

Digital Signal Processing for Music

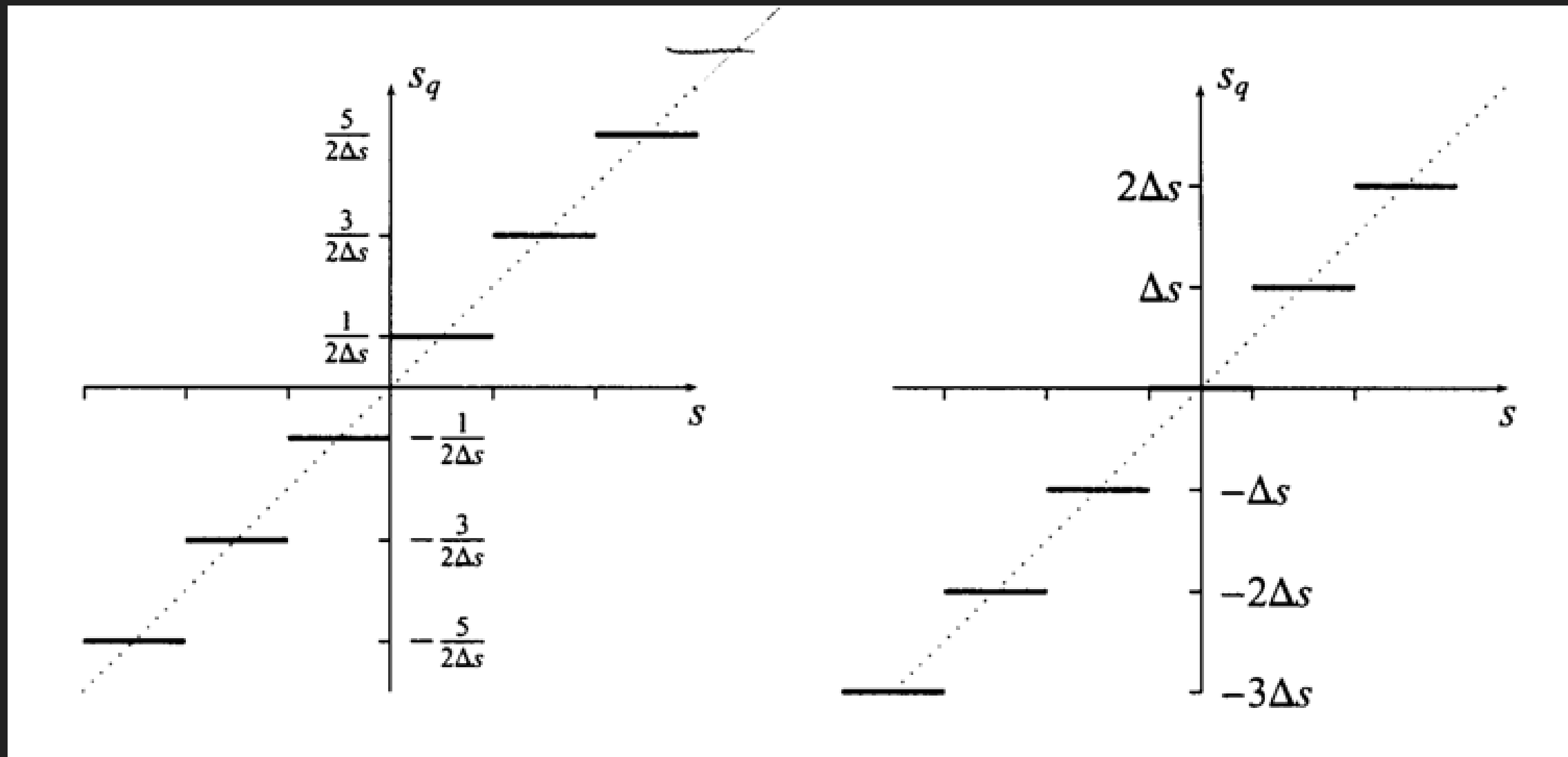
Part 10: Discretization, Part 2 - Quantization

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Quantizer:

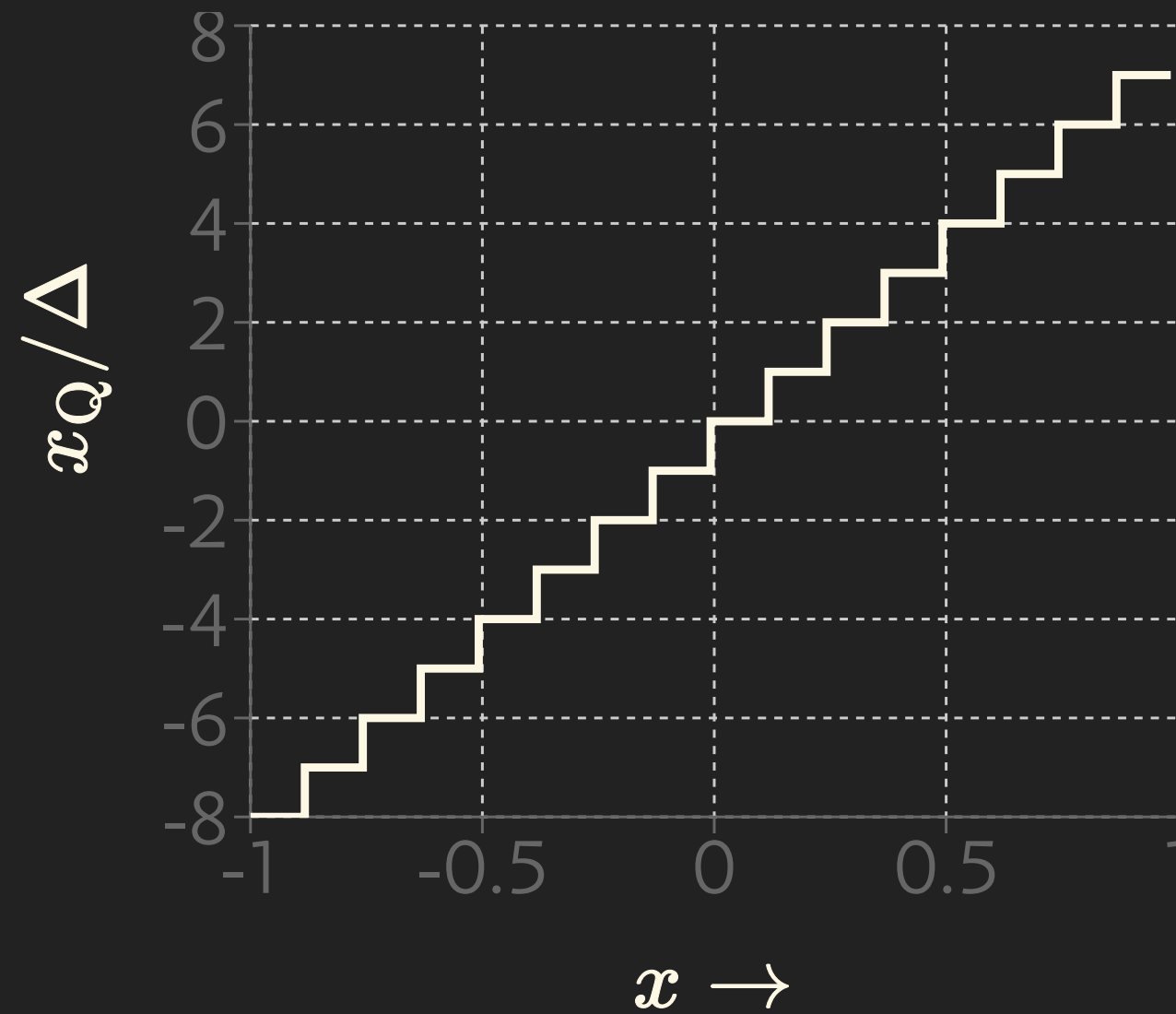
Continuous \mapsto Discrete (pre-defined set of allowed values)

- » Quantization is **non-linear**
- » Quantization is **irreversible**



Mid-Rise

Mid-Tread

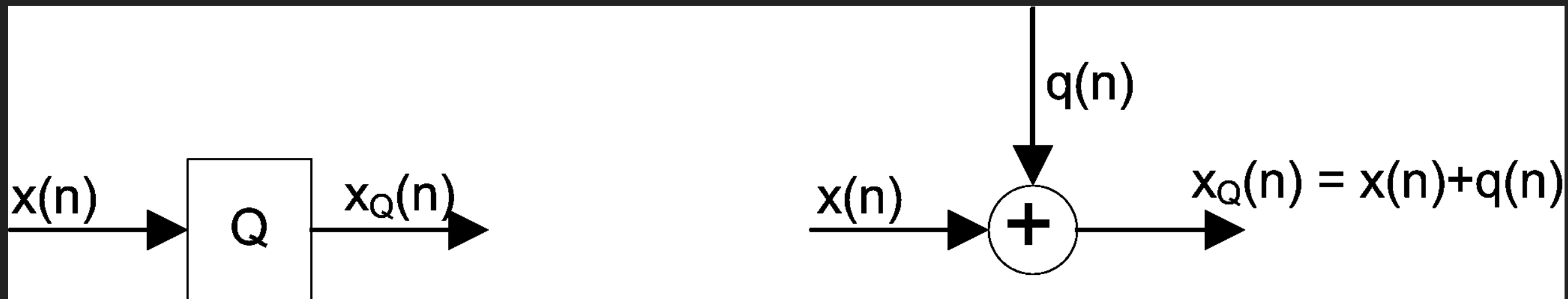


- » Number of quantization steps: $\mathcal{M} = 16$
- » Word Length (bits): $w = \log_2(\mathcal{M}) = 4\text{bit}$

Quantization: Word Length & Number of Steps

w	$\mathcal{M} = 2^w$
1	2
2	4
4	16
8	256
12	4096
16	65536
20	1048576
24	16777216

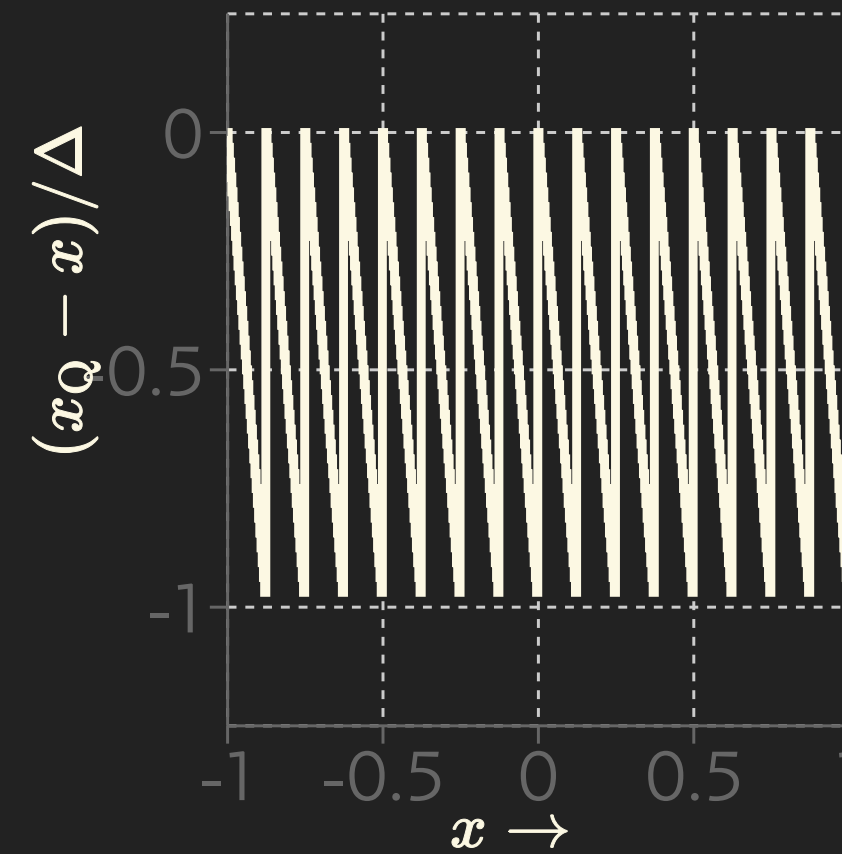
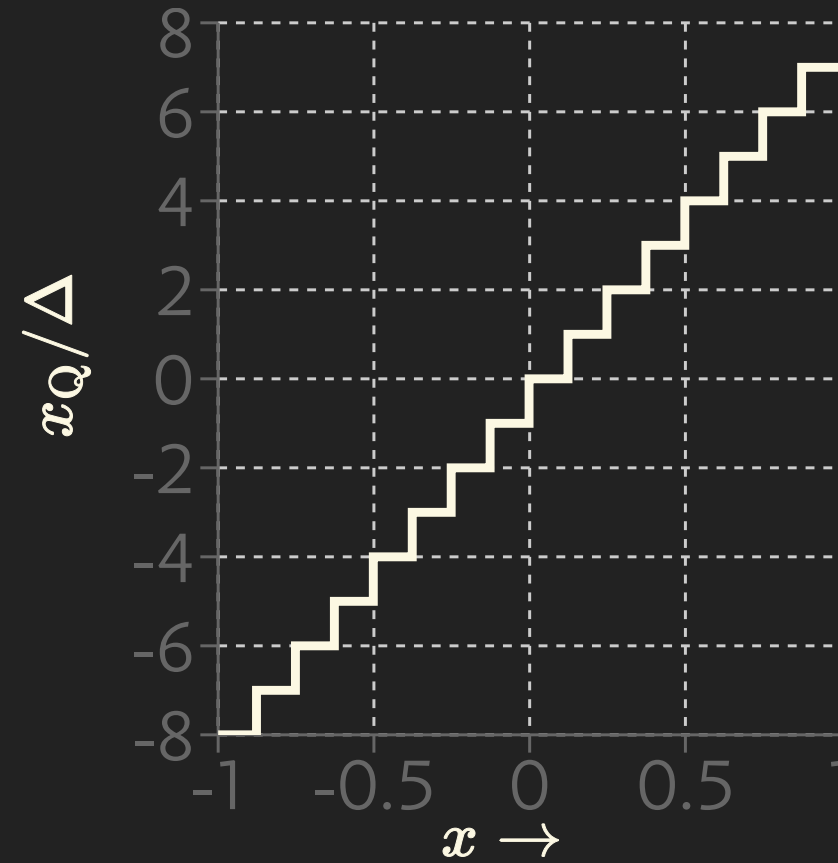
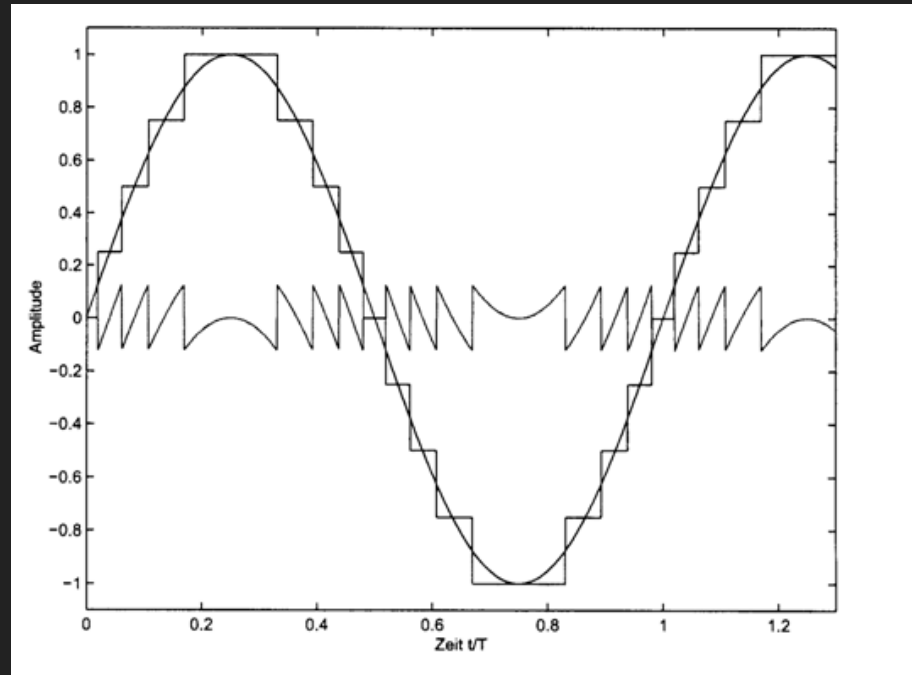
Quantization Error: Definition



$$q(i) = x_Q(i) - x(i)$$

What is the maximum amplitude of the quantization error?

Maximum Amplitude of Quantization Error

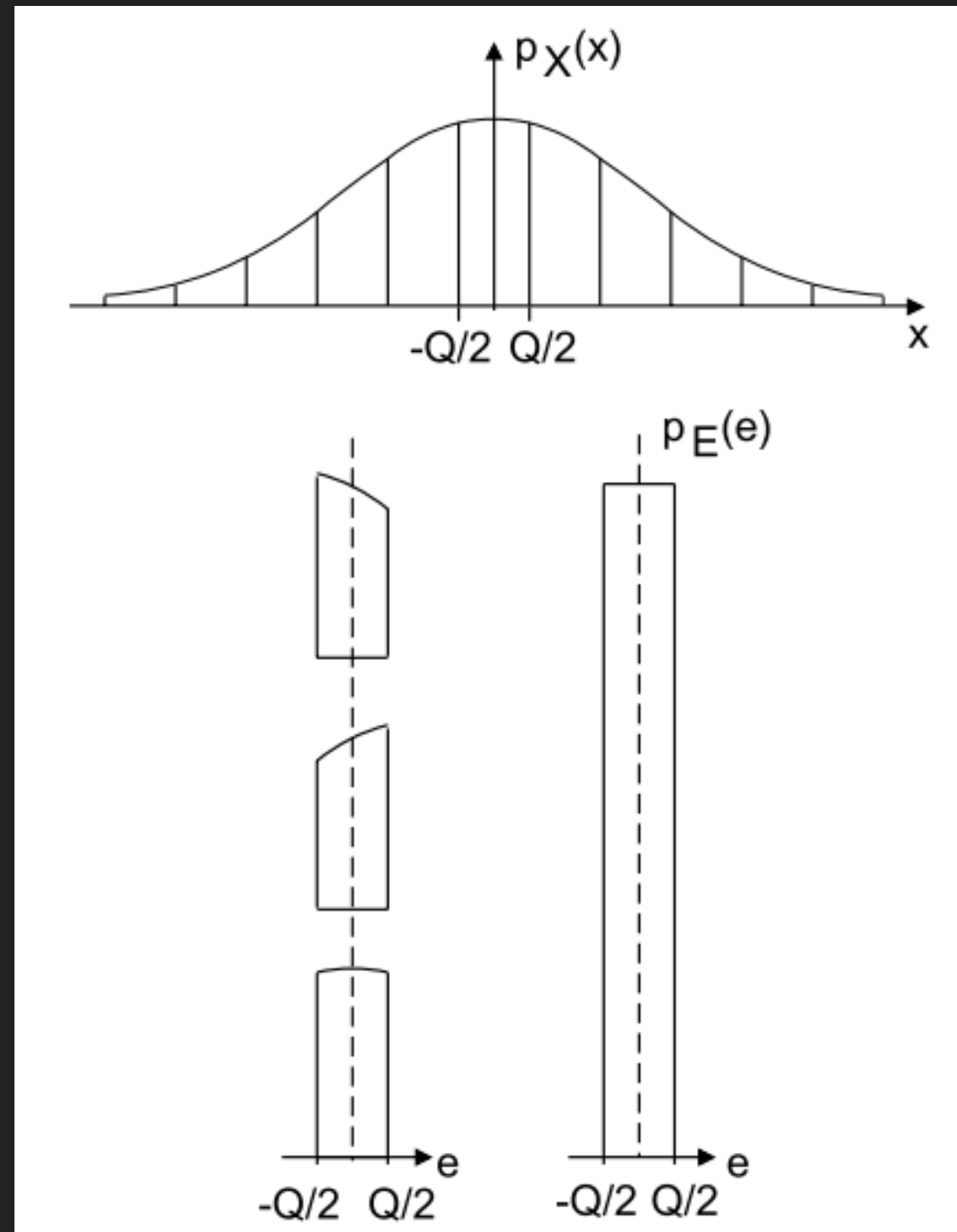


$$|q(i)| \leq \frac{\Delta}{2}$$

What is the PDF of the quantization error?

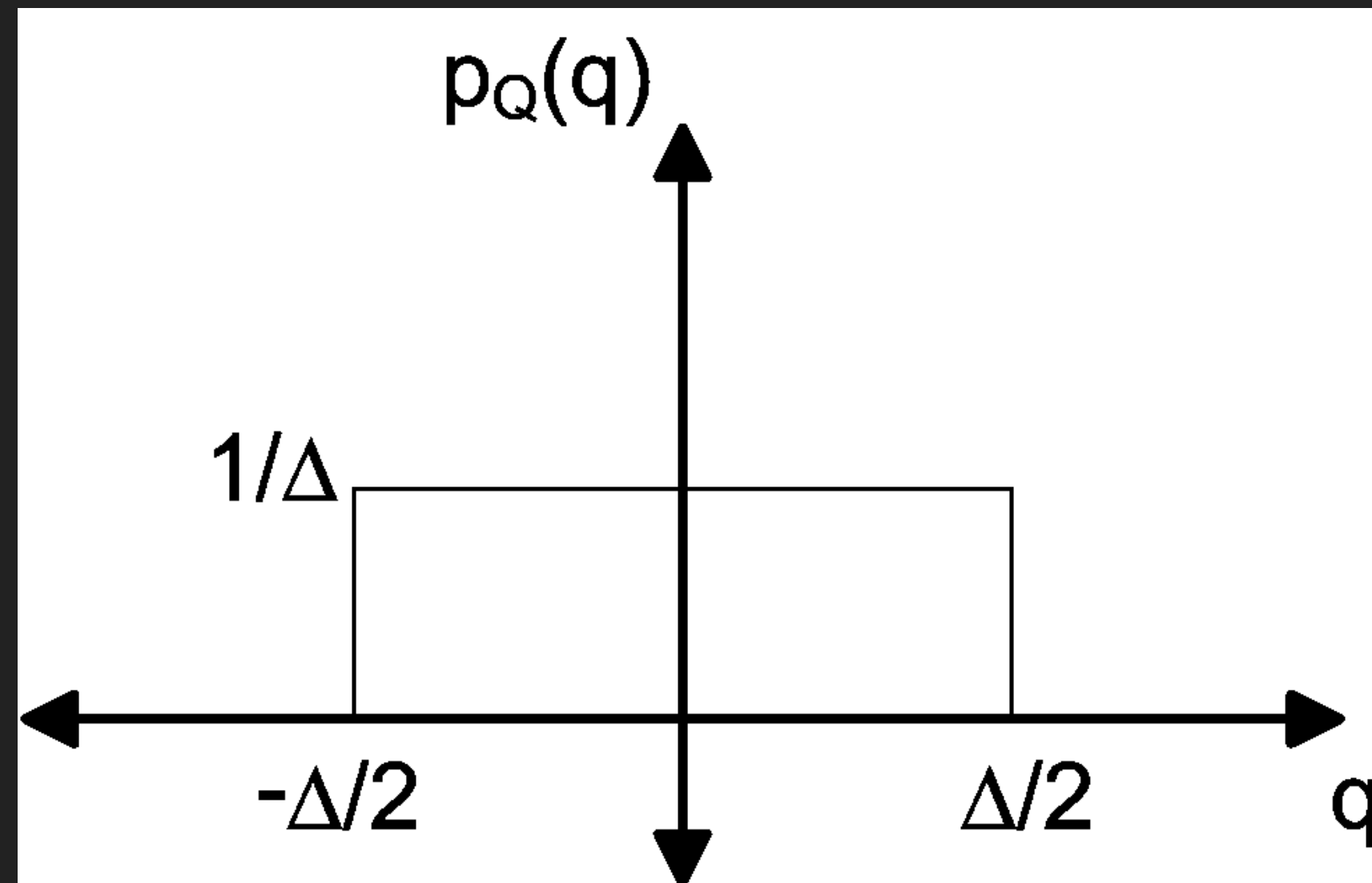
PDF of Quantization Error

Assuming $\Delta \ll \max(|x(i)|)$



PDF of Quantization Error

Assuming $\Delta \ll \max(|x(i)|)$



It can be shown that the PDF of the quantization error depends
(without derivation)

» on the **variance of the input** signal in relation to the step size

» on the **pdf of the input** signal

→ will be **uniform** for large values of $\frac{\sigma_X}{\Delta}$

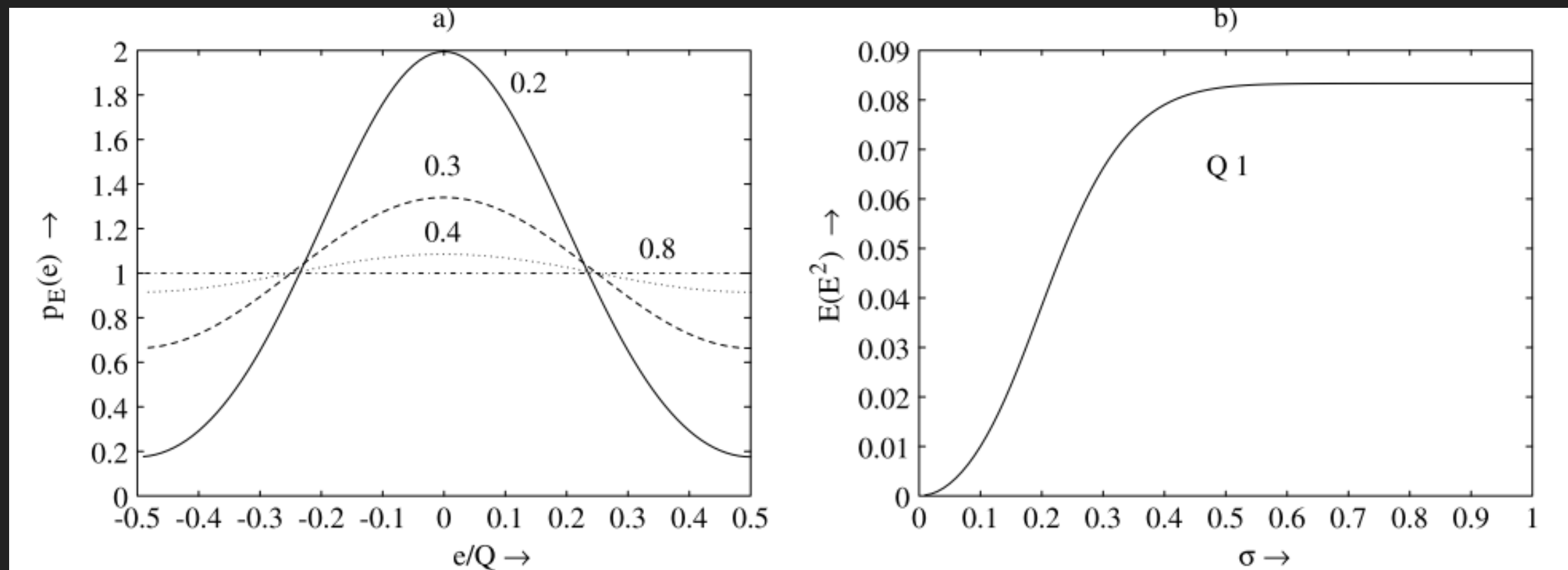


Figure 2.16 (a) PDF of quantization error for different standard deviations of a Gaussian PDF input. (b) Variance of quantization error for different standard deviations of a Gaussian PDF input.

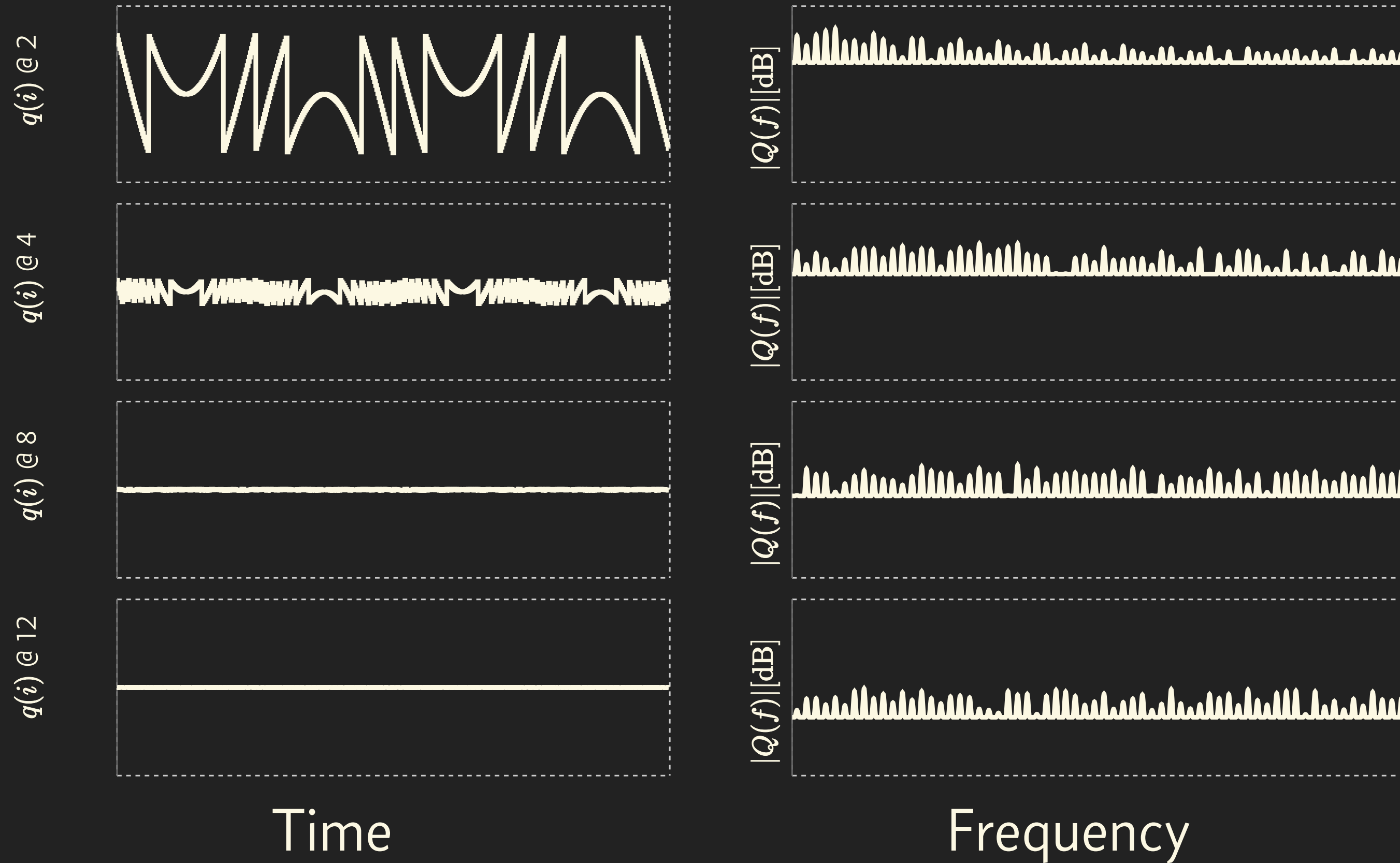
How to computer the power W_Q
of Quantization Error?

Computing power W_Q of Quantization Error

From PDF:

$$\begin{aligned} W_Q &= \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} q^2 \cdot \underbrace{p_Q(q)}_{\frac{1}{\Delta}} dq \\ &= \frac{1}{\Delta} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} q^2 dq \\ &= \frac{1}{\Delta} \left[\frac{1}{3} q^3 \right]_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \\ &= \frac{1}{3\Delta} \left(\frac{\Delta^3}{8} + \frac{\Delta^3}{8} \right) \\ &= \frac{\Delta^2}{12} \end{aligned}$$

Quantization Error of Full-Scale Sinusoidal



Audio Examples

w	$x_{Q,\text{sine}}(i)$	$q_{\text{sine}}(i)$	$x_{Q,\text{speech}}(i)$	$q_{\text{speech}}(i)$	$x_{Q,\text{music}}(i)$	$q_{\text{music}}(i)$
16						
12						
8						
6						
4						
2						

Quality Assessment of a Quantizer: Signal-to-Noise Ratio (SNR)

- » Power of the signal in relation to power of the (quantization) noise.

$$SNR' = \frac{\text{signal energy}}{\text{noise energy}} = \frac{W_S}{W_Q}$$

- » Often in decibel

$$SNR = 10 \cdot \log_{10} \left(\frac{W_S}{W_Q} \right) [\text{dB}]$$

- » SNR grows by:
 - » Reducing the noise power
 - » Increasing the signal power

Derive the SNR of quantized full-scale sinusoidal

$$SNR = 10 \cdot \log_{10} \left(\frac{W_S}{W_Q} \right) [\text{dB}]$$

$$\text{Use } \sin^2(t) = \frac{1 - \cos(2t)}{2}$$

$$W_S = \frac{A^2}{2} \xrightarrow{\text{full-scale}} W_S = \frac{(\Delta \cdot 2^{w-1})^2}{2}$$

$$W_Q = \frac{\Delta^2}{12}$$

$$\frac{W_S}{W_Q} = \frac{3}{2} \cdot 2^{2w}$$

$$SNR = w \cdot 20 \log_{10}(2) + 10 \cdot \log_{10} \left(\frac{3}{2} \right) [\text{dB}]$$

Derive the SNR of full-scale square wave

$$SNR = 10 \cdot \log_{10} \left(\frac{W_S}{W_Q} \right) [\text{dB}]$$

$$W_S = A^2 \xrightarrow{\text{full-scale}} W_S = (\Delta \cdot 2^{w-1})^2$$

$$W_Q = \frac{\Delta^2}{12}$$

$$\frac{W_S}{W_Q} = 3 \cdot 2^{2w}$$

$$SNR = w \cdot 20 \log_{10} (2) + 10 \cdot \log_{10} (3) [\text{dB}]$$

Signal-to-Noise Ratio

$$SNR = 6.02 \cdot w + c_s \quad [\text{dB}]$$

- »» Every additional bit adds ~6 dB SNR
- »» Constant C_s depends on signal (scaling and PDF shape)

SNR for different input signal examples:

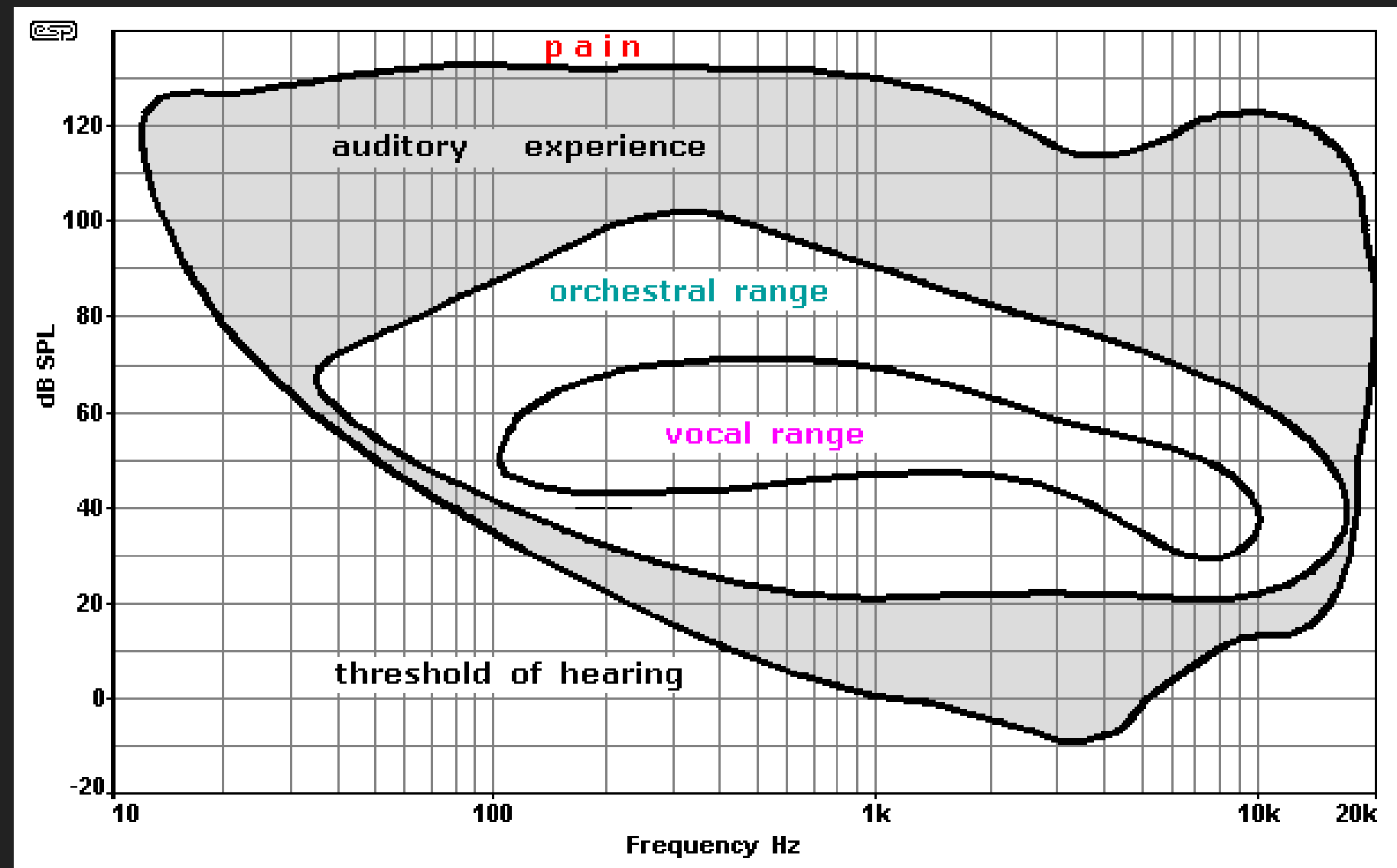
- »» Square wave (full scale): $C_s = 4.77\text{dB}$
- »» Sinusoidal wave (full scale): $C_s = 1.76\text{dB}$
- »» Rectangular PDF (full scale): $C_s = 0\text{dB}$
- »» Gaussian PDF (full scale = $4\sigma_g$): $C_s = -7.27\text{dB}$

Quantization: Word Length and SNR

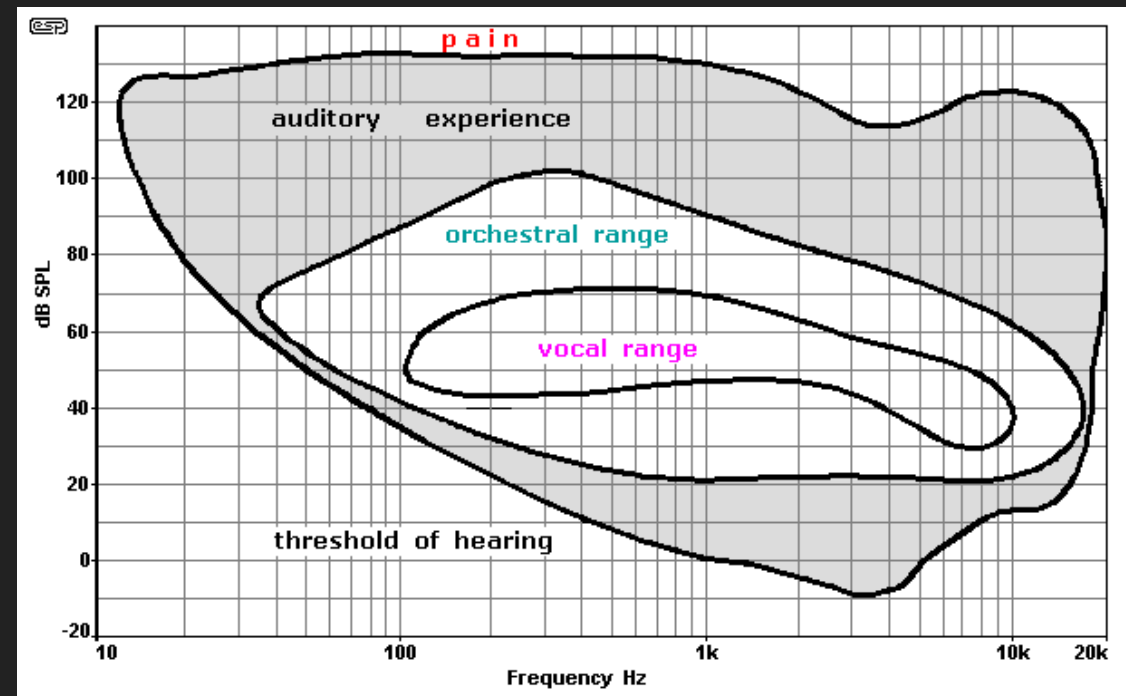
w	Δ	Max. Amp	theo. SNR
8 (Int)	± 1	0 ... 255	≈ 48 dB
16 (Int)	± 1	-32768 ... 32767	≈ 96 dB
20 (Int)	± 1	-524288 ... 524287	≈ 120 dB
24 (Int)	± 1	-16777216 ... 16777215	≈ 144 dB
32 (Float)	$\pm 1.175 \cdot 10^{-38}$	$\pm 3.403 \cdot 10^{1038}$	1529 dB
64 (Float)	$\pm 2.225 \cdot 10^{-308}$	$\pm 1.798 \cdot 10^{10308}$	12318 dB

SNR and Auditory Sensation Area

How many bits do we need?

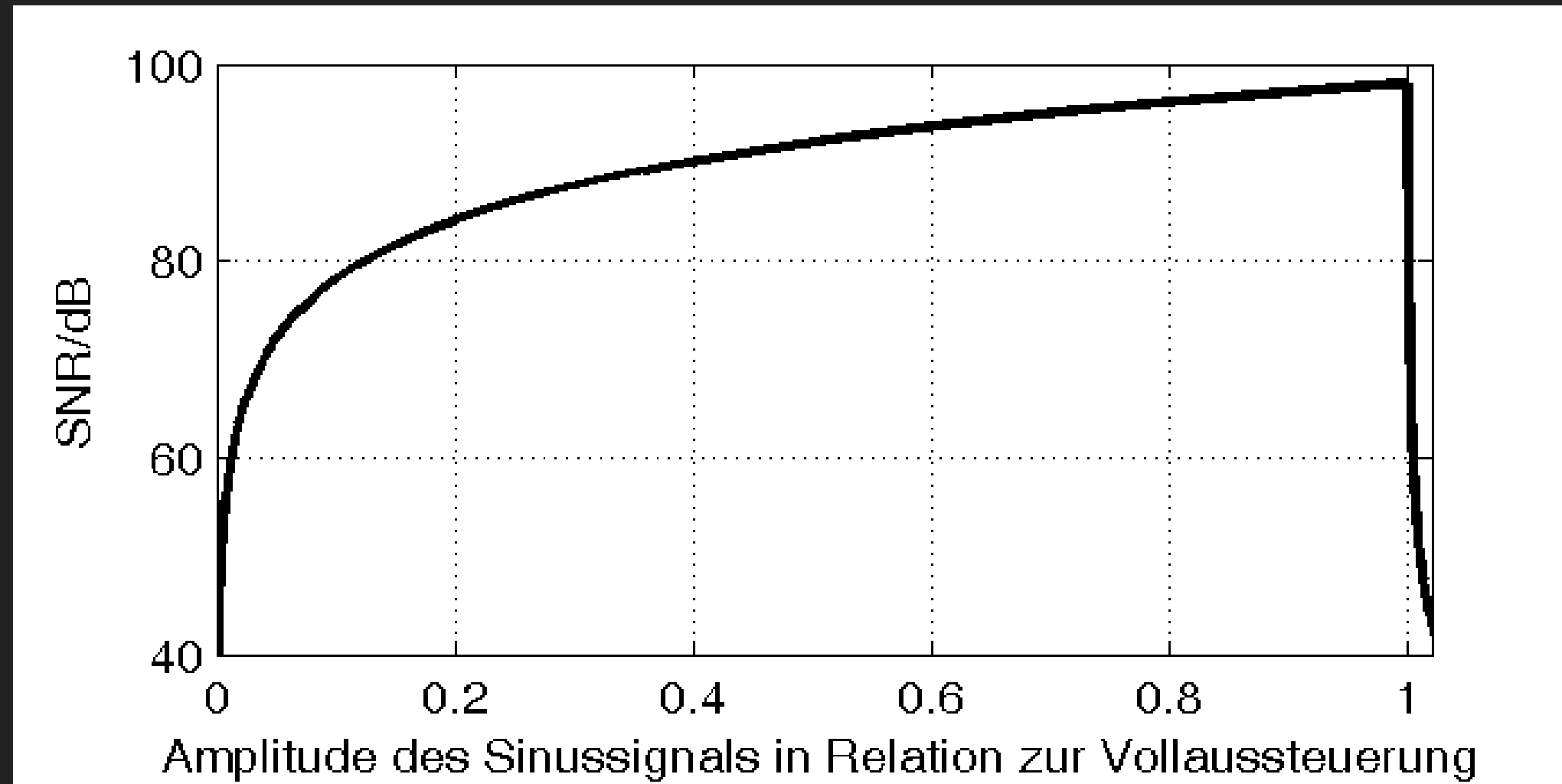


How many bits do we need?



- » To cover the whole range of hearing: 20-24 bit
- » Practically, a lower range is sufficient as the dynamic range of recordings has to be much lower
- » In production with many processing and possible requantization steps, high resolution (if possible floating point) is recommended

SNR and Signal Scaling



Full Scale:

- » Absolute maximum before clipping
- » Usually 1 (in floating point systems)
- » Marks 0 dbFS

- » Quantization is **non-linear & irreversible**
 - » Information is lost
 - » Error is introduced
- » Quantization **error**
 - » Power is determined by number of bits (word length)
 - » Is approximately white noise (flat spectrum and uncorrelated to signal) when the signal power is much higher than the quantization step size
 - » Special severe case: clipping
- » **SNR** is used to assess quantizer quality
 - » Depends on both signal power and quant error power (ratio)
 - » Each additional bit gains 6 dB SNR
 - » Different signals with identical maximum amplitude yield different SNRs
- » **Typical word lengths** include
 - » 8 bit: Phone
 - » 16 bit: Consumer audio
 - » 24 bit and higher: Production audio