

Universidad Autónoma de Madrid Escuela Politécnica Superior Departamento de Ingeniería Informática

Recommender System Performance Evaluation and Prediction: An Information Retrieval Perspective

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Abstract

Personalised recommender systems aim to help users access and retrieve relevant information or items from large collections, by automatically finding and suggesting products or services of likely interest based on observed evidence of the users' preferences. For many reasons, user preferences are difficult to guess, and therefore recommender systems have a considerable variance in their success ratio in estimating the user's tastes and interests. In such a scenario, self-predicting the chances that a recommendation is accurate before actually submitting it to a user becomes an interesting capability from many perspectives. Performance prediction has been studied in the context of search engines in the Information Retrieval field, but there is little if any prior research of this problem in the recommendation domain.

This thesis investigates the definition and formalisation of performance prediction methods for recommender systems. Specifically, we study adaptations of search performance predictors from the Information Retrieval field, and propose new predictors based on theories and models from Information Theory and Social Graph Theory. We show the instantiation of information-theoretical performance prediction methods on both rating and access log data, and the application of social-based predictors to social network structures.

Recommendation performance prediction is a relevant problem per se, because of its potential application to many uses. Thus, we primarily evaluate the quality of the proposed solutions in terms of the correlation between the predicted and the observed performance on test data. This assessment requires a clear recommender evaluation methodology against which the predictions can be contrasted. Given that the evaluation of recommender systems is an open area to a significant extent, the thesis addresses the evaluation methodology as a part of the researched problem. We analyse how the variations in the evaluation procedure may alter the apparent behaviour of performance predictors, and we propose approaches to avoid misleading observations.

In addition to the stand-alone assessment of the proposed predictors, we research the use of the predictive capability in the context of one of its common applications, namely the dynamic adjustment of recommendation methods and components. We research approaches where the combination leans towards the algorithm or the component that is predicted to perform best in each case, aiming to enhance the performance of the resulting dynamic configuration. The thesis reports positive empirical evidence confirming both a significant predictive power for the proposed methods in different experiments, and consistent improvements in the performance of dynamic recommenders employing the proposed predictors.

Resumen

Los sistemas de recomendación personalizados tienen como objetivo ayudar a los usuarios en el acceso y recuperación de información u objetos relevantes en vastas colecciones mediante la sugerencia automática de productos o servicios de potencial interés, basándose en la evidencia observada de las preferencias de los usuarios. Las preferencias de usuario son difíciles de predecir por muchos motivos y, por tanto, los sistemas de recomendación tienen una variabilidad considerable en su tasa de acierto al intentar estimar los gustos e intereses de cada usuario. En este escenario la autopredicción de las probabilidades de que una recomendación sea acertada antes de proporcionarla al usuario se convierte en una capacidad interesante desde múltiples perspectivas. La predicción de eficacia ha sido estudiada en el contexto de los motores de búsqueda en el campo de la Recuperación de Información, pero apenas se ha investigado en el dominio de la recomendación.

Esta tesis investiga la definición y formalización de métodos de predicción de eficacia para sistemas de recomendación. Concretamente, se estudian adaptaciones de predictores de eficacia de búsqueda en el campo de la Recuperación de Información, y se proponen nuevos predictores basados en modelos y técnicas de la Teoría de la Información y la Teoría de Grafos Sociales. Se propone la instanciación de métodos de teoría de información para predicción de eficacia tanto en datos de valoraciones de usuario explícitas como en registros de accesos, así como la aplicación de predictores sociales sobre estructuras de red social.

La predicción de eficacia de recomendación es un problema relevante per se dados sus múltiples usos y aplicaciones potenciales. Por ello, en primer lugar se evalúa la calidad de las soluciones propuestas en términos de la correlación entre la eficacia estimada y la observada en los datos de test. Esta valoración requiere una metodología clara de evaluación de sistemas de recomendación con la que las predicciones puedan ser contrastadas. Dado que la evaluación de los sistemas de recomendación es aún un área de investigación en buena medida abierta, la tesis aborda la metodología de evaluación como parte del problema a investigar. Se analizan entonces cómo las variaciones en el procedimiento de evaluación pueden alterar la percepción del comportamiento de los predictores de eficacia, y se proponen aproximaciones para evitar observaciones engañosas.

Además de las valoraciones independientes de los predictores propuestos, investigamos el uso de su capacidad predictiva en el contexto de una de sus aplicaciones comunes, a saber, el ajuste dinámico de métodos híbridos para combinar algoritmos y componentes de recomendación. Se investigan aproximaciones donde la combinación se inclina hacia el algoritmo o la componente que se predice va a tener mejor eficacia en cada caso, a fin de mejorar la eficacia de la configuración dinámica resultante. La tesis presenta resultados empíricos positivos que confirman tanto un poder predictivo significativo para los métodos propuestos, como consistentes mejoras en la eficacia de recomendaciones dinámicas que utilizan los predictores propuestos.

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