HW5

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① a) We will use this matrix:

$$C_{view\text{-}to\text{-}world} = \begin{bmatrix} \hat{v}_{side} & \hat{v}_{up} & \hat{v}_{fwd} & C_{pos} \\ 0 & 0 & 0 & 1 \end{bmatrix} = C_{v2w}$$

We calculate $\hat{v}_{fwd}$ by using the equation:

$$\hat{v}_{dir} = \text{Normalize}(T_{pos} - C_{pos})$$

$(1, -463, -522) - (1, 5, -2) = (0, -468, -520)$

$\text{Normalize}(0, -468, -520) = (0, -.669, -.7433)$

Since we are working w/ a Right-hand camera then we take $-(\hat{v}_{dir})$:

$$\hat{v}_{fwd} = -\hat{v}_{dir} = (0, .669, .7433)$$

Next, In order to calculate $\hat{v}_{side}$ we use this equation:

$$\hat{v}_{side} = \text{Normalize}(up \times \hat{v}_{fwd})$$

$(0, 1, 0) \times (0, .669, .7433) = (.7433, 0, 0)$

$\text{Normalize}(.7433, 0, 0) = (1, 0, 0) = \hat{v}_{side}$

In order to calculate $\hat{v}_{up}$, we use the following equation:

$$\hat{v}_{up} = \text{Normalize}(\hat{v}_{fwd} \times \hat{v}_{side})$$

$$(0, .669, .7433) \times (1, 0, 0) = (0, .7433, -.669)$$

The normalization is the same

Thus:

$$C_{v2w} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & .7433 & .669 & 5 \\ 0 & -.669 & .7433 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) In order to calculate C_{w2v}, we calculate the inverse of C_{v2w}:

$$(C_{v2w})^{-1} = \begin{bmatrix} R^T & -(R^T C_{pos}) \\ 0^T & 1 \end{bmatrix} = C_{w2v}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & .7433 & -.669 \\ 0 & .669 & .7433 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 5.0545 \\ 1.8584 \end{bmatrix}$$

Thus:

$$C_{w2v} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & .7433 & -.669 & -5.0545 \\ 0 & .669 & .7433 & -1.8584 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\left(\frac{2}{1200}\right)426$

c) We use the following matrix:

$$M_{ortho} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{1200} & 0 & 0 & 0 \\ 0 & \frac{2}{675} & 0 & 0 \\ 0 & 0 & -\frac{2}{6999} & -\frac{7001}{6999} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} .00167 & 0 & 0 & 0 \\ 0 & .00296 & 0 & 0 \\ 0 & 0 & -.00029 & -1.00029 \\ 0 & 0 & 0 & 1 \end{bmatrix} = M_{ortho}$$

d) We use the following matrix:

$$M_{NDC2Scr} = \begin{bmatrix} w/2 & 0 & 0 & w/2 \\ 0 & -h/2 & 0 & h/2 \\ 0 & 0 & dz/2 & dz/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1920}{2} & 0 & 0 & \frac{1920}{2} \\ 0 & -\frac{1080}{2} & 0 & \frac{1080}{2} \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 960 & 0 & 0 & 960 \\ 0 & -540 & 0 & 540 \\ 0 & 0 & .5 & .5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = M_{NDC2Scr}$$

e) In order to calculate this, we must multiply each point C_{w2v} then take this new point and multiply it by M_{ortho}, and finally multiply this value by $M_{NDC2Scr}$.

P1:

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & .7433 & -.669 & -5.0545 \\ 0 & .669 & .7433 & -1.8584 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -90 \\ -1352 \\ -1573 \\ 1 \end{bmatrix} = \begin{bmatrix} -91 \\ 65.0869 \\ -2100.8295 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} .00167 & 0 & 0 & 0 \\ 0 & .00296 & 0 & 0 \\ 0 & 0 & -.00029 & -1.00029 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -91 \\ 65.0869 \\ -2100.8295 \\ 1 \end{bmatrix} = \begin{bmatrix} -.15197 \\ .19266 \\ -.39105 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 960 & 0 & 0 & 960 \\ 0 & -540 & 0 & 540 \\ 0 & 0 & .5 & .5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -.15197 \\ .19266 \\ -.39105 \\ 1 \end{bmatrix} = \begin{bmatrix} 814.1088 \\ 435.9651 \\ .3045 \\ 1 \end{bmatrix}$$

(From here on out I wont copy all matrices, In order to save time and space)

P2:

$C_{W2V}(-65,-1389,-1573,1)=(-66,14.8388,$
$-2100.3103,1)$

$M_{ortho}(-66,14.8388,-2100.3103,1)=$
$(-.1102,.0439,-.3912,1)$

$M_{NDC2Scr}(-.1102,.0439,-.3912,1)=$
$(854.1888,516.2817,.3044,1)$

P3:

$C_{W2V}(-140,-1333,-1624,1)=(-141,90.5826,$
$-2100.7546,1)$

$M_{ortho}(-141,90.5826,-2100.7546,1)=$
$(-.2355,.2681,-.3911,1)$

$M_{NDC2Scr}(-.2355,.2681,-.3911,1)=$
$(733.9488,395.2128,.3045,1)$

	C_{W2V}	M_{ortho}	$M_{NDC2Scr}$
P1	-91	-.15197	814.1088
	65.0869	-.19266	435.9651
	-2100.8295	-.39105	.3045
P2	-66	-.1102	845.1888
	14.8388	.0439	516.2817
	-2100.3103	-.3912	.3044
P3	-141	-.2355	733.9488
	90.5826	.2681	395.2128
	-2100.7546	-.3911	.3045

g) We begin by setting the points in near and far clip planes

$$near = \begin{bmatrix} 1871 \\ 56 \\ 0 \\ 1 \end{bmatrix} \quad far = \begin{bmatrix} 1871 \\ 56 \\ 1 \\ 1 \end{bmatrix}$$

Next, we apply $M_{scr2NDC} = (M_{NDC2scr})^{-1}$:

$$(M_{NDC2scr})^{-1} = \begin{bmatrix} .00104 & 0 & 0 & -1 \\ 0 & -.00185 & 0 & 1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = M_{scr2NDC}$$

Near: $M_{scr2NDC}(1871, 56, 0, 1) = (.94896, .8963, -1, 1)$

Far: $M_{scr2NDC}(1871, 56, 1, 1) = (.94896, .8963, 1, 1)$

Next we apply $(M_{ortho})^{-1}$ to get points in camera space:

$$(M_{ortho})^{-1} = \begin{bmatrix} 598.8024 & 0 & 0 & 0 \\ 0 & 337.8378 & 0 & 0 \\ 0 & 0 & -3448.2759 & -3448.2759 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Near: $(M_{ortho})^{-1}(.94896, .8963, -1, 1) = (568.2385, 302.8028, -1, 1)$

Far: $(M_{ortho})^{-1}(.94896, .8963, 1, 1) =$

$(567.2385, 302.8028, -6897.5517, 1)$

And finally, we apply C_{v2w} to get the points in World space:

Near: $C_{v2w}(568.2385, 302.8028, -1, 1) =$

$(569.2385, 229.4043, -205.3184, 1) = World_{near}$

Far: $C_{v2w}(567.2385, 302.8028, -6897.5517, 1) =$

$(568.2385, -4384.3888, -5331.5253, 1) = World_{far}$

② a) In order to calculate this we will use the following matrix:

$$C_{v2w} = \begin{bmatrix} \hat{v}_{side} & \hat{v}_{up} & \hat{v}_{fwd} & C_{pos} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We calculate $\hat{v}_{fwd}$ by using this equation:

$$\hat{v}_{fwd} = -\hat{v}_{dir} = -(\text{Normalize}(T_{pos} - C_{pos}))$$

$(83, -40, -13) - (-9, -6, -2) = (92, -34, -11)$

$\text{Normalize}(92, -34, -11) = (.9322, -.3445, -.1115)$

$$\hat{v}_{fwd} = (-.9322, .3445, .1115)$$

Next, we use this equation to calculate $\hat{v}_{side}$:

$$\hat{v}_{side} = \text{Normalize}(up \times \hat{v}_{fwd})$$

$(0,1,0) \times (-.9322, .3445, .1115) = (.1115, 0, .9322)$
Normalize$(.1115, 0, .9322) = (.1188, 0, .9929) = \hat{v}_{side}$

To calculate $\hat{v}_{up}$, we use:
$\hat{v}_{up} = $ Normalize$(\hat{v}_{fwd} \times \hat{v}_{side})$

$(-.9322, .3445, .1115) \times (.1188, 0, .9929) =$
$(.3421, .9388, -.0409)$
Normalize$(.3421, .9388, -.0409) =$
$(.3421, .9388, -.0409)$

Thus:
$$C_{V2W} = \begin{bmatrix} .1188 & .3421 & -.9322 & -9 \\ 0 & .9388 & .3445 & -6 \\ .9929 & -.0409 & .1115 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) In order to calculate C_{W2V}, we calculate the inverse of C_{V2W}:
$$(C_{V2W})^{-1} = \begin{bmatrix} R^T & -(R^T C_{pos}) \\ 0^T & 1 \end{bmatrix} = C_{W2V}$$

$$\begin{bmatrix} .1188 & 0 & .9929 \\ .3421 & .9388 & -.0409 \\ -.9322 & .3445 & .1115 \end{bmatrix} \begin{bmatrix} -9 \\ -6 \\ -2 \end{bmatrix} = \begin{bmatrix} -3.055 \\ -8.6299 \\ 6.0998 \end{bmatrix}$$

Thus:

$$C_{w2v} = \begin{bmatrix} .1188 & 0 & .9929 & 3.055 \\ .3421 & .9388 & -.0409 & 8.6299 \\ -.9322 & .3445 & .1115 & -6.0998 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) We will use the following matrix:

$$M_{persp} = \begin{bmatrix} d/a & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

where $d = \cot(\frac{\theta}{2})$ and a is the aspect ratio

$d = \cot(\frac{\pi/3}{2}) = 1.7321$

$a = 1920/1080 = 1.7778$

Thus:

$$M_{persp} = \begin{bmatrix} .9743 & 0 & 0 & 0 \\ 0 & 1.7321 & 0 & 0 \\ 0 & 0 & -1.002 & -2.002 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

d) We will use the following matrix:

$$M_{NDC2Scr} = \begin{bmatrix} W/2 & 0 & 0 & W/2 \\ 0 & -H/2 & 0 & H/2 \\ 0 & 0 & d_z/2 & d_z/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1920}{2} & 0 & 0 & \frac{1920}{2} \\ 0 & -\frac{1080}{2} & 0 & \frac{1080}{2} \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 960 & 0 & 0 & 960 \\ 0 & -540 & 0 & 540 \\ 0 & 0 & .5 & .5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = M_{NDC2Scr}$$

e) We calcute this by multiplying each point by C_{W2V}, then take this and multiply this by M_{persp}, and finally take this and multiply this by $M_{NDC2scr}$. As before, I will not write out all matrices:

P1:

$C_{W2V}(140, -42, -141, 1) = (-120.3119, 22.8612, -166.7983, 1)$

$M_{persp}(-120.3119, 22.8612, -166.7983, 1) = (-117.2199, 39.5979, 165.1299, 166.7983)$

Divide by w: $(-.7028, .2374, .99, 1)$

$M_{NDC2scr}(-.7028, .2374, .99, 1) = (285.312, 411.804, .995, 1)$

P2:

$C_{W2V}(139, -37, -141, 1) = (-120.4307, 27.2131, -164.1436, 1)$

$M_{persp}(-120.4307, 27.2131, -164.1436, 1) = (-117.3356, 47.1358, 162.4699, 164.1436)$

Divide by w: $(-.7148, .2872, .9898, 1)$

$M_{NDC2scr}(-.7148, .2872, .9898, 1) = (273.792, 384.912, .9949, 1)$