

Tamarin: Concolic Disequivalence for MIPS

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Abstract. TODO

1 Introduction

We are staring at two opaque black boxes laying at our feet. Each box has a narrow slot through which we can place items in the box, but we cannot quite see what is inside. They look approximately like this:



We know each box contains an animal, but we do not know which specific animal is in each one. We would like to find out if both boxes contain the same species of animal. Our solution is simple: we take two carrots, and drop one in each box through the slots.

After a while, a chewing sound emerges from the boxes. We peer into them and, indeed, it looks like the carrots were successfully eaten. Triumphant, we declare that the boxes contain the same species of animal. The truth is altogether different:



The boxes are assembly programs. The animals are the functions those programs compute. The carrot is unit testing. The task was to determine whether the programs were equivalent. And we failed at it. In this paper, we show a technique that is better than the carrot.

Program equivalence. The program is the specification. The complications of assembly language.

2 Program Equivalence for MIPS

Let us set up the problem a bit more formally. Consider the set P of MIPS-assembly programs that satisfy two restrictions: they take as inputs only the values of registers \$1 and \$2, and when they stop executing we define their

output to be (exclusively) the value of \$3. Other side effects, such as printing values to the screen, or system calls, are disallowed.

We can now define a relation $\text{equiv} \subseteq P \times P$ (and its complement, $\neg\text{equiv}$) of equivalent programs. Given $P_1, P_2 \in P$, we say that $P_1 \text{ equiv } P_2$ (read “ P_1 is equivalent to P_2 ”) if, for all inputs \$1 and \$2, one of the following holds:

- Both P_1 and P_2 fail during execution (for example, due to a divide-by-zero error).
- P_1 and P_2 stop with the same output in \$3.

For example, the two programs in Figure 1 are equivalent.

<pre># P_1 add \$3, \$1, \$2</pre>	<pre># P_2 add \$4, \$1, \$1 lis \$5 42 sw \$4, 0, \$5 add \$3, \$1, \$2</pre>
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Fig. 1. $P_1 \text{ equiv } P_2$

Notice that $P_1 \text{ equiv } P_2$ even though P_2 modifies the contents of the memory and an additional register (\$4), because:

- Both P_1 and P_2 terminate without errors.
- The value of \$3 will be the same when they do so.

Unfortunately, even though equiv captures an already-simplified notion of equivalence¹, a decision procedure for it does not exist, due to Rice’s theorem.

To get decidability back, we define a new class of relations $\text{equiv}_S \subseteq P \times P$ (whose complement is $\neg\text{equiv}_S$). We say that $P_1 \text{ equiv}_S P_2$ (read “ P_1 is S -equivalent to P_2 ”) if, for all inputs, one of the following holds:

- Either P_1 or P_2 does not stop within S steps (we can think of each CPU cycle as one step).
- Both P_1 and P_2 fail.
- Both P_1 and P_2 stop with the same output.

The equiv_S relation captures the notion that we cannot tell P_1 and P_2 apart by running them for at most S steps. Figure 2 shows an example of two programs that are S -equivalent for $S = 10$, but not equivalent. This is the case because P_2 loops while the counter is less than 42, so with 10 steps in our “budget” we will have to stop P_2 before the loop is over and we can observe the different result.

¹ For example, equiv has a very narrow notion of output that excludes side effects.

```

# P_1
add $3, $1, $2

# P_2
add $4, $0, 1 # counter
add $5, $0, 42 # upper bound
loop:
    slt $6, $4, $5
    beq $6, $0, end
    add $4, $4, 1
    beq $0, $0, loop
end:
    add $3, $1, $1

```

Fig. 2. $P_1 \text{ equiv}_{10} P_2$, but $P_1 \not\text{equiv} P_2$

Given a fixed S , the equiv_S relation is decidable because there is a finite number of inputs to try, and for each input we only need to run the programs a finite number of steps.

We already saw that equivalence not always implies S -equivalence. However, the converse always holds. The following lemma shows that equiv_S over-approximates equiv .

Lemma 1. $\forall S, P_1, P_2, P_1 \text{ equiv} P_2 \implies P_1 \text{ equiv}_S P_2$.

Proof. Let $P_1 \text{ equiv} P_2$. Then we have one of two cases:

- Either P_1 or P_2 (or both) do not stop within S steps. Then by definition $P_1 \text{ equiv}_S P_2$.
- Both P_1 and P_2 stop within S steps. Then because they are equivalent, we know that they either fail with an error, or both stop with the same output. In either case, $P_1 \text{ equiv}_S P_2$.

Corollary 1. $P_1 \not\text{equiv}_S P_2 \implies P_1 \not\text{equiv} P_2$.

Proof. This is just the contrapositive of Lemma 1.

Corollary 1 can be used to argue the soundness (with respect to equiv) of any decision procedure that under-approximates equiv_S . In the next section we will show one such under-approximation based on concolic execution.

3 Concolic Disequivalence

We know from Corollary 1 that any relation that under-approximates equiv_S is sound. Figure 3 shows why we want an under-approximation: efficiency. equiv captures the class of programs that are disequivalent, but is undecidable. equiv_S is decidable, but likely cannot be computed efficiently. Therefore, we look for a subset of equiv_S (an under-approximation) that can be efficiently computed.



Fig. 3. Hierarchy of disequivalence relations

To fill the missing relation in Figure 3 we propose concolic disequivalence. Abstractly, concolic disequivalence is a function $\text{compare}(P_1, P_2, S)$ that takes as inputs two MIPS programs and returns one of two answers:

- “disequivalent”, in which case $P_1 \not\text{equiv}_S P_2$.
- “possibly equivalent”, meaning that P_1 and P_2 might or might not be S -equivalent.

Figure 3 shows pseudocode for compare . The algorithm alternately executes P_1 and P_2 . At every step, one of the programs is labelled as the “driver” and the other one as the “verifier”. The driver program is then concolically executed, yielding a set of inputs that exercise a new program path (of the driver). The inputs can then be fed to the verifier, and the results of both driver and verifier compared. If the results are different, then we know P_1 and P_2 are disequivalent. Otherwise, the driver becomes the verifier, and vice-versa. Eventually, we will traverse all explorable paths, at which point P_1 and P_2 can be declared possibly equivalent.

We now give an example of how compare operates. Consider the sample programs below:

```
# P_1                                # P_2
bne $1, 42, end                      add $3, $1, $2
add $3, $0, $0                       bne $2, 100, end
end:                                  add $3, $3, $2
add $3, $1, $2                       end:
```

Figure 5 summarizes the state of the algorithm as it compares P_1 and P_2 . First, notice how the driver and verifier roles flip between P_1 and P_2 in con-

```

function COMPARE( $P_1, P_2, S$ )
   $b \leftarrow true$ 
  while either  $P_1$  or  $P_2$  has unexplored paths do
    if  $b$  then                                      $\triangleright$  Select driver and verifier
       $D \leftarrow P_1$ 
       $V \leftarrow P_2$ 
    else
       $D \leftarrow P_2$ 
       $V \leftarrow P_1$ 
    end if
    if  $D$  has unexplored paths then
       $I \leftarrow$  new inputs that exercise an unexplored path
       $R_1 \leftarrow \text{run}(P_1, I, S)$ 
       $R_2 \leftarrow \text{run}(P_2, I, S)$ 
      if both  $P_1$  and  $P_2$  stopped then
        if both  $P_1$  and  $P_2$  stopped with an error then
           $\triangleright$  do nothing
        else if either  $P_1$  or  $P_2$  stopped with an error then
          return “disequivalent”
        else
          if  $R_1 \neq R_2$  then
            return “disequivalent”
          end if
        end if
      end if
      mark the path discovered by  $I$  as explored
       $b \leftarrow \neg b$ 
    end if
  end while
  return “possibly equivalent”
end function

```

Fig. 4. Concolic disequivalence algorithm

secutive runs. Every row indicates the input values, as well as the outputs R_D and R_V of the driver and verifier, respectively. At every run, we also record the path taken by the driver. Path conditions are negated to make sure we explore new paths in every iteration. In the fourth iteration, we can see of compare finds that the input pair $\$1 = 1, \$2 = 100$ leads to different outputs in the driver and verifier. At this point, P_1 and P_2 are declared as disequivalent.

Notice that in order to uncover the different, it is necessary to concolically explore the paths in both P_1 and P_2 , and not only of P_1 . In Figure 5, runs 1 and 3 explore both branches of the conditional jump in P_1 , but they exercise the same path in P_2 .

Run	Driver	Verifier	\$1	\$2	Path	R_D	R_V
1	P_1	P_2	1	1	$\$1 \neq 42$	2	2
2	P_2	P_1	1	1	$\$2 \neq 100$	2	2
3	P_1	P_2	42	1	$\$1 = 42$	2	2
4	P_2	P_1	1	100	$\$2 = 100$	201	2

Fig. 5. A sample execution of `compare`

4 Tamarin

Overview.

4.1 Trace Collection

CPU instrumentation, PC concretization, error boxing, and fuel.

4.2 Transformations

Desugaring, simplification, trimming, and conversion to SSA.

4.3 Query Representation

Memory, jumps, arithmetic operators.

4.4 Concolic Execution Redux

Alternation. Compatibility. Soundness/Completeness. Efficiency.

5 Evaluation

6 Related Work

7 Conclusions