Digital Signal Processing for Music

Part 11: Non-Linear Quantization

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Review: Linear Quantization SNR

$$SNR = 6.02 \cdot w + c_S [\mathrm{dB}]$$

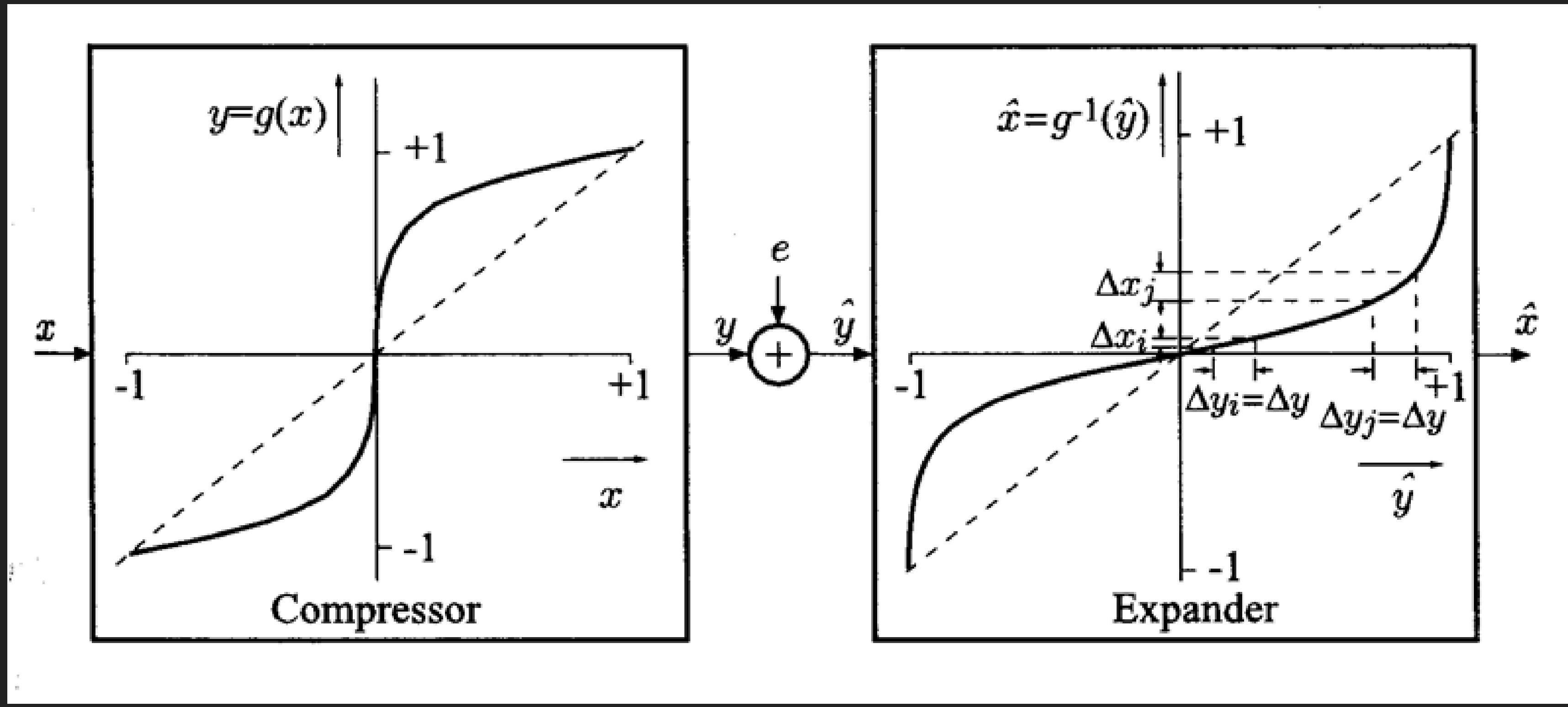
c_S depends on signal's PDF (and scaling)'

PDF	SNR
Square Wave	$c_S = 4.8$
Sine Wave	$c_S = 1.8$
Rectangle	$c_S = 0$
Triangle	$c_S \approx -3$
Gaussian	$c_S \approx -7$
LaPlace	$c_S \approx -9$
Speech	$c_S \approx -10 \ldots -15$

How can we quantize frequent signal values at higher resolution?

Approach 1

1. Flatten PDF (companding)
2. Linear quantization
3. Extract signal (expanding)



How can we quantize frequent signal values at higher resolution?

Approach 1

1. Flatten PDF (companding)
2. Linear quantization
3. Extract signal (expanding)

Approach 2

1. Adapt quantization step size to PDF

Both approaches are equivalent in their result

A-Law Quantization (ITU-T G.711)

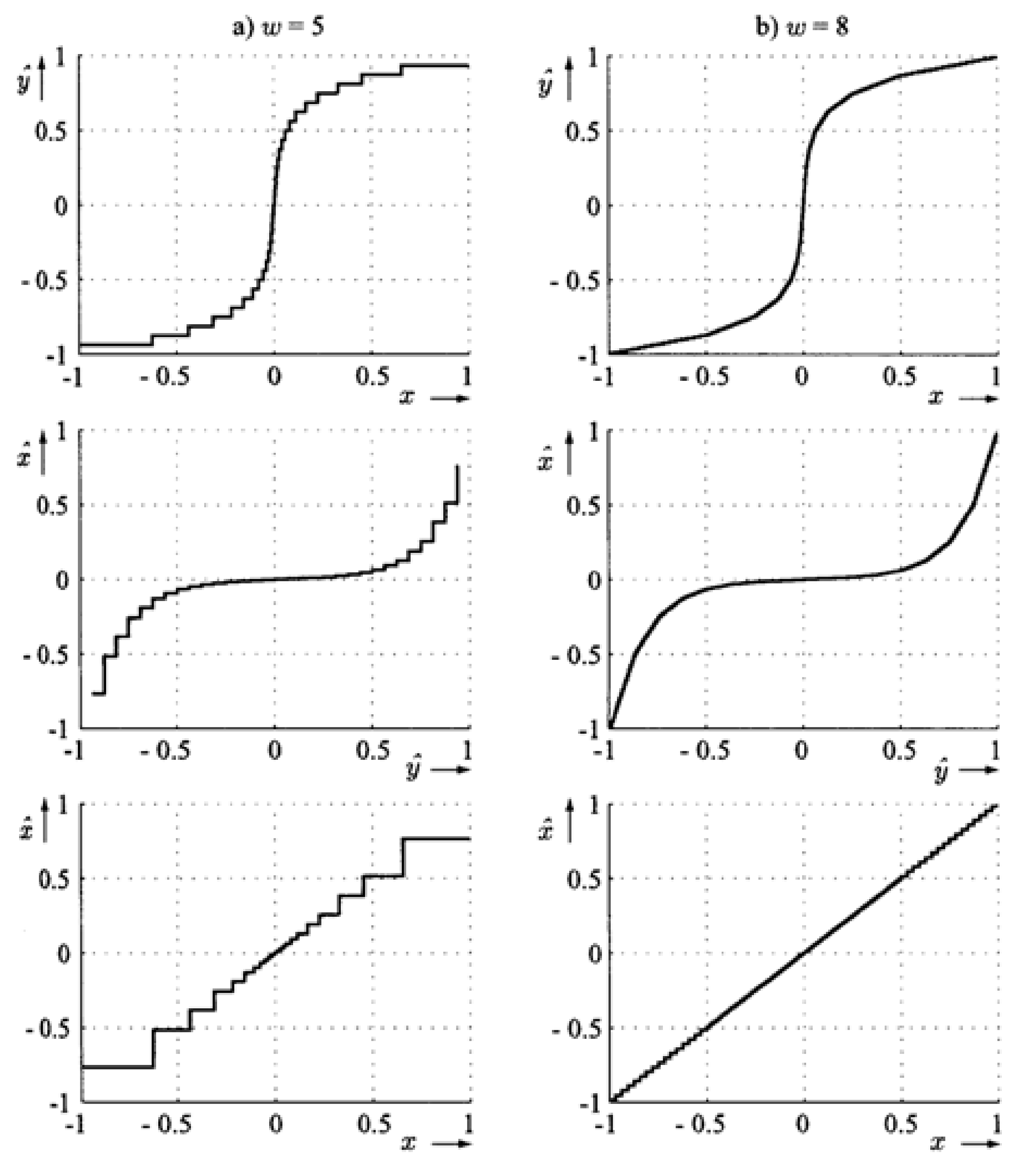
$$F(x) = sign(x) \begin{cases} \frac{A|x|}{1+\log(A)}, & |x| \leq \frac{1}{A} \\ \frac{1+\log(A|x|)}{1+\log(A)}, & \frac{1}{A} \leq |x| \leq 1 \end{cases}$$

$$F^{-1}(y) = sign(y) \begin{cases} \frac{|y|(1+\log(A))}{A}, & |y| \leq \frac{1}{1+\log(A)} \\ \frac{\exp\left(|y|(1+\log(A))-1\right)}{A}, & \frac{1}{1+\log(A)} \leq |y| \leq 1 \end{cases}$$

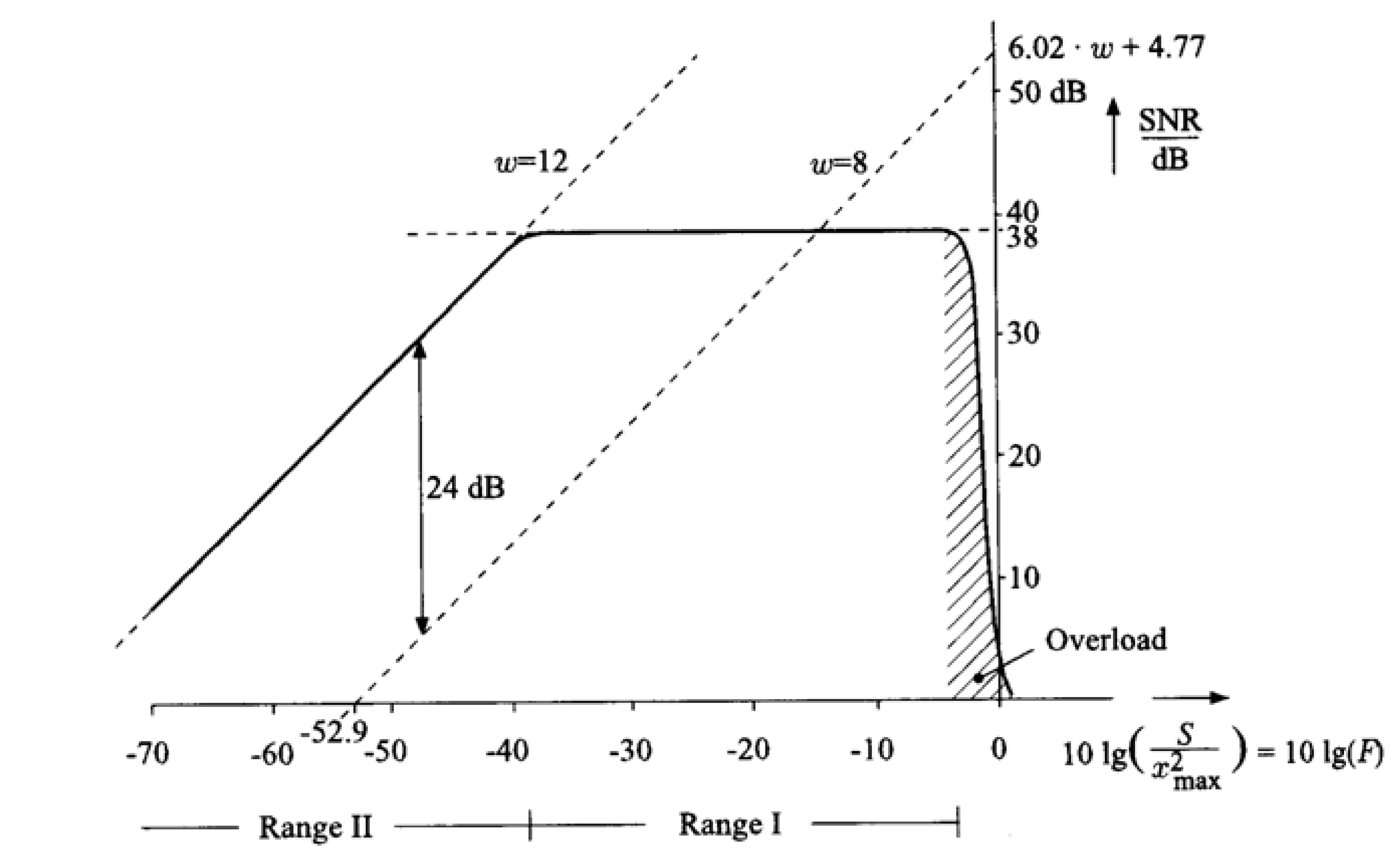
with $A = 87.7$

- Linear and high resolution for small amplitudes
- Log and increasingly low resolution for high amplitudes

A-Law Quantization: Visualization



A-Law Quantization: SNR



- **Range I**: SNR is linear regardless of input level
- **Range II**: SNR increases with input level

µ-Law Quantization (ITU-T G.711)

$$F(x) = sign(x)\frac{\log(1+\mu|x|)}{\log(1+\mu)}$$

$$F^{-1}(y) = sign(y)\frac{1}{\mu}\left((1+\mu)^{|y|} - 1\right)$$

Compared to A-Law:

- Higher dynamic range
- Higher error at small amplitudes

Summary

- **Advantages** of non-linear quantization
  - Takes advantage of non-uniform distriubtion of input
  - In line with non-linear loudness perception of the ear
  - → Similar perceptual quality as higher resolution linear quantization

- **Disadvantages**
  - Processing not easily implemented in non-linear amplitude space
  - → Only used for transmission