Program	B. Tech. (SoCS & SoAE)	Semester	I
Course	Advanced Engineering Mathematics I	Course Code	MATH 1059
Session	July-Dec 2025	Unit	Prerequisite

1. Find the domain of the following functions:

(i) $f(x) = \sqrt{x+4} + \sqrt{9-x}$ (ii) $f(x) = \sqrt{\frac{1-|x|}{2-|x|}}$

2. Find the range of the following functions:

(i) $f(x) = \frac{1}{1-cos3x}$ (ii) $f(x) = \frac{|x|}{x}$

3. Evaluate the following limits:

(i) $\lim_{x \to 0} \frac{1-\cos x}{x^2}$

(ii) $\lim_{x \to 0} \frac{x \tan 2x - 2x\, tanx}{(1-Cos\ 2x)^2}$

(iii) $\lim_{x \to \infty} \left(\frac{x-3}{x+2}\right)^x$

(iv) $\lim_{x \to \infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$

4. Find the value of a, b, and c, such that $\lim_{\theta \to 0} \frac{ae^{\theta} - b\cos\theta + ce^{\theta}}{\theta \sin\theta} = 2$.

5. Let $f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$, $x \neq 0$. If $f(x)$ is continuous at $x = 0$, then find the value of $f(0)$.

6. A function $f(x)$ is defined as follows

$$f(x) = \begin{cases} -x^2, & when\ x \leq 0 \\ 5x^2, & when\ 0 < x < 1 \\ 4 + x^2, & when\ x \geq 1 \end{cases};$$

Then show that the function is continuous at x = 0 and x = 1.

7. Discuss the continuity and differentiability of the function at $x = 0$ defined on $\mathbb{R}$,

$$f(x) = \begin{cases} x \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$$

8. . Find the derivative of the function:

(i) $Sin^{-1}x + Sin^{-1}\sqrt{1 - x^2}$

(ii) $\frac{x^2-4x+1}{x^3-2x^2-x-2}$

(iii) $Sin\, y = x\, Sin\,(a + y)$

(iv) 7^{x^2+3x}

(v) x^{1+x+x^2}

(vi) $x = at^2,\ y = 2at.$

(vii) $(1 - x)(1 - 2x)(1 - 3x)$

9. Evaluate the Integration

(i) $\int_0^\infty \frac{dx}{1+x^2}$

(ii) $\int_{-1}^{1} \sqrt{\frac{1+x}{1-x}}\ dx$

(iii) $\int_1^2 \frac{dx}{2+x}$

(iv) $\int_0^\infty x^2 e^{-x^2} dx$

(v) $\int_0^\pi \frac{a^2}{(Sin\,\theta+Cos\,\theta)^2}\ d\theta$

10. Find the area above the x − axis, included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = a^2$.

11. Find the area bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

12. Find the area bounded by the curve $y = x^3$, and the ordinates $x = 1, x = 4$.

13. Evaluate the area of the curve $xy^2 = a^2(a - x)$ and the y − axis.

14. Show that the expression $(1 + a_1 + a_2 + \cdots + a_n)$ obtained from the determinant,

$$\begin{vmatrix} 1+a_1 & a_2 & a_3 & \cdots & a_n \\ a_1 & 1+a_2 & a_3 & \cdots & a_n \\ a_1 & a_2 & 1+a_3 & \cdots & a_n \\ \vdots & & & \ddots & \\ a_1 & a_2 & a_3 & \cdots & 1+a_n \end{vmatrix}.$$

15. (a) Find the derivative of the determinant $\begin{vmatrix} x & 1 & 2 \\ x^2 & 2x+1 & x^3 \\ 0 & 3x-2 & x^2+1 \end{vmatrix}$ and expand it.

(b) Find the value of x, for which the determinant $\begin{vmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{vmatrix}$ is nonzero.

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