Using Iris for Program Verification

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1 Monotone Counter

▷ **Coq code:** https://github.com/abeln/iris-practice/blob/master/counter.v ◁

This is the counter example from Section 7.7 of the notes. The module exports three methods:

- `newCounter` creates a new counter. Counters are represented simply with an int reference.

  ```
  Definition newCounter : val := \lambda: <>, ref #0.
  ```

- `read` returns the value currently stored in a counter.

  ```
  Definition read : val := \lambda: "c", !"c".
  ```

- `incr` takes a counter, increments it, and returns unit.

  ```
  Definition incr : val :=
    rec: "incr" "c" :=
      let: "n" := !"c" in
      let: "m" := #1 + "n" in
      if: CAS "c" "n" "m" then #() else "incr" "c".
  ```

The client for the counter is a program that instantiates a counter, spawns two threads each incrementing the counter, and then reads the value off the counter:

```
Definition client : val :=
 \lambda: <>,
   let: "c" := newCounter #() in
   ((incr "c") ||| (incr "c")) ;;
   read "c".
```

1.1 Specs

We can prove two different specs for the code above:

- Authoritative RA

  In the first spec, our resource algebra is $\textsc{Auth}(\mathbb{N})$. Elements of this RA are of the form either $\bullet n$ (authoritative) or $\circ n$ (non-authoritative), where $n \in \mathbb{N}$.

  The important properties of this RA (for the counter example) are:

  – $\bullet 0 \cdot \circ 0 \in \mathcal{V}$
  – $\bullet m \cdot \circ n \in \mathcal{V}$ implies $m \geq n$
  – $\bullet m \cdot \circ n \rightsquigarrow \bullet(m+1) \cdot \circ(n+1)$
  – $|\circ n| = \circ n$

  Then we can define our predicate for the counter:

  $$\text{isCounter}(l, n, \gamma) = \ulcorner \circ n \urcorner^{\gamma} * \exists s. \boxed{\exists m. l \hookrightarrow m \land \ulcorner \bullet m \urcorner^{\gamma}}^{s}$$

  This predicate is persistent because the invariant component is persistent and $\circ n$ is also persistent (via the $\textsc{Persistently-core}$ rule), because the core of $\circ n$ is $\circ n$ itself .

  The intuition behind the predicate is that if we own $\text{isCounter}(l, n, \gamma)$ then, at an atomic step (e.g. a load), we can know that l is allocated and it points to a value that's $\geq n$.

  Additionally, after an increment, we can update the resource to $\text{isCounter}(l, n + 1, \gamma)$.

  We then prove the following specs for the counter methods:

  – $\{\top\}$ newCounter () $\{l.\exists\gamma.\text{isCounter}(l, 0, \gamma)\}$
  – $\{\text{isCounter}(l, n, \gamma)\}$ read l $\{v.v \geq n\}$. Notice how we don't have to give the counter back, because it's persistent.
  – $\{\text{isCounter}(l, n, \gamma)\}$ incr l $\{\text{isCounter}(l, n + 1, \gamma)\}$

  We can also prove the client spec:

  $$\{\top\}\text{client }()\{n.n \geq 1\}$$

  The "problem" with this specification *isCounter* doesn't tell us anything about what other threads are doing with the counter. So in the verification of the client code, after both threads return, we know

  $$\text{isCounter}(l, 1, \gamma) * \text{isCounter}(l, 1, \gamma) \equiv \text{isCounter}(l, 1, \gamma)$$

  So we know that the counter must be ≥ 1, but we don't know that it's 2.

- Authoritative RA + Fractions

  The second spec we can give is more precise, and allows us to conclude that the client reads exactly 2 once the two threads finish.

  The resource algebra we use is $\text{AUTH}((\mathbb{Q}_p \times \mathbb{N})_?)$:

  - $\mathbb{Q}_p$ is the RA of positive fractions, where the valid elements are those ≤ 1.
  - The ? is the *optional RA* construction. In this case, it's needed because to use the authoritative RA we need the argument algebra to be *unital*. And $\mathbb{Q}_p \times \mathbb{N}$ is not unital because 0 is not an element of $\mathbb{Q}_p$.

  The important properties of this RA are:

  - $\circ(1,0) \cdot \bullet(1,0) \in \mathcal{V}$
  - $\circ(p,n) \cdot \bullet(1,m) \in \mathcal{V}$ implies $m \geq n$
  - $\circ(1,n) \cdot \bullet(1,m) \in \mathcal{V}$ implies $m = n$. This is the rule that will allows us to give a more precise specification.
  - $\circ(p,n) \cdot \bullet(q,m) \rightsquigarrow \circ(p,n+1) \cdot \bullet(q,m+1)$

  With this RA, we get the new counter resource

$$\text{isCounter}(l,n,p,\gamma) = \boxed{\circ(p,n)}^{\gamma} * \exists s. \boxed{\exists m. l \hookrightarrow m \wedge \boxed{\bullet(1,m)}^{\gamma}}^{s}$$

This resource is no longer persistent, because the core $|p|$ is undefined. However, we can show the following

$$\text{isCounter}(l, n+m, p+q, \gamma) \dashv\vdash \text{isCounter}(l,n,p,\gamma) * \text{isCounter}(l,m,q,\gamma)$$

We then prove the following specs for the counter methods:

- $\{\top\}$ newCounter () $\{l. \exists \gamma. \text{isCounter}(l, 0, 1, \gamma)\}$
- $\{\text{isCounter}(l,n,p,\gamma)\}$ read l $\{v. v \geq n * \text{isCounter}(l,n,p,)\}$. Notice how we need to give back the counter in this case.
- $\{\text{isCounter}(l,n,1,\gamma)\}$ read l $\{v. v = n * \text{isCounter}(l,n,p,)\}$. Notice that since we have the entire fraction, we can get the exact value of the counter.
- $\{\text{isCounter}(l,n,p,\gamma)\}$ incr l $\{\text{isCounter}(l,n+1,p,\gamma)\}$

With this, we get the more precise client spec

$$\{\top\}\text{client }()\{n. n = 2\}$$