Notes on Coxeter Matroids

Abel Doñate Muñoz
abel.donate@estudiantat.upc.edu

Contents

1 Matroids

Definition (Matroid)**.** *A base of a matroid M over a given a ground set $[n]$ is $\binom{\mathcal{B}(M)\subseteq}{[n],r}$, where r is the rank of the matroid. The set $\mathcal{B}$ must fulfill:*

- $A, B \in \mathcal{B}, a \in A - B \Rightarrow \; \exists b \in B - A : (A - \{a\}) \cup \{b\} \in \mathcal{B}$

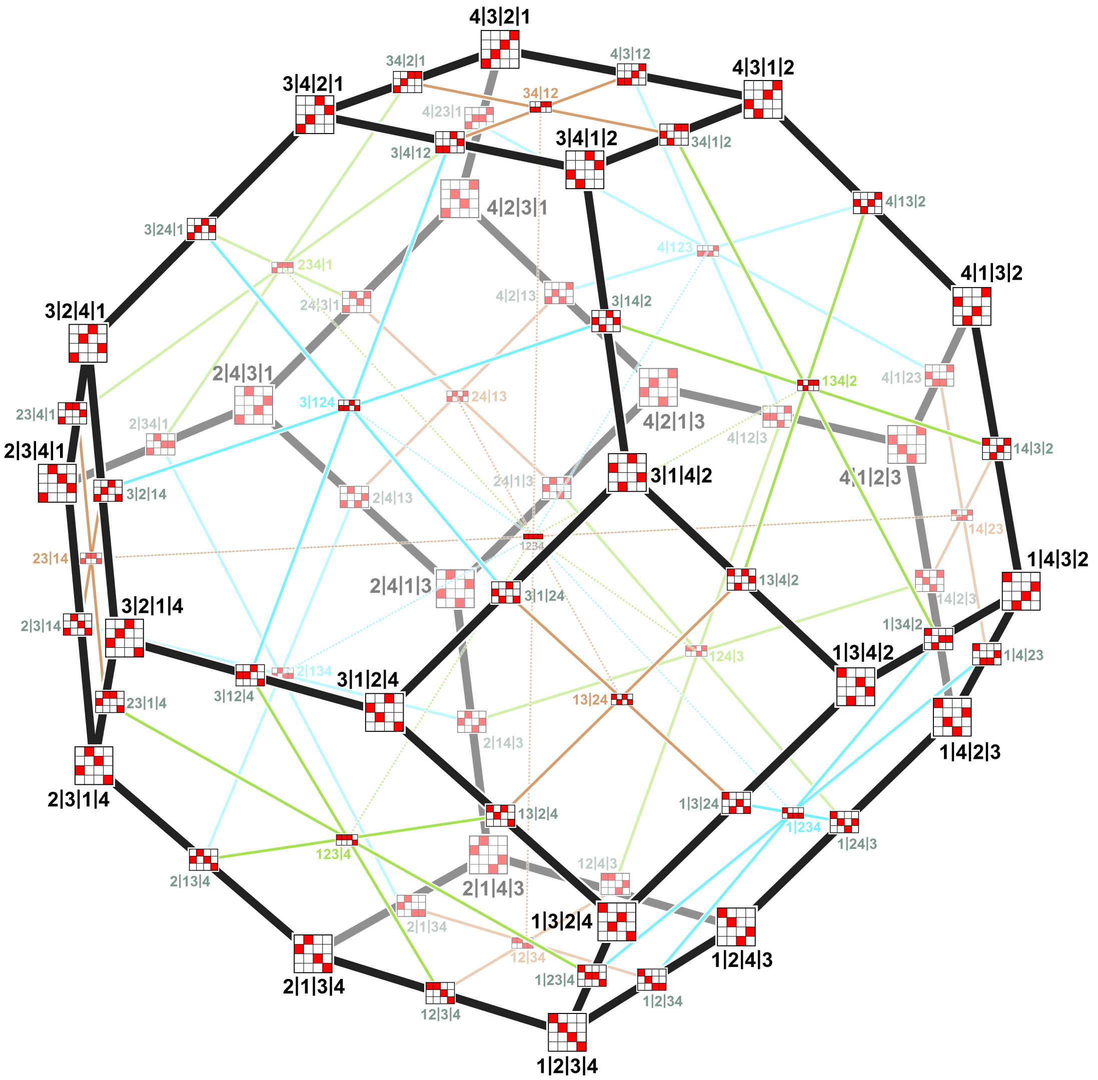
2 Permutahedron

2.1 Regular permutahedron

The permutahedron Π_n is generated by the convex hull of the vertices $V = \{(\sigma(1), \ldots \sigma(n)) : \sigma \in S_n\}$

There is a (fancy) bijection between the flags of $[n]$ and the faces of permutahedron Π_n as shown in the picture.

Flags could be interpreted as ordered partitions. One example of the three points of view as follows: $F = \{\{3\}, \{1, 2, 3, 4\}\} \iff 3|124 \iff$ "the face whose vertices have a 3 in the first position and the other three are free permutations".



2.2 Generalized permutahedra

Definition (Hypersimplex)**.** $\Delta(n,k) = \{(x_1,\ldots,x_n) : x_1 + \ldots + x_n = k\}$

The basis of $\Delta(n,k)$ (vertices of the polytope) is formed by vectors with k ones and $n-k$ zeroes.

Definition (Generalized Permutahedron)**.** *Convex polytope with all the edges parallel to* $e_i - e_j$

Permutahedron vertices came from a subset of the vertices of $\Delta(n,k)$

Definition (Matroid polytope)**.** *Matroid generated by the permutahedron whose vertices are a subset of* $\Delta(n,k)$

3 Tropical geometry

The idea behind tropical geometry is