CSCI 3104 Fall 2021
Instructors: Profs. Grochow and Waggoner

Midterm 1- Standard 10

Due Date ..... TODO
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

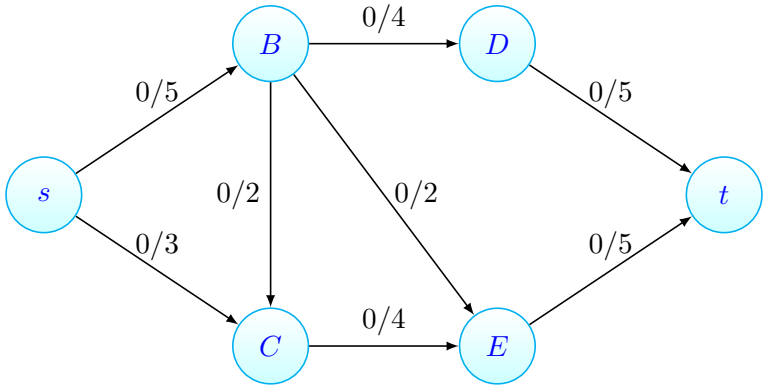
Problem 1.

- My submission is in my own words and reflects my understanding of the material.
- I have not collaborated with any other person.
- I have not posted to external services including, but not limited to Chegg, Discord, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (Abeal Sileshi). □

3 Standard 10- Network Flows: Ford-Fulkerson

Problem 2. Consider the following flow network, with no flow being pushed across the edges.

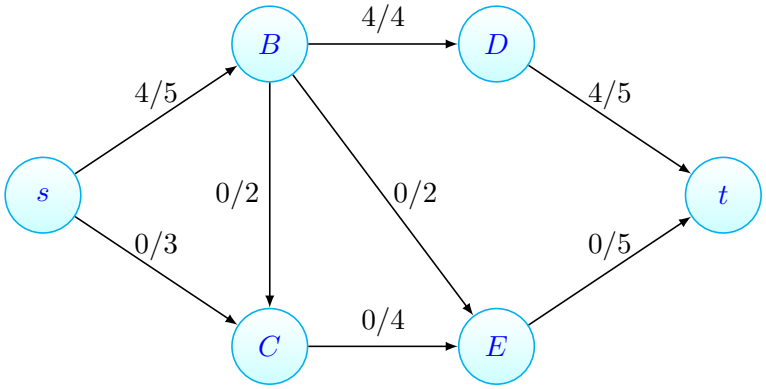


Do the following.

3.1 Problem 2(a)

(a) Consider the flow-augmenting path $s \to B \to D \to t$. Push as much flow as possible through the flow-augmenting path and draw the updated flow network below.

Answer.



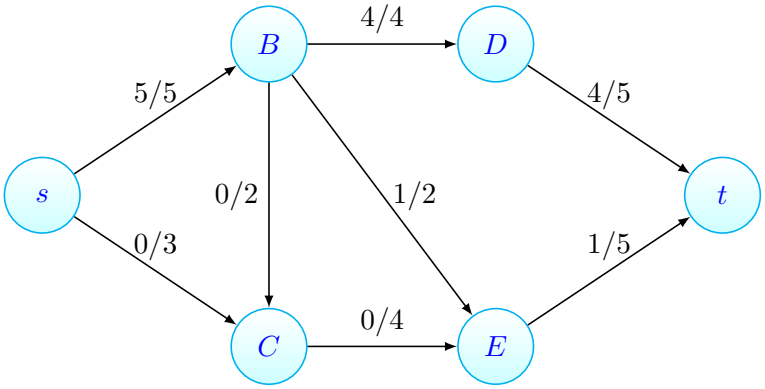
□

3.2 Problem 2(b)

(b) Find a flow-augmenting path using the updated flow configuration from part (a). Then do the following: (i) clearly identify both the flow-augmenting path and the maximum amount of flow that can be pushed through said path; and then (ii) push as much flow through the flow-augmenting path and draw the updated flow network below.

Answer.

A flow-augmenting path is: $s \to B \to E \to t$. The maximum amount of flow that can be pushed is 1 because among the 3 edges the smallest remaining capacity is $s \to B$ with a remaining capacity of 1.



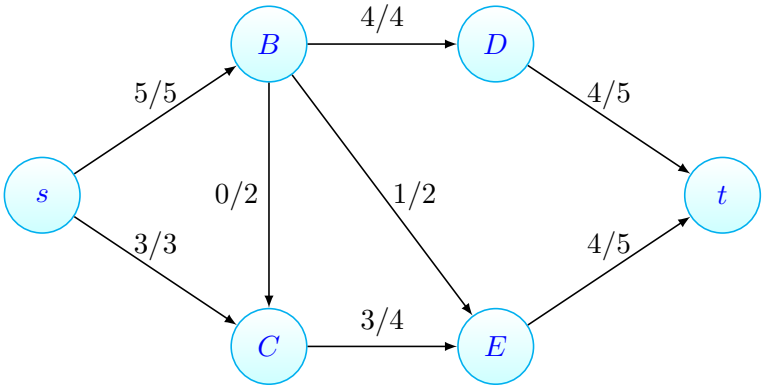
□

3.3 Problem 2(c)

(c) Find a flow-augmenting path using the updated flow configuration from part (b). Then do the following: (i) clearly identify both the flow-augmenting path and the maximum amount of flow that can be pushed through said path; and then (ii) push as much flow through the flow-augmenting path and draw the updated flow network below.

Answer.

A flow-augmenting path is: $s \to C \to E \to t$. The maximum amount of flow that can be pushed is 3 because among the 3 edges the smallest remaining capacity is $s \to C$ with a remaining capacity of 3.



□

3.4 Problem 2(d)

(d) Using the flow configuration from part ((c)), finish executing the Ford-Fulkerson algorithm. Include the following here: (i) your flow network, reflecting the maximum-valued flow configuration you found, and (ii) the corresponding minimum capacity cut. There may be multiple minimum capacity cuts, but you should identify the one corresponding to your maximum-valued flow configuration. Then (iii) finally, compare the value of your flow to the capacity of the cut.

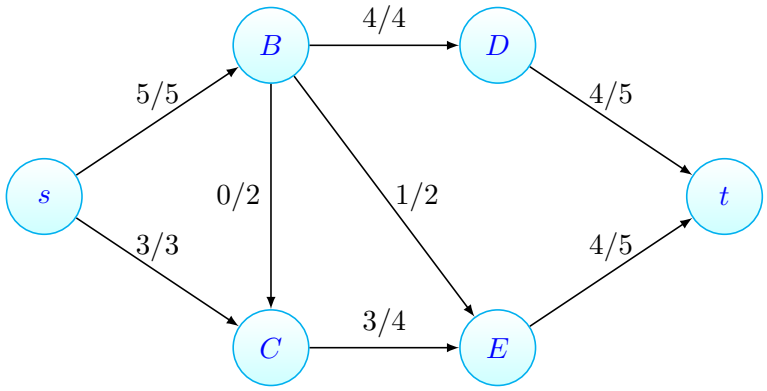
Note: You do **not** need to include the remaining steps of the Ford-Fulkerson algorithm. We will not check these steps when grading.

Answer.

Our flow doesn't have any flow-augmenting networks so this is our final flow network. The maximum-valued flow is 8, going from $s \to t$.

The minimum corresponding capacity cut $X = \{s\}$ and $Y = \{B, C, D, E, T\}$
$c(X, Y) = c(s, B) + c(s, C)$
$c(X, Y) = 5 + 3$
$= 8$

We observe that we also pushed 8 units of flow from $s \to t$. So the value of our flow $f\, val(f) = c(X, Y)$



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