F-Modules

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1 Introduction

We will work in positive characteristic. Let R commutative unital ring of prime characteristic p.

2 The Frobenius functor

Definition (Frobenius endomorphism)**.** The map

$$f : R \to R \quad \text{such that} \quad f(r) = r^p$$

defines a ring morphism in a ring of characteristic p known as Frobenius endomorphism.

Notice that the application is, in fact, a morphism. The behaviour for the product $f(ab) = (ab)^p = a^p b^p = f(a)f(b)$, and for the sum we have to make use of binomial expansion

$$f(a+b) = (a+b)^p = a^p + \binom{p}{1} a^{p-1} b^1 + \cdots + \binom{p}{p-1} a^1 b^{p-1} + b^p = a^p + b^p = f(a) + f(b)$$

since $p | \binom{p}{k}$ $\forall k = 1, \ldots, p-1$.

Observation. f is not necessarily injective nor surjective. Some counterexamples are

1.