Laboratory 1A: Pre-SiSy Math Exercises

Thema-1 : Functions with fractions

(1) Simplify the compound fraction (“Doppelbruch”) in the expression below:

$$f(x) = \frac{1}{1 + \frac{1}{1+x}}$$

Solution: $f(x) = \frac{1+x}{2+x}$

(2) Calculate the following limit cases for the function f(x) from exercise (1):

$$for \quad 0 < x << 1 \Rightarrow f(x) \approx \frac{1}{2}$$

$$for \quad x >> 1 \Rightarrow f(x) \approx 1$$

Write a Matlab script generating a plot of f(x) for $x \in \left[10^{-3}, 10^{+3}\right]$

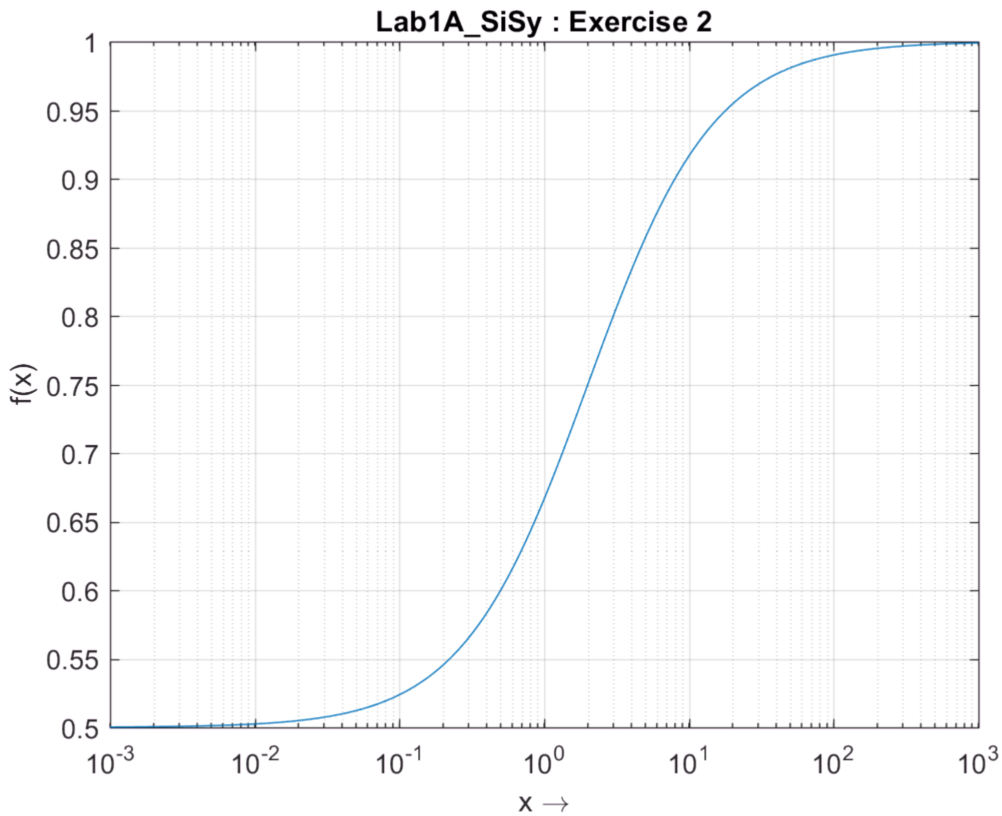
Use the function *logspace* to define a vector x.

Check the syntax of this function with the help in Matlab.

```
clear all,close all, clc
display('Lab1A:exercise 2')
```

```
Lab1A:exercise 2
```

```

% PARAMETERS
x = logspace(-3,+3,200);
f_x = (1+x)./(2+x);

% PLOT
figure(1)
semilogx(x,f_x),grid on
    title('Lab1A\_SiSy : Exercise 2')
    xlabel('x \rightarrow')
    ylabel('f(x)')
```



Thema-2 : Complex Numbers (specially polar notation with Euler's identity)

(3) Determine the Cartesian notation of the following complex numbers:

$$z_1 = 2 \cdot exp(j\frac{\pi}{2}) = 2 \cdot e^{+j\frac{\pi}{2}} = +2 \cdot j$$

$$z_2 = 1 \cdot exp(-j\pi) = -1$$

$$z_3 = \sqrt{2} \cdot exp(-j\frac{\pi}{4}) = 1 - j$$

(4) Determine the polar notation of the following complex numbers:

Hint: draw them as a vector in a complex plane

$$z_4 = 1 + j \cdot 1 = \sqrt{2} \cdot exp(+j\frac{\pi}{4})$$

$$z_5 = -1 + j \cdot 1 = \sqrt{2} \cdot exp(+j\frac{3\pi}{4})$$

$$z_6 = -j \cdot 3 = +3 \cdot exp(-j\frac{\pi}{2})$$

Find out the Matlab conversion functions that allow you to verify your results from exercises 3 & 4.

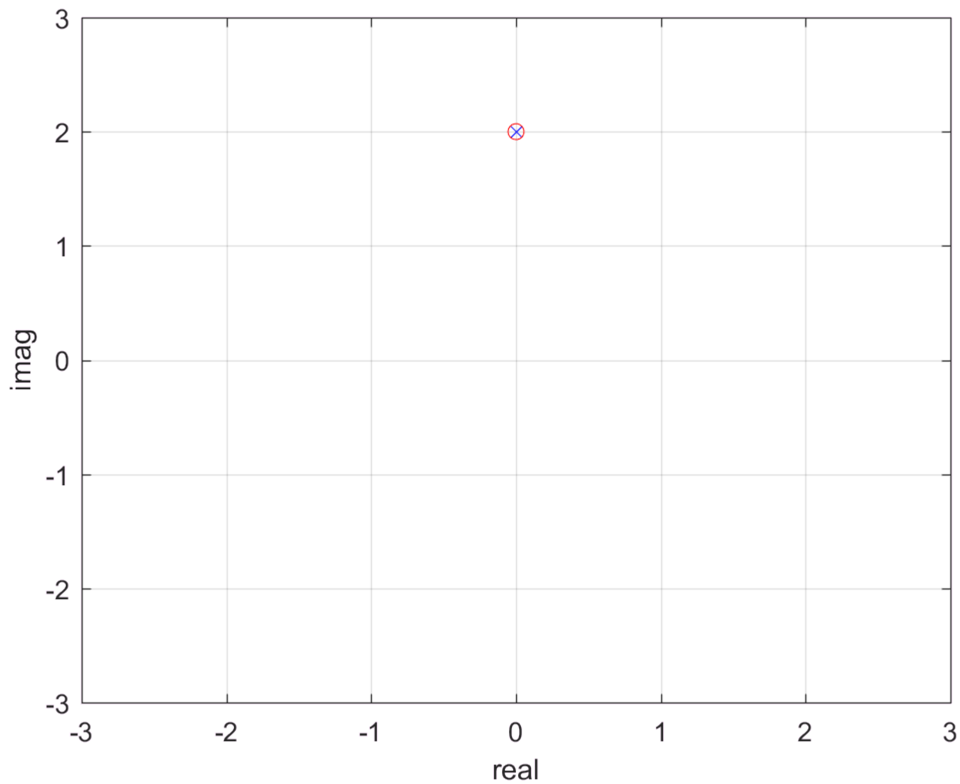
```
clear all,close all, clc
display('Lab1A:exercise 3&4')
```

```
Lab1A:exercise 3&4
```

```

% SYNTAX (checked with help) :   [X,Y] = pol2cart(TH,R)
[x1,y1] = pol2cart(pi/2,2)   % check numerical error approx to "zero"
```

```
x1 = 1.2246e-16
y1 = 2
```

```
z1 = x1+j*y1;
plot(x1,y1,'xb'), grid on, hold on, xlabel('real'),ylabel('imag')
plot(z1,'or'), hold off, axis([-3 3 -3 3])
```




```
% similar to other complex numbers
```

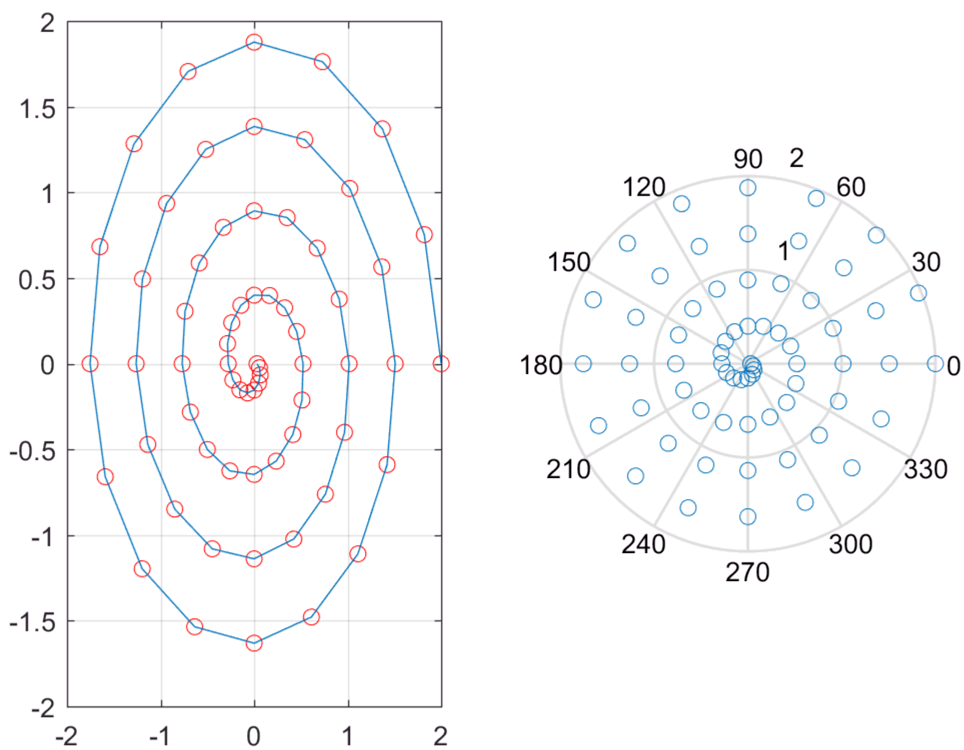
(5) Open, execute and understand the code of the Matlab script presisy_auf5.m

Make the necessary changes, such that you generate a plot showing a spiral with four windings (Spiralwicklung).

```
clear all,close all, clc
display('Lab1A:exercise 5')
```

```
Lab1A:exercise 5
```

```
N = 4;                  % number of windings
theta=0:pi/8:N*2*pi;

M = length(theta);
aux = 1:-1/M:1/M;
rho = 2*aux;
z1 = rho.*exp(j*theta);

subplot(121),plot(z1,'ro'),grid on, hold on
              plot(z1),grid on, hold off
              subplot(122),polar(angle(z1),abs(z1),'o'),grid on
```



(6) Determine the polar notation of the following complex numbers:

Hint: Please do not use the method of multiplication with the conjugated complex,

but rather calculate the polar notation for both numerator and denominator

Solution

$$z_7 = \frac{1}{-j \cdot 3} = \frac{1}{3} \cdot \frac{1}{exp(-j\frac{\pi}{2})} = \frac{1}{3} \cdot exp(j\frac{\pi}{2}) = j \cdot \frac{1}{3}$$

$$z_8 = \frac{1}{-1+j} = \frac{1}{\sqrt{2}} \cdot e^{-j\frac{3\pi}{4}} = \frac{-1-j}{2}$$

$$z_9 = \frac{-1+j}{1+j} = e^{j\frac{\pi}{2}} = j$$

(7) Given the complex function $f(x) = 1 + j \cdot x$, determine the value of x for which:

a. $|f(x)| = abs\{f(x)\} = \sqrt{2}$

b. $\langle f(x) = phase\{f(x)\} = +45°$

(7) *Solution*

a) $x = 1$

b) $x = 1$

8) What is the value of the magnitude and phase of the complex function $g(x) = \dfrac{1}{1 + j \cdot x}$, when x=1:

a. $|g(x)| = abs\{g(x)\} = \ldots$

b. $\langle g(x) = phase\{g(x)\} = \ldots$

8) *Solution*

a) $\dfrac{1}{\sqrt{2}}$

b) $\dfrac{-\pi}{4}$

Thema-3: Trigonometric Functions

(9) & (10) *Solution*

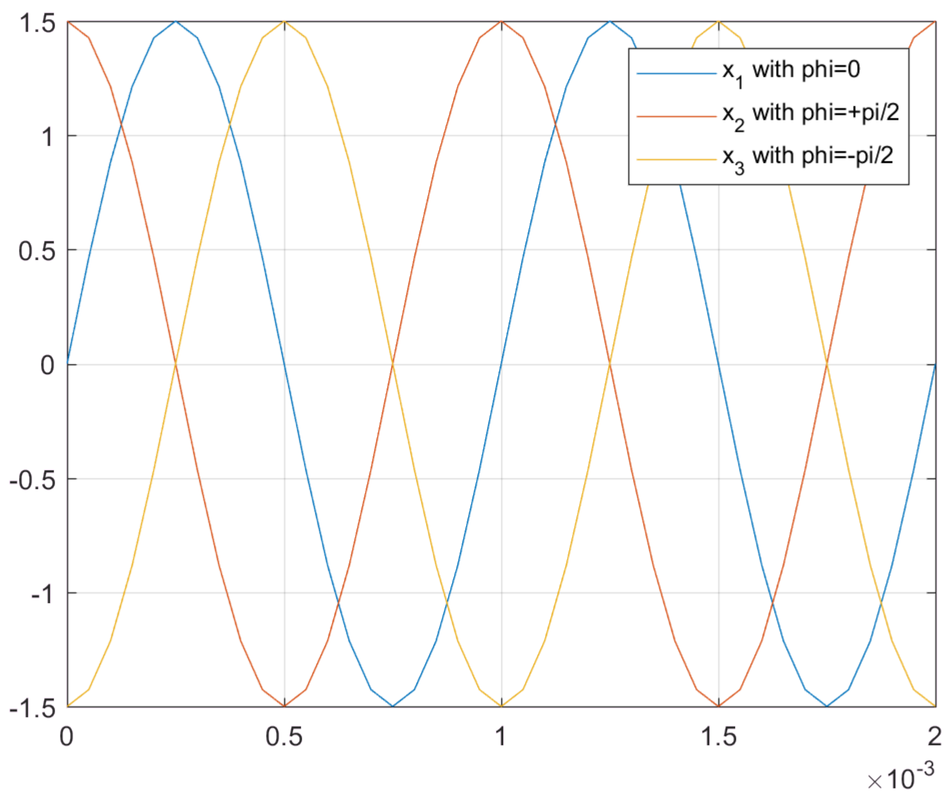
Verify your sketch by generating the same plot in Matlab.

```
clear all,close all, clc
display('Lab1A:exercise 9&10')
```

```
Lab1A:exercise 9&10
```

```
% PARAMETERS
A = 1.5;
```

```
f0 = 1e3;
phi = [0 +pi/2 -pi/2];

T0 = 1/f0;
tstep = T0/20;
t = 0:tstep:2*T0;

x1_t = A*sin(2*pi*f0*t + phi(1));
x2_t = A*sin(2*pi*f0*t + phi(2));
x3_t = A*sin(2*pi*f0*t + phi(3));

% PLOTS
plot(t,x1_t,t,x2_t,t,x3_t),grid on
    legend('x_1 with phi=0','x_2 with phi=+pi/2','x_3 with phi=-pi/2')
```



Thema-4 : Function description using the sum-sign

(11) *Solution* $x(t) = 2 + \frac{1}{2} \cdot cos(2\pi f_0 t) + \frac{3}{2} \cdot cos(2\pi 3 f_0 t + \pi)$

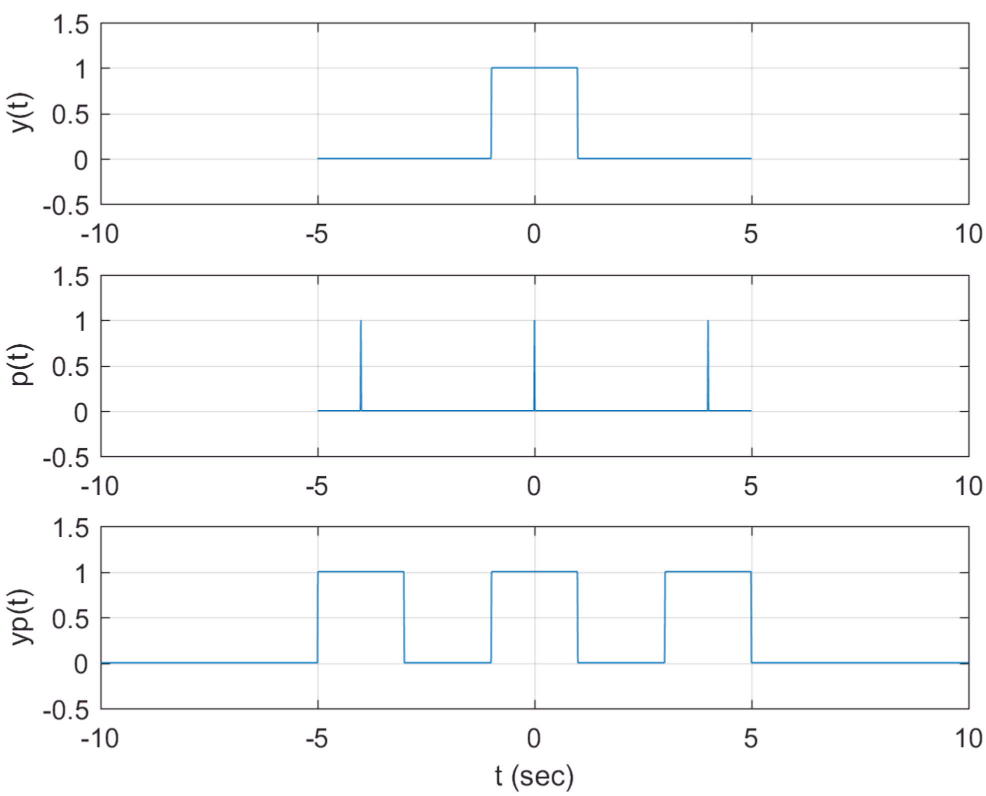
(12) and (13) Try it out the conv operation with the comb-function to get shifted copies of the original y(t) !

```
clear all, close all
```

```
display('Lab-1A: Exercise 12 & 13')
```

```
Lab-1A: Exercise 12 & 13
```

```
tstep = 1e-2;
t = -5:tstep:5;
t_long = -10:tstep:10;

y_t = double( abs(t)<1 );
p_t = double( (t==-4) | (t==0) | (t==4) );
yp_t = conv(y_t,p_t);

subplot(311), plot (t,y_t),grid on, ylim([-0.5 1.5]), xlim([-10 10]), ylabel('y(t)')
subplot(312), plot (t,p_t),grid on, ylim([-0.5 1.5]), xlim([-10 10]), ylabel('p(t)')
subplot(313), plot (t_long,yp_t),grid on, ylim([-0.5 1.5]), xlim([-10 10]), ylabel('yp(t)')
                xlabel('t (sec)')
```



Thema-5 : Integration and Differentiation of exponential-function

(14) *Solution*

a. $\int e^{at} dt = \frac{1}{a} e^{at} + C$

b. $\frac{d\left(e^{at}\right)}{dt} = a \cdot e^{at}$

c. $\int_0^t e^{a\lambda} d\lambda = \frac{1}{a}\left[e^{at} - 1\right]$

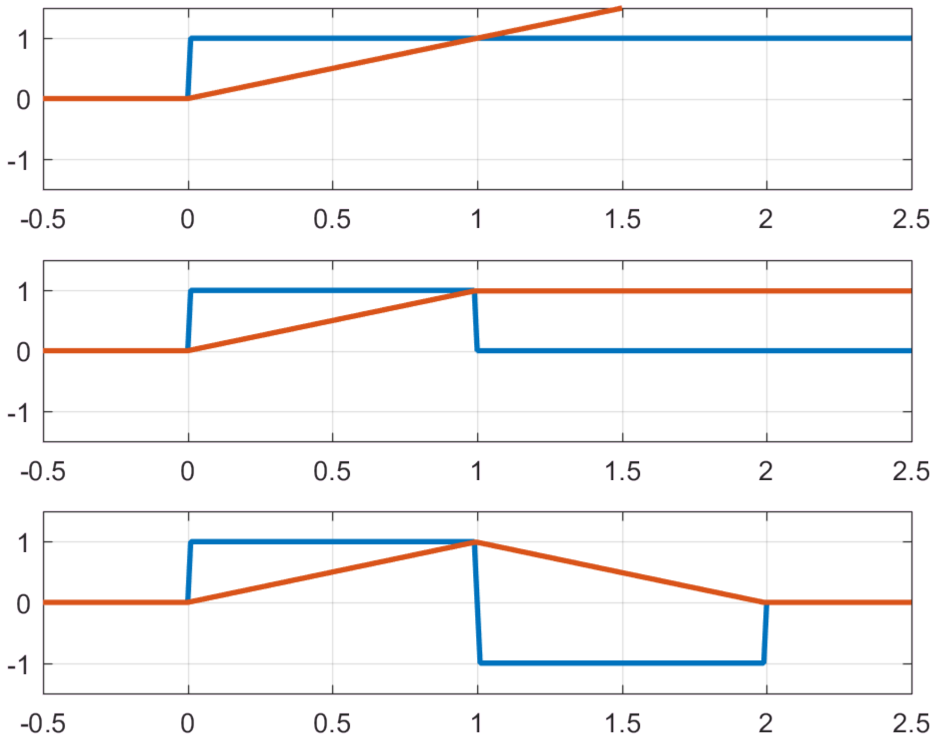
Thema-6 : Graphical solution for Integrals (as area under the curve)

(15) *Solution: integral approximation with cumsum*tstep*

```
clear all,close all, clc
display('Lab1A:exercise 15')
```

```
Lab1A:exercise 15
```

```

% PARAMETERS
tstep = 1/100;
t = -0.5:tstep:2.5;

x1_t = double(t>0);
y1_t = cumsum(x1_t*tstep);

x2_t = double( (t>0) & (t<1) );
y2_t = cumsum(x2_t*tstep);

x3_t = x2_t - double( (t>1) & (t<2) );
y3_t = cumsum(x3_t*tstep);

figure()
subplot(311)
    plot(t,x1_t,'LineWidth',2),grid on, hold on
    plot(t,y1_t,'LineWidth',2),grid on
    ylim([-1.5 1.5])
subplot(312)
    plot(t,x2_t,'LineWidth',2),grid on, hold on
    plot(t,y2_t,'LineWidth',2),grid on
    ylim([-1.5 1.5])
subplot(313)
    plot(t,x3_t,'LineWidth',2),grid on, hold on
    plot(t,y3_t,'LineWidth',2),grid on
    ylim([-1.5 1.5])
```



Thema-7 : Plots in log-log scale

(16) Solution

for $0 < x << 1 \Rightarrow f(x) \approx 1$

for $x >> 1 \Rightarrow f(x) \approx \frac{1}{x}$

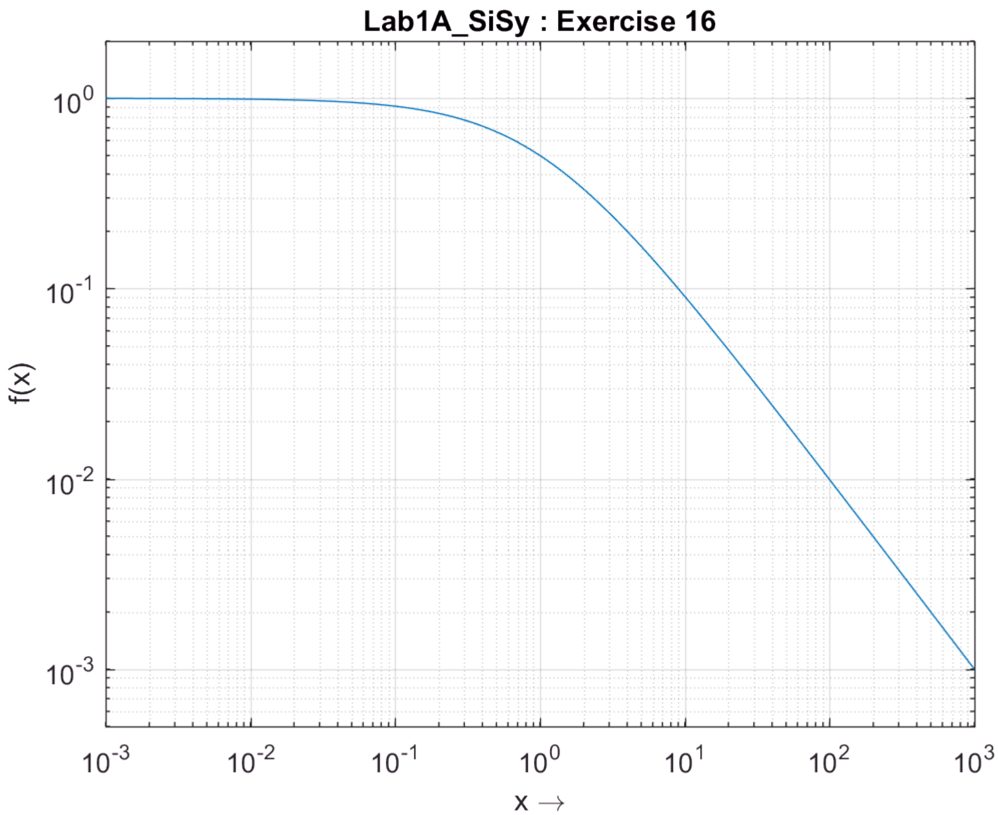
possibly $f(x) = \frac{1}{1+x}$

```
display('Lab1A:exercise 16')
```

```
Lab1A:exercise 16
```

```
% PARAMETERS
x = logspace(-3,+3,200);
f_x = 1./(1+x);

% PLOT
figure()
loglog(x,f_x),grid on
    title('Lab1A\_SiSy : Exercise 16'), ylim([5e-4 2])
```

```
    xlabel('x \rightarrow'), ylabel('f(x)')
```



Thema-8 : Logarithm of basis 10

(17) Determine the value of the following logarithmic expressions:

a. $log_{10}(10^n) = n$

b. $log_{10}\left(\frac{10^n}{10^p}\right) = n - p$

c. $log_{10}(2^n) = n \cdot (0.3)$ given $log_{10}(2) \cong 0.3$

d. $20 \cdot log_{10}(2) \cong 6.0$

e. $20 \cdot log_{10}\left(\frac{1}{2}\right) \cong -6.0$

Check your results in Matlab, using the function log10 which calculates the logarithm with basis 10.

```
% log10(X) is the base 10 logarithm of the elements of X
```