CS 491: Introduction to Machine Learning
Spring 2015
Final Exam

Name: ____________________

User ID: ____________________

Instructions:

1. Write your name and user ID above. Do not begin the exam (look at other pages) until told to do so.
2. There should be 7 pages. Count the pages (without looking at the questions).
3. Do not discuss the exam with students who have not taken the exam!

Some useful formulas:

- $P(\mathbf{x}) = \sum_{\mathbf{y}} P(\mathbf{x}, \mathbf{y})$ (marginalization)
- $P(\mathbf{x}|\mathbf{y}) = \frac{P(\mathbf{x},\mathbf{y})}{P(\mathbf{y})}$ (conditioning)
- $P(\mathbf{x}|\mathbf{y}) = \frac{P(\mathbf{y}|\mathbf{x})P(\mathbf{x})}{P(\mathbf{y})}$ (Bayes theorem)
- $\mathbb{E}_{x\sim P}[f(X)] = \sum_x P(x)f(x)$ (Expectation)
- $X \sim \text{Normal}(\mu, \sigma) \implies P(X = x|\mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
- $X \sim \text{Multinoulli}(\theta) \implies P(\mathbf{x}) = \prod_{i=1}^{K} \theta_i^{x_i}$

	Points
Q1	/20
Q2	/20
Q3	/20
Q4	/20
Q5	/20
Total	

Q1. True-False questions (5 questions, 20 points total)

Circle correct answer. Give one sentence explanation or small picture.

Q1.1: (4 points) X independent of Y ($X \perp Y$) and Y independent of Z ($Y \perp Z$) implies that X is independent of Z ($X \perp Z$).

True / False. Explanation: $X = Z$ for example

Q1.2: (4 points) Maximum a posteriori (MAP) estimation averages over the posterior parameter distribution to make predictions for new data.

True / False. Explanation: Full Bayesian $\int_\theta P(\theta | D)\, P(D' | \theta)\, d\theta$

MAP $\arg\max_\theta P(\theta | D)$

Q1.3: (4 points) A logistic regression model with L_1 regularization will have lower training log loss / higher log likelihood than the same logistic regression model without regularization.

True / False. Explanation: $\max_\theta \ell(\theta; D) - \lambda \|\theta\|_1$

Q1.4: (4 points) The normalization term of a Markov random field, $\sum_{\mathbf{x}} e^{\psi(\mathbf{x})}$, with a tree graph structure connecting all n random variables can be computed in time linear in n.

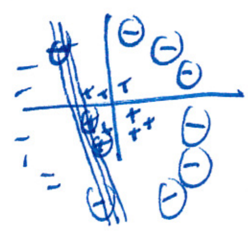
True / False. Explanation:

we didn't cover this semester

Q1.5: (4 points) Consider a linear SVM with k support vectors. The SVM using polynomial kernel with degree $n \geq 2$ trained from the same data must have at least k support vectors.

True / False. Explanation:

$\begin{pmatrix} x_1 \\ x_2 \\ \cancel{x_3} \end{pmatrix}$ many support vectors



$\begin{pmatrix} x_1^4 \\ x_2^4 \end{pmatrix}$ few support vectors

Q2. Short Answer (3 questions, 20 points total)

Q2.1: (6 points) Which model is more prone to overfitting: a decision tree of depth k or a logistic regression model with k features? Why?

$K=10$

Decision Trees, much greater complexity

Q2.2: (8 points) Consider a binary logistic regression model, $P(Y=1|x) = \frac{2^{w_2x_2+w_1x_1+w_0}}{2^{w_2x_2+w_1x_1+w_0}+1}$, that provides predictions of: $P(Y=1|X_1=2, X_2=0) = 0.5$, $P(Y=1|X_1=0, X_2=3) = 0.5$, and $P(Y=1|X_1=0, X_2=0) = \frac{2}{3}$. What are the values of w_0, w_1, and w_2?

$$w_2 \cdot 0 + w_1 \cdot 2 + w_0 = 0$$
$$w_2 \cdot 3 + w_1 \cdot 0 + w_0 = 0$$
$$w_2 \cdot 0 + w_1 \cdot 0 + w_0 = 1$$

$w_0 =$ 1

$w_1 =$ $-1/2$

$w_2 =$ $-1/3$

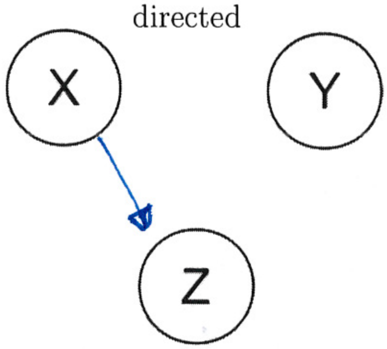
Q2.3: (6 points) Consider the expectation-maximization algorithm for Naïve Bayes with m unlabeled examples and a single input x. Let $\alpha_i(y)$ represent the expectation step's computation of $P(Y=y|x)$. What is the M step's conditional probability estimate for $\hat{P}(x|y)$ in terms of $\alpha_i(y)$ and training data? (Hint: for labeled data, $\hat{P}(x|y) = \frac{\sum_{i=1}^{m} I(X_i=x_i \wedge Y_i=y_i)}{\sum_{i=1}^{m} I(Y_i=y_i)}$, where $I(\cdot)$ is an indicator function returning 1 when the function is true and 0 otherwise.)

$$\hat{P}(x|y) = \frac{\sum_{i=1}^{m} I(X_i=x)\,\alpha_i(y)}{\sum_{i=1}^{m} \alpha_i(y)}$$

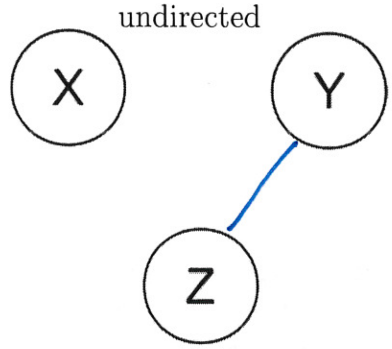
Q3. Graphical Models (20 points total)

Q3.1: **(5 points)** Given three random variables, X, Y, and Z, draw a **directed graphical model (Bayesian network)** (left) and an **undirected graphical model (Markov random field)** (right) that share at least one (conditional) independence property and each have at least one unique (conditional) independence property.

directed



undirected



Q3.2: **(4 points)** Write a (conditional) independence property that both graphical models share (e.g., $X \perp Z|Y$):

$X \perp Y$

Q3.3: **(4 points)** Write a (conditional) independence property that the directed graphical model possesses that the undirected graphical model does not:

$Y \perp Z$

Q3.4: **(4 points)** Write a (conditional) independence property that the undirected graphical model possesses that the directed graphical model does not:

$X \perp Z$

Q3.5: **(4 points)** Assuming that X, Y, and Z are binary-valued. Write a joint probability distribution that corresponds to your undirected graphical model above and has no additional independence properties.

$$P(X, Y, Z) = P(X)\, P(Y, Z)$$

X	Y	Z	P(X)	P(Y, Z)	P(X, Y, Z)
0	0	0	.4	.1	.04
0	0	1	.4	.2	.08
0	1	0	.4	.3	.12
0	1	1	.4	.4	.16
1	0	0	.6	.1	.06
1	0	1	.6	.2	.12
1	1	0	.6	.3	.18
1	1	1	.6	.4	.24

Q4. Support Vector Machines (20 points)

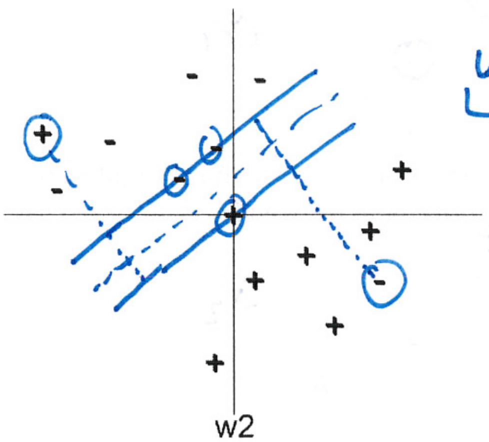
Q4.1 Linear SVM Consider the linear support vector machine,

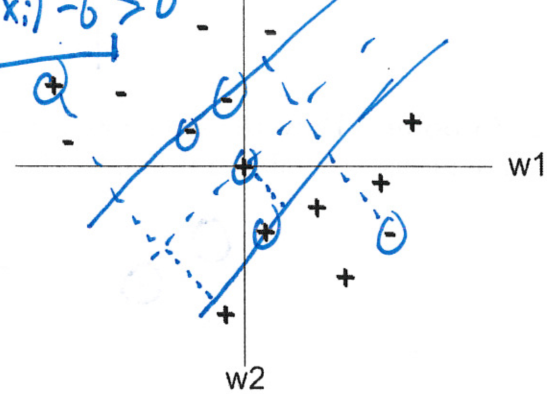
$$\min_{\mathbf{w},b,\xi\geq 0} ||\mathbf{w}||^2 + C\sum_{i=1}^{n} \xi_i \text{ such that: } \underbrace{y_i(\mathbf{w}\cdot\mathbf{x}_i - b)} \geq 1-\xi_i \qquad \forall i \in \{1, ..., n\},$$

on the following dataset:

$w \cdot x_i - b > 0$ → $\hat{y} = 1$, → $\hat{y} = -1$

$\underbrace{w \cdot \phi(x_i) - b} > 0$





(a) **(5 points)** On the left, plot the decision boundary and margin boundaries when C is very large (as C goes to infinity) and <u>circle</u> the examples serving as support vectors.

(b) **(5 points)** On the right, plot the decision boundary and margin boundaries when C is more moderate and <u>circle</u> the examples serving as support vectors.

Q4.2 Kernelization Consider using a feature function ϕ that transforms from the original input space (size $|X|$) to a feature space of size $|F|$ with a corresponding kernel function K that requires k time to evaluate. Further, assume that the size of the training data is m and the number of support vectors is $|S|$.

(a) **(5 points)** What is the computational complexity of making a prediction for a new example $\mathbf{x}$ using the linear support vector machine <u>without</u> the kernel trick?

$\underbrace{\vec{w} \cdot \phi(\vec{x}_i)} - b$

$O(|F|)$ time

(b) **(5 points)** What is the computational complexity of making a prediction for a new example $\mathbf{x}$ using the linear support vector machine <u>with</u> the kernel trick?

$\vec{w} = \sum_j \alpha_j \phi(\vec{x}_j)$ $\qquad$ $\vec{w} \cdot \phi(\vec{x}_i) - b$

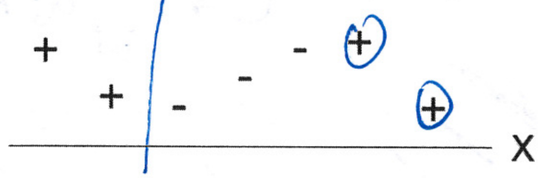
$= \sum_j \alpha_j \phi(\vec{x}_j) \cdot \phi(\vec{x}_i) - b$

$O(k|S|)$ $\qquad$ $= \sum_j \alpha_j K(\vec{x}_j, \vec{x}) - b$

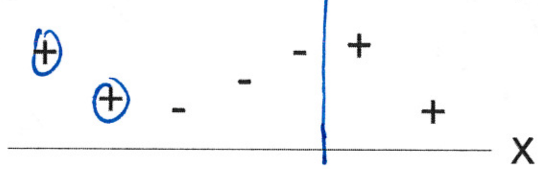
Q5. Boosting (20 points)

Consider the AdaBoost algorithm with threshold functions ($x_i > c \implies x_i =$ '+' and '-' otherwise or with labels reversed) as weak classifiers.

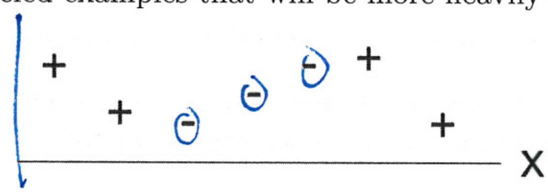
Q5.1: (5 points) Draw the first weak classifier learned on the following dataset. Circle the incorrectly labeled examples that will be more heavily weighted.



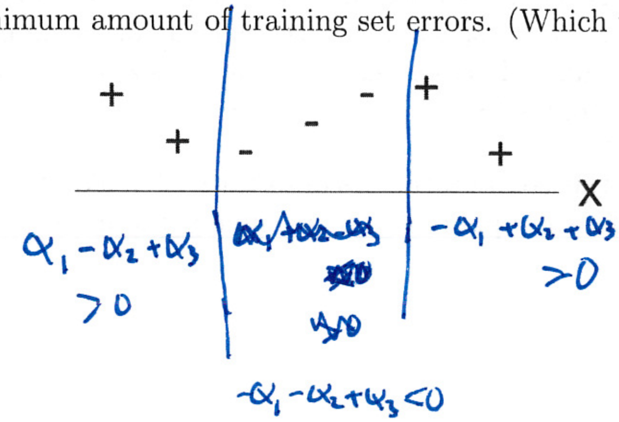
Q5.2: (5 points) Draw the second weak classifier learned on the resulting weighted dataset. Circle the incorrectly labeled examples that will be more heavily weighted.



Q5.3: (5 points) Draw the third weak classifier learned on the resulting weighted dataset. Circle the incorrectly labeled examples that will be more heavily weighted.



Q5.3: (5 points) Draw the decision functions and combined weights for each region that would produce a minimum amount of training set errors. (Which will be larger, α_1 or α_2?)



$\alpha_1 = .3$

$\alpha_2 = .4$ (for example)

$\alpha_3 = .2$