Digital Signal Processing for Music

Part 13: Improving (Re-)Quantization Quality

Andrew Beck

Introduction

Quantization error properties are fixed, so there is no way of improving the quality

Or is there????

"Cheating" for Better Quality

Improving perceptual quality of errors due to:

- Quantization
  - Oversampling
  - Noise Shaping
- Re-Quantization / Word Length Reduction
  - Dither
  - Noise Shaping

Oversampling

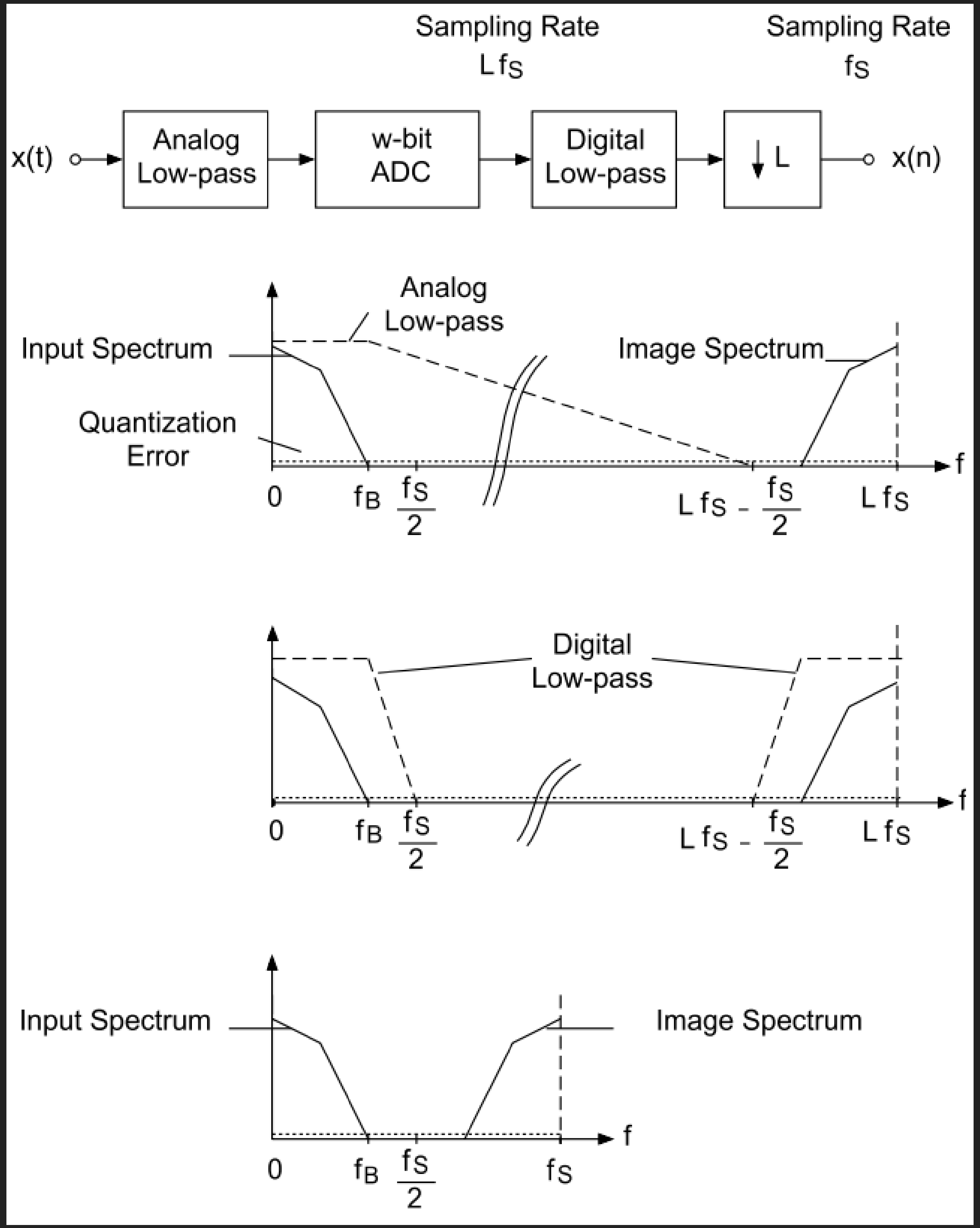
- Recording at higher sample rates and downsampling
- Allows use of less steep anti-aliasing filters
- Also improves quantization error

Quantization error properties

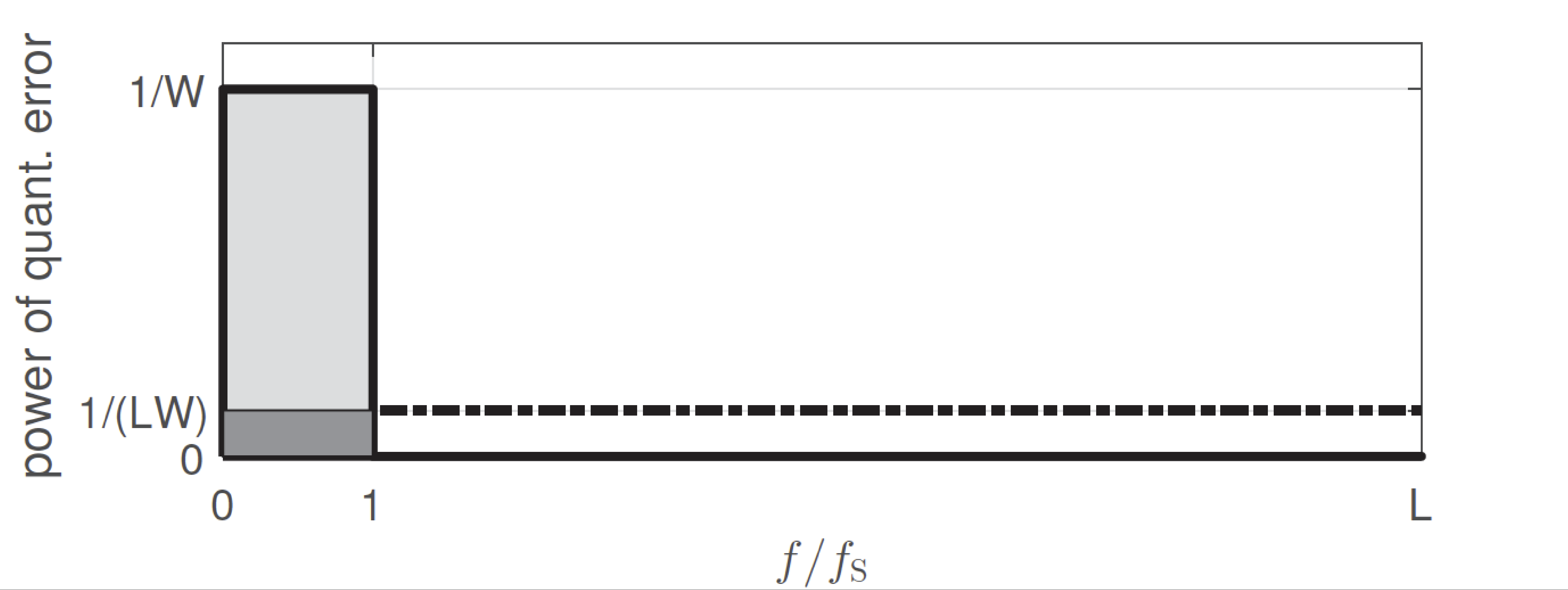
- *White* noise (ie **flat spectrum**)
- Noise power *sample rate independent*

$$|Q(\mathrm{j}\omega)|^2 \sim \frac{\Delta^2}{12 \cdot \omega_\mathrm{S}}$$

Oversampling Process



Quantization Noise Spectrum for Oversampling Amount



SNR Gain from Oversampling

$$
\begin{aligned}
|Q(\mathrm{j}\omega)|^2 &= \frac{\Delta^2}{12 \cdot \omega_S^*} \\
&= \frac{\Delta^2}{12 \cdot L \cdot \omega_S} \\
W_{\mathrm{Q,LP}}^* &= \frac{\Delta^2}{12 \cdot L} \\
\Rightarrow \\
SNR^* &= 6.02 \cdot w + 10 \log_{10}(L) + c_S
\end{aligned}
$$

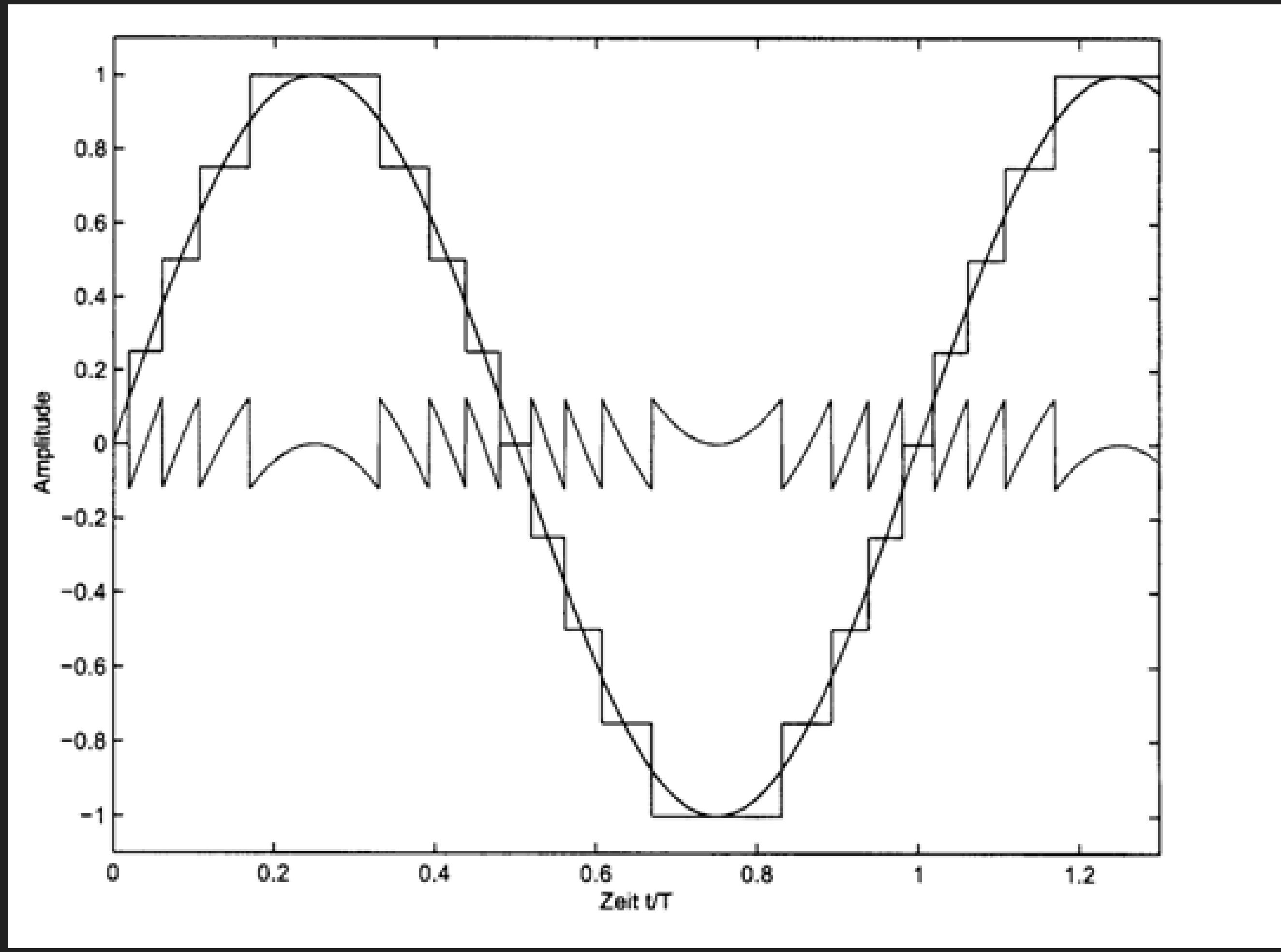
Oversampling Summary

$$SNR = 6.02 \cdot w + c_S + 10\log_{10}(L)$$

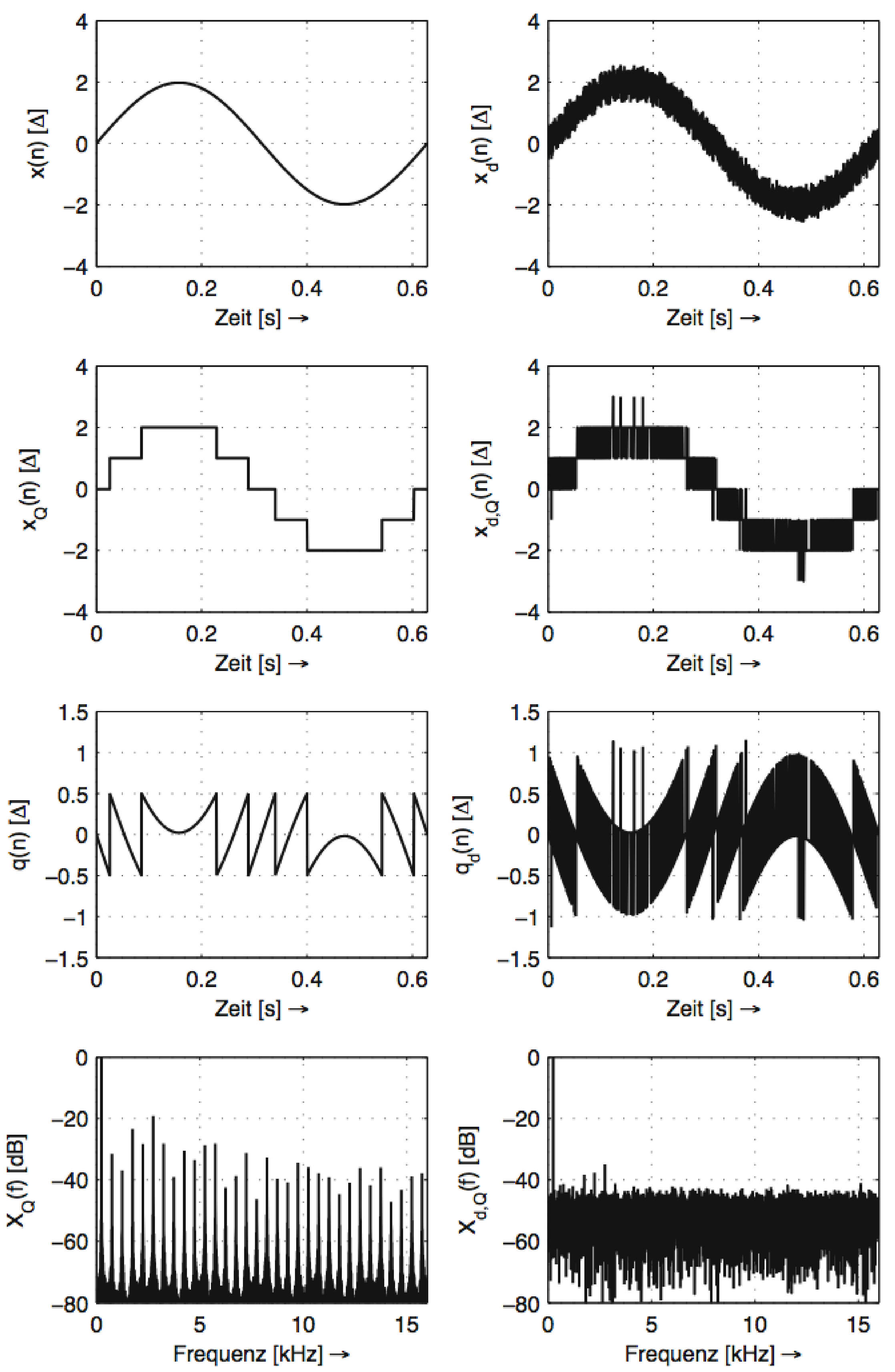
Every doubling of f_S **adds ~3dB SNR**

Dither

- Previous assumption: **Quantization error is white noise** (ie, rect)
  - **No correlation** between signal and quantization error
- **Not true for:**
  - Low signal level
  - Low signal frequency

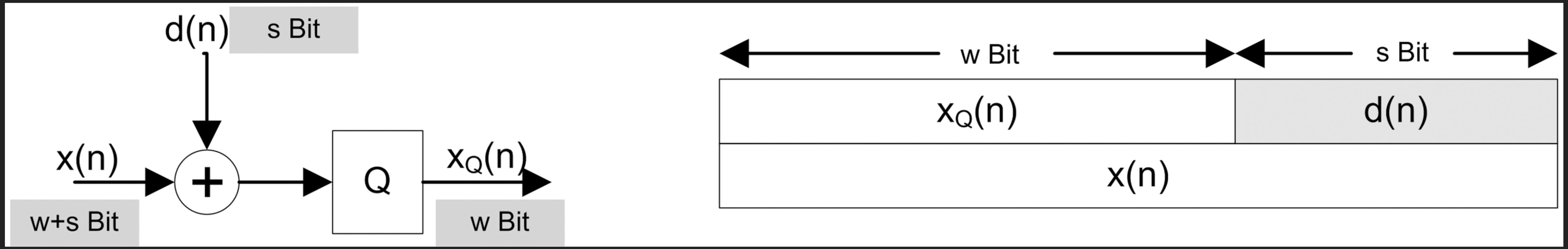


Solution: Add noise before quantization (dither)



x(n) [Δ]
Zeit [s] →
x_d(n) [Δ]
Zeit [s] →
x_Q(n) [Δ]
Zeit [s] →
x_d,Q(n) [Δ]
Zeit [s] →
q(n) [Δ]
Zeit [s] →
q_d(n) [Δ]
Zeit [s] →
X_Q(f) [dB]
Frequenz [kHz] →
X_d,Q(f) [dB]
Frequenz [kHz] →

Dither Process

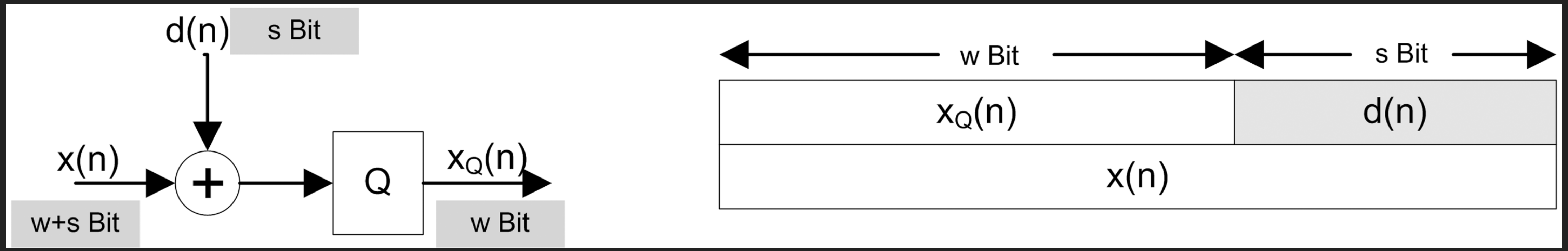


Dither Simple Example

Input signal: DC at $1.3 \cdot \Delta$

- Without dither:
  - Output value: Δ
  - *Quantization error constant*: $0.3 \cdot \Delta$
- With dither:
  - Output range: $-\Delta/2 \ldots \Delta/2$
  - Signal is most frequently quantized to Δ(p = 0.7), but sometimes to $2 \cdot \Delta$(p = 0.3)
  - *Average* output value: $1.3 \cdot \Delta$
  - *Quantization error varying* between $0.3 \cdot \Delta$ and $0.7 \cdot \Delta$

Dither Properties



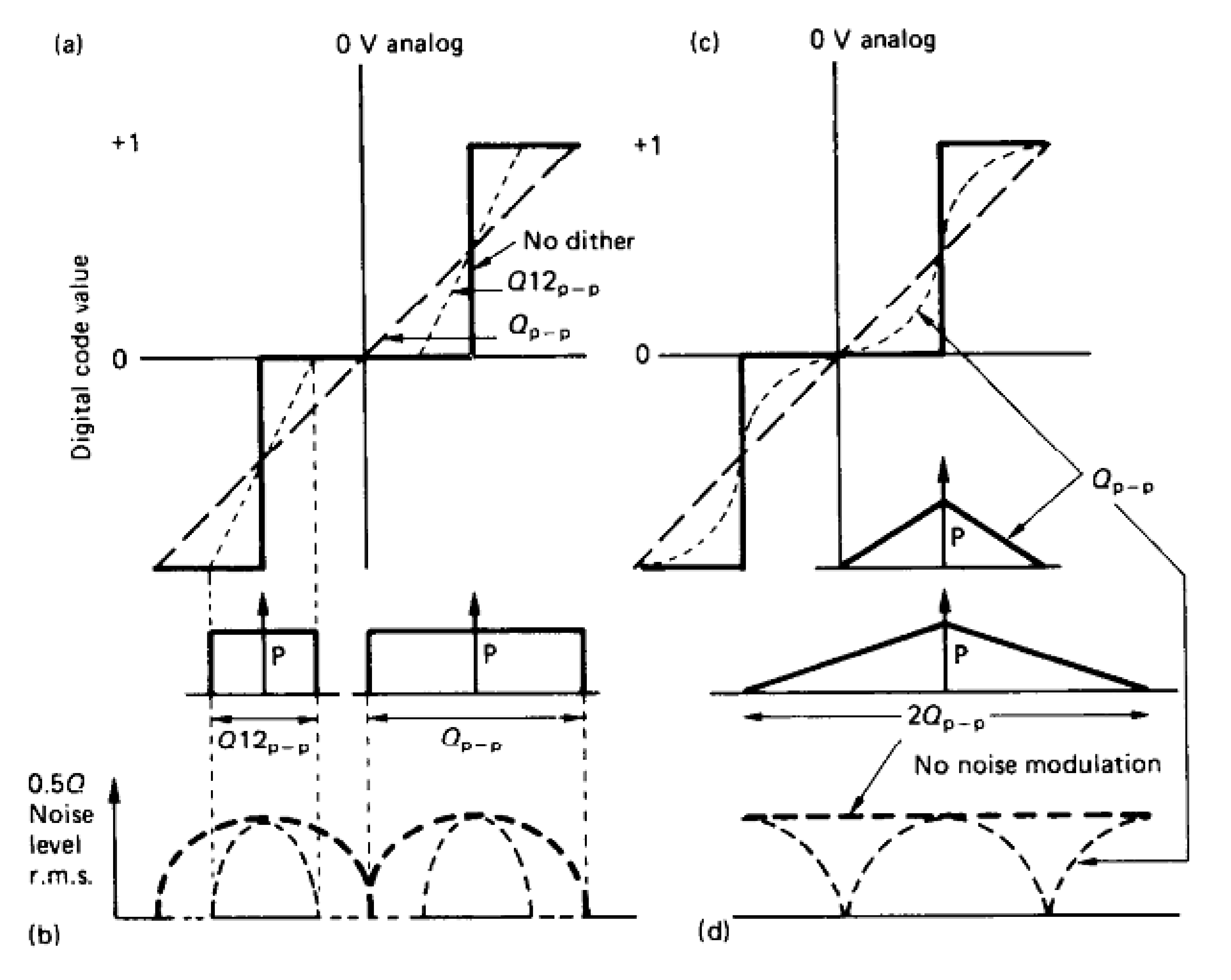
Dither with Rectangular PDF, $-\frac{\Delta}{2} \ldots \frac{\Delta}{2}$, Not Quantized

$$
\begin{aligned}
x &= 0 \cdot \Delta & & \rightarrow \bar{x}_\mathrm{Q} = 0, \\
\sigma_R(x) & & = \Delta\sqrt{(-0)^2 \cdot 1.0} & = 0.0\Delta \\
x &= 0.1 \cdot \Delta & & \rightarrow \bar{x}_\mathrm{Q} = 0.1\Delta, \\
\sigma_R(x) &= \Delta\sqrt{(-0.1)^2 \cdot 0.9 + (0.9)^2 \cdot 0.1} & & = 0.3\Delta \\
x &= 0.3 \cdot \Delta & & \rightarrow \bar{x}_\mathrm{Q} = 0.3\Delta, \\
\sigma_R(x) &= \Delta\sqrt{(-0.3)^2 \cdot 0.7 + (0.7)^2 \cdot 0.3} & & = 0.46\Delta \\
x &= 0.5 \cdot \Delta & & \rightarrow \bar{x}_\mathrm{Q} = 0.5\Delta, \\
\sigma_R(x) &= \Delta\sqrt{(-0.5)^2 \cdot 0.5 + (0.5)^2 \cdot 0.5} & & = 0.5\Delta \\
x &= 0.7 \cdot \Delta & & \rightarrow \bar{x}_\mathrm{Q} = 0.7\Delta, \\
\sigma_R(x) &= \Delta\sqrt{(-0.7)^2 \cdot 0.3 + (0.3)^2 \cdot 0.7} & & = 0.46\Delta \\
x &= 0.9 \cdot \Delta & & \rightarrow \bar{x}_\mathrm{Q} = 0.9\Delta, \\
\sigma_R(x) &= \Delta\sqrt{(-0.9)^2 \cdot 0.1 + (0.1)^2 \cdot 0.9} & & = 0.3\Delta \\
x &= 1 \cdot \Delta & & \rightarrow \bar{x}_\mathrm{Q} = 0, \\
\sigma_R(x) &= 0
\end{aligned}
$$

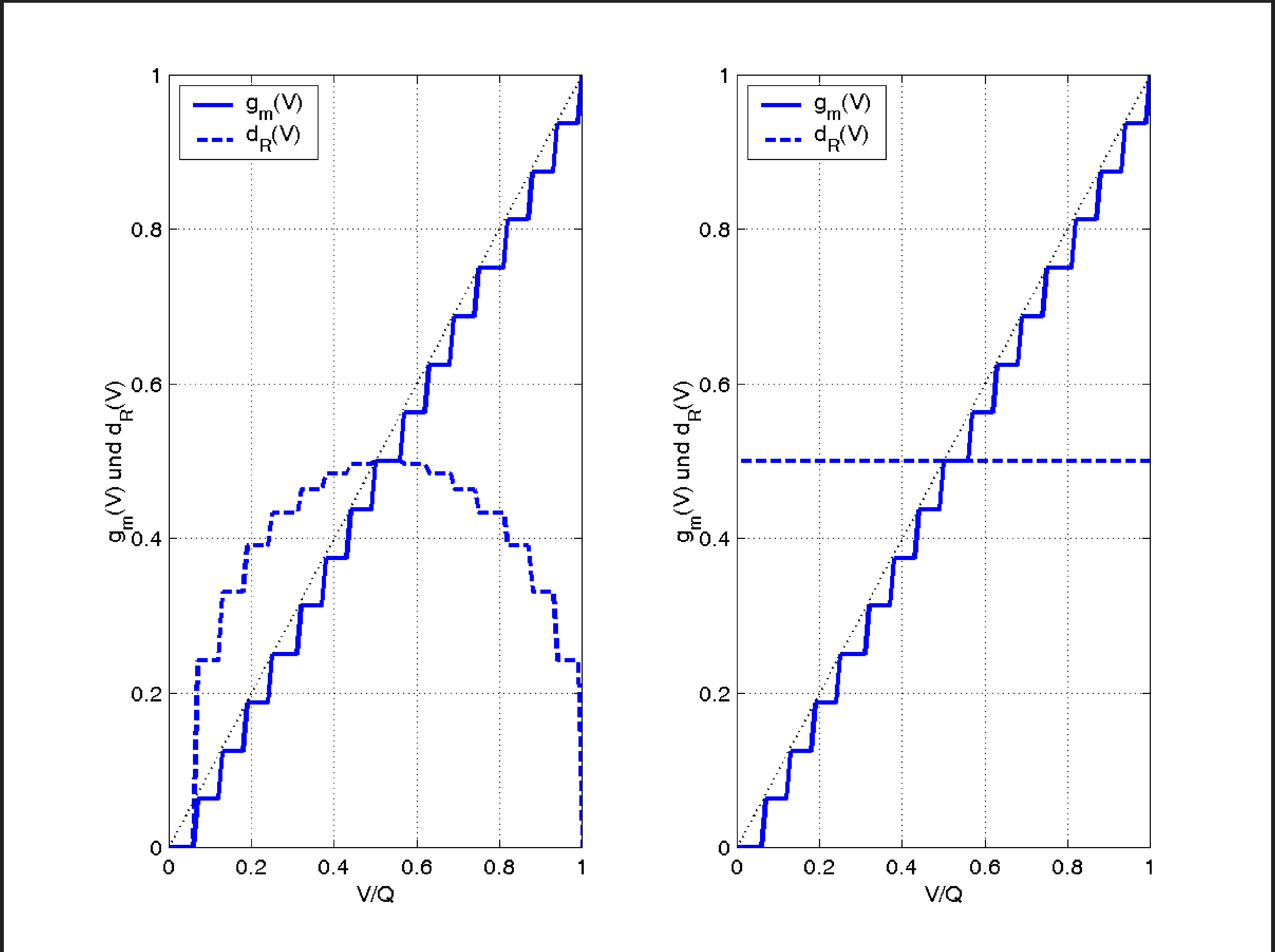
Dither with Triangular PDF, $-\Delta \ldots \Delta$, Not Quantized

$$
\begin{aligned}
x &= 0 \cdot \Delta & \rightarrow \bar{x_Q} = 0, \\
\sigma_R(x) &= 0.5\Delta \\
x &= 0.1 \cdot \Delta & \rightarrow \bar{x_Q} = 0.1\Delta, \\
\sigma_R(x) &= 0.5\Delta \\
x &= 0.3 \cdot \Delta & \rightarrow \bar{x_Q} = 0.3\Delta, \\
\sigma_R(x) &= 0.5\Delta \\
x &= 0.5 \cdot \Delta & \rightarrow \bar{x_Q} = 0.5\Delta, \\
\sigma_R(x) &= 0.5\Delta \\
x &= 0.7 \cdot \Delta & \rightarrow \bar{x_Q} = 0.7\Delta, \\
\sigma_R(x) &= 0.5\Delta \\
x &= 0.9 \cdot \Delta & \rightarrow \bar{x_Q} = 0.9\Delta, \\
\sigma_R(x) &= 0.5\Delta \\
x &= 1 \cdot \Delta & \rightarrow \bar{x_Q} = 0, \\
\sigma_R(x) &= 0.5\Delta
\end{aligned}
$$

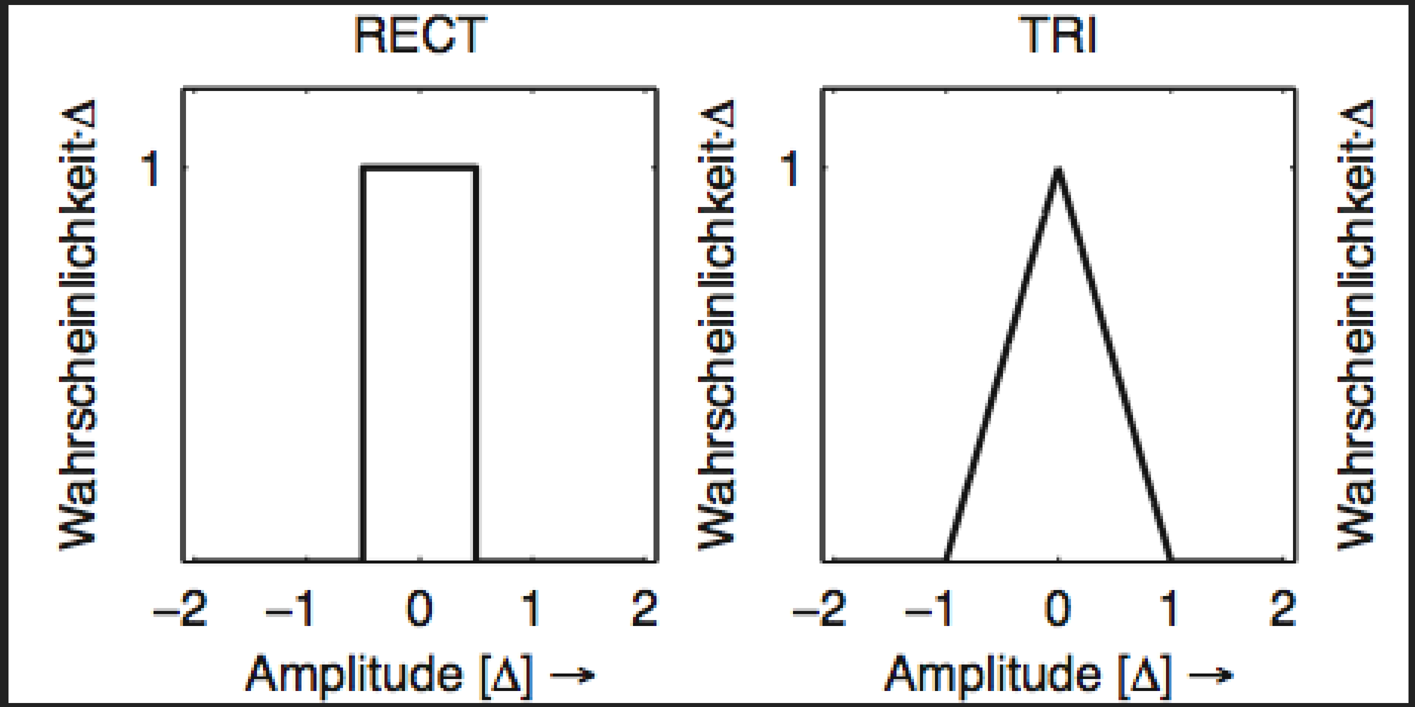
Linearization and Noise Modulation



Linearization and Noise Modulation



Noise Properties

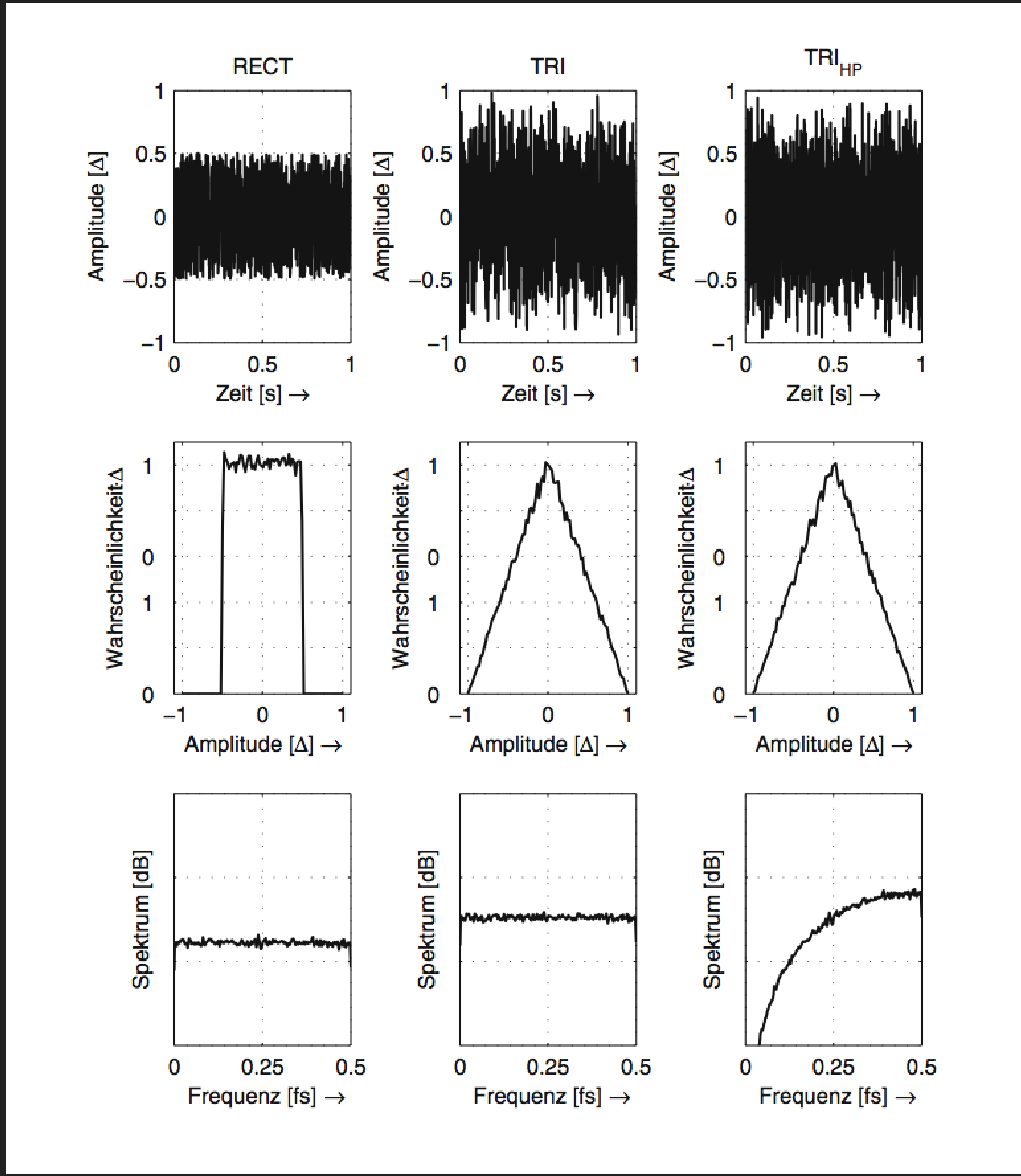


$$d_{\mathrm{RECT}}(n) = d(n)$$

$$d_{\mathrm{TRI}}(n) = d_{\mathrm{RECT},1}(n) + d_{\mathrm{RECT},2}(n)$$

$$d_{\mathrm{HP}}(n) = d(n) - d(n-1)$$

Noise Properties



How Does the SNR Change by Adding Dither?

Noise power of d_{RECT} & d_{TRI}

$$W_{\mathrm{RECT}} = \frac{\Delta^2}{12}$$
$$W_{\mathrm{TRI}} = \frac{\Delta^2}{6}$$

SNR of dithered full scale signal

$$SNR_{\mathrm{RECT}} = SNR_{normal} - 3.01\ [dB]$$
$$SNR_{\mathrm{TRI}} = SNR_{normal} - 4.77\ [dB]$$

		Sine	Speech	Music
8-Bit	Trunc			
	Rect			
	Tri			
4-Bit	Trunc			
	Rect			
	Tri			
2-Bit	Trunc			
	Rect			
	Tri			