Digital Signal Processing for Music

Part 14: Digital Filters I

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Filter - Broad Description

System that amplifies or attenuates certain components/aspects of a signal

Filter - Narrow

Linear time-invariant system for changing the magnitude and phase of specific frequency regions

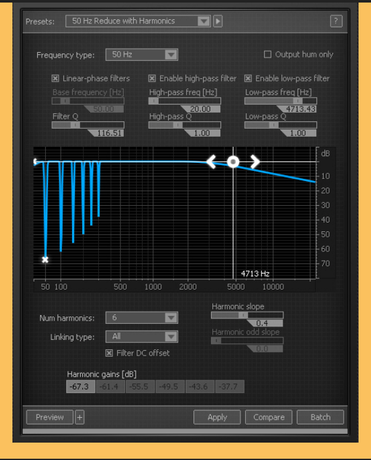
- Example for other type of filters:
  - Adaptive and time-variant (e.g., denoising)
- Examples for "real-world" filters:
  - Reverberation
  - Absorption
  - Echo

Audio Equalization

- Parametric EQs
- Graphic EQs

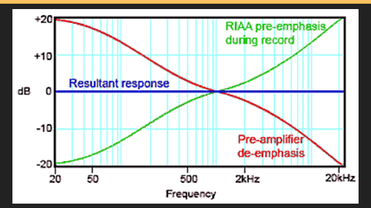
Removal of Unwanted Components

- Remove DC, rumble
- Remove hum
- Remove hiss



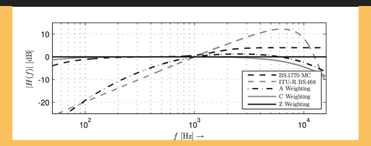
Pre-emphasis / De-emphasis

- Vinyl
- Old Dolby noise reduction systems



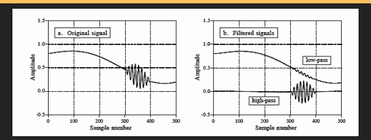
Weighting Function

- dBA, dBC, ...



(Parameter) **Smoothing**

- Smooth sudden changes



Reminder: System Theory

- Output of a system (filter) *y* computed by **convolution** of input *x* and impulse response *h*

$$y(t) = x(t) * h(t)$$

- This is equivalent to a frequency domain multiplication

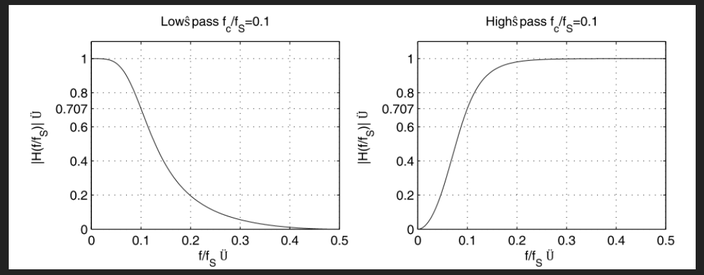
$$Y(\mathrm{j}\omega) = X(\mathrm{j}\omega) \cdot H(\mathrm{j}\omega)$$

$$H(\mathrm{j}\omega) = \frac{Y(\mathrm{j}\omega)}{X(\mathrm{j}\omega)}$$

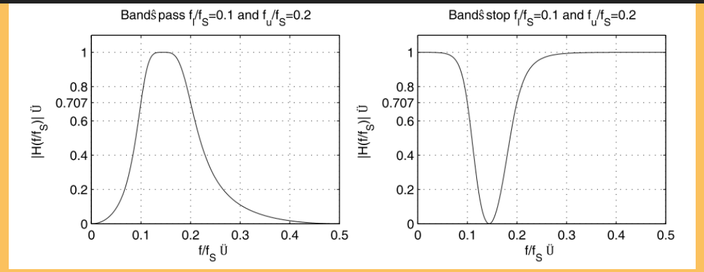
- **Transfer function** $H(\mathrm{j}\omega)$ is complex, often represented as:
  - **Magnitude** $|H(\mathrm{j}\omega)|$
  - **Phase** $\Phi_H(\mathrm{j}\omega)$

Common Transfer Function Shapes

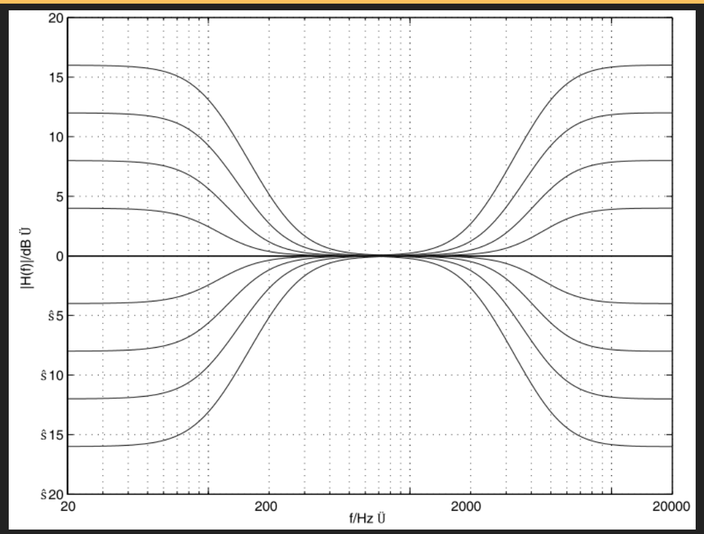
Low/high pass filters



Band pass/band stop filters

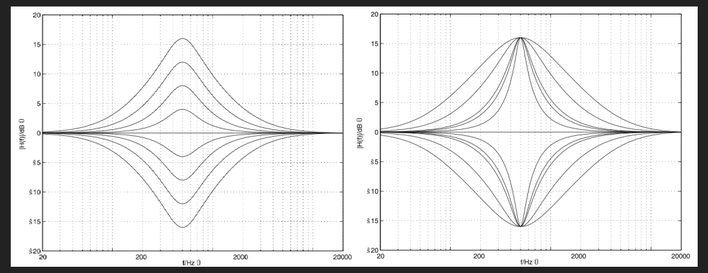


Low/high shelving filters

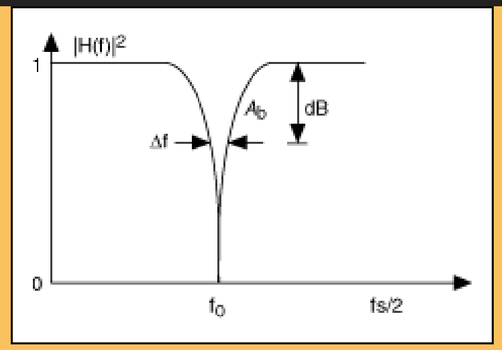


Common Transfer Function Shapes

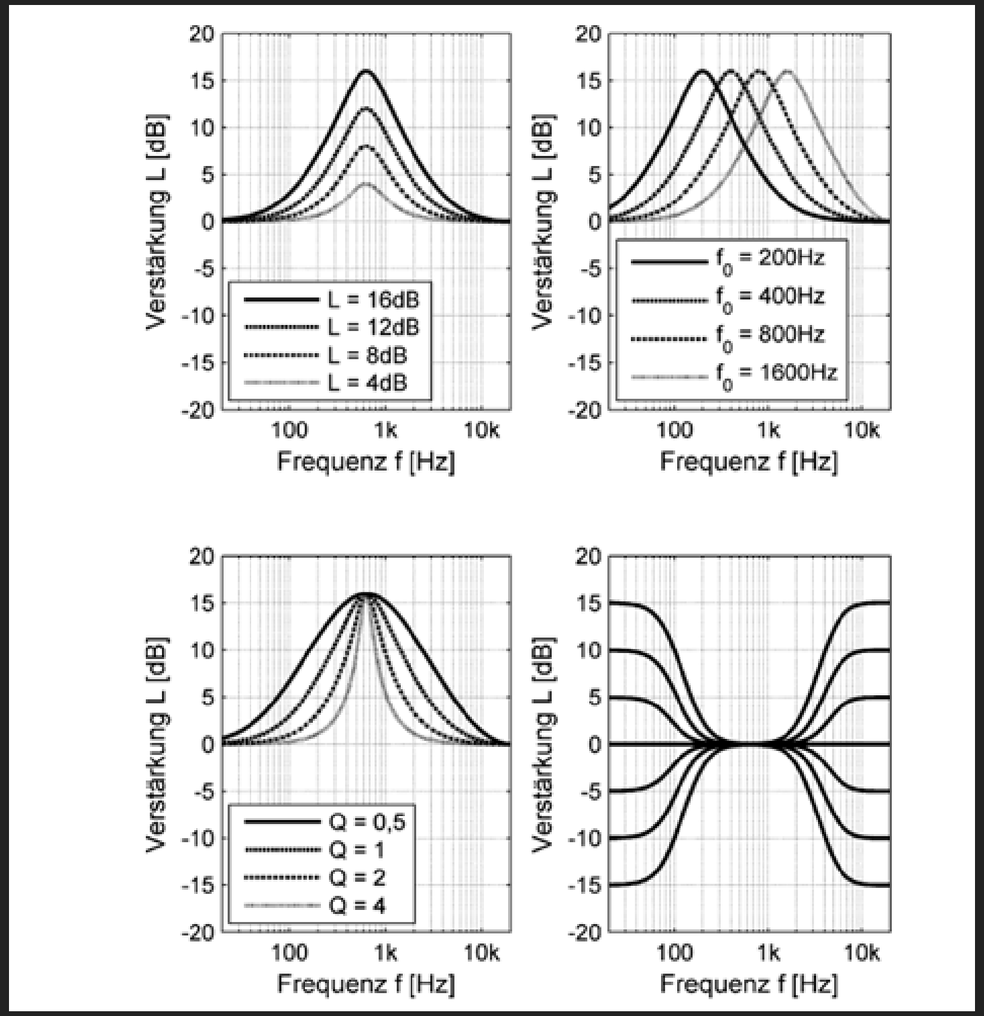
Peak filters



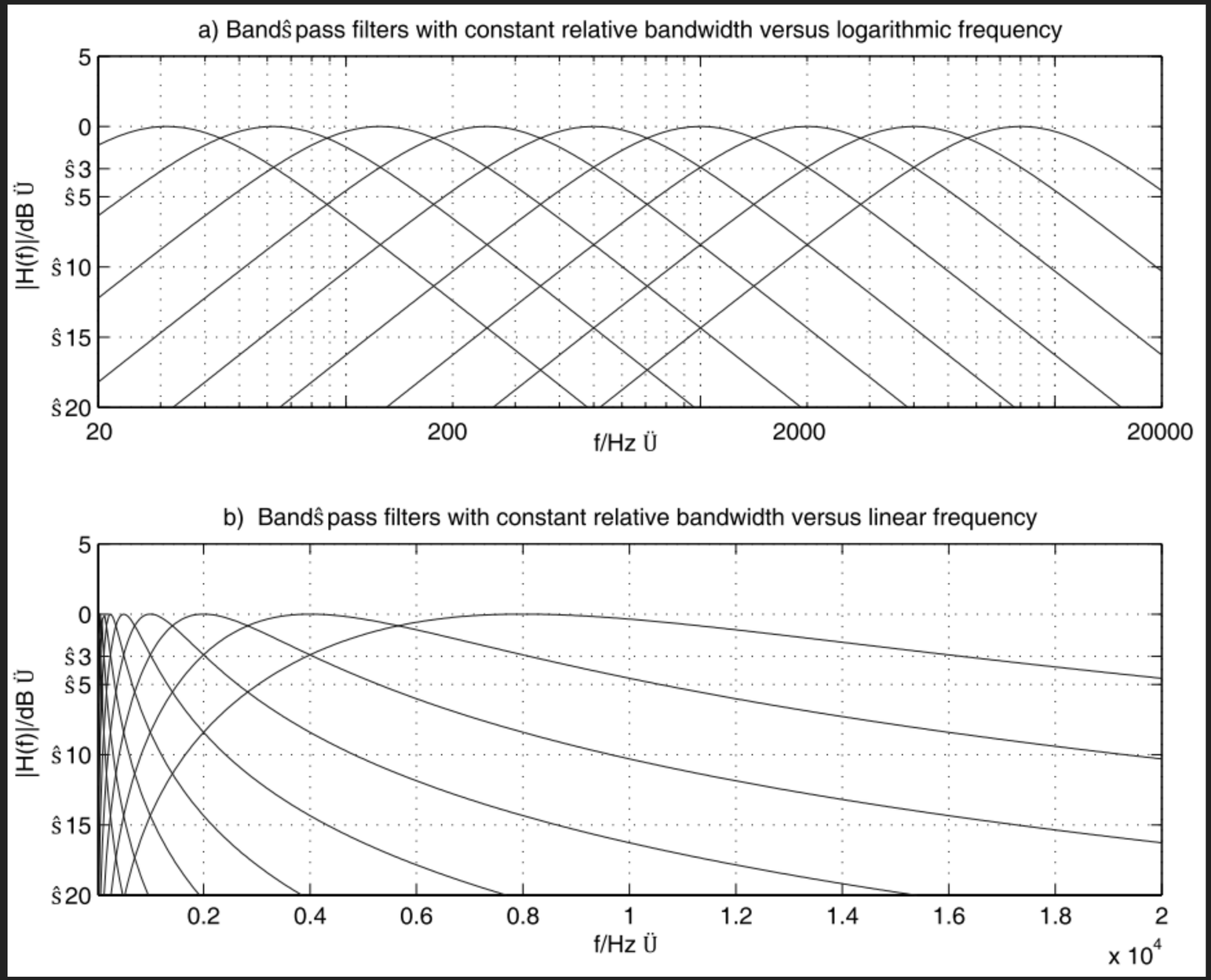
Resonanace/notch filters



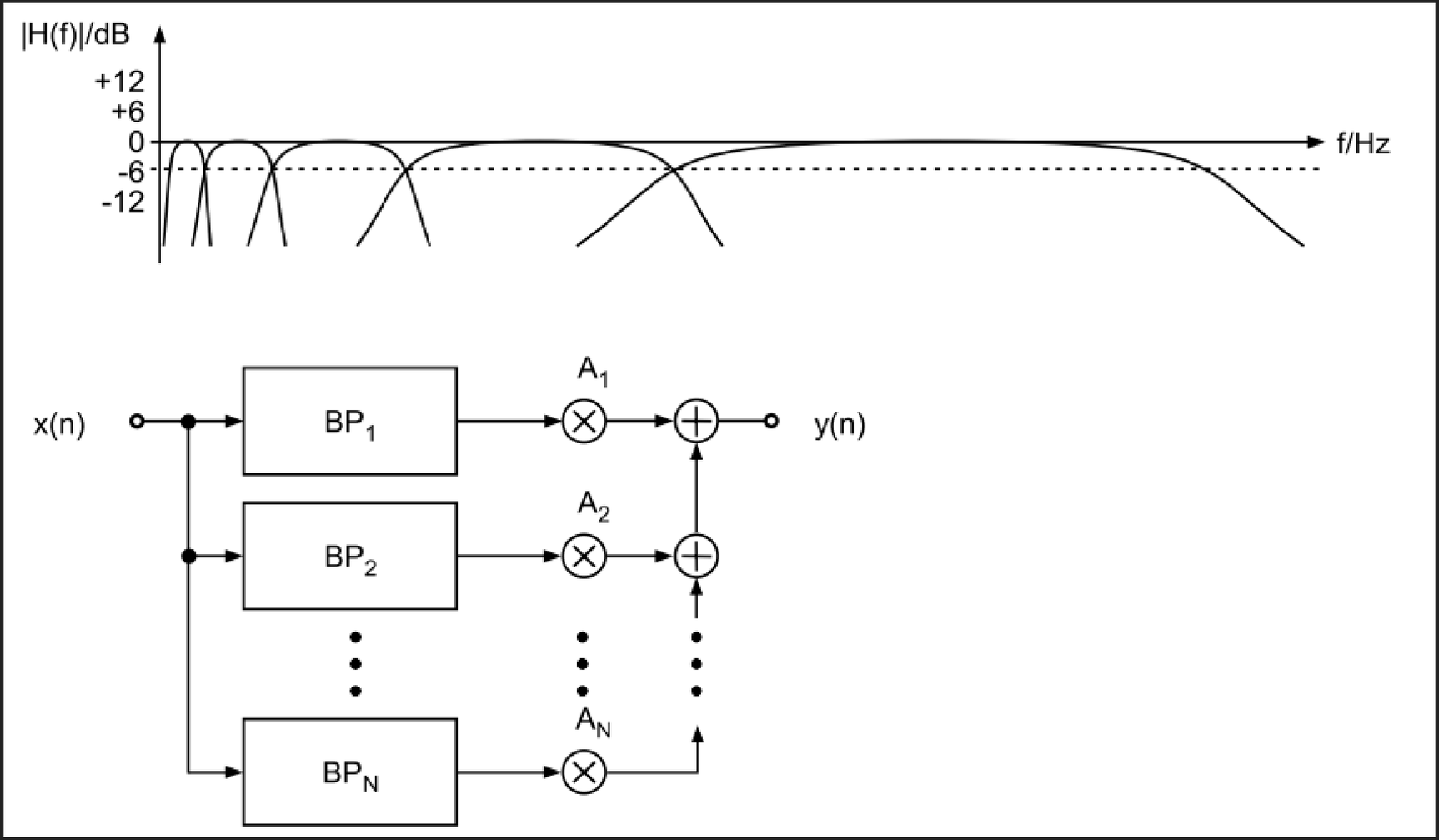
Common Transfer Function Shapes



Filter Banks

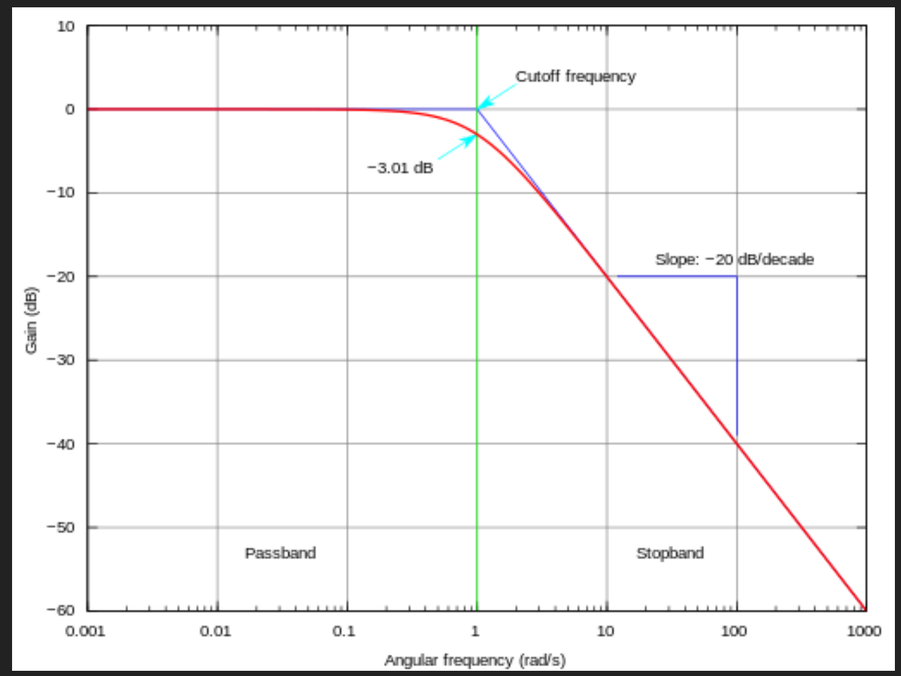


Filter Banks



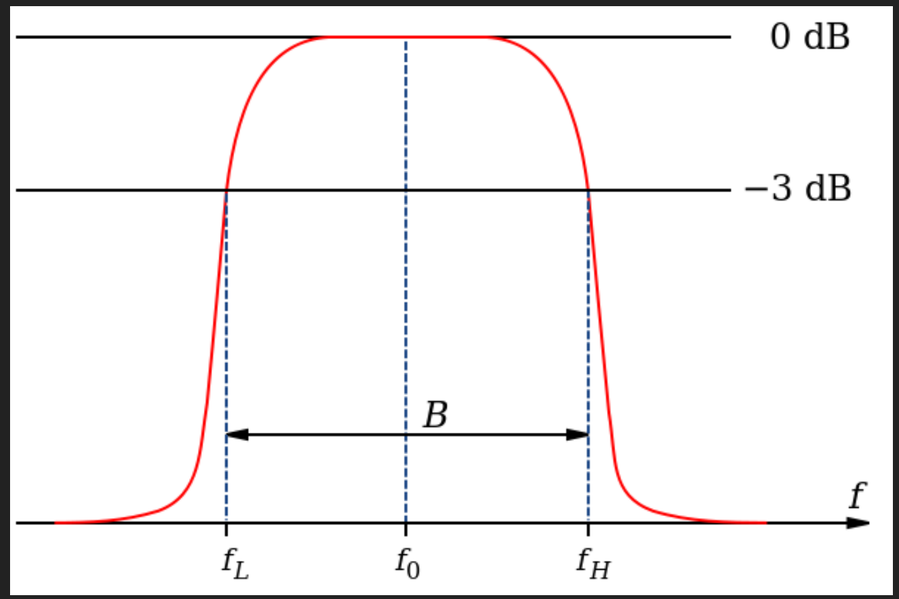
Filter Parameters - Lowpass/Highpass

- **Cut-off** frequency f_c
  - Frequency marking the transition of pass to stop band
  - -3dB of pass band level
- **Slope**/steepness
  - Measured in dB/octave or dB/decade
  - Typically directly related to filter order
- Sometimes: **resonance**
  - Level increase in narrow band around cut-off frequency



Filter Parameters - Bandpass/Bandstop

- **Center** frequency f_c
  - Frequency marking the center of the pass or stop band
- **Bandwidth** ΔB
  - Width of the pass band
  - at -3dB of max pass band level
- Possibly: **slope**
  - Typically directly related to filter order

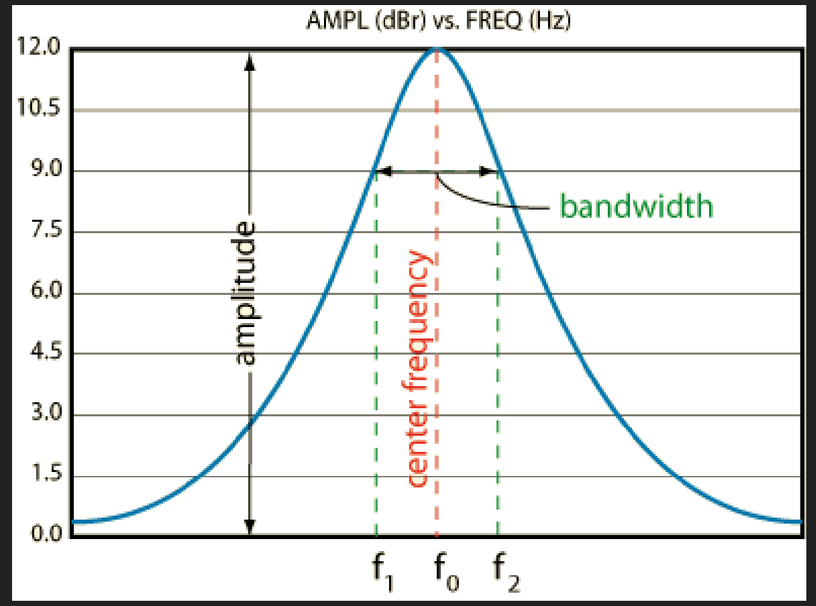


Filter Parameters - Peak

- **Center** frequency f_c
  - Frequency marking the center of the peak
- **Q factor** or **bandwidth** ΔB
  - Width of the bell
  - at -3dB of max gain

$$Q = \frac{f_c}{\Delta B}$$

- **Gain**
  - Amplification / attenuation in dB



Filter Parameters - Overview

Parameter	**Lowpass**	**Low Shelving**	**Band Pass**	**Peak**	**Resonance**
Frequency	Cut-off	Cut-off	Center	Center	Center
Bandwidth/Q	Res. gain	--	ΔB	Q	--
Gain	--	Yes	--	Yes	--

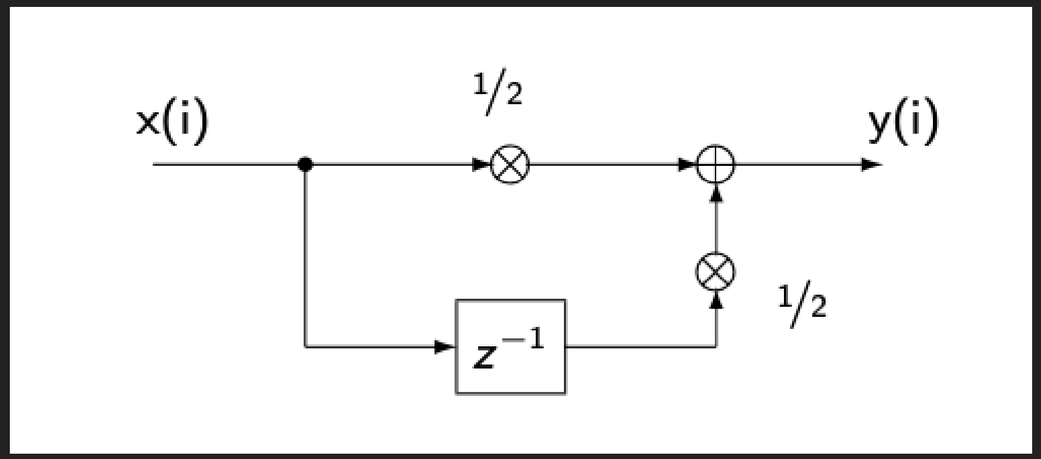
Digital Filter Description

A filter is defined by its

- **Complex transfer function** $H(\mathrm{j}\omega)$
- Or … its **Impulse response** $h(t)$
- Or … its **List of pole and zero positions in the Z-Plane**

$$H(\mathrm{j}\omega) = \mathfrak{F}\{h(t)\}$$

Example Filter 1



$$y(i) = 0.5 \cdot x(i) + 0.5 \cdot x(i-1)$$

Example Filter 1: Transfer Function

$$y(i) = 0.5 \cdot x(i) + 0.5 \cdot x(i-1)$$

$$H(z) = 0.5 + 0.5 \cdot z^{-1}$$

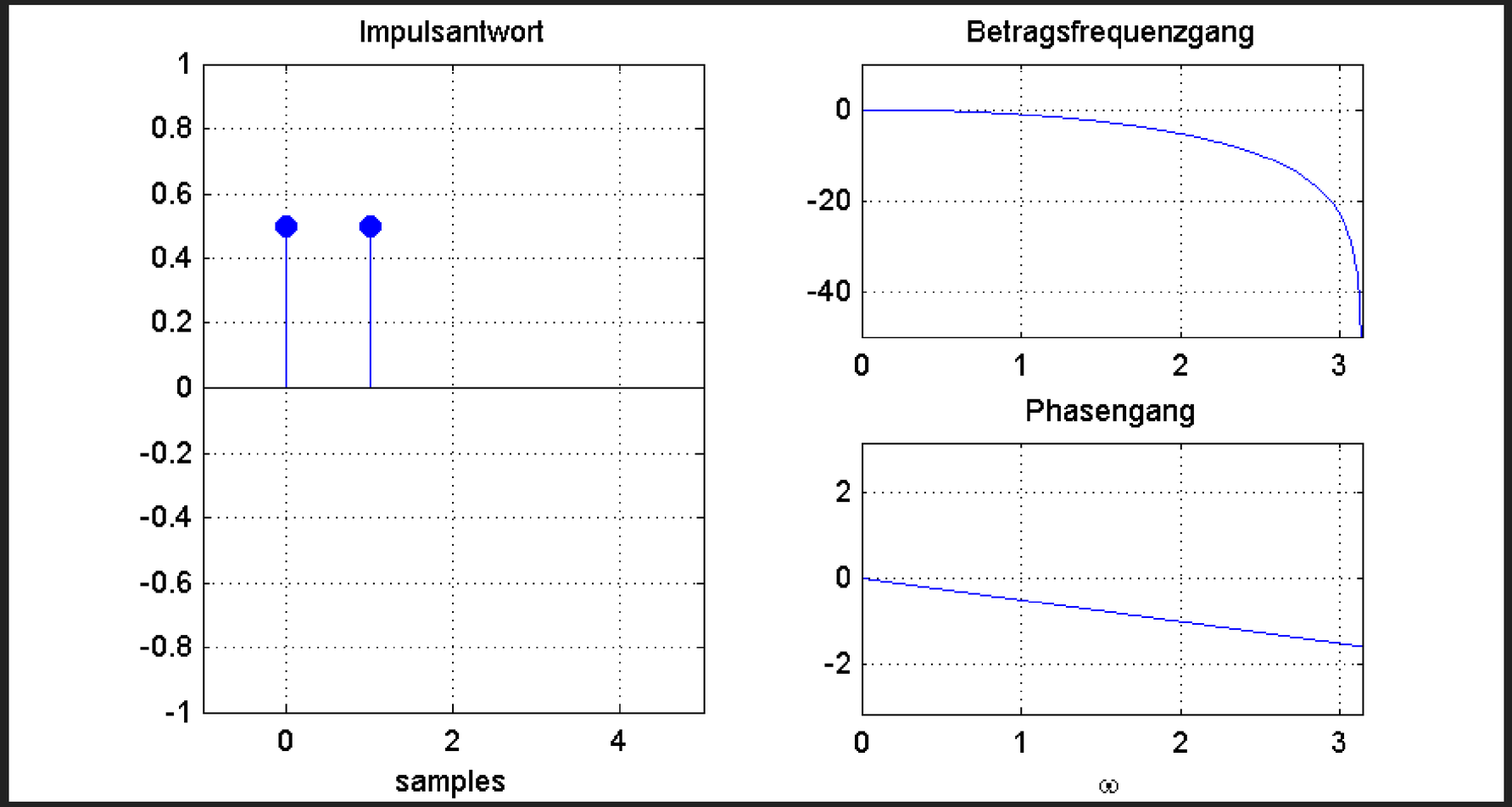
$$H(\mathrm{j}\omega) = 0.5 + 0.5 \cdot e^{-\mathrm{j}\omega}$$

$$|H(\mathrm{j}\omega)| = 0.5 \cdot \left| e^{-\mathrm{j}\frac{\omega}{2}} \cdot \left(e^{\mathrm{j}\frac{\omega}{2}} + e^{-\mathrm{j}\frac{\omega}{2}} \right) \right|$$

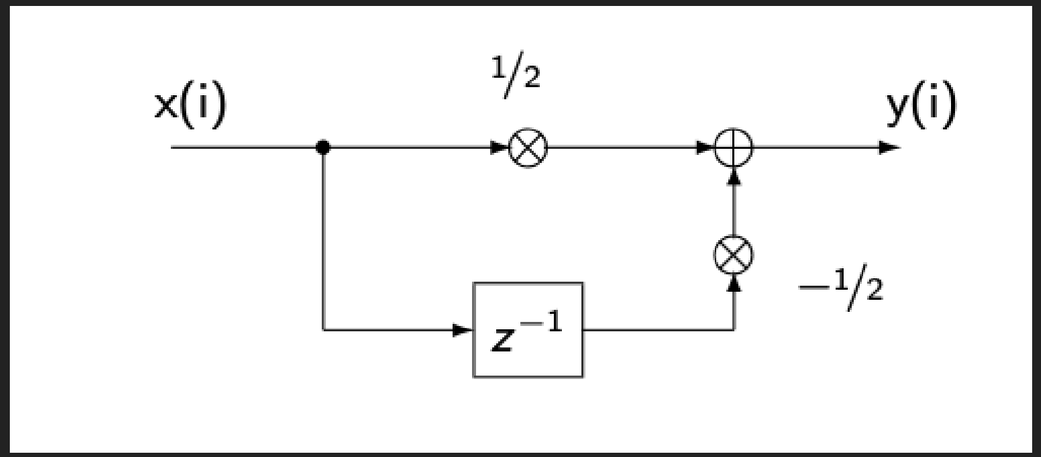
$$= 0.5 \cdot \underbrace{\left| e^{-\mathrm{j}\frac{\omega}{2}} \right|}_{1} \cdot \underbrace{\left| \left(e^{\mathrm{j}\frac{\omega}{2}} + e^{-\mathrm{j}\frac{\omega}{2}} \right) \right|}_{\left| 2\cos\left(\frac{\omega}{2}\right) \right|}$$

$$= \left| \cos\left(\frac{\omega}{2}\right) \right|$$

Example Filter 1: Visualization



Example Filter 2

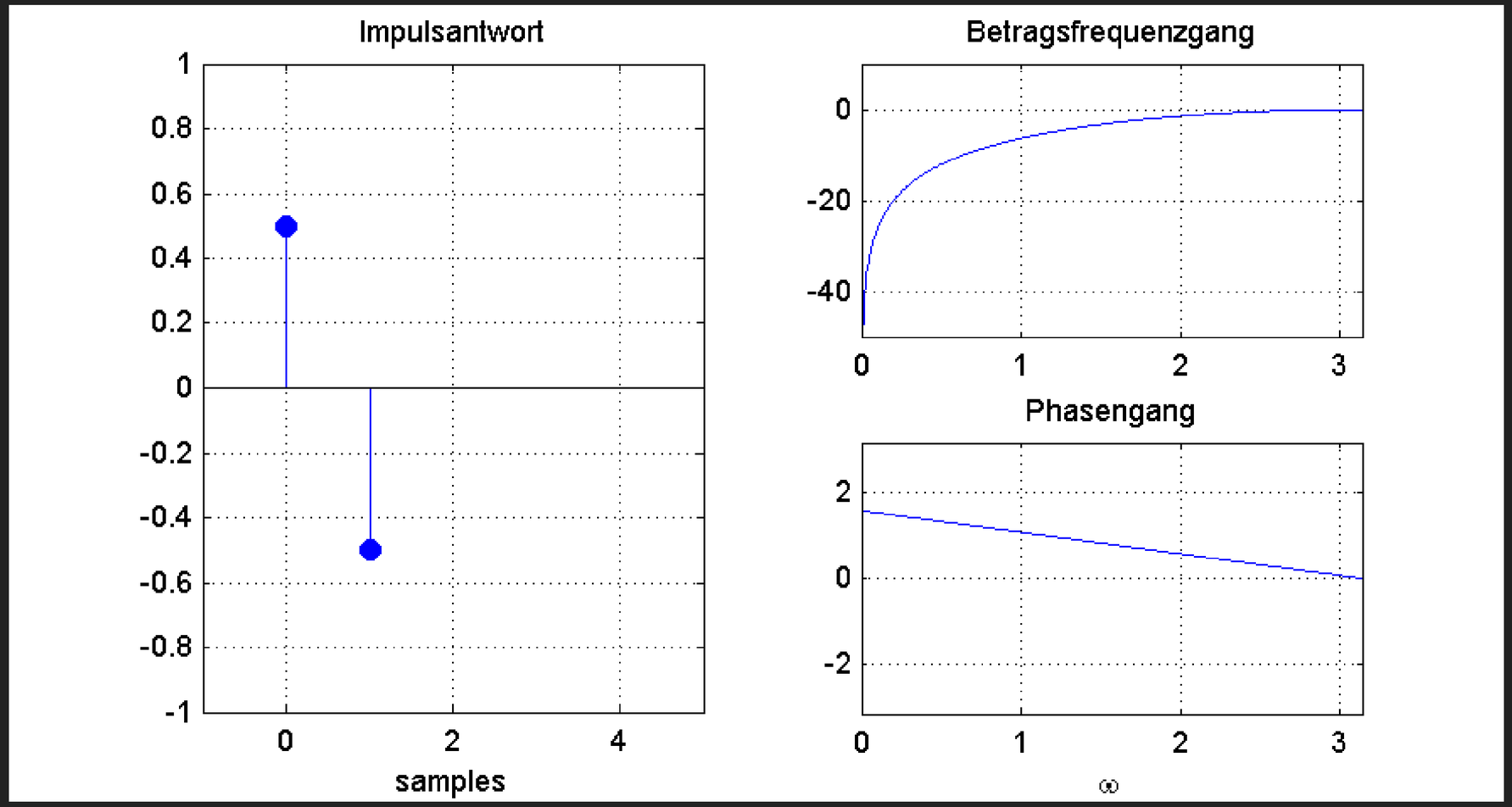


$$y(i) = 0.5 \cdot x(i) - 0.5 \cdot x(i-1)$$

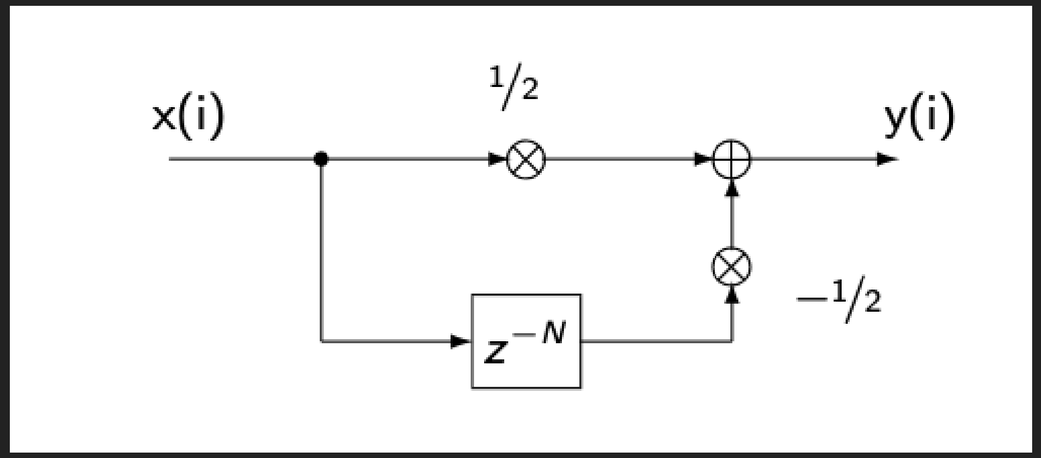
$$H(z) = 0.5 - 0.5 \cdot z^{-1}$$

$$|H(\mathrm{j}\omega)| = \left|\sin\left(\frac{\omega}{2}\right)\right|$$

Example Filter 2: Visualization



Example Filter 3



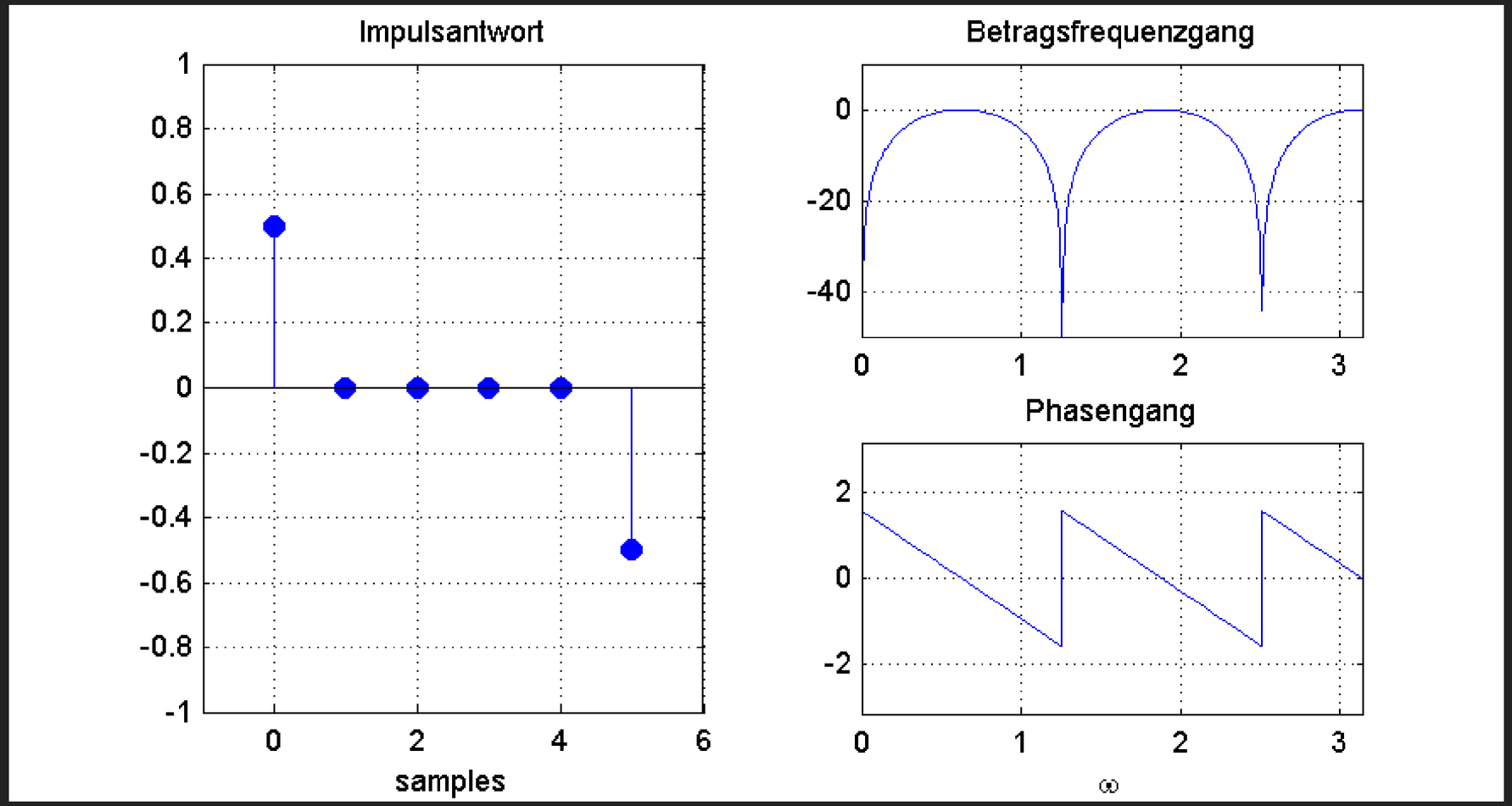
$$y(i) = 0.5 \cdot x(i) - 0.5 \cdot x(i - N)$$

$$H(z) = 0.5 - 0.5 \cdot z^{-N}$$

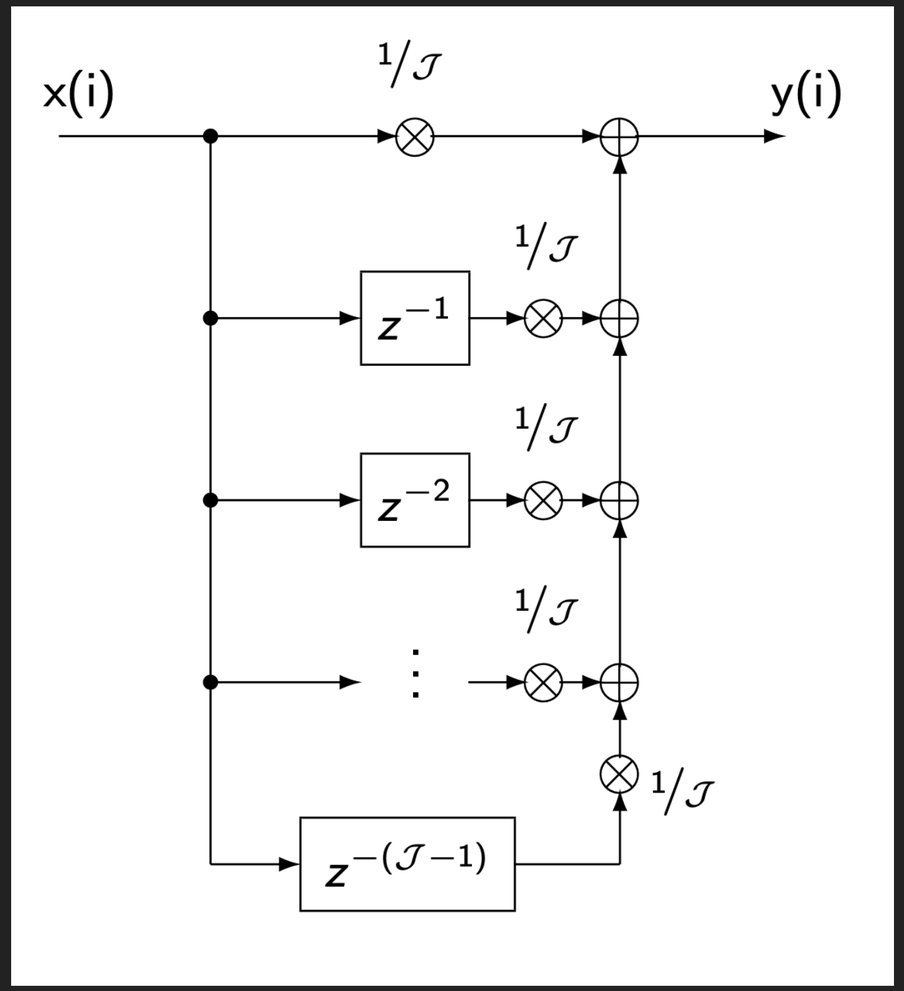
$$|H(\mathrm{j}\omega)| = 0.5 \cdot \left| e^{-\mathrm{j}\frac{N\omega}{2}} \cdot \left(e^{\mathrm{j}\frac{N\omega}{2}} - e^{-\mathrm{j}\frac{N\omega}{2}} \right) \right|$$

$$= \left| \sin\left(\frac{N\omega}{2} \right) \right|$$

Example Filter 3: Visualization

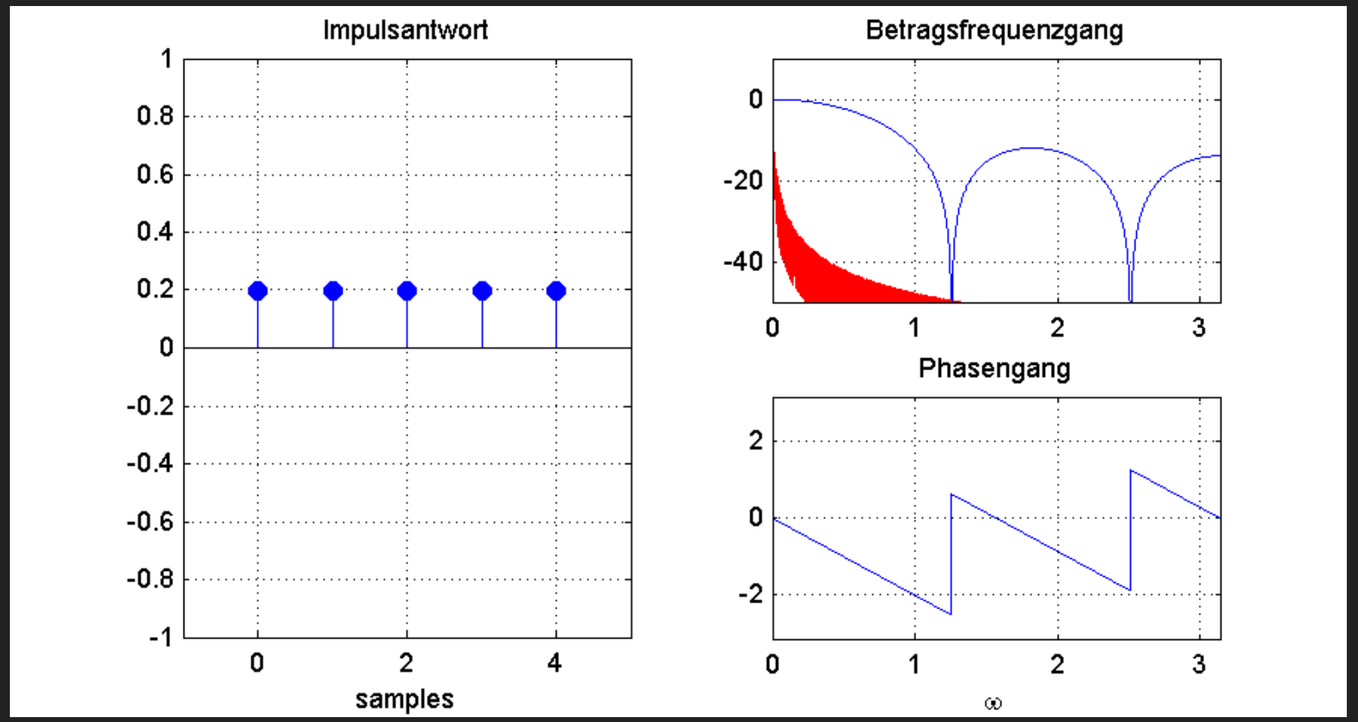


Example Filter 4



$$y(i) = \frac{1}{\mathcal{J}} \sum_{j=0}^{\mathcal{J}-1} x(i-j)$$

Example Filter 4: Transfer Function/Visualization



$$H(\mathrm{j}\omega) = e^{-\mathrm{j}\mathcal{J}\frac{\omega}{2}} \frac{\sin\left(\mathcal{J}\cdot\frac{\omega}{2}\right)}{\mathcal{J}\cdot\sin\left(\frac{\omega}{2}\right)}$$

Example Filter 4: Recursive Implementation

Not applicable with windowed coefficients!