Digital Signal Processing for Music

Part 14: Digital Filters I

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Filter - Broad Description

System that amplifies or attenuates certain components/aspects of a signal

Filter - Narrow

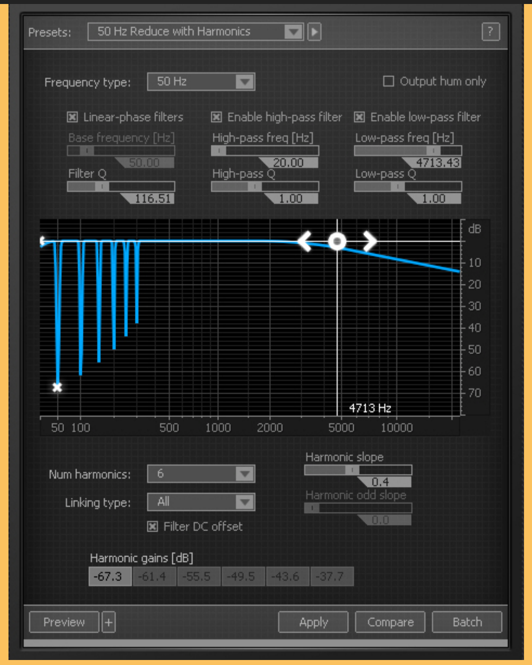
Linear time-invariant system for changing the magnitude and phase of specific frequency regions

- Example for other type of filters:
  - Adaptive and time-variant (e.g., denoising)
- Examples for "real-world" filters:
  - Reverberation
  - Absorption
  - Echo

Audio Equalization

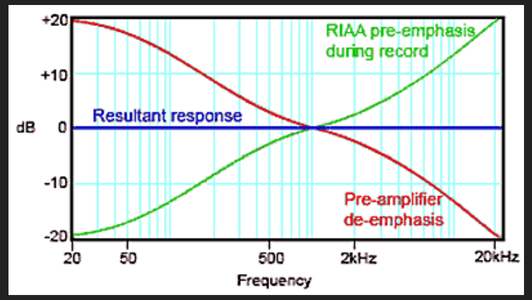
- Parametric EQs
- Graphic EQs

Removal of Unwanted Components



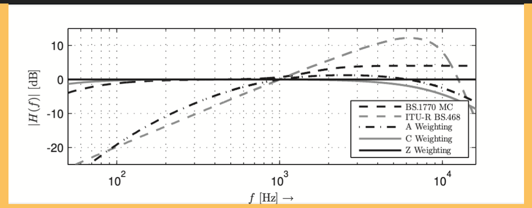
- Remove DC, rumble
- Remove hum
- Remove hiss

Pre-emphasis / De-emphasis



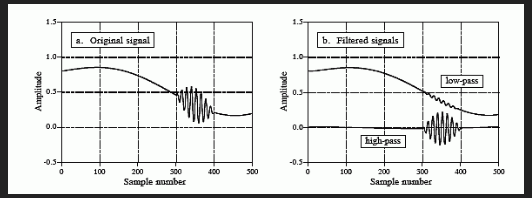
- Vinyl
- Old Dolby noise reduction systems

Weighting Function



- dBA, dBC, ...

(Parameter) **Smoothing**



- Smooth sudden changes

Reminder: System Theory

- Output of a system (filter) y computed by **convolution** of input x and impulse response h

$$y(t) = x(t) * h(t)$$

- This is equivalent to a frequency domain multiplication

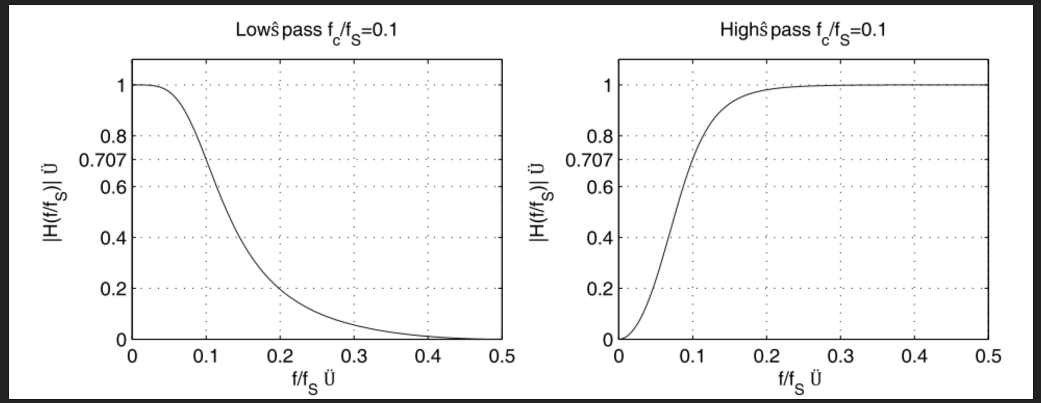
$$Y(\mathrm{j}\omega) = X(\mathrm{j}\omega) \cdot H(\mathrm{j}\omega)$$

$$H(\mathrm{j}\omega) = \frac{Y(\mathrm{j}\omega)}{X(\mathrm{j}\omega)}$$

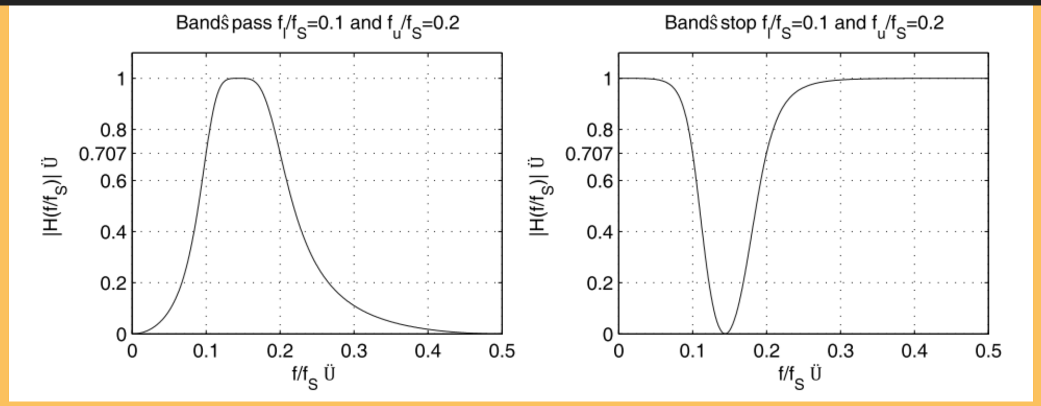
- **Transfer function** $H(\mathrm{j}\omega)$ is complex, often represented as:
  - **Magnitude** $|H(\mathrm{j}\omega)|$
  - **Phase** $\Phi_H(\mathrm{j}\omega)$

Common Transfer Function Shapes

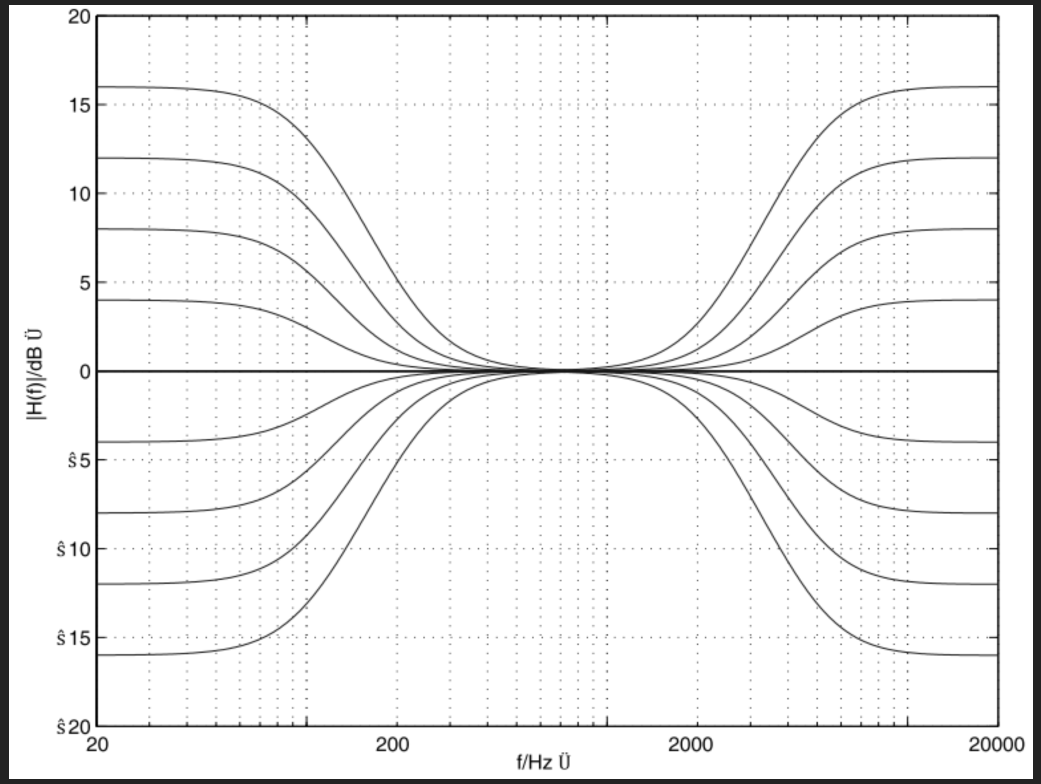
Low/high pass filters



Band pass/band stop filters

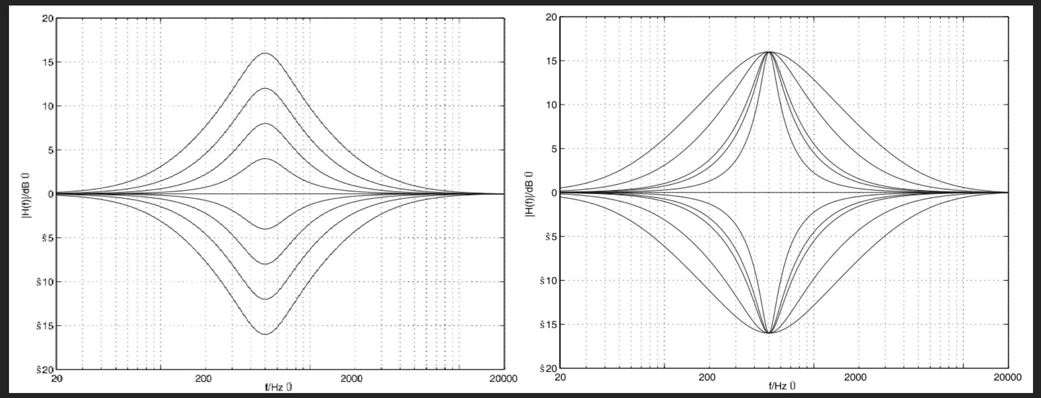


Low/high shelving filters

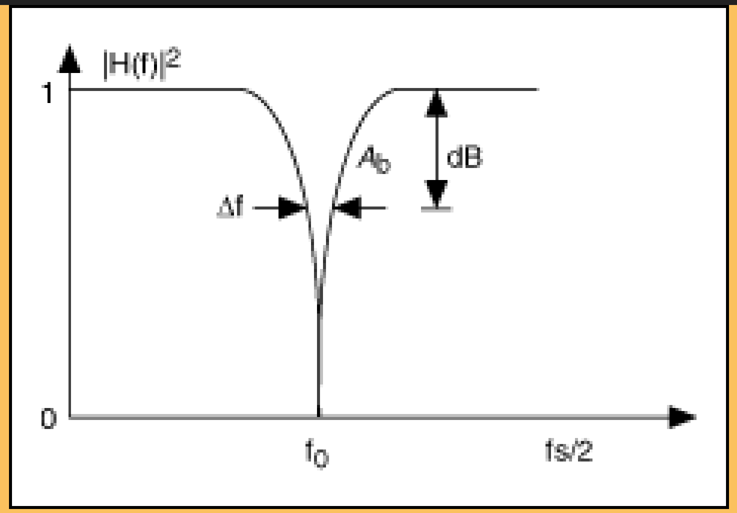


Common Transfer Function Shapes

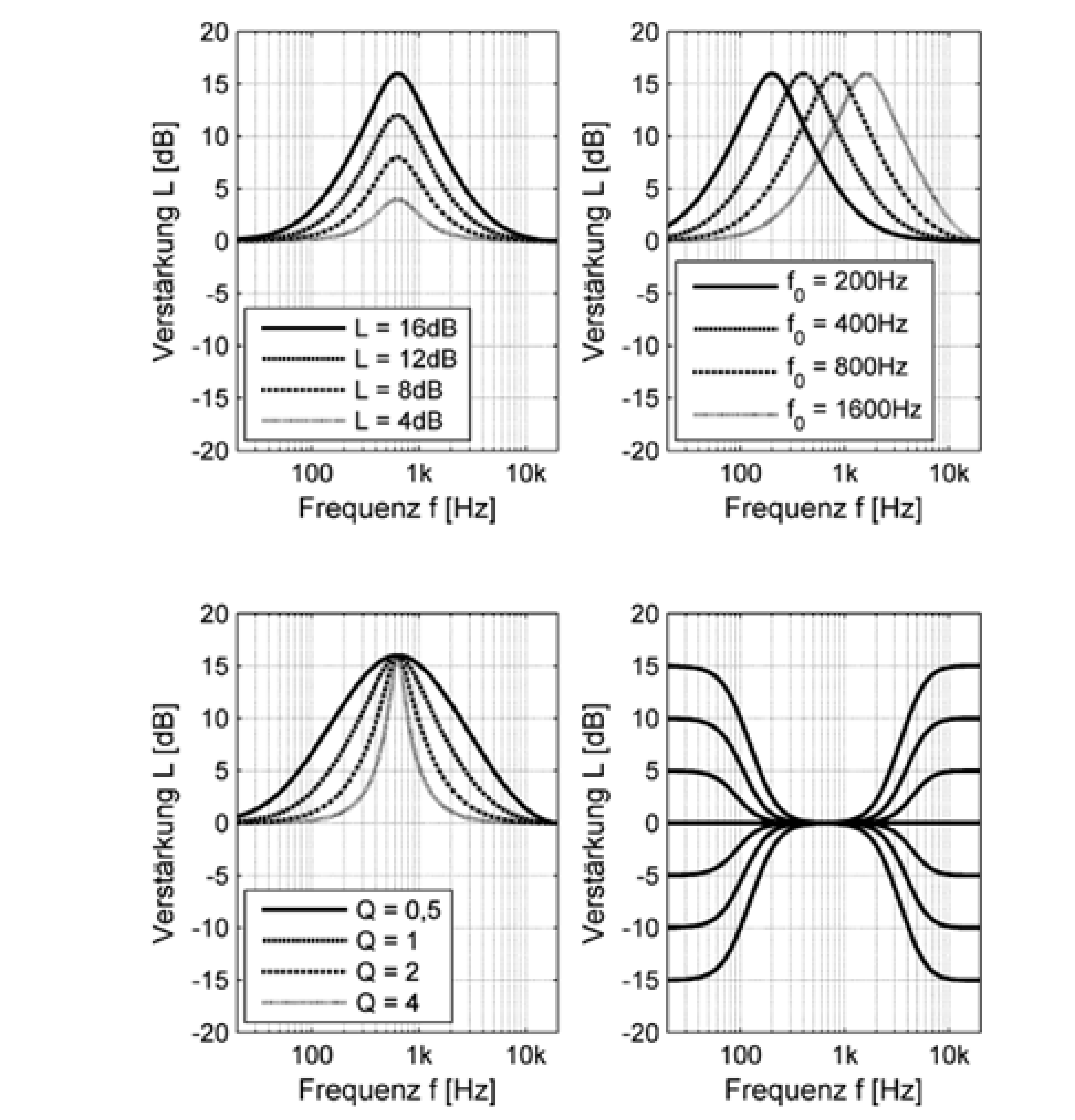
Peak filters



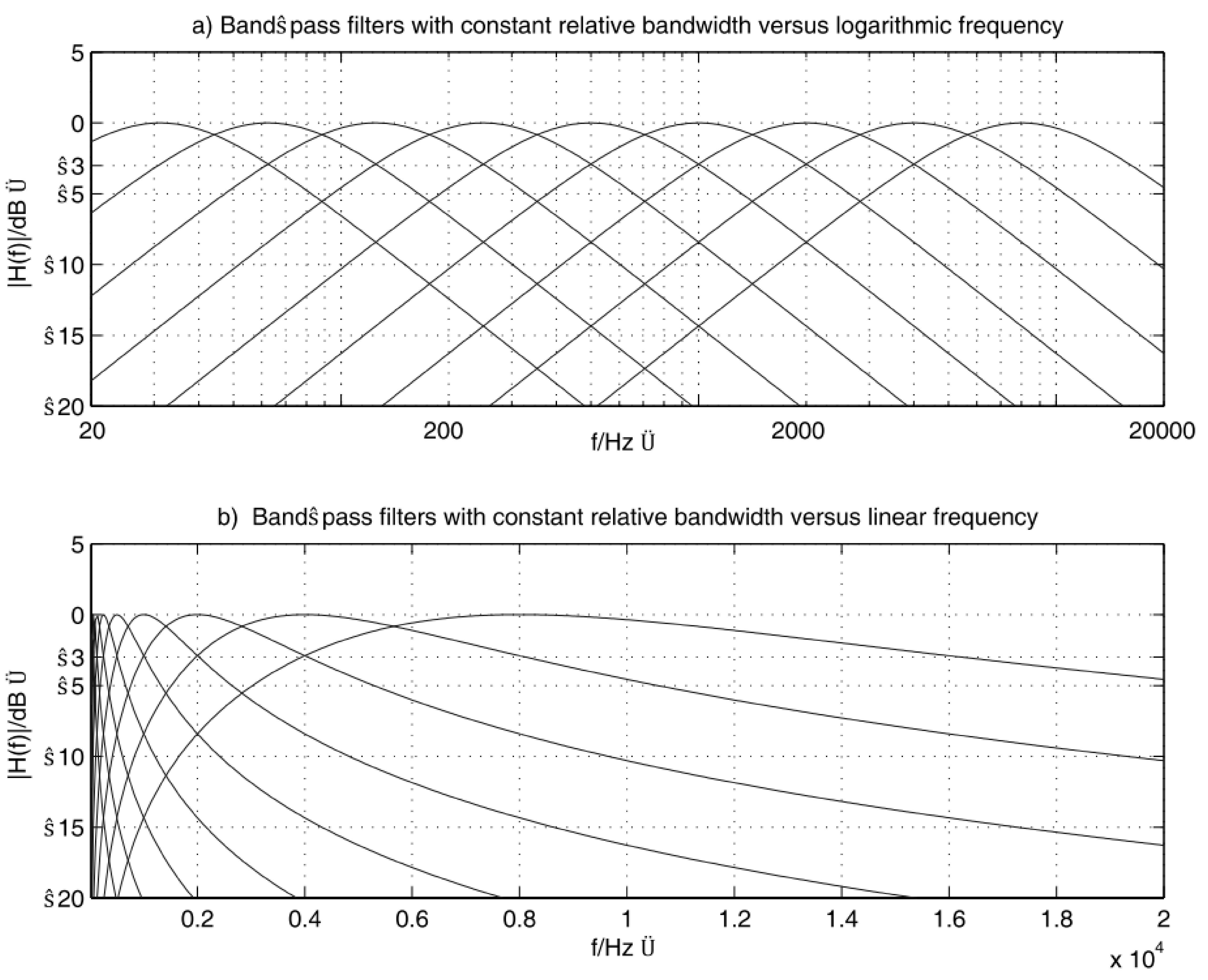
Resonanace/notch filters



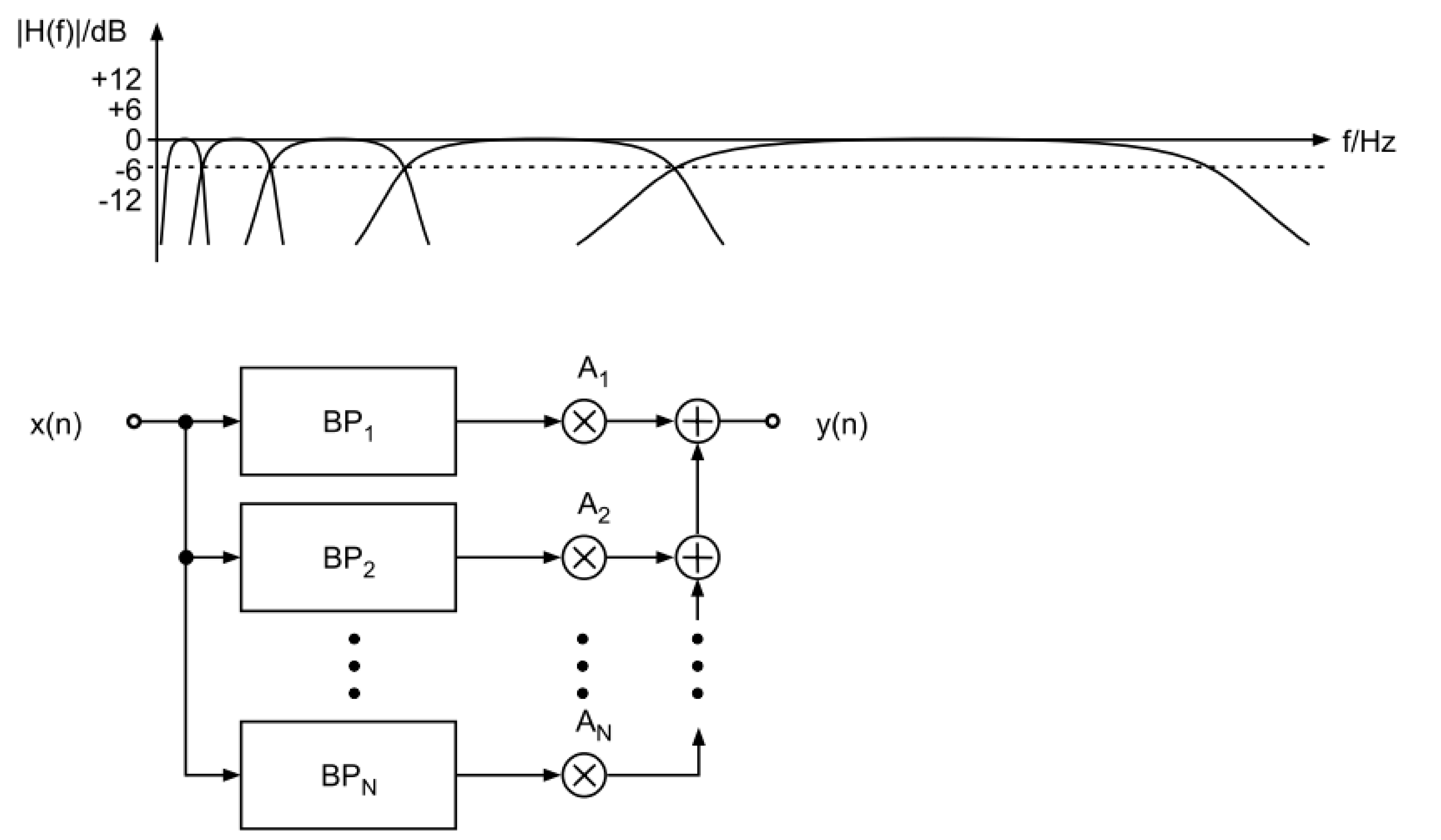
Common Transfer Function Shapes



Filter Banks

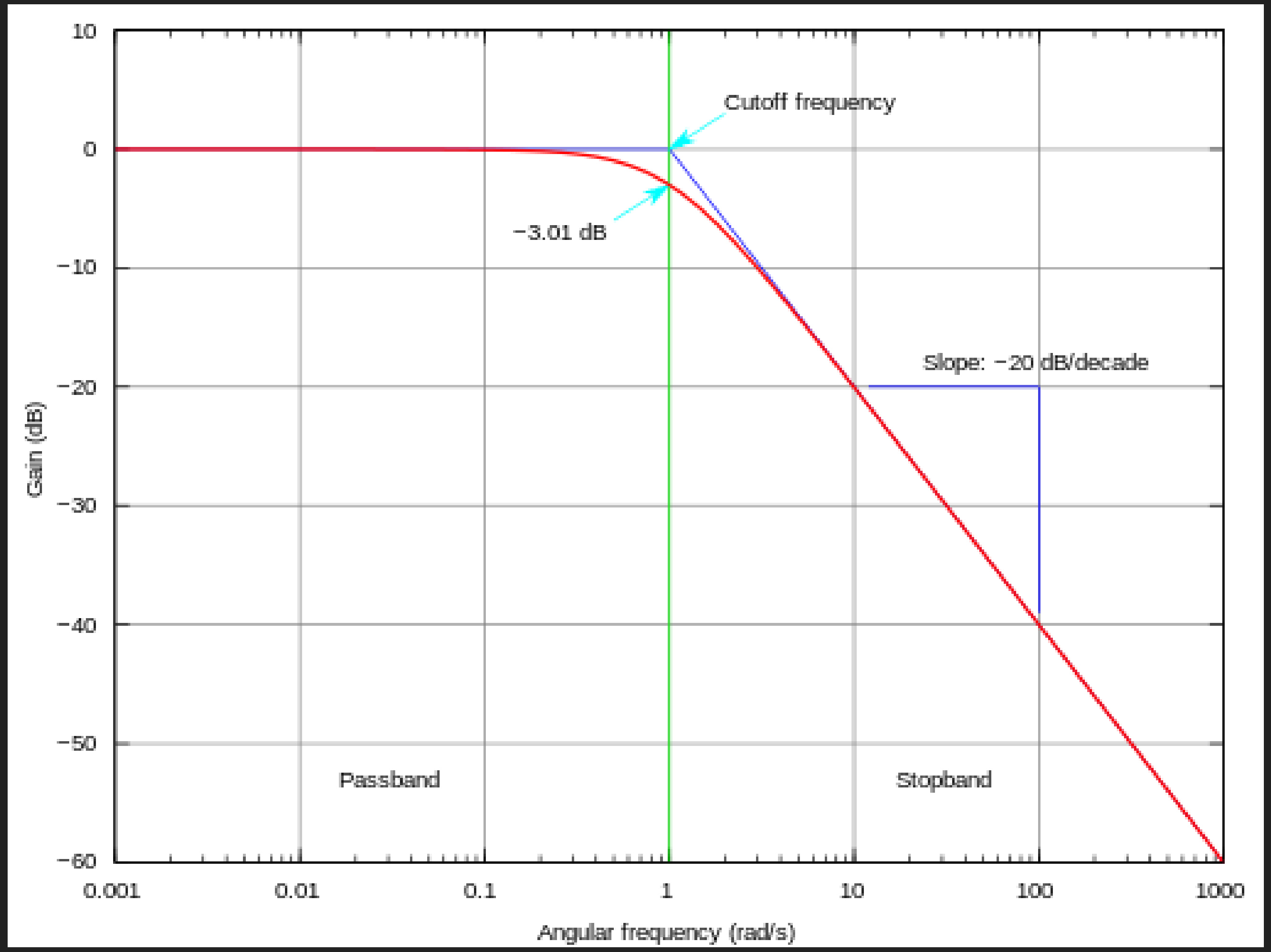


Filter Banks



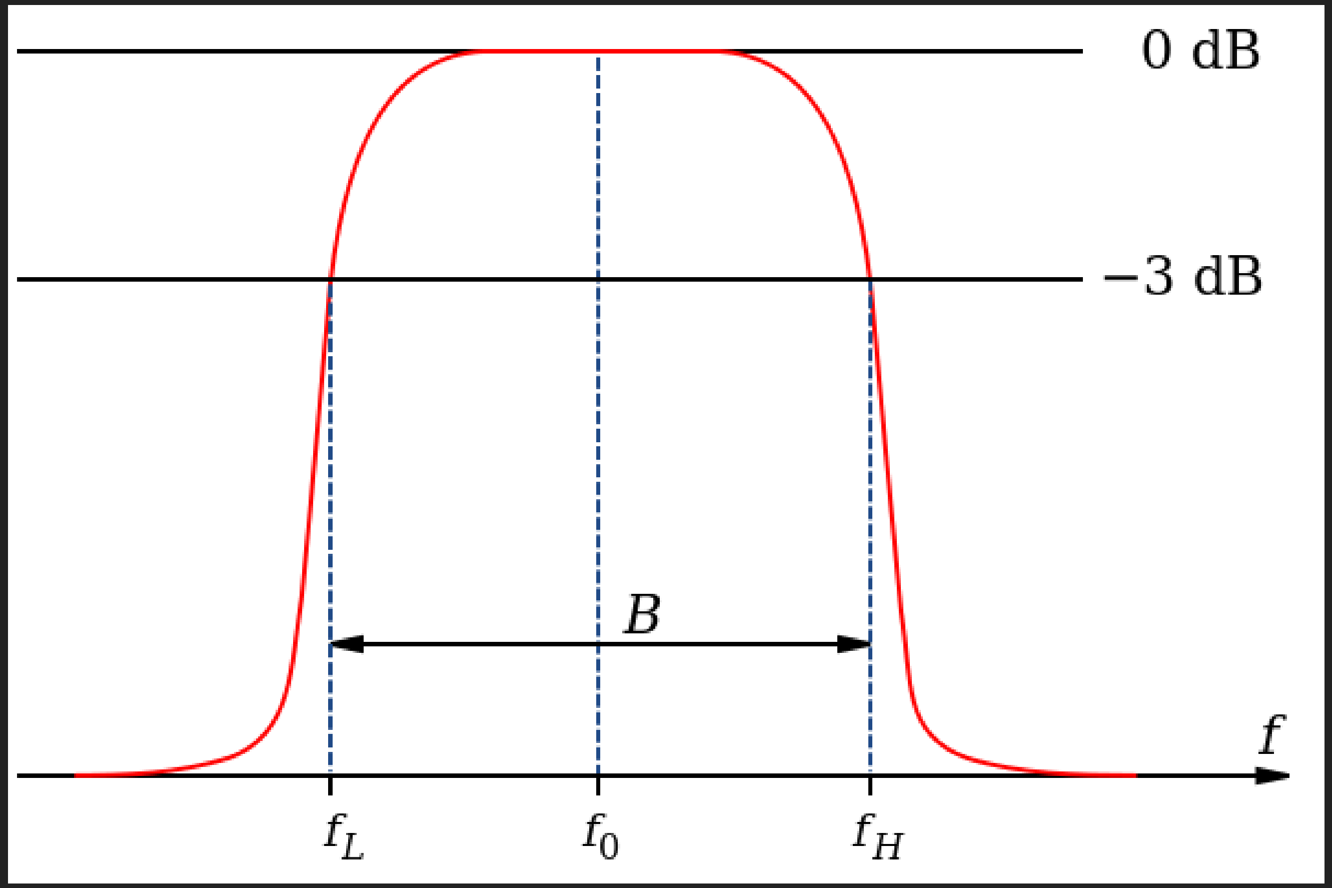
Filter Parameters - Lowpass/Highpass

- **Cut-off** frequency f_c
  - Frequency marking the transition of pass to stop band
  - -3dB of pass band level
- **Slope**/steepness
  - Measured in dB/octave or dB/decade
  - Typically directly related to filter order
- Sometimes: **resonance**
  - Level increase in narrow band around cut-off frequency



Filter Parameters - Bandpass/Bandstop

- **Center** frequency f_c
  - Frequency marking the center of the pass or stop band
- **Bandwidth** ΔB
  - Width of the pass band
  - at -3dB of max pass band level
- Possibly: **slope**
  - Typically directly related to filter order

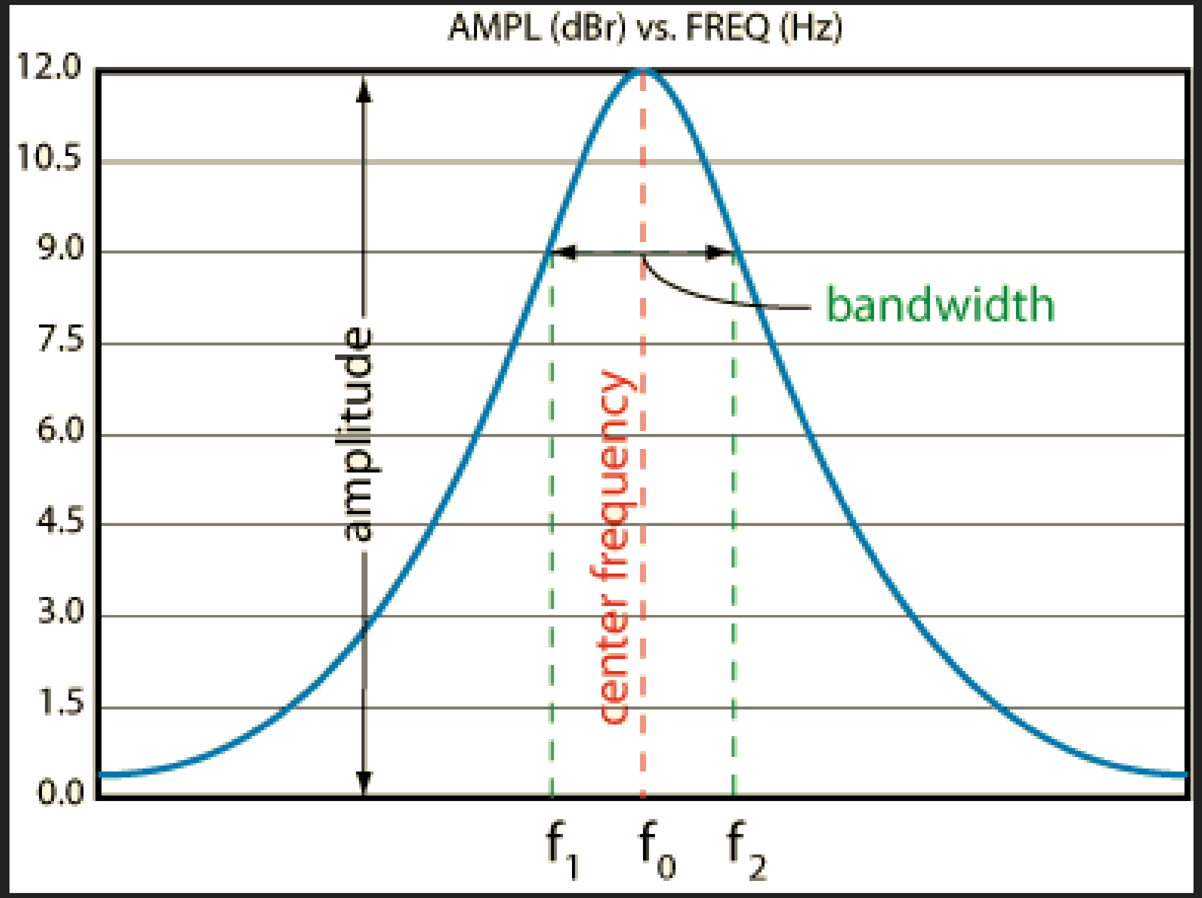


Filter Parameters - Peak

- **Center** frequency f_c
  - Frequency marking the center of the peak
- **Q factor** or **bandwidth** ΔB
  - Width of the bell
  - at -3dB of max gain

$$Q = \frac{f_c}{\Delta B}$$

- **Gain**
  - Amplification / attenuation in dB



Filter Parameters - Overview

Parameter	**Lowpass**	**Low Shelving**	**Band Pass**	**Peak**	**Resonance**
Frequency	Cut-off	Cut-off	Center	Center	Center
Bandwidth/Q	Res. gain	--	ΔB	Q	--
Gain	--	Yes	--	Yes	--

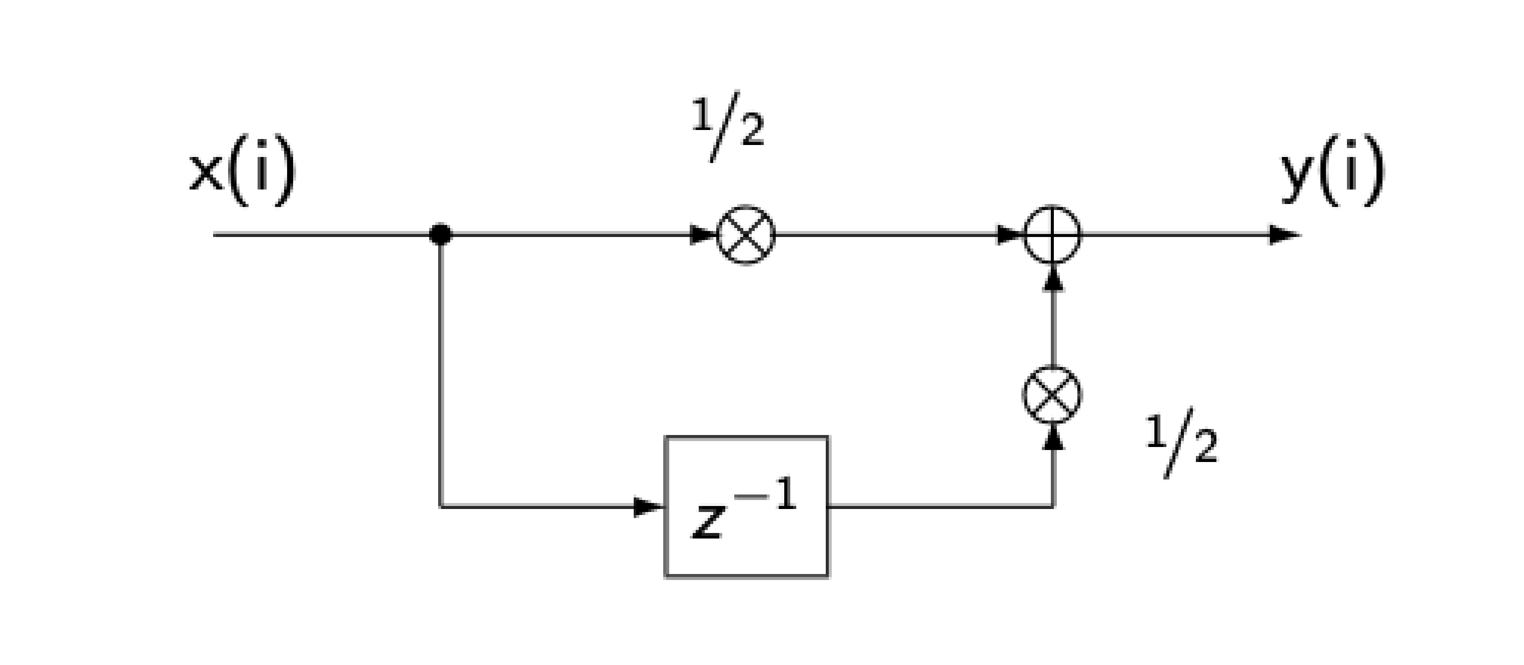
Digital Filter Description

A filter is defined by its

- **Complex transfer function** $H(\mathrm{j}\omega)$
- Or … its **Impulse response** $h(t)$
- Or … its **List of pole and zero positions in the Z-Plane**

$$H(\mathrm{j}\omega) = \mathfrak{F}\{h(t)\}$$

Example Filter 1



$$y(i) = 0.5 \cdot x(i) + 0.5 \cdot x(i-1)$$

Example Filter 1: Transfer Function

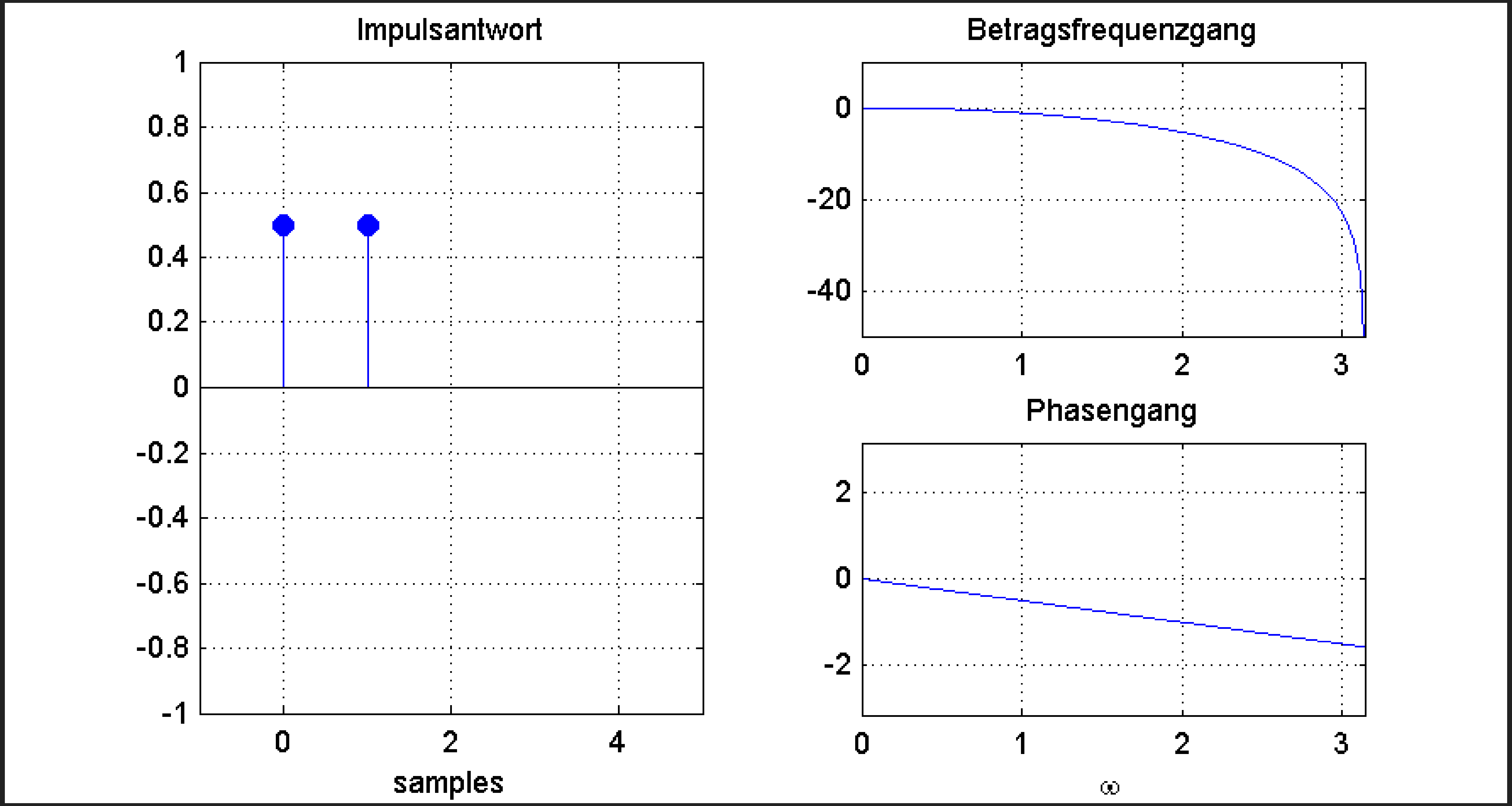
$$y(i) = 0.5 \cdot x(i) + 0.5 \cdot x(i-1)$$

$$H(z) = 0.5 + 0.5 \cdot z^{-1}$$

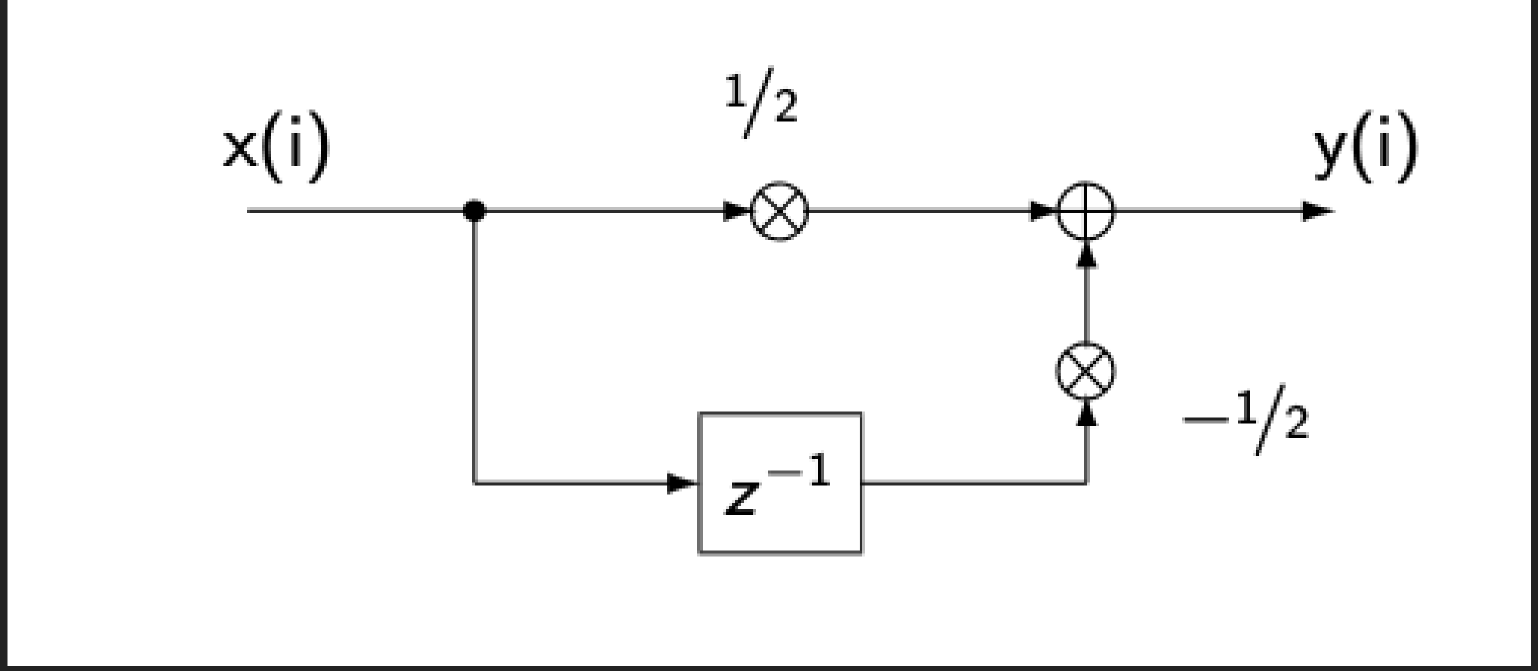
$$H(\mathrm{j}\omega) = 0.5 + 0.5 \cdot e^{-\mathrm{j}\omega}$$

$$\begin{aligned}
\left|H(\mathrm{j}\omega)\right| &= 0.5 \cdot \left|e^{-\mathrm{j}\frac{\omega}{2}} \cdot \left(e^{\mathrm{j}\frac{\omega}{2}} + e^{-\mathrm{j}\frac{\omega}{2}}\right)\right| \\
&= 0.5 \cdot \underbrace{\left|e^{-\mathrm{j}\frac{\omega}{2}}\right|}_{1} \cdot \underbrace{\left|\left(e^{\mathrm{j}\frac{\omega}{2}} + e^{-\mathrm{j}\frac{\omega}{2}}\right)\right|}_{\left|2\cos\left(\frac{\omega}{2}\right)\right|} \\
&= \left|\cos\left(\frac{\omega}{2}\right)\right|
\end{aligned}$$

Example Filter 1: Visualization



Example Filter 2

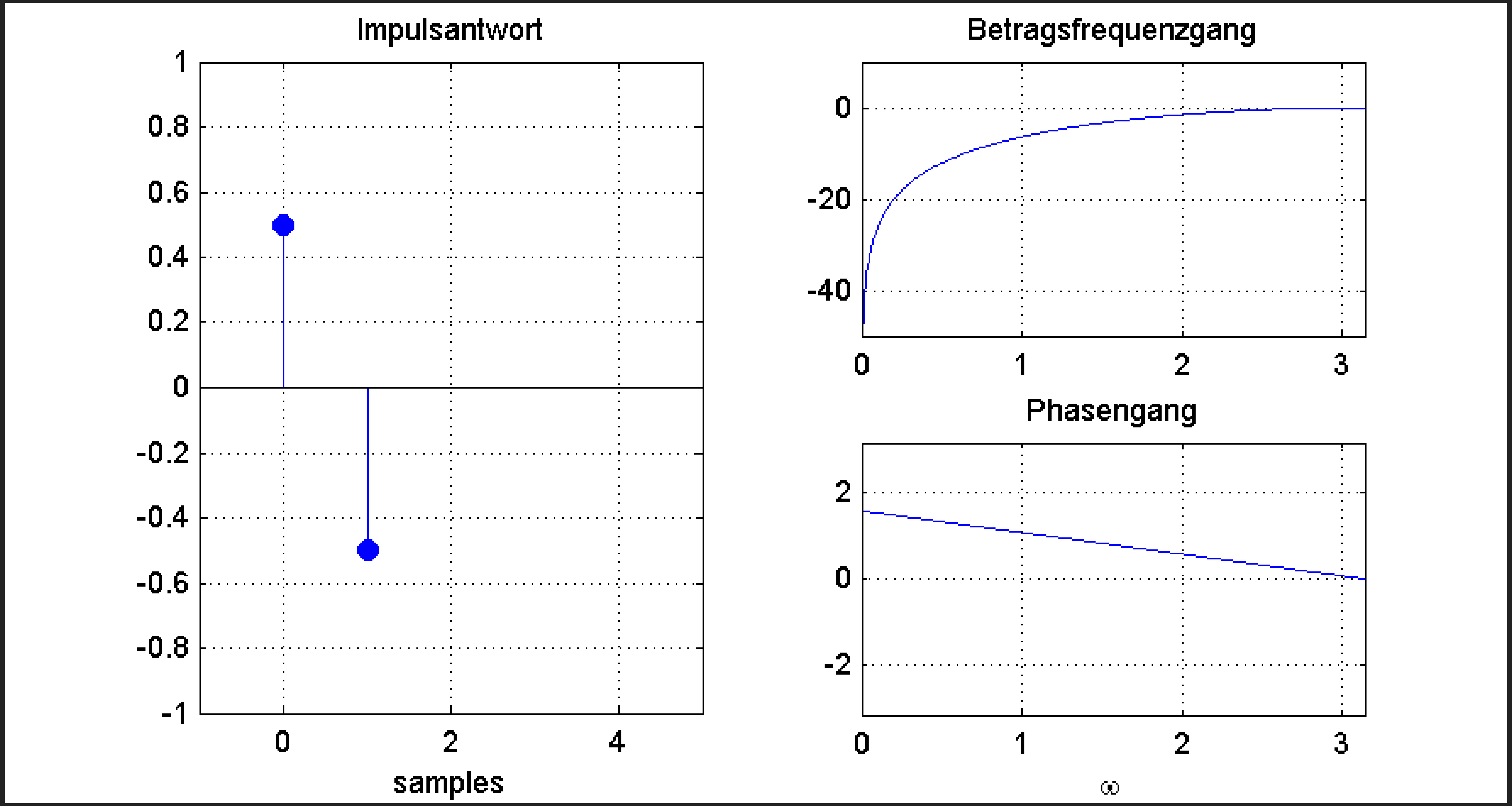


$$y(i) = 0.5 \cdot x(i) - 0.5 \cdot x(i-1)$$

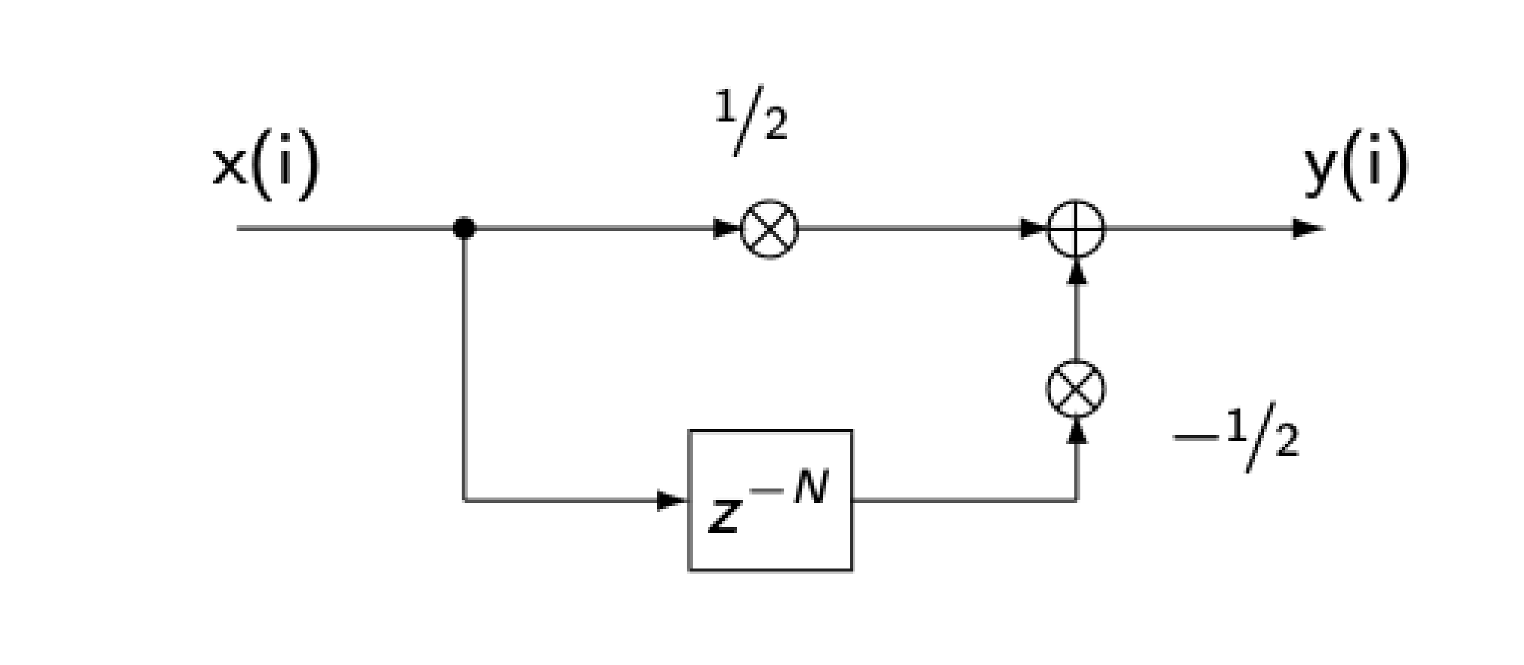
$$H(z) = 0.5 - 0.5 \cdot z^{-1}$$

$$|H(\mathrm{j}\omega)| = \left|\sin\left(\frac{\omega}{2}\right)\right|$$

Example Filter 2: Visualization



Example Filter 3



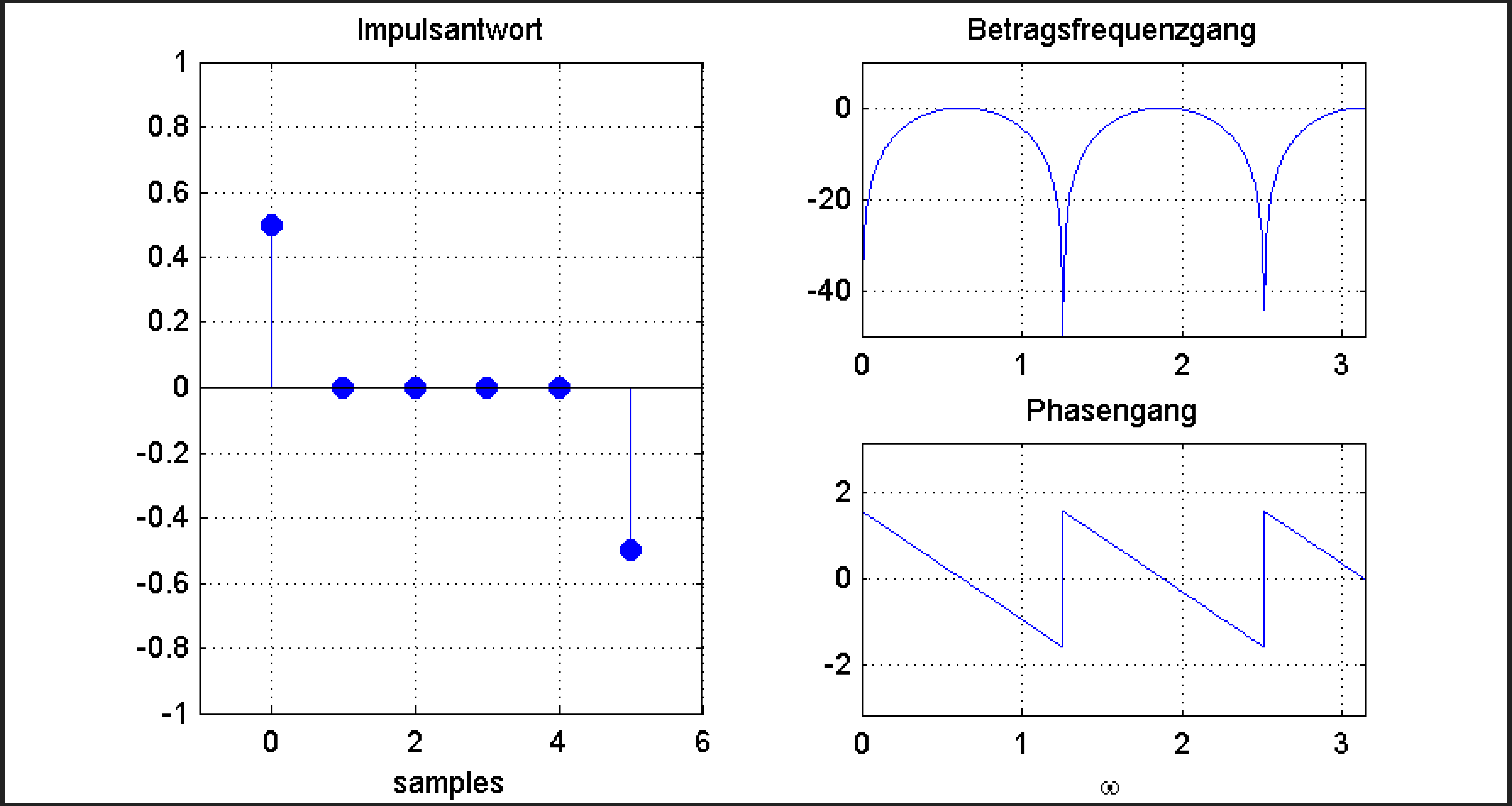
$$y(i) = 0.5 \cdot x(i) - 0.5 \cdot x(i-N)$$

$$H(z) = 0.5 - 0.5 \cdot z^{-N}$$

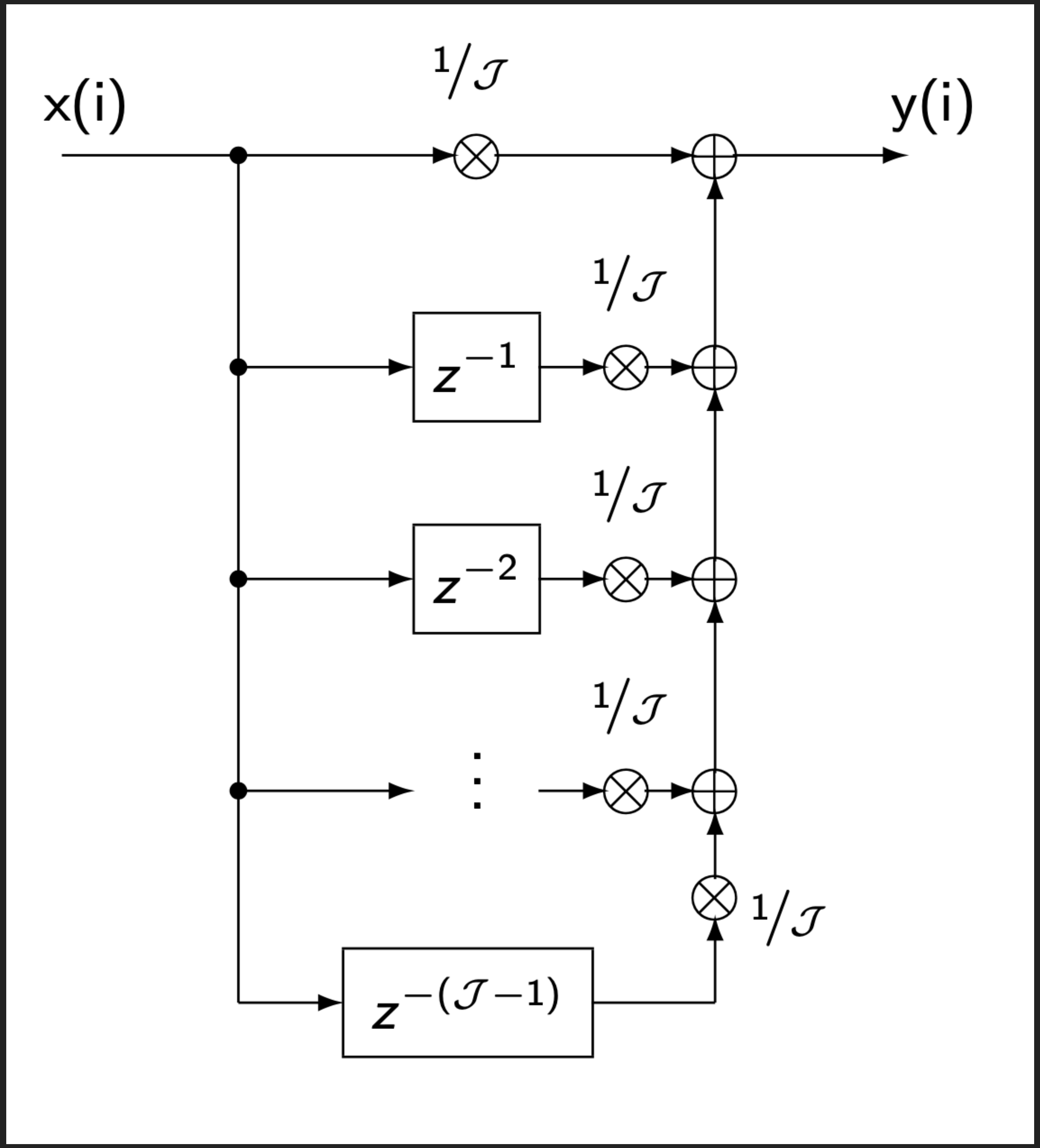
$$|H(\mathrm{j}\omega)| = 0.5 \cdot \left| e^{-\mathrm{j}\frac{N\omega}{2}} \cdot \left(e^{\mathrm{j}\frac{N\omega}{2}} - e^{-\mathrm{j}\frac{N\omega}{2}} \right) \right|$$

$$= \left| \sin\left(\frac{N\omega}{2} \right) \right|$$

Example Filter 3: Visualization

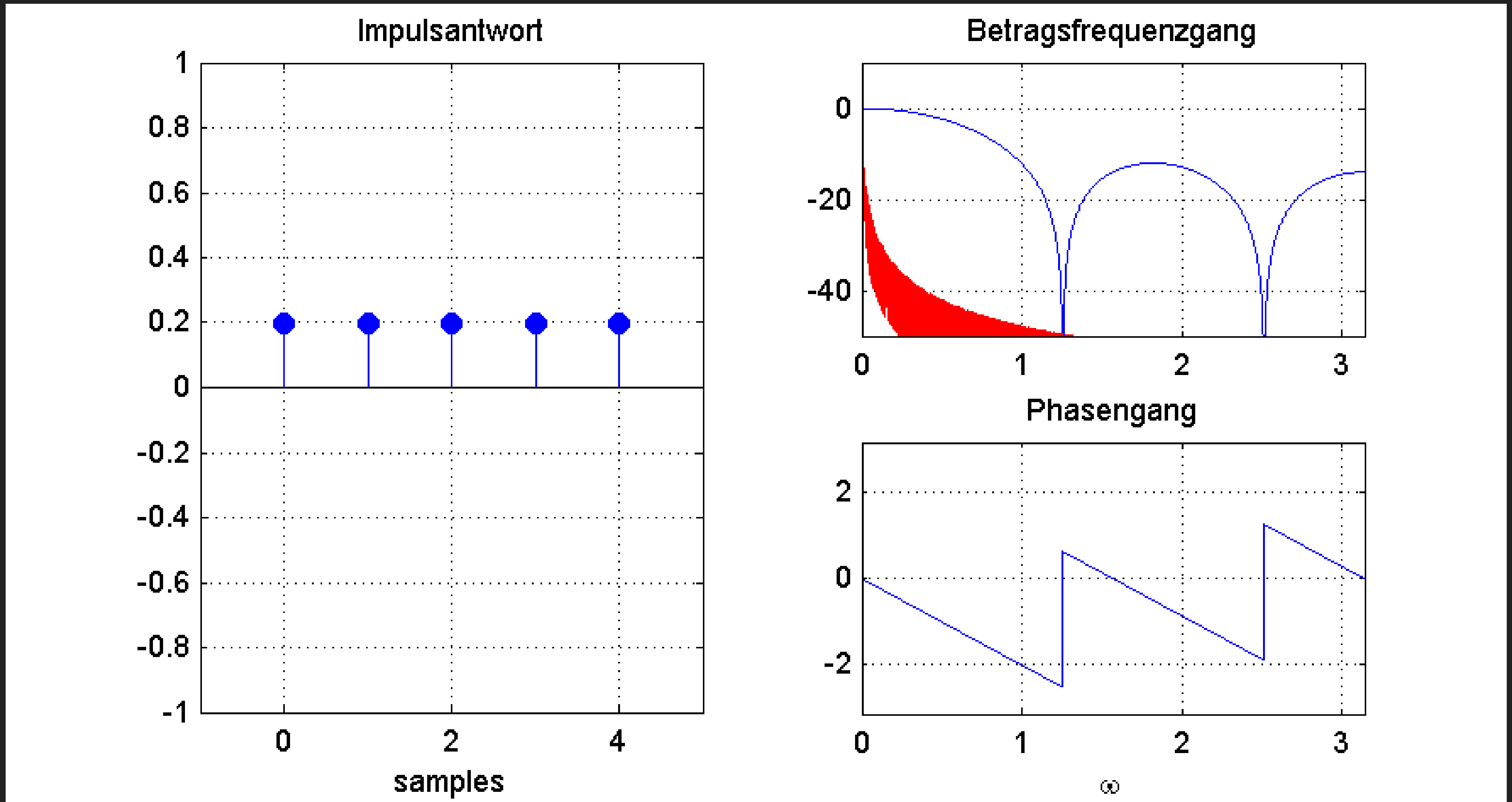


Example Filter 4



$$y(i) = \frac{1}{\mathcal{J}} \sum_{j=0}^{\mathcal{J}-1} x(i-j)$$

Example Filter 4: Transfer Function/Visualization



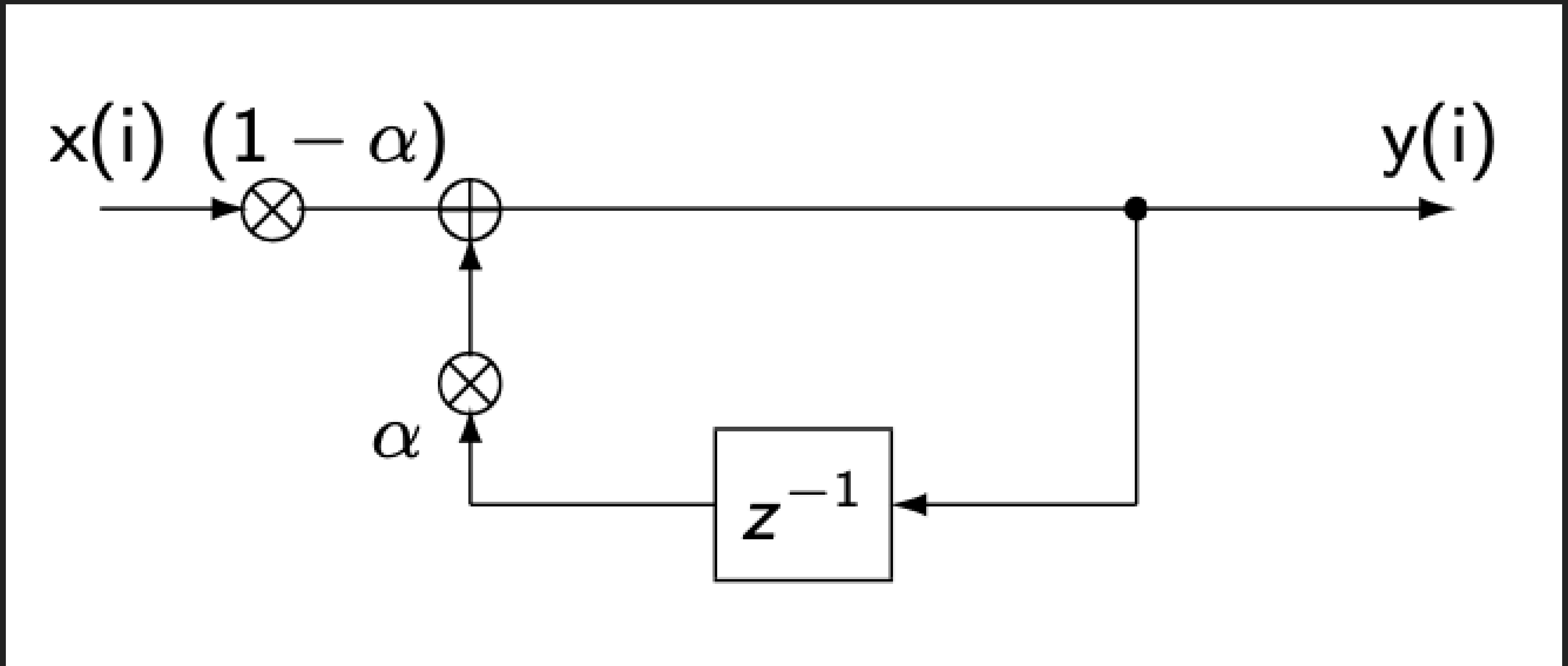
$$H(\mathrm{j}\omega) = e^{-\mathrm{j}\mathcal{J}\frac{\omega}{2}} \frac{\sin\left(\mathcal{J}\cdot\frac{\omega}{2}\right)}{\mathcal{J}\cdot\sin\left(\frac{\omega}{2}\right)}$$

Example Filter 4: Recursive Implementation

$$
\begin{aligned}
y(i) &= \sum_{j=0}^{\mathcal{J}-1} \frac{1}{\mathcal{J}} \cdot x(i-j) \\
&= \frac{1}{\mathcal{J}} \cdot \big(x(i) - x(i-\mathcal{J})\big) + \underbrace{\sum_{j=1}^{\mathcal{J}} \frac{1}{\mathcal{J}} \cdot x(i-j)}_{y(i-1)} \\
&= \frac{1}{\mathcal{J}} \cdot \big(x(i) - x(i-\mathcal{J})\big) + y(i-1)
\end{aligned}
$$

Not applicable with windowed coefficients!

Example Filter 5



$$
\begin{aligned}
y(i) &= (1-\alpha)\cdot x(i) + \alpha \cdot y(i-1) \\
&= x(i) + \alpha \cdot (y(i-1) - x(i))
\end{aligned}
$$

Example Filter 5: Transfer Function

$$y(i) = (1-\alpha)\cdot x(i) + \alpha \cdot y(i-1)$$

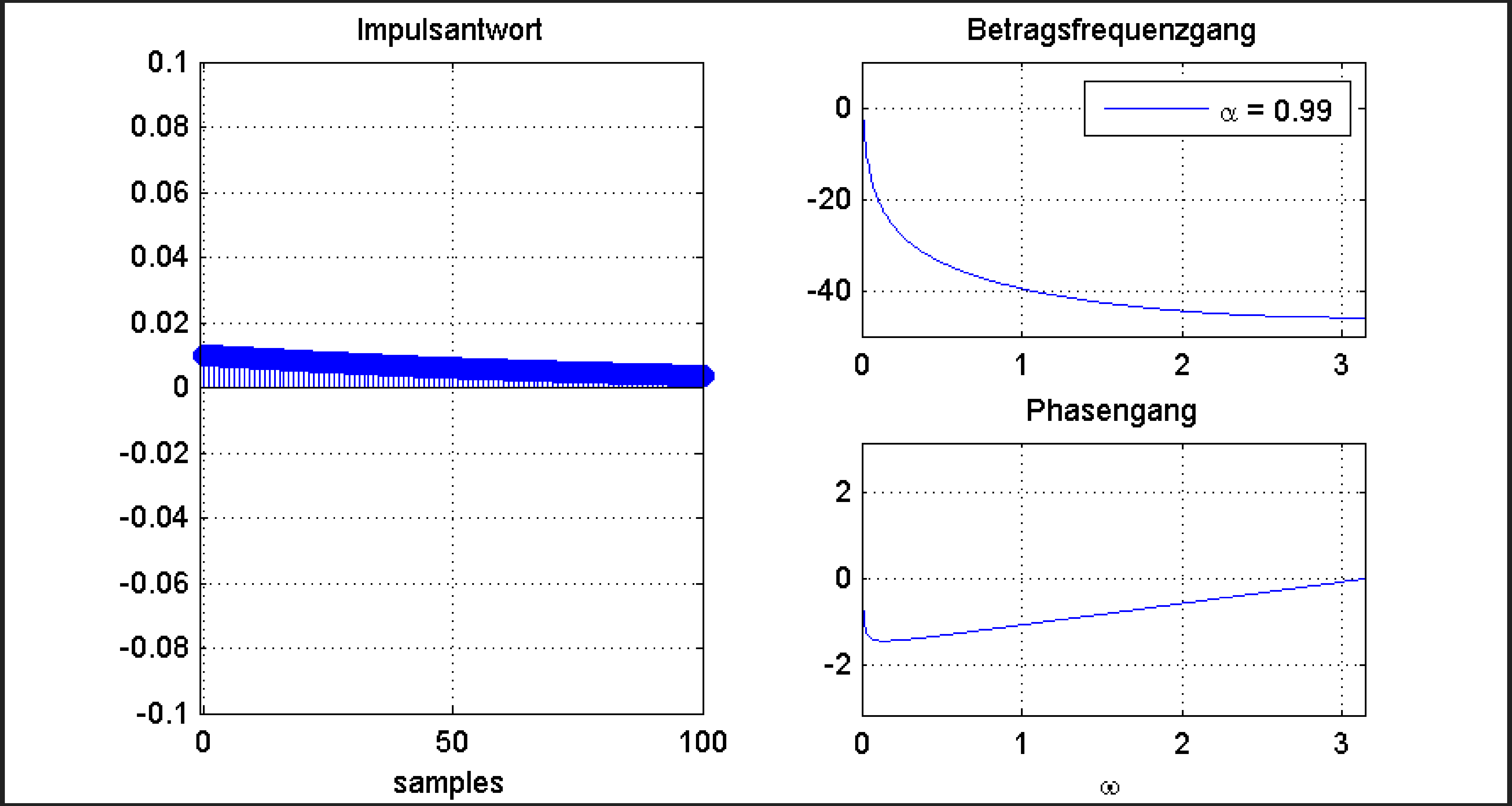
$$H(z) = \frac{1-\alpha}{1-\alpha z^{-1}}$$

$$H(\mathrm{j}\omega) = \frac{1-\alpha}{1-\alpha e^{-\mathrm{j}\omega}}$$

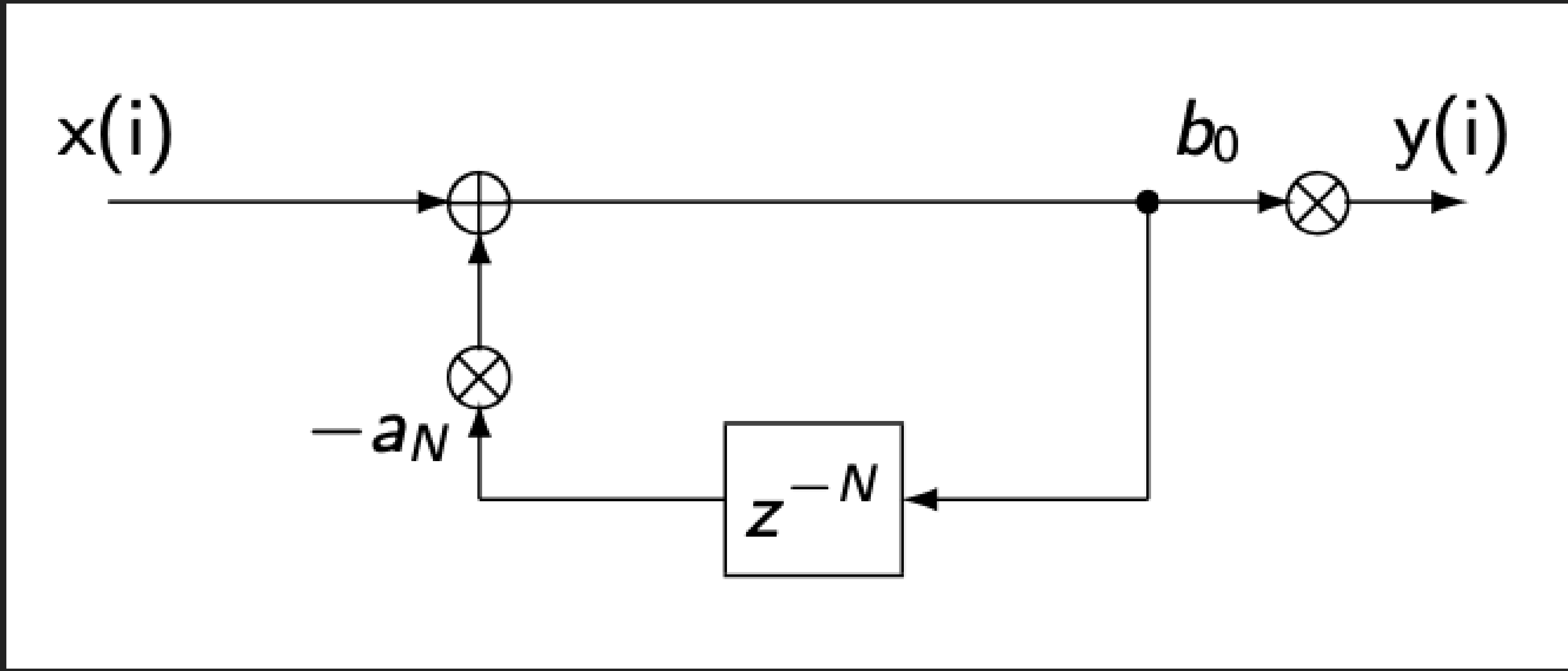
$$|H(\mathrm{j}\omega)| = \left|\frac{1-\alpha}{1-\alpha e^{-\mathrm{j}\omega}}\right|$$

$$= \frac{1-\alpha}{\sqrt{(1+\alpha^2 - 2\alpha\cos(\omega))}}$$

Example Filter 5: Visualization

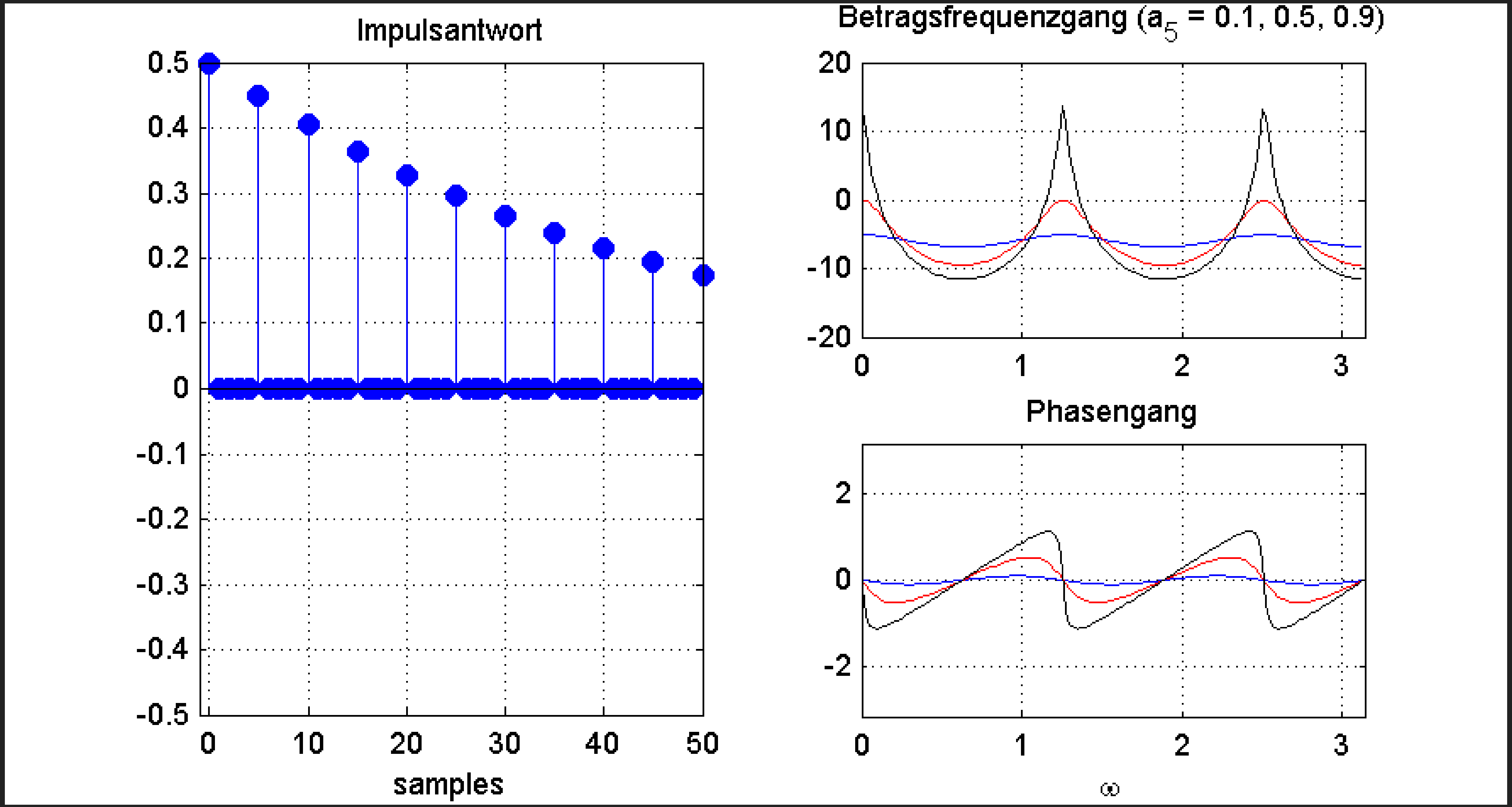


Example Filter 6



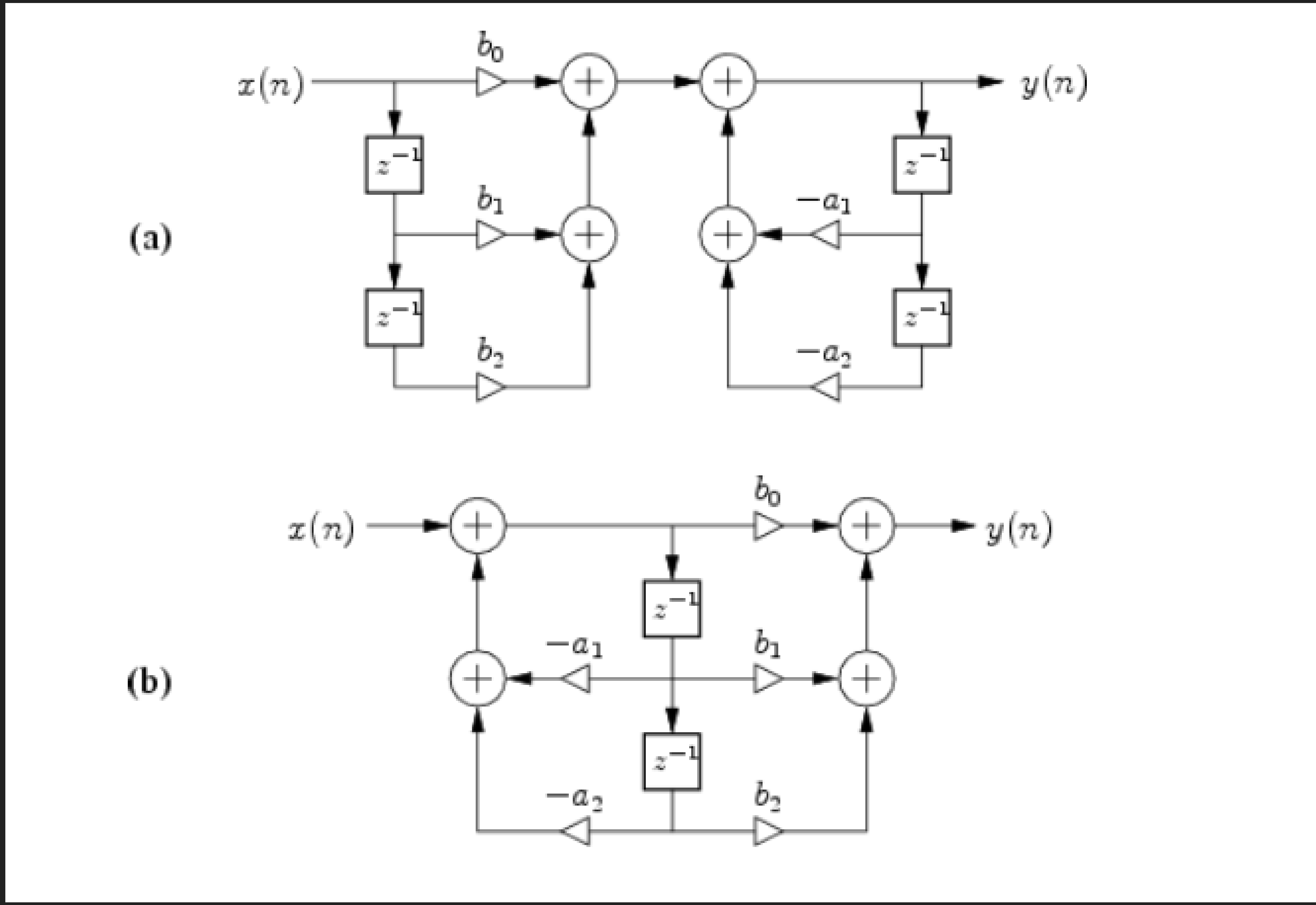
$$y(i) = b_0 \cdot x(i) - a_N \cdot y(i - N)$$

Example Filter 6: Transfer Function/Visualization

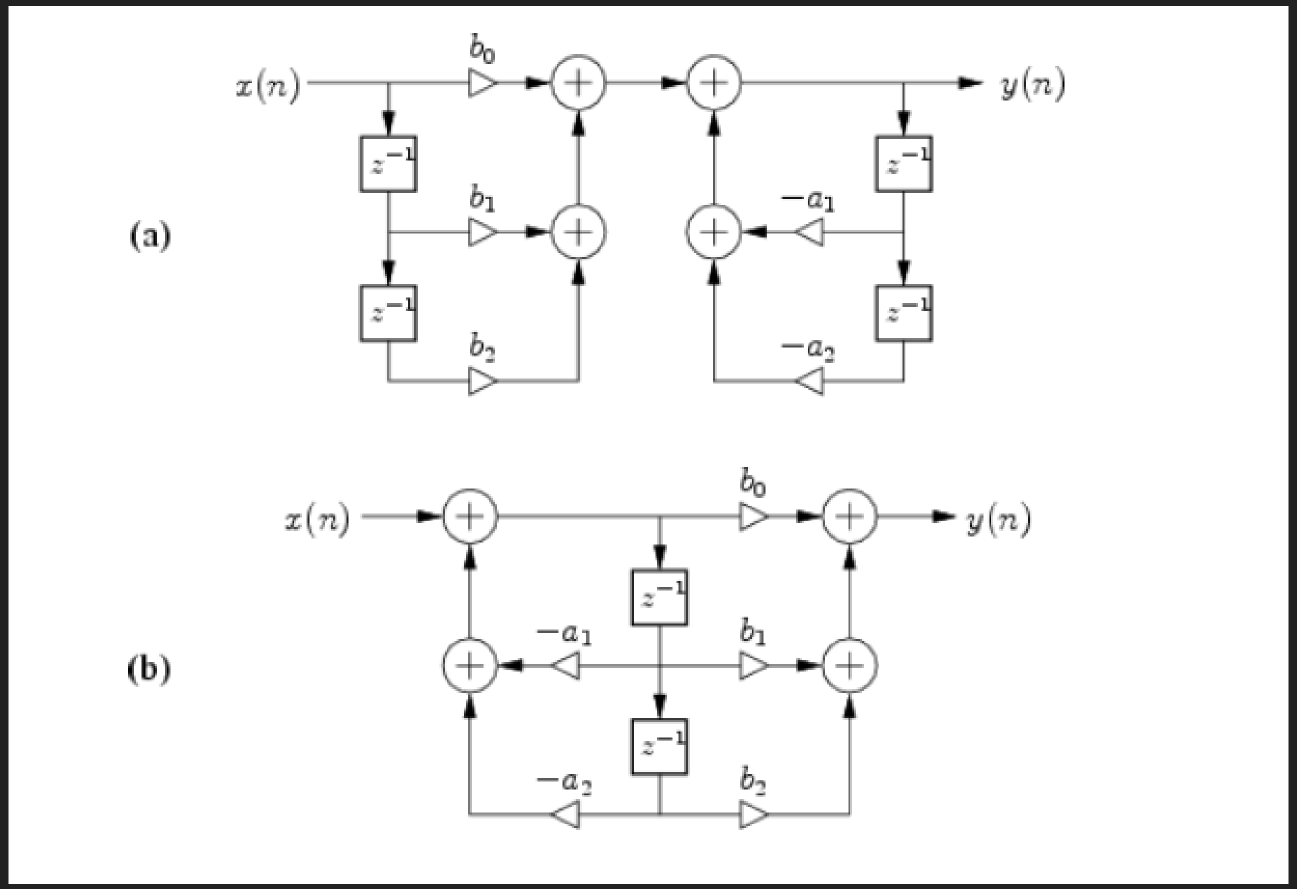


$$H(\mathrm{j}\omega) = \frac{b_0}{1 - a_N \cdot e^{-\mathrm{j}\omega N}}$$

Biquad: Structure



Biquad: Structure



$$\text{diff eq} : y(i) = \sum_{k=0}^{K_1} b_k \cdot x(i-k) + \sum_{k=1}^{K_2} -a_k \cdot y(i-k)$$

$$\text{trans. fct} : H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{K_1} b_k \cdot z^{-k}}{1 + \sum_{k=1}^{K_2} a_k \cdot z^{-k}}$$

Summary

- Filter (equalization) can be used for various tasks
  - Changing the sound quality of a signal
  - Hiding unwanted frequency components
  - Smoothing
  - Processing for measurement and transmission
- Most common audio filter types are:
  - Low/high pass
  - Peak
  - Shelving

Summary

- Filter parameters include:
  - Frequency (mid, cutoff)
  - Bandwidth or Q
  - Gain
- Filter Orders:
  - Typical orders are 1st, 2nd, maybe 4th
  - Higher order give more flexibility wrt transfer function
  - Higher orders are difficult to design and control