Formulario de Física

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Cinemática

•a constante
$\bar{x} = \bar{x_0} + \bar{v_0}t + \frac{1}{2}\bar{a}t^2$

•MCUA
$a_t = \frac{v^2}{R} = \omega^2 R$

•$|a|$ constante
$\bar{a} = a_t\hat{\tau} + a_n\hat{n}$

•Comparación de variables
$x = r\phi$
$v = r\omega$
$a = r\alpha$
$m \Rightarrow I = mr^2$
$p = mv \Rightarrow L = I\omega = r \times p$
$F = ma \Rightarrow \tau = I\alpha = r \times F$

Fricción

•Estática y dinámica
$F_{fmax} = \mu_e N$
$F_{fd} = \mu_d N$
$\mu_d < \mu_e$

•En fluídos v
$F_D = -bv \Rightarrow ma = mg - bv$
$\gamma = \frac{b}{m}$
$v_z = \frac{g}{\gamma} - \left(\frac{g}{\gamma} - v_{z0}\right) e^{-\gamma t}$
$z = z_0 + \frac{g}{\gamma}t - \frac{1}{\gamma}\left(\frac{g}{\gamma} - v_0\right)(1 - e^{-\gamma t})$

•En fluídos v^2
$F_D = -bv^2 \Rightarrow ma = -bv^2$
$v = \frac{v_0}{1+v_0\gamma t}$
$x = x_0 + \frac{ln(1+v_0\gamma t)}{\gamma}$

Energía

•Trabajo
$W = \int_{r_a}^{r_b} \bar{F} \cdot d\bar{r}$

•Energías
Cinética
$K = \frac{1}{2}mv^2$
Pot. gravitatoria
$U = mgh$
Pot. elástica
$U = \frac{1}{2}kx^2$

•Impulso
$I = \int_{t_1}^{t_2} F dt = \Delta p$

•Potencia
$P = \frac{dW}{dt} = Fv$

•En un campo conservativo
$\bar{F} = -\nabla U$

•Sistema no conserva-

Oscilaciones

•Variables

- $m\ddot{x} + b\dot{x} + kx = F_0 cos(\omega t)$
- $\omega_0 = \sqrt{\frac{k}{m}}$
- $\gamma = \frac{b}{2m}$
- $\omega_1 = \sqrt{\omega_0^2 - \gamma^2}$
- $D = m\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}$
- $A^2 = x_0^2 + \frac{v_0^2}{\omega_0^2}$
- $tan(\phi) = -\frac{v_0^2}{x_0\omega_0}$

Tipos de Osciladores

•Débilmente amortiguado ($\omega_0 > \gamma$)
$x = Ae^{-\gamma t} sin(\omega_1 t + \phi)$
$A_0^2 = x_0^2 + (\frac{\gamma x_0 + v_0}{\omega_1})^2$
$tan(\phi) = \frac{\omega_1 x_0}{\gamma x_0 + v_0}$
Muy débil $|\Delta E/E|_T \approx T/\tau_E$

•Críticamente amortiguado ($\omega_0 = \gamma$)
$x = (A + Bt)e^{-\gamma t}$

•Fuertemente amortiguado ($\omega_0 < \gamma$)
$x = A_1 e^{-\gamma_1 t} + A_2 e^{-\gamma_2 t}$
$\gamma_1 = \gamma - \sqrt{\gamma^2 - \omega_0^2}$
$\gamma_2 = \gamma + \sqrt{\gamma^2 - \omega_0^2}$
$A_1 = \frac{v_0 + \gamma_2 x_0}{\gamma_2 - \gamma_1}$
$A_2 = \frac{v_0 + \gamma_1 x_0}{\gamma_1 - \gamma_2}$

•Forzado
$x = x_h + x_p$
$x_p = A_p cos(\omega t - \beta)$
$tan\beta = \frac{2\gamma\omega}{\omega_0^2 - \omega^2} = \frac{b\omega}{k - m\omega^2}$
$A_p = \frac{F_0}{D} = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}}$
(Impedancia mecánica)
$Z = \frac{D}{\omega} = \sqrt{b^2 + (mk - k/\omega)^2}$
$cos(\beta - \pi) = \frac{b}{Z},\ sin(\beta - \pi) = \frac{m\omega - k/\omega}{Z}$

Energía y potencia

•Energía
Si $\gamma << \omega_0$:
$E = \frac{1}{2}kA(t)^2 = \frac{1}{2}kA_0^2 e^{-2\gamma t}$

•Potencia
$P = -F\dot{v} = F_0\omega A_p cos(\omega t) sin(\omega t - \beta)$
$< P > = \frac{1}{T}\int F_{ext} v dt = \frac{bF_0^2}{2Z^2}$

Resonancia

•Frecuencia
$\omega_A = \omega_0 \rightarrow D_{min} = b\omega \rightarrow A_{max}$
Velocidad máxima
$V_0 = A\omega = \frac{F_0}{Z} = \frac{F_0}{b} \Rightarrow \omega = \omega_0$
$< P > = \frac{F_0^2}{4m\gamma}$

•Banda
$\omega = \omega_0 \pm \gamma$
$< P > = \frac{F_0^2}{8m\gamma}$

Variables de calidad

•Constante de tiempo (Energía)
$\tau_E = \frac{1}{2\gamma}$
el tiempo que pasa de tener E a E/e

•Teorema del trabajo-energía
$n = \frac{\tau_E}{T}$ las oscilaciones antes de τ_E

•Factor de calidad
$Q = \frac{2\pi}{|\frac{\Delta E}{E}|_T} = \frac{\omega_1}{2\gamma} = 2\pi n$

Péndulo

•Periodo del péndulo
$T = 2\pi\sqrt{\frac{l}{g}}$
$\omega = \sqrt{\frac{g}{l}}$

Campo gravitatorio

•Ley de grav. universal
$F(\bar{r}) = -\frac{GMm}{r^2}\hat{r}$

•Energía potencial grav.
$U(r) = -\frac{GMm}{r}$

•Momento angular y fuerza
$\bar{L} = \bar{r} \times \bar{p}$ constante
$\bar{\tau} = \bar{r} \times \bar{F} = \frac{d\bar{L}}{dt} = 0$

•Leyes de Kepler

1) Órbitas elípticas casi circulares
2) Áreas = en tiempos =
3) $T^2 \propto R^3$

Coordenadas polares

•Vectores unitarios
$\bar{r} = x\hat{i} + y\hat{j} = r\hat{r}$
$\hat{r} = cos(\theta)\hat{i} + sin(\theta)\hat{j}$
$\hat{\theta} = -sin(\theta)\hat{i} + cos(\theta)\hat{j}$

•Velocidad y L
$\bar{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} =$ radial + angular
$L_0 = mr^2\dot{\theta}\hat{k} \Rightarrow r^2\dot{\theta}$ const

•Energía
$E = K + U = \frac{1}{2}m\dot{r}^2 + V_{eff}$
$v_{eff} = \frac{L_0^2}{2mr^2} - \frac{GMm}{r}$
Dependiendo de V_{eff} la órbita es:

- $E > 0 \Rightarrow$ Hiperbólica
- $E = 0 \Rightarrow$ Parabólica
- $V_{effmin} < E < 0 \Rightarrow$ Elíptica
- $E = V_{effmin} \Rightarrow$ Circular

Órbitas

•Ecuación diferencial
$\dot{\theta} = \frac{L_0}{mr^2}$
$\dot{r} = \sqrt{\frac{2}{m}(E - V_{eff}(r))}$
$r(\theta) = \frac{\alpha}{1+\varepsilon cos(\theta)}$

•Variables
$\alpha = \frac{L_0^2}{GMm^2} = \frac{b^2}{a} = a(1-\varepsilon^2)$
$\varepsilon = \sqrt{1 + \frac{2\alpha E}{GMm}} = \frac{c}{a}$
$T = 2\pi\sqrt{\frac{a^3}{GM}}$ Perihelio $r_p = \frac{\alpha}{1+\varepsilon}$
Afelio $r_a = \frac{\alpha}{1-\varepsilon}$

•Órbitas circulares
$V_{orb} = \sqrt{\frac{GM}{R_{orb}}}$
$V_{esc} = \sqrt{\frac{2GM}{R_T}}$

•Órbitas circulares
$2a = b + c = r_p + r_a$
$E = -\frac{GMm}{2a}$ $V_{esc} = \sqrt{\frac{2GM}{R_T}}$

Sistemas de partículas

•Ecuaciones centro masa
$\bar{r}_{CM} = \frac{\sum m_i \bar{r}_i}{M}$

•Ecuaciones
$\bar{p} = M\bar{v}_{CM}$
$\bar{F} = M\bar{a}_{CM}$
$F^{ext} = \frac{dp}{dt}$

•Conservación de variables
$\bar{L} = m_i\bar{r}_i \times \bar{v}_i$
$\tau_i = \frac{dL}{dt}$
$\tau^{ext} = \frac{dL}{dt}$
$E = K + U$ const. $\iff$ F_i conserv.

Choques

•Choques elásticos
p, K se conservan
$m_1(v_1 - u_1) = m_2(u_2 - v_2)$
$v_2 - v_1 = -(u_2 - u_1)$
$u_1 = \frac{m_1 - m_2}{m_1 + m_2}v_1 + \frac{2m_2}{m_1 + m_2}v_2$
$u_2 = \frac{2m_1}{m_1 + m_2}v_1 + \frac{m_2 - m_1}{m_1 + m_2}v_2$

•Choques inelásticos
p se conserva, K no
$m_1(v_1 - u_1) = m_2(u_2 - v_2)$
$v_2 - v_1 = -e(u_2 - u_1);\ 0 \leq e < 1$

Masa variable

•Conservación de p)
$dp = vdm + mdv - udm = 0$
$ma = -v_{rel}\frac{dm}{dt}$
$v = v_0 + v_{rel}ln(\frac{m(t)}{m_0})$

Vectores

$F_x = F\cos\theta$
$F_y = F\sin\theta$
$\vec{F} = \vec{F_x} + \vec{F_y} = F\cos\theta_{\hat{i}} + F\sin\theta_{\hat{j}}$
$F = \sqrt{F_x^2 + F_y^2}$
$\theta = \tan^{-1}\frac{F_y}{F_x}$

•Distancia entre dos puntos en el espacio
$|P_1P_2| =$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

•Vector unitario
$\hat{u} = \frac{\vec{QP}}{\|\vec{QP}\|} = \frac{P - Q}{\|P - Q\|}$
$\vec{T} = T\hat{u}$

•Cosenos directores
$\theta_x = \cos^{-1}\frac{F_x}{F}$

$\theta_y = \cos^{-1}\frac{F_y}{F}$

$\theta_z = \cos^{-1}\frac{F_z}{F}$