Chapter Three
Data Representation and Computer Arithmetic

- Number Systems and Conversion
- Units of Data Representation
- Coding Methods
- Binary Arithmetic
- Complements
- Fixed and Floating points representation
- Boolean Algebra and Logic Circuits *

Number systems and conversion

- Number Systems
  - Decimal
  - Binary
  - Octal
  - Hexadecimal
- Conversion

Decimal systems

- **The decimal system**
  - Base 10 with ten distinct digits (0, 1, 2, …, 9)
  - Any number greater than 9 is represented by a combination of these digits
  - The weight of a digit based on power of 10

Example:

- The number 81924 is actually the sum of: $(8X10^4)+(1X10^3)+(9X10^2)+(2X10^1)+(4X10^0)$

Binary systems

Computers use the binary system to store and compute numbers.

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1
0 or 1	0 or 1	0 or 1	0 or 1	0 or 1	0 or 1	0 or 1	0 or 1

To represent any decimal number using the binary system, each place is simply assigned a value of either 0 or 1. To convert binary to decimal, simply add up the value of each place.

Example:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	0	1	1	0	0	1
128	0	0	16	8	0	0	1

128 + 0 + 0 + 16 + 8 + 0 + 0 + 1 = 153

10011001 = 153

Binary systems

- **The binary system (0 & 1)**
  - The two digits representation is called a binary system
  - Two electrical states – **on (1) & off (0)**
  - The position weights are based on the **power of 2**
  - The various combination of the two digits representation gives us the final value

Examples :

i) 1011011 in binary = 91 in decimal

ii) 1101.01 in binary = 13.25 in decimal

Binary Fractions

Binary fractions can also be represented:

Position Value:	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	etc.
Fractions:	**1/2**	**1/4**	**1/8**	**1/16**	**1/32**	
Decimal:	**.5**	**.25**	**.125**	**.0625**	**.03125**	

Binary into Decimal conversion

$5^{th}4^{th}3^{rd}2^{nd}1^{st}0^{th}$

$1\ 0\ 1\ 1\ 1\ 1_2 = (1X2^5)+(0X2^4)+(1X2^3)+(1X2^2)+(1X2^1)+(1X2^0)$

$= 32+8+4+2+1$

$= 47_{10}$

$1011001_2 = 89_{10}$

Exercise :

Convert the following binary numbers into their decimal equivalent

- $1110100_2 = (?)_{10}$
- $101101.1101_2 = (?)_{10}$

Conversion of Decimal to Binary

Divide by 2 (remainder division) till the dividend is zero and read remainders in reverse order. The right column shows result of integer division

mod 2	637/2
1	318
0	159
1	79
1	39
1	19
1	9
1	4
0	2
0	1
1	0

Read answer in this direction, Write answer left to right

$637_{10} = 1001111101_2$

- Convert 789_{10} to base 2

To Binary Fractions - Conversions

Multiply by 2 till enough digits are obtained, say 8, or a product is zero.

$\overline{.637_{10}}$

1.274

Read answer 0.548

in this direction 1.096

write it left 0.192 Ans= 0.10100011_2

to right 0.384

0.768

1.536

1.072

- Convert 0.325_{10} to base 2

Octal system

- Octal system
  - Base 8 systems (0, 1, 2, ..., 7)
  - Used to give shorthand ways to deal with the long strings of 1 & 0 created in binary
  - Numbers 0 .. 7 can be represented by three binary digits

❖ Examples :-

i) $(3137)_8 = 1631_{10}$

ii) 134 in octal = 1011100 in binary

iii) $(6)_8 = (110)_2$

iv) 432.2 in octal = 282.25 in decimal

v) 123.45 in octal = 001010011.100101 in binary

Hexadecimal systems

- The Hexadecimal system
  - Base 16 system
  - 0 .. 9 and letters A .. F for sixteen place holders needed
  - A = 10, B = 11, …, F = 15
  - Used in programming as a short cut to the binary number systems
  - Can be represented by four binary digits

❖ Examples :-

i) $(1D7F)_{16} = 7551_{10}$

ii) 6B2 in hexadecimal = 011010110010 in binary

iii) 101000010111 in binary = A17 in hexadecimal

Exercise – Convert …

Decimal	Binary	Octal	Hexa-decimal
29.8			
	101.1101		
		3.07	
			C.82

Exercise – Convert ...

Answer

Decimal	Binary	Octal	Hexa-decimal
29.8	11101.110011…	35.63…	1D.CC…
5.8125	101.1101	5.64	5.D
3.109375	11.000111	3.07	3.1C
12.5078125	1100.10000010	14.404	C.82

Bits

- How many bits does a computer use to store an integer?
  - Intel Pentium PC = 32 bits
  - Alpha = 64 bits
- What if we try to compute or store a larger integer?
  - If we try to compute a value larger than the computer can store, we get an <u>arithmetic overflow</u> error.

Representing Unsigned Integers

- How does a 16-bit computer represent the value 14?

0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0

- What is the largest 16-bit integer?

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

$= 1 \text{x} 2^{15} + 1 \text{x} 2^{14} + \ldots + 1 \text{x} 2^{1} + 1 \text{x} 2^{0} = 65{,}535$

Representing Signed Integers

- How does a 16 bit computer represent the value -14?

1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0

Sign bit.

- What is the largest 16-bit signed integer?

0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

$$= 1x2^{14} + 1x2^{13} + \ldots + 1x2^{1} + 1x2^{0} = 32{,}767$$

- Problem ➔ the value 0 is represented twice!
  - Most computers use a different representation, called <u>two's complement</u>.

Signed-magnitude representation

- Also called, “sign-and-magnitude representation”
- A number consists of a magnitude and a symbol representing the sign
- Usually 0 means positive, 1 negative
  - Sign bit
  - The number is represented with 1 sign bit to the left, followed by magnitude bits

Machine arithmetic with signed-magnitude representation

- Takes several steps to add a pair of numbers
  - Examine signs of the addends
  - If same, add magnitudes and give the result the same sign as the operands
  - If different, must…
    - Compare magnitude of the two operands
    - Subtract smaller number from larger
    - Give the result the sign of the larger magnitude operand
  - If magnitudes are equal and sign is different; two representations of zero problem
- For this reason the signed-magnitude representation is not as popular as one might think because of its "naturalness"

Complement number systems

- Negates a number by *taking its complement* instead of negating the sign
- Not natural for humans, but better for machine arithmetic
- Will describe 2 complement number systems
  - *Radix complement – very popular in real computers*
    - Must first decide how many bits to represent the number – say n.
    - Complement of a number = r^n – number
    - Example: 2's Complement
  - *Diminished radix-complement – not very useful, may skip it or not spend much time on it*
    - *Example: 1's Complement:* r^n - number - 1

Two's-complement representation

- Just radix-complement when radix = 2
- The most used representation of integers in computers and other digital arithmetic circuits
- 0 and positive numbers: leftmost bit = 0
- Negative numbers: leftmost bit = 1
- Representation of zero
  - i.e. 0 is represented as 0000 using 4-bit binary sequence.
- To find a number's 2's-complement – just flip all the bits and add 1

Properties of Two's Complement Notation

- Relationship between +n and –n.

  - 0 1 0 0 +4 0 0 0 1 0 0 1 0 +18
  - 1 1 0 0 -4 1 1 1 0 1 1 1 0 -18

Two’s Complement Notation with 4-bits

Binary Pattern	Value in 2’s complement.
0 1 1 1	7
0 1 1 0	6
0 1 0 1	5
0 1 0 0	4
0 0 1 1	3
0 0 1 0	2
0 0 0 1	1
0 0 0 0	0
1 1 1 1	-1
1 1 1 0	-2
1 1 0 1	-3
1 1 0 0	-4
1 0 1 1	-5
1 0 1 0	-6
1 0 0 1	-7
1 0 0 0	-8

Advantages of Two's Complement Notation

- It is easy to add two numbers.

	0 0 0 1	+1		1 0 0 0	-8
+	0 1 0 1	+5	+	0 1 0 1	+5
	0 1 1 0	**+6**		**1 1 0 1**	**-3**

- Subtraction is 2's complement addition
- Multiplication is just a repeated addition
- Division is just a repeated 2's complement addition
- Two's complement is widely used in ***ALU***

Comparison of decimal and 4-bit numbers Complements and other Notations

Decimal	Two's Complement	Ones' Complement	Signed Magnitude	Excess 2^{m-1}
−8	1000	—	—	0000
−7	1001	1000	1111	0001
−6	1010	1001	1110	0010
−5	1011	1010	1101	0011
−4	1100	1011	1100	0100
−3	1101	1100	1011	0101
−2	1110	1101	1010	0110
−1	1111	1110	1001	0111
0	0000	1111 or 0000	1000 or 0000	1000
1	0001	0001	0001	1001
2	0010	0010	0010	1010
3	0011	0011	0011	1011
4	0100	0100	0100	1100
5	0101	0101	0101	1101
6	0110	0110	0110	1110
7	0111	0111	0111	1111

Excess is $2^{4-1} = 8$; Thus, retrieved value=stored value-8

Decimal numbers, their two's complements, ones' complements, signed magnitude and excess 2^{m-1} binary codes

Existence of two zeros!

EXPLAIN

Two's-Comp Addition and Subtraction Rules

- Starting from 1000 (-8) on up, each successive 2's comp number all the way to 0111 (+7) can be obtained by adding 1 to the previous one, ignoring any carries beyond the 4^{th} bit position
- Since addition is just an extension of ordinary counting, 2's comp numbers can be added by ordinary binary addition!
- No different cases based on operands' signs!
- Overflow possible
  - Occurs if result is out of range
  - Happens if operands are the same sign but **sum is a different sign** of that of the operands

Storing an integer in two’s complement format:

- Convert the integer to an n-bit binary.
- If it is **positive** or **zero**, it is stored as it is. If it is **negative**, take the two’s complement and then store it.

Retrieving an integer in two’s complement format:

- If the **leftmost bit** is 1, the computer applies the two’s complement operation to the n-bit binary. If the leftmost bit is 0, no operation is applied.
- The computer changes the binary to decimal (integer) and corresponding sign is added.

1000	1001	1010	1011	1100	1101	1110	1111	0000	0001	0010	0011	0100	0101	0110	0111
−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7

Example:

Retrieve the integer that is stored as 11100110 in memory using two's complement format.

Solution:

The leftmost bit is 1, so the integer is negative. The integer needs to be two's complemented before changing to decimal.

Leftmost bit is 1. The sign is negative	1	1	1	0	0	1	1	0
	↓	↓	↓	↓	↓	↓	↓	↓
Apply two's complement operation	**0**	**0**	**0**	**1**	**1**	**0**	1	0
Integer changed to decimal								26
Sign is added								−26

Comparison

Summary of integer representations

Contents of memory	*Unsigned*	*Sign-and-magnitude*	*Two's complement*
0000	0	0	+0
0001	1	1	+1
0010	2	2	+2
0011	3	3	+3
0100	4	4	+4
0101	5	5	+5
0110	6	6	+6
0111	7	7	+7
1000	8	−0	−8
1001	9	−1	−7
1010	10	−2	−6
1011	11	−3	−5
1100	12	−4	−4
1101	13	−5	−3
1110	14	−6	−2
1111	15	−7	−1

STORING REAL NUMBERS

A **number** is changed to binary before being stored in computer memory, as described earlier. There are two issues that need to be handled:

1. How to store the sign of the number (we already know this).
2. How to show the (radix) point.

For the (radix) point, computers use two different representations: **fixed-point** and **floating-point**. The first is used to store a number as an integer, without a fraction part, the second is used to store a number as a real number, with a fractional part.

0	1	1	0	1	1	1	0	0	0	1	0	0	1	1	0

Memory location

Decimal point / Radix point
(assumed position)

Fixed point representation of integers

An integer is normally stored in memory using fixed-point representation.

Applications of unsigned integers:
Counting- Addressing- storing other data types (text, images, audio and video)

Storing real numbers Continued

A real number is a number with an integral part and a fractional part. For example, 23.7 is a real number—the integral part is 23 and the fractional part is 7/10. Although a fixed-point representation can be used to represent a real number, the result may not be accurate or it may not have the required precision. The next two examples explain why.

Real numbers with very large integral parts or very small fractional parts should not be stored in fixed-point representation.

Example 1:

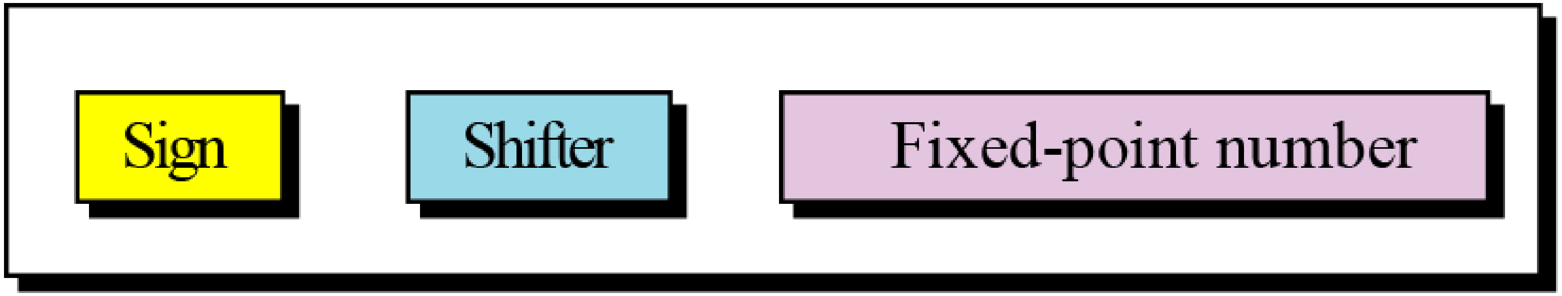
In the decimal system, assume that we use a fixed-point representation with **two digits to the right** of the decimal point and **fourteen digits to the left** of the decimal point, for a total of sixteen digits. The precision of a real number in this system is lost if we try to represent a decimal number such as 1.00234: the system stores the number as 1.00

Example 2:

In the decimal system, assume that we use a fixed-point representation with **six digits to the right** of the decimal point and **ten digits to the left** of the decimal point, for a total of sixteen digits. The accuracy of a real number in this system is lost if we try to represent a decimal number such as 236154302345.00 The system stores the number as 6154302345.00 The integral part is much smaller than it should be.

Floating-point representation

The solution for maintaining accuracy or precision is to use **floating-point representation**.



Floating-point pepresentation

A floating point representation of a number is made up of three parts: a sign, a shifter and a fixed-point number.

Floating-point representation is used in science to represent very small or very large decimal numbers. In this representation called **scientific notation**, the fixed-point section has only one digit to the left of point and the shifter is the power of 10.

Example:

The following shows the decimal number

7,425,000,000,000,000,000,000.00

in scientific notation (floating-point representation).

Actual number	$\rightarrow$	+	7,425,000,000,000,000,000,000.00
Scientific notation	$\rightarrow$	+	7.425×10^{21}

The three sections are the sign (+), the shifter (21) and the fixed-point part (7.425). Note that the shifter is the exponent.

Some programming languages and calculators shows the number as +7.425E21

Example:

Show the number -0.0000000000000232

in scientific notation (floating-point representation).

Solution

We use the same approach as in the previous example—we move the decimal point after the digit 2, as shown below:

Actual number	$\rightarrow$	$-$	0.0000000000000232
Scientific notation	$\rightarrow$	$-$	2.32×10^{-14}

The three sections are the sign ($-$), the shifter (-14) and the fixed-point part (2.32). Note that the shifter is the exponent.

Example:

Show the number,

$$(101001000000000000000000000000000.00)_2$$

in floating-point representation.

Solution

We use the same idea, keeping only one digit to the left of the radix (decimal) point.

Actual number	$\rightarrow$	$+$	$(101001000000000000000000000000000.00)_2$
Scientific notation	$\rightarrow$	$+$	1.01001×2^{32}

Example:

Show the number

$$-(0.00000000000000000000000101)_2$$

in floating-point representation.

Solution

We use the same idea, keeping only one digit to the left of the decimal point.

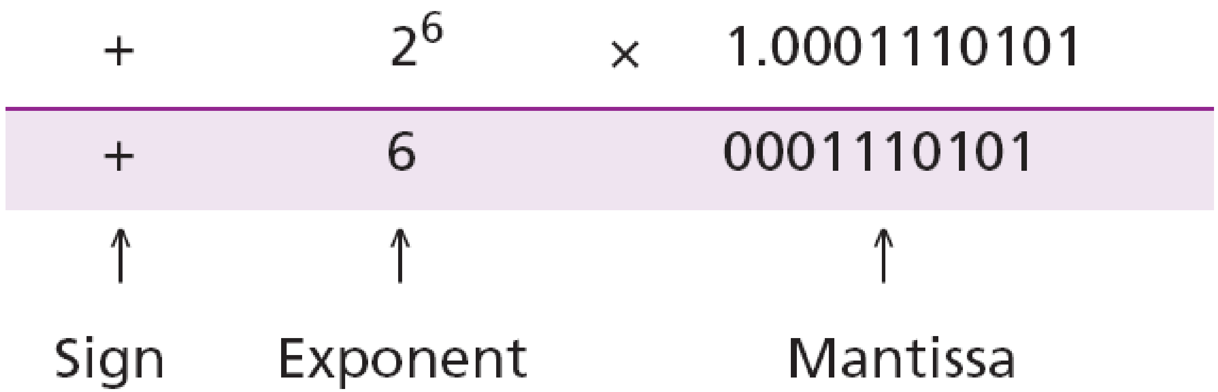
Actual number	$\rightarrow$	$-$	$(0.00000000000000000000000101)_2$
Scientific notation	$\rightarrow$	$-$	1.01×2^{-24}

Normalization

To make the **fixed part** of the representation **uniform**, both the scientific method (for the decimal system) and the floating-point method (for the binary system) use only one non-zero digit on the left of the decimal point. This is called **normalization**. In the decimal system this digit can be 1 to 9, while in the binary system it can only be 1. In the following, *d* is a non-zero digit, *x* is a digit, and *y* is either 0 or 1.

Decimal	→	±	d.xxxxxxxxxxxxxx	Note: *d* is 1 to 9 and each *x* is 0 to 9
Binary	→	±	1.yyyyyyyyyyyyyy	Note: each *y* is 0 or 1

To store 1000111.0101_2 in memory, using floating point representation; First we put it in normalized form $1.0001110101 * 2^6$, and then store it as shown below



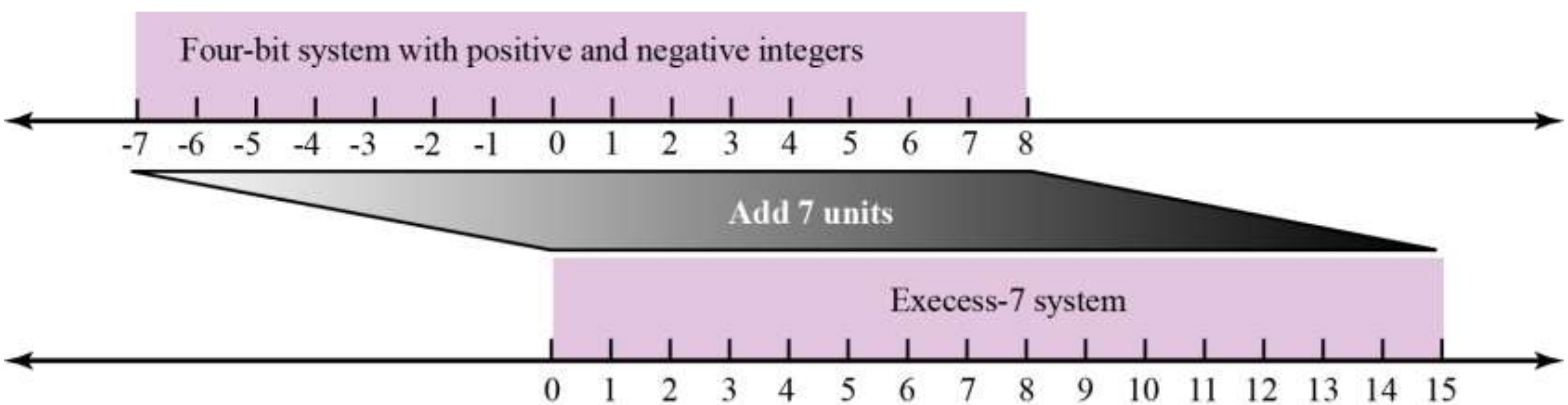
Note that the point and the bit 1 to the left of the fixed-point section (Mantissa) are not stored; They are implicit (hidden or not shown).

Excess Notation

- The **exponent**, the power that shows how many bits the decimal point should be moved to the left or right, is a signed number.
- Although this could have been stored using two's complement representation, a new representation, called the Excess notation, is used instead.
- In the Excess notation, **both positive and negative integers are stored as unsigned integers**.
- To represent a positive or negative integer, a positive integer (called a bias) is added to each number to shift them uniformly to the non-negative side. The value of this bias is $\mathbf{2^{m-1} - 1}$, where m is the size of the memory to store the exponent.

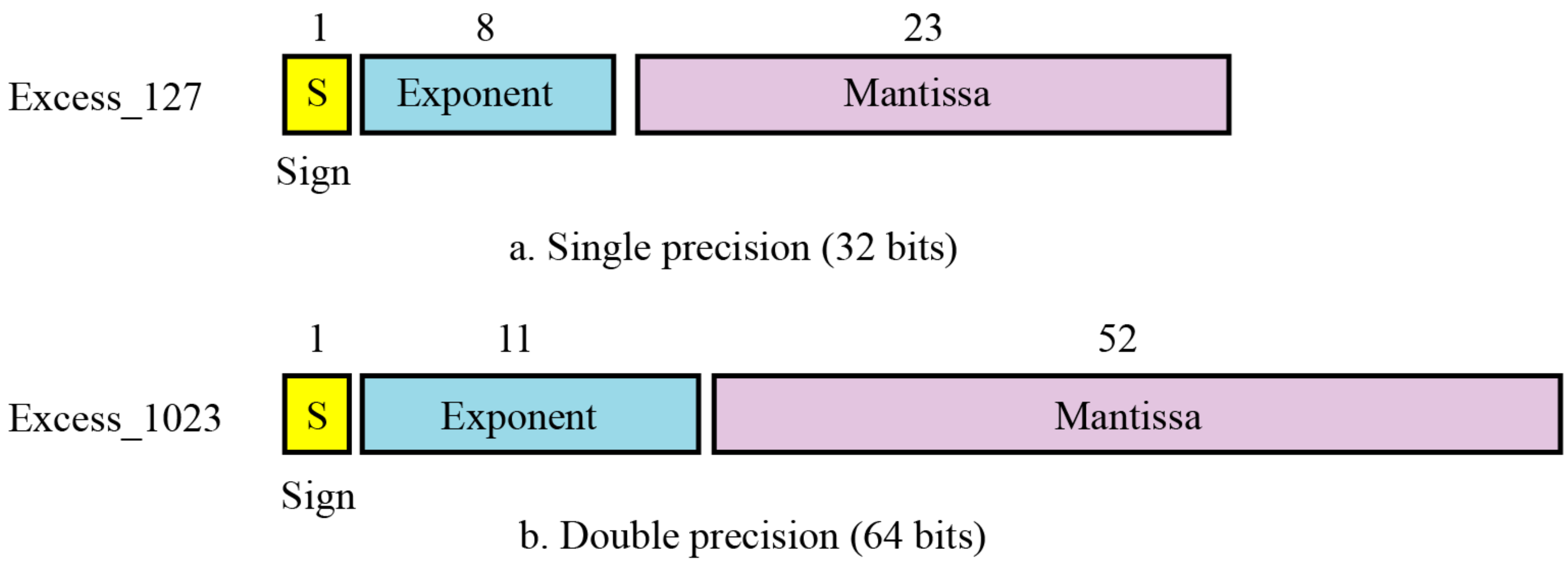
Example:

We can express sixteen integers in a number system with 4-bit allocation. By adding seven units to each integer in this range, we can uniformly translate all integers to the right and make all of them positive without changing the relative position of the integers with respect to each other, as shown in the figure. The new system is referred to as Excess-7, or biased representation with biasing value of 7.



Shifting in Excess representation

IEEE Standard



IEEE standards for floating-point representation

IEEE Specifications

Specifications of the two IEEE floating-point standards

Parameter	*Single Precision*	*Double Precision*
Memory location size (number of bits)	32	64
Sign size (number of bits)	1	1
Exponent size (number of bits)	8	11
Mantissa size (number of bits)	23	52
Bias (integer)	127	1023

Storage of IEEE standard floating point numbers:

1. Store the sign in S (0 or 1).
2. Change the number to binary.
3. Normalize.
4. Find the values of E and M.
5. Concatenate S, E, and M.

Example 1:

Show the Excess_127 (single precision) representation of the decimal number 5.75

Solution

a. The sign is positive, so S = 0.
b. Decimal to binary transformation: $5.75 = (101.11)_2$.
c. Normalization: $(101.11)_2 = (1.0111)_2 \times 2^2$.
d. $E = 2 + 127 = 129 = (10000001)_2$, $M = 0111$. We need to add nineteen zeros at the right of M to make it 23 bits.
e. The representation is shown below:

0	10000001	01110000000000000000000
S	E	M

The number is stored in the computer as

01000000101110000000000000000000

Example 2:

Show the Excess_127 (single precision) representation of the decimal number –161.875

Solution

a. The sign is negative, so S = 1.
b. Decimal to binary transformation: 161.875= $(10100001.111)_2$.
c. Normalization: $(10100001.111)_2 = (1.0100001111)_2 \times 2^7$.
d. E = 7 + 127 = 134 = $(10000110)_2$ and M = $(0100001111)_2$.
e. Representation:

1	10000110	01000011110000000000000
S	E	M

The number is stored in the computer as

11000011001000011110000000000000

Example 3:

Show the Excess_127 (single precision) representation of the decimal number –0.0234375

Solution

a. S = 1 (the number is negative).

b. Decimal to binary transformation: 0.0234375 = $(0.0000011)_2$.

c. Normalization: $(0.0000011)_2 = (1.1)_2 \times 2^{-6}$.

d. E = –6 + 127 = 121 = $(01111001)_2$ and M = $(1)_2$.

e. Representation:

1	01111001	10000000000000000000000
S	E	M

The number is stored in the computer as

10111100110000000000000000000000

Retrieving numbers stored in IEEE standard floating point format:

1. Find the value of S,E, and M.
2. If S=0, set the sign to positive, otherwise set the sign to negative.
3. Find the shifter (E-127).
4. De-normalize the mantissa.
5. Change the de-normalized number and find the absolute value (in decimal).
6. Add sign.

Example 4:

The bit pattern $(11001010000000000111000100001111)_2$ is stored in Excess_127 format, in memory. Show the retrieved value in decimal.

Solution

a. The first bit represents S, the next eight bits, E and the remaining 23 bits, M.

S	E	M
1	10010100	00000000111000100001111

b. The sign is negative.

c. The shifter = E − 127 = 148 − 127 = 21.

d. This gives us $(1.00000000111000100001111)_2 \times 2^{21}$

e. The binary number is $(1000000001110001000011.11)_2$

f. The absolute value is 2,104,387.75

g. The number is −2,104,387.75

Example 5

- Single precision

0 10000010 11000000000000000000000

1.11_2

$130 - 127 = 3$

0 = positive mantissa

$$+1.11_2 \times 2^3 = 1110.0_2 = 14.0_{10}$$

Exercise

1. Represent +0.8 in the following floating-point representation:
   - 1-bit sign
   - 4-bit exponent
   - 6-bit normalised mantissa (significand).
2. Convert the value represented back to decimal.
3. Calculate the relative error of the representation.

Binary Codes

- Computers also use binary numbers to represent non-numeric information, such as text or graphics.
- Binary representations of text, (letters, textual numbers, punctuation symbols, etc.) are called codes.
- In a binary code, the binary number is a symbol and does not represent an actual number.
- A code normally cannot be "operated on" in the usual fashion – mathematical, logical, etc. That is, one can not usually add up, for example, two binary codes. It would be like attempting to add text and graphics!

Character representation- ASCII

- ASCII (American Standard Code for Information Interchange) - **Binary Codes**
- It is the scheme used to represent characters.
- Each character is represented using 7-bit binary code.
- If 8-bits are used, the first bit is always set to 0

Numeric and Alphabetic Codes

- **ASCII code**
  - **A**merican **S**tandard **C**ode for **I**nformation **I**nterchange
  - an **alphanumeric code**
  - each character represented by a 7-bit code
    - gives 128 possible characters
    - codes defined for upper and lower-case alphabetic characters,
      digits 0 – 9, punctuation marks and various non-printing control characters (such as carriage-return and backspace)

ASCII – examples

Symbol	decimal	Binary
7	55	00110111
8	56	00111000
9	57	00111001
:	58	00111010
;	59	00111011
<	60	00111100
=	61	00111101
>	62	00111110
?	63	00111111
@	64	01000000
A	65	01000001
B	66	01000010
C	67	01000011

Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char
0	00	Null	32	20	Space	64	40	@	96	60	`
1	01	Start of heading	33	21	!	65	41	A	97	61	a
2	02	Start of text	34	22	"	66	42	B	98	62	b
3	03	End of text	35	23	#	67	43	C	99	63	c
4	04	End of transmit	36	24	$	68	44	D	100	64	d
5	05	Enquiry	37	25	%	69	45	E	101	65	e
6	06	Acknowledge	38	26	&	70	46	F	102	66	f
7	07	Audible bell	39	27	'	71	47	G	103	67	g
8	08	Backspace	40	28	(	72	48	H	104	68	h
9	09	Horizontal tab	41	29	)	73	49	I	105	69	i
10	0A	Line feed	42	2A	*	74	4A	J	106	6A	j
11	0B	Vertical tab	43	2B	+	75	4B	K	107	6B	k
12	0C	Form feed	44	2C	,	76	4C	L	108	6C	l
13	0D	Carriage return	45	2D	-	77	4D	M	109	6D	m
14	0E	Shift out	46	2E	.	78	4E	N	110	6E	n
15	0F	Shift in	47	2F	/	79	4F	O	111	6F	o
16	10	Data link escape	48	30	0	80	50	P	112	70	p
17	11	Device control 1	49	31	1	81	51	Q	113	71	q
18	12	Device control 2	50	32	2	82	52	R	114	72	r
19	13	Device control 3	51	33	3	83	53	S	115	73	s
20	14	Device control 4	52	34	4	84	54	T	116	74	t
21	15	Neg. acknowledge	53	35	5	85	55	U	117	75	u
22	16	Synchronous idle	54	36	6	86	56	V	118	76	v
23	17	End trans. block	55	37	7	87	57	W	119	77	w
24	18	Cancel	56	38	8	88	58	X	120	78	x
25	19	End of medium	57	39	9	89	59	Y	121	79	y
26	1A	Substitution	58	3A	:	90	5A	Z	122	7A	z
27	1B	Escape	59	3B	;	91	5B	[	123	7B	{
28	1C	File separator	60	3C	<	92	5C	\	124	7C	|
29	1D	Group separator	61	3D	=	93	5D	]	125	7D	}
30	1E	Record separator	62	3E	>	94	5E	^	126	7E	~
31	1F	Unit separator	63	3F	?	95	5F	_	127	7F	□

- Representation schemes:
  - **Top layers - Character string to character sequence**: Write each letter separately, enclosed in quotes. End string with '\0'.

    Notation: enclose strings in double quotes

    "Hello world"

    `'H' 'e' 'l' 'l' 'o' ' ' 'W' 'o' 'r' 'l' 'd' '\0'`

  - **Bottom layer - Character to bit-string**: Represent a character using the binary equivalent according to the ASCII table provided.

"SI"

'S' 'I' '\0'

010100110100100100000000

The colors are intended to help you read it; computers don't care that all the bits run together.

exercise

- Use the ASCII table to write the ASCII code for the following:
  - CIS110
  - 6=2*3
  - Write your name in hexadecimal.

Unicode - representation

- **ASCII** code can represent only $128 = 2^7$ characters.
- It only represents the English Alphabet, numeric characters, few other characters plus some control characters.
- **Unicode** is designed to represent the worldwide printable and non printable characters.
- It uses 16 bits (or more) and can represent 65536 characters (or more).
- For compatibility, the first 128 Unicode are the same as that of the **ASCII**.

Unicode cont'd..

- Let's consider how Ethiopia's character sets are represented
- The character set is called Ethiopic
- Range: 1200-1378 (in hexadecimal)
- Example character sets

Syllables

1200	ሀ	ETHIOPIC SYLLABLE HA
1201	ሁ	ETHIOPIC SYLLABLE HU
1202	ሂ	ETHIOPIC SYLLABLE HI
1203	ሃ	ETHIOPIC SYLLABLE HAA
1204	ሄ	ETHIOPIC SYLLABLE HEE
1205	ህ	ETHIOPIC SYLLABLE HE
1206	ሆ	ETHIOPIC SYLLABLE HO
1207	ሇ	ETHIOPIC SYLLABLE HOA
1208	ለ	ETHIOPIC SYLLABLE LA
1209	ሉ	ETHIOPIC SYLLABLE LU
120A	ሊ	ETHIOPIC SYLLABLE LI
120B	ላ	ETHIOPIC SYLLABLE LAA
120C	ሌ	ETHIOPIC SYLLABLE LEE
120D	ል	ETHIOPIC SYLLABLE LE
120E	ሎ	ETHIOPIC SYLLABLE LO
120F	ሏ	ETHIOPIC SYLLABLE LWA
1210	ሐ	ETHIOPIC SYLLABLE HHA
1211	ሑ	ETHIOPIC SYLLABLE HHU
1212	ሒ	ETHIOPIC SYLLABLE HHI
1213	ሓ	ETHIOPIC SYLLABLE HHAA
1214	ሔ	ETHIOPIC SYLLABLE HHEE
1215	ሕ	ETHIOPIC SYLLABLE HHE
1216	ሖ	ETHIOPIC SYLLABLE HHO
1217	ሗ	ETHIOPIC SYLLABLE HHWA
1218	መ	ETHIOPIC SYLLABLE MA
1219	ሙ	ETHIOPIC SYLLABLE MU
1242	ቂ	ETHIOPIC SYLLABLE QI
1243	ቃ	ETHIOPIC SYLLABLE QAA
1244	ቄ	ETHIOPIC SYLLABLE QEE
1245	ቅ	ETHIOPIC SYLLABLE QE
1246	ቆ	ETHIOPIC SYLLABLE QO
1247	ቇ	ETHIOPIC SYLLABLE QOA
1248	ቈ	ETHIOPIC SYLLABLE QWA
1249		<reserved>
124A	ቊ	ETHIOPIC SYLLABLE QWI
124B	ቋ	ETHIOPIC SYLLABLE QWAA
124C	ቌ	ETHIOPIC SYLLABLE QWEE
124D	ቍ	ETHIOPIC SYLLABLE QWE
124E		<reserved>
124F		<reserved>
1250	ቐ	ETHIOPIC SYLLABLE QHA
1251	ቑ	ETHIOPIC SYLLABLE QHU
1252	ቒ	ETHIOPIC SYLLABLE QHI
1253	ቓ	ETHIOPIC SYLLABLE QHAA
1254	ቔ	ETHIOPIC SYLLABLE QHEE
1255	ቕ	ETHIOPIC SYLLABLE QHE
1256	ቖ	ETHIOPIC SYLLABLE QHO
1257		<reserved>
1258	ቘ	ETHIOPIC SYLLABLE QHWA
1259		<reserved>
125A	ቚ	ETHIOPIC SYLLABLE QHWI
125B	ቛ	ETHIOPIC SYLLABLE QHWAA
125C	ቜ	ETHIOPIC SYLLABLE QHWEE
125D	ቝ	ETHIOPIC SYLLABLE QHWE

exercise

- Use UNICODE character representation to write the following:
  - ሀ ሁ ሂ ሃ ሄ ህ ሆ

Boolean Algebra & Digital Logic

- Boolean algebra is a mathematical system for the manipulation of variables that can have one of two values.
  - In formal logic, these values are “true” and “false.”
  - In digital systems, these values are “on” and “off,” 1 and 0, or “high” and “low.”
- Boolean expressions are created by performing operations on Boolean variables.
  - Common Boolean operators include **AND**, **OR**, and **NOT**.

Boolean Algebra

- A Boolean operation can be completely described using a truth table.
- The truth table for the Boolean operators AND and OR are shown at the right.
- The AND operation is also known as a Boolean product. The OR operation is the Boolean sum.

X AND Y

X	Y	XY
0	0	0
0	1	0
1	0	0
1	1	1

X OR Y

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

Boolean Algebra

- The truth table for the Boolean NOT operator is shown at the right.
- The NOT operation is most often designated by an overbar. It is sometimes indicated by a prime mark (‘) or an “elbow” (¬).

NOT X

x	$\overline{x}$
0	1
1	0

Boolean Algebra

- A Boolean function has:
  - At least one Boolean variable,
  - At least one Boolean operator, and
  - At least one input from the set {0,1}.
- It produces an output that is also a member of the set {0,1}.

Most modern programming Languages include the **Boolean** data type.

Boolean Algebra

- The truth table for the Boolean function:

  $$F(x,y,z) = x\bar{z}+y$$

  is shown at the right.
- To make evaluation of the Boolean function easier, the truth table contains extra (shaded) columns to hold evaluations of subparts of the function.

$$F(x,y,z) = x\bar{z}+y$$

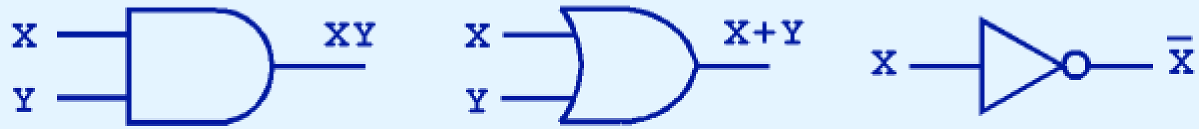
x	y	z	$\bar{z}$	$x\bar{z}$	$x\bar{z}+y$
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	1	0	1
0	1	1	0	0	1
1	0	0	1	1	1
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	0	0	1

Logic Gates

- We have looked at Boolean functions in abstract terms.
- In this section, we see that Boolean functions are implemented in digital computer circuits called gates.
- A gate is an electronic device that produces a result based on two or more input values.
  - In reality, gates consist of one to six transistors, but digital designers think of them as a single unit.
  - Integrated circuits contain collections of gates suited to a particular purpose.

Logic Gates

- The three simplest gates are the AND, OR, and NOT gates.



X AND Y

X	Y	XY
0	0	0
0	1	0
1	0	0
1	1	1

X OR Y

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

NOT X

X	$\overline{X}$
0	1
1	0

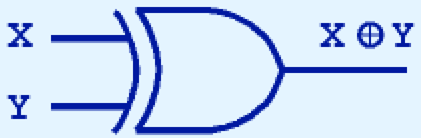
- They correspond directly to their respective Boolean operations, as you can see by their truth tables.

Logic Gates

- Another very useful gate is the exclusive OR (XOR) gate.
- The output of the XOR operation is true only when the values of the inputs differ.

X XOR Y

X	Y	X ⊕ Y
0	0	0
0	1	1
1	0	1
1	1	0



Note the special symbol ⊕ for the XOR operation.

Adders

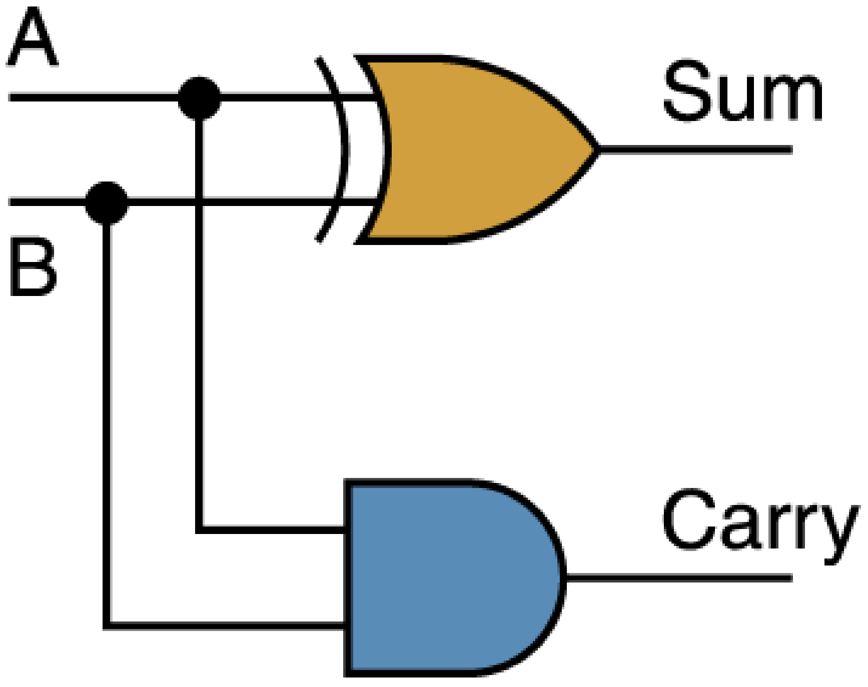
- At the digital logic level, addition is performed in binary
- Addition operations are carried out by special circuits called, appropriately, **adders**

Adders

- The result of adding two binary digits could produce a *carry value*
- Recall that 1 + 1 = 10 in base two
- A circuit that computes the **sum** of two bits and produces the correct carry bit is called a **half adder**

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Adders



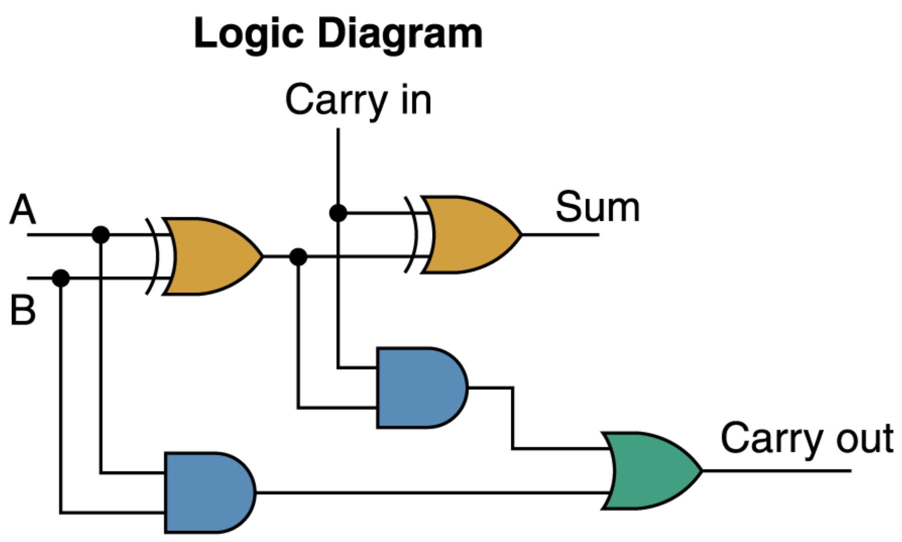
- Circuit diagram representing a **half adder**
- Two Boolean expressions:

$$\text{sum} = A \oplus B$$

$$\text{carry} = AB$$

Adders

- A circuit called a **full adder** takes the carry-in value into account

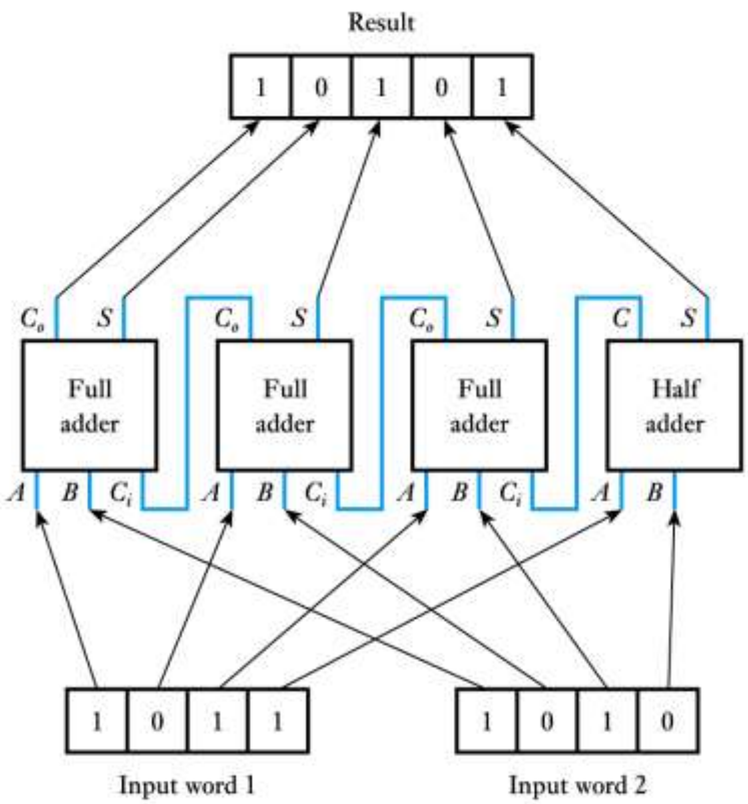


Truth Table

A	B	Carry-in	Sum	Carry-out
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Full Adder

- More complex circuits can add digital words



- Similar circuits can be constructed to perform subtraction
- More complex arithmetic (such as multiplication and division) *can* be done by dedicated hardware but is more often performed using a microcomputer or complex logic device

Assignment:

- Construct a digital circuit that takes a 4-bit binary number as an input and outputs the 2’s complement of the entered number.