CSCI 3104 Fall 2021
Instructor: Profs. Grochow and Waggoner

# Problem Set 3

Due Date ..... **September 21, 2021**
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Collaborators .....

## Contents



## 1 Instructions

- The solutions **must be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You are welcome and encouraged to collaborate with your classmates, as well as consult outside resources. You must **cite your sources in this document. Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

## 2 Honor Code (Make Sure to Virtually Sign)

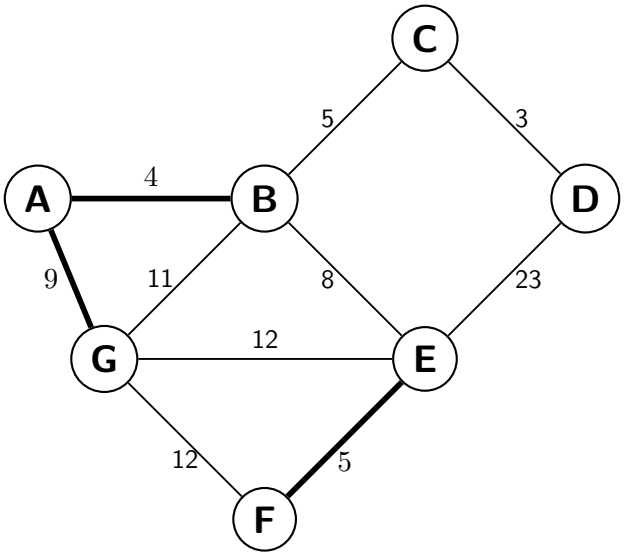
**Problem 1.**

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

*Agreed (signature here).* I agree to the above, Abeal Sileshi □

## 3 Standard 6 - MST: safe and useless edges

**Problem 2.** Consider the weighted graph $G(V, E, w)$ below. Let $\mathcal{F} = \{\{A, B\}, \{A, G\}, \{E, F\}\}$ be an intermediate spanning forest (indicated by the thick edges below). Label each edge that is **not** in $\mathcal{F}$ as safe, useless, or undecided. Provide a 1-2 sentence explanation for each such edge.

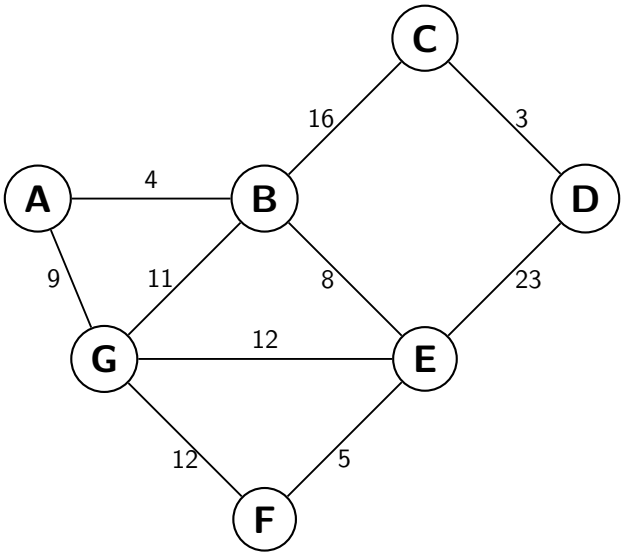



*Answer.* $\{G, B\}$ is useless because it would create a cycle in the subgraph
$\{B, C\}$ is safe because it is the least weighted adjacent edge to tree AGB
$\{B, E\}$ is safe because it's the least weighing adjacent edge to tree EF
$\{G, F\}$ at this point this edge is undecided because it is neither useless nor light
$\{G, E\}$ at this point this edge is undecided because it is neither useless nor light
$\{E, D\}$ at this point this edge is undecided because it is neither useless nor light
$\{C, D\}$ at this point this edge is undecided because it is neither useless nor light

□

# 4 Standard 7- Kruskal's Algorithm

**Problem 3.** Consider the weighted graph $G(V, E, w)$ below. Clearly list the order in which Kruskal's algorithm adds edges to a minimum-weight spanning tree for $G$. Additionally, clearly articulate the steps that Kruskal's algorithm takes as it selects the first **three** edges.




*Answer.* To begin, an empty global set A is created
$A = \emptyset$
$V$ number of disjoint sets are made
$\{A\}\{B\}\{C\}\{D\}\{E\}\{F\}\{G\}$
Then the edges of Graph G, $(G.E)$, are sorted into nondecreasing order
$\{C, D\} : 3$, $\{A, B\} : 4$, $\{E, F\} : 5$, $\{B, E\} : 8$, $\{A, G\} : 9$, $\{B, G\} : 11$, $\{F, G\} : 12$, $\{G, E\} : 12$, $\{B, C\} : 16$, $\{D, E\} : 23$

CD is chosen first because it has the minimum edge weight in the graph
We then check if C and D are in the same tree, which they're not, because starting off all vertices are disjoint
Then we add edge CD to our global set A
$A = \{\{C, D\}\}$
and CD becomes an edge in our MST
Next, we go to the 2nd least weight edge $\{A, B\} : 4$
We check if A and B belong to the same tree, and they don't, so they're added to our MST
This is the new solution set
$A = \{\{C, D\}, \{A, B\}\}$
Then we move on to the next least weighing edge, $\{E, F\} : 5$
We check if E and F belong to the same tree, and they don't, so they're added to our MST
$A = \{\{C, D\}, \{A, B\}\{E, F\}\}$
Next least weight edge is $\{B, E\} : 8$
Vertices B and E don't belong to the same tree, so they're added to our MST
$A = \{\{C, D\}, \{A, B\}\{E, F\}\{B, E\}\}$
Next least weight edge is $\{A, G\} : 9$
Vertices A and G don't belong to the same tree, so they're added to our MST
$A = \{\{C, D\}, \{A, B\}\{E, F\}\{B, E\}, \{A, G\}\}$
Next least weight edge is $\{B, G\} : 9$
Vertices A and G **do** belong to the same tree, so we skip to the next least weight edge
Next least weight edge is $\{G, E\} : 12$
Vertices G and E **do** belong to the same tree, so we skip to the next least weight edge
Next least weight edge is $\{B, C\} : 16$

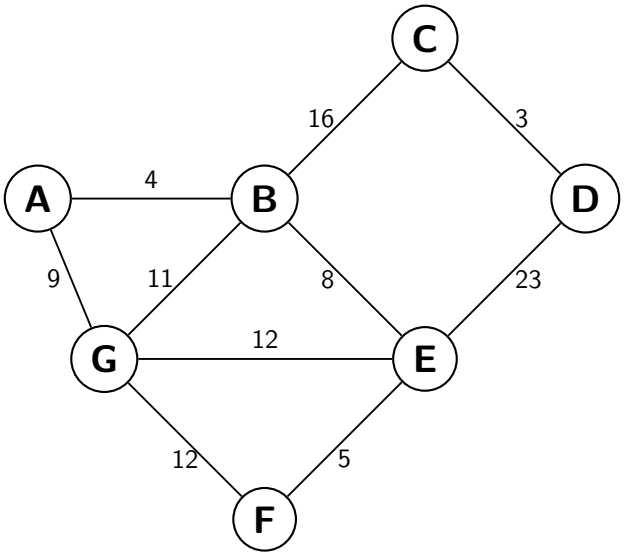
Vertices B and C don't belong to the same tree, so they're added to our MST
$A = \{\{C, D\}, \{A, B\}\{E, F\}\{B, E\}, \{A, G\}$,{B,C}}
The last and heaviest edge is $\{D, E\} : 23$
Vertices D and E **do** belong to the same tree, so we skip to the next least weight edge

Our final MST produced by Kruskal's Algorithm is: $A = \{\{C, D\}, \{A, B\}\{E, F\}\{B, E\}, \{A, G\}$,{B,C}}

□

## 5 Standard 8- Prim's Algorithm

**Problem 4.** Consider the weighted graph $G(V, E, w)$ below. Clearly list the order in which Prim's algorithm, **using the source vertex** $A$, adds edges to a minimum-weight spanning tree for $G$. Additionally, clearly articulate the steps that Prim's algorithm takes as it selects the first **three** edges.




*Answer.* □