Digital Signal Processing for Music

Part 12: Digital Number Formats

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Word length and SNR

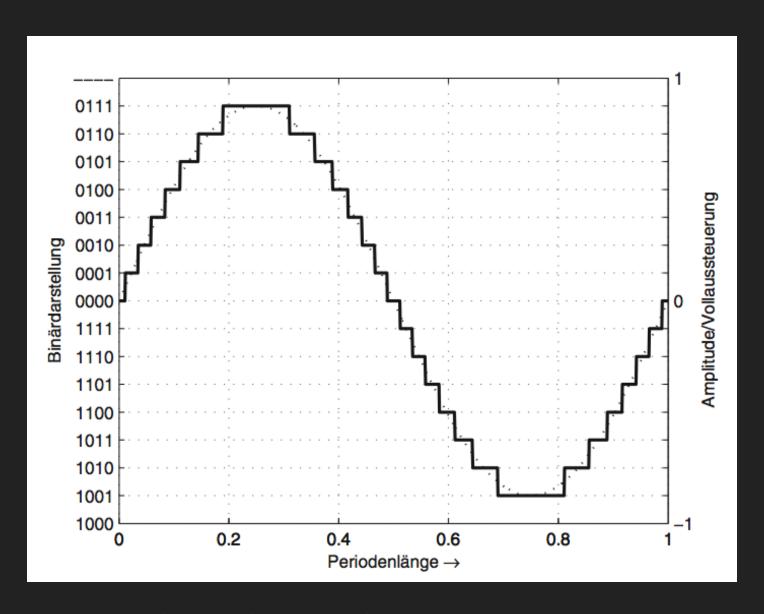
Δ	Max. Amp	theo. SNR
±1	0255	≈48 dB
± 1	$-32768 \dots 32767$	\approx 96 dB
± 1	$-524288 \dots 524287$	pprox120 dB
± 1	$-16777216 \dots 16777215$	\approx 144 dB
$\pm 1.175 \cdot 10^{-38}$	$\pm 3.403 \cdot 10^{1038}$	1529 dB
$\pm 2.225 \cdot 10^{-308}$	$\pm 1.798\cdot 10^{10308}$	12318 dB
	$egin{array}{c} \pm 1 \ \pm 1 \ \pm 1 \ \pm 1 \ \end{array}$	$egin{array}{cccccccccccccccccccccccccccccccccccc$

How do we represent this in bits?

Number Formats: Value Range

- >> Unnormalized: $-2^{w-1} \dots 2^{w-1} 1$
 - >> Integer representation
 - >> Non-symmetric step count for positive and negative values
 - >> Used for transmission, etc.
- \rightarrow Normalized: -1...1
 - >> Used for floating point representations
 - >> Word length independent
 - >> Used for processing

Number Representation



- \blacktriangleright Least Significant Bit (LSB): b_0 (usually on the right)
- \longrightarrow Most Significant Bit (MSB): b_{w-1} (usually on the left)



Amplitude

Range (normalized)

Amplitude
$$x_Q=-b_{w-1}+\sum\limits_{i=0}^{w-2}b_i2^{-(w-i-1)}$$
 $x_Q=\sum\limits_{i=0}^{w-1}b_i2^{-(w-1)}$

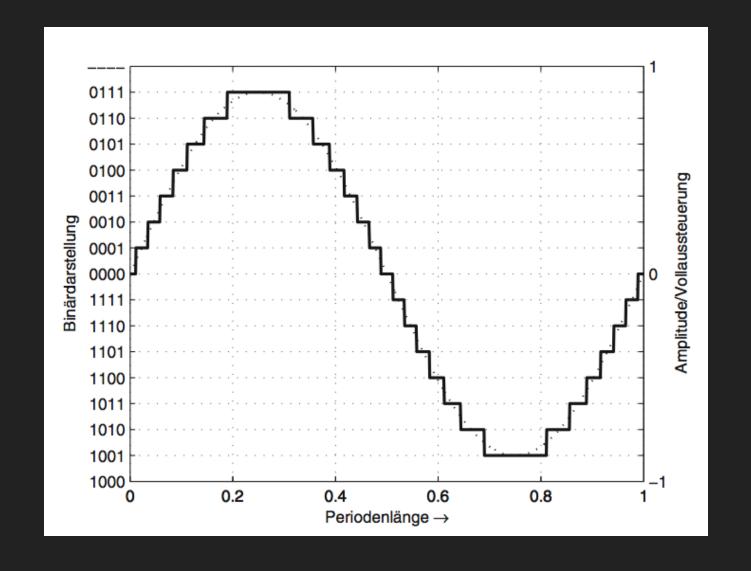
$$-1 \leq x_Q \leq 1 - 2^{-(w-1)}$$

Unsigned

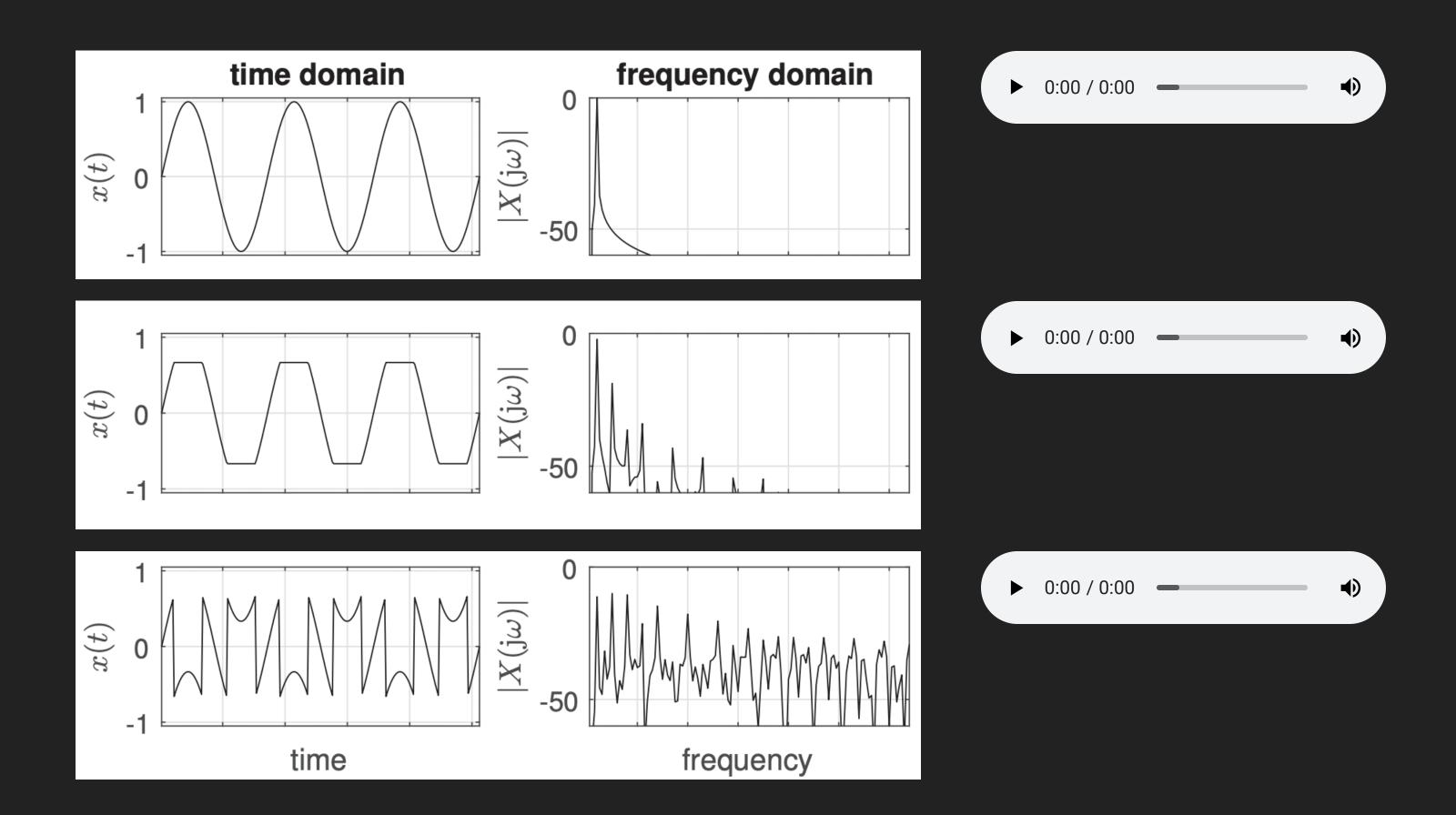
$$x_Q = \sum_{i=0}^{w-1} b_i 2^{-(w-1)}$$

$$0 \leq x_Q \leq 1-2^{-w}$$

- \Rightarrow w: word length
- \Rightarrow b_i : ith bit



Clipping & Wrap-Around



Fixed Point and Floating Point: Number Formats and their Most Frequent Uses

- >> Unsigned Format: Small word lengths (4...8 bit)
- >> 2's Complement': File formats with higher word lengths (16...24 bit), some DSPs
- >> Floating Point: Internal representation for processing

Floating Point

$$x_Q = M_G \cdot 2^{E_G}$$

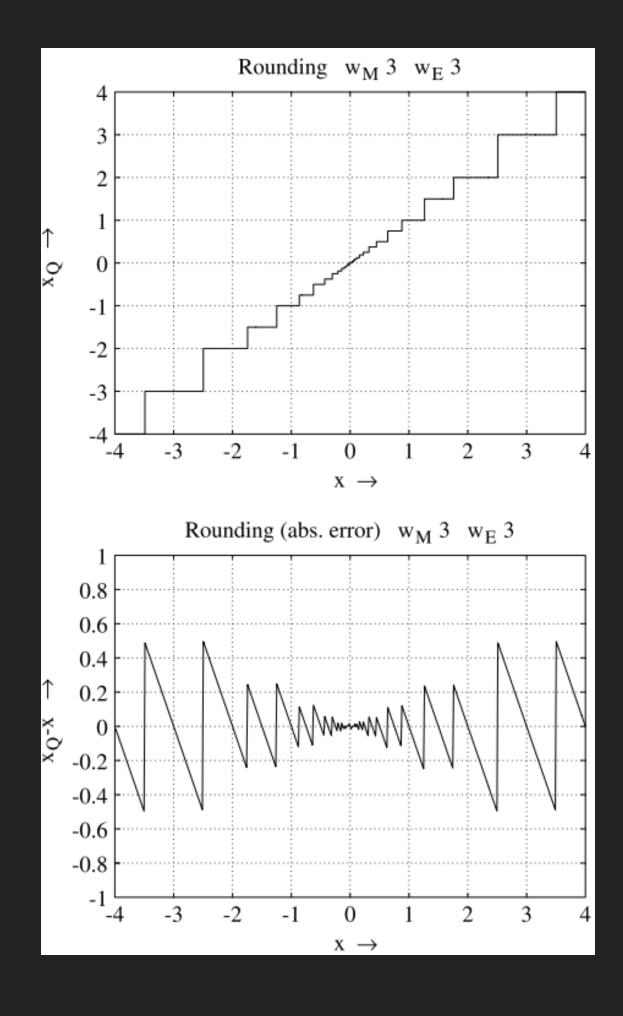
- $\blacktriangleright \blacktriangleright M_G$: Normalized Mantissa $0.5 \leq M_G < 1$
- $\blacktriangleright \triangleright E_G$: Exponent

32 Bit IEEE 754 Floating Format

Bit 31: Sign	Bits 30-23: Exponent	Bits 22-0: Mantissa
\boldsymbol{s}	$e_7 \dots e_0$	$m_{22}m_0$

Exceptions

Type	E_G	M_G	Value
Normal	$1 \leq E_G \leq 254$	Any	$(-1)^s(0.m)2^{E_G-127}$
NaN (Not a Number)	255	$\neq 0$	Undefined
Infinity	255	= 0	∞
Zero	0	0	0



>> High Exponent:

Large quantization error energy

>> Low Exponent:

Small quantization error energy

>> Linear quantization:

Within one exponent

Summary

- >> Most common number representations
 - >> 2-Complement for high quality audio storage
 - >> Floating point for high quality audio processing (non-linear quantization)