Digital Signal Processing for Music

Part 13: Improving (Re-)Quantization Quality

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Introduction

Quantization error properties are fixed, so there is no way of improving the quality

Or is there????

"Cheating" for Better Quality

Improving perceptual quality of errors due to:

- Quantization
  - Oversampling
  - Noise Shaping
- Re-Quantization / Word Length Reduction
  - Dither
  - Noise Shaping

Oversampling

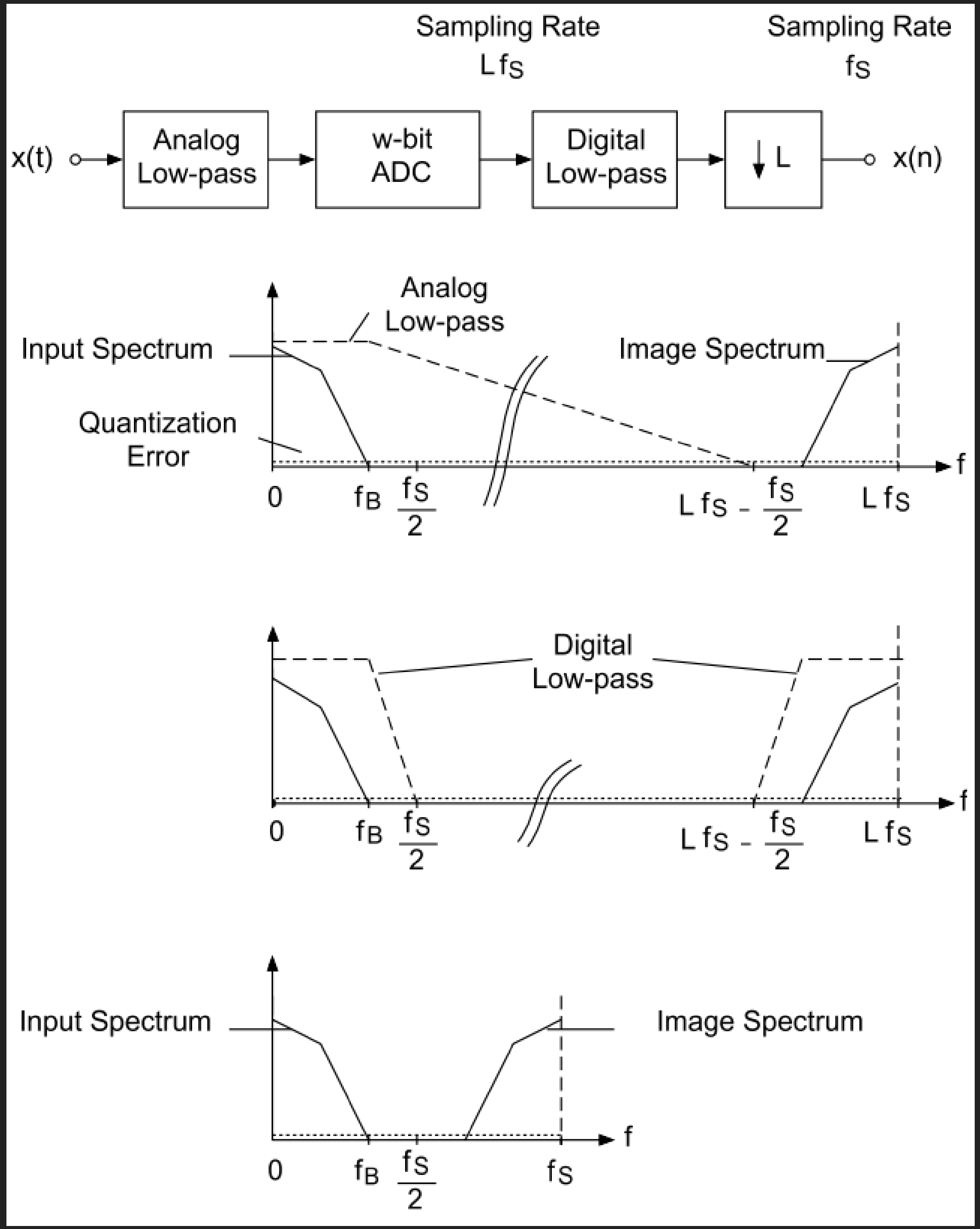
- Recording at higher sample rates and downsampling
- Allows use of less steep anti-aliasing filters
- Also improves quantization error

Quantization error properties

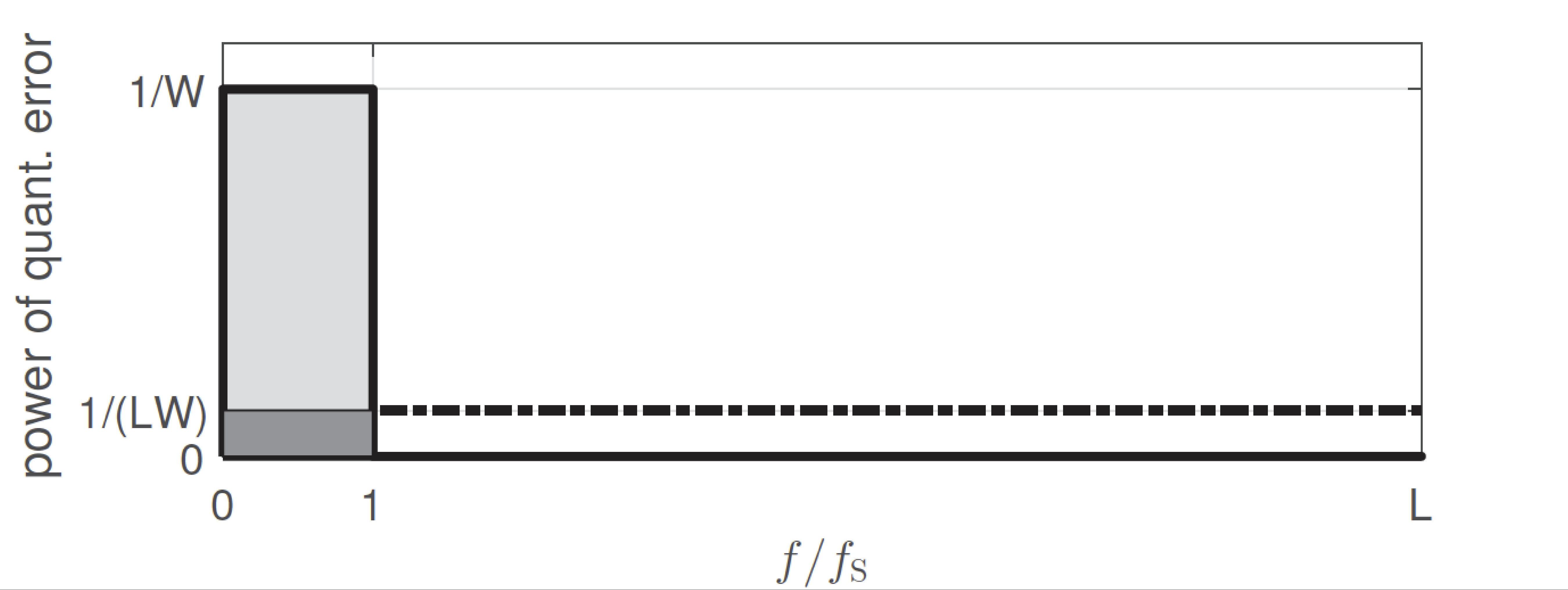
- *White* noise (ie **flat spectrum**)
- Noise power *sample rate independent*

$$|Q(\mathrm{j}\omega)|^2 \sim \frac{\Delta^2}{12 \cdot \omega_\mathrm{S}}$$

Oversampling Process



Quantization Noise Spectrum for Oversampling Amount



SNR Gain from Oversampling

$$\begin{aligned}
|Q(\mathrm{j}\omega)|^2 &= \frac{\Delta^2}{12 \cdot \omega_S^*} \\
&= \frac{\Delta^2}{12 \cdot L \cdot \omega_S} \\
W_{\mathrm{Q,LP}}^* &= \frac{\Delta^2}{12 \cdot L} \\
&\Rightarrow \\
SNR^* &= 6.02 \cdot w + 10 \log_{10}(L) + c_S
\end{aligned}$$

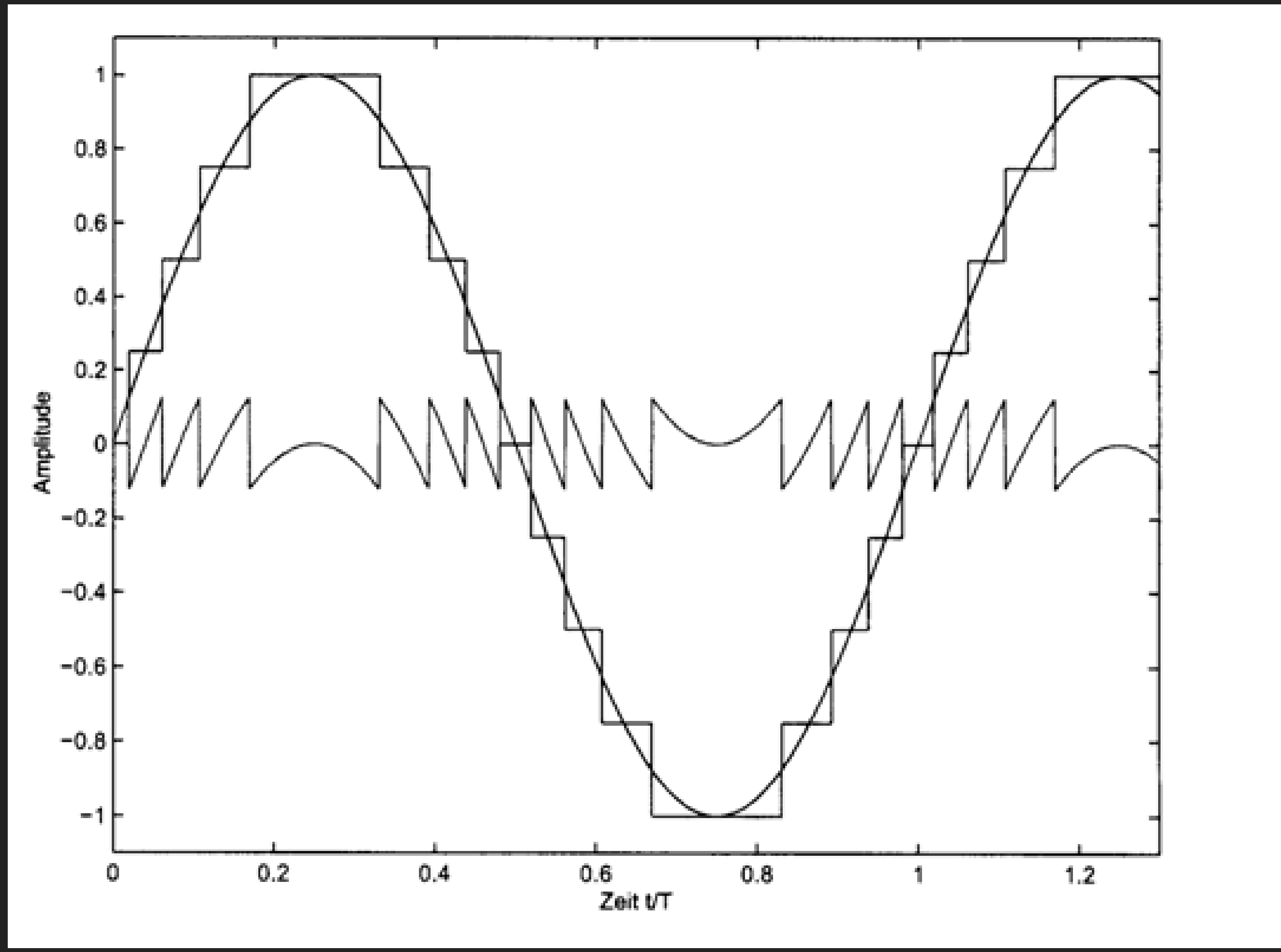
Oversampling Summary

$$SNR = 6.02 \cdot w + c_S + 10\log_{10}(L)$$

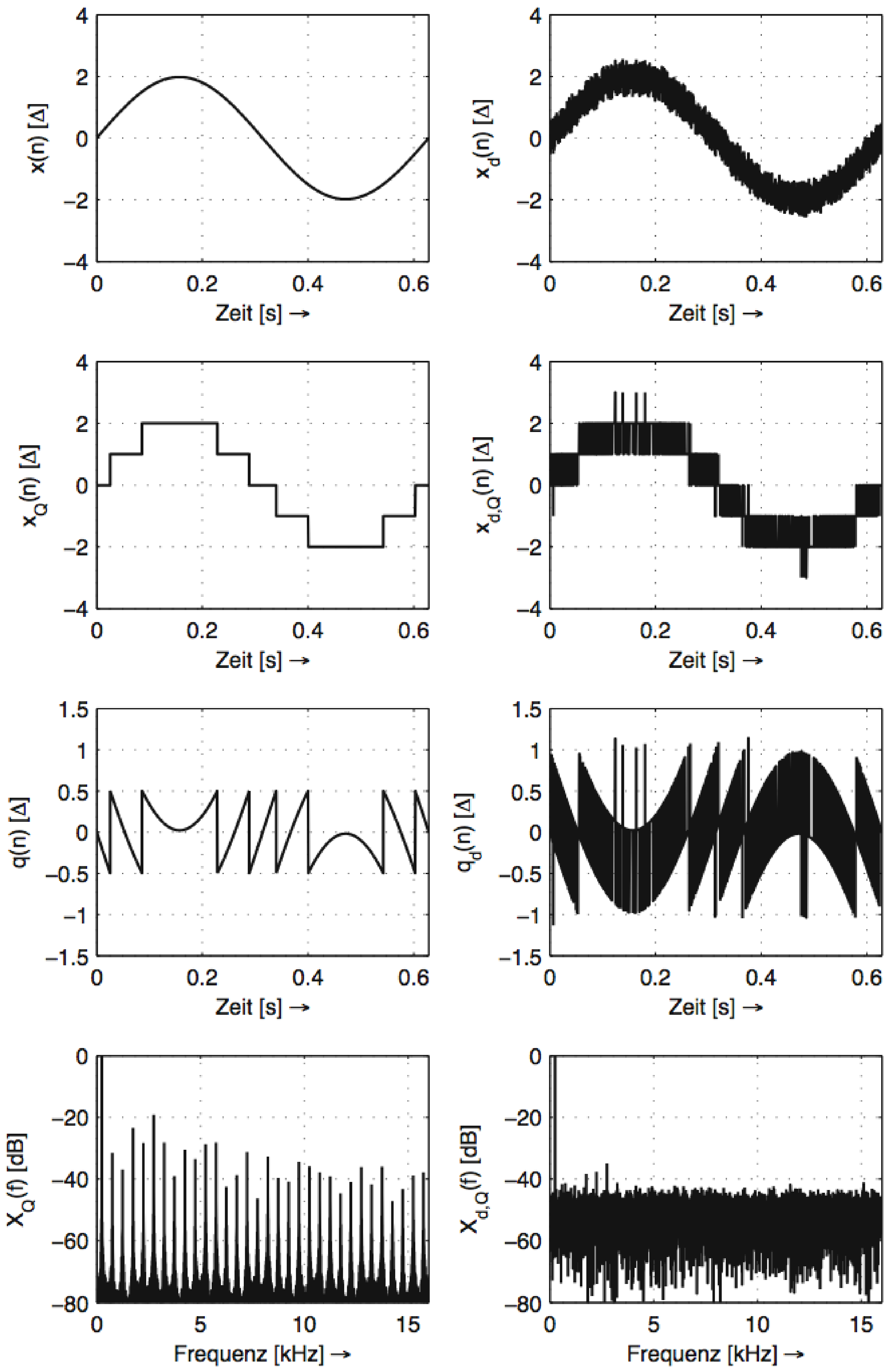
Every doubling of f_S **adds ~3dB SNR**

Dither

- Previous assumption: **Quantization error is white noise** (ie, rect)
  - **No correlation** between signal and quantization error
- **Not true for:**
  - Low signal level
  - Low signal frequency

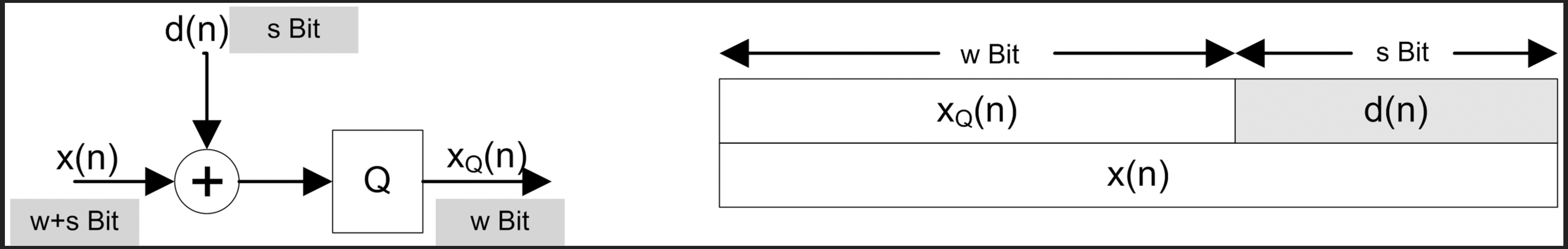


Solution: Add noise before quantization (dither)



x(n) [Δ]
Zeit [s] →
$x_d(n)$ [Δ]
Zeit [s] →
$x_Q(n)$ [Δ]
Zeit [s] →
$x_{d,Q}(n)$ [Δ]
Zeit [s] →
q(n) [Δ]
Zeit [s] →
$q_d(n)$ [Δ]
Zeit [s] →
$X_Q(f)$ [dB]
Frequenz [kHz] →
$X_{d,Q}(f)$ [dB]
Frequenz [kHz] →
4
2
0
−2
−4
0
0.2
0.4
0.6
1.5
1
0.5
−0.5
−1
−1.5
−20
−40
−60
−80
5
10
15

Dither Process



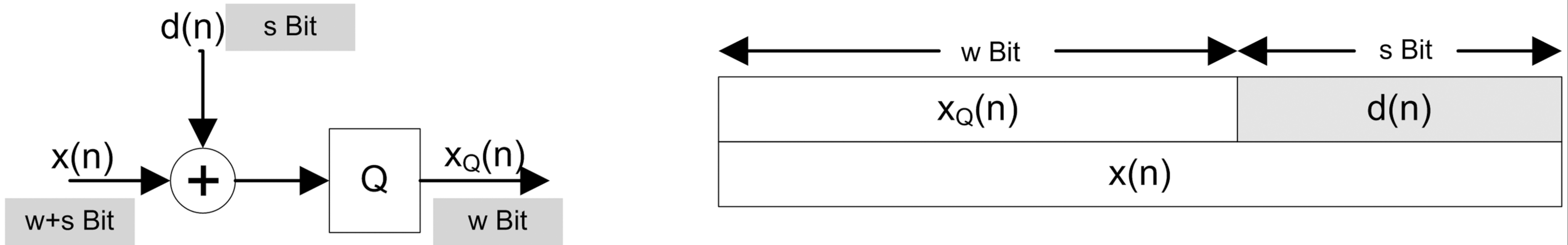
Dither Simple Example

Input signal: DC at $1.3 \cdot \Delta$

- Without dither:
  - Output value: Δ
  - *Quantization error constant*: $0.3 \cdot \Delta$
- With dither:
  - Output range: $-\Delta/2 ... \Delta/2$
  - Signal is most frequently quantized to $\Delta(p = 0.7)$, but sometimes to $2 \cdot \Delta(p = 0.3)$
  - *Average* output value: $1.3 \cdot \Delta$
  - *Quantization error varying* between $0.3 \cdot \Delta$ and $0.7 \cdot \Delta$

≡

Dither Properties



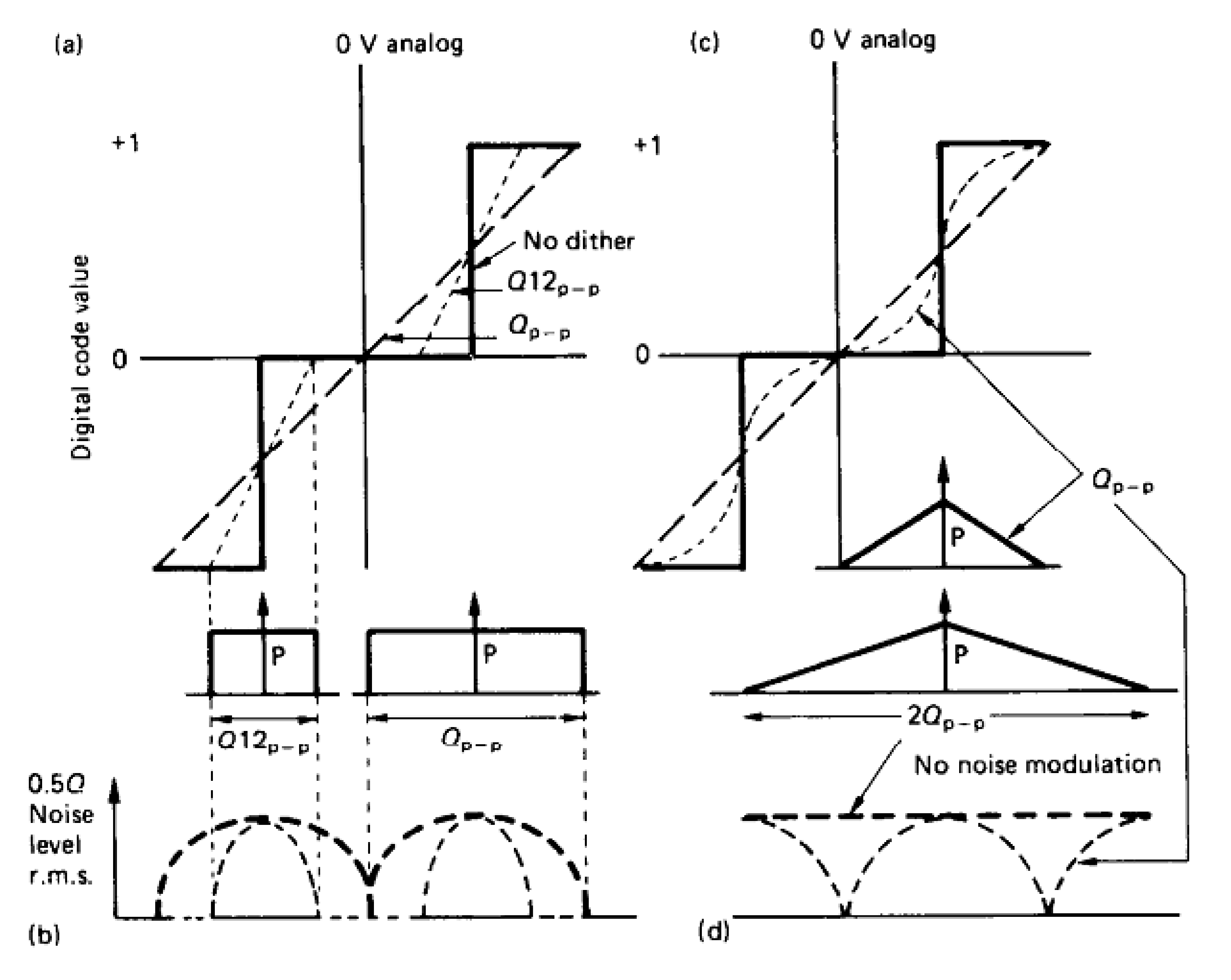
Dither with Rectangular PDF, $-\frac{\Delta}{2}\ldots\frac{\Delta}{2}$, Not Quantized

$$
\begin{aligned}
x &= 0\cdot\Delta & &\rightarrow \bar{x_Q} = 0,\\
\sigma_R(x) &= \Delta\sqrt{(-0)^2\cdot 1.0} & &= 0.0\Delta\\
x &= 0.1\cdot\Delta & &\rightarrow \bar{x_Q} = 0.1\Delta,\\
\sigma_R(x) &= \Delta\sqrt{(-0.1)^2\cdot 0.9+(0.9)^2\cdot 0.1} & &= 0.3\Delta\\
x &= 0.3\cdot\Delta & &\rightarrow \bar{x_Q} = 0.3\Delta,\\
\sigma_R(x) &= \Delta\sqrt{(-0.3)^2\cdot 0.7+(0.7)^2\cdot 0.3} & &= 0.46\Delta\\
x &= 0.5\cdot\Delta & &\rightarrow \bar{x_Q} = 0.5\Delta,\\
\sigma_R(x) &= \Delta\sqrt{(-0.5)^2\cdot 0.5+(0.5)^2\cdot 0.5} & &= 0.5\Delta\\
x &= 0.7\cdot\Delta & &\rightarrow \bar{x_Q} = 0.7\Delta,\\
\sigma_R(x) &= \Delta\sqrt{(-0.7)^2\cdot 0.3+(0.3)^2\cdot 0.7} & &= 0.46\Delta\\
x &= 0.9\cdot\Delta & &\rightarrow \bar{x_Q} = 0.9\Delta,\\
\sigma_R(x) &= \Delta\sqrt{(-0.9)^2\cdot 0.1+(0.1)^2\cdot 0.9} & &= 0.3\Delta\\
x &= 1\cdot\Delta & &\rightarrow \bar{x_Q} = 0,\\
\sigma_R(x) &= 0
\end{aligned}
$$

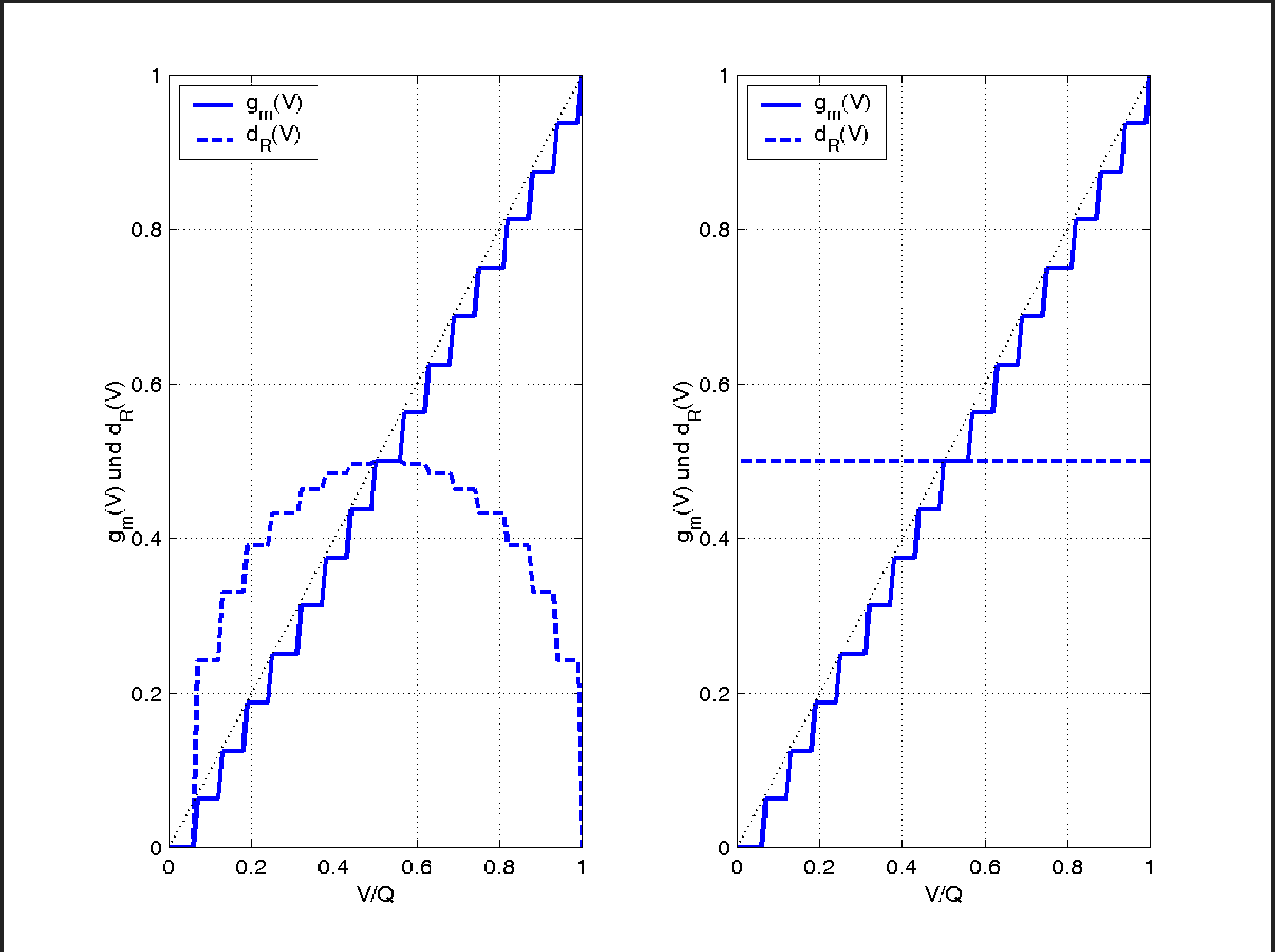
Dither with Triangular PDF, $-\Delta \ldots \Delta$, Not Quantized

$$
\begin{aligned}
x &= 0 \cdot \Delta && \rightarrow \bar{x_{\mathrm{Q}}} = 0, \\
\sigma_R(x) &= 0.5\Delta \\
x &= 0.1 \cdot \Delta && \rightarrow \bar{x_{\mathrm{Q}}} = 0.1\Delta, \\
\sigma_R(x) &= 0.5\Delta \\
x &= 0.3 \cdot \Delta && \rightarrow \bar{x_{\mathrm{Q}}} = 0.3\Delta, \\
\sigma_R(x) &= 0.5\Delta \\
x &= 0.5 \cdot \Delta && \rightarrow \bar{x_{\mathrm{Q}}} = 0.5\Delta, \\
\sigma_R(x) &= 0.5\Delta \\
x &= 0.7 \cdot \Delta && \rightarrow \bar{x_{\mathrm{Q}}} = 0.7\Delta, \\
\sigma_R(x) &= 0.5\Delta \\
x &= 0.9 \cdot \Delta && \rightarrow \bar{x_{\mathrm{Q}}} = 0.9\Delta, \\
\sigma_R(x) &= 0.5\Delta \\
x &= 1 \cdot \Delta && \rightarrow \bar{x_{\mathrm{Q}}} = 0, \\
\sigma_R(x) &= 0.5\Delta
\end{aligned}
$$

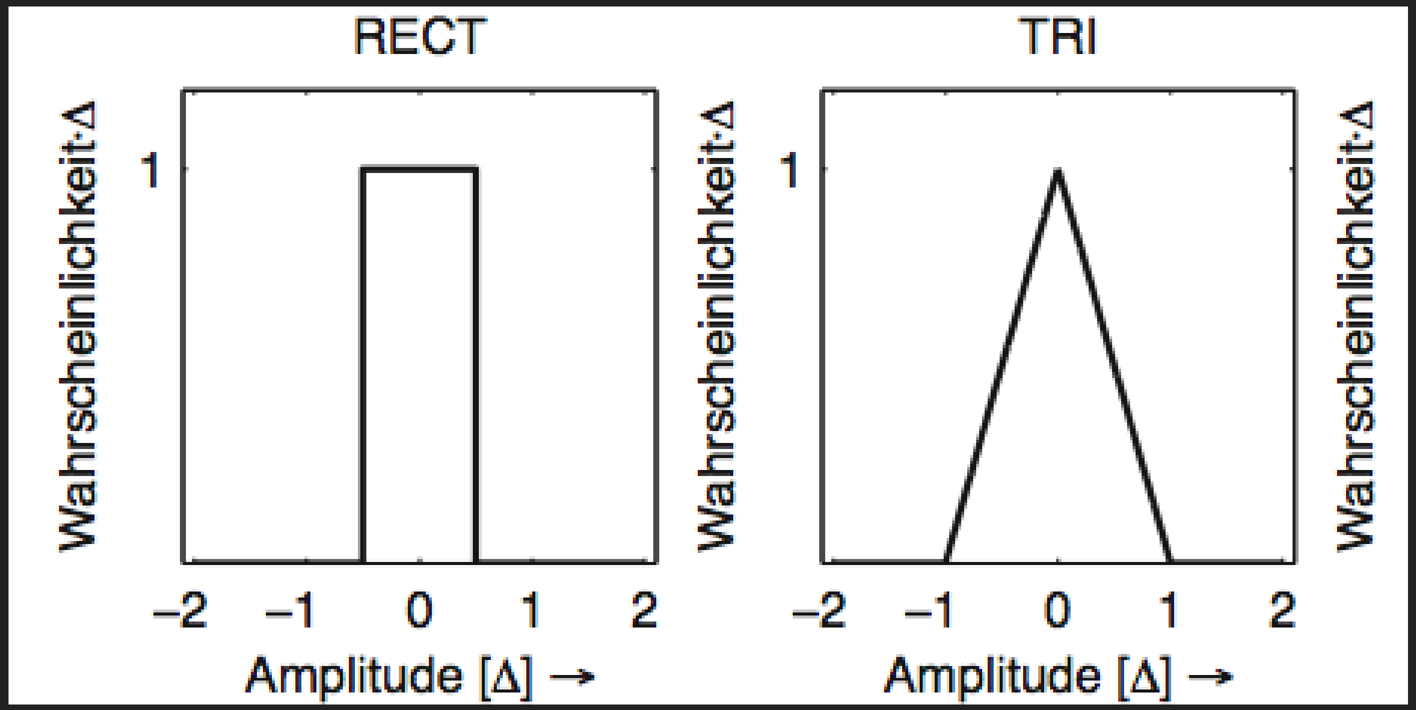
Linearization and Noise Modulation



Linearization and Noise Modulation

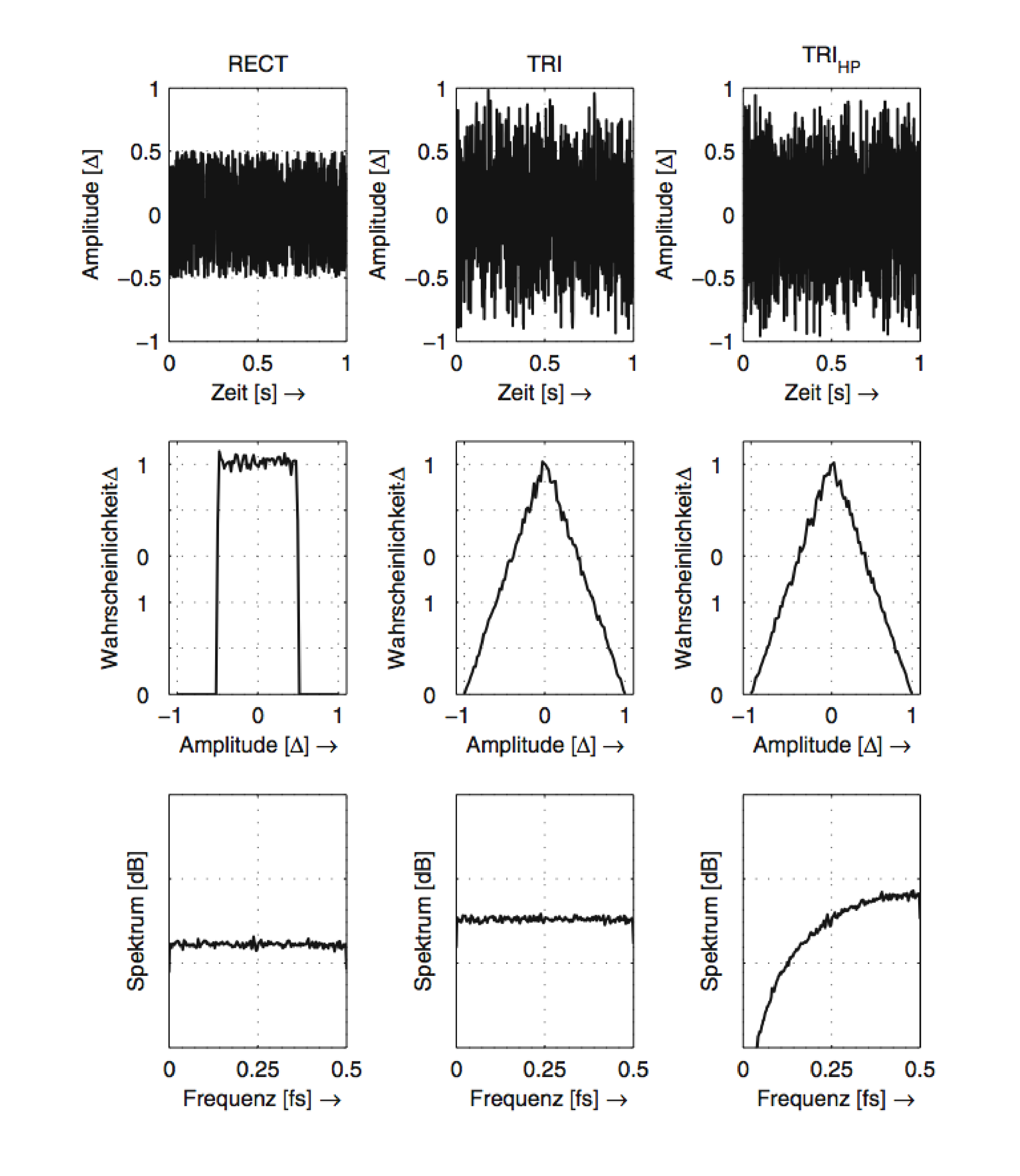


Noise Properties



$$
\begin{aligned}
d_{\mathrm{RECT}}(n) &= d(n)\\
d_{\mathrm{TRI}}(n) &= d_{\mathrm{RECT},1}(n) + d_{\mathrm{RECT},2}(n)\\
d_{\mathrm{HP}}(n) &= d(n) - d(n-1)
\end{aligned}
$$

Noise Properties



How Does the SNR Change by Adding Dither?

Noise power of d_{RECT} & d_{TRI}

$$W_{\mathrm{RECT}} = \frac{\Delta^2}{12}$$
$$W_{\mathrm{TRI}} = \frac{\Delta^2}{6}$$

SNR of dithered full scale signal

$$SNR_{\mathrm{RECT}} = SNR_{normal} - 3.01\ [dB]$$
$$SNR_{\mathrm{TRI}} = SNR_{normal} - 4.77\ [dB]$$

		Sine	Speech	Music
8-Bit	Trunc			
	Rect			
	Tri			
4-Bit	Trunc			
	Rect			
	Tri			
2-Bit	Trunc			
	Rect			
	Tri			

Z-Transform (Quick and Dirty)

Z		Time
$X(z)$	$\leftrightarrow$	$x(n)$
$X(z) \cdot z^{-k}$	$\leftrightarrow$	$x(n-k)$

Transfer Function:

$$H(z) = \frac{out}{in} = \frac{Y(z)}{X(z)}$$

Spectrum:

$$H(\mathrm{j}\Omega) = H(z\big|_{z=e^{\mathrm{j}\Omega}})$$

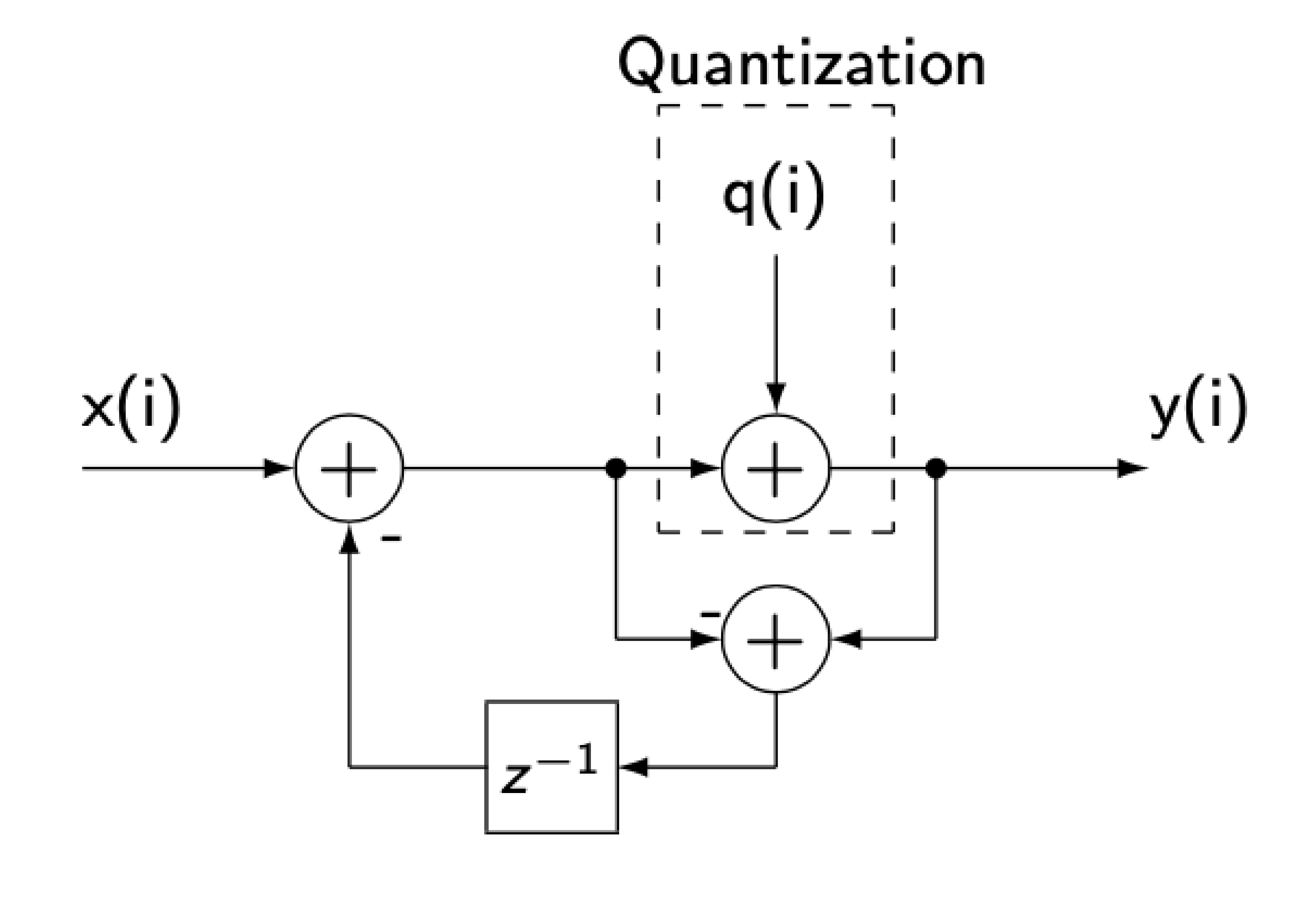
Noise Shaping

Idea

Filter quantization error, shape its frequency response

- Move power to high frequencies
- Less recognizable in lower frequencies

First Order Noise Shaping



$$
\begin{aligned}
y(i) &= [x(i) - q(i-1)]_Q \\
&= x(i) - q(i-1) + q(i)
\end{aligned}
$$

$$y(i) = x(i) - q(i-1) + q(i)$$

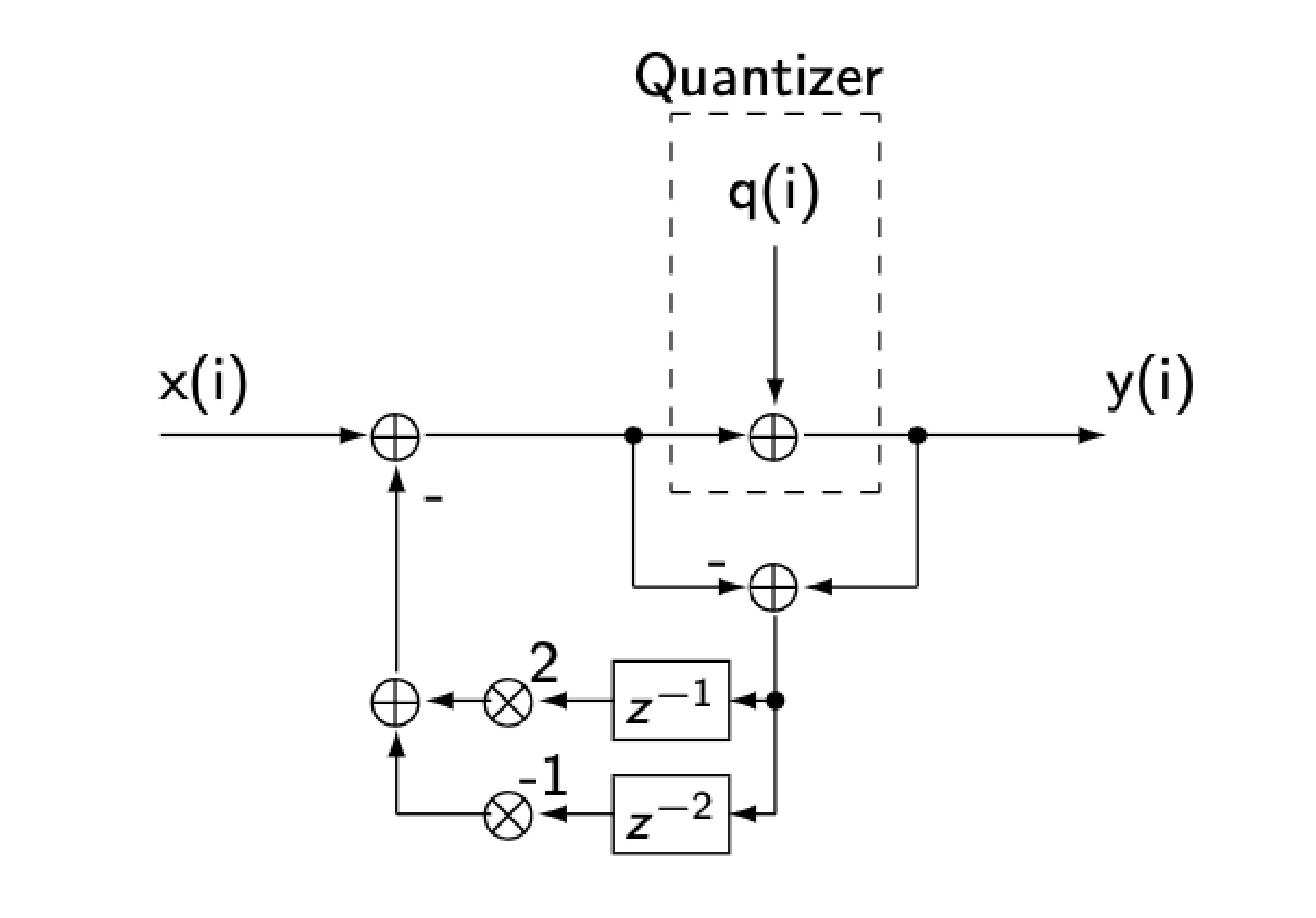
$$\begin{aligned} Y(z) &= X(z) - z^{-1} \cdot Q(z) + Q(z) \\ &= X(z) + \underbrace{\left(1 - z^{-1}\right)}_{H_{\mathrm{Q}}(z)} \cdot Q(z) \end{aligned}$$

$$\Rightarrow$$

$$H_{\mathrm{Q}}(z) = 1 - z^{-1}$$

$$\begin{aligned} \left|H_{\mathrm{Q}}(\mathrm{j}\Omega)\right| &= \left|1 - e^{-\mathrm{j}\Omega}\right| \\ &= 2 \cdot \left|\sin\left(\frac{\Omega}{2}\right)\right| \end{aligned}$$

Second Order Noise Shaping



$$
\begin{aligned}
y(i) &= [x(i) - 2 \cdot q(i-1) + q(i-2)]_Q \\
&= x(i) - 2 \cdot q(i-1) + q(i-2) + q(i)
\end{aligned}
$$

$$y(i) = x(i) - 2 \cdot q(i-1) + q(i-2) + q(i)$$

$$\begin{aligned} Y(z) &= X(z) - 2 \cdot z^{-1} \cdot Q(z) + z^{-2} \cdot Q(z) + Q(z) \\ &= X(z) + (1 - z^{-1})^2 \cdot Q(z) \end{aligned}$$

$$\Rightarrow$$

$$H_{\mathrm{Q}}(z) = (1 - z^{-1})^2$$

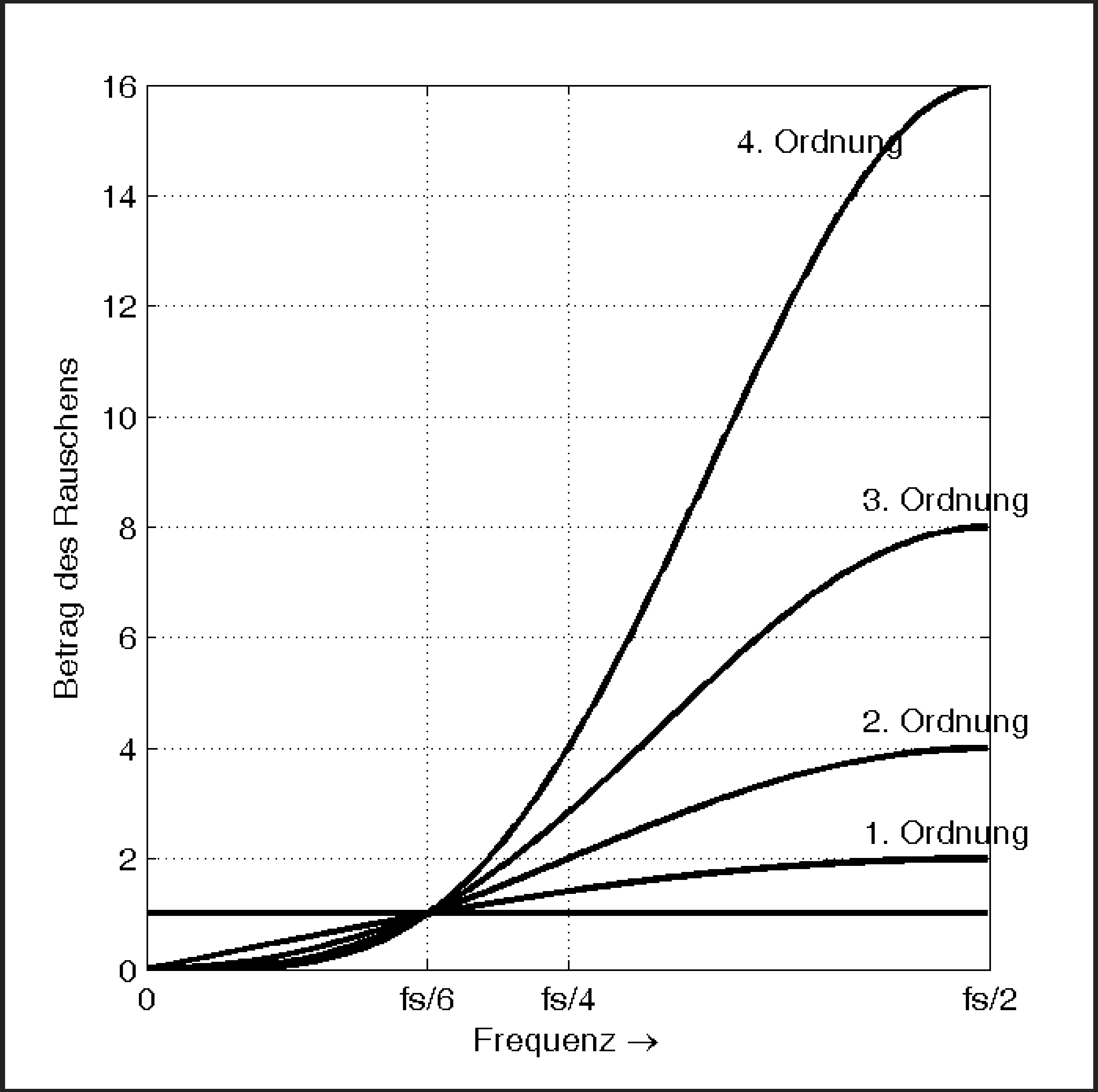
Without derivation: *n*th order noise shaping

$$Y(z) = X(z) + (1 - z^{-1})^n \cdot Q(z)$$

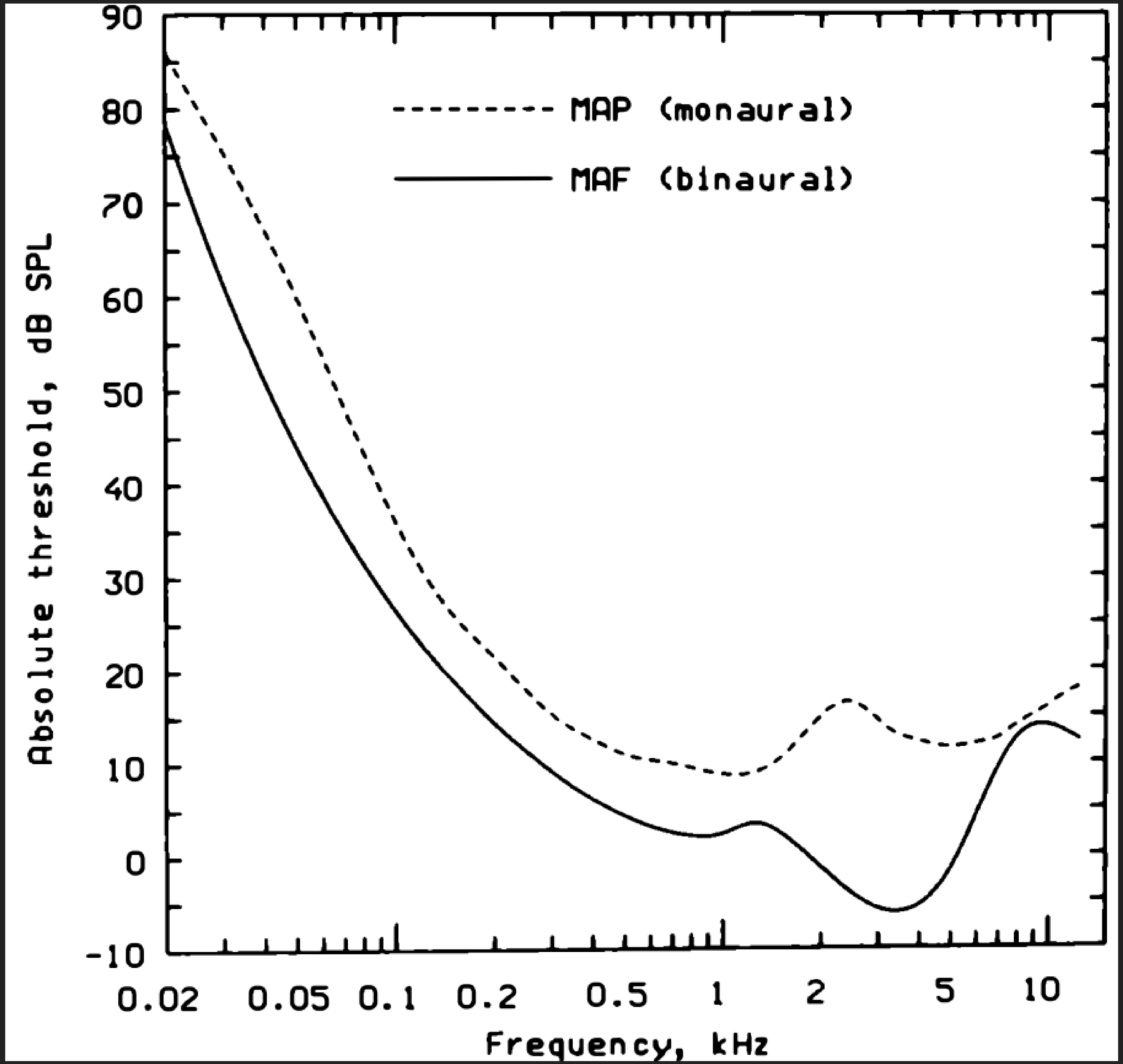
$$\Rightarrow$$

$$H_{\mathrm{Q}}(z) = (1 - z^{-1})^n$$

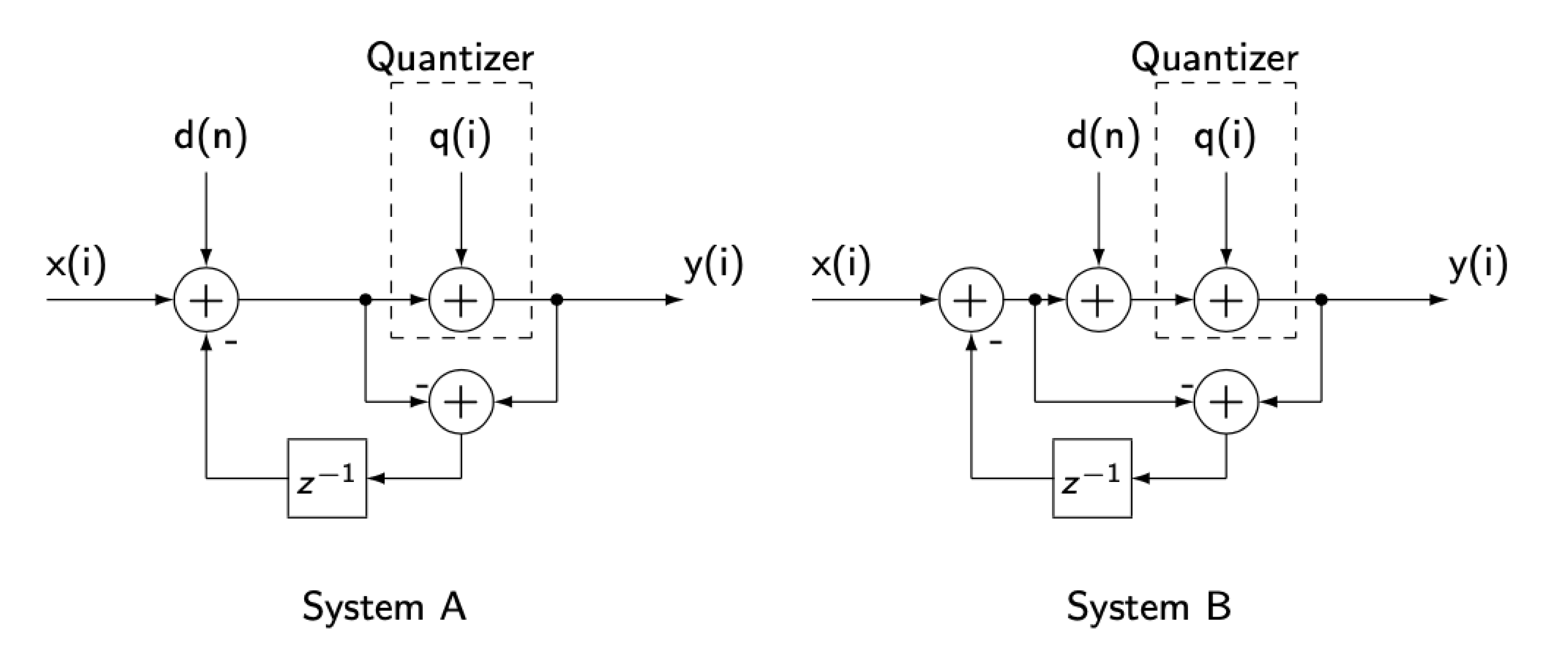
Higher Order Noise Shaping



Arbitrary Noise Shaping Transfer Functions



Dither & Noise Shaping



Dither & Noise Shaping: System A

$$
\begin{aligned}
y(i) &= [x(i) + d(n) - q(i-1)]_Q \\
&= x(i) + d(n) - q(i-1) + q(i)
\end{aligned}
$$

$$
\begin{aligned}
Y(z) &= X(z) - z^{-1} \cdot Q(z) + Q(z) + D(z) \\
&= X(z) + (1 - z^{-1}) \cdot Q(z) + D(z)
\end{aligned}
$$

Dither & Noise Shaping: System B

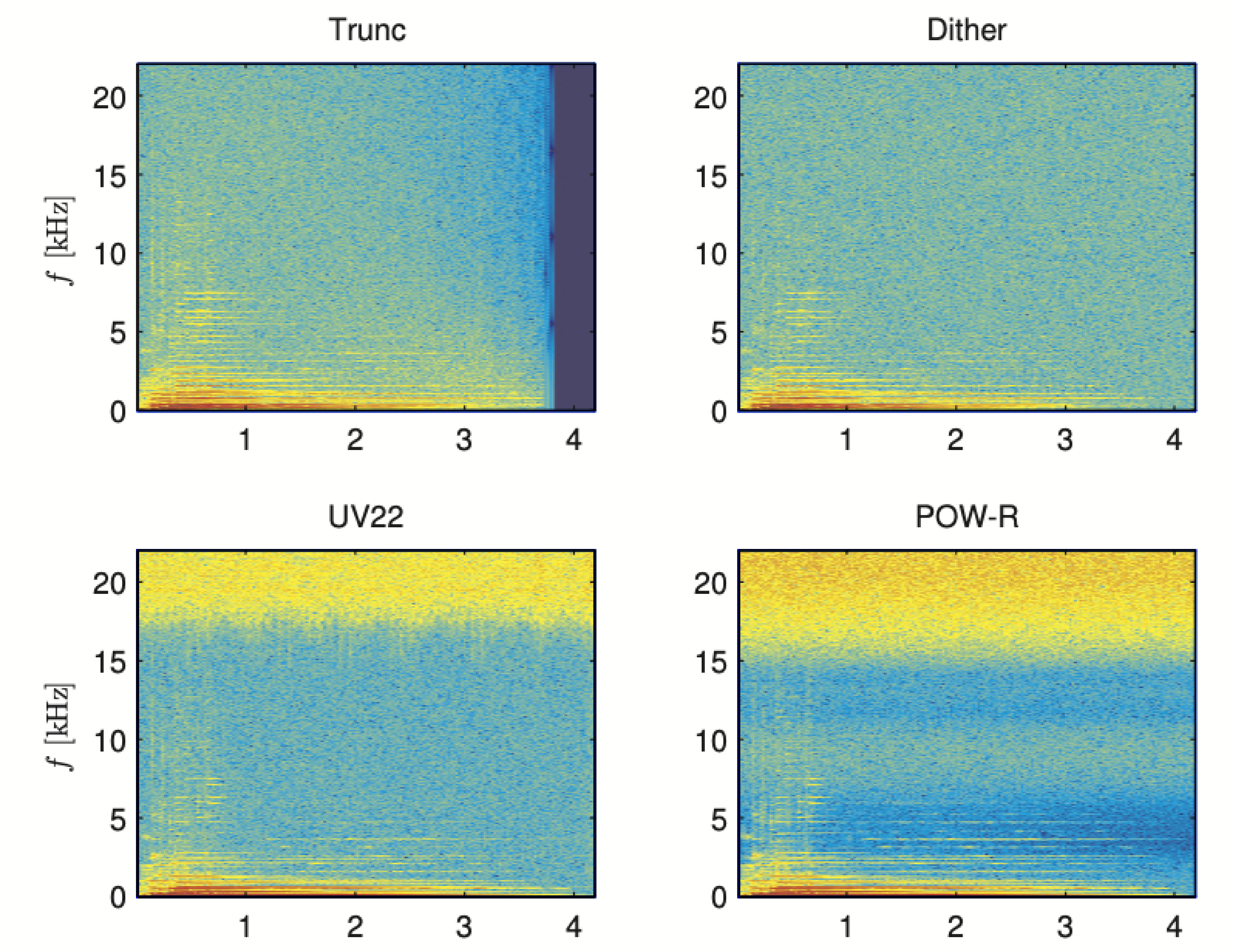
$$
\begin{aligned}
y(i) &= [x(i) + d(n) - q(i-1) - d(n-1)]_Q \\
&= x(i) - q(i-1) + q(i) - d(n-1) + d(n)
\end{aligned}
$$

$$
\begin{aligned}
Y(z) &= X(z) - z^{-1} \cdot Q(z) + Q(z) - z^{-1} \cdot D(z) + D(z) \\
&= X(z) + (1 - z^{-1}) \cdot (Q(z) + D(z))
\end{aligned}
$$

Noise Shaping Audio Example

- 16 Bit: 0:00 / 0:04
- 8 Bit: 0:00 / 0:04
- 8 Bit Dither: 0:00 / 0:04
- 8 Bit Standard Noise Shaping: 0:00 / 0:04
- 8 Bit Powerful Noise Shaping: 0:00 / 0:04

Noise Shaping Spectrograms



Summary

- **Oversampling**
  - Reduces quantization error power in the audible band
  - *Process*: oversampling $\rightarrow$ filtering $\rightarrow$ downsampling
- **Dither**
  - Reduces correlation of error and signal for low amplitude signals
  - Increases the power of the quantization error slightly
  - *Process*: Add triangular shaped low-level noise before word-length reduction
- **Noise Shaping**
  - Reduces the audibility of the quantization error by shifting it to high frequencies
  - Works best at high sample rates
  - *Process*: Feedback the quantization error