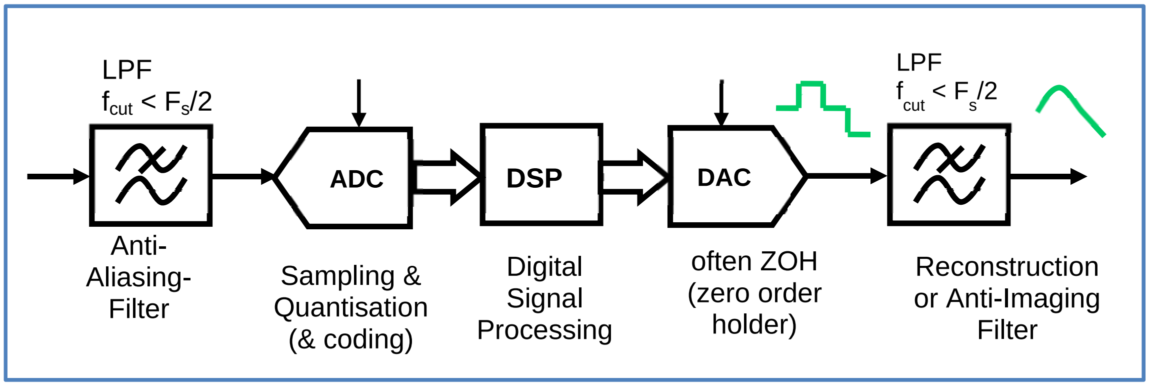
Chapter 4:

AD & DA Conversion

In the Time and Frequency Domains

Overview of AD-DA Chain



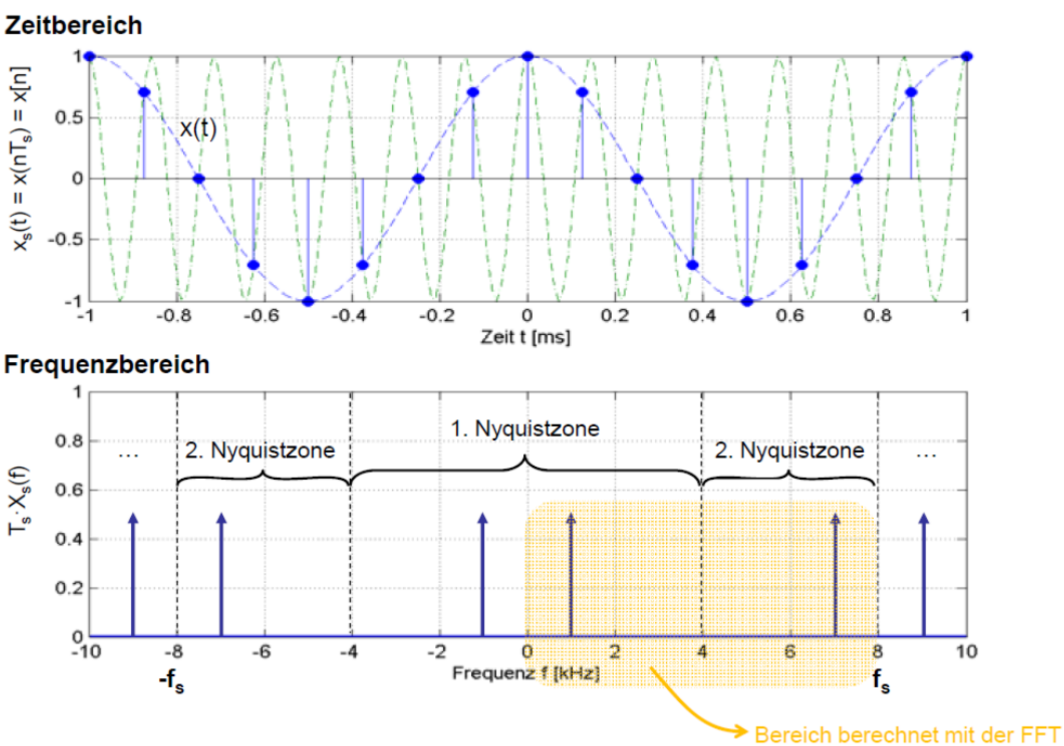


Figure 4 : A continuous sinusoidal time signal with fundamental frequency 1kHz, is sampled with Fs = 8KHz. Visualization of the periodic spectrum of the discrete signal, containing the original, plus the image spectra.

Zürcher Hochschule für Angewandte Wissenschaften

School of Engineering

Chapter 3:

Fourier Transformation

Signals in Frequency Domain

Definition as limit case of the Fourier Series

$$\lim_{T_0\to\infty}(c_k\cdot T_0)=\lim_{T_0\to\infty}\left(c_k\cdot\frac{1}{f_0}\right)=X(f)$$

Fourier Transformation (1a)

$$X(f)=\int_{-\infty}^{+\infty}x(t)\cdot e^{-j2\pi f t}dt \quad ; \quad X(\omega)=\int_{-\infty}^{+\infty}x(t)\cdot e^{-j\omega t}dt$$

Inverse Fourier Transformation (1b)

$$x(t)=\int_{-\infty}^{+\infty}X(f)\cdot e^{+j2\pi f t}df \quad ; \quad x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(\omega)\cdot e^{+j\omega t}d\omega$$

Pair of associated functions in time and frequency domain

$$x(t)\xleftrightarrow{FT}X(f) \qquad (2)$$

The complex valued spectrum density is often split into amplitude and phase components:

$$X(f)=|X(f)|\cdot e^{j\cdot phase\{X(f)\}} \qquad (3)$$

Some FT Reference Signals

Sketch Time-D	Equation Time-D	Equation Freq-D	Sketch Freq-D
	δ(t)	1	
	1	δ(f)	
	A.rect(t / τ)	A. τ .sinc(f.τ)	
	A. τ .sinc(t.τ)	A.rect(f / τ)	
	$e^{j2\pi f0t}$	$\delta(f-f_0)$	
	$\cos(2\pi f_0 t)$	$1/2 . [\delta(f-f_0) + \delta(f+f_0)]$	

Properties of Fourier Transformation

Table 3-1 below gives an overview of some properties of the Fourier transformation. For a complete list of properties and the proof of them please refer to the bibliography reference [1].

Property	Time Domain	Frequency Domain
Linearity or superposition	$y(t)=A\cdot x_1(t)+B\cdot x_2(t)$	$Y(f)=A\cdot X_1(f)+B\cdot X_2(f)$
Time-Shift	$y(t)=x(t-\lambda)$	$Y(f)=X(f)\cdot e^{-j2\pi f\lambda}$
Time-Scaling or Time-Bandwidth Product	$y(t)=x(a\cdot t)$	$Y(f)=\frac{1}{\lvert a\rvert}\cdot X\left(\frac{f}{a}\right)$
Duality or Symmetry between time and frequency domain	$y(t)=X(f)$	$Y(f)=x(-t)$
Frequency-Shift	$y(t)=x(t)\cdot e^{+j2\pi f_0 t}$	$Y(f)=X(f-f_0)$
Derivation in time domain	$y(t)=\frac{dx(t)}{dt}=\dot{x}(t)$	$Y(f)=(j2\pi f)\cdot X(f)$
	$y(t)=\frac{d^n x(t)}{dt^n}$	$Y(f)=(j2\pi f)^n\cdot X(f)$
Convolution vs Multiplication	$y(t)=x_1(t)*x_2(t)$	$Y(f)=X_1(f)\cdot X_2(f)$
	$y(t)=x_1(t)\cdot x_2(t)$	$Y(f)=X_1(f)*X_2(f)$
Symmetry in the frequency domain	$y(t)\in\Re$	Amplitude $\lvert Y(f)\rvert$ is even Phase $\langle Y(f)$ is odd
Symmetry in the time domain	$y(t)\in\Re$ and even	$Y(f)$ is purely real
	$y(t)\in\Re$ and odd	$Y(f)$ is purely imaginary
Parseval Theorem or Signal Energy	$E_x=\int_{-\infty}^{+\infty}\lvert x(t)\rvert^2 dt$	$E_x=\int_{-\infty}^{+\infty}\lvert X(f)\rvert^2 df$

Table 3-1 Properties of the Fourier Transformation

Chapter 2:

Fourier Series and DFT

Periodic Signals in the Frequency Domain

$$x(t) = c_0 + \sum_{k=1}^{\infty}\left[c_k \cdot \exp(+jk\omega_0 t) + c_{-k} \cdot \exp(-jk\omega_0 t)\right] = \sum_{k=-\infty}^{\infty}\left[c_k \cdot \exp(jk\omega_0 t)\right] \quad (4a)$$

with
$$c_k = \frac{1}{T_0}\int_{T_0} x(t) \cdot e^{-jk\omega_0 t}\, dt \quad (4b)$$

Reference Signal: Periodic Square

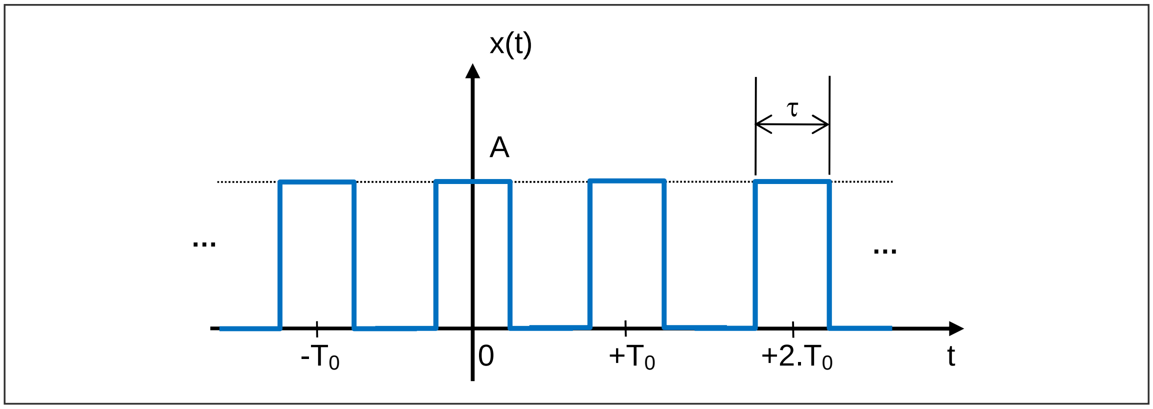


Figure 2-3 **Periodic square pulse function with duty cycle (τ / T_0).100%**

So that the coefficients c_k can be rewritten as:

$$c_k = \frac{A\tau}{T_0} \cdot \frac{\sin\left(\pi k \frac{\tau}{T_0}\right)}{\left(\pi k \frac{\tau}{T_0}\right)} = \frac{A\tau}{T_0} \cdot sinc\left(k \frac{\tau}{T_0}\right) \quad (6)$$

Properties of Fourier Series

Property	Observation
Symmetry	Because $c_k = c_{-k}^*$ the amplitude spectrum is always an even function, and the phase spectrum an odd function
Discrete Spectrum	Functions which are periodic in the time domain are discrete in the frequency domain (because they can be represented with Fourier series).
DC-Offset	Adding a constant value (DC-offset) to a time function only affects its c_0 coefficient.
Time-Shift	Shifting a time function only affects the phase spectrum (phase of c_k). $$x(t-\lambda) = \sum_{k=-\infty}^{+\infty} c_k \cdot e^{jk\omega_0 t} \cdot e^{-jk\omega_0 \lambda} = \sum_{k=-\infty}^{+\infty} \lvert c_k \rvert \cdot e^{j(\varphi_k - k\omega_0 \lambda)} \cdot e^{jk\omega_0 t} \quad (7)$$
Parseval Theorem	The power of a periodic function can be calculated as the sum of the power of its harmonics. $$\frac{1}{T_0} \cdot \int_{T_0} [x(t)]^2 dt = \sum_{k=-\infty}^{+\infty} \lvert c_k \rvert^2 \quad (8)$$

Table 2-1 Selected Properties of the Fourier Series

Numerical Approximation with DFT