1 Fórmulas matemáticas

Fórmula: *Ecuación o regla que relaciona objetos matemáticos o cantidades.*
Cambio de coordenadas
Cilíndricas

$$\begin{cases} x = scos\phi \\ y = ssin\phi \\ z = z \end{cases}, \begin{cases} s = \sqrt{x^2+y^2} \\ \phi = arctan(\frac{y}{x}) \\ z = z \end{cases},$$

$$\begin{pmatrix} v_s \\ v_\phi \\ v_z \end{pmatrix} = \begin{pmatrix} cos\phi & sin\phi & 0 \\ -sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}, \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} cos\phi & -sin\phi & 0 \\ sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_s \\ v_\phi \\ v_z \end{pmatrix}$$

Esféricas

$$\begin{cases} x = rsin\theta cos\phi \\ y = ssin\theta sin\phi \\ z = rcos\theta \end{cases}, \begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = arccos(\frac{z}{r}) \\ \phi = arctan(\frac{y}{x}) \end{cases},$$

$$\begin{pmatrix} v_r \\ v_\theta \\ v_\phi \end{pmatrix} = \begin{pmatrix} sin\theta cos\phi & sin\theta sin\phi & cos\theta \\ cos\theta cos\phi & cos\theta sin\phi & -sin\theta \\ -sin\phi & cos\phi & 0 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}, \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} sin\theta cos\phi & cos\theta cos\phi & -sin\phi \\ sin\theta sin\phi & cos\theta sin\phi & cos\phi \\ cos\theta & -sin\theta & 0 \end{pmatrix} \begin{pmatrix} v_r \\ v_\theta \\ v_\phi \end{pmatrix}$$

Identidades trigonométricas:

$$\begin{aligned}
\sin x + \sin y &= 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) \\
\sin x - \sin y &= 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right) \\
\cos x + \cos y &= 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) \\
\cos x - \cos y &= -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right) \\
\tan x + \tan y &= \frac{\sin(x+y)}{\cos x\cos y} \\
\tan x - \tan y &= \frac{\sin(x-y)}{\cos x\cos y} \\
\sin x\sin y &= \frac{1}{2}\big[\cos(x-y) - \cos(x+y)\big] \\
\cos x\cos y &= \frac{1}{2}\big[\cos(x-y) + \cos(x+y)\big] \\
\sin x\cos y &= \frac{1}{2}\big[\sin(x+y) + \sin(x-y)\big] \\
\tan x\tan y &= \frac{\tan x + \tan y}{\cot x + \cot y} \\
\tan x\cot y &= \frac{\tan x + \cot y}{\cot x + \tan y}
\end{aligned}
\quad \leftrightarrow \quad
\begin{aligned}
\sin(2x) &= 2\sin x\cos x \\
\cos(2x) &= \cos^2 x - \sin^2 x \\
&= 2\cos^2 x - 1 \\
&= 1 - 2\sin^2 x \\
\tan(2x) &= \frac{2\tan x}{1-\tan^2 x} \\
\sin\frac{x}{2} &= \pm\sqrt{\frac{1-\cos x}{2}} \\
\cos\frac{x}{2} &= \pm\sqrt{\frac{1+\cos x}{2}} \\
\tan\frac{x}{2} &= \frac{1-\cos x}{\sin x} \\
&= \frac{\sin x}{1+\cos x} \\
\nabla\cdot(\nabla\times\mathbf{A}) &= 0 \\
\nabla\times(\nabla\times\mathbf{A}) &= \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A} \\
\nabla\cdot(\mathbf{A}\times\mathbf{B}) &= (\nabla\times\mathbf{A})\cdot\mathbf{B} - \mathbf{A}\cdot(\nabla\times\mathbf{B})
\end{aligned}$$

$$\oiint_S E\cdot dS = \frac{q_{int}}{\varepsilon_0} \qquad \nabla\cdot E = \frac{\rho}{\varepsilon_0} \tag{1}$$

$$\oiint_S B\cdot dS = 0 \qquad \nabla\cdot B = 0 \tag{2}$$

$$\oint_C E\cdot dl = -\frac{d}{dt}\int B\cdot dS \qquad \nabla\times E = -\frac{dB}{dt} \tag{3}$$

$$\oint_C B\cdot dl = \mu_0 I_c + \mu_0\varepsilon_0\frac{d\Phi_E}{dt} \qquad \nabla\times B = \mu_0 J + \mu_0\varepsilon_0\frac{dE}{dt} \tag{4}$$

$$dU = -F_x dx - F_y dy - F_z dz \implies \nabla U = -\bar{F}, \quad \begin{cases} \nabla = \left(\frac{\partial}{\partial s}, \frac{1}{s}\frac{\partial}{\partial\phi}, \frac{\partial}{\partial z}\right) \\ \nabla = \left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial\theta}, \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\right) \end{cases}, \quad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

2 Física

Tema 1 - Ondas
Función de onda armónica

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2}, \quad v^2 = \frac{T}{\mu}, \quad y(x,t) = A\sin(kx - \omega t + \phi_0) \quad \begin{cases} k = \frac{2\pi}{\lambda} \\ \omega = \frac{2\pi}{\tau} = 2\pi f \\ v = \frac{\lambda}{\tau} \\ \Delta\phi = k(d_1 - d_2) \text{ ondas en fase} \end{cases}$$

Energía de una onda

$$\mu_E = \mu_c + \mu_p = 2\mu_c = \frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2 + \frac{1}{2}T\left(\frac{\partial y}{\partial x}\right)^2$$

Potencia de una onda

$$P(t) = \frac{\partial E}{\partial t} = \mu_E v, \quad <P> = \frac{1}{\tau}\int_t^{t+\tau} P(t)dt = \frac{1}{2}\mu A^2\omega^2 v, \quad A_T^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\Delta\phi)$$

Ondas de sonido

$$\phi(x,t) = \phi_0\sin(kx - \omega t + \varphi_0), \quad P(x,t) = \rho_0 v\frac{\partial\phi}{\partial t}, \quad v^2 = \frac{B}{\rho_0}, \quad \rho_E = \rho_0\left(\frac{\partial\phi}{\partial t}\right)^2, \quad I = \frac{dE}{Adt} = \rho_E v$$

$$<I> = \frac{1}{2}v\rho_0\phi_m^2\omega^2, \quad I_{dB} = 10\log\left(\frac{I}{I_0}\right), \quad I = \frac{P}{4\pi r^2} = \frac{P}{2\pi r} = I_0 e^{-\beta x}$$

Ondas estacionarias

$$\begin{cases} y_1 = A\sin(kx - \omega t) \\ y_1 = A\sin(kx - \omega t) \end{cases} \implies y = 2A\sin(kx)\cos(\omega t), \quad Modos: \begin{cases} \text{Extremos fijos } \lambda_n = \frac{2L}{n} \\ \text{Un ecxtremo suelto } \lambda_n = \frac{2L}{n+\frac{1}{2}} \end{cases}$$

Efecto Doppler

$$f = f\left(\frac{v \pm v_m \pm v_0}{v \pm v_m \mp v_F}\right) \begin{cases} v_0 \text{ velocidad del observador (+ se acerca)}, & v \text{ velocidad de la onda} \\ v_F \text{ velocidad de la fuente (- se acerca)}, & v_m \text{ velocidad del medio} \end{cases}$$

Tema 2 - Campos Elestrostáticos. Potencial y Energía
Partículas

$$F_{12} = K\frac{q_1q_2}{r_{12}^2}\hat{u}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}^2}\hat{u}_{12} = q\bar{E}, \quad V(r) = \frac{KQ}{r} + V_0$$

Campo creado por:

$$\bar{E} = \text{(Varilla): } \frac{2K\lambda}{s}\hat{s} = \text{(Anillo): } K\frac{2\pi r\lambda z}{(r^2+z^2)^{3/2}}\hat{z} = \text{(Plano): } \frac{\sigma}{2\varepsilon_0}\hat{z} = \text{(Cort. esf.): } \begin{cases} 0 & \text{si } r < R \\ K\frac{Q}{r^2}\hat{r} & \text{si } r > R \end{cases} =$$

$$= \text{(Esf. mac.): } \begin{cases} K\frac{Qr}{R^3}\hat{r} & \text{si } r < R \\ K\frac{Q}{r^2}\hat{r} & \text{si } r > R \end{cases} = \text{(Cort. cil.): } \begin{cases} 0 & \text{si } s < R \\ K\frac{R\sigma}{\varepsilon_0 s}\hat{s} & \text{si } s > R \end{cases} = \text{(Cil. mac.): } \begin{cases} \frac{\rho s}{2\varepsilon_0}\hat{s} & \text{si } s < R \\ \frac{\rho R^2}{2\varepsilon_0 s}\hat{s} & \text{si } r > R \end{cases}$$

Flujo:

$$\text{Ley de Gauss } \Phi = \int_S d\Phi = \int_S \bar{E}\cdot d\bar{S} = \frac{Q}{\varepsilon_0}$$

Trabajo y Potencial:

$$\Delta U = U(B) - U(A) = -\int_A^B \bar{F}\cdot d\bar{l} = q(V(A) - V(B)) = -q\int_A^B \bar{E}\cdot d\bar{l}$$

Tema 3: Conductores y energía

$$\text{Energías} \implies \begin{cases} \text{Formación} & U_f = \frac{1}{2}\int_{dist} V dq \\ \text{Total} & U = U_{12} + U_{f1} + U_{f2} \\ \text{Densidad} & \eta_E = \frac{1}{2}\varepsilon_0 E^2, \quad \eta_B = \frac{1}{2\mu_0} B^2 \end{cases}$$

Conductores

$$\text{sin cavidad} \implies E_{s+} = \frac{\sigma}{\varepsilon_0}\hat{n}, \quad \text{con cavidad y q dentro} \implies Q_{Sint} = -q, \quad Q_{Sext} = Q_0 + q$$

Capacidad y Condensadores

$$\begin{cases} C = \frac{Q}{V}(F) \\ U = \frac{1}{2}QV \end{cases} = \text{(Planos):} \begin{cases} = \frac{\varepsilon_0 S}{d} \\ = \frac{1}{2}QV \end{cases} = \text{(Cilíndricos):} \begin{cases} = \frac{2\pi\varepsilon_0 L}{\ln(\frac{R_2}{R_1})} \\ = \frac{1}{2}QV \end{cases} = \text{(Planos):} \begin{cases} = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1} \\ = \frac{1}{2}QV \end{cases}$$

Corriente eléctrica

$$\frac{dI}{dS} = \bar{J} = q n_v \bar{v}_d = \sigma \bar{E} = nq\mu E, \quad \Delta V = E \cdot l, \quad R = \frac{l}{\sigma A} = \frac{\rho l}{A}, \quad v_{cm} = \sqrt{\frac{\sum v_i^2}{N}} = \sqrt{\frac{3KT}{m_e}}$$

Circuitos

$$P = \int_V \bar{J} \cdot \bar{E} dV (= \varepsilon I), \quad \Delta V = \varepsilon - Ir \text{ (r en batería)}, \quad \varepsilon = IR + Ir$$

Circuitos RC (condensador)

$$\tau = RC, \quad \text{Sin f.e.m. } q = q_0 e^{-\frac{t}{\tau}}, \quad i = i_0 e^{-\frac{t}{\tau}}, \quad \text{Con f.e.m. } q = \varepsilon C(1 - e^{-\frac{t}{\tau}}), \quad i = \frac{\mathcal{E}}{R} e^{-\frac{t}{\tau}}$$

Tema 4: Magnetostática

Fuerza:

$$F = \int_L I d\bar{l} \times \bar{B}, \quad \text{Ef. Hall: } B = \frac{nqa}{I} V_H, \quad \text{mom. mag } \mathbf{m}\text{: } \tau = \bar{m} \times \bar{B} = (IS\hat{n}) \times \bar{B}, \quad \text{sens } = \frac{V_H}{B}$$

Biot-Savart:

$$d\bar{B} = \frac{\mu_0 I (d\bar{l} \times \hat{r})}{4\pi r^2} = \text{(Centro esp. circ.) } \frac{\mu_0 I}{2R} = \text{(Eje esp.) } \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}} = \text{(Hilo cond.) } \frac{\mu_0 I}{2\pi r}$$

Teorema de Gauss y ley de Ampere:

$$\oiint_S \bar{B} \cdot d\bar{S} = 0, \quad \oint_C \bar{B} \cdot d\bar{l} = \mu_0 I \implies B = \text{(Cil.)} \begin{cases} \frac{\mu_0 I}{2\pi r}, r > R \\ \frac{\mu_0 I r}{2\pi R^2}, r < R \end{cases} = \text{(solen.) } \mu_0 m I = \text{(solen. toro) } \frac{\mu_0 I N}{2\pi R}$$

Autoinducción:

$$\Phi_1 = L_1 I_1 + M_{12} I_2, \quad \text{dos sol. conc.} \begin{cases} L_1 = \mu_0 n_1^2 l \pi r_1^2 \\ L_2 = \mu_0 n_2^2 l \pi r_2^2 \\ M_{12} = M_{21} = \mu_0 n_1 n_2 l \pi r_1^2 \end{cases}, r_1 < r_2$$

Tema 5: Campos eléctricos y magnéticos no estacionarios

Ley de Faraday-Lenz:

$$\varepsilon_{ind} = -\frac{d\Phi}{dt} = I_{ind} R$$

Circuitos RL (Bobina)

$$\Phi = LI, \quad \varepsilon_{ind} = -\frac{d(LI)}{dt}, \quad \frac{dU}{dt} = \varepsilon I = RI^2 + LI\frac{dI}{dt} \quad , I(t) = \frac{\varepsilon}{R}(1 - e^{-\frac{t}{\tau}}), \quad \tau = \frac{L}{R}$$

Vector de Poynting

$$\bar{P} = \frac{1}{\mu_0}(E \times B), \quad \oint_S \bar{P} \cdot d\bar{S} = -VI = -P$$