MMDM Lab Work 2

Problem 1

$$\int_0^1 \left[\left(\frac{du}{dx}\right)^2 - 2\lambda e^u\right] dx \to \min_u$$
$$u(0) = u(1) = 0$$

How to do it:

1. (1 points) Variate funcitional with respect to unknown function and obtain Euler-Lagrange equation. Euler-Lagrange equation + given boundary conditions = boundary value problem of ODEs.
2. (3 points) Solve given boundary value problem using the Netwon method[1] for $\lambda = 1$.
3. (* 4 points) Find bifurcation point = value of λ where the determinant of the matrix of derivatives in Newton method is equal to 0^2.
4. (* 2 point) Plot the graph of a bifurcation path: graph where $x = \|u\|_\infty, y = \lambda$.

Problem 2

Use IGKM method with only 1 term to solve the given minimisation problem.

$$\int_0^1 \int_0^1 \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 - 4\pi^2 \sin(\pi x)\sin(\pi y) u(x,y)\right] dxdy \to \min_u$$
$$u(0,y) = u(1,y) = u(x,0) = u(x,1) = 0$$

1. (2 points) Take $u(x,y) = h(x)g(y)$. Variate functional with respect to $h(x)$ and obtain boundary value problem of ODEs for $h(x)$ with integral coefficients containing function $g(y)$. Variate functional with respect to $g(y)$ and obtain boundary value problem of ODEs for $g(y)$ with integral coefficients containing function $h(x)$.
2. (4 points) Find functions $h(x)$ and $g(y)$ using iterative algorithm:
   (a) Define $h^{(0)}(x), g^{(0)}(y)$ such that functions satisfy given boundary conditions[3]
   (b) Solve boundary value problems of ODEs for $h(x)$ using the Newton method: so you find $h^{(1)}(x)$.
   (c) Solve boundary value problems of ODEs for $g(y)$ using the Newton method: so you fund $g^{(1)}(y)$.
   (d) Repeate (b), (c) until convergence.
3. (*4 points) Apply full IGKM algorithm. So, you define $u(x,y) = \sum_{i=1}^{M} h_i(x)g_i(y)$ and then you have two options:

[1] aka Shooting method or method of reducing boundary value problem to initial value problem using the Newton method
[2] Continuation of the parameter method or method of changing the leading parameter can be useful.
[3] you can define them as quadratic functions passing through points (0,0), (1,0)

(a) Fix $M = 2$: derive system for $h_i(x), i = 1, 2$; derive system for $g_i(y), i = 1, 2$. Solve them iteratively using algorithm 2.2. Check convergence: you solution for $M = 1$ is "same" as for $M = 2$. If not take $M = 3$ and so on.

(b) Organise another iterative algorithm to iteratively add new terms $h_2(x)g_2(y), ...$