9. No.

11. (a) $c^2\dfrac{\partial^2 f}{\partial x^2} = -c^2\operatorname{sen}(x-ct) = \dfrac{\partial^2 f}{\partial t^2}$.

(b) $c^2\dfrac{\partial^2 f}{\partial x^2} = -c^2\operatorname{sen}(x)\operatorname{sen}(ct) = \dfrac{\partial^2 f}{\partial t^2}$.

(c) $c^2\dfrac{\partial^2 f}{\partial x^2} = 30c^2(x-ct)^4 + 30c^2(x+ct)^4 = \dfrac{\partial^2 f}{\partial t^2}$.

13. (a) $\partial^2 z/\partial x^2 = 6, \partial^2 z/\partial y^2 = 4,$.

$\partial^2 z/\partial x\,\partial y = \partial^2 z/\partial y\,\partial x = 0$

(b) $\partial^2 z/\partial x^2 = 0, \partial^2 z/\partial y^2 = 4x/3y^3,$.

$\partial^2 z/\partial x\,\partial y = \partial^2 z/\partial y\,\partial x = -2/3y^2$

15. $f_{xy} = 2x+2y, f_{yz} = 2z, f_{zx} = 0, f_{xyz} = 0$.

17. Dado que f y $\partial f/\partial z$ son ambas de clase C^2, tenemos

$$\frac{\partial^3 f}{\partial x\,\partial y\,\partial z} = \frac{\partial^2}{\partial x\,\partial y}\frac{\partial f}{\partial z} = \frac{\partial^2}{\partial y\,\partial x}\frac{\partial f}{\partial z} = \frac{\partial}{\partial y}\left(\frac{\partial^2 f}{\partial x\,\partial z}\right) = \frac{\partial}{\partial y}\left(\frac{\partial^2 f}{\partial z\,\partial x}\right) = \frac{\partial^3 f}{\partial y\,\partial z\,\partial x}.$$

19. $f_{xzw} = f_{zwx} = e^{xyz}[2xy\cos(xw) + x^2y^2z\cos(xw) - x^2yw\operatorname{sen}(xw)]$.

21. (a) $\dfrac{\partial f}{\partial x} = \arctan\dfrac{x}{y} + \dfrac{xy}{x^2+y^2}$,

$\dfrac{\partial f}{\partial y} = \dfrac{-x^2}{x^2+y^2}$,

$\dfrac{\partial^2 f}{\partial x^2} = \dfrac{2y^3}{(x^2+y^2)^2}, \dfrac{\partial^2 f}{\partial y^2} = \dfrac{2x^2y}{(x^2+y^2)^2}$,

$\dfrac{\partial^2 f}{\partial x\,\partial y} = \dfrac{\partial^2 f}{\partial y\,\partial x} = \dfrac{-2xy^2}{(x^2+y^2)^2}$.

(b) $\dfrac{\partial f}{\partial x} = \dfrac{-x\operatorname{sen}\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}$,

$\dfrac{\partial f}{\partial y} = \dfrac{-y\operatorname{sen}\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}$,

$$\frac{\partial^2 f}{\partial x^2} = \frac{x^2\operatorname{sen}\sqrt{x^2+y^2}}{(x^2+y^2)^{3/2}} - \frac{x^2\cos\sqrt{x^2+y^2}}{x^2+y^2} - \frac{\operatorname{sen}\sqrt{x^2+y^2}}{(x^2+y^2)^{1/2}},$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{y^2\operatorname{sen}\sqrt{x^2+y^2}}{(x^2+y^2)^{3/2}} - \frac{y^2\cos\sqrt{x^2+y^2}}{x^2+y^2} - \frac{\operatorname{sen}\sqrt{x^2+y^2}}{(x^2+y^2)^{1/2}},$$

$$\frac{\partial^2 f}{\partial x\,\partial y} = \frac{\partial^2 f}{\partial y\,\partial x} = xy\left[\frac{\operatorname{sen}\sqrt{x^2+y^2}}{(x^2+y^2)^{3/2}} - \frac{\cos\sqrt{x^2+y^2}}{x^2+y^2}\right].$$

(c) $\dfrac{\partial f}{\partial x} = -2x\exp(-x^2-y^2)$,

$\dfrac{\partial f}{\partial y} = -2y\exp(-x^2-y^2)$,

$\dfrac{\partial^2 f}{\partial x^2} = (4x^2-2)\exp(-x^2-y^2)$,

$\dfrac{\partial^2 f}{\partial y^2} = (4y^2-2)\exp(-x^2-y^2)$,

$\dfrac{\partial^2 f}{\partial x\,\partial y} = \dfrac{\partial^2 f}{\partial y\,\partial x} = 4xy\exp(-x^2-y^2)$.

23.
$$\frac{\partial^2 f}{\partial x^2}\left(\frac{dx}{dt}\right)^2 + 2\frac{\partial^2 f}{\partial x\,\partial y}\frac{dx}{dt}\frac{dy}{dt} + \frac{\partial^2 f}{\partial y^2}\left(\frac{dy}{dt}\right)^2 + \frac{\partial f}{\partial x}\frac{d^2x}{dt^2} + \frac{\partial f}{\partial y}\frac{d^2y}{dt^2},$$

donde $\mathbf{c}(t) = (x(t), y(t))$.

25. Calcular las derivadas $\partial^2 u/\partial x^2$ y $\partial^2 u/\partial y^2$ y sumar.

27. (a) La primera función es armónica, la segunda no lo es.

(b) Cualquier polinomio de grado 1 o 0 es armónico.

29. (a) Calcular las derivadas y comparar.

(b) Véase la figura de la página siguiente.

31. $V = -GmM/r = -GmM(x^2+y^2+z^2)^{-1/2}$. Comprobar que

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = GmM(x^2+y^2+z^2)^{-3/2}$$
$$[3 - 3(x^2+y^2+z^2)(x^2+y^2+z^2)^{-1}] = 0.$$