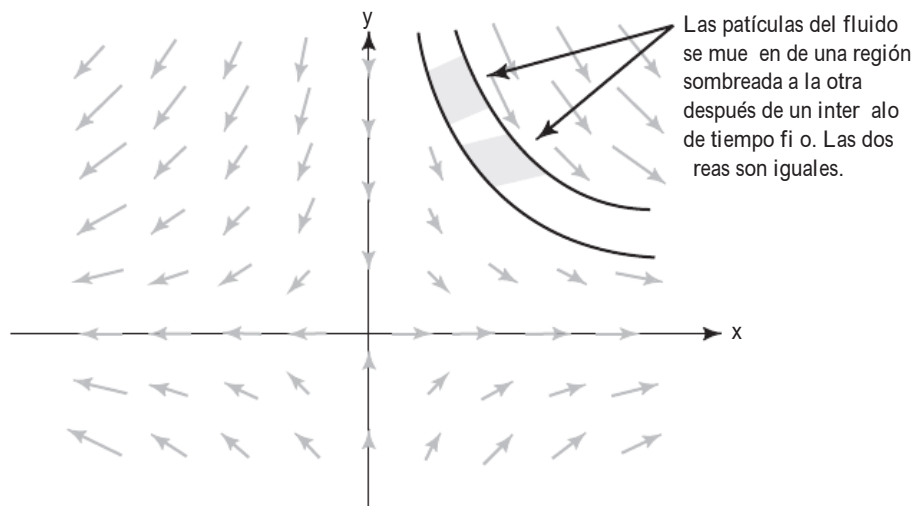


Figura 4.4.6 Campo vectorial $\mathbf{F}(x, y) = x\mathbf{i} - y\mathbf{j}$.



Rotacional

Para calcular el rotacional, la segunda operación básica para campos vectoriales, tomamos el *producto vectorial* de ∇ con \mathbf{F} .

Rotacional de un campo vectorial Si $\mathbf{F} = F_1\mathbf{i} + F_2\mathbf{j} + F_3\mathbf{k}$, el *rotacional* de \mathbf{F} es el *campo vectorial*

$$\begin{aligned} \text{rot } \mathbf{F} = \nabla \times \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} \\ &= \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z} \right) \mathbf{i} + \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x} \right) \mathbf{j} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) \mathbf{k}. \end{aligned}$$

Si escribimos $\mathbf{F} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$, que es una notación alternativa, la misma fórmula para el rotacional sería

$$\begin{aligned} \text{rot } \mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} \\ &= \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) \mathbf{i} - \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z} \right) \mathbf{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \mathbf{k}. \end{aligned}$$

Ejemplo 7

Sea $\mathbf{F}(x, y, z) = x\mathbf{i} + xy\mathbf{j} + \mathbf{k}$. Hallar $\nabla \times \mathbf{F}$.

Solución

Utilizamos la fórmula anterior:

$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & xy & 1 \end{vmatrix} = (0 - 0)\mathbf{i} - (0 - 0)\mathbf{j} + (y - 0)\mathbf{k}.$$

Luego, $\nabla \times \mathbf{F} = y\mathbf{k}$. ▲