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# Methods and Models for Combinatorial Optimization project

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# Indice

# 1 Abstract

The project is composed by two parts:

- Implementing an Integer Linear Programming model with the Cplex API
- Implementing a meta-heuristic solution method

For the second part my choice of method is a genetic algorithm, details on implementation and decisions in section 3. Both solution methods will be applied to the following problem:

> A company produces boards with holes used to build electric frames. Boards are positioned over a machines and a drill moves over the board, stops at the desired positions and makes the holes. Once a board is drilled, a new board is positioned and the process is iterated many times. Given the position of the holes on the board, the company asks us to determine the hole sequence that minimizes the total drilling time, taking into account that the time needed for making an hole is the same and constant for all the holes.

The two different approaches are then tested using some benchmark tests from literature and compared to spot differences in results (optimality, execution time, reliability, ...)

# 2 Integer Linear Programming Model

## 2.1 Network Flow Model Representation

The problem can be represented on a complete weighted graph $G = (N, A)$ where $N$ is the set of nodes (holes on the board) and $A$ is the set of the arcs $(i, j), \forall i, j \in N$ (trajectory of the drill moving from hole $i$ to hole $j$).To each arc will be associated a weight $c_{ij}$ that represents the time needed to move from starting hole to destination.
Using this representation the problem is equivalent to determine the minimum weight hamiltonian cycle on G, so it is like the very popular Travelling Salesman Problem (TSP).
To solve the TSP problem we can formulate it as a network flow model on G. Given a starting node $0 \in N$, let $|N|$ be the amount of its output flow, the problem can be solved by finding the path for which:

- Each node receives 1 unit of flow
- Each node is visited once
- The sum of costs in selected arcs is minimum

## 2.2 Variables and Constraints

The problem can then be represented like this:

**SETS:**

$N$ = the graph nodes (holes)
$A$ = arcs in the form $(i, j), \forall i, j \in N$ (trajectories between holes)

**PARAMETERS:**

$c_{ij}$ = time taken by the drill to move from $i$ to $j$, $\forall (i, j) \in A$ $0$ = starting hole, $0 \in N$

**DECISION VARIABLES:**

$x_{ij}$ = amount of the flow shipped from $i$ to $j$, $\forall (i,j) \in A$ $y_{ij} = 1$ if arc $(i,j)$ ships some flow, 0 otherwise, $\forall (i,j) \in A$

**OBJECTIVE FUNCTION:**

$$min \sum_{i,j|(i,j)\in A} c_{ij} \cdot y_{ij}$$

**CONSTRAINTS:**

$$\sum_{j|(0,j)\in A} x_{0j} = |N|$$

$$\sum_{i|(i,k)\in A} x_{ik} - \sum_{j|(k,j)\in A} x_{kj} = 1 \qquad \forall k \in N \setminus \{0\}$$

$$\sum_{j|(i,j)\in A} y_{ij} = 1 \qquad \forall i \in N$$

$$\sum_{i|(i,j)\in A} y_{ij} = 1 \qquad \forall j \in N$$

$$x_{ij} \leq |N| \cdot y_{ij} \qquad \forall (i,j) \in A$$

$$x_{ij} \in \mathbb{Z}_+ \qquad \forall (i,j) \in A$$

$$y_{ij} \in \{0,1\} \qquad \forall (i,j) \in A$$

# A Bibliography


- Project pt. 1: http://www.math.unipd.it/~luigi/courses/metmodoc/z01.eserc.lab.01.en.pdf
- Project pt. 2: http://www.math.unipd.it/~luigi/courses/metmodoc/z02.eserc.lab.01.en.pdf