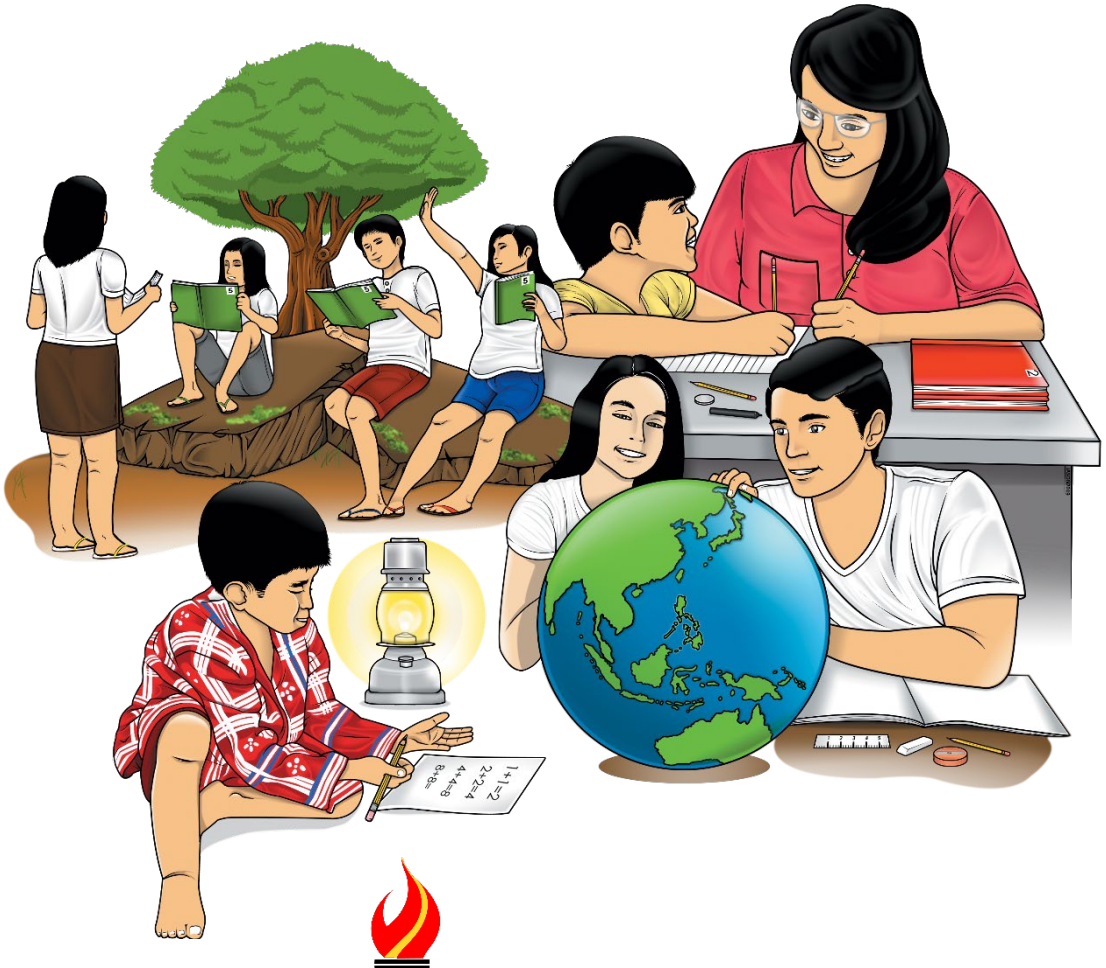
Senior High School

DepED
DEPARTMENT OF EDUCATION

General Mathematics

Quarter 1 – Module 13:

Representations of an Inverse Functions



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Quarter 1 – Module 13: Representations of Inverse Functions

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed and written with you in mind. It is here to help you understand the inverse function. Particularly, this will provide you guide on how to find the inverse of a one-to-one function. Enjoy as you immerse yourself in solving for the inverse function intuitively or using a set of more established steps.

The module is composed of one lesson, namely:

- Lesson 1 – Representing an Inverse Function through Table of Values, and Graph

After going through this module, you are expected to:

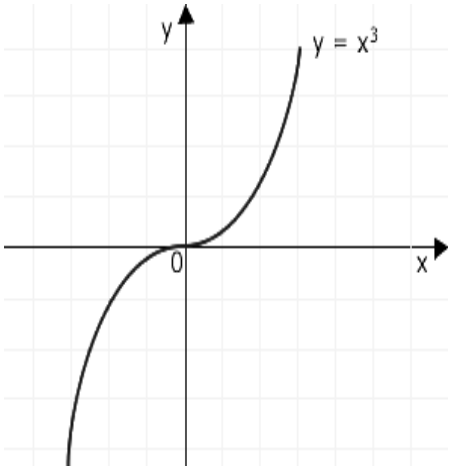
1. represent an inverse function through its: (a) table of values and (b) graph.

What I Know

Read the given questions carefully and choose the correct options. Write the letter of your choice on a separate sheet of paper.

1. Which of the following graphs do not belong to the group?

a.



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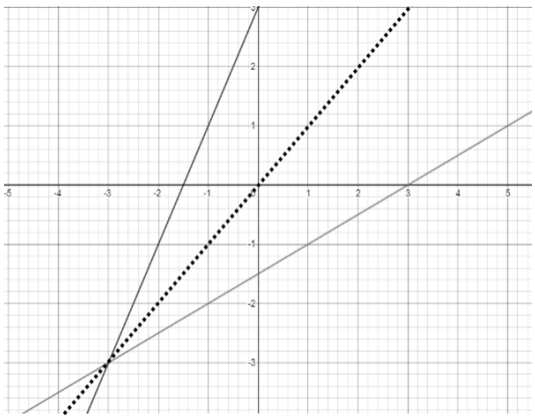

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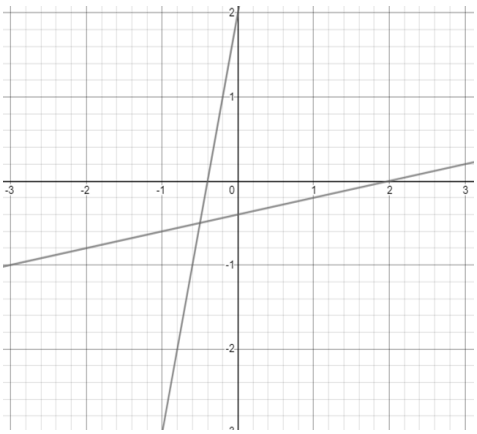

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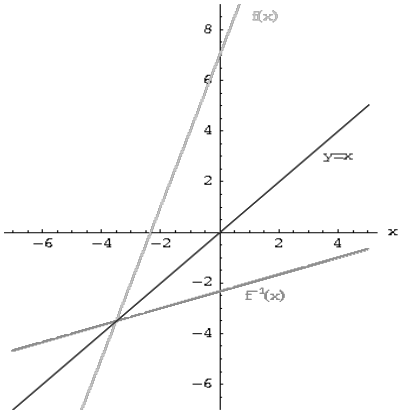


2. The graph of a one-to-one function and its inverse function is _____

a. Hyperbolic b. Parabola c. Parallel d. Symmetric

3. Which of the following terms deals with inverse?

a. alternate b. delete c. eliminate d. interchange

4. Which of the following equations is used to test if the graph of a one-to-one function and its inverse function?

a. f(x) = y b. y = x c. f(x) = x d. f(x) = x+y

5. Which of the following ordered pairs of an inverse function has its one-to-one function indicated in the table below?

f(x)	3	4	5	6	7
y	-2	-1	0	1	2

a. $f^{-1}(x)$ = {(-3,-2), (-4,-1), (0,5), (-1,6), (-2, 7)}

b. $f^{-1}(x)$ = {(3,2), (4,1), (0,-5), (1,-6), (2,-7)}

c. $f^{-1}(x)$ = {(3,-2), (4,-1), (0,5), (1,6), (2, 7)}

d. $f^{-1}(x)$ = {(-2,3), (-1,4), (0,5), (1,6), (2,7)}

6. What test is used to determine if a function is one-to-one function?

a. Diagonal line test

b. Horizontal line test

c. Straight line test

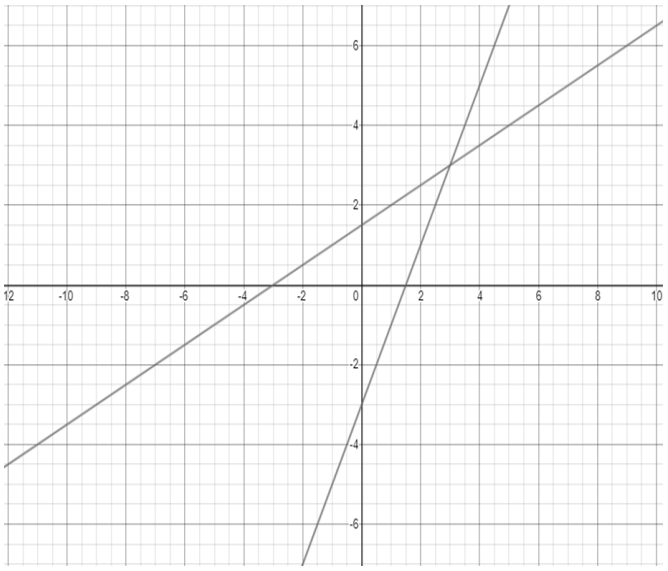
d. Vertical line test

7. What is the missing value in the table below with the equation, f(x) = x -3?

f(x)	3	5	7	9	11
y	0	2	____	6	8

a. 3 b. 4 c. 5 d. 6

8. The illustration shows the one-to-one function and its inverse, which pair has the correct functions?



a. $f(x) = 2x - 3$ and $f^{-1}(x) = \frac{x+3}{2}$

b. $f(x) = 3x - 2$ and $f^{-1}(x) = \frac{x+2}{3}$

c. $f(x) = x - 3$ and $f^{-1}(x) = \frac{x+1}{3}$

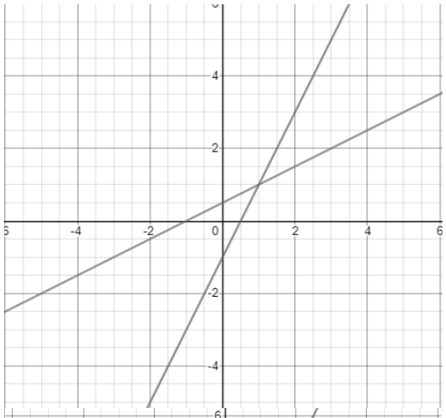
d. $f(x) = 2x + 3$ and $f^{-1}(x) = \frac{x-3}{2}$

9. Which term was used to represent the x-values of the function are the y-values of its inverse, and the y-values of the function are the x-values of its inverse?
   a. Coordinating values
   b. Intersecting values
   c. Table of values
   d. True values

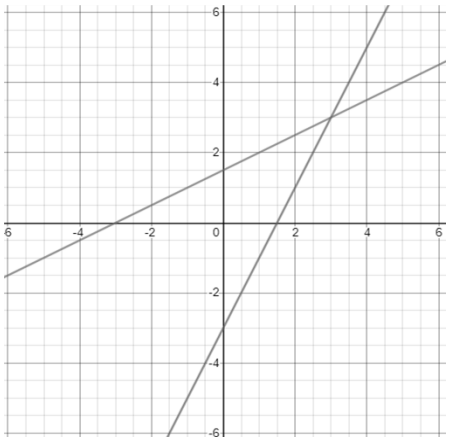
10. Which is true about f(x) = 2x - 1 and $y = \frac{x+1}{2}$?
   a. They are inverse functions.
   b. They are not one-to-one functions.
   c. The graphs are parallel to each other.
   d. The graphs intersect at two or more points.

11. Which of the following graphs best described f(x) = 2x - 1 and $y = \frac{x+1}{2}$?

a.



c.



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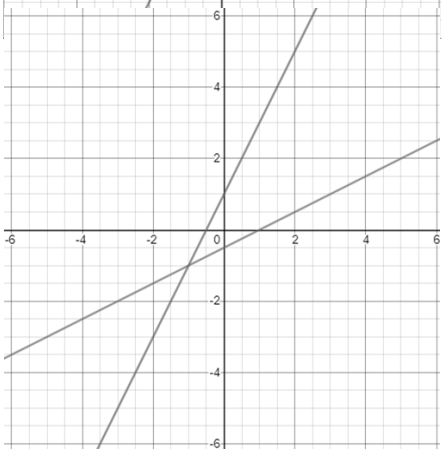
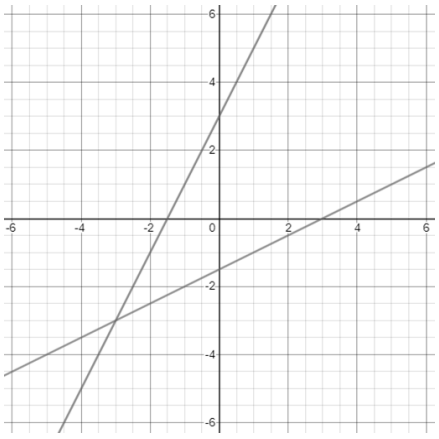


d.



12. If the function is one-to-one and has x and y-values of (2, -6), what is its inverse values?

a. (-2, 6) b. (6,-2) c. (-6,2) d. (6, 2)

13. Which term best complete the sentence: To graph the inverse all you need to do in the coordinates of each ordered pair is to _____.

a. delete b. investigate c. switch d. replace

14. All of the following are used in the representation of inverse function, **EXCEPT**.

a. $f^{-1}(x)$ b. graph c. range d. table of values

15. Which of the following table of values represent the correct inverse function of $f(x) = 2x + 3$?

a.

$f^{-1}(x)$	-2	-1	0	1	2
y	-1	1	3	5	7

b.

$f^{-1}(x)$	-1	1	3	5	7
y	-2	-1	0	1	2

c.

$f^{-1}(x)$	1	-1	-3	-5	-7
y	-2	-1	0	1	2

d.

$f^{-1}(x)$	-1	1	3	5	7
y	2	1	0	-1	-2

Lesson 1 Representing an inverse function through table of values, and graph

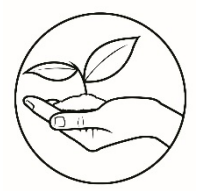
Among the functions, only a one-to-one function has an inverse which is a function also which can be represented in table of values and graphs.

What's In

You have learned different types of functions in the previous lessons. Do you ever wonder if inverses of functions are functions as well? That is, both the original equation and its inverse are both functions. In this lesson, you will delve into these functions with function inverses.

Notes to the Teacher

Enable learners to perform each task or activity in this module. Solutions should be provided for exercises which will not be successfully answered by the learners especially for "Additional Activities" Part.



What's New

You Complete Me!

Below is a table with function f(x) and its inverse $f^{-1}(x)$; and possible values. To enjoy this activity, you need to review the concept of finding the inverse of a one-to-one function and investigate on the possible values of the function and its inverse by completing the statements below the table as many as you can.

Functions			Possible Values		
$f(x)=2x+3$	$f^{-1}(x)=\frac{x+3}{5}$	$f(x)=3x-1$	$f(0)=-3$	$f^{-1}(-1)=-2$	$f^{-1}(1)=\frac{2}{3}$
$f(x)=3x-2$	$f^{-1}(x)=\frac{x-1}{3}$	$f^{-1}(x)=\frac{x-3}{2}$	$f(1)=5$	$f(-1)=-4$	$f(5)=13$
$f(x)=5x-3$	$f^{-1}(x)=\frac{x+2}{3}$	$f^{-1}(x)=\frac{x+1}{3}$	$f^{-1}(1)=1$	$f^{-1}(0)=-\frac{3}{2}$	$f^{-1}(0)=\frac{3}{5}$

1. The function _____ has an inverse function of _____

   with possible values of the function as _____ and _____.

2. The function _____ has an inverse function of _____

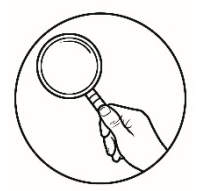
   with possible values of the function as _____ and _____.

3. The function _____ has an inverse function of _____

   with possible values of the function as _____ and _____.

4. The function _____ has an inverse function of _____

   with possible values of the function as _____ and _____.



What is It

Inverse Function Defined with table of values and graph

The inverse of a function is a function with domain B and range A given that the original function has domain A and range B.

This inverse function of function f is denoted by f^{-1}. It is defined by the equation $f^{-1}(y) = x$ if and only if $f(x) = y$ for any y in range B. Since both are functions, then a function has to be one-to-one for its inverse to be a function at the same time. If it is a many-to-one function, its inverse is one-to-many which is not a function.

In using **table of values** of the functions, first we need to ascertain that the given function is a one-to-one function wherein no x-values are repeated. It is represented as the x-values of the function resulted as the y-values of its inverse, and the y-values of the function are the x-values of its inverse. Also, the graph should correspond to a one to one function by applying the **Horizontal Line test**. If it passes the test, the corresponding function is one-to-one. Given the graph of a one-to-one function, the **graph of its inverse** can be obtained by reflecting the graph about the line **y = x.**

Example 1

In the given function f(x) = 2x + 3, with an inverse function of $f^{-1}(x) = \frac{x-3}{2}$ as discussed in the previous activity. Let us use the x-values to complete the table of values in y-values for the f(x) = 2x + 3.

f(x)	-2	-1	0	1	2
y					

In order to complete the y-values, let us substitute each x-value from the function, f(x) = 2x + 3.

If x = -2, f(-2) = 2(-2) + 3, by solving it, f(-2) = -4 + 3, then, f(-2) = -1.

If x = -1, f(-1) = 2(-1) + 3, by solving it, f(-1) = -2 + 3, then, f(-1) = 1.

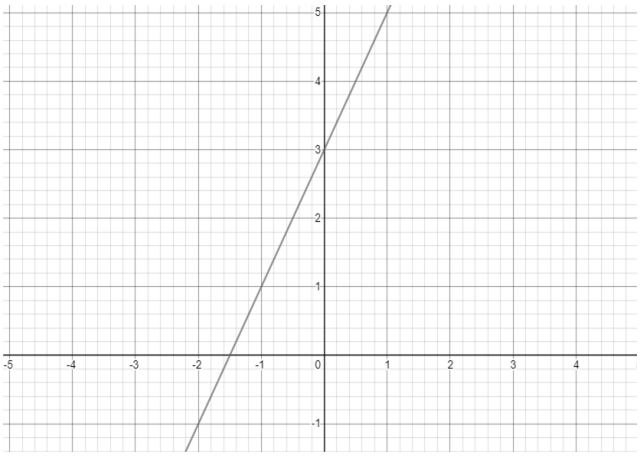
If x = 0, f(0) = 2(0) + 3, by solving it, f(0) = 0 + 3, then, f(0) = 3.

If x = 1, f(1) = 2(1) + 3, by solving it, f(1) = 2 + 3, then, f(1) = 5.

If x = 2, f(2) = 2(2) + 3, by solving it, f(2) = 4 + 3, then, f(2) = 7.

Thus, the table of values for f(x) = 2x + 3 is presented below with its corresponding graph.

f(x)	-2	-1	0	1	2
y	-1	1	3	5	7



At this point, let us investigate on the inverse function $f^{-1}(x) = \frac{x-3}{2}$ by using the y-values from the original function as x-values of the inverse function. Observe the same process in completing the table of values by substituting the x-values to the given inverse function. Now the table of values will be as follows:

$f^{-1}(x)$	-1	1	3	5	7
y					

In order to complete the y-values, let us substitute each x-value from the given inverse function, $f^{-1}(x) = \frac{x-3}{2}$.

If $f^{-1}(-1)$, $f^{-1}(-1) = \frac{(-1)-3}{2}$, by solving it, $f^{-1}(-1) = \frac{-4}{2}$, then $f^{-1}(-1) = -2$.

If $f^{-1}(1)$, $f^{-1}(1) = \frac{(1)-3}{2}$, by solving it, $f^{-1}(1) = \frac{-2}{2}$, then $f^{-1}(1) = -1$.

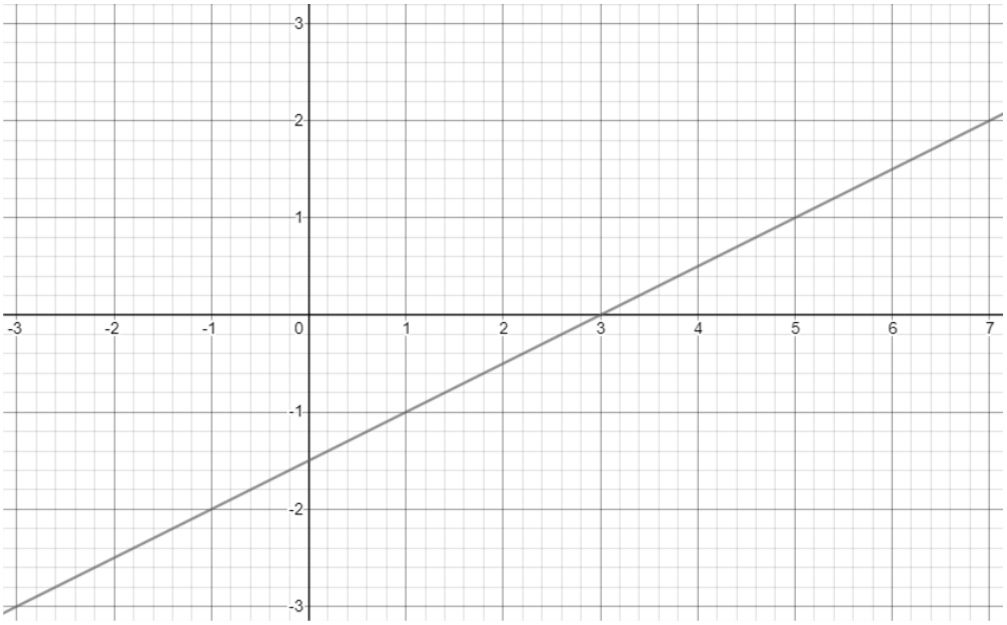
If $f^{-1}(3)$, $f^{-1}(3) = \frac{(3)-3}{2}$, by solving it, $f^{-1}(3) = \frac{0}{2}$, then $f^{-1}(3) = 0$.

If $f^{-1}(5)$, $f^{-1}(5) = \frac{(5)-3}{2}$, by solving it, $f^{-1}(5) = \frac{2}{2}$, then $f^{-1}(5) = 1$.

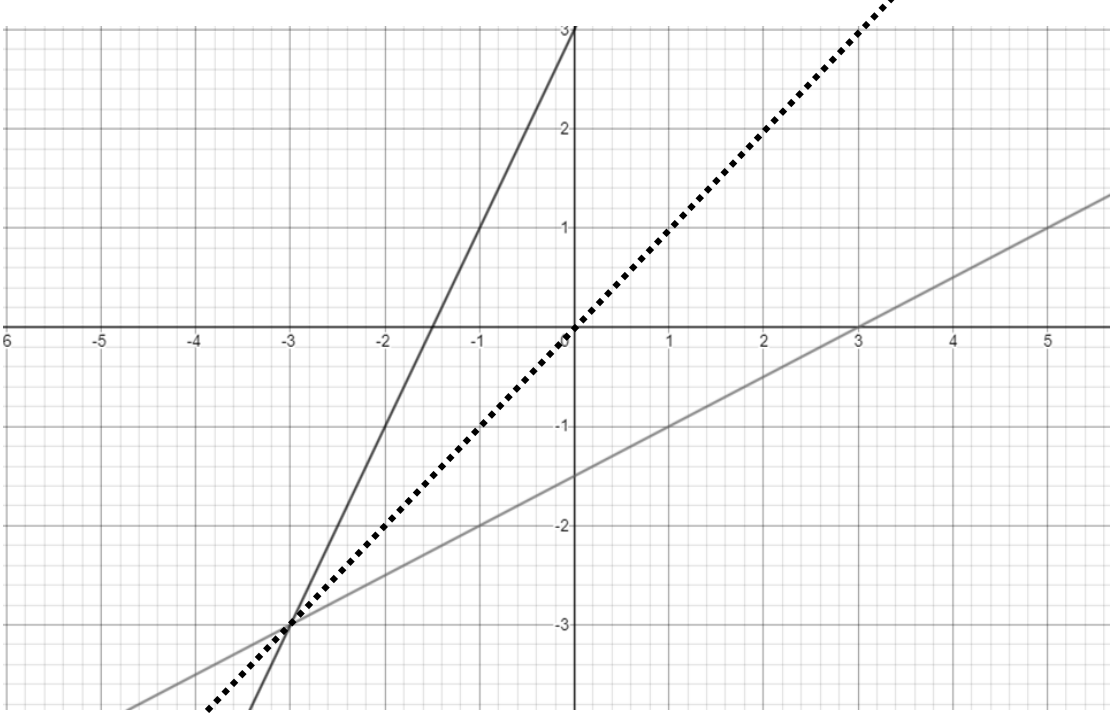
If $f^{-1}(7)$, $f^{-1}(7) = \frac{(7)-3}{2}$, by solving it, $f^{-1}(7) = \frac{4}{2}$, then $f^{-1}(7) = 2$.

Thus, the table of values for $f^{-1}(x) = \frac{x-3}{2}$ is presented below.

$f^{-1}(x)$	-1	1	3	5	7
y	-2	-1	0	1	2

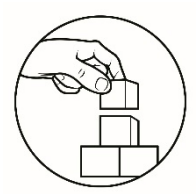


This is the graph of the inverse function, $f^{-1}(x) = \frac{x-3}{2}$. In the next page, the graphs of the two functions will be presented to you. Let us see how it looks like!



As you observed, there is an diagonal line (represented as broken line) across the origin to the point of intersection of the line f(x) = 2x + 3 and $f^{-1}(x) = \frac{x-3}{2}$.

However, for easy steps, if you're asked to graph a function and its inverse, all you must do is graph the function and then switch all x and y values in each point to graph the inverse. Just look at all those values switching places from the f(x) function to its inverse $f^{-1}(x)$ (and back again). Furthermore, the two graphs will be **symmetric** about the line y = x.



What's More

Activity 13.1

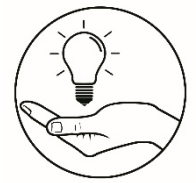
Intuitively, give the table of values of each of the following functions. (Use the values from -2 to 2).

1. $f(x) = x + 2$
2. $g(x) = 12x - 1$
3. $h(x) = -\frac{x}{4}$
4. $f(x) = x$
5. $g(x) = \frac{3x+5}{8}$

Activity 13.2

Illustrate the graph of the given one-to-one function and its inverse.

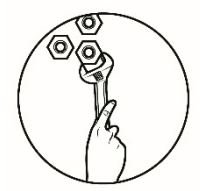
1. $f(x) = 3x - 4$
2. $f(x) = 5x + 3$
3. $f(x) = 7x - 5$
4. $f(x) = \dfrac{x-2}{3}$
5. $f(x) = \dfrac{x+3}{2}$
6. $f(x) = \dfrac{x+5}{4}$



What I Have Learned

Now, answer the following questions.

1. Describe the graph of a one-to-one function and its inverse?
2. What will you do to graph the inverse function?
3. How important it is to present table of values of a function?



What I Can Do

In real life, temperature conversions provide a rich source of linear functions which are encountered not only in science but also in our everyday lives when we travel abroad. This time try to explore the task below, the first part of this task provides an opportunity to construct a linear function given two input-output pairs. The second part investigates the inverse of a linear function while the third part requires reasoning about quantities and/or solving a linear equation.

Let f be the function that assigns to a temperature in degrees Celsius its equivalent in degrees Fahrenheit.

A. The freezing point of water in degrees Celsius is 0 while in degrees Fahrenheit it is 32. The boiling point of water is 100 degrees Celsius and 212 degrees Fahrenheit. Given that the function f is linear, use this information to find an equation for f.

B. Find the inverse of the function f and explain its meaning in terms of temperature conversions.

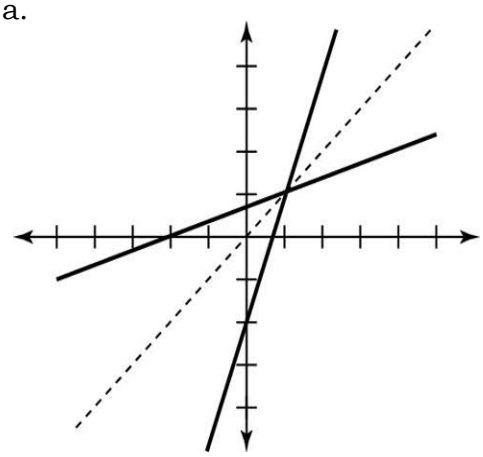
C. Is there a temperature which is the same in degrees Celsius and in degrees Fahrenheit? Explain how you know.

Assessment

Multiple Choice. Choose the letter of the best answer. Write the letter of your choice on a separate sheet of paper.

1. Which of the following graphs do not belong to the group?

a.



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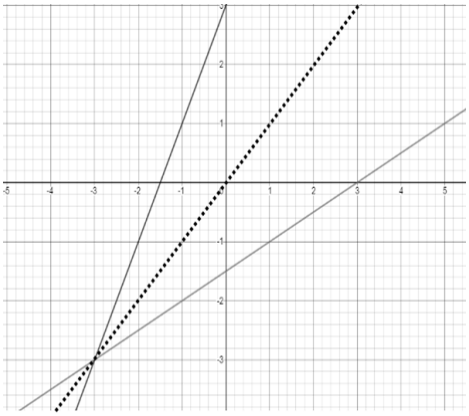

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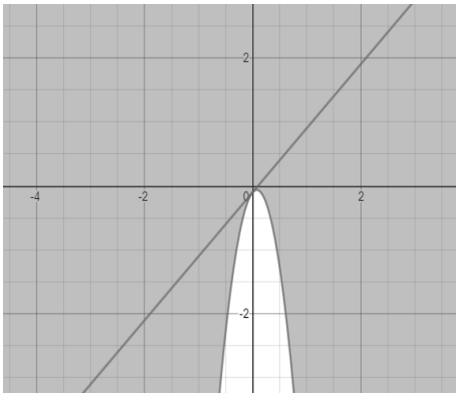

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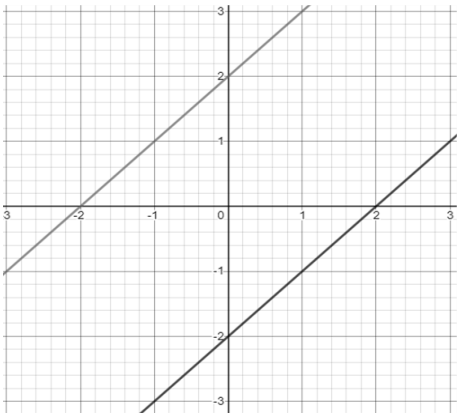


2. The following are the behaviors of a graph of a one-to-one function and its inverse function, **EXCEPT.**

a. intersecting b. Parabola c. Parallel d. Symmetric

3. Which of the following terms do not deals with inverse?

a. alternate b. interchange c. switch d. reverse

4. Which of the following equations is used to test if the graph of a one-to-one function and its inverse function?

a. f(x) = y b. f(x) = x c. f(x) = x+y d. y = x

5. Which of the following ordered pairs of an inverse function has its one-to-one function indicated in the table below?

f(x)	-2	-1	0	1	2
y	0	1	2	3	4

a. $f^{-1}(x) = \{(-2,0), (-1,1), (0,2), (1,3), (2, 4)\}$

b. $f^{-1}(x) = \{(-2,0), (-1,-1), (0,-2), (1,-3), (2,-4)\}$

c. $f^{-1}(x) = \{(2,0), (1,1), (0,2), (-1,3), (-2, 4)\}$

d. $f^{-1}(x) = \{(0,-2), (1,-1), (2,0), (3,1), (4,2)\}$

6. What is the missing ordered pair in the table below with a f(x) = 2x +9?

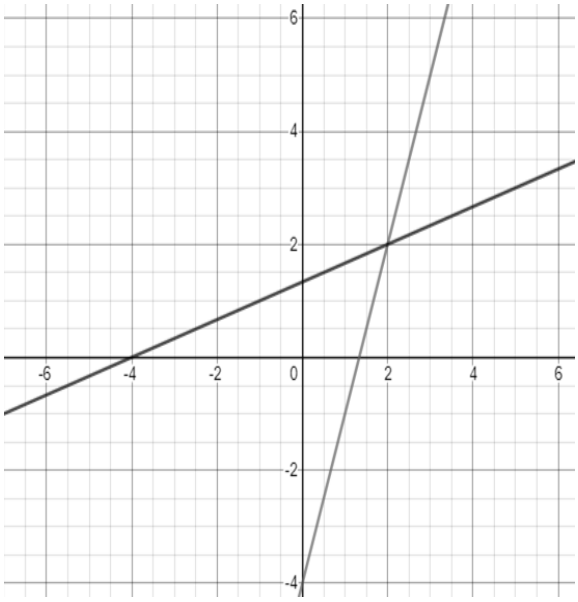
f(x)	3	5	___	9	11
y	15	19	____	27	31

a. (6,20) b. (7,22) c. (7,23) d. (8,25)

7. In item number 8, which is the missing inverse ordered pair?

a. (20, 6) b. (22,7) c. (23,7) d. (25,8)

8. The illustration shows the one-to-one function and its inverse, which pair has the correct functions?



a.. $f(x) = 3x - 4$ and $f^{-1}(x) = \dfrac{x+4}{3}$

b. $f(x) = 3x - 2$ and $f^{-1}(x) = \dfrac{x+2}{3}$

c. $f(x) = 4x - 3$ and $f^{-1}(x) = \dfrac{x+3}{4}$

d. $f(x) = 2x + 3$ and $f^{-1}(x) = \dfrac{x-3}{2}$

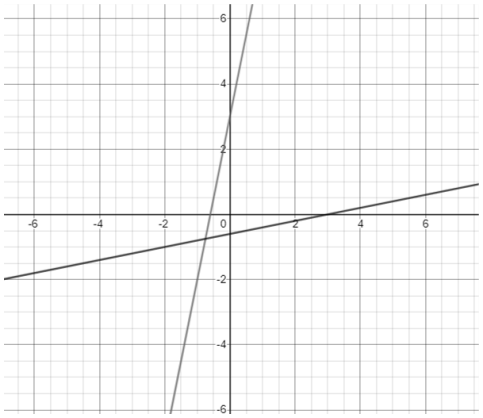
9. Which term was used to represent the x and y-values of the functions.
   a. Coordinating values
   b. Intersecting values
   c. Table of values
   d. True values

10. Which is true about f(x) = 5x - 3 and $y = \frac{x+5}{3}$?

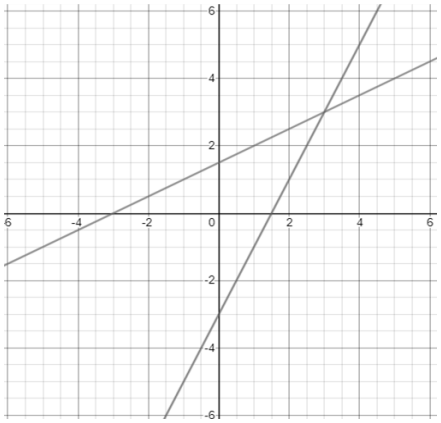
a. They are inverse functions.
b. They are not inverse functions.
c. The graphs are parallel to each other.
d. The graphs intersect at two or more points.

11. Which of the following graphs best descirbed $f(x) = 5x + 3$ and $y = \frac{x-3}{5}$?

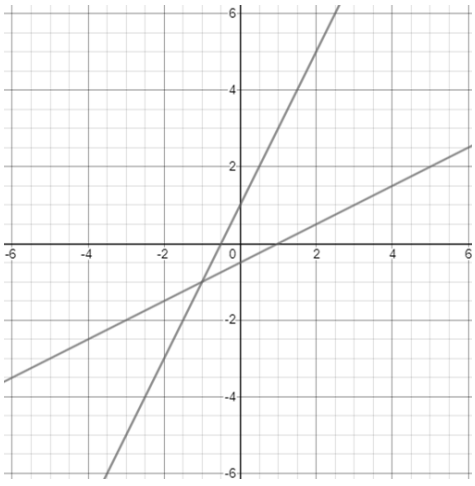
a.



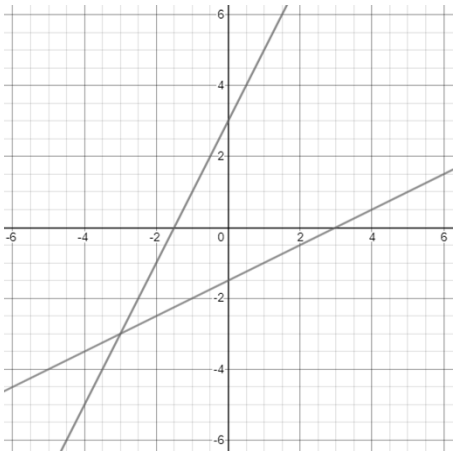
c.



b.



d.



12. If the function is one-to-one and has x and y-values of (-2, 7), what is its inverse values?

a. (-2, 7) b. (7,-2) c. (-7,2) d. (7, 2)

13. If the function is one-to-one and has x and y-values of (5, 0), what is its inverse values?

a. (5, 0) b. (0,-5) c. (0, 5) d. (-5, 0)

14. All of the following are used in the representation of inverse function, EXCEPT.

a. domain b. $f^{-1}(x)$ c. graph d. table of values

15. Which of the following table of values best represents the correct inverse function of f(x) = 3x + 7?

a.

$f^{-1}(x)$	1	-1	-3	-5	-7
y	10	4	-2	-8	-14

b.

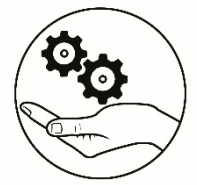
$f^{-1}(x)$	-2	-1	0	1	2
y	1	4	7	10	13

c.

$f^{-1}(x)$	1	4	7	10	13
y	-2	-1	0	1	2

d.

$f^{-1}(x)$	10	4	-2	-8	-14
y	1	4	7	10	13



Additional Activities

Show the inverse function and construct a table of values for each of the following function.

1. $f(x) = \frac{x+7}{3}$

2. $f(x) = \frac{x-2}{5}$

Answer Key

What I Know

1. A
2. D
3. D
4. B
5. D
6. D
7. B
8. A
9. C
10. A
11. A
12. C
13. C
14. C
15. B

What's More

Activity 12.1

1. f(x)=x+2

$f(x)$	-2	-1	0	1	2
y	0	1	2	3	4

2. f(x)=12x - 1

$f(x)$	-2	-1	0	1	2
y	-25	-13	-1	11	23

3. h(x)= - x/4

$f(x)$	-2	-1	0	1	2
y	1/2	1/4	0	1/4	1/2

4. f(x) = x

$f(x)$	-2	-1	0	1	2
y	-2	-1	0	1	2

5. g(x) = (3x+5)/8

$f(x)$	-2	-1	0	1	2
y	-1/8	1/4	5/8	1	11/8

Additional Activities

1. $f(x) = \frac{x+7}{3}$;

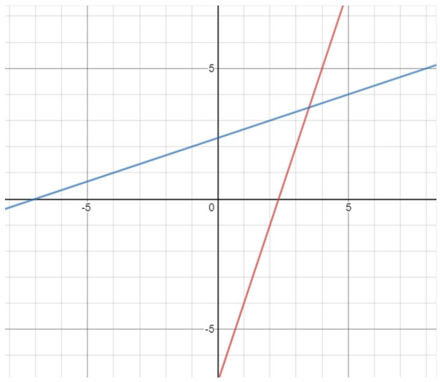
$f(x)$	-4	-1	2	5	8
y	1	2	3	4	5

$f^{-1}(x) = 3x - 7$

$f^{1}(x)$	1	2	3	4	5
y	-4	-1	2	5	8

The graph of $f(x) = \frac{x+7}{3}$ and

$f^{-1}(x) = 3x - 7$



2. $f(x) = \frac{x-2}{5}$

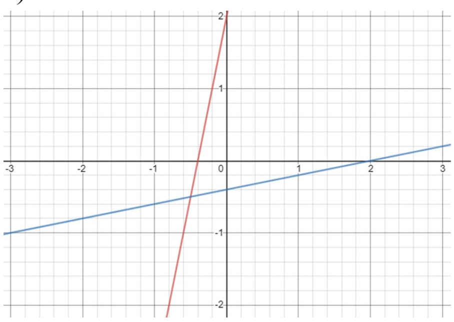
$f(x)$	-3	2	7	12	17
y	-1	0	1	2	3

$f^{-1}(x) = 5x + 2$

$f^{1}(x)$	-1	0	1	2	3
y	-3	2	7	12	17

The graph of $f(x) = \frac{x-2}{5}$ and

$f^{-1}(x) = 5x + 2$



What I Can Do

Solution

A. Since ***f*** is a linear function of ***x***, the temperature in degrees Celsius, we can write $f(x) = ax + b$. We are given that zero degrees Celsius converts to 32 degrees Fahrenheit so $32 = f(0) = b$.

In order to find the slope, ***a***, we can use the second piece of data given, namely that 100 degrees Celsius converts to 212 degrees Fahrenheit: $212 = f(100) = a(100) + 32$

Solving for a gives $a = \frac{9}{5}$. So to convert the temperature in degrees Celsius to degrees Fahrenheit, the appropriate linear function is $f(x) = \frac{9}{5}(x + 32)$.

B. The function ***f*** multiplies the input, ***x***, by $\frac{9}{5}$ and then adds ***32***. Let's call the inverse ***g***. Then an equation describing ***g*** could first subtract ***32*** from its input and then divide by $\frac{9}{5}$. Dividing by $\frac{9}{5}$ is the same as multiplying by $\frac{5}{9}$ so we find $g(x) = \frac{9}{5}(x - 32)$.

The function ***g*** takes as inputs the temperature in degrees Fahrenheit and gives as output the corresponding temperature in degrees Celsius.

C. If there is a temperature x which is the same in degrees Celsius and in degrees Fahrenheit then we would have $f(x) = x$

If we solve the equation $\frac{9}{5}x + 32 = x$ for ***x***, we find that $x = -40$. So ***−40*** is the temperature which registers the same on the Fahrenheit and Celsius scales. For temperatures ***above* −40**, the temperature in degrees Fahrenheit will be greater than the corresponding temperature in degrees Celsius while for temperatures ***below* −40** the temperature in degrees Fahrenheit is less than the corresponding temperature in degrees Celsius.

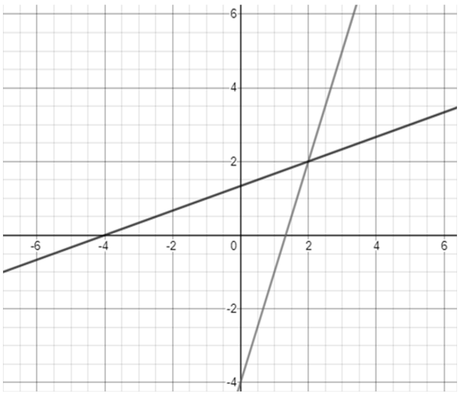
Assessment

1. c
2. b
3. a
4. d
5. d
6. c
7. c
8. a
9. c
10. b
11. a
12. b
13. c
14. a
15. b

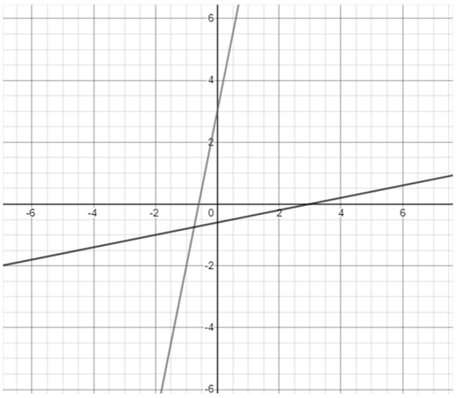
What's More

Activity 12.2

1.



2.



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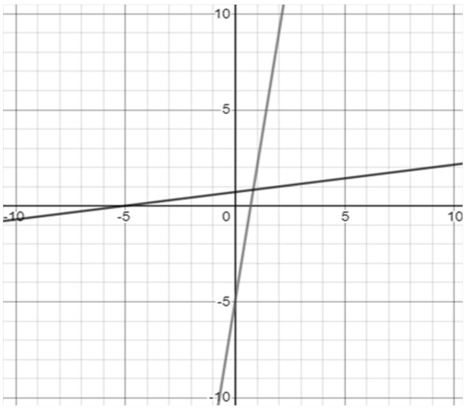

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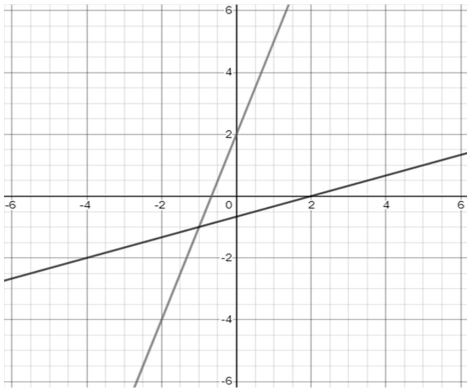

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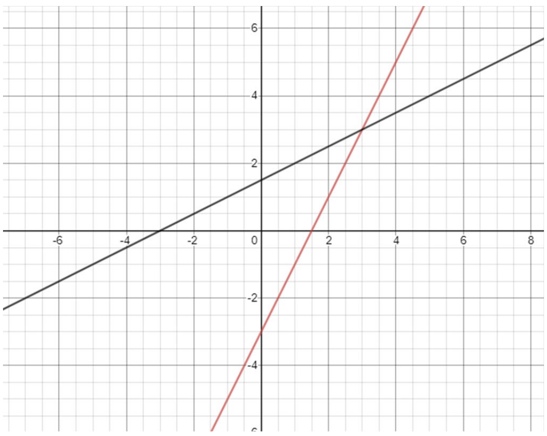

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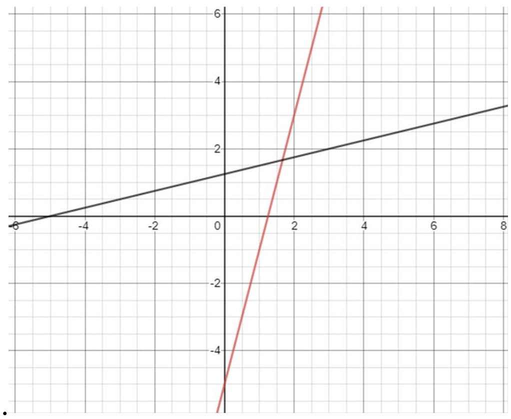

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Verzosa, Debbie Marie, et.al. 2016. *General Mathematics: Learner's Material, First Edition.* Philippines: Lexicon Press Inc.

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