### Constantes y aclaraciones

$$G(x) = \frac{dN}{dx} = \text{dens. de est.}; \quad g(x) = \frac{dn}{dx} = \frac{\text{dens. de est.}}{V}$$

$k_B = 1.381 \times 10^{-23} JK^{-1} = 8.62 \times 10^{-5} eVK^{-1}$

$m_e = 9.11 \times 10^{-31} kg = 0.511 MeVc^{-2}$

$m_p = 1.67 \times 10^{-27} kg = 938 MeVc^{-2}$

$\varepsilon_0 = \dfrac{1}{4\pi K} = 8.85 \times 10^{-12} Fm^{-1}$

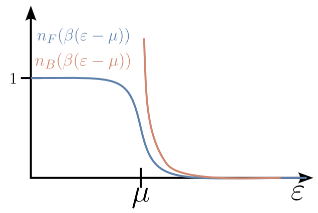
$\hbar = 1.055 \times 10^{-34} Js = 6.58 \times 10^{-16} eVs$

$e = 1.602 \times 10^{-19} C$

Fermions: $e^-, p, n \quad (n_F(x) = \dfrac{1}{e^x + 1})$

Bosons: phonon, photon $\quad (n_B(x) = \dfrac{1}{e^x - 1})$

$n \sim 10^{22} cm^{-3}; \tau \sim 10^{-15} s; v \sim 10^{-5} \dfrac{m}{s}$




$\underline{T \text{ bajas}}$

$n_F(x) \approx n_B(x) \approx e^{-x}$

$\underline{T \text{ altas}}$

$n_F(x) \approx \frac{1}{2+x}$

$n_B(x) \approx \frac{1}{x}$

# 1 Estructura cristalina

## 1.1 Redes de Bravais

| | |
|---|---|
| $a$ | triclínica |
| $m$ | monoclínica |
| $o$ | ortorómbica |
| $t$ | tetragonal |
| $h$ | hexagonal |
| $c$ | cúbica |

| | |
|---|---|
| $P$ | Primitiva |
| $S$ | Centrada en una cara |
| $I$ | Centrada en el cuerpo |
| $R$ | Centrada romboidal |
| $F$ | Centrada en las caras |

14 posibles redes de Bravais

| Tric. | Monoc. | Ortor. | Tetra. | Hex. | Cúbico |
|---|---|---|---|---|---|
| $aP$ | $mP, mS$ | $oP, oS, oF, oI$ | $tP, tI$ | $hP, hR$ | $cP, cF, cI$ |

## 1.2 Cosas

Base dual y matriz métrica

$$a^* = \frac{b \times c}{V}, \quad b^* = \frac{c \times a}{V}, \quad c^* = \frac{a \times b}{V}, \quad V = \det(\bar{a}, \bar{b}, \bar{c})$$

$$(\bar{a}^*, \bar{b}^*, \bar{c}^*) = \begin{pmatrix} \bar{a}^T \\ \bar{b}^T \\ \bar{c}^T \end{pmatrix}^{-1}, G = \begin{pmatrix} a \cdot a & a \cdot b & a \cdot c \\ b \cdot a & b \cdot b & b \cdot c \\ c \cdot a & c \cdot b & c \cdot c \end{pmatrix}, G^* = G^{-1}$$

Cambio de base

$$(\bar{a}', \bar{b}', \bar{c}') = (\bar{a}, \bar{b}, \bar{c})P, \quad \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = P^{-1} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$(x, y, z) = (x^*, y^*, z^*)P, \quad \begin{pmatrix} a'^* \\ b'^* \\ z'^* \end{pmatrix} = P^{-1} \begin{pmatrix} a^* \\ b^* \\ c^* \end{pmatrix}$$

Distancia interplanar $g_{hkl} = \frac{1}{d_{hkl}}; \quad g_{hkl}^2 = (hkl)G^* \begin{pmatrix} h \\ k \\ l \end{pmatrix}$

Transferencia de momento $Q = \frac{4\pi \sin\theta}{\lambda}$

Condiciones de Laue $\overline{Q} = 2\pi \overline{g}_{hkl}$

Ley de Bragg $g_{hkl} = \frac{2 \sin\theta_{hkl}}{\lambda}$

Módulo de Young $\nu_s = \sqrt{\frac{\gamma}{\rho}}$

Factor de estructura $F_{hkl} = \sum_p f_p e^{-i2\pi \overline{g}_{hkl} \cdot \overline{r}_p}; I \propto |F_{hkl}|^2$

## 1.3 Estructuras comunes

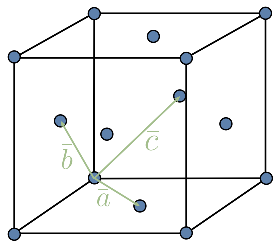
FCC (primitiva volumen 1/4)

$$\begin{cases} \bar{a} = \frac{1}{2}(1\ 1\ 0) \\ \bar{b} = \frac{1}{2}(0\ 1\ 1) \\ \bar{c} = \frac{1}{2}(1\ 0\ 1) \end{cases} \qquad \begin{cases} \bar{a}^* = (1\ 1\ -1) \\ \bar{b}^* = (-1\ 1\ 1) \\ \bar{c}^* = (1\ -1\ 1) \end{cases}$$

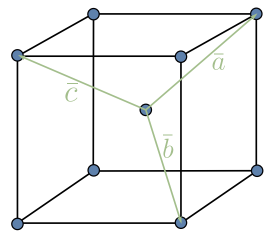
BCC (primitiva volumen 1/2)

$$\begin{cases} \bar{a} = \frac{1}{2}(1\ 1\ -1) \\ \bar{b} = \frac{1}{2}(-1\ 1\ 1) \\ \bar{c} = \frac{1}{2}(1\ -1\ 1) \end{cases} \qquad \begin{cases} \bar{a}^* = (1\ 1\ 0) \\ \bar{b}^* = (0\ 1\ 1) \\ \bar{c}^* = (1\ 0\ 1) \end{cases}$$

FCC



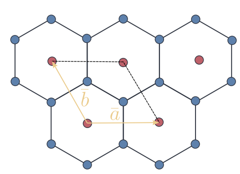

BCC

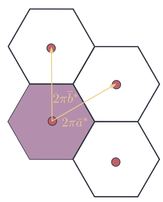


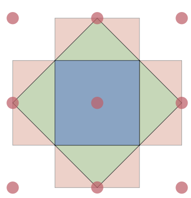

Hexagonal

$$\begin{cases} \bar{a} = (1, 0) \\ \bar{b} = (-\frac{1}{2}, \frac{\sqrt{3}}{2}) \end{cases} \qquad \begin{cases} \bar{a}^* = \frac{2\sqrt{3}}{3}(\frac{\sqrt{3}}{2}, \frac{1}{2}) \\ \bar{b}^* = \frac{2\sqrt{3}}{3}(0, 1) \end{cases}$$

$$G = \begin{pmatrix} a^2 & -\frac{a^2}{2} & 0 \\ -\frac{a^2}{2} & a^2 & 0 \\ 0 & 0 & c^2 \end{pmatrix}, \quad G^* = \begin{pmatrix} \frac{4}{3a^2} & \frac{2}{3a^2} & 0 \\ \frac{2}{3a^2} & \frac{4}{3a^2} & 0 \\ 0 & 0 & \frac{1}{c^2} \end{pmatrix}$$



En una hcp $c = 1.633a$

## 1.4 Grupos

$$m_{100} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; n_{001} = \begin{pmatrix} \cos\left(\frac{360}{n}\right) & -\sin\left(\frac{360}{n}\right) & 0 \\ \sin\left(\frac{360}{n}\right) & \cos\left(\frac{360}{n}\right) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Cambio de base a $\mathcal{B} = \{\overline{u}, \overline{v}, \overline{w}\}$

$$M_{\mathcal{C}} = M_{\mathcal{B}\to\mathcal{C}} M_{\mathcal{B}} M_{\mathcal{B}\to\mathcal{C}}^{-1}, \quad M_{\mathcal{B}\to\mathcal{C}} = (\overline{u}, \overline{v}, \overline{w})$$

Reflexión vector director $(a, b, c)$

$$M = \frac{1}{a^2+b^2+c^2}\begin{pmatrix} -a^2+b^2+c^2 & -2ab & -2ac \\ -2ab & a^2-b^2+c^2 & -2bc \\ -2ac & -2bc & a^2+b^2-c^2 \end{pmatrix}$$

Rotación respecto $\hat{u} = (u_x, u_y, u_z)$ $(c = \cos\theta, \quad s = \sin\theta)$.
$R =$

$$\begin{pmatrix} c + u_x^2(1-c) & u_x u_y(1-c) - u_z s & u_x u_z(1-c) + u_y s \\ u_y u_x(1-c) + u_z s & c + u_y^2(1-c) & u_y u_z(1-c) - u_x s \\ u_z u_x(1-c) - u_y s & u_z u_y(1-c) + u_x s & c + u_z^2(1-c) \end{pmatrix}$$

Centrosimétricos $(x, y, z) \to (-x, -y, -z)$ no tienen polarización espontánea

# 2 Dinámica de cristales

## 2.1 Densidad de estados

$$\overline{k} = \left(\tfrac{2\pi}{L}n \quad \tfrac{2\pi}{L}m \quad \tfrac{2\pi}{L}l\right) \ \forall n, m, l \in \mathbb{Z}$$

Número de estados hasta $k$

$$N(k) = \int_{(\frac{2\pi}{L})^2(n^2+m^2+l^2)\le k^2} dV = \frac{L^3}{6\pi^2}k^3 = \frac{V}{6\pi^2}k^3$$

1, 2 y 3 dimensiones respectivamente (y se cumple $\omega = \nu_s k$)

$$\begin{cases} G(k) = \frac{L}{\pi} \\ G(\omega) = \frac{L}{\pi\nu} \end{cases} \quad \begin{cases} G(k) = \frac{L^2}{2\pi}k \\ G(\omega) = \frac{L^2}{2\pi\nu^2}\omega \end{cases} \quad \begin{cases} G(k) = \frac{V}{2\pi^2}k^2 \\ G(\omega) = \frac{V}{2\pi^2\nu_s^3}\omega^2 \end{cases}$$

## 2.2 Dispersión

Oscilador con masa $m$ y constante $k_s$

$$F_n = m\ddot{x}_n = k_s(x_{n+1} + x_{n-1} - 2x_n)$$

$$-m\omega^2 A e^{i(kna-\omega t)} = k_s A e^{i(kna-\omega t)}(e^{ika} + e^{-ika} - 2) =$$
$$= -4k_s \sin^2\left(\frac{ka}{2}\right) \Rightarrow \boxed{\omega = 2\sqrt{\frac{k_s}{m}}\left|\sin\left(\frac{ka}{2}\right)\right|}$$

Oscilador con masa $m$ y constantes alternadas $k_1, k_2$

$$\begin{cases} m\ddot{x}_n = k_1(y_{n-1} - x_n) + k_2(y_n - x_n) \\ m\ddot{y}_n = k_1(x_{n+1} - y_n) + k_2(x_n - y_n) \end{cases}$$

Ansatz

$$x_n = Ae^{i(kna-\omega t)} \qquad y_n = Be^{i(kna-\omega t)}$$

Ecuaciones

$$\begin{cases} -m\omega^2 A = -A(k_1+k_2) + B(k_1 e^{ika} + k_2) \\ -m\omega^2 B = -A(k_1 e^{ika} + k_2) + B(-k_1 - k_2) \end{cases}$$

Forma matricial

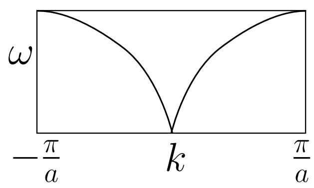
$$m\omega^2 \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} (k_1+k_2) & -k_2 - k_1 e^{ika} \\ -k_2 - k_1 e^{ika} & (k_1+k_2) \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = K\begin{pmatrix} A \\ B \end{pmatrix}$$

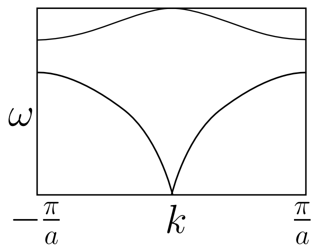
$$0 = \det(K - m\omega^2 I) = |(k_1+k_2) - m\omega^2|^2 - |k_2 + k_1 e^{ika}|^2$$

$$\boxed{\omega_\pm(k) = \sqrt{\frac{k_1+k_2}{m} \pm \frac{1}{m}\sqrt{(k_1+k_2)^2 - 4k_1k_2\sin^2(ka/2)}}}$$

Si $m_1 \neq m_2$ y $k_s$ es la misma, sea $K_i = \frac{k}{m_i}$, entonces

$$\boxed{\omega_\pm(k) = \sqrt{(K_1+K_2) \pm \sqrt{(K_1+K_2)^2 - 4K_1K_2\sin^2(ka/2)}}}$$




Si hay $N$ átomos / celda: $3N$ ramas:

- 3 acústicas (2 trans. < 1 long.)
- $3N - 3$ ópticas

## 2.3 Modelo de Einstein

$$E_n = \hbar\omega(n + \frac{1}{2}) \quad \Rightarrow \quad Z_1 = \frac{1}{2\sinh(\frac{\beta\hbar\omega}{2})}$$

$$\langle E_1 \rangle = -\frac{\partial}{\partial\beta}\ln Z_1 = \frac{\hbar\omega}{2}\coth\left(\frac{\beta\hbar\omega}{2}\right)$$

Energía y capacidad calorífica

$$\langle E \rangle = \frac{3}{2}N\hbar\omega\coth\left(\frac{\beta\hbar\omega}{2}\right)$$

$$C_v = \frac{\partial\langle E\rangle}{\partial T} = 3Nk_B(\beta\hbar\omega)^2\frac{e^{\beta\hbar\omega}}{(e^{\beta\hbar\omega}-1)^2}$$

Definimos ahora $T_E = \frac{\hbar\omega_E}{k_B}$. En los límites

- Si $T \gg T_E \quad \Rightarrow \quad C_v = 3Nk_b$
- Si $T \ll T_E \quad \Rightarrow \quad C_v = 3Nk_b(\frac{T_E}{T})^2\frac{1}{\sinh^2(\frac{T_E}{2T})}$

## 2.4 Modelo de Debye

Aproximamos la ecuación de dispersión para $k$ baja como $\omega = \nu k$

$$3N = \int_0^{\omega_D} 3G(\omega)d\omega = \frac{V}{2\pi^2\nu^3}\omega_D^3 \Rightarrow \boxed{\omega_D = \sqrt[3]{\frac{6\pi^2\nu^3 N}{V}}}$$

donde hemos contado cada partícula y cada estado 3 veces y hemos usado

$$\omega = \nu k, \qquad G(k) = \frac{V}{2\pi^2}k^2, \qquad G(\omega) = \frac{V}{2\pi^2\nu^3}\omega^2 = 3N\frac{\omega^2}{\omega_D^3}$$

La energía y la capacidad calorífica

$$\langle E\rangle = \int_0^{\omega_D} \hbar\omega 3G(\omega)\left(\frac{1}{e^{\beta\hbar\omega}-1}+\frac{1}{2}\right)d\omega =$$
$$= E_0 + \frac{3V\hbar}{2\pi^2\nu^3}\int_0^{\omega_D}\frac{\hbar\omega^3}{e^{\beta\hbar\omega}-1}d\omega \qquad (x=\frac{\hbar\omega}{k_BT})$$
$$T_D := \frac{\hbar\omega_D}{k_B} \quad \Rightarrow \boxed{\langle E\rangle = \frac{3Vk_B^4T^4}{2\pi^2\nu^3\hbar^3}\int_0^{\frac{T_D}{T}}\frac{x^3}{e^x-1}dx}$$

La capacidad calorífica $C_v = \frac{\partial\langle E\rangle}{\partial T}$ en los extremos:

- Si $T \gg T_D \quad \Rightarrow \quad \langle E\rangle \sim 3Nk_BT \quad \Rightarrow \quad C_v \sim 3Nk_B$
- Si $T \ll T_D \quad \Rightarrow \quad \langle E\rangle \sim \frac{3\pi^4Nk_BT^4}{5T_D^3} \quad \Rightarrow \quad C_v \sim \frac{12\pi^4}{5}Nk_B\left(\frac{T}{T_D}\right)^3$

# 3 Electrones en los sólidos

Modelo de Drude

$$n=\frac{N}{V};\quad \frac{dp}{dt}=-\frac{p}{\tau}+F;\quad \overline{j}=-ne\overline{v}=\sigma\overline{E}$$
$$\sigma_0=\frac{e^2\tau n}{m};\quad \text{si } F-e\mathrm{Re}[E_0e^{i\omega t}]\Rightarrow\sigma(\omega)=\frac{\sigma_0}{1-i\omega t}$$
$$mv=p=-e\tau E;\quad R_H=\frac{-1}{ne}=\frac{\rho_{yx}}{|B|}$$
$$\overline{E}=\tilde{\rho}\overline{j};\quad \rho_{xx}=\rho_{yy}=\rho_{zz}\frac{m}{ne^2\tau};\quad \frac{1}{2}mv_0^2=\frac{3}{2}k_BT$$

Efecto Hall (2 portadores con la misma carga opuesta)

$$\mu_i=\frac{\tau_i}{m_i};\quad \sigma=ne^2(\mu_1+\mu_2);\quad R_H=\frac{\mu_2^2-\mu_1^2}{n_0e(\mu_1+\mu_2)^2}e^{\beta E}$$

Hall resistivity $\rho_{xy}=-\rho_{yx}=\frac{B}{ne}$ ($\overline{B}\propto\hat{z}$)

Peltier coefficient $\Pi=-\frac{k_BT}{2e}=\frac{-c_vT}{3e}$

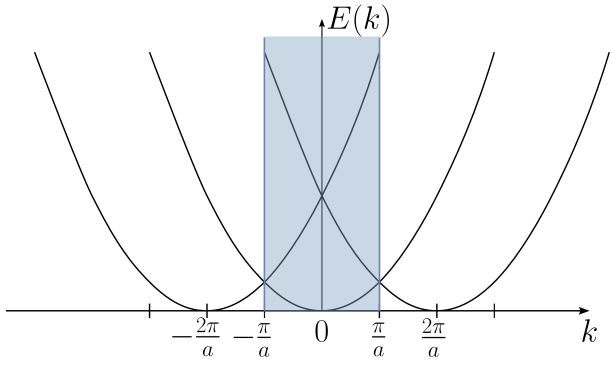
Seebeck coefficient $S=\frac{\Pi}{T}$

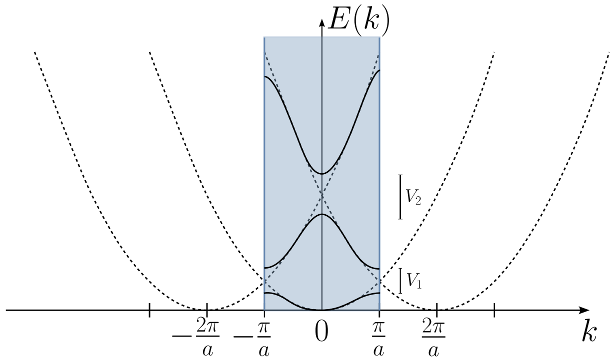
$$\langle v\rangle_{gasid.}=\sqrt{\frac{8k_BT}{\pi m}};\quad \kappa=\frac{1}{3}nc\langle v\rangle^2\tau=\frac{4}{\pi}\frac{n\tau k_B^2T}{m}$$

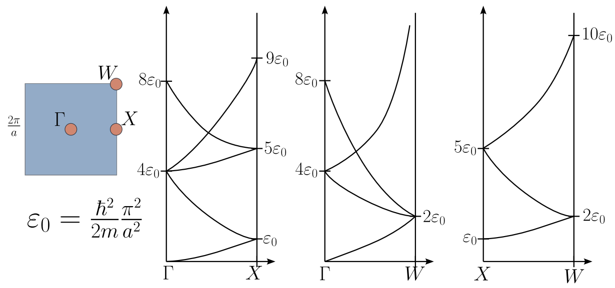
Capacidad Calorífica

$$g(\varepsilon)=\frac{3n}{2(E_F)^{\frac{3}{2}}}\varepsilon^{\frac{1}{2}}=\frac{(2m)^{\frac{3}{2}}}{2\pi^2\hbar^3}\varepsilon^{\frac{1}{2}},\quad k=\sqrt{\frac{2\varepsilon m}{\hbar^2}}$$
$$n=\int_0^\infty d\varepsilon g(\varepsilon)n_F(\beta(\varepsilon-\mu)),\quad \frac{E_T}{V}=\int_0^\infty d\varepsilon\varepsilon g(\varepsilon)n_F(\beta(\varepsilon-\mu))$$
$$C=\frac{\pi^2}{3}\left(\frac{3Nk_B}{2}\right)\left(\frac{T}{T_F}\right)$$

$$\overline{M}=g(E_F)\mu_B^2\overline{B};\quad \mu_B=0.67\left(\frac{K}{Tesla}\right)k_B$$




$$\varepsilon_0=\frac{\hbar^2}{2m}\frac{\pi^2}{a^2}$$

**Teorema de Bloch** ($V(\overline{r})$ periódico)

$$\psi_{\overline{k}}(\overline{r})=u_{\overline{k}}(\overline{r})e^{i\overline{k}\cdot\overline{r}},\quad E(\overline{k})=E(\overline{k}+\overline{G})$$

(1D) Fourier del potencial de dos formas

$$V(x)=V_0+\sum_{j=1}^{\infty}V_j\cos\left(\frac{2\pi j}{a}x\right) \quad \text{ó} \quad V(x)=\sum_{j=-\infty}^{\infty}V_{\frac{2\pi j}{a}}e^{i\frac{2\pi j}{a}x}$$

Con las relaciones $V_j=2V_{\frac{2\pi j}{a}}$, y donde los coeficientes son

$$V_j=\frac{2}{a}\int_{-\frac{a}{2}}^{\frac{a}{2}}dxV(x)\cos\left(\frac{2\pi j}{a}x\right);\quad V_{\frac{2\pi j}{a}}=\frac{1}{a}\int_{-\frac{a}{2}}^{\frac{a}{2}}dxV(x)e^{-i\frac{2\pi j}{a}x}$$

Gas de electrones libres

$$\overline{k}=\frac{2\pi}{L}(n_x,n_y,n_z),\quad E(\overline{k})=\frac{\hbar^2}{2m}|\overline{k}|^2,\quad n_F(x)=\frac{1}{e^x+1}$$
$$N=2\sum_{\overline{k}}n_F(\beta(E(\overline{k})-\mu))=2\frac{V}{(2\pi)^3}\int d\overline{k}n_F(\beta(E(\overline{k})-\mu))$$

Fermi energy ($E_F=\mu(T\to 0)$) (d numero de dimensiones)

$$\varepsilon_F=\frac{\hbar^2k_F^2}{2m}=k_BT_F;\quad p_F=\hbar k_F;\quad U_T=\frac{3}{5}\varepsilon_FN$$
$$N=2\frac{V}{(2\pi)^d}\int_{|k|<k_F}dk\Rightarrow k_F=(3\pi^2n)^{\frac{1}{3}},\quad \varepsilon_F=\frac{\hbar^2(3\pi^2n)^{\frac{2}{3}}}{2m}$$

Considerando los dos espines (multiplicamos por 2)

$$N_T=2\cdot\left(\frac{4}{3}\pi(n_x^2+n_y^2+n_z^2)^{3/2}\right) \quad \Rightarrow \quad k_{max}^2=k_F^2=(3n\pi^2)^{\frac{2}{3}}$$

$e^-$ excitados por encima de $\varepsilon_F$

$$n_{e^-} = 2\cdot\frac{1}{2}\cdot(2k_BT)(\frac{1}{2}g(\varepsilon_F)) = k_BTg(\varepsilon_F); \qquad \frac{n_{e^-}}{n} = \frac{3}{4}\frac{k_BT}{\varepsilon_F}$$

Electrones casi-libres

$$\psi_+ \sim \cos(\pi\frac{x}{a}), \quad \psi_- \sim \sin(\pi\frac{x}{a})$$

$$E^{\pm} = \frac{1}{2}(E^0_{\bar{k}-\bar{G}} + E^0_{\bar{k}}) \pm \sqrt{\frac{1}{4}(E^0_{\bar{k}-\bar{G}} - E^0_{\bar{k}})^2 + |V_{\bar{G}}|^2}$$

Enlace fuerte, celda primitiva cúbica ($B=\gamma, A=\beta$)

$$\varepsilon(\bar{k}) = E - \left(\frac{\beta + \sum_{r_i\neq 0}\gamma(r_i)e^{i\bar{k}\cdot\bar{r}_i}}{1+\sum_{r_i\neq 0}\alpha(r_i)e^{i\bar{k}\cdot\bar{r}_i}}\right)$$

$$E(\bar{k}) \approx E_i - A - 2B(\cos k_xa + \cos k_ya + \cos k_za)$$

$$A = -\langle\varphi_{i,n}|v|\varphi_{i,n}\rangle, \quad B = -\langle\varphi_{i,m}|v|\varphi_{i,n}\rangle$$

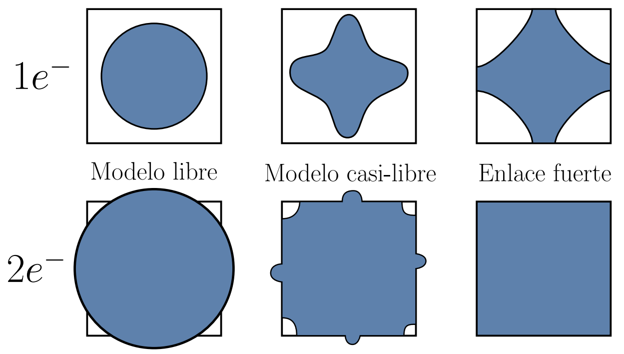
$$\bar{v} = \nabla_{\bar{k}}\omega(\bar{k}) = \frac{1}{\hbar}\nabla_{\bar{k}}E(\bar{k})$$

Carga de un campo $\bar{\mathcal{E}}$

$$\dot{v}_i = \frac{1}{\hbar^2}\sum_j \frac{\partial^2 E}{\partial k_i\partial k_j}(-e\mathcal{E}_j), \quad \left(\frac{1}{m^*}\right)_{ij} = \frac{1}{\hbar^2}\frac{\partial^2 E(\bar{k})}{\partial k_i\partial k_j}$$

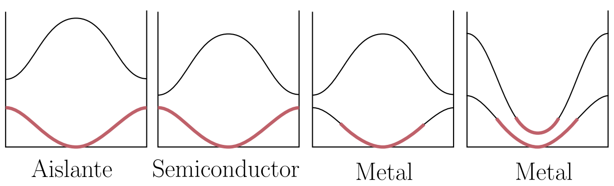
Caso totalmente degenerado

$$m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}, \quad E(\bar{k}) = E_0 + \frac{\hbar^2}{2m^*}|k|^2, \quad \sigma \simeq \frac{e^2\tau(E_F)n}{m^*}$$




Tipos de materiales

**Aislante:** Banda llena ($2e^-$). $V_g > 4eV$
**Semiconductor** Banda llena ($2e^-$). $V_g < 4eV$.
**Metal** Banda semillena ($1e^2$ ó $2e^-$ con bandas solapantes).




## 4 Semiconductores

extrínseco = dopado
Opacos si $h\nu > E_g$
Nivel de Fermi $E_F = \mu$
Densidad de estados ($n$ electrones, $p$ holes)
Energía de donadores / impurezas $E_D$

| $n$ (negativo) | $N_C$ (conducción) | $N_D$ (donadores) |
|---|---|---|
| $p$ (positivo) | $N_V$ (valencia) | $N_A$ (aceptores) |

$$g_C(\varepsilon) = \frac{(2m_n^*)^{2/3}}{2\pi^2\hbar^3}\sqrt{\varepsilon-\varepsilon_C}; g_V(\varepsilon) = \frac{(2m_p^*)^{2/3}}{2\pi^2\hbar^3}\sqrt{\varepsilon_V-\varepsilon}$$

$$n = \int_{\varepsilon_C}^{\infty} d\varepsilon g_C(\varepsilon)n_F(\beta(\varepsilon-\mu)) \approx \int_{\varepsilon_C}^{\infty} d\varepsilon g_C(\varepsilon)e^{\beta(\mu-\varepsilon)}$$

$$p = \int_{-\infty}^{\varepsilon_V} d\varepsilon g_V(\varepsilon)(1-n_F(\beta(\varepsilon-\mu))) \approx \int_{-\infty}^{\varepsilon_V} d\varepsilon g_V(\varepsilon)e^{\beta(\varepsilon-\mu)}$$

$$n = \frac{1}{4}\left(\frac{2m_n^*k_BT}{\pi\hbar^2}\right)^{3/2}e^{\beta(\mu-\varepsilon_C)} = N_Ce^{\beta(\mu-\varepsilon_C)}$$

$$p = \frac{1}{4}\left(\frac{2m_p^*k_BT}{\pi\hbar^2}\right)^{3/2}e^{\beta(\varepsilon_V-\mu)} = N_Ve^{\beta(\varepsilon_V-\mu)}$$

$$np = N_CN_Ve^{-\beta E_g} = 4\left(\frac{k_BT}{2\pi\hbar^2}\right)^3(m_n^*m_p^*)^{3/2}e^{-\beta E_g}$$

$$e^{2\beta\mu} = \frac{N_V}{N_C}e^{\beta(\varepsilon_V+\varepsilon_C)}, \quad \mu = \frac{\varepsilon_C+\varepsilon_V}{2} + \frac{3}{4}k_BT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$\mu = \frac{e\tau}{m^*}, \quad \sigma = e(n\mu_n + p\mu_p), \quad E_g = \varepsilon_C - \varepsilon_V$$

$$n = p = \sqrt{N_CN_V}e^{-\frac{\beta E_g}{2}} \quad \text{si intrínseco}$$

Semiconductores dopados ($n=1$ ionización) $\varepsilon = \varepsilon_0\varepsilon_r$

$$E_n = \frac{m^*e^4}{2(4\pi\varepsilon\hbar)^2}\frac{1}{n^2}, \quad r_n = \varepsilon\frac{4\pi\hbar^2}{m^*e^2}n^2$$

$$n_n \approx \frac{2N_D}{1+\sqrt{1+4\frac{N_D}{N_C}e^{\beta E_d}}}$$

Unión p-n

$$n_n = N_Ce^{\beta(\mu-\varepsilon_C^n)}; \quad p_p = N_Ve^{\beta(\varepsilon_V^p-\mu)}$$

$$d_n^0 = \sqrt{\frac{2\varepsilon V_D}{e}\frac{N_A/N_D}{N_A+N_D}}; \quad d_p^0 = \sqrt{\frac{2\varepsilon V_D}{e}\frac{N_D/N_A}{N_A+N_D}}$$

$$d_n(V) = d_n^0\sqrt{\frac{V_D-V}{V_D}}; \quad d_p(V) = d_p^0\sqrt{\frac{V_D-V}{V_D}}$$

ancho de zona de carga espacial $= d_n(V) + d_p(V)$

$$eV_D = k_BT\ln\left(\frac{n_np_p}{n_i^2}\right); \quad I(V) = (I_n^{gen}+I_p^{gen})(e^{\beta eV}-1)$$

p-Dopado ($N_A \gg N_D$):

- $T$ bajas $\Rightarrow p \approx N_Ve^{-\beta E_A}$
- $T$ intermedias $\Rightarrow p \approx N_A - N_D$
- $T$ alta $\Rightarrow p = n = \sqrt{N_CN_V}e^{-\beta\frac{E_g}{2}}$

## 5 Mates

$$\sin^2\left(\frac{x}{2}\right) = \frac{1-\cos a}{2}$$

$$\int_0^\infty \frac{1}{e^x-1}dx = +\infty, \quad \int_0^\infty \frac{1}{e^x+1}dx = \ln(2)$$

$$\int_0^\infty \frac{x}{e^x-1}dx = \frac{\pi^2}{6}, \quad \int_0^\infty \frac{x}{e^x+1}dx = \frac{\pi^2}{12}$$

$$\int_0^\infty \frac{x^2}{e^x-1}dx = 2\zeta(3), \quad \int_0^\infty \frac{x^2}{e^x+1}dx = \frac{3}{2}\zeta(3)$$

$$\int_0^\infty \frac{x^3}{e^x-1}dx = \frac{\pi^4}{15}, \quad \int_0^\infty \frac{x^3}{e^x+1}dx = \frac{7\pi^4}{120}$$