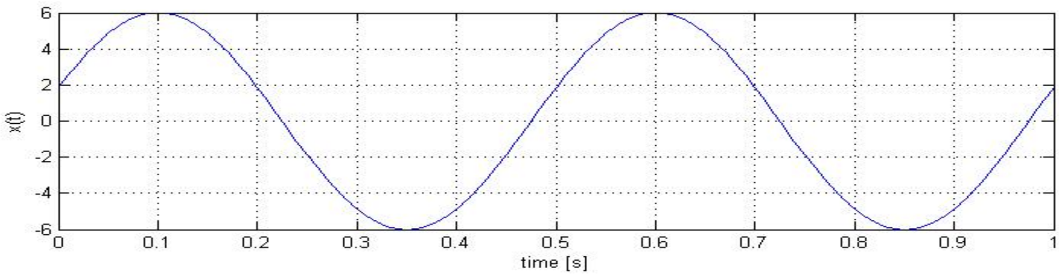
List 1
Signals and Systems

Exercise 1 *Sinusoidal Signal.*

Describe the signal x(t) shown below with three different mathematical expressions using:

a) A single cosine function.
b) A sum of sine and cosine functions.
c) The real part of a complex exponential function.
d) The sum of 2 complex exponential functions.



Exercise 2 *Phasors (vector rotating in a complex plane).*

Use the phasor notation to solve the following equations:
Hint: express the cosine functions as a sum of 2 complex exponentials.

a) $x(t) = \left[3 \cdot cos\left(6\pi t + \frac{\pi}{2}\right)\right] \times \left[2 \cdot cos\left(6\pi t + \frac{\pi}{4}\right)\right] = ?$

b) $x(t) = A \cdot cos(\omega t + \theta)$; $\frac{dx(t)}{dt} = ?$

Exercise 3 *Signal Description.*

Describe the following signals with a mathematical equation:

a) A sine signal with period T_0=0.5s, which is time limited between t_1=1s and t_2=5s . Outside of this interval the signal is equal to 0 .
Hint: Use the unit step function (also called Heaviside function).

b) An infinite sequence of equidistant unit impulses (Dirac deltas) with amplitude A and spacing T_s.

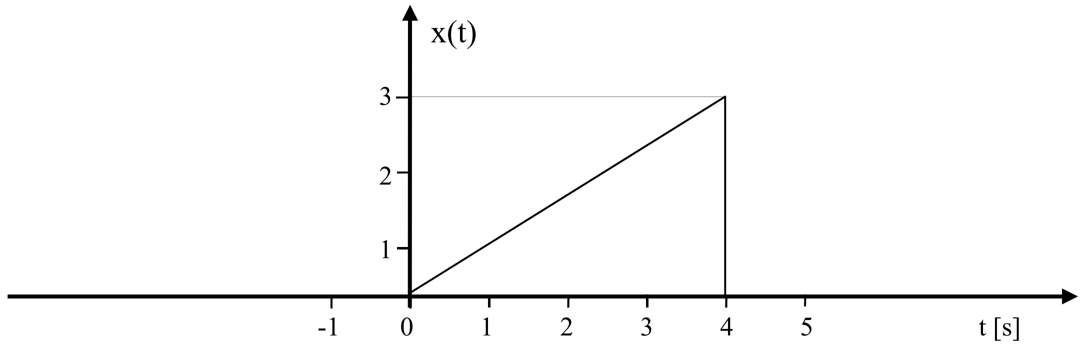
c) Determine for the signals above (items a and b) the following characteristics: power/energy signals, symmetric (even/odd) or not, periodic or not.

d) Which kind of signal do you get in Matlab with the command:
➢ sig_d = randn(1 , 100)
Calculate the average value and the standard deviation of sig_d .

Exercise 4 *Signal Manipulations (Operations with the time variable).*

Given the continuous time signal x(t) shown below, prepare a sketch for each of the following signals (conversions of x(t)):

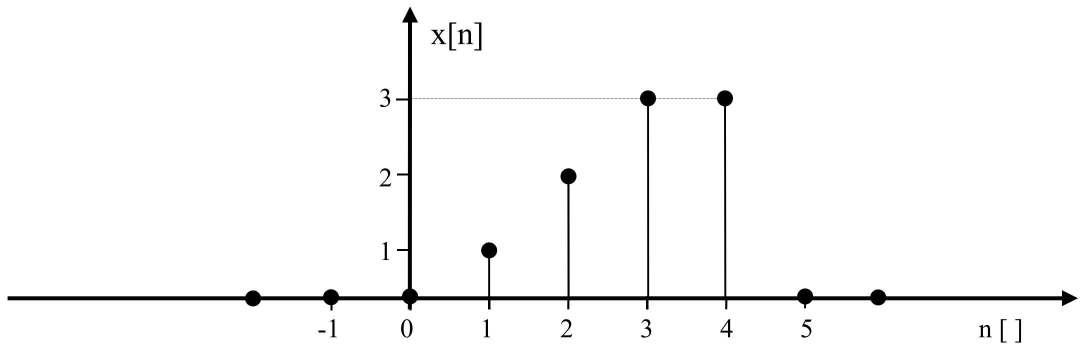
a) x(t-2)
b) x(2.t)
c) x(t/2)
d) x(-t)



Exercise 5 *Signal Manipulations.*

Given the discrete time signal x[n] shown below, prepare a sketch for each of the following signals (conversions of x[n]):

a) x[n-2]
b) x[2.n]
c) x[-n]
d) x[-n+2]



e) Describe the signal x[n] as a sum of weighted and shifted unit impulses $\delta[n]$.

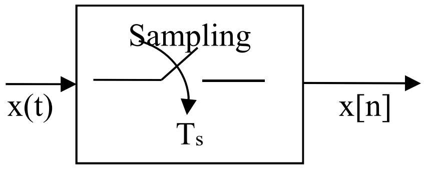
Exercise 6 *Sampling and Discrete Signals.*

The time continuous signal x(t) is described as:

$$x(t) = \begin{cases} 1-|t| & \text{for} \quad -1 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Draw a sketch representing x(t) .

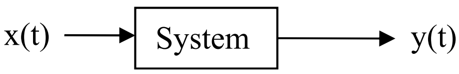
The signal x(t) is then sampled with different sampling intervals (also called sampling period T_s). Prepare sketches of the resulting discrete signals x[n] for the following values of T_s :



b) T_s = 0.25 s
c) T_s = 0.5 s
d) T_s = 1 s

Exercise 7 *System Classification.*

Consider a system with a single input signal x(t) and a single output signal y(t) as shown below:



Which of the following functions describe then a linear system? Justify your answer, testing if the equation defining the system fulfills the superposition principle.

a) $y(t) = 0.2 \cdot x(t) - 1.5$

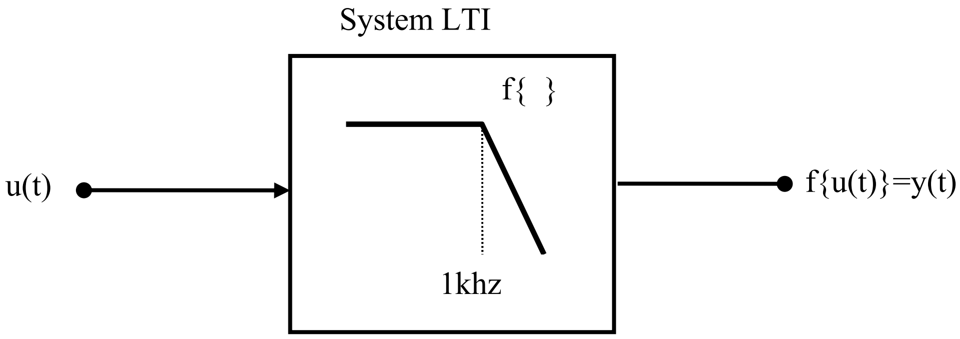
b) $y(t) = x(t) + \int x(t)dt$

c) $y(t) = 0.4 \cdot x(t) + 0.2 \cdot \dot{x}(t)$

d) $y(t) = 0.4 \cdot x(t) + 0.2 \cdot x^2(t)$

Exercise 8 *Linear Time Invariant System (LTI).*

The following LTI system is a low pass filter, it let through low frequencies unchanged and attenuates high frequencies. Check its effect on test signals $u_1(t)$ und $u_2(t)$ and determine then the output signals $y_3(t)$ und $y_4(t)$ for the input signals $u_3(t)$ und $u_4(t)$.
Hint: consider the properties of a LTI system.



a) Simplified LTI (only amplitude effect)

$$u_1(t) = 3 \cdot \cos(2\pi \cdot 100t)$$
$$u_2(t) = 3 \cdot \cos(2\pi \cdot 10kt)$$

$$y_1(t) = 3 \cdot \cos(2\pi \cdot 100t)$$
$$y_2(t) = 0{,}3 \cdot \cos(2\pi \cdot 10kt)$$

$$u_3(t) = 2 \cdot \sin\left(2\pi \cdot 100t + \frac{\pi}{4}\right)$$
$$u_4(t) = 8 \cdot \sin\left(2\pi \cdot 10kt + \frac{\pi}{10}\right) + 4 \cdot \cos\left(2\pi \cdot 100t + \frac{\pi}{6}\right)$$

$$y_3(t) = ?$$
$$y_4(t) = ?$$

b) LTI with Amplitude and Phase Effect

$$u_1(t) = 3 \cdot \cos(2\pi \cdot 100t)$$
$$u_2(t) = 3 \cdot \cos(2\pi \cdot 10kt)$$

$$y_1(t) = 3 \cdot \cos(2\pi \cdot 100t)$$
$$y_2(t) = 0{,}3 \cdot \cos\left(2\pi \cdot 10kt - \frac{\pi}{2}\right)$$

$$u_3(t) = 2 \cdot \sin\left(2\pi \cdot 100t + \frac{\pi}{4}\right)$$
$$y_3(t) = ?$$

$$u_4(t) = 8 \cdot \sin\left(2\pi \cdot 10kt + \frac{\pi}{10}\right) + 4 \cdot \cos\left(2\pi \cdot 100t + \frac{\pi}{6}\right)$$
$$y_4(t) = ?$$