Quantum Mechanics

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Contents

1 Hilbert Spaces

Theorem (Gram-Schmidt)**.** *Given a basis $|a_i\rangle$ we can always make and orthonormal basis (bon)*

$$|e'_{m+1}\rangle = |a_{m+1}\rangle - \sum |e_i\rangle\langle e_i|a_{n+1}\rangle, \quad |e_{m+1}\rangle = \frac{|e'_{m+1}\rangle}{\langle e'_{m+1}|e'_{m+1}\rangle}$$

Definition (Hermitic Operator)**.** *A is hermitic $\iff A^\dagger = A$*

Definition (Unitary Operator)**.** *U is unitary $\iff U^\dagger U = I$*

Definition (Expected value)**.** *The expected value of the operator A in the state $|\psi\rangle$ is $\langle A\rangle = \langle\psi|A|\psi\rangle$*

Definition (Uncertainty)**.** *Define $\overline{A} = A - \langle A\rangle I$. Then we define uncertainty as $\Delta A = \sqrt{\langle\overline{A}^2\rangle}$*

Theorem (Cauchy-Schwarz)**.** $\langle\psi|\psi\rangle\langle\varphi|\varphi\rangle \geq |\langle\psi|\varphi\rangle|^2 \quad \Rightarrow \quad \Delta A\Delta B \geq \frac{1}{2}|\langle[A,B]\rangle|$

Theorem (Glauber's formula)**.**

$$\begin{cases}[A,[A,B]] = 0 \\ [B,[A,B]] = 0\end{cases} \Rightarrow e^A e^B = e^{A+B} e^{\frac{1}{2}[A,B]}$$

Theorem (Propiedades del operador unitario)**.** *Sea U un unitary operator*

1. *Their VEPs form a bon*
2. *Every VAP is of norm one*

Definition (Unitary transform)**.** $A' = UAU^\dagger \Rightarrow \psi' = U\psi, \quad Uf(A)U^\dagger = f(UAU^\dagger)$

Proposition. *Change of basis.* $A_{new} = UA_{old}U^\dagger \Rightarrow A_{ik} = \langle u_i|A|u_k\rangle$

Proposition. *Spectral decomposition.* $A = \sum_{n=1}^{\infty} a_n \sum_{i=1}^{g_n} |\psi_n^i\rangle\langle\psi_n^i|$

2 Postulates of Quantum mechanics

First postulate: The state of an isolated physical system is defined by specifying a ket $|\psi(t_0)\rangle$ in the Hilbert space

Second postulate: Every measurable physical quantity $\mathcal{A}$ is described by an operator A acting in the Hilbert space. This operator is an observable.

Third postulate: The only possible result of the measurement of a physical quantity $\mathcal{A}$ is one of the VAPs of A

Fourth postulate: (discrete spectrum) When $\mathcal{A}$ is measured on ψ (normalized), the probability of obtaining $\mathcal{A} = a_n$ is $P(a_n) = \sum_{i=1}^{g_n} |\langle u_n|\psi\rangle|^2$ (continuous non-degenerate spectrum) $dP(\alpha) = P(\alpha \leq x \leq \alpha + d\alpha) = |\langle v_\alpha|\psi\rangle|^2 d\alpha$

Fifth postulate: If the measurement of the quantity $\mathcal{A}$ on the state $|\psi\rangle$ gives as a result a_n, the state of the system immediately after the measurement is the normalized projection $\dfrac{P_n|\psi\rangle}{\sqrt{\langle\psi|P_n|\psi\rangle}}$

Sixth postulate: The time evolution of the state vector $|\psi(t)\rangle$ follows the Schrödinger equation:

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle$$