HW7

1) In order to calculate this, we will begin by extending S to a line and then finding the closest point on the line. This will be when:

$$t = \left(\frac{v \cdot (Q - P_0)}{v \cdot v} \right) \quad \text{for } v = P_1 - P_0$$

If $t < 0$ then the nearest point is P_0
If $t > 1$ then the nearest point is P_1
Else we compute the nearest point using this equation:

$$S(t) = P_0 - t(P_1 - P_0)$$

$P_1 - P_0 = (-63, 21, -105) - (-57, 19, -95) = (-6, 2, -10) = v$
$v \cdot v = 140$

a) $Q - P_0 = (-75, 27, -114) - (-57, 19, -95) = (-18, 8, -19)$
$v \cdot (Q - P_0) = 314$

$$\frac{v \cdot (Q - P_0)}{v \cdot v} = \frac{314}{140} = 2.2429 = t$$

Clamped $t = 1$
Thus:

$$S(t) = (-63, 21, -105) = P_1$$

b) $Q - P_0 = (-69, 12, -120) - (-57, 19, -95) = (-12, -7, -25)$
$v \cdot (Q - P_0) = 308$

$$\frac{v \cdot (Q - P_0)}{v \cdot v} = \frac{308}{140} = 2.2$$ Clamped $t = 1$

Thus:

$$s(t) = (-63, 21, -105) = P_1$$

c) $Q - P_0 = (-69, 15, -102) - (-57, 19, -95) = (-12, -4, -7)$

$V \cdot (Q - P_0) = 134$

$\frac{V \cdot (Q - P_0)}{V \cdot V} = \frac{134}{140} = .9571$ clamped $t = .9571$

$s(t) = (-57, 19, -95) - .9571(-6, 2, -10)$

$= (-57, 19, -95) - (-5.7426, 1.9142, -9.571)$

$= (-51.2574, 17.0858, -85.429)$

Thus:

$$s(t) = (-51.2574, 17.0858, -85.429)$$

d) $Q - P_0 = (-57, 9, -96) - (-57, 19, -95) = (0, -10, -1)$

$V \cdot (Q - P_0) = -10$

$\frac{V \cdot (Q - P_0)}{V \cdot V} = \frac{-10}{140} = -.0714$ clamped $t = 0$

Thus:

$$s(t) = (-57, 19, -95) = P_0$$

2) In order to calculate this, we will use the following algorithm:

Given line $L(P_0, V)$ and AABB(Max, Min)

If $V_x = 0$

  If ($P_{0x} < Min_x$ || $P_{0x} > Max_x$) return false

  else $[s_x, t_x] = [-\infty, \infty]$

else

$a = (Min_x - P_{0x}) / V_x$
$b = (Max_x - P_{0x}) / V_x$
S_x = smallest of (a, b)
t_x = largest of (a, b)
(Repeat for y and z axis)

If $[s_x, t_x]$ overlaps $[s_y, t_y]$ AND
$[s_y, t_y]$ overlaps $[s_z, t_z]$ AND
$[s_z, t_z]$ overlaps $[s_x, t_x]$ return true

a) $a_x = (-22 - {}^{-}12) / -3 = 3.3333$
$b_x = (-18 - {}^{-}12) / -3 = 2$
$S_x = 2 \quad t_x = 3.3333$
$a_y = (8 - {}^{-}31) / 15 = 2.6$
$b_y = (32 - {}^{-}31) / 15 = 4.2$
$S_y = 2.6 \quad t_y = 4.2$
$a_z = (-18 - {}^{-}38) / 8 = 2.8$
$b_z = (-2 - {}^{-}38) / 8 = 4.5$
$S_z = 2.8 \quad t_z = 4.5$

$[2, 3.3333]$ vs $[2.6, 4.2]$ ✓
$[2.6, 4.2]$ vs $[2.8, 4.5]$ ✓
$[2.8, 4.5]$ vs $[2, 3.3333]$ ✓

L(t) does intersect AABB

b) $a_x = (-22 - 3)/4 = -6.25$

$b_x = (-18 - 3)/4 = -5.25$

$s_x = -6.25, \quad t_x = -5.25$

$a_y = (8 - {-57})/{-19} = -3.4211$

$b_y = (32 - {-57})/{-19} = -4.6842$

$s_y = -4.6842 \quad t_y = -3.4211$

$a_z = (-18 - {-8})/{-2} = 5$

$b_z = (-2 - {-8})/{-2} = -3$

$s_z = -3 \quad t_z = 5$

X $[-6.25, -5.25]$ vs $[-4.6842, -3.4211]$

X $[-4.6842, -3.4211]$ vs $[-3, 5]$

X $[-3, 5]$ vs $[-6.25, -5.25]$

L(t) Does Not Intersect AABB

3) $a = v_1 \cdot v_1 = (10,1,15)\cdot(10,1,15) = 326$

$b = v_1 \cdot v_2 = (10,1,15)\cdot(-5,-7,-14) = -267$

$c = v_2 \cdot v_2 = (-5,-7,-14)\cdot(-5,-7,-14) = 270$

$d = (P_1 - P_2)\cdot v_1 = (-25, 28, 1)\cdot(10,1,15) = -207$

$e = (P_1 - P_2)\cdot v_2 = (-25, 28, 1)\cdot(-5,-7,-14) = -85$

$b^2 - ac = -16731$ NOT PARALLEL

$$C_1 = (-40, 6, -36) + \left(\frac{(270)(-207) - (-267)(-85)}{-16731}\right)(10, 1, 15)$$
$$= (-40, 6, -36) + 4.697(10, 1, 15)$$
$$= (-40, 6, -36) + (46.97, 4.697, 70.455)$$
$$= (6.97, 10.697, 34.45)$$

$$C_2 = (-15, -22, -37) + \left(\frac{(-207)(-267) - (326)(-85)}{-16731}\right)(-5, -7, -14)$$
$$= (-15, -22, -37) + -4.9596(-5, -7, -14)$$
$$= (-15, -22, -37) + (24.798, 34.7172, 69.4344)$$
$$= (9.798, 12.7172, 32.4344)$$

$$C_1 - C_2 = (-2.828, -2.0202, 2.0156)$$
$$\|C_1 - C_2\| = 4.0176$$

Thus the distance is: 4.0176

(4) $d = \left|\frac{(C - P_0)\cdot n}{\|n\|}\right|$ If $d < r$ return true, else return false. If $d = 0$ C in plane. If $d > 0$ C ↑ plane. If $d < 0$ C ↓ plane

BS1: $(C - P_0)\cdot n = (-7, -15, -36)\cdot n = -30$
$\|n\| = 3.3166$

$$\frac{(C - P_0)\cdot n}{\|n\|} = -9.0454 \quad \left|\frac{(C - P_0)\cdot n}{\|n\|}\right| = 9.0454$$

$9.0454 \nless 6$

Thus: a) They do not intersect
b) Center is below the plane

BS2: $(C-P_0)\cdot n=(-22,-10,-31)\cdot n = 25$
$||n|| = 3.3166$
$$\frac{(C-P_0)\cdot n}{||n||} = 7.5378 = \left|\frac{(C-P_0)\cdot n}{||n||}\right|$$
$7.5378 < 8$

Thus: a) They do intersect
b) Center is above the plane

(5) I will use the second method.
Seg1) $d_1 = (S_1-P_0)\cdot n = (-2,-26,8)\cdot n = -4$
$d_2 = (S_2-P_0)\cdot n = (-3,-24,7)\cdot n = -1$
$d_1 d_2 = 4 \not< 0$

Seg1 does not intersect the plane

Seg2) $d_1 = (S_1-P_0)\cdot n = (-6,-25,7)\cdot n = 7$
$d_2 = (S_2-P_0)\cdot n = (0,-23,7)\cdot n = -9$
$d_1 d_2 = -63 < 0$

Seg2 does intersect the plane

⑥ a) $C = (Max + min)/2$

$C_x = (3.75 + {}^{-}4.25)/2 = -.25$

$C_y = (4.5 + {}^{-}5.5)/2 = -.5$

$C_z = (6.38 + {}^{-}5.63)/2 = .38$

$C = (-.25, -.5, .38)$

b) $d = Max - C = (3.75, 4.5, 6.38) - (-.25, -.5, .38)$

$d = (4, 5, 6)$

c) $s = ||v||$

$s_x = ||(1.29987, 0, 3.24967)|| = 3.5$

$s_y = ||(-1.9377, 2.80971, .77509)|| = 3.5$

$s_z = ||(-2.6087, -2.087, 1.04335)|| = 3.5$

$s = 3.5$

d) $C' = WC$

$$\begin{bmatrix} 1.29987 & -1.9377 & -2.6087 & -3 \\ 0 & 2.80971 & -2.087 & -7 \\ 3.24967 & .77509 & 1.04335 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -.25 \\ -.5 \\ .38 \\ 1 \end{bmatrix} = \begin{bmatrix} -3.3474 \\ -9.1979 \\ 8.1966 \\ 1 \end{bmatrix}$$

$C' = (-3.3474, -9.1979, 8.1966)$

e) We begin by computing OBB half-proj on n:

$$W^{-1} = \begin{bmatrix} .1061 & 0 & .2653 & -2.0692 \\ -.1582 & .2294 & .0633 & .5615 \\ -.213 & -.1704 & .0852 & -2.5981 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$n' = W^{-1} n = (-1.6448, .6196, -.1278, 0)$

$$\text{projmax} = \frac{(|n'_x a| + |n'_y b| + |n'_z c|)}{||n||} = \frac{10.444}{6.1644} = 1.6942$$

$$h = (3.5)^2 1.6942 = 20.754$$

Next, we compute a segment that span's the OBB's projection on n:

$$S_1 = c' - h\frac{n}{||n||} = c' - 20.754 \frac{(-3, 2, -5)}{6.1644}$$

$$= c' - 20.754(-.4867, .3244, -.8111)$$

$$= c' - (-10.101, 6.7326, -16.8336)$$

$$= (6.7536, -15.9305, 25.0302)$$

$$S_2 = c' + (-10.101, 6.7326, -16.8336)$$

$$= (-13.4484, -2.4653, -8.637)$$

$$d_1 = (S_1 - P_0) \cdot n = (-5.2464, -12.9305, 16.0302) \cdot n$$

$$= -90.2728$$

$$d_2 = (S_2 - P_0) \cdot n = (-25.4484, .5347, -17.637) \cdot n$$

$$= 165.5996$$

$$d_1 = -90.2728 < 0$$

$$d_2 = 165.5996 > 0$$

Thus, the OBB intersects the Plane.