CSCI 3104 Fall 2021
Instructor: Profs. Grochow and Waggoner

Requizzing Period 1- Standard 3

Due Date .... TODO
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

Problem 1.

- My submission is in my own words and reflects my understanding of the material.
- I have not collaborated with any other person.
- I have not posted to external services including, but not limited to Chegg, Discord, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (Abeal Sileshi agrees). □

3 Standard Dijkstra

3.1 Problem 2

Problem 2. Suppose we are given a finite, connected, and weighted graph $G(V, E, w)$, where the edge weights are non-negative. We define the *weight* of a path P to be the *product* (note: product, *not* sum!) of the edge weights along P. Fix vertices s, t. Our goal is to find a minimum-weight path from s to t.

(a) Suppose we construct a new graph $H(V, E, w')$ that is identical to G, with the exception that $w'((x, y)) = \log(w((x, y)))$ for all edges (x, y). That is, $V(H) = V(G)$ and $E(H) = E(G)$. So we have the same underlying graph, with the only difference being the edge weights. You may take as fact that P is a minimum-weight s to t path in G if and only if P is a shortest s to t path in H.

Suppose now that we run Dijkstra's algorithm on H, in order to find a shortest path from s to t in G. Is this approach valid? Justify your reasoning.

Answer. This approach is valid because like the description says $V(H) = V(G)$ and $E(H) = E(G)$, the the only difference being the edge weights. And seeing that all the edge weights are changed by a constant amount $w'((x, y)) = \log(w((x, y)))$. Not $w'((x, y)) = \log 5(w((x, y)))$ and $w'((x, z)) = \log(2w((x, z)))$ where the amount from w prime to w changes based on the edge. The shortest path P will be the shortest path in H as well, the only difference will be the quantity of the distance traveled. $\square$

(b) Suppose now that the edge weights of G are all positive. That is, $w((x, y)) > 0$ for all edges $(x, y) \in E(G)$. Let $H(V, E, w')$ be the graph corresponding to G, as defined in part (a). Is it now a valid approach to run Dijkstra's algorithm on H, in order to find a shortest path from s to t in G? Justify your reasoning.

Answer. It isn't valid anymore because in order for Dijkstra's algorithm to work all edges need to be non-negative and with $w > 0$, the logarithm of a number between 0 and 1 could be negative which would be detrimental to Dijkstra's algorithm. $\square$

(c) Give conditions on the edge weights of G, so that it suffices to run Dijkstra's algorithm on H, in order to find a minimum-weight path from s to t in G. Clearly explain why your conditions are correct.

Answer. w ¿ 1, because all edges on H need to be positive. For example if you have an edge weight of 1/2 in G, and then H takes a log(1/2) this will produce a negative number. The weight also can't be zero because if there are any edges with weight zero and you multiply this with the other edge weights you'll have a total distance of zero. $\square$