

Digital Signal Processing for Music

Part 14: Digital Filters I

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Filter - Broad Description

System that amplifies or attenuates certain components/aspects of a signal

Filter - Narrow

Linear time-invariant system for changing the magnitude and phase of specific frequency regions

- » Example for other type of filters:
 - » Adaptive and time-variant (e.g., denoising)
- » Examples for "real-world" filters:
 - » Reverberation
 - » Absorption
 - » Echo

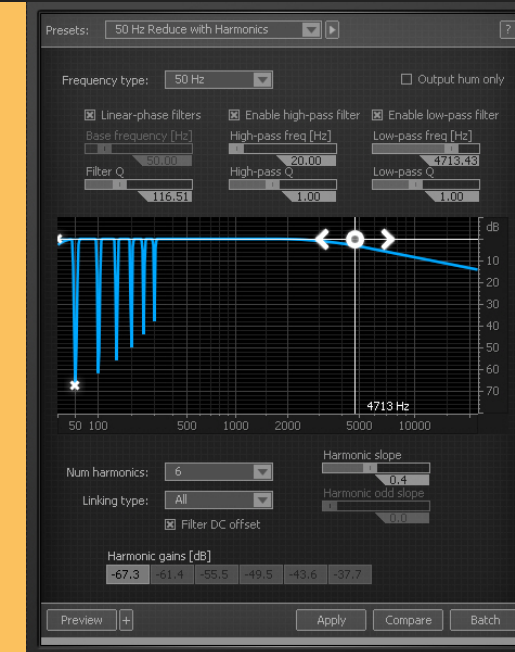
Audio Equalization

- » Parametric EQs
- » Graphic EQs



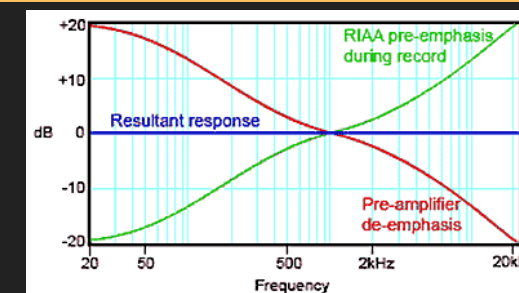
Removal of Unwanted Components

- » Remove DC, rumble
- » Remove hum
- » Remove hiss



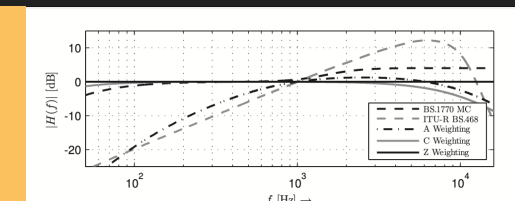
Pre-emphasis / De-emphasis

- » Vinyl
- » Old Dolby noise reduction systems



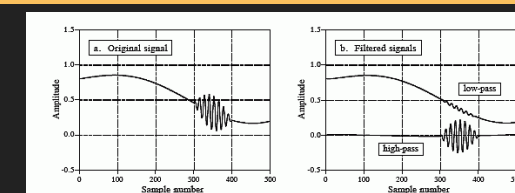
Weighting Function

- » dBA, dBC, ...



(Parameter) Smoothing

- » Smooth sudden changes



Reminder: System Theory

- » Output of a system (filter) y computed by **convolution** of input x and impulse response h

$$y(t) = x(t) * h(t)$$

- » This is equivalent to a frequency domain multiplication

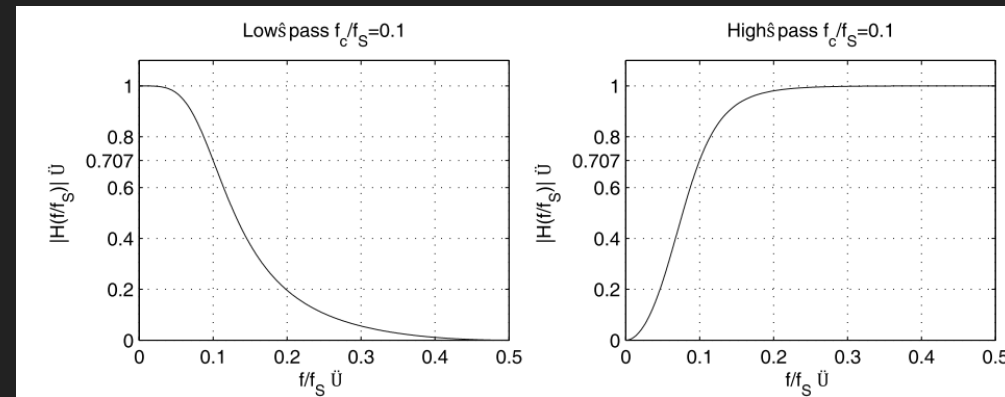
$$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

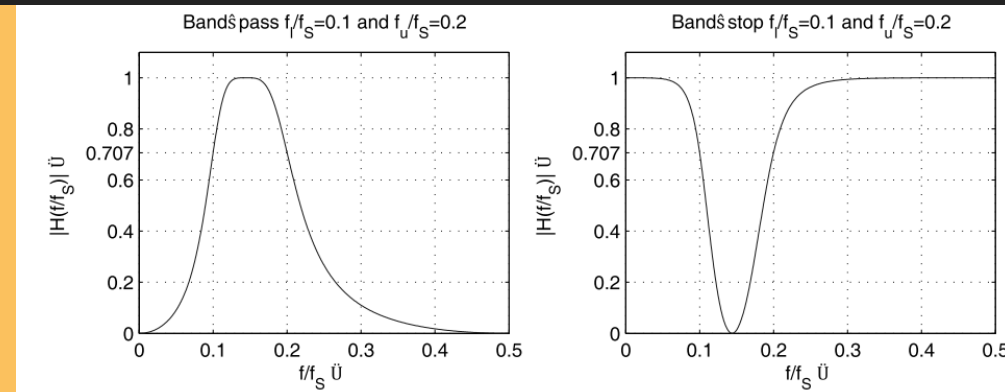
- » **Transfer function** $H(j\omega)$ is complex, often represented as:
 - » **Magnitude** $|H(j\omega)|$
 - » **Phase** $\Phi_H(j\omega)$

Common Transfer Function Shapes

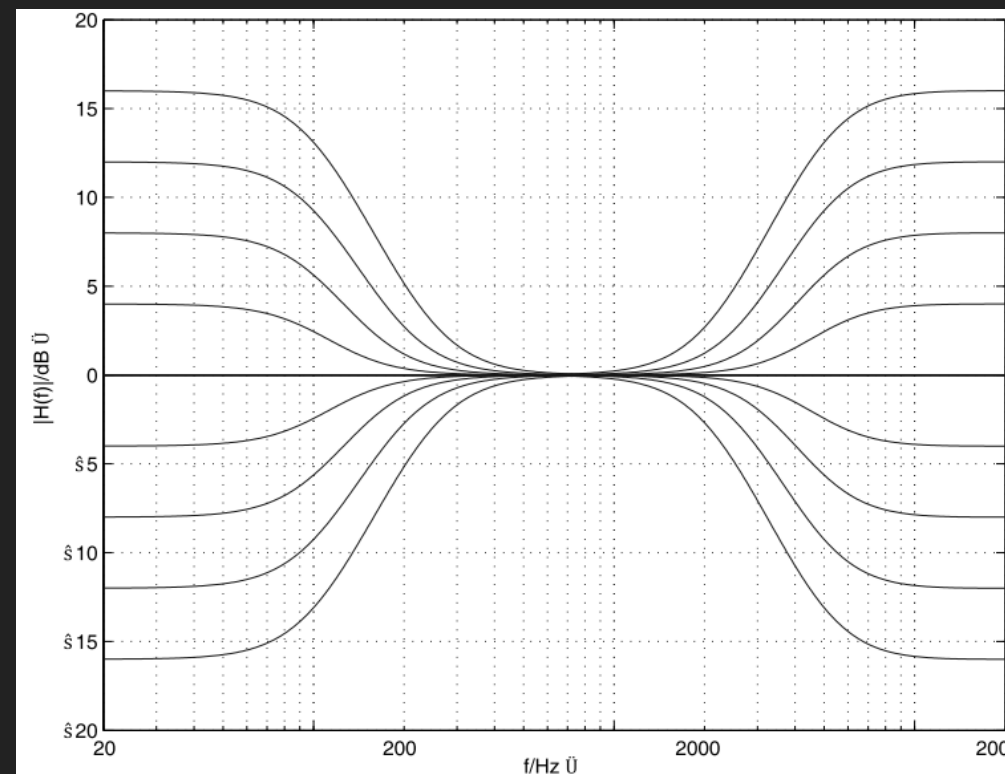
Low/high pass filters



Band pass/band stop filters

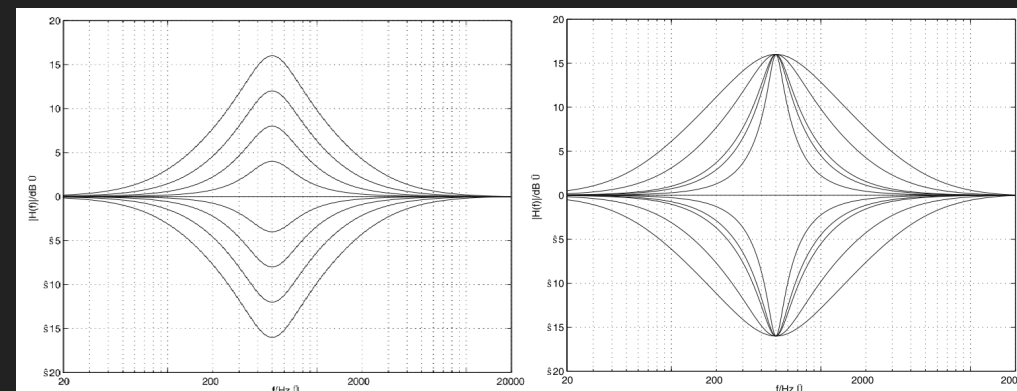


Low/high shelving filters

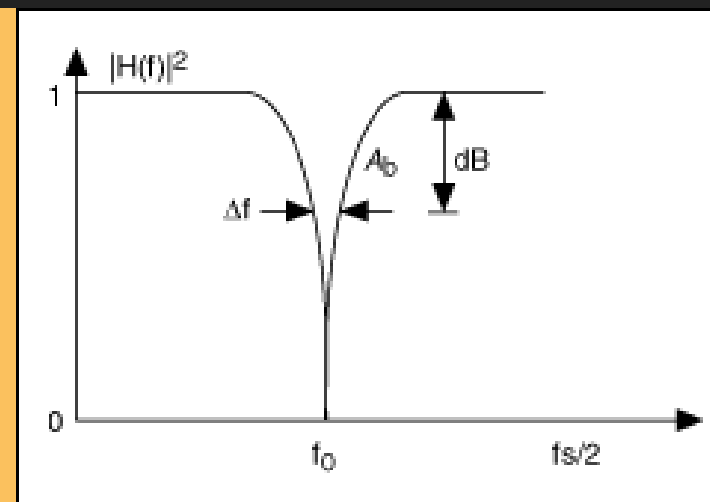


Common Transfer Function Shapes

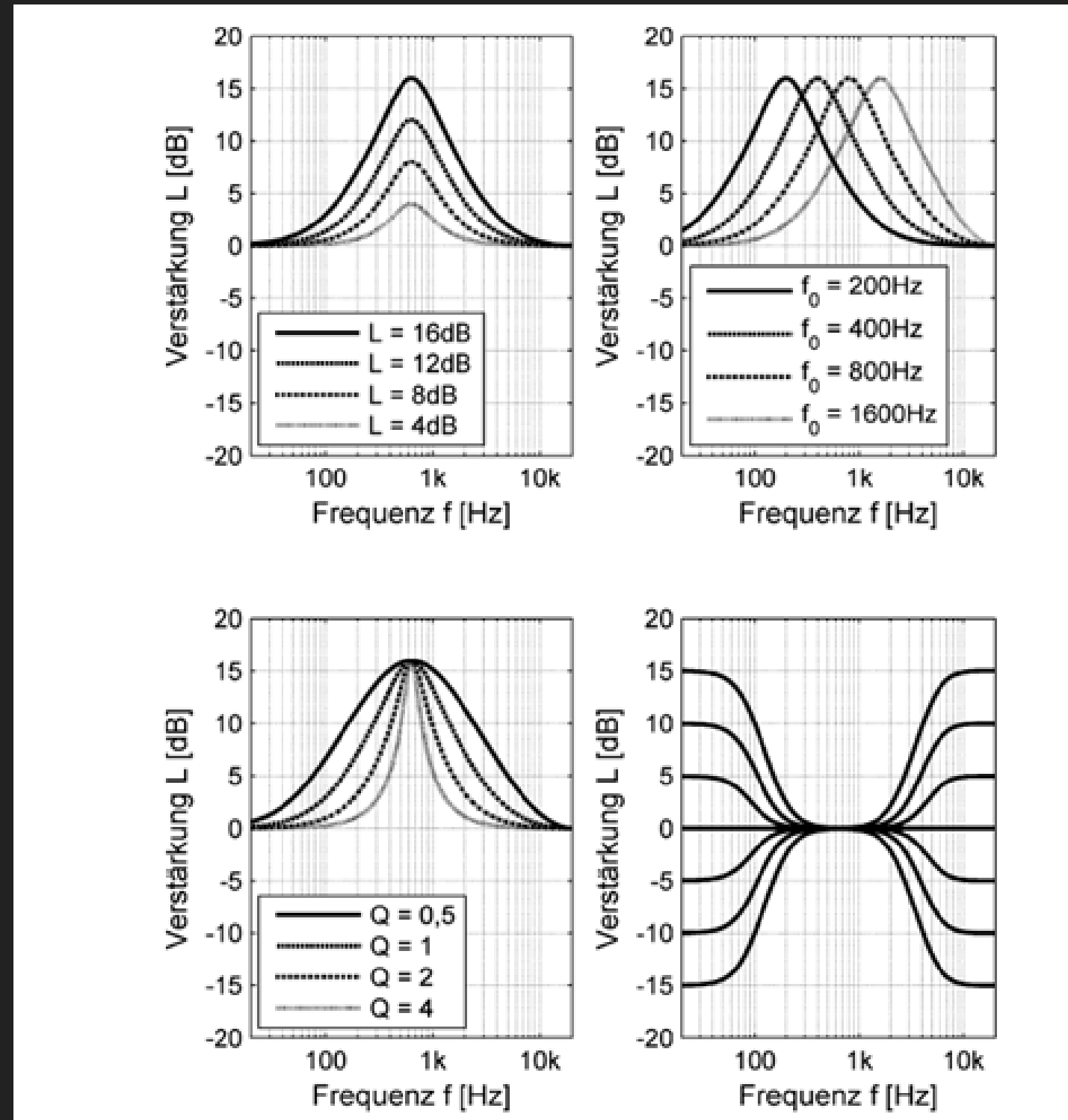
Peak filters



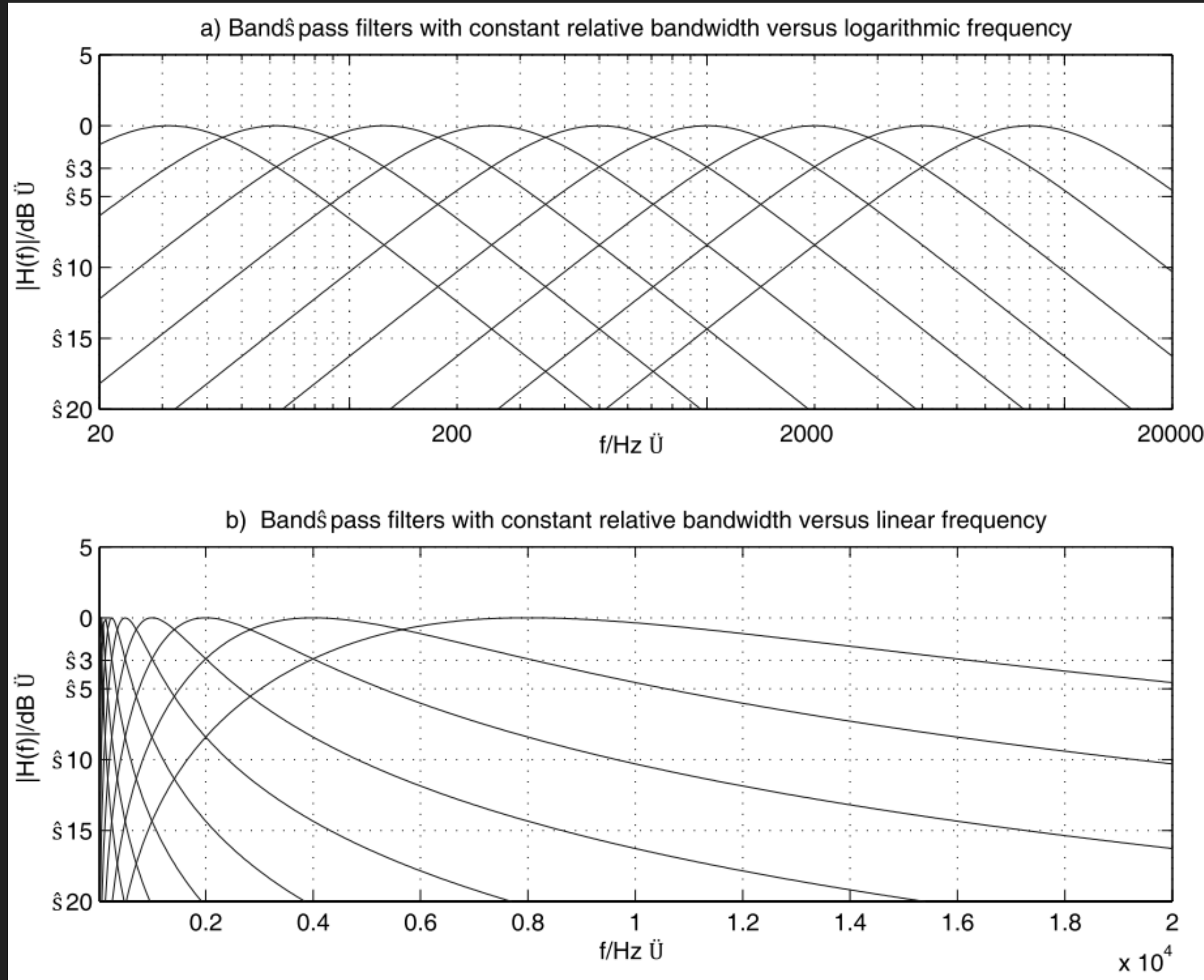
Resonance/notch filters



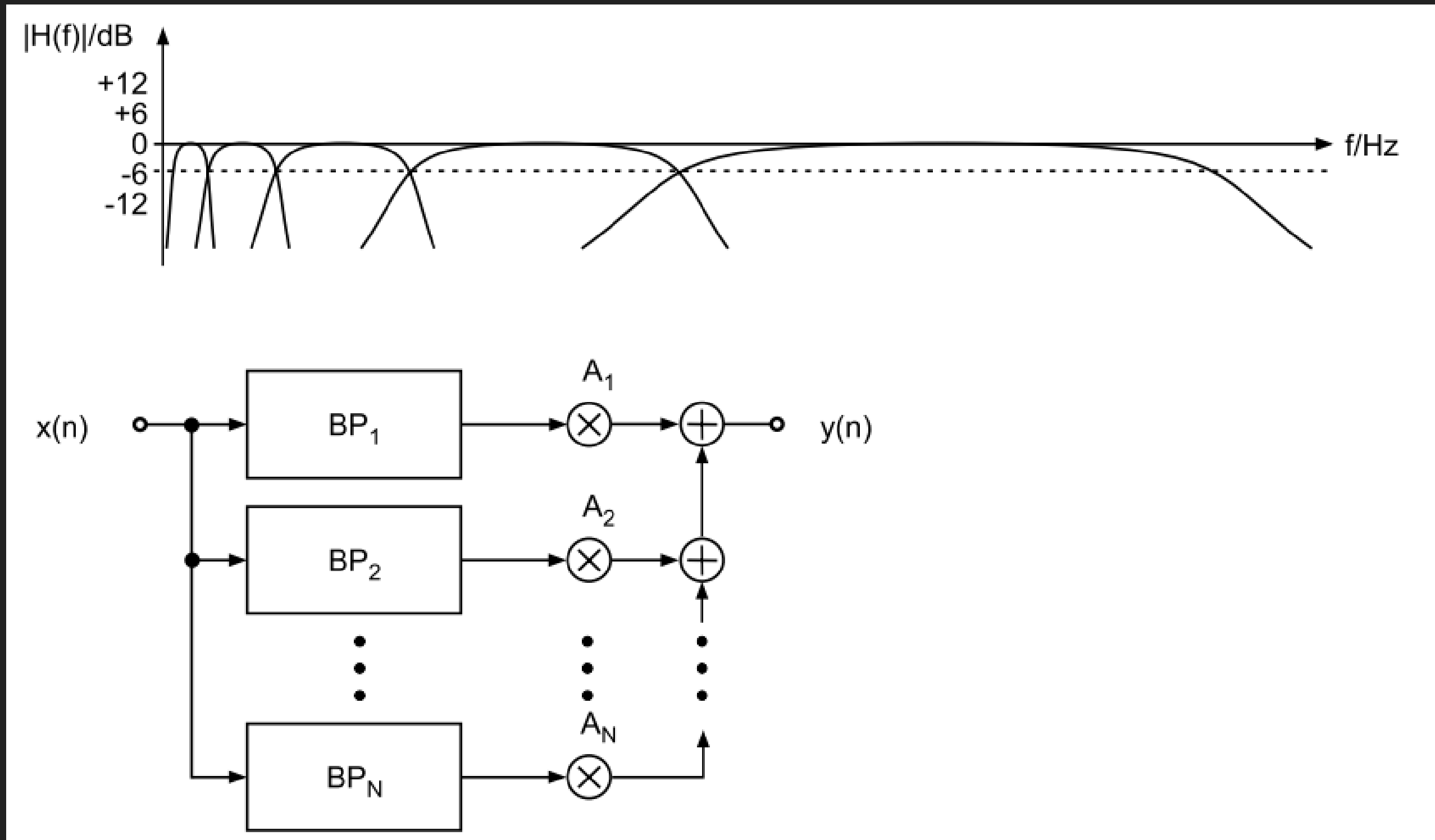
Common Transfer Function Shapes



Filter Banks

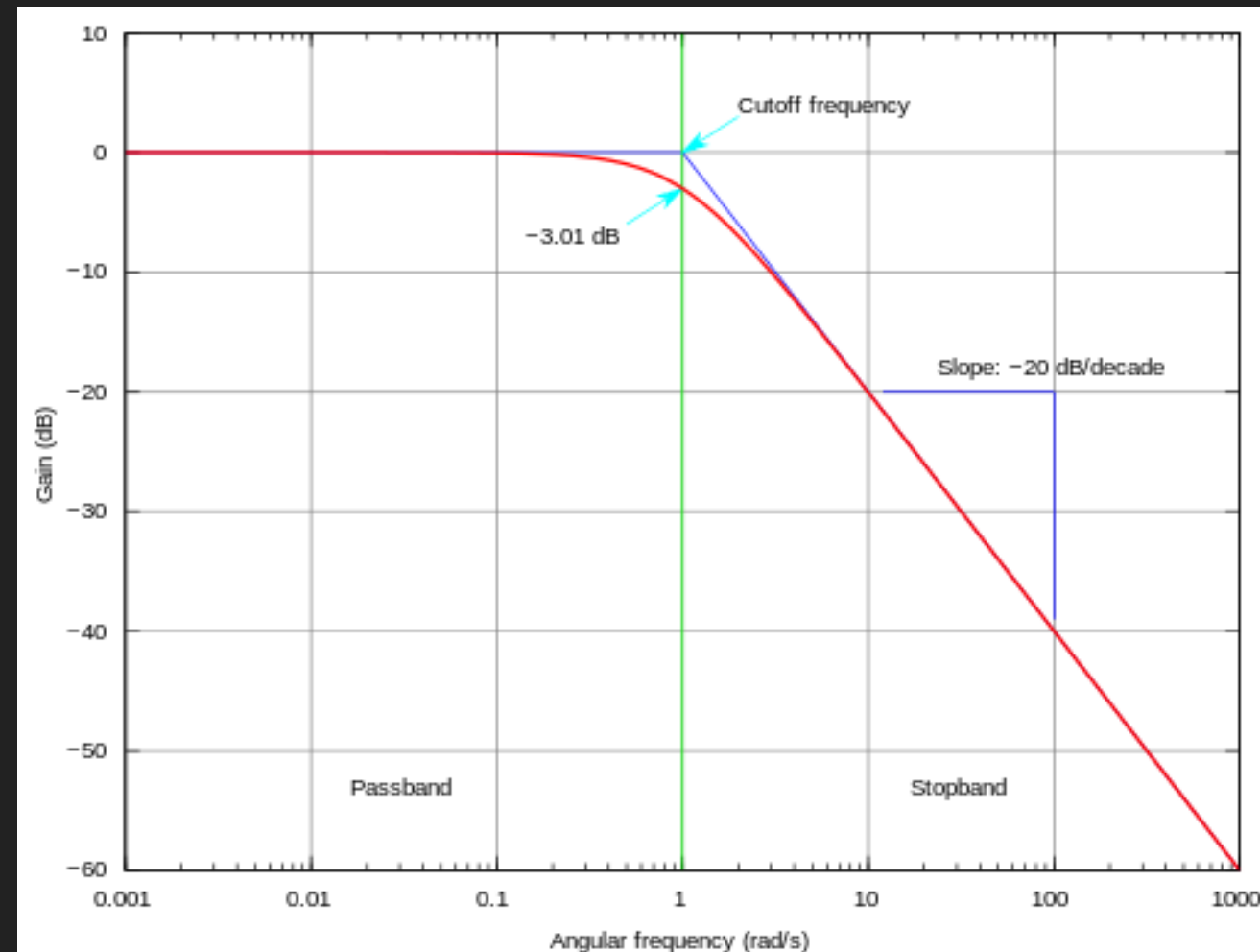


Filter Banks



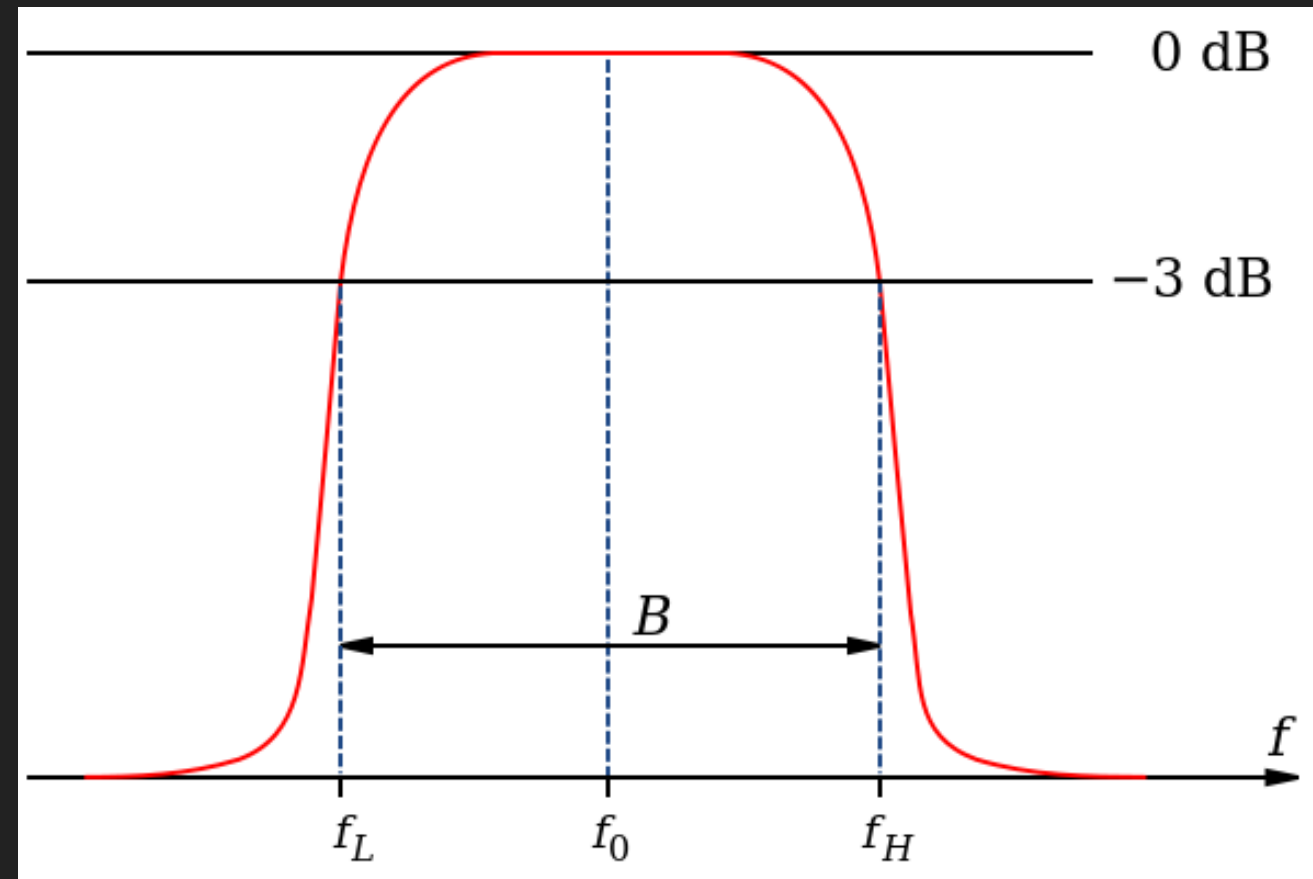
Filter Parameters - Lowpass/Highpass

- » **Cut-off frequency f_c**
 - » Frequency marking the transition of pass to stop band
 - » -3dB of pass band level
- » **Slope/steepness**
 - » Measured in dB/octave or dB/decade
 - » Typically directly related to filter order
- » Sometimes: **resonance**
 - » Level increase in narrow band around cut-off frequency



Filter Parameters - Bandpass/Bandstop

- » **Center frequency f_c**
 - » Frequency marking the center of the pass or stop band
- » **Bandwidth ΔB**
 - » Width of the pass band
 - » at -3dB of max pass band level
- » Possibly: **slope**
 - » Typically directly related to filter order

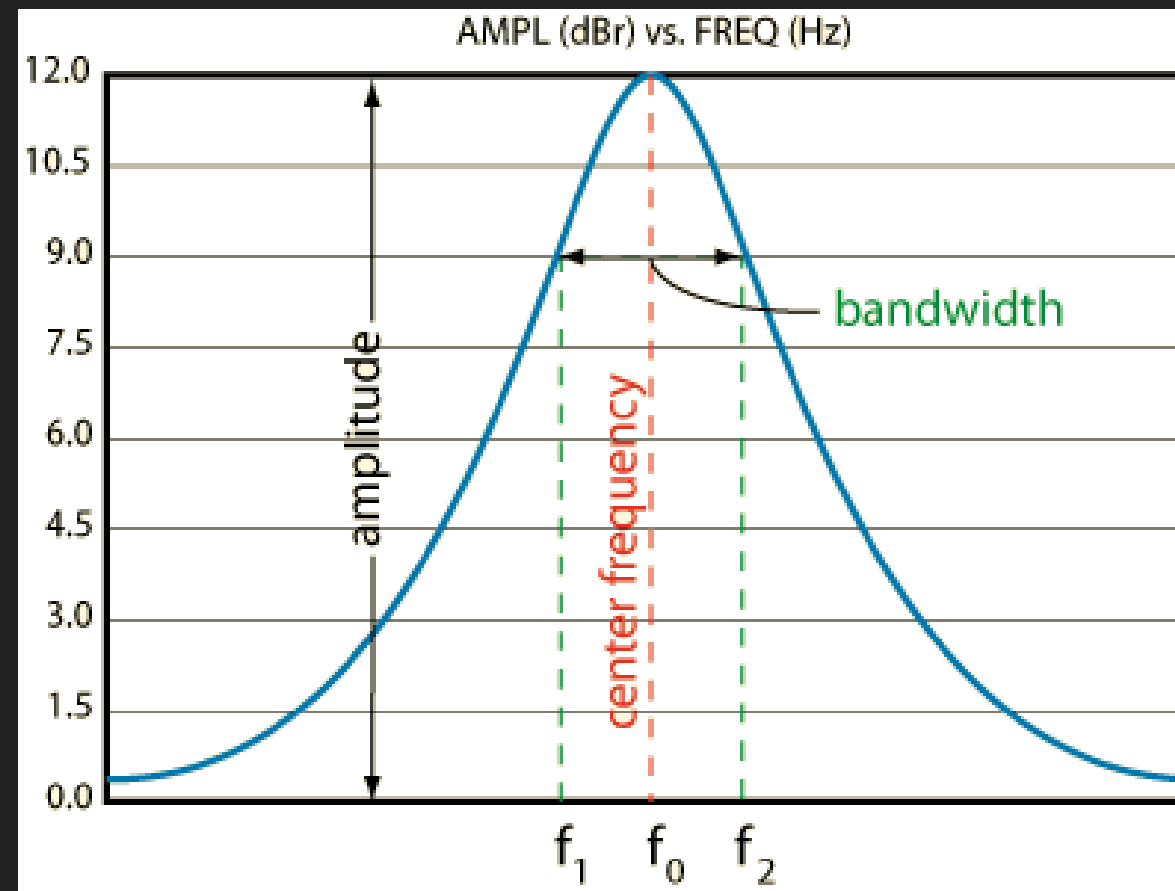


Filter Parameters - Peak

- » **Center frequency f_c**
 - » Frequency marking the center of the peak
- » **Q factor or bandwidth ΔB**
 - » Width of the bell
 - » at -3dB of max gain

$$Q = \frac{f_c}{\Delta B}$$

- » **Gain**
 - » Amplification / attenuation in dB



Filter Parameters - Overview

<i>Parameter</i>	Lowpass	Low Shelving	Band Pass	Peak	Resonance
<i>Frequency</i>	Cut-off	Cut-off	Center	Center	Center
<i>Bandwidth/Q</i>	Res. gain	--	ΔB	Q	--
<i>Gain</i>	--	Yes	--	Yes	--

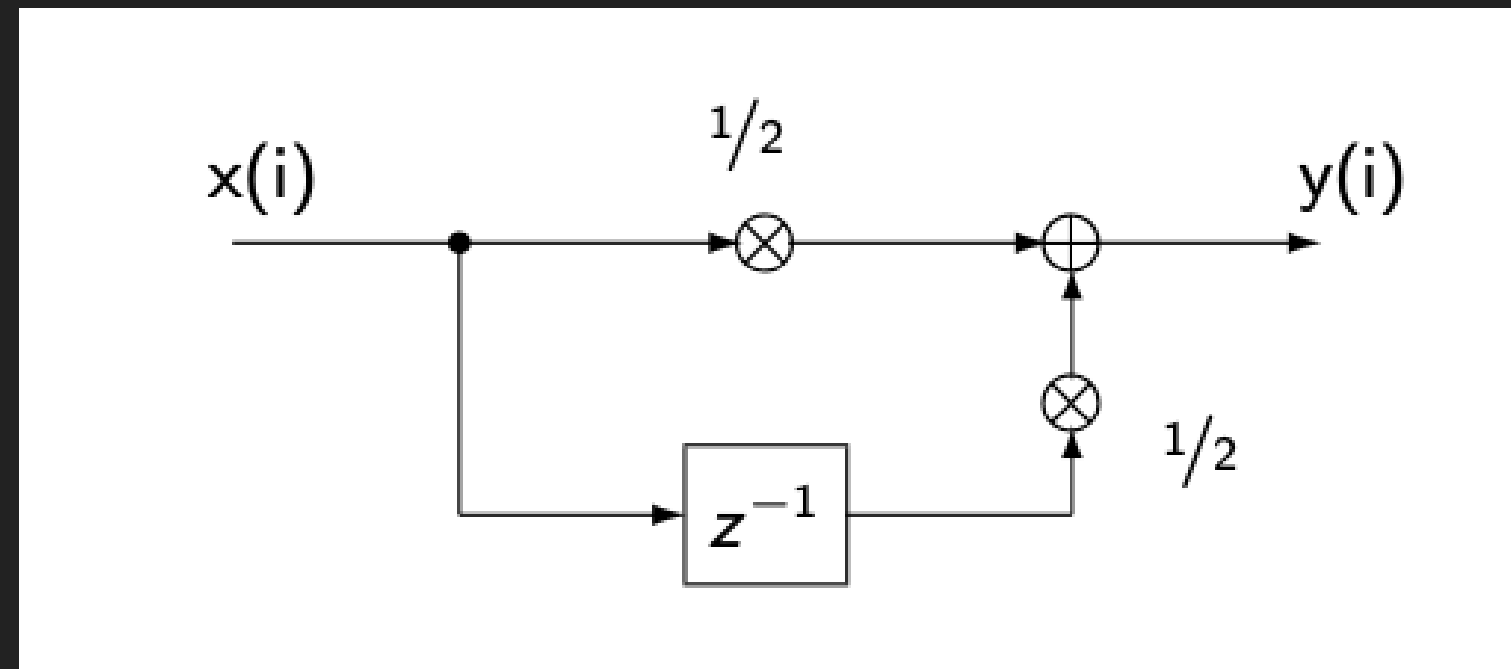
Digital Filter Description

A filter is defined by its

- » **Complex transfer function $H(j\omega)$**
- » Or ... its **Impulse response $h(t)$**
- » Or ... its **List of pole and zero positions in the Z-Plane**

$$H(j\omega) = \mathcal{F}\{h(t)\}$$

Example Filter 1



$$y(i) = 0.5 \cdot x(i) + 0.5 \cdot x(i - 1)$$

Example Filter 1: Transfer Function

$$y(i) = 0.5 \cdot x(i) + 0.5 \cdot x(i - 1)$$

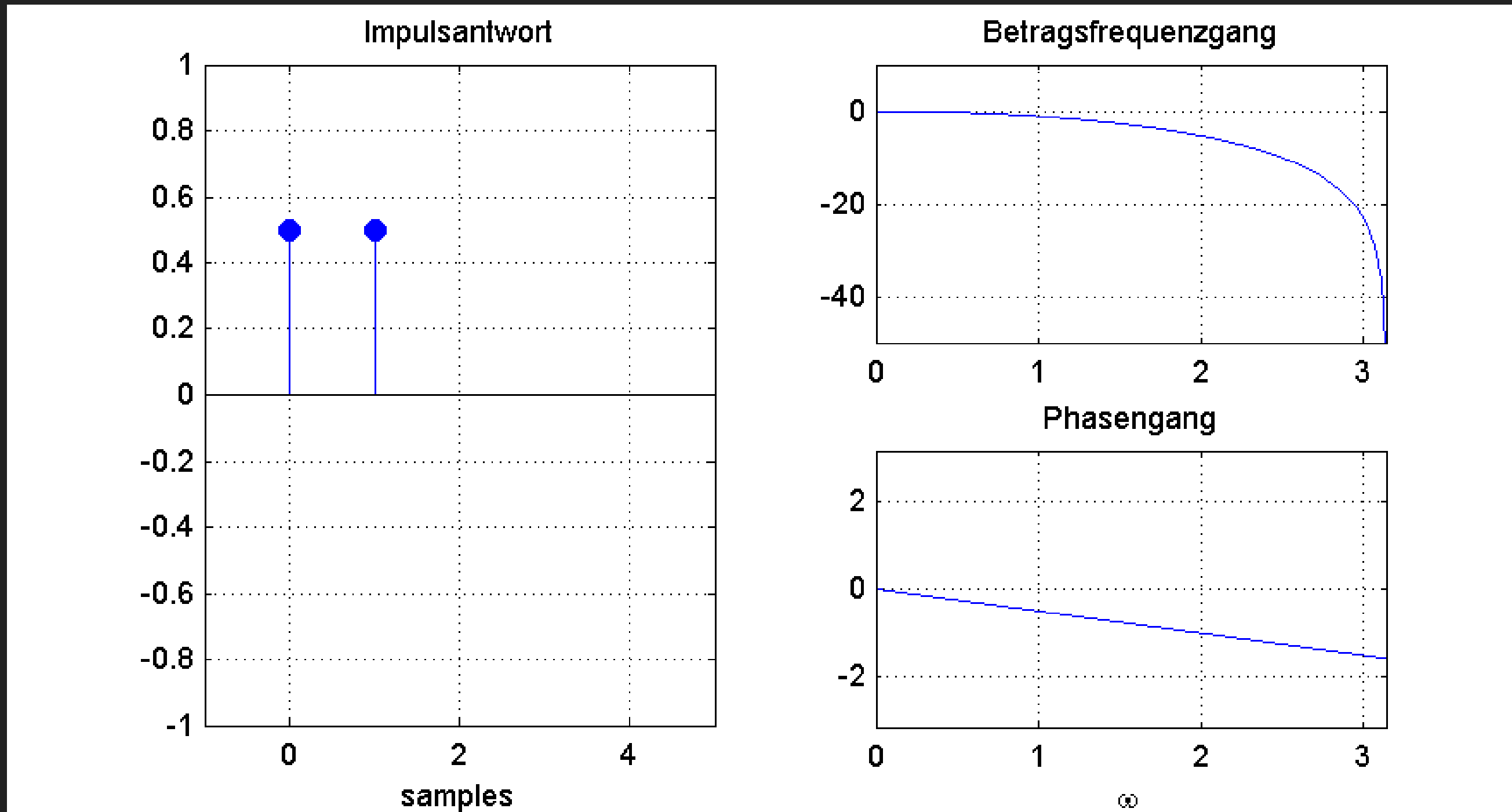
$$H(z) = 0.5 + 0.5 \cdot z^{-1}$$

$$H(j\omega) = 0.5 + 0.5 \cdot e^{-j\omega}$$

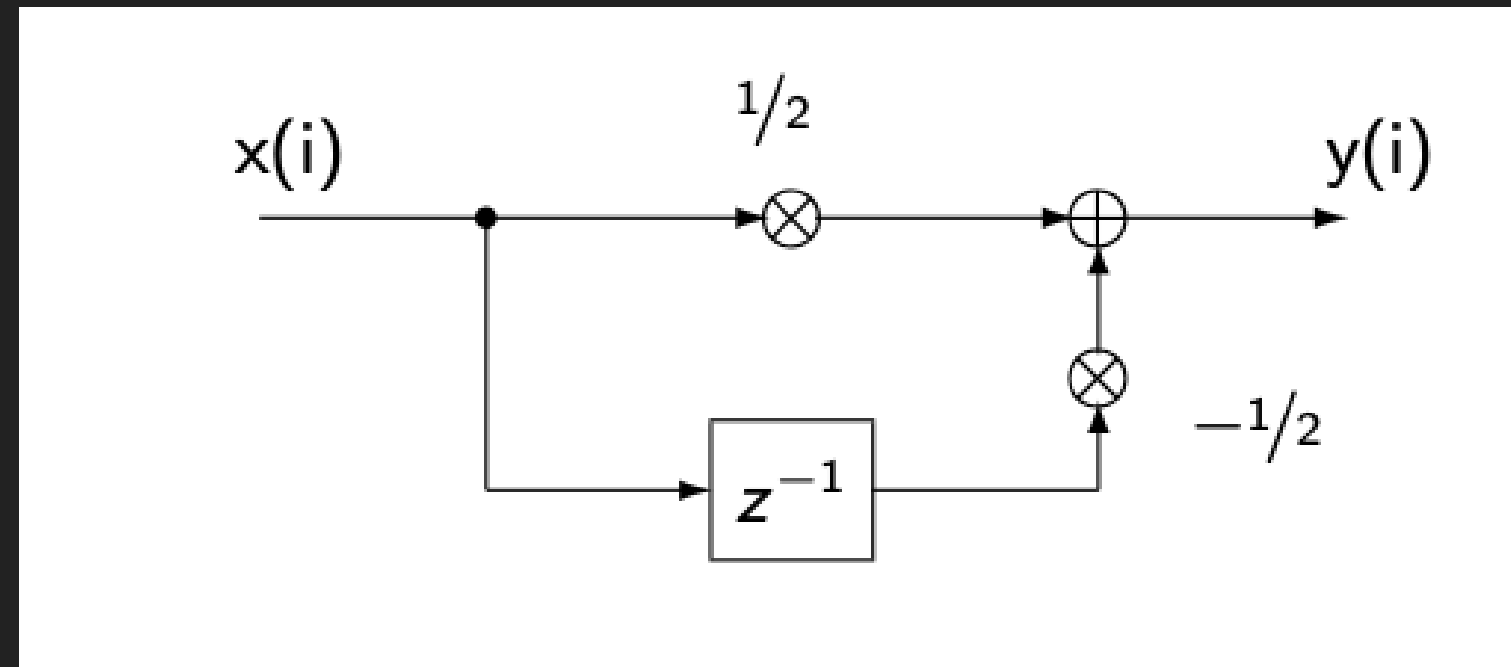
$$\begin{aligned} |H(j\omega)| &= 0.5 \cdot \left| e^{-j\frac{\omega}{2}} \cdot \left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}} \right) \right| \\ &= 0.5 \cdot \underbrace{\left| e^{-j\frac{\omega}{2}} \right|}_1 \cdot \underbrace{\left| \left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}} \right) \right|}_{\left| 2 \cos\left(\frac{\omega}{2}\right) \right|} \end{aligned}$$

$$= \left| \cos\left(\frac{\omega}{2}\right) \right|$$

Example Filter 1: Visualization



Example Filter 2

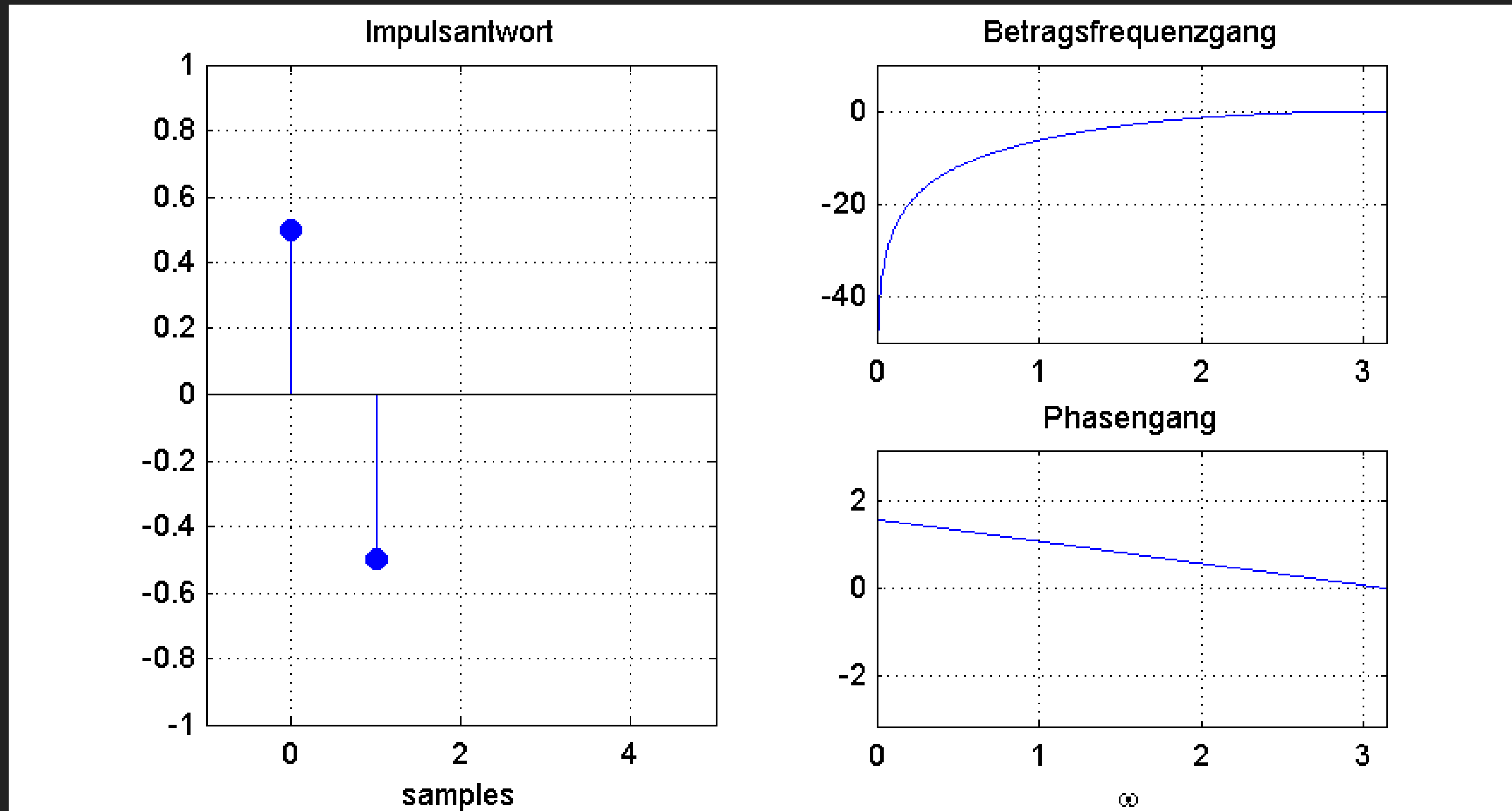


$$y(i) = 0.5 \cdot x(i) - 0.5 \cdot x(i - 1)$$

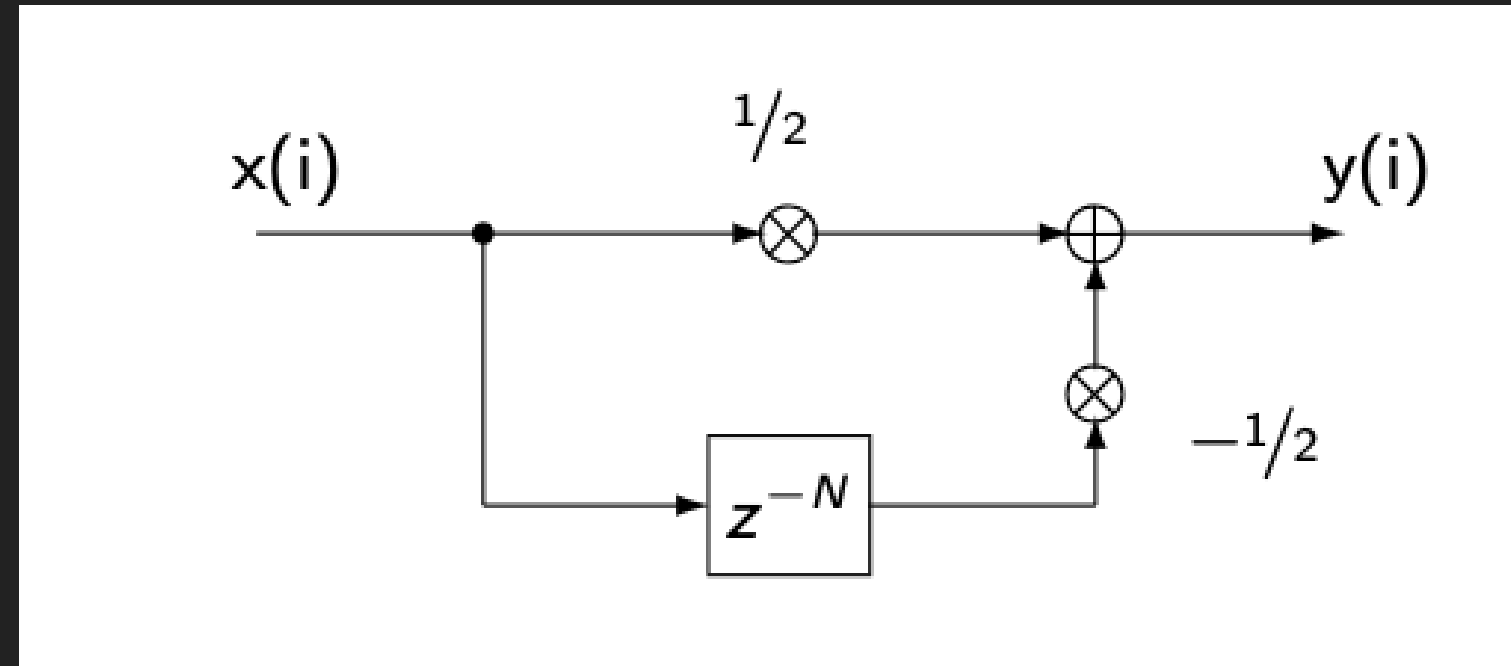
$$H(z) = 0.5 - 0.5 \cdot z^{-1}$$

$$|H(j\omega)| = \left| \sin\left(\frac{\omega}{2}\right) \right|$$

Example Filter 2: Visualization



Example Filter 3



$$y(i) = 0.5 \cdot x(i) - 0.5 \cdot x(i - N)$$

$$H(z) = 0.5 - 0.5 \cdot z^{-N}$$

$$|H(j\omega)| = 0.5 \cdot \left| e^{-j\frac{N\omega}{2}} \cdot \left(e^{j\frac{N\omega}{2}} - e^{-j\frac{N\omega}{2}} \right) \right|$$

$$= \left| \sin \left(\frac{N\omega}{2} \right) \right|$$

Example Filter 3: Visualization

