Digital Signal Processing for Music

Part 9: Discretization, Part 1 - Sampling

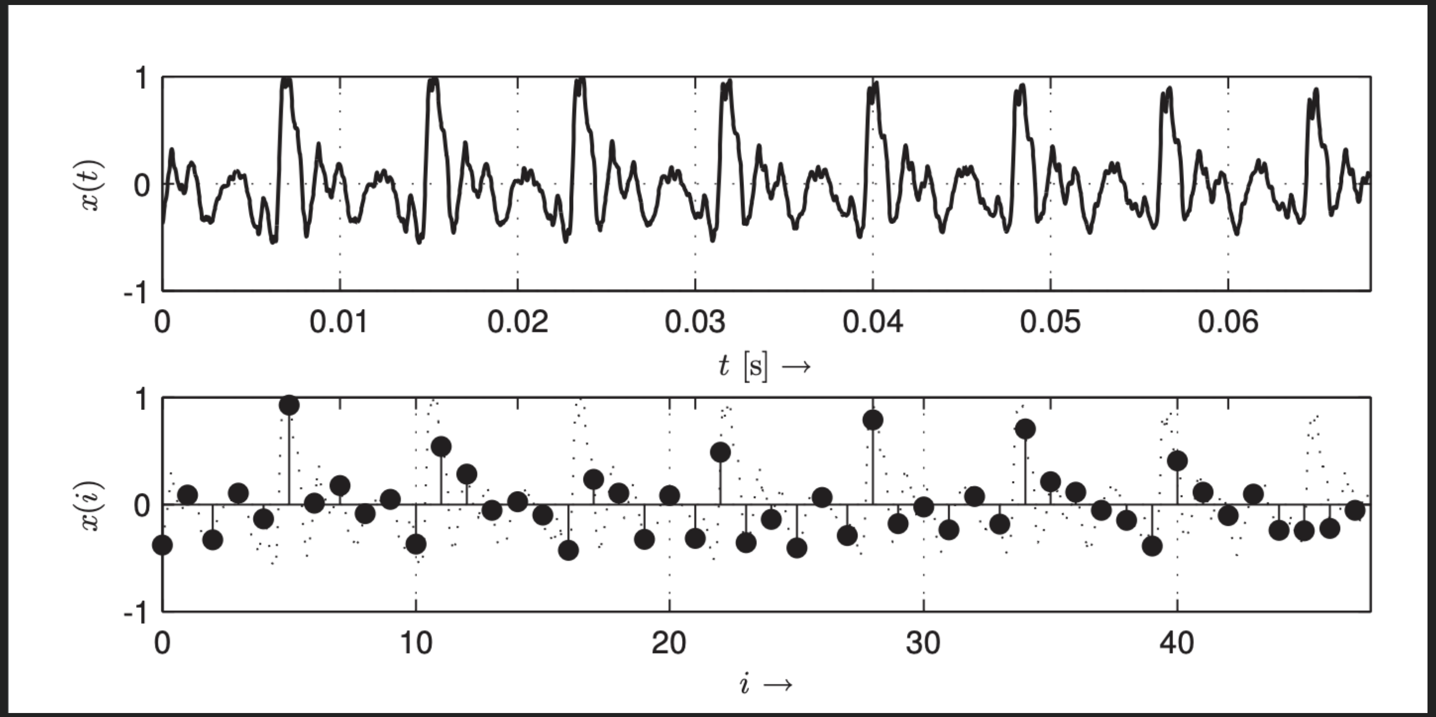
Andrew Beck

Digital signals can only be represented with a limited number of values

- Time discretization:
  Sampling
- Amplitude discretization:
  Quantization

Sampling

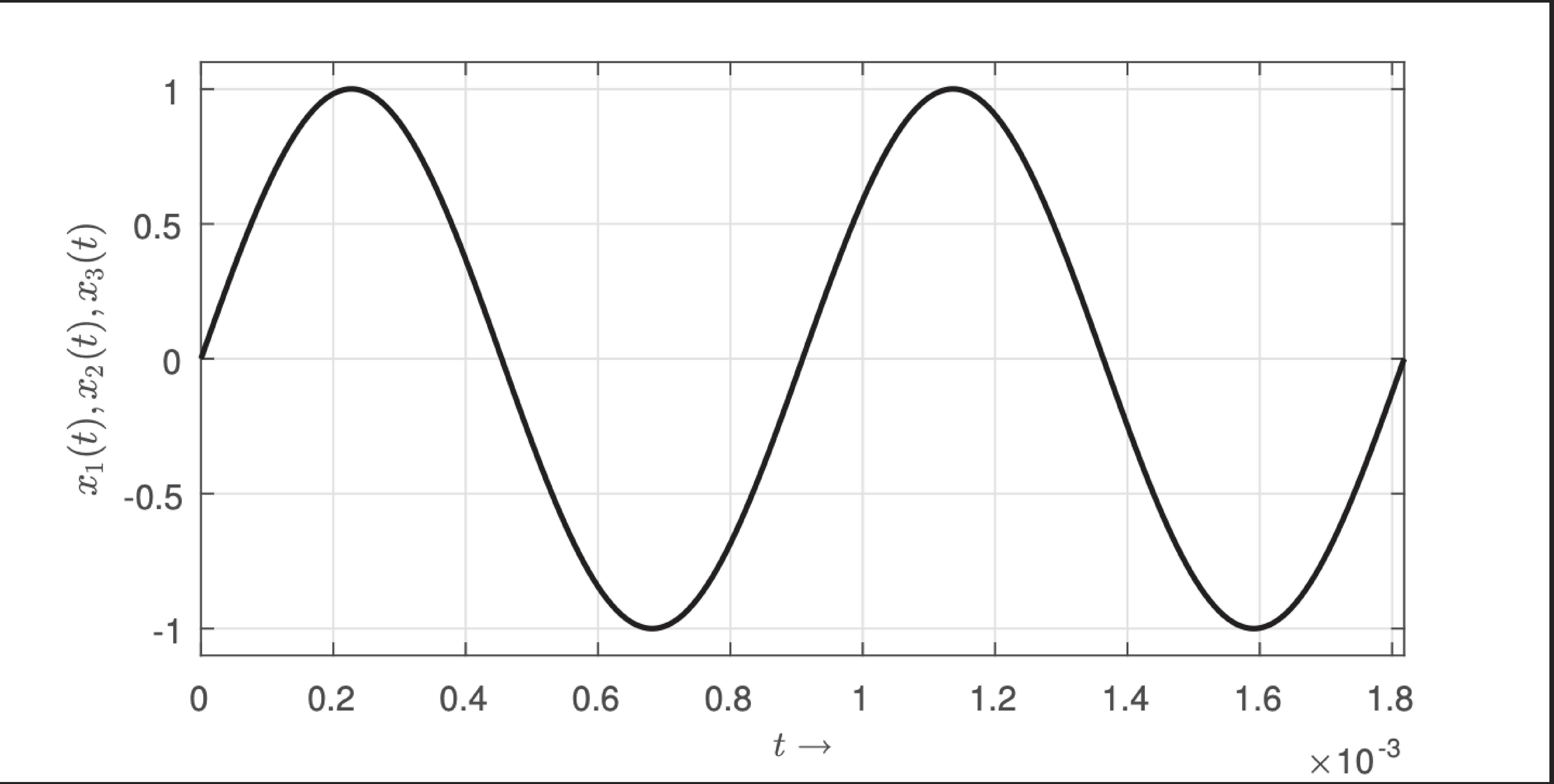
$$T_S = \frac{1}{f_S}$$



Typical Sample Rates

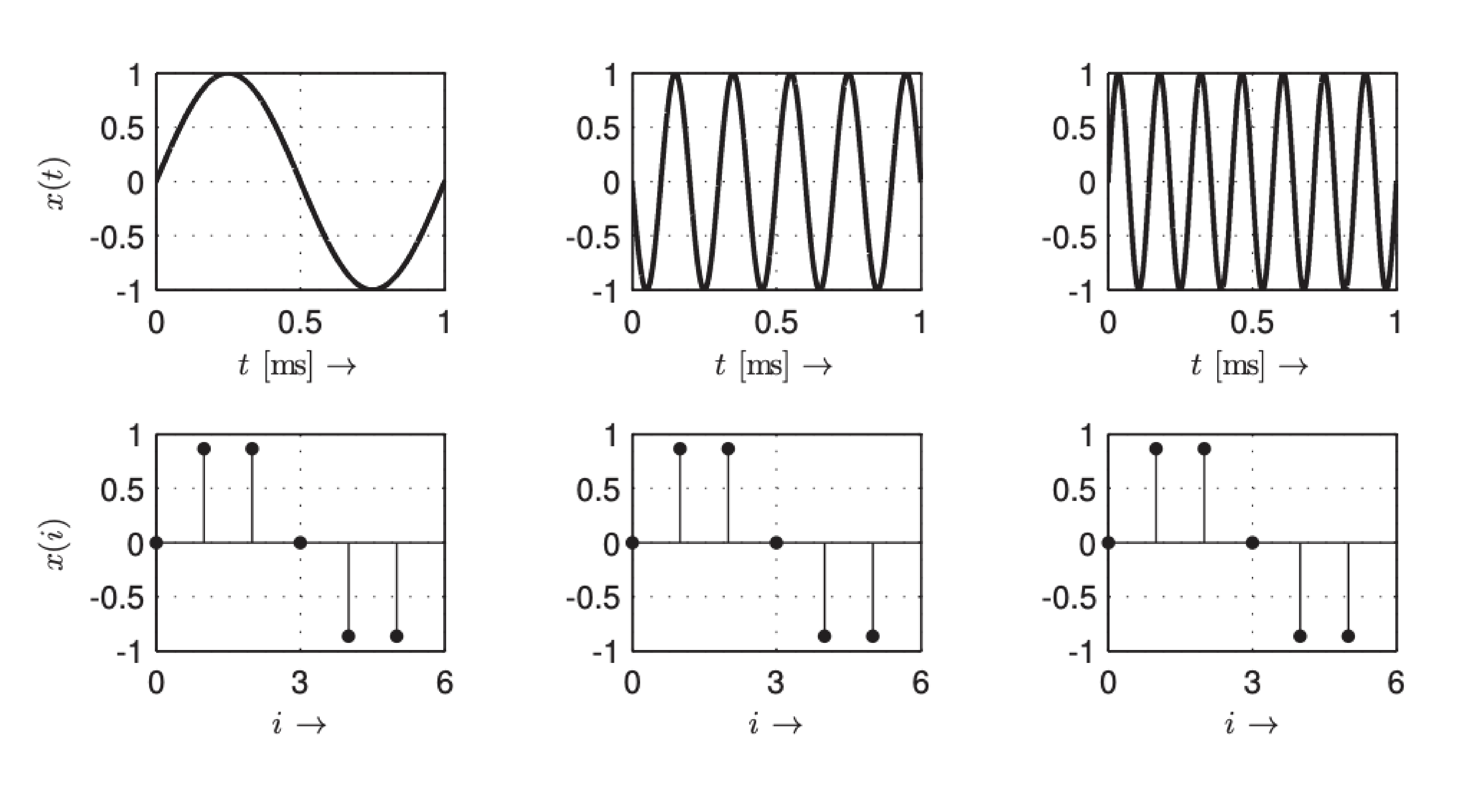
- 8-16kHz: Speech (phone)
- 44.1-48kHz: Consumer audio/music
- Higher: Production audio

Sampling Ambiguity



$$f_0 = [1, 5, 7\text{kHz}]$$

$$f_S = 6\text{kHz}$$



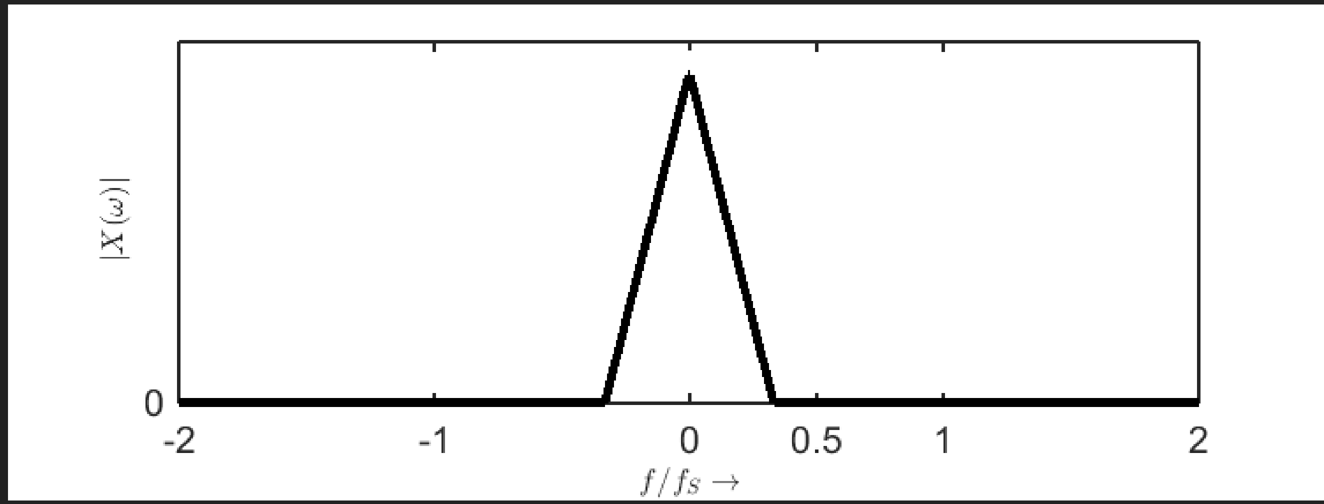
Sampling Ambiguity: Wagon Wheel Effect

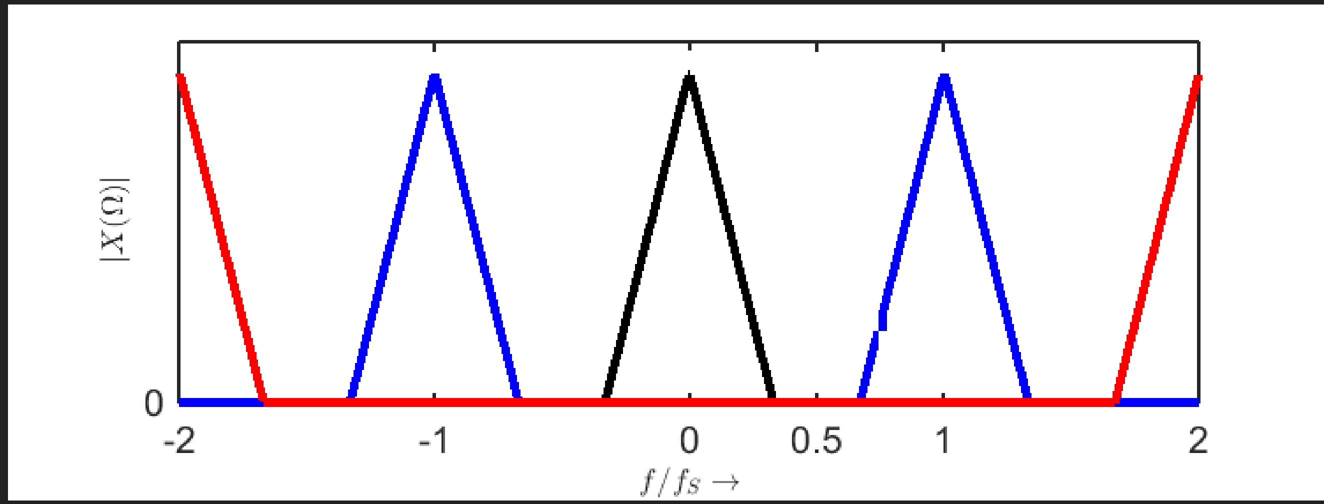
- $f_{wheel} < \frac{f_s}{2}$
  Speeding up
- $\frac{f_s}{2} < f_{wheel} < f_s$
  Slowing down
- $f_{wheel} = f_s$
  Standing still
- f_{wheel} far from f_s
  No effect

http://youtu.be/uENITui5_jU

All this ambiguity is simply the intuitive understanding of aliasing.

$$x(t) \mapsto X(\mathrm{j}\omega) \qquad x(t) \cdot \delta_T \mapsto X(\mathrm{j}\omega) * \delta_{\omega_T}$$

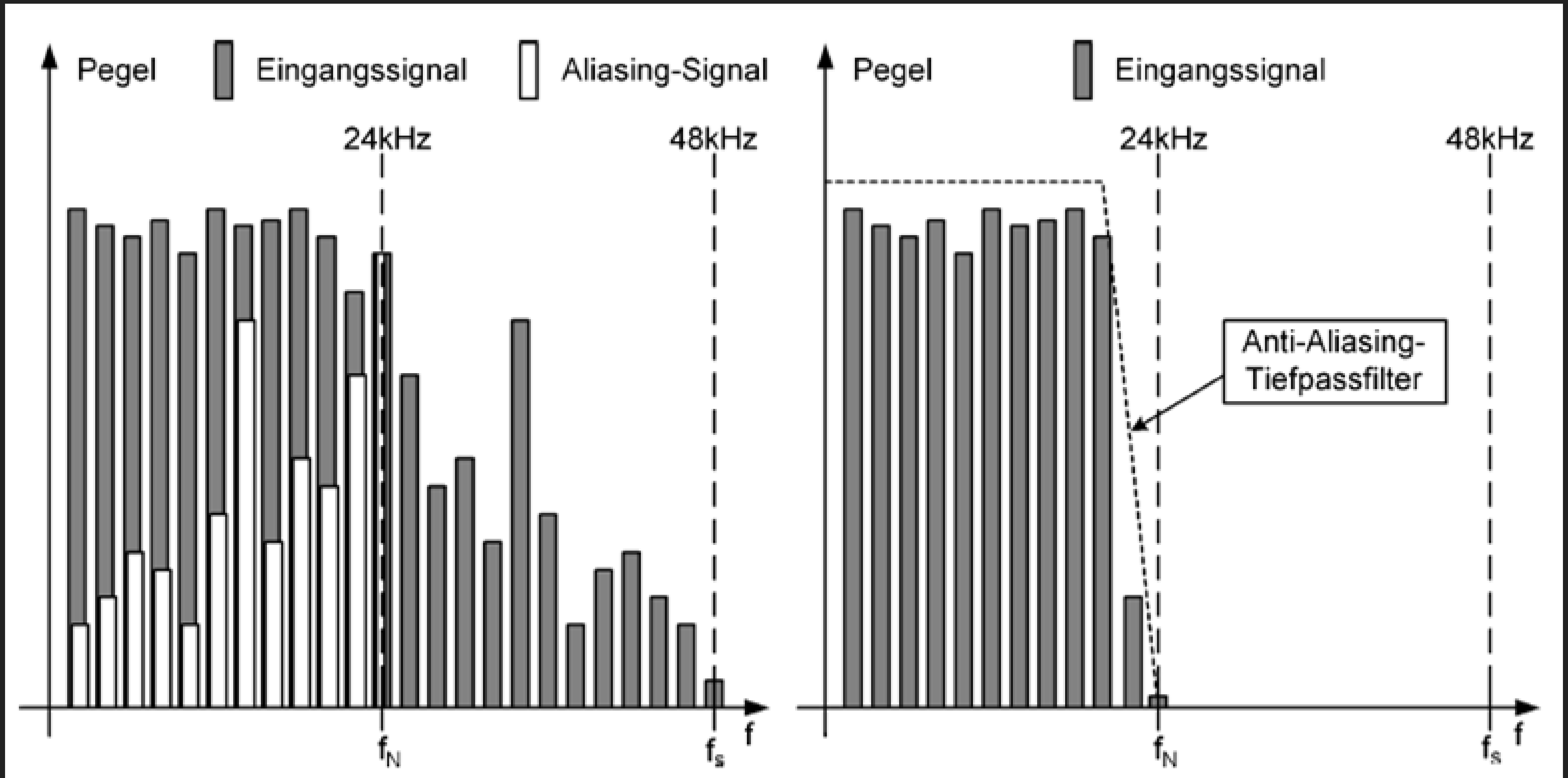




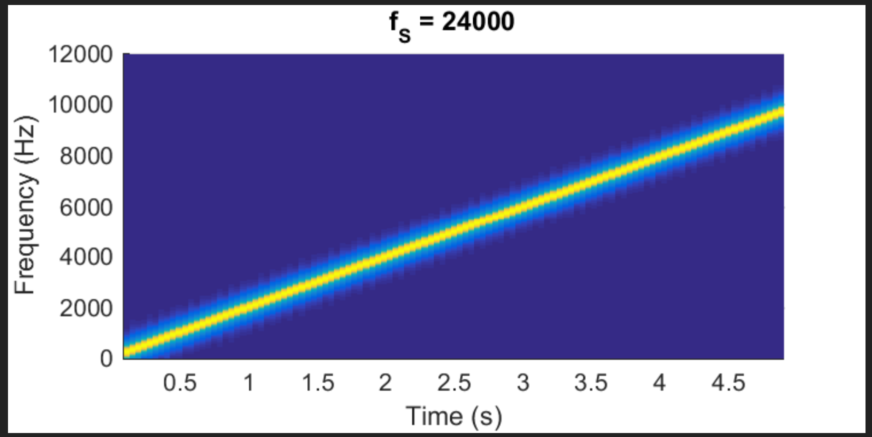
Sampling Theorem

A sampled audio signal can be reconstructed **without loss of information** if the sample rate f_S is higher than twice the bandwidth $f_{\max}$ of the signal.

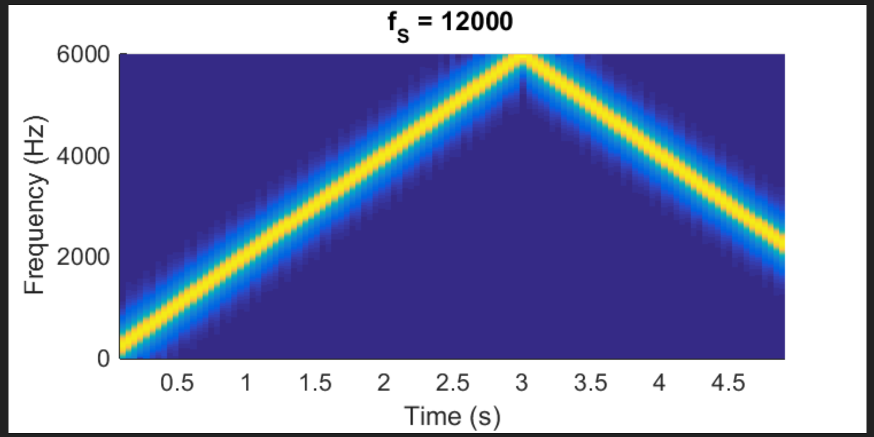
$$f_S > 2 \cdot f_{\max}$$


Pegel
Eingangssignal
Aliasing-Signal
24kHz
48kHz
f_N
f_s
f
Pegel
Eingangssignal
24kHz
48kHz
Anti-Aliasing-
Tiefpassfilter
f_N
f_s
f

Aliasing Examples: Sine sweep 100-10k @ 24, 12, 6k



0:00 / 0:05



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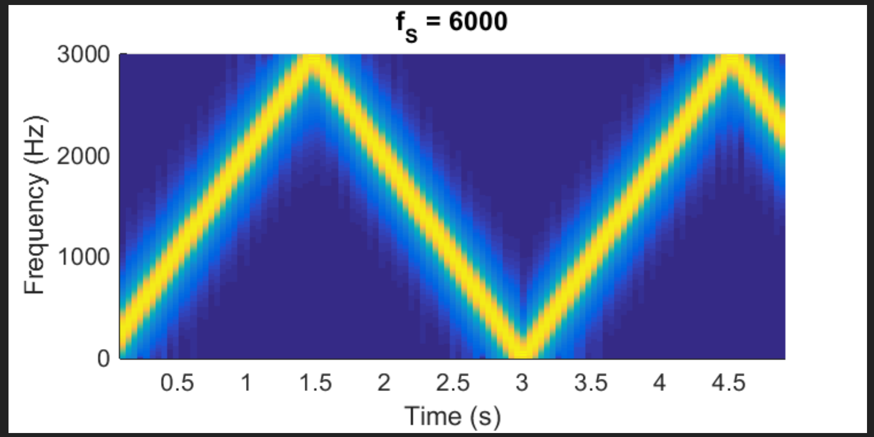

0:00 / 0:05

Aliasing Examples: Music

Big Band

- Original (48 kHz): 0:00 / 0:17
- Samples discarded (6 kHz): 0:00 / 0:17
- Downsampled w/ Anti-aliasing filter (6 kHz): 0:00 / 0:17

Sax

- Original (48 kHz): *Sorry, don't have it :(*
- Samples discarded (6 kHz): 0:00 / 0:27
- Downsampled w/ Anti-aliasing filter (6 kHz): 0:00 / 0:27

Summary

Continuous Input Signal

↓

1. **Anti-Aliasing Filter**

   Filtered continuous input signal

2. **Sampling**

   Sampled input signal

3. **Reconstruction Filter**

↓

Continuous Output Signal

Summary

Sampling Theorem

A Sampled audio signal can be reconstructed without loss of information if the sample rate f_S is higher than twice the bandwidth $f_{\max}$ of the signal

- Perfect reconstruction!
- Ensure accordance through filtering, otherwise aliasing (mirror frequencies)

Band of interest does not have to be base band $(0 \dots \frac{f_S}{2})$, but any band $(k \cdot \frac{f_S}{2} \dots (k+1) \cdot \frac{f_S}{2}$ as long as the **bandwidth** is not wider, and unwanted frequencies are filtered out.