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Exercise 1

$$P(t) = 3\times10^{-9}\, t^2(100-t^2) \quad 0 \le t \le 100 \text{ years}$$

$P(t) = 0$ outside of the range.

a) $$P(60<t<70) = \int_{60}^{70} P(t)\,dt$$

$$= 3\times10^{-9}\int_{60}^{70} t^2(100-t)^2\,dt$$

$$= 3\times10^{-9}\int_{60}^{70} t^4 - 200t^3 + 10000t^2\,dt$$

$$= 3\times10^{-9}\left[\frac{t^5}{5} - 200\frac{t^4}{4} + 10000\frac{t^3}{3}\right]_{60}^{70}$$

$$= 3\times10^{-9}\left[\frac{t^5}{5} - 50t^4 + 10000\frac{t^3}{3}\right]_{60}^{70}$$

$$= \cancel{3}\times10^{-9} \times \frac{154360000}{\cancel{3}}$$

$$= 154360000\times10^{-9}$$

$$= \underline{\underline{0.15436}}$$

b) $\dfrac{P(60<t<70)}{P(t\geq 60)}$

$= \dfrac{P(60<t<70)}{P(60<t<100)}$

$P(60<t<70) = 0.15436$ [from (a)]

$P(60<t<100) = \int_{60}^{100} p(t)\,dt$

$= 3\times10^{-9}\int_{60}^{100} t^2(100-t)^2\,dt$

$= 3\times10^{-9}\left[\dfrac{t^5}{5} - 50t^4 + 10000\dfrac{t^3}{3}\right]_{60}^{100}$

$= \cancel{3}\times10^{-9} \times \dfrac{317440000}{\cancel{3}}$

$= 317440000\times10^{-9}$

$= \underline{\underline{0.31744}}$

$\therefore \dfrac{P(60<t<70)}{P(60<t<100)} = \dfrac{0.15436}{0.31744} = \underline{\underline{0.4863}}$

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Exercise 2 :

A set of events $\{A_i\}$ are said to be independent if for every finite sub collection $A_{i_1}, A_{i_2}, A_{i_3} \dots A_{i_n}$ we have

$$P\left(\bigcap_{k=1}^{n} A_{i_k}\right) = \prod_{k=1}^{n} P(A_{i_k}) \longrightarrow (1)$$

Let $A = A_1 \cup A_2 \cup A_3 \dots\dots \cup A_n$ a union of n independent events. Then by De-Morgan's law:

$$\bar{A} = \bar{A}_1 \, \bar{A}_2 \dots \bar{A}_n \longrightarrow (2)$$

Using their independence ~~events~~

$$P(\bar{A}) = P(\bar{A}_1 \, \bar{A}_2 \dots \bar{A}_n) = \prod_{i=1}^{n} P(\bar{A}_i) = \prod_{i=1}^{n} (1 - P(A_i)) \longrightarrow (3)$$

Thus for any A as in (2)

$$P(A) = 1 - P(\bar{A})$$

$$P(A) = 1 - \prod_{i=1}^{n} (1 - P(A_i)) \longrightarrow (4)$$

Exercise 2 (cont)

$P(S1) = P(S2) = P(S3) = P$

$\therefore P(\overline{S1}) = P(\overline{S2}) = P(\overline{S3}) = 1-P$

a) Probability of receiving an input signal at the output

$P(S1 \cup S2 \cup S3) = 1 - P(\overline{S1} \cap \overline{S2} \cap \overline{S3})$

$= 1 - P(\overline{S1}) \cdot P(\overline{S2}) \cdot P(\overline{S3})$

$= 1 - (1-P) \cdot (1-P) \cdot (1-P)$

$= 1 - (1-P)^3 = 1 - (1 - P^3 - 3P + 3P^2)$

$= 1 - 1 + P^3 + 3P - 3P^2$

$= \underline{\underline{P^3 - 3P^2 + 3P}}$

b) Probability that switch S_i is open given that an input signal is received at the output

$$P(\overline{S1} \mid S1 \cup S2 \cup S3)$$

$$= \frac{P(\overline{S1} \cap (S1 \cup S2 \cup S3))}{P(S1 \cup S2 \cup S3)}$$

$$\frac{P(\overline{S1} \cap (S2 \cup S3))}{1-(1-p)^3} = \frac{P(\overline{S1}) \cdot P(S2 \cup S3)}{1-(1-p)^3}$$

$$= \frac{(1-p)\left[1-P(\overline{S2} \cap \overline{S3})\right]}{p^3-3p^2+3p}$$

$$= \frac{(1-p)\left[1-(1-p)^2\right]}{p^3-3p^2+3p} \Rightarrow \begin{cases} P(\overline{S2}) \cdot P(\overline{S3}) \\ (1-p) \cdot (1-p) \end{cases}$$

$$= \frac{(1-p)(1-(1-2p+p^2))}{p^3-3p^2+3p}$$

$$= \frac{(1-p)(2p-p^2)}{p^3-3p^2+3p} = \frac{2p-p^2-2p^2+p^3}{p^3-3p^2+3p}$$

$$= \frac{2p-3p^2+p^3}{p^3-3p^2+3p} = \frac{\cancel{p}(2-3p+p^2)}{\cancel{p}(p^2-3p+3)} = \underline{\underline{\frac{2-3p+p^2}{3-3p+p^2}}}$$