Homework 3

(1) i) Before we begin, we must normalize the rotation axis:

$$\|r\| = \|(-1, 10, 3)\| = \sqrt{(-1,10,3)\cdot(-1,10,3)} = 10.488$$

$$\hat{r} = \frac{(-1, 10, 3)}{10.488} = (-.095, .953, .286)$$

Next, we must plug $\hat{r}$ and $\pi/8$ into the Rodrigues rotation formula

$$R_{\hat{r}, \pi/8}(v) = v\cos\left(\frac{\pi}{8}\right) + [1 - \cos\left(\frac{\pi}{8}\right)](v\cdot\hat{r})\hat{r} + [\hat{r}\times v]\sin\left(\frac{\pi}{8}\right)$$

where $v = (-1, -1, 3)$

$\cos\left(\frac{\pi}{8}\right) = .924 \quad \sin\left(\frac{\pi}{8}\right) = .383$

$v(.924) = (-.924, -.924, 2.772)$

$1 - .924 = .076$

$(v\cdot\hat{r}) = .095 + -.953 + .858 = 0$

$(v\cdot\hat{r})\hat{r} = (0, 0, 0)$

$[1 - \cos\left(\frac{\pi}{8}\right)](v\cdot\hat{r})\hat{r} = (0, 0, 0)$

$\hat{r} \times V = (3.145, -.001, 1.048)$

$[\hat{r} \times V] \sin(\frac{\pi}{8}) = (1.205, 0, .398)$

$V\cos(\frac{\pi}{8}) + [1 - \cos\frac{\pi}{8}](V \cdot \hat{r})\hat{r} + [\hat{r} \times V]\sin(\frac{\pi}{8}) =$

$(-.924, -.924, 2.772) + (0,0,0) +$

$(1.205, 0, .398) =$

$(.281, -.924, 3.17)$.

Thus: $R_{\hat{r}, \frac{\pi}{8}}(V) = (.281, -.924, 3.17)$

2) In order to rotate the forward vector by the left axis, we must first find the left axis. This can be done by cross multiplying the up vector by the forward vector (this would be 2 from the Note):

$(-1, 10, 3) \times (-1, -1, 3) = (33, 0, 11)$

Next, we normalize this vector:

$\|r\| = \|(33, 0, 11)\| = \sqrt{(33,0,11) \cdot (33,0,11)} = 34.785$

$\hat{r} = \frac{(33, 0, 11)}{34.785} = (.949, 0, .316)$

Next, Rodrigues rotation formula:

$$R_{\hat{r},\frac{\pi}{4}}(v) = v\cos\left(-\frac{\pi}{4}\right) + \left[1-\cos\left(-\frac{\pi}{4}\right)\right](v\cdot\hat{r})\hat{r} + [\hat{r}\times v]\sin\left(-\frac{\pi}{4}\right)$$

Where $v=(-1,-1,3)$

$\cos\left(\frac{\pi}{4}\right)=1.000 \quad \sin\left(\frac{\pi}{4}\right)=-.014$

$V(1)=(-1,-1,3)$

$1-1=0$

$[0](v\cdot\hat{r})\hat{r}=(0,0,0)$

$\hat{r}\times v=(.316,-3.163,-.949)$

$[\hat{r}\times v]\sin\left(-\frac{\pi}{4}\right)=(-.004,.044,-.013)$

$v\cos\left(\frac{\pi}{4}\right)+\left[1-\cos\left(-\frac{\pi}{4}\right)\right](v\cdot\hat{r})\hat{r}+[\hat{r}\times v]\sin\left(-\frac{\pi}{4}\right)=$

$(-1,-1,3)+(0,0,0)+(-.004,.044,-.013)$

$=(-1.004,-.956,2.987)$

Thus: $R_{\hat{r},-\frac{\pi}{4}}(v)=(-1.004,-.956,2.987)$

(3B) Step 1: We begin by translating each point fwd by 9

$$P_1 = \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 9 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 8 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 9 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 11 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 9 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 8 \end{pmatrix}$$

Now rotate about y axis $\frac{\pi}{4}$