The results confirm that the machine is not producing bolts with the correct mean diameter and needs adjustment.



5 Week 5

5.1 Finding Confidence Interval

Given

- Population Standard Deviation (σ)
- Sample Size (N)
- Sample Mean ($\bar{x}$)
- Desired Confidence Level (e.g., 90%, 95%, 99%)

Steps

1. Find Z-score for the desired confidence level
    - Common Z-scores:

Confidence Level	Z-score
90%	1.645
95%	1.96
98%	2.326
99%	2.576

    - Or calculate using Z-table

    $$\text{Z-score} = Z_{\alpha/2} = \text{Z-table value}\left[\frac{1 - \text{Confidence Level}}{2}\right]$$

2. Calculate Standard Error (SE)

    $$\text{SE} = \frac{\sigma}{\sqrt{N}}$$

3. Calculate Margin of Error (ME)

    $$\text{ME} = Z_{\alpha/2} \times \text{SE}$$

4. Calculate Confidence Interval (CI)

    $$\text{CI} = \bar{x} \pm \text{ME}$$
    $$\text{CI} = (\bar{x} - \text{ME}, \bar{x} + \text{ME})$$

CI In one formula

$$\text{CI} = \bar{x} \pm Z_{\alpha/2} \times \frac{\sigma}{\sqrt{N}}$$

5.2 Finding Simple Linear Regression

Steps

1. Create a table like this:

	x	y	xy	x^2
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\sum$	$\sum x$	$\sum y$	$\sum xy$	$\sum x^2$

2. Find Slope (m)

    $$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

3. Find Y-Intercept (b)

    $$b = \frac{\sum y - m(\sum x)}{n}$$

4. Plug-in m and b

$$y = mx + b$$

> **NOTE**
> - You might be asked to find a value of y for a given x or vice versa after finding the regression line.
> - Just plug-in the value in the equation $y = mx + b$ and solve for the unknown.

5.3 Finding R-Squared (Goodness of Fit)

Steps

1. Use the steps above to find the regression line: $y = mx + b$
2. Create a table like this:

	x	y_i (Actual)	$\hat{y}_i$ (Predicted)	$(y_i - \hat{y}_i)^2$	$(y_i - \bar{y})^2$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\sum$				$SS_{RES} = \sum(y_i - \hat{y}_i)^2$	$SS_{TOT} = \sum(y_i - \bar{y})^2$

3. Calculate R-Squared

$$R^2 = 1 - \frac{SS_{RES}}{SS_{TOT}} = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2}$$

> **NOTE**
> - R-Squared value ranges from 0 to 1.
> - 1 indicates perfect fit, while 0 indicates no fit.

5.4 Finding Correlation Coefficient (r)

Steps

1. Create a table like this:

	x	y	xy	x^2	y^2
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\sum$	$\sum x$	$\sum y$	$\sum xy$	$\sum x^2$	$\sum y^2$

2. Calculate Correlation Coefficient

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}}$$

5.5 Finding Gradient Descent

Steps

1. Start with an initial guess for m and b (e.g., m=0, b=0).
2. Create a table like this:

	x_i	y_i	$\hat{y}_i$	$y_i - \hat{y}_i$	$x_i(y_i - \hat{y}_i)$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\sum$				$\sum(y_i - \hat{y}_i)$	$\sum x_i(y_i - \hat{y}_i)$

3. Compute the gradients

   - Gradient with respect to m

$$\frac{\partial J}{\partial m} = -\frac{2}{n}\sum_{i=1}^{n} x_i(y_i - \hat{y}_i)$$

   - Gradient with respect to b

$$\frac{\partial J}{\partial b} = -\frac{2}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)$$

   Where $\hat{y}_i = mx_i + b$

4. Update m and b

$$m = m - \alpha\frac{\partial J}{\partial m}$$

$$b = b - \alpha\frac{\partial J}{\partial b}$$

   Where α is the learning rate.

   > **NOTE**
   > - A small value like 0.01 is often used for α to ensure stable convergence.
   > - See the Gradient Descent Example Animation

5. Repeat steps 2-4 until convergence (i.e., until changes in m and b become very small)

5.6 Finding Local Minima

Derivative Formulas

Given

- A differentiable function $f(x)$

Steps

1. Find the first derivative of the function $f'(x)$.
2. Set the first derivative equal to zero and solve for x to find critical points.

$$f'(x) = 0$$

3. Find the second derivative of the function $f''(x)$.
4. Evaluate the second derivative at the critical points:

   - If $f''(x) > 0$, then x is a local minimum.
   - If $f''(x) < 0$, then x is a local maximum.
   - If $f''(x) = 0$, then the test is inconclusive.

5. Find the local minimum value by substituting the local minimum x back into the original function $f(x)$.

$$f(x) = \text{local minimum value}$$