

Tamarin: Concolic Disequivalence for MIPS

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Abstract. TODO

1 Introduction

We are staring at two opaque black boxes laying at our feet. Each box has a narrow slot through which we can place items in the box, but we cannot quite see what is inside. They look approximately like this:



We know each box contains an animal, but we do not know which specific animal is in each one. We would like to find out if both boxes contain the same species of animal. Our solution is simple: we take two carrots, and drop one in each box through the slots.

After a while, a chewing sound emerges from the boxes. We peer into them and, indeed, it looks like the carrots were successfully eaten. Triumphant, we declare that the boxes contain the same species of animal. The truth is altogether different:



The boxes are assembly programs. The animals are the functions those programs compute. The carrot is unit testing. The task was to determine whether the programs were equivalent. And we failed at it. In this paper, we show a technique that is better than the carrot.

Program equivalence. The program is the specification. The complications of assembly language.

2 Program Equivalence for MIPS

Let us set up the problem a bit more formally. Consider the set P of MIPS-assembly programs that satisfy two restrictions: they take as inputs only the values of registers \$1 and \$2, and when they stop executing we define their

output to be (exclusively) the value of \$3. Other side effects, such as printing values to the screen, or system calls, are disallowed.

We can now define a relation $\text{equiv} \subseteq P \times P$ (and its complement, $\neg\text{equiv}$) of equivalent programs. Given $P_1, P_2 \in P$, we say that $P_1 \text{ equiv } P_2$ (read “ P_1 is equivalent to P_2 ”) if, for all inputs \$1 and \$2, one of the following holds:

- Both P_1 and P_2 fail during execution (for example, due to a divide-by-zero error).
- P_1 and P_2 stop with the same output in \$3.

For example, the two programs in Figure 1 are equivalent.

<pre># P_1 add \$3, \$1, \$2</pre>	<pre># P_2 add \$4, \$1, \$1 lis \$5 42 sw \$4, 0, \$5 add \$3, \$1, \$2</pre>
------------------------------------	--

Fig. 1. $P_1 \text{ equiv } P_2$

Notice that $P_1 \text{ equiv } P_2$ even though P_2 modifies the contents of the memory and an additional register (\$4), because:

- Both P_1 and P_2 terminate without errors.
- The value of \$3 will be the same when they do so.

Unfortunately, even though equiv captures an already-simplified notion of equivalence¹, a decision procedure for it does not exist, due to Rice’s theorem.

To get decidability back, we define a new class of relations $\text{equiv}_S \subseteq P \times P$ (whose complement is $\neg\text{equiv}_S$). We say that $P_1 \text{ equiv}_S P_2$ (read “ P_1 is S -equivalent to P_2 ”) if, for all inputs, one of the following holds:

- Either P_1 or P_2 does not stop within S steps (we can think of each CPU cycle as one step).
- Both P_1 and P_2 fail.
- Both P_1 and P_2 stop with the same output.

The equiv_S relation captures the notion that we cannot tell P_1 and P_2 apart by running them for at most S steps. Figure 2 shows an example of two programs that are S -equivalent for $S = 10$, but not equivalent. This is the case because P_2 loops while the counter is less than 42, so with 10 steps in our “budget” we will have to stop P_2 before the loop is over and we can observe the different result.

¹ For example, equiv has a very narrow notion of output that excludes side effects.

```

# P_1
add $3, $1, $2

# P_2
add $4, $0, 1 # counter
add $5, $0, 42 # upper bound
loop:
    slt $6, $4, $5
    beq $6, $0, end
    add $4, $4, 1
    beq $0, $0, loop
end:
    add $3, $1, $1

```

Fig. 2. $P_1 \text{ equiv}_{10} P_2$, but $P_1 \not\text{equiv} P_2$

Given a fixed S , the equiv_S relation is decidable because there is a finite number of inputs to try, and for each input we only need to run the programs a finite number of steps.

We already saw that equivalence not always implies S -equivalence. However, the converse always holds. The following lemma shows that equiv_S over-approximates equiv .

Lemma 1. $\forall S, P_1, P_2, P_1 \text{ equiv} P_2 \implies P_1 \text{ equiv}_S P_2$.

Proof. Let $P_1 \text{ equiv} P_2$. Then we have one of two cases:

- Either P_1 or P_2 (or both) do not stop within S steps. Then by definition $P_1 \text{ equiv}_S P_2$.
- Both P_1 and P_2 stop within S steps. Then because they are equivalent, we know that they either fail with an error, or both stop with the same output. In either case, $P_1 \text{ equiv}_S P_2$.

Corollary 1. $P_1 \not\text{equiv}_S P_2 \implies P_1 \not\text{equiv} P_2$.

Proof. This is just the contrapositive of Lemma 1.

Corollary 1 can be used to argue the soundness (with respect to equiv) of any decision procedure that under-approximates equiv_S . In the next section we will show one such under-approximation based on concolic execution.

3 Concolic Disequivalence

We know from Corollary 1 that any relation that under-approximates equiv_S is sound. Figure 3 shows why we want an under-approximation: efficiency. equiv captures the class of programs that are disequivalent, but is undecidable. equiv_S is decidable, but likely cannot be computed efficiently. Therefore, we look for a subset of equiv_S (an under-approximation) that can be efficiently computed.



Fig. 3. Hierarchy of disequivalence relations

To fill the missing relation in Figure 3 we propose concolic disequivalence. Abstractly, concolic disequivalence is a function $\text{compare}(P_1, P_2, S)$ that takes as inputs two MIPS programs and returns one of two answers:

- “disequivalent”, in which case $P_1 \not\text{equiv}_S P_2$.
- “possibly equivalent”, meaning that P_1 and P_2 might or might not be S -equivalent.

Figure 3 shows pseudocode for compare . The algorithm alternately executes P_1 and P_2 . At every step, one of the programs is labelled as the “driver” and the other one as the “verifier”. The driver program is then concolically executed, yielding a set of inputs that exercise a new program path (of the driver). The inputs can then be fed to the verifier, and the results of both driver and verifier compared. If the results are different, then we know P_1 and P_2 are disequivalent. Otherwise, the driver becomes the verifier, and vice-versa. Eventually, we will traverse all explorable paths, at which point P_1 and P_2 can be declared possibly equivalent.

We now give an example of how compare operates. Consider the sample programs below:

```
# P_1                                # P_2
bne $1, 42, end                       add $3, $1, $2
add $3, $0, $0                         bne $2, 100, end
end:                                   add $3, $3, $2
add $3, $1, $2                         end:
```

Figure 5 summarizes the state of the algorithm as it compares P_1 and P_2 . First, notice how the driver and verifier roles flip between P_1 and P_2 in con-

```

function COMPARE( $P_1, P_2, S$ )
   $b \leftarrow true$ 
  while either  $P_1$  or  $P_2$  has unexplored paths do
    if  $b$  then ▷ Select driver and verifier
       $D \leftarrow P_1$ 
       $V \leftarrow P_2$ 
    else
       $D \leftarrow P_2$ 
       $V \leftarrow P_1$ 
    end if
    if  $D$  has unexplored paths then
       $I \leftarrow$  new inputs that exercise an unexplored path
       $R_1 \leftarrow \text{run}(P_1, I, S)$ 
       $R_2 \leftarrow \text{run}(P_2, I, S)$ 
      if both  $P_1$  and  $P_2$  stopped then
        if both  $P_1$  and  $P_2$  stopped with an error then ▷ do nothing
          else if either  $P_1$  or  $P_2$  stopped with an error then
            return “disequivalent”
          else
            if  $R_1 \neq R_2$  then
              return “disequivalent”
            end if
          end if
        end if
        end if
        mark the path discovered by  $I$  as explored
         $b \leftarrow \neg b$ 
      end if
    end while
    return “possibly equivalent”
  end function

```

Fig. 4. Concolic disequivalence algorithm

secutive runs. Every row indicates the input values, as well as the outputs R_D and R_V of the driver and verifier, respectively. At every run, we also record the path taken by the driver. Path conditions are negated to make sure we explore new paths in every iteration. In the fourth iteration, we can see of compare finds that the input pair $\$1 = 1, \$2 = 100$ leads to different outputs in the driver and verifier. At this point, P_1 and P_2 are declared as disequivalent.

Notice that in order to uncover the different, it is necessary to concolically explore the paths in both P_1 and P_2 , and not only of P_1 . In Figure 5, runs 1 and 3 explore both branches of the conditional jump in P_1 , but they exercise the same path in P_2 . Only after we also execute P_2 do we find a counterexample to equivalence.

Run	Driver	Verifier	\$1	\$2	Path	R_D	R_V
1	P_1	P_2	1	1	$\$1 \neq 42$	2	2
2	P_2	P_1	1	1	$\$2 \neq 100$	2	2
3	P_1	P_2	42	1	$\$1 = 42$	2	2
4	P_2	P_1	1	100	$\$2 = 100$	201	2

Fig. 5. A sample execution of `compare`

4 Tamarin

Tamarin² is a Scala implementation of the `compare` algorithm from Section 3. We first give an overview of the major modules in Tamarin, shown in Figure 6, and then describe them in more detail in subsequent sections:

- `Concolic` is the entry point to Tamarin. It implements the top-level loop that visits unexplored paths, alternating between the two programs being compared. `Concolic` uses the other modules as helpers (in Figure 6, requests made by a module appear as solid lines, and responses to prior requests are shown with dashed lines).
- `CPU` is a MIPS emulator that is instrumented to record symbolic traces containing path conditions, which can later be negated to explore new paths.
- `Trace` consumes raw traces coming from `CPU` and transforms them in multiple ways so that they can be handed over to the `Z3` solver.
- `Query` translates (modified) `CPU` traces into equivalent logical formulae. The formulae are then solved by the `Z3` solver, producing new inputs that, if fed to the program under test, will lead to traversing unexplored paths.

² Tamarins are small-sized monkeys from Central and South America. They are related to marmosets, which are also New World monkeys, and less-importantly give name to the black-box submission and testing server in use at the University of Waterloo as of Fall 2017 [6].

- Z3 is an SMT solver developed at Microsoft [4]. We use it as black box for solving queries, over the theories of bitvectors and arrays, that result from program traces.

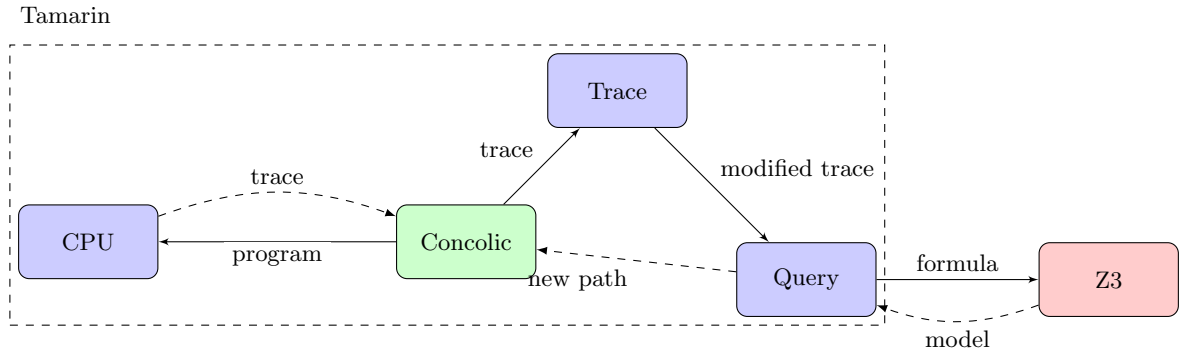


Fig. 6. Overview of Tamarin’s modules

4.1 Trace Collection

The CPU module is in charge of running MIPS programs and collecting symbolic traces from executions. It is based on the MIPS emulator written by Ondřej Lhoták for CS241E at the University of Waterloo [5].

The interface to the module consists of a single function:

```
def run(prog: Seq[Word], r1: Word, r2: Word, fuel: Long): RunRes
```

The `run` function takes as input a program represented as a sequence of words, the values of registers \$1 and \$2 (the inputs to the program) and a `fuel` value (explained below).

The output is an algebraic data type `RunRes` that can take one of three forms:

```
trait RunRes
case class Done(state: State, trace: Trace) extends RunRes
case class NotDone(trace: Trace) extends RunRes
case class Error(ex: RuntimeException) extends RunRes
```

- If the program executes without error, then `Done(state, trace)` is returned. `state` is the state of the CPU after execution, including the contents of memory (which are ignored) and of register \$3, the output register. `trace` is the symbolic trace captured during the program’s execution, and is described below.

- If the program ran for more than `fuel` CPU cycles without stopping, then the result is `NotDone(trace)`. Notice that even though the program did not stop we can still return a trace recording the execution right until the moment we stopped it. The `fuel` argument to `run` plays the same role as the `S` argument in Figure 3.
- Finally, if there was an error during program execution (for example, an attempted division-by-zero), then we return `Error`.

Notice that due to `fuel` parameter and error boxing, the augmented emulator in Tamarin, unlike a vanilla MIPS emulator, is “hardened” in the sense that it can execute MIPS programs that do not stop (the emulator itself will stop) or throw errors (will be caught at the top level by the emulator).

While the emulator is executing a program, it also records a symbolic trace of (most of) the executed instructions. We support a subset of the MIPS instruction set, containing 18 instruction types [1]. Notably, unlike in full MIPS, there are no system calls in our supported subset.

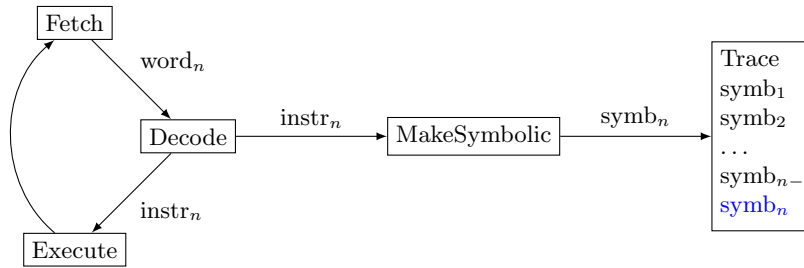


Fig. 7. Instrumented CPU that records symbolic traces

The fetch-decode-execute cycle in our emulator is modified to record a symbolic representation of each instruction as it is executed (Figure 7). The symbolic instructions are then stored in a trace. There are two types of instructions in a trace: assignments and path conditions.

- Assignments are instructions that mutate the CPU state, but do not affect the control flow. The symbolic form of most assignments is very similar to the concrete instruction that is executed. For example, the symbolic representation of the instruction `add $3, $1, $2` is $r_3 \leftarrow r_1 + r_2$.
- Path conditions are instructions that modify the control flow of the program. This set potentially contains both conditional and unconditional jumps, but we track only conditional ones. Specifically, we record the branch taken at each conditional jump, and symbolically record it as an (in)equality. For example, if the branch `beq $1, $2, foolabel` is not taken, then we will add $r_1 \neq r_2$ to our trace.

One additional point of note: the program counter (`pc`) is not symbolically tracked. This means Tamarin cannot reason about unconditional jumps, so only a

single path is explored for the program in Figure 8. There, Tamarin will execute P with the initial inputs (a hard-coded constant) and error out, because the jump will lead to an invalid instruction. Tamarin will miss the fact that $\$1$ and $\$2$ can be given values such that we could jump to either of the labels, leading to a successful execution.

```
# P
  add $4, $1, $2
  jr $4
good:
  add $3, $1, $2
  jr $31 # $31 contains the termination pc
bad:
  add $3, $1, $1
  jr $31
```

Fig. 8. Not tracking the pc leads to under-approximations

Since the pc is not tracked, whenever it is used in another instruction we need to substitute it by its concrete value. This technique is called concretization [3]. For example, the load immediate and skip instruction has the following semantics:

`lis $d` $d \leftarrow Mem[pc]; pc \leftarrow pc + 4$

Instead of the (precise) interpretation above, we use concretization to generate the (under-approximating) symbolic instruction $d \leftarrow v$, where v is the value at the address following the `lis` instruction.

4.2 Transformations

The Trace module also exposes just a single function:

```
def transform(trace: Trace, depth: Int): Trace
```

`transform` takes as input a trace that was produced by CPU and a `depth` parameter (explained below), and modifies the input trace so that it can be later converted into a logical formula. The `depth` parameter limits the number of path conditions in the outputted trace: this is so we can bound the depth of our DFS as we explore new paths (see Section 4.4 for more details). In effect, we can think of traces as the intermediate representation (IR) for queries to the SMT solver. The `Transform` module can be then thought of as lowering the semantic complexity of traces.

Internally, `Transform` consists of multiple phases, each of type `Trace => Trace`, which are sequentially applied to the input. They are described below.

Input	Output
<i>Jalr</i> (<i>concretePC</i>)	<i>Add</i> (\$31, <i>concretePC</i>)
<i>Mult</i> (<i>s</i> , <i>t</i>)	<i>Mult64</i> (<i>tmp</i> , <i>s</i> , <i>t</i>); <i>Low32</i> (<i>lo</i> , <i>tmp</i>); <i>High32</i> (<i>hi</i> , <i>tmp</i>)
<i>Div</i> (<i>s</i> , <i>t</i>)	<i>Quot</i> (<i>lo</i> , <i>s</i> , <i>t</i>); <i>Rem</i> (<i>hi</i> , <i>s</i> , <i>t</i>)
<i>Mflo</i> (<i>d</i>)	<i>Add</i> (<i>d</i> , <i>lo</i> , 0)

Fig. 9. Desugared instructions

Desugaring The desugaring phase removes “complicated” instructions by replacing them with simpler ones. Table 9 summarizes some of the transformations.

A couple of points of interest:

- \$31 holds by convention the return address of the current procedure. Since the *pc* is not tracked, its concretized value is used.
- Notice how some instructions (e.g. *Mult*) are desugared into multiple instructions: this is because they in fact have more than one side effect in the CPU state.
- While desugaring some instructions, we introduce “helper” instructions like *Mult64* and *Quot* that are not part of the MIPS instruction set. These instructions help communicate the intended semantics to the SMT solver. For example, later on we will see that the contents of registers are represented with 32-bit vectors, but multiplication of two 32-bit integers can take up to 64 bits of storage, so we need a way to let the solver know that it should upcast the product of two registers (so we can later decompose it into the low and high 32-bits): the *Mult64* instruction accomplishes this.

Simplification The simplification phase removes trivial path conditions, like *beq* 0, 0, *fooLabel*, which might be common (the specific *beq* example is a common idiom for jumping to a loop header) but do not need to be considered by the SMT solver.

Trimming The trimming phase just shortens the trace so that it contains at most *depth* (an argument to *transform*) path conditions. This is so we can do a bounded DFS (again, see Section 4.4) to limit the search space.

SSA Conversion The SSA phase converts traces into static single assignment (SSA) form [2]. SSA form is crucial to be able to transform a trace into the corresponding logical formula. Consider the example in Figure 10, which shows a small trace together with incorrect and correct translations.

The incorrect translation represents assignment through equality. This is intuitive, but misguided, because assignment mutates state, but equality does not. For example, suppose we append to our trace the path condition $\$3 = 9 \wedge \$1 = 3$. This gets translated to the equality $z = 9 \wedge x = 3$. But the equalities in the incorrect translation imply that $y = 0$ (because $x = x - y$) and so $z = x = y$.

The path condition is then declared as unsatisfiable, but this is clearly wrong, because $\$1 = 6 \wedge \$2 = 3$ is a solution.

What went wrong, and how do we fix it? As we pointed, assignment mutates the value of registers, so we need to somehow encode that the “\$1” in the last *Add* instruction is “not the same” as the one in the first *Add*. This behaviour is precisely captured by SSA form, where each variable is assigned to exactly once. This means that two reads of the same register that are separated by a write to the same register will be encoded as different variables. The correct translation in Figure 10 uses subscripts to refer to the “versions” of each variable after writes. If we add the same path condition as before, we can see that it gets translated as $z_1 = 9 \wedge x_2 = 3$, which is now satisfiable because $x_2 = x_1 - y_1$ does not imply that $x_1 = y_1$.

Trace	Incorrect	Correct
<i>Add</i> (\$3, \$1, \$2)	$z = x + y$	$z_1 = x_1 + y_1$
<i>Sub</i> (\$1, \$1, \$2)	$x = x - y$	$x_2 = x_1 - y_1$
<i>Add</i> (\$2, \$1, \$2)	$y = x + y$	$y_2 = x_2 + y_1$

Fig. 10. A trace together with an incorrect translation to a logical formula and a correct translation using SSA form

The algorithms for efficiently converting code into SSA form are sophisticated [2], requiring the calculation of so-called dominators and ϕ -functions. However, we are only interested in converting traces, which do not contain any jumps. In this case, a simple linear-time pass over the trace suffices for converting to SSA. The algorithm is shown in Figure 11.

```

function SSACONVERT(trace)
  trace'  $\leftarrow \emptyset$ 
   $\forall x. last(x) \leftarrow x_1$ 
  for all instr  $\in$  trace do
    if instr = Add(d, s, t) then
      s'  $\leftarrow last(s)$ 
      t'  $\leftarrow last(t)$ 
      di  $\leftarrow last(d)$ 
      last(d)  $\leftarrow d_{i+1}$ 
      instr'  $\leftarrow Add(d_{i+1}, s', t')$ 
      append(trace', instr')
    else ... ▷ Other cases
    end if
  end for
  return trace'
end function

```

Fig. 11. SSA conversion of traces

4.3 Queries

The Query module is responsible for interfacing with the Z3 solver to determine the satisfiability of path constraints, leading to unexplored program paths.

The API for the module is the function

```
def solve(trace: Trace): Option[Soln]
```

The input to `solve` is a trace that has been processed by `Trace`, likely containing a recently-negated path condition (negated by the Concolic module from Section 4.4). `solve`'s task is to convert the trace into an equivalent logical formula, hand it to Z3, and then return the solution found by the solver.

A `Soln` is just a `Seq[RegVal]` (`solve` returns an `Option[Soln]` in case the formula is unsatisfiable), and a `RegVal` is an algebraic datatype of the form

```
case object Unbound extends RegVal
case class Fixed(v: Long) extends RegVal
```

A `RegVal` represents the value of a register in the model returned by the solver. `Unbound` represents the case when the solver does not constrain a register in its solution. By contrast, `Fixed` is used when the solver requires a specific value.

program:	trace:	formula:
<pre>add \$3, \$1, \$2 slt \$4, \$1, \$2 lis \$5 1 beq \$4, \$5, skip add \$6, \$3, \$1 skip:</pre>	<pre>add \$3, \$1, \$2 slt \$4, \$1, \$2 lis \$5 1 \$4 == \$5</pre>	<pre>(declare-const r1 (_ BitVec 32)) (declare-const r2 (_ BitVec 32)) (declare-const r3 (_ BitVec 32)) (declare-const r4 (_ BitVec 32)) (declare-const r5 (_ BitVec 32)) (assert (= r3 (bvadd r1 r2))) (assert (= r4 (ite (bvslt r1 r2) (_ bv1 32) (_ bv0 32)))) (assert (= r5 (_ bv1 32))) (assert (not (= r4 r5))) (check-sat) (get-model)</pre>

Fig. 12. A sample program, a trace in it, and the trace's representation as a logical formula

Translation

Memory Representation

4.4 Concolic Execution Redux

Alternation. Compatibility. Soundness/Completeness. Efficiency.

5 Evaluation

6 Related Work

7 Conclusions

References

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