ALGUNAS SUMATORIAS

1) $\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$

2) $\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$

3) $\sum_{j=m}^{n} j = \sum_{j=1}^{n} j - \sum_{j=1}^{m-1} j = \frac{n(n+1)}{2} - \frac{m(m-1)}{2}$

4) $\sum_{i=2}^{n} \lfloor \log_2 i \rfloor = \left(n - 2^{\lfloor \log_2 n \rfloor} + 1\right) \lfloor \log_2 n \rfloor + \sum_{j=1}^{\lfloor \log_2 n \rfloor - 1} j * 2^j$

5) $\sum_{j=0}^{n} 2^j = 2^{n+1} - 1$

6) $\sum_{j=0}^{n} a^j = \frac{a^{n+1} - 1}{a - 1}$

7) $\sum_{j=1}^{n} j2^j = (n-1)2^{n+1} + 2$

8) $\sum_{j=0}^{m} \sum_{i=j}^{k} aj = a\left((k+1)\frac{m(m+1)}{2} - \frac{m(m+1)(2m+1)}{6} \right)$

9) $\sum_{i=0}^{n} \left\lfloor \frac{i}{2} \right\rfloor = \frac{n^2 - 1}{4}$

10) $\sum_{j=0}^{n} \frac{1}{2^j} = 2 - \frac{1}{2^n}$

11) $\sum_{j=0}^{n} ax^j = a\left(\frac{1 - x^{n+1}}{1 - x} \right)$

12)

$$S = \sum_{j=1}^{n} jx^j = x + \sum_{j=2}^{n} jx^j$$

$$S = x + x\sum_{j=2}^{n} jx^{j-1} = x + x\sum_{j=1}^{n-1} (j+1)x^j$$

$$S = x + x\sum_{j=1}^{n} jx^j + x\sum_{j=1}^{n-1} x^j - x^{n+1}n$$

$$S = x + xS + x\left(\frac{1-x^n}{1-x}\right) - x^{n+1}n$$

$$S - xS = x + x\left(\frac{1-x^n}{1-x}\right) - x^{n+1}n$$

$$S(1-x) = x + x\left(\frac{1-x^n}{1-x}\right) - x^{n+1}n$$

$$S = \frac{x + x\left(\frac{1-x^n}{1-x}\right) - x^{n+1}n}{(1-x)}$$