Chapter 2: Association Rules & Sequential Patterns

Road map

- Basic concepts of Association Rules
- Apriori algorithm
- Different data formats for mining
- Mining with multiple minimum supports
- Mining class association rules
- Sequential pattern mining
- Summary

Association rule mining

- Proposed by Agrawal et al in 1993.
- It is an important data mining model studied extensively by the database and data mining community.
- Assume all data are categorical.
- No good algorithm for numeric data.
- Initially used for Market Basket Analysis to find how items purchased by customers are related.

Bread $\rightarrow$ Milk [sup = 5%, conf = 100%]

The model: data

- $I = \{i_1, i_2, \ldots, i_m\}$: a set of *items*.
- Transaction *t* :
  - *t* a set of items, and $t \subseteq I$.
- Transaction Database *T*: a set of transactions $T = \{t_1, t_2, \ldots, t_n\}$.

Transaction data: supermarket data

- Market basket transactions:

  t1: {bread, cheese, milk}

  t2: {apple, eggs, salt, yogurt}

  … …

  tn: {biscuit, eggs, milk}

- Concepts:
  - An *item*: an item/article in a basket
  - *I*: the set of all items sold in the store
  - A *transaction*: items purchased in a basket; it may have TID (transaction ID)
  - A *transactional dataset*: A set of transactions

Transaction data: a set of documents

- **A text document data set. Each document is treated as a "bag" of keywords**

doc1:	Student, Teach, School
doc2:	Student, School
doc3:	Teach, School, City, Game
doc4:	Baseball, Basketball
doc5:	Basketball, Player, Spectator
doc6:	Baseball, Coach, Game, Team
doc7:	Basketball, Team, City, Game

The model: rules

- A transaction *t* contains *X*, a set of items (itemset) in *I*, if $X \subseteq t$.
- An association rule is an implication of the form:

$$X \to Y, \text{ where } X, Y \subset I, \textit{ and } X \cap Y = \emptyset$$

- An itemset is a set of items.
  - E.g., X = {milk, bread, cereal} is an itemset.
- A *k*-itemset is an itemset with *k* items.
  - E.g., {milk, bread, cereal} is a 3-itemset

Rule strength measures

- Support: The rule holds with support *sup* in *T* (the transaction data set) if sup % of transactions contain $X \cup Y$.
- Confidence: The rule holds in *T* with confidence *conf* if *conf* % of transactions that contain *X* also contain *Y.*
- An association rule is a pattern that states when *X* occurs, *Y* occurs with certain probability.

Support and Confidence

- Support count: The support count of an itemset *X*, denoted by *X.count*, in a data set *T* is the number of transactions in *T* that contain *X*. Assume *T* has *n* transactions.
- Then,

$$support = \frac{(X \cup Y).count}{n}$$

$$confidence = \frac{(X \cup Y).count}{X.count}$$

Goal and key features

- **Goal:** Find all rules that satisfy the user-specified *minimum support* (minsup) and *minimum confidence* (minconf).
- **Key Features**
  - Completeness: find all rules.
  - No target item(s) on the right-hand-side
  - Mining with data on hard disk (not in memory)

An example

t1:	Beef, Chicken, Milk
t2:	Beef, Cheese
t3:	Cheese, Boots
t4:	Beef, Chicken, Cheese
t5:	Beef, Chicken, Clothes, Cheese, Milk
t6:	Chicken, Clothes, Milk
t7:	Chicken, Milk, Clothes

- Transaction data
- Assume:

  minsup = 30%

  minconf = 80%
- An example frequent *itemset*:

  {Chicken, Clothes, Milk}   [sup = 3/7]
- Association rules from the itemset:

  $\text{Clothes} \rightarrow \text{Milk, Chicken}$ [sup = 3/7, conf = 3/3]

  …   …

  $\text{Clothes, Chicken} \rightarrow \text{Milk,}$ [sup = 3/7, conf = 3/3]

Transaction data representation

- A simplistic view of shopping baskets,
- Some important information not considered. E.g,
  - the quantity of each item purchased and
  - the price paid.

Many mining algorithms

- There are a large number of them!!
- They use different strategies and data structures.
- Their resulting sets of rules are all the same.
  - Given a transaction data set *T*, and a minimum support and a minimum confident, the set of association rules existing in *T* is uniquely determined.
- Any algorithm should find the same set of rules although their computational efficiencies and memory requirements may be different.
- We study only one: the Apriori Algorithm

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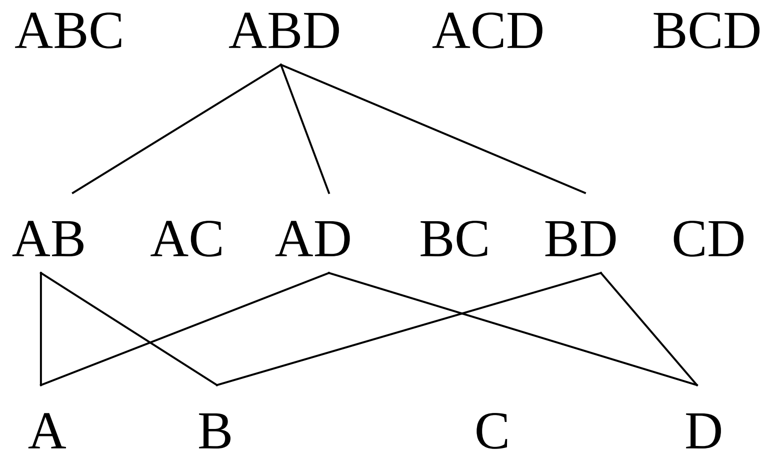
The Apriori algorithm

- **The best known algorithm**
- **Two steps**:
  - Find all itemsets that have minimum support (*frequent itemsets*, also called large itemsets).
  - Use frequent itemsets to generate rules.

- E.g., a frequent itemset
  {Chicken, Clothes, Milk} [sup = 3/7]
  and one rule from the frequent itemset
  Clothes $\rightarrow$ Milk, Chicken [sup = 3/7, conf = 3/3]

Step 1: Mining all frequent itemsets

- A frequent *itemset* is an itemset whose support is ≥ minsup.
- Key idea: The apriori property (downward closure property): any subset of a frequent itemset is also a frequent itemset



The Algorithm

- Iterative algo. (also called level-wise search): Find all 1-item frequent itemsets; then all 2-item frequent itemsets, and so on.
  - In each iteration *k*, only consider itemsets that contain some *k*-1 frequent itemset.

- Find frequent itemsets of size 1: F_1
- From $k = 2$
  - C_k = candidates of size *k*: those itemsets of size *k* that could be frequent, given F_{k-1}
  - F_k = those itemsets that are actually frequent, $F_k \subseteq C_k$ (need to scan the database once).

Example – Finding frequent itemsets

Dataset T

minsup=0.5

TID	Items
T100	1, 3, 4
T200	2, 3, 5
T300	1, 2, 3, 5
T400	2, 5

itemset:count

1. scan T ➔ C_1: {1}:2, {2}:3, {3}:3, {4}:1, {5}:3

➔ F_1: {1}:2, {2}:3, {3}:3, {5}:3

➔ C_2: {1,2}, {1,3}, {1,5}, {2,3}, {2,5}, {3,5}

2. scan T ➔ C_2: {1,2}:1, {1,3}:2, {1,5}:1, {2,3}:2, {2,5}:3, {3,5}:2

➔ F_2: **{1,3}**:2, **{2,3}**:2, **{2,5}**:3, **{3,5}**:2

➔ C_3: {2, 3,5}

3. scan T ➔ C_3: **{2, 3, 5}**:2 ➔ F_3: **{2, 3, 5}**

Details: ordering of items

- The items in *I* are sorted in lexicographic order (which is a total order).
- The order is used throughout the algorithm in each itemset.
- {$w[1]$, $w[2]$, …, $w[k]$} represents a k-itemset w consisting of items $w[1]$, $w[2]$, …, $w[k]$, where $w[1] < w[2] < … < w[k]$ according to the total order.

Details: the algorithm

```
Algorithm Apriori($T$)
    $C_1 \leftarrow$ init-pass($T$);
    $F_1 \leftarrow \{f \mid f \in C_1, f.\text{count}/n \geq minsup\}$;    // n: no. of transactions in T
    for ($k = 2$; $F_{k-1} \neq \emptyset$; $k$++) do
    $C_k \leftarrow$ candidate-gen($F_{k-1}$);
    for each transaction $t \in T$ do
        for each candidate $c \in C_k$ do
    if $c$ is contained in $t$ then
       $c.count$++;
        end
    end
          $F_k \leftarrow \{c \in C_k \mid c.count/n \geq minsup\}$
    end
return $F \leftarrow \cup_k F_k$;
```

Apriori candidate generation

- The candidate-gen function takes F_{k-1} and returns a superset (called the candidates) of the set of all frequent k-itemsets. It has two steps
  - *join* step: Generate all possible candidate itemsets C_k of length k
  - *prune* step: Remove those candidates in C_k that cannot be frequent.

Candidate-gen function

```
Function candidate-gen($F_{k-1}$)
    $C_k \leftarrow \emptyset$;
    forall $f_1, f_2 \in F_{k-1}$
    with $f_1 = \{i_1, \ldots, i_{k-2}, i_{k-1}\}$
    and $f_2 = \{i_1, \ldots, i_{k-2}, i'_{k-1}\}$
    and $i_{k-1} < i'_{k-1}$ do
        $c \leftarrow \{i_1, \ldots, i_{k-1}, i'_{k-1}\}$;          // join $f_1$ and $f_2$
        $C_k \leftarrow C_k \cup \{c\}$;
        for each ($k$-1)-subset $s$ of $c$ do
    if ($s \notin F_{k-1}$) then
        delete $c$ from $C_k$;  // prune
        end
    end
    return $C_k$;
```

An example

- F_3 = {{1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {1, 3, 5}, {2, 3, 4}}

- After join
  - C_4 = {{1, 2, 3, 4}, {1, 3, 4, 5}}
- After pruning:
  - C_4 = {{1, 2, 3, 4}}

    because {1, 4, 5} is not in F_3 ({1, 3, 4, 5} is removed)

Step 2: Generating rules from frequent itemsets

- Frequent itemsets $\neq$ association rules
- One more step is needed to generate association rules
- For each frequent itemset *X*,

  For each proper nonempty subset *A* of *X*,
  - Let *B* = X - *A*
  - A $\rightarrow$ B is an association rule if
    - Confidence(A $\rightarrow$ B) $\geq$ minconf,

      support(A $\rightarrow$ B) = support(A$\cup$B) = support(X)

      confidence(A $\rightarrow$ B) = support(A $\cup$ B) / support(A)

Generating rules: an example

- Suppose {2,3,4} is frequent, with sup=50%
  - Proper nonempty subsets: {2,3}, {2,4}, {3,4}, {2}, {3}, {4}, with sup=50%, 50%, 75%, 75%, 75%, 75% respectively
  - These generate these association rules:
    - 2,3 $\rightarrow$ 4, confidence=100%
    - 2,4 $\rightarrow$ 3, confidence=100%
    - 3,4 $\rightarrow$ 2, confidence=67%
    - 2 $\rightarrow$ 3,4, confidence=67%
    - 3 $\rightarrow$ 2,4, confidence=67%
    - 4 $\rightarrow$ 2,3, confidence=67%
    - All rules have support = 50%

Generating rules: summary

- To recap, in order to obtain A → B, we need to have support(A ∪ B) and support(A)
- All the required information for confidence computation has already been recorded in itemset generation. No need to see the data *T* any more.
- This step is not as time-consuming as frequent itemsets generation.

On Apriori Algorithm

Seems to be very expensive

- Level-wise search
- K = the size of the largest itemset
- It makes at most K passes over data
- In practice, K is bounded (10).
- The algorithm is very fast. Under some conditions, all rules can be found in linear time.
- Scale up to large data sets

More on association rule mining

- Clearly the space of all association rules is exponential, $O(2^m)$, where m is the number of items in *I*.
- The mining exploits sparseness of data, and high minimum support and high minimum confidence values.
- Still, it always produces a huge number of rules, thousands, tens of thousands, millions, ...

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Different data formats for mining

- The data can be in transaction form or table form

Transaction form:

a, b
a, c, d, e
a, d, f

Table form:

Attr1	Attr2	Attr3
a,	b,	d
b,	c,	e

- Table data need to be converted to transaction form for association mining

From a table to a set of transactions

Table form:

Attr1	Attr2	Attr3
a,	b,	d
b,	c,	e

⇒ Transaction form:

(Attr1, a), (Attr2, b), (Attr3, d)

(Attr1, b), (Attr2, c), (Attr3, e)

candidate-gen can be slightly improved. Why?

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Problems with the association mining

- Single minsup: It assumes that all items in the data are of the same nature and/or have similar frequencies.
- Not true: In many applications, some items appear very frequently in the data, while others rarely appear.

  E.g., in a supermarket, people buy *food processor* and *cooking pan* much less frequently than they buy *bread* and *milk*.

Rare Item Problem

- If the frequencies of items vary a great deal, we will encounter two problems
  - If minsup is set too high, those rules that involve rare items will not be found.
  - To find rules that involve both frequent and rare items, minsup has to be set very low. This may cause combinatorial explosion because those frequent items will be associated with one another in all possible ways.

Multiple minsups model

- The minimum support of a rule is expressed in terms of *minimum item supports* (MIS) of the items that appear in the rule.
- Each item can have a minimum item support.
- By providing different MIS values for different items, the user effectively expresses different support requirements for different rules.
- To prevent very frequent items and very rare items from appearing in the same itemsets, we introduce a **support difference constraint**.

  $max_{i\in s}\{sup(\mathrm{i})\} - min_{i\in s}\{sup(i)\} \leq \varphi,$

Minsup of a rule

- Let MIS(*i*) be the MIS value of item *i*. The *minsup* of a rule *R* is the lowest MIS value of the items in the rule.
- I.e., a rule *R*: $a_1, a_2, \ldots, a_k \rightarrow a_{k+1}, \ldots, a_r$ satisfies its minimum support if its actual support is $\geq$

  $\min(\text{MIS}(a_1), \text{MIS}(a_2), \ldots, \text{MIS}(a_r))$.

An Example

- Consider the following items:

  bread, *shoes*, *clothes*

  The user-specified MIS values are as follows:

  MIS(*bread*) = 2%   MIS(*shoes*) = 0.1%

  MIS(*clothes*) = 0.2%

  The following rule doesn't satisfy its minsup:

  clothes → *bread* [sup=0.15%,conf =70%]

  The following rule satisfies its minsup:

  clothes → *shoes* [sup=0.15%,conf =70%]

Downward closure property

- In the new model, the property no longer holds (?)

E.g., Consider four items 1, 2, 3 and 4 in a database. Their minimum item supports are

MIS(1) = 10%	MIS(2) = 20%
MIS(3) = 5%	MIS(4) = 6%

{1, 2} with support 9% is infrequent, but {1, 2, 3} and {1, 2, 4} could be frequent.

To deal with the problem

- We sort all items in *I* according to their MIS values (make it a total order).
- The order is used throughout the algorithm in each itemset.
- Each itemset *w* is of the following form:

  {w[1], w[2], …, w[k]}, consisting of items,

  w[1], w[2], …, w[k],

  where MIS(w[1]) $\leq$ MIS(w[2]) $\leq$ … $\leq$ MIS(w[k]).

The MSapriori algorithm

```
Algorithm MSapriori(T, MS, φ)          // φ is for support difference constraint
    M ← sort(I, MS);
    L ← init-pass(M, T);
    $F_1 \leftarrow \{\{i\} \mid i \in L, i.count/n \geq MIS(i)\}$;
    for ($k = 2$; $F_{k-1} \neq \emptyset$; $k$++) do
    if $k$=2 then
       $C_k \leftarrow$ level2-candidate-gen($L$, $\varphi$)
    else $C_k \leftarrow$ MScandidate-gen($F_{k-1}$, $\varphi$);
    end;
    for each transaction $t \in T$ do
        for each candidate $c \in C_k$ do
             if $c$ is contained in $t$ then
     c.count++;
             if $c - \{c[1]\}$ is contained in $t$ then
    c.tailCount++
        end
    end
           $F_k \leftarrow \{c \in C_k \mid c.count/n \geq MIS(c[1])\}$
    end
    return $F \leftarrow \cup_k F_k$;
```

Candidate itemset generation

- Special treatments needed:
  - Sorting the items according to their MIS values
  - First pass over data (the first three lines)
    - Let us look at this in detail.
  - Candidate generation at level-2
    - Read it in the handout.
  - Pruning step in level-*k* (*k* > 2) candidate generation.
    - Read it in the handout.

First pass over data

- It makes a pass over the data to record the support count of each item.
- It then follows the sorted order to find the first item *i* in *M* that meets MIS(*i*).
  - *i* is inserted into *L*.
  - For each subsequent item *j* in *M* after *i*, if *j.count*/*n* $\geq$ MIS(*i*) then *j* is also inserted into *L*, where *j.count* is the support count of *j* and *n* is the total number of transactions in *T*. Why?
- *L* is used by function level2-candidate-gen

First pass over data: an example

- Consider the four items 1, 2, 3 and 4 in a data set. Their minimum item supports are:

  MIS(1) = 10%   MIS(2) = 20%

  MIS(3) = 5%   MIS(4) = 6%

- Assume our data set has 100 transactions. The first pass gives us the following support counts:

  {3}.*count* = 6, {4}.*count* = 3,

  {1}.*count* = 9, {2}.*count* = 25.

- **Then** L = {3, 1, 2}, and F_1 = {{3}, {2}}
- Item 4 is not in L because 4.*count*/*n* < MIS(3) (= 5%),
- {1} is not in F_1 because 1.*count*/*n* < MIS(1) (= 10%).

Rule generation

- The following two lines in MSapriori algorithm are important for rule generation, which are not needed for the Apriori algorithm

```
if c – {c[1]} is contained in t then
    c.tailCount++
```

- Many rules cannot be generated without them.
- Why?

On multiple minsup rule mining

- Multiple minsup model subsumes the single support model.
- It is a more realistic model for practical applications.
- The model enables us to find rare item rules yet without producing a huge number of meaningless rules with frequent items.
- By setting MIS values of some items to 100% (or more), we effectively instruct the algorithms not to generate rules only involving these items.

Project 1: Implementation

- Implement: Msapriori (excluding rule generation)
  - Consider: multiple minimum supports, support difference constraint, and item constraints
- Item constraints: Two types
  - Cannot–be-together: sets of items cannot be in the same itemset,
    - e.g., {1, 2} and {6, 7}
  - Must-have: every itemset must have,
    - e.g., (1 or 2)
- Deadline: Sept. 28, 2017

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Mining class association rules (CAR)

- Normal association rule mining does not have any target.
- It finds all possible rules that exist in data, i.e., any item can appear as a consequent or a condition of a rule.
- However, in some applications, the user is interested in some targets.
  - E.g, the user has a set of text documents from some known topics. He/she wants to find out what words are associated or correlated with each topic.

Problem definition

- Let T be a transaction data set consisting of n transactions.
- Each transaction is also labeled with a class y.
- Let I be the set of all items in T, Y be the set of all class labels and $I \cap Y = \emptyset$.
- A **class association rule** (**CAR**) is an implication of the form

  $X \to y$, where $X \subseteq I$, and $y \in Y$.
- The definitions of **support** and **confidence** are the same as those for normal association rules.

An example

- **A text document data set**

doc 1:	Student, Teach, School	: Education
doc 2:	Student, School	: Education
doc 3:	Teach, School, City, Game	: Education
doc 4:	Baseball, Basketball	: Sport
doc 5:	Basketball, Player, Spectator	: Sport
doc 6:	Baseball, Coach, Game, Team	: Sport
doc 7:	Basketball, Team, City, Game	: Sport

- Let *minsup* = 20% and *minconf* = 60%. The following are two examples of class association rules:

  Student, School → Education [sup= 2/7, conf = 2/2]

  game → Sport [sup= 2/7, conf = 2/3]

Mining algorithm

- Unlike normal association rules, CARs can be mined directly in one step.
- The key operation is to find all **ruleitems** that have support above *minsup*. A **ruleitem** is of the form:

  $(condset, y)$

  where **condset** is a set of items from I (*i.e.*, $condset \subseteq I$), and $y \in Y$ is a class label.
- Each ruleitem basically represents a rule:

  $condset \rightarrow y$,
- The Apriori algorithm can be modified to generate CARs

Multiple minimum class supports

- The multiple minimum support idea can also be applied here.
- The user can specify different minimum supports to different classes, which effectively assign a different minimum support to rules of each class.
- For example, we have a data set with two classes, Yes and No. We may want
  - rules of class Yes to have the minimum support of 5% and
  - rules of class No to have the minimum support of 10%.
- By setting minimum class supports to 100% (or more for some classes), we tell the algorithm not to generate rules of those classes.
  - This is a very useful trick in applications.

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Sequential pattern mining

- Association rule mining does not consider the order of transactions.
- In many applications such orderings are significant. E.g.,
  - in market basket analysis, it is interesting to know whether people buy some items in sequence,
    - e.g., buying bed first and then bed sheets some time later.
  - In Web usage mining, it is useful to find navigational patterns of users in a Web site from sequences of page visits of users

Basic concepts

- Let $I = \{i_1, i_2, \ldots, i_m\}$ be a set of items.
- **Sequence:** An ordered list of itemsets.
- **Itemset/element**: A non-empty set of items $X \subseteq I$. We denote a sequence s by $\langle a_1 a_2 \ldots a_r \rangle$, where a_i is an itemset, which is also called an **element** of s.
- An element (or an itemset) of a sequence is denoted by $\{x_1, x_2, \ldots, x_k\}$, where $x_j \in I$ is an item.
- We assume without loss of generality that items in an element of a sequence are in **lexicographic order**.

Basic concepts (contd)

- **Size**: The **size** of a sequence is the number of elements (or itemsets) in the sequence.
- **Length**: The **length** of a sequence is the number of items in the sequence.
  - A sequence of length *k* is called ***k*-sequence**.
- A sequence $s_1 = \langle a_1 a_2 \ldots a_r \rangle$ is a **subsequence** of another sequence $s_2 = \langle b_1 b_2 \ldots b_v \rangle$, or s_2 is a **supersequence** of s_1, if there exist integers $1 \leq j_1 < j_2 < \ldots < j_{r-1} < j_r \leq v$ such that $a_1 \subseteq b_{j1}$, $a_2 \subseteq b_{j2}$, …, $a_r \subseteq b_{jr}$. We also say that s_2 **contains** s_1.

An example

- Let *I* = {1, 2, 3, 4, 5, 6, 7, 8, 9}.
- Sequence ‹{3}{4, 5}{8}› is **contained** in (or is a **subsequence** of) ‹{6} {3, 7}{9}{4, 5, 8}{3, 8}›
  - because {3} ⊆ {3, 7}, {4, 5} ⊆ {4, 5, 8}, and {8} ⊆ {3, 8}.
  - However, ‹{3}{8}› is not contained in ‹{3, 8}› or vice versa.
  - The size of the sequence ‹{3}{4, 5}{8}› is 3, and the length of the sequence is 4.

Objective

- Given a set *S* of input data sequences (or sequence database), the problem of mining sequential patterns is to find all the sequences that have a user-specified minimum support.
- Each such sequence is called a **frequent sequence**, or a **sequential pattern**.
- The **support** for a sequence is the fraction of total data sequences in *S* that contains this sequence.

Example

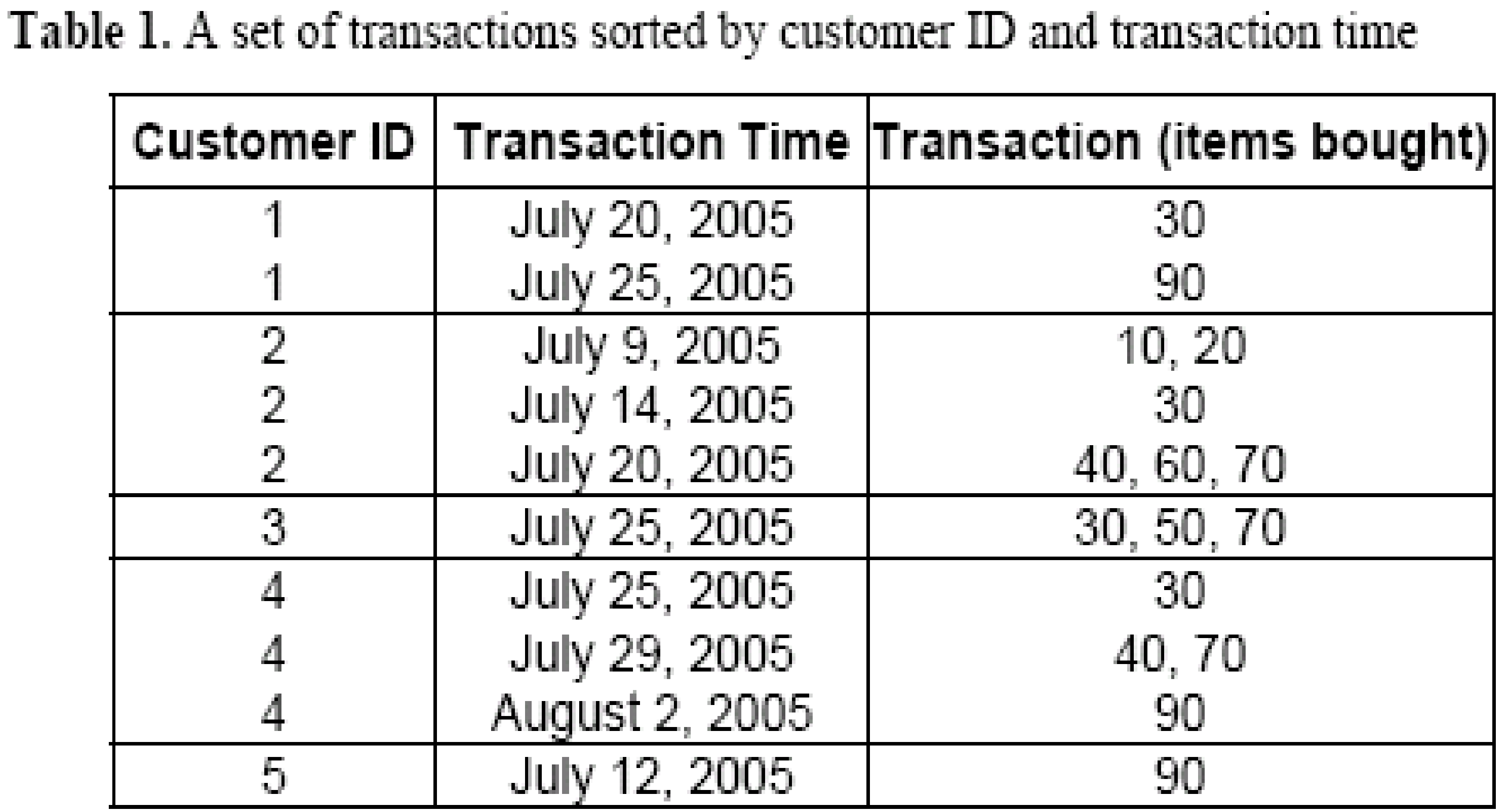
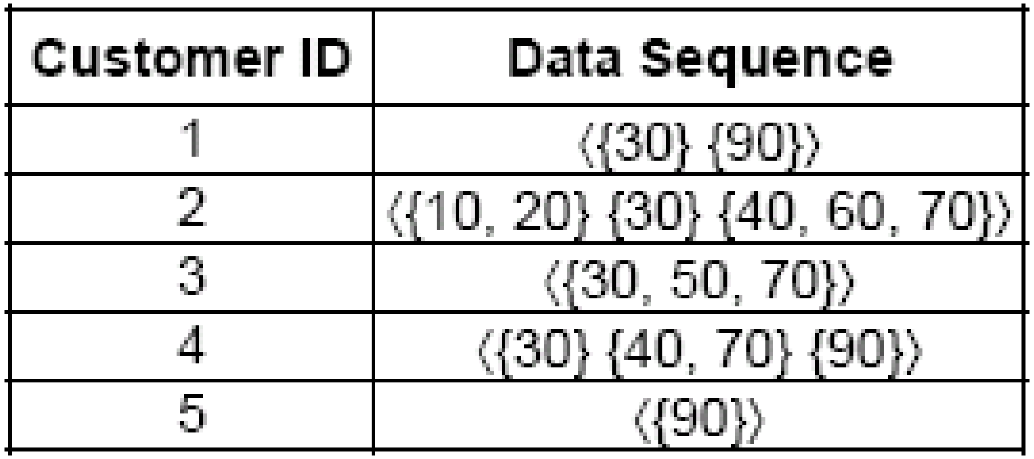


Table 1. A set of transactions sorted by customer ID and transaction time

Customer ID	Transaction Time	Transaction (items bought)
1	July 20, 2005	30
1	July 25, 2005	90
2	July 9, 2005	10, 20
2	July 14, 2005	30
2	July 20, 2005	40, 60, 70
3	July 25, 2005	30, 50, 70
4	July 25, 2005	30
4	July 29, 2005	40, 70
4	August 2, 2005	90
5	July 12, 2005	90

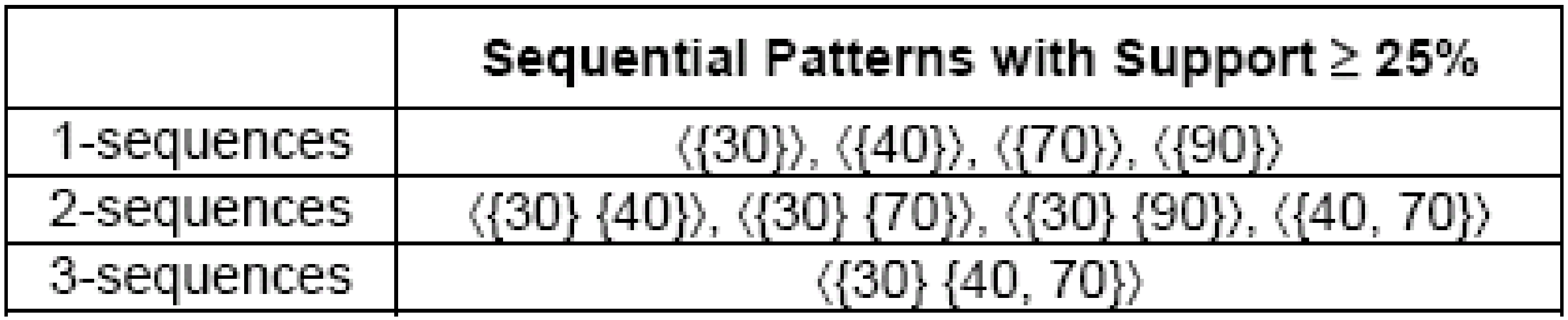
Example (cond)

Table 2. Data sequences produced from the transaction database in Table 1.



Customer ID	Data Sequence
1	⟨{30} {90}⟩
2	⟨{10, 20} {30} {40, 60, 70}⟩
3	⟨{30, 50, 70}⟩
4	⟨{30} {40, 70} {90}⟩
5	⟨{90}⟩

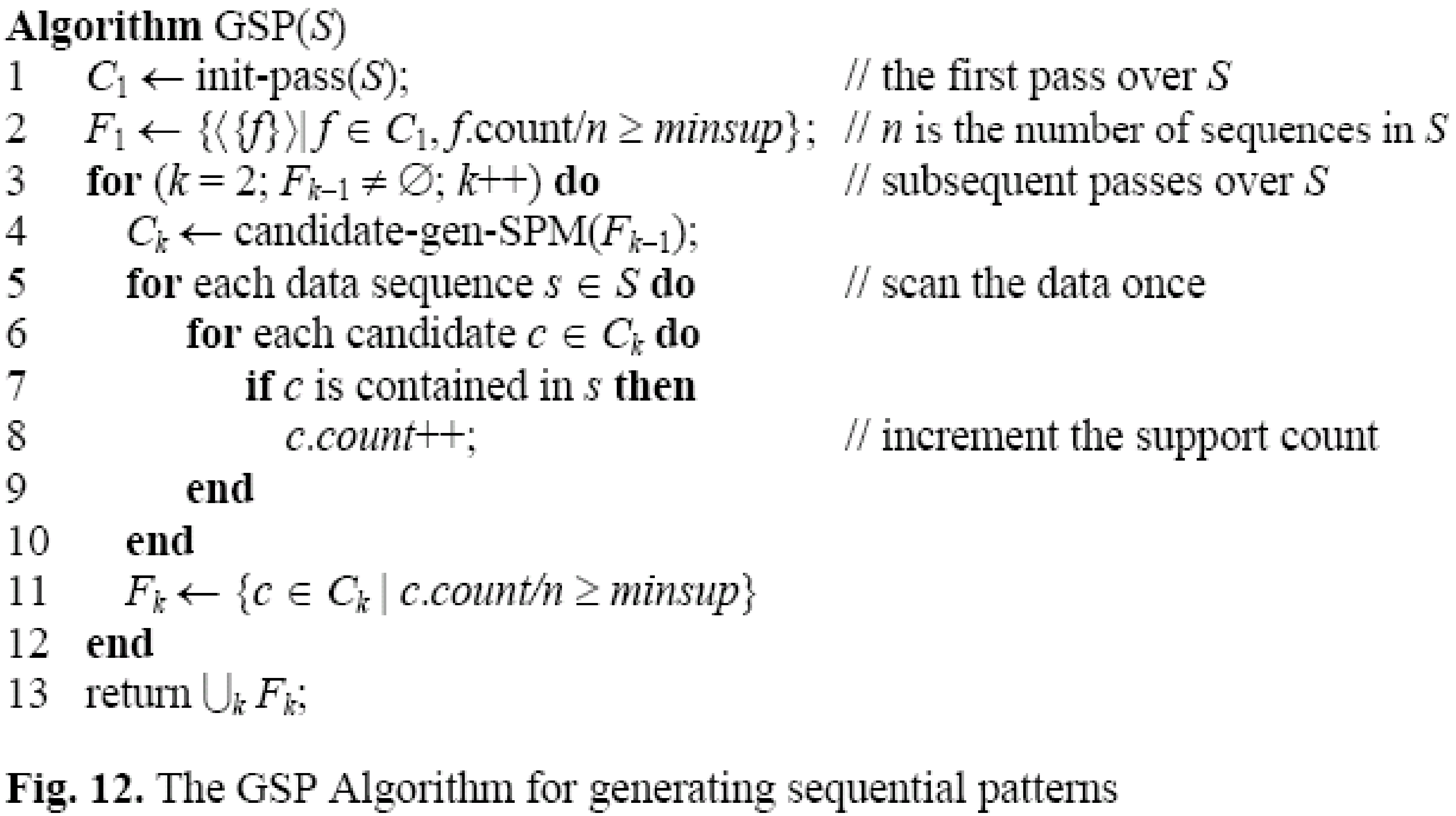
Table 3. The final output sequential patterns



	Sequential Patterns with Support ≥ 25%
1-sequences	⟨{30}⟩, ⟨{40}⟩, ⟨{70}⟩, ⟨{90}⟩
2-sequences	⟨{30} {40}⟩, ⟨{30} {70}⟩, ⟨{30} {90}⟩, ⟨{40, 70}⟩
3-sequences	⟨{30} {40, 70}⟩

GSP mining algorithm

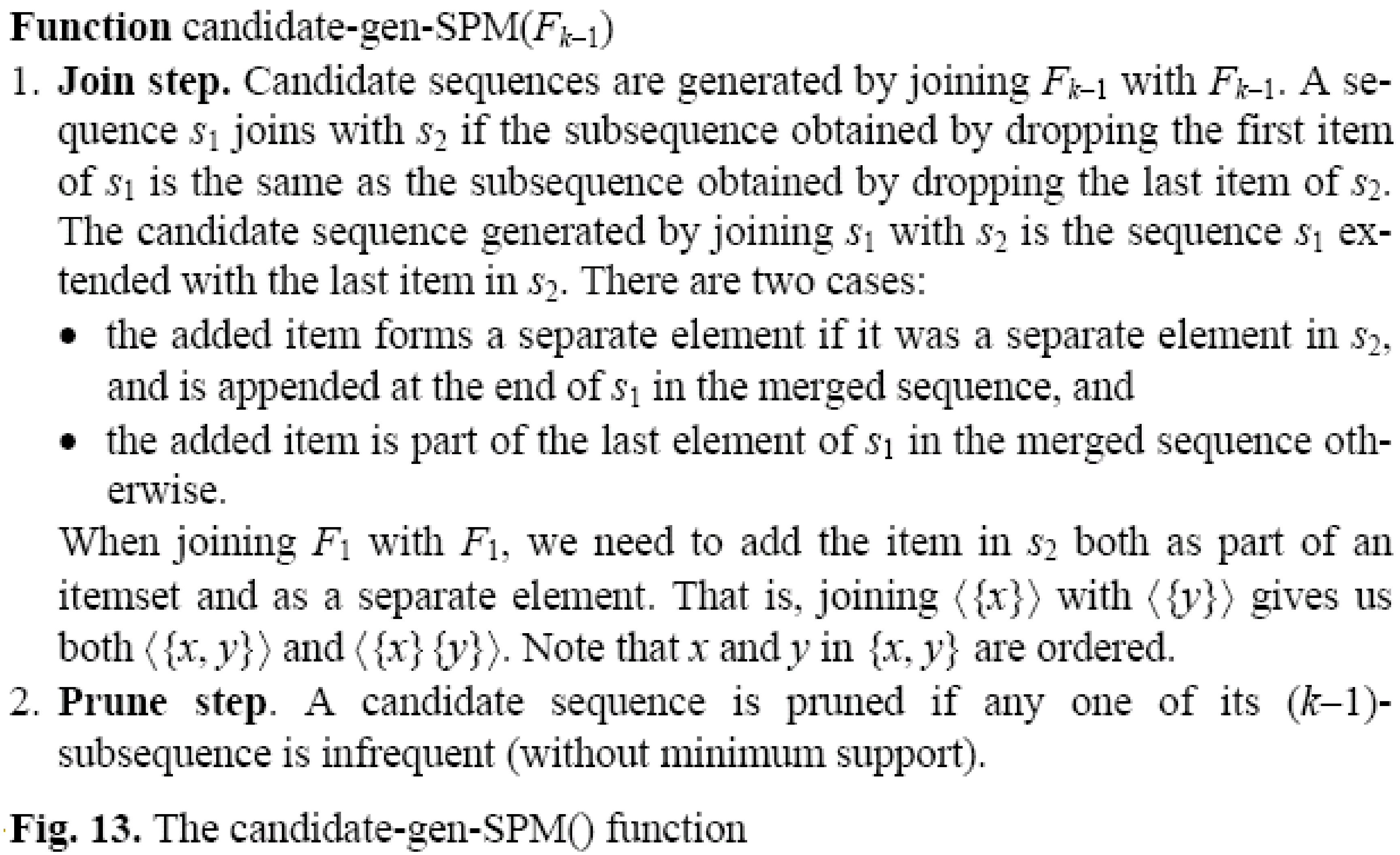
- Very similar to the Apriori algorithm



```
Algorithm GSP($S$)
$C_1 \leftarrow$ init-pass($S$);                                   // the first pass over $S$
$F_1 \leftarrow \{\langle\{f\}\rangle \mid f \in C_1, f.\text{count}/n \geq minsup\}$;   // $n$ is the number of sequences in $S$
for ($k = 2$; $F_{k-1} \neq \emptyset$; $k$++) do                    // subsequent passes over $S$
    $C_k \leftarrow$ candidate-gen-SPM($F_{k-1}$);
    for each data sequence $s \in S$ do                    // scan the data once
        for each candidate $c \in C_k$ do
            if $c$ is contained in $s$ then
                $c.count$++;                               // increment the support count
        end
    end
    $F_k \leftarrow \{c \in C_k \mid c.count/n \geq minsup\}$
end
return $\bigcup_k F_k$;
```

Fig. 12. The GSP Algorithm for generating sequential patterns

Candidate generation



Function candidate-gen-SPM(F_{k-1})

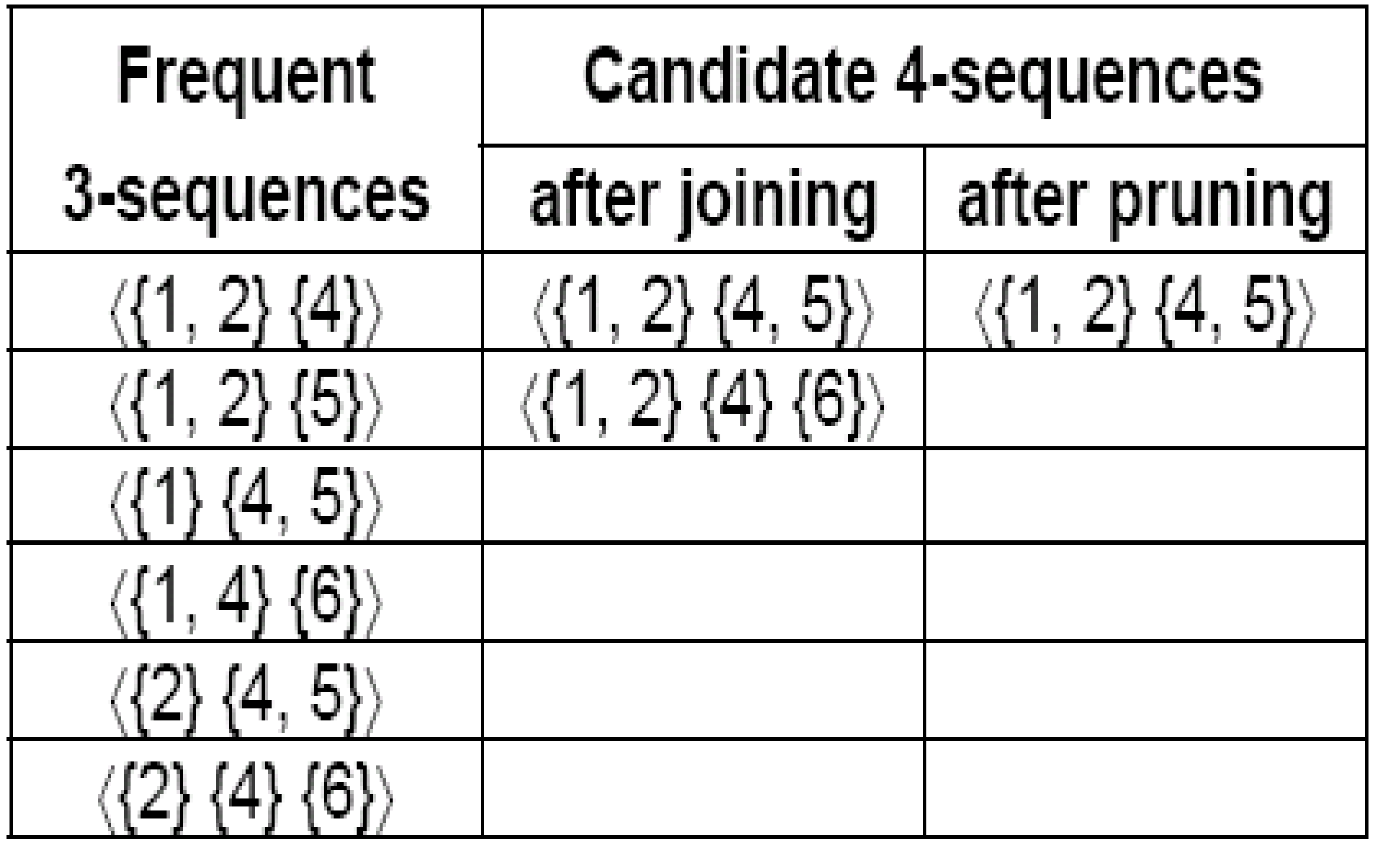
1. **Join step.** Candidate sequences are generated by joining F_{k-1} with F_{k-1}. A sequence s_1 joins with s_2 if the subsequence obtained by dropping the first item of s_1 is the same as the subsequence obtained by dropping the last item of s_2. The candidate sequence generated by joining s_1 with s_2 is the sequence s_1 extended with the last item in s_2. There are two cases:
   - the added item forms a separate element if it was a separate element in s_2, and is appended at the end of s_1 in the merged sequence, and
   - the added item is part of the last element of s_1 in the merged sequence otherwise.

   When joining F_1 with F_1, we need to add the item in s_2 both as part of an itemset and as a separate element. That is, joining $\langle\{x\}\rangle$ with $\langle\{y\}\rangle$ gives us both $\langle\{x, y\}\rangle$ and $\langle\{x\}\{y\}\rangle$. Note that x and y in $\{x, y\}$ are ordered.
2. **Prune step**. A candidate sequence is pruned if any one of its (k–1)-subsequence is infrequent (without minimum support).

Fig. 13. The candidate-gen-SPM() function

An example

Table 4. Candidate generation: an example



Frequent 3-sequences	Candidate 4-sequences	
	after joining	after pruning
⟨{1, 2} {4}⟩	⟨{1, 2} {4, 5}⟩	⟨{1, 2} {4, 5}⟩
⟨{1, 2} {5}⟩	⟨{1, 2} {4} {6}⟩	
⟨{1} {4, 5}⟩		
⟨{1, 4} {6}⟩		
⟨{2} {4, 5}⟩		
⟨{2} {4} {6}⟩		

Road map

- Basic concepts of Association Rules
- Apriori algorithm
- Different data formats for mining
- Mining with multiple minimum supports
- Mining class association rules
- Sequential pattern mining
- Summary

Summary

- Association rule mining has been extensively studied in the data mining community.
- So is sequential pattern mining
- There are many efficient algorithms and model variations.
- Other related work includes
  - Multi-level or generalized rule mining
  - Constrained rule mining
  - Incremental rule mining
  - Maximal frequent itemset mining
  - Closed itemset mining
  - Rule interestingness and visualization
  - Parallel algorithms
  - …