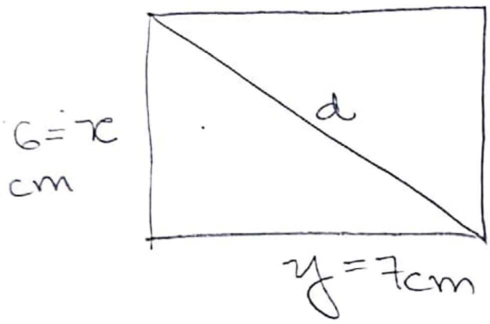
CM 202 B - Gabarito - Prova 2A

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
GRR:	9	9	9	9	3	4	5	6

Questão 1



$$D(x,y) = \sqrt{x^2 + y^2}$$

$$\frac{\partial D}{\partial x} = \frac{1}{2}\frac{1}{\sqrt{x^2+y^2}}2x = \frac{x}{\sqrt{x^2+y^2}}$$

$$\frac{\partial D}{\partial y} = \frac{1}{2}\frac{1}{\sqrt{x^2+y^2}}2y = \frac{y}{\sqrt{x^2+y^2}}$$

Diferencial:

$$dD = \frac{\partial D}{\partial x}dx + \frac{\partial D}{\partial y}dy$$

$$dD = \frac{x}{\sqrt{x^2+y^2}}dx + \frac{y}{\sqrt{x^2+y^2}}dy$$

$$dD = \frac{6}{\sqrt{36+49}}dx + \frac{7}{\sqrt{36+49}}dy \Rightarrow \boxed{dD = \frac{6}{\sqrt{85}}dx + \frac{7}{\sqrt{85}}dy}$$

Questão 2

$$f(x,y) = xe^y$$

$$\frac{\partial f}{\partial x} = e^y$$

$$\frac{\partial f}{\partial y} = xe^y$$

Plano Tangente:

$$z - z_0 \cong \underbrace{\left(\frac{\partial f}{\partial x}\right)\underbrace{(x - x_0)}_{dx} + \left(\frac{\partial f}{\partial y}\right)\underbrace{(y - y_0)}_{dy}}_{dz}$$

$$z - z_0 \cong dz \Rightarrow z \cong z_0 + dz$$

Temos $x_0 = 2$, $dx = 0{,}01$

$y_0 = 0$, $dz = 0{,}017$

$z_0 = f(x_0, y_0) = 2e^0 = 2$

$dz = e^{y_0}dx + x_0 e^{y_0} dy = e^0 \cdot 0{,}01 + 2 \cdot e^0 \cdot 0{,}017$

$= 0{,}01 + 2{,}017$

$= 0{,}01 + 0{,}034$

$= 0{,}044$

$\underline{z} \cong z_0 + dz$

$f(2{,}01; 0{,}017) \cong 2 + 0{,}044 = 2{,}044$

Questão 3

a) $f(x,y) = e^{x^2+y^2}$

Teste da 1ª Derivada:

$\frac{\partial f}{\partial x} = (e^{x^2+y^2})(2x) = 0 \Rightarrow 2x \underbrace{e^{x^2+y^2}}_{>0} = 0 \Rightarrow x = 0$ ✓

$\frac{\partial f}{\partial y} = (e^{x^2+y^2})(2y) = 0 \Rightarrow 2y \underbrace{e^{x^2+y^2}}_{>0} = 0 \Rightarrow y = 0$ ✓

Teste da 2ª Derivada:

$$\frac{\partial^2 f}{\partial x^2} = 2e^{x^2+y^2} + 2x \cdot 2x \cdot e^{x^2+y^2} = (2+4x^2)e^{x^2+y^2}$$

$$\frac{\partial^2 f}{\partial y^2} = 2e^{x^2+y^2} + 2y \cdot 2ye^{x^2+y^2} = (2+4y^2)e^{x^2+y^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = 2x \cdot 2ye^{x^2+y^2} = 4xye^{x^2+y^2}$$

$$H = \begin{bmatrix} (2+4x^2)e^{x^2+y^2} & 4xye^{x^2+y^2} \\ 4xye^{x^2+y^2} & (2+4y^2)e^{x^2+y^2} \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} \underline{2} \,|> 0 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow D = 4 > 0.$$

O pto (0,0) é pto de mínimo.

b) $f(x,y) = 5x^2 + 4xy + y^2 - 18x - 8y + 10$

Teste 1ª Derivada:

$$\frac{\partial f}{\partial x} = 10x + 4y - 18 = 0 \Rightarrow 5x + 2y = 9$$

$$\frac{\partial f}{\partial y} = 4x + 2y - 8 = 0 \Rightarrow 4x + 2y = 8$$

$$x = 1, \; y = 2$$

Teste da 2ª Derivada:

$$H = \begin{bmatrix} 10 & 4 \\ 4 & 2 \end{bmatrix} \quad 10 > 0 \Rightarrow D = 20 - 16 > 0$$

O pto (1,2) é pto de mínimo.

Questão 4

$u = y \ln(x^2 + y^2)$

$$\frac{\partial u}{\partial x} = \frac{y(2x)}{x^2+y^2} = \frac{2xy}{x^2+y^2}$$

$$\frac{\partial u}{\partial y} = \ln(x^2+y^2) + \frac{y(2y)}{x^2+y^2}$$

$x = 4s + 3t$

$x = 4(-1) + 3 \cdot 1 = -1$

$y = 5t - 2s$

$y = 5(1) - 2(-1)$
$= 7$

$$\frac{\partial x}{\partial s} = 4 \qquad \frac{\partial x}{\partial t} = 3$$

$$\frac{\partial y}{\partial s} = -2 \qquad \frac{\partial y}{\partial t} = 5$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t}$$

$$\frac{\partial u}{\partial t} = \frac{2(-1)(7)}{1+49} \cdot 3 + \left(\ln(1+49) + \frac{2 \cdot 7 \cdot 7}{1+49}\right) \cdot 5$$

$$\frac{\partial u}{\partial t} = -\frac{42}{50} + 5\ln 50 + \frac{490}{50} = \frac{448}{50} + 5\ln 50$$

$$\frac{\partial u}{\partial t} = \frac{224}{25} + 5\ln 50$$

$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial u}{\partial s} = \frac{2(-1)(7)}{1+49} \cdot 4 + \left(\ln(1+49) + \frac{2 \cdot 7 \cdot 7}{1+49}\right) \cdot (-2)$$

$$\frac{\partial u}{\partial s} = \frac{-56}{50} - 2\ln 50 - \frac{196}{50} = -\frac{252}{50} - 2\ln 50$$

$$\frac{\partial u}{\partial s} = -\frac{126}{25} - 2\ln 50$$

b) $e^{y}\,\mathrm{sen}(xy) = x + 2xy + 3$

$$F(x,y) = e^{y}\,\mathrm{sen}(xy) - x - 2xy - 3$$

$$\frac{\partial F}{\partial x} = e^{y} \cdot y \cdot \cos xy - 1 - 2y$$

$$\frac{\partial F(3,0)}{\partial x} = \cancel{1 \cdot 0 \cdot \cos 0} - 1 - 0 = -1$$

$$\frac{\partial F}{\partial y} = e^{y}\,\mathrm{sen}(xy) + e^{y} \cdot x \cdot \cos xy - 2x$$

$$\frac{\partial F(3,0)}{\partial y} = \cancel{1 \cdot \mathrm{sen}\, 0} + e^{0} \cdot 3 \cdot \cos 0 - 6 = 1 \cdot 3 \cdot 1 - 6 = -3$$

$$\frac{dy(3)}{dx} = -\frac{\frac{\partial F(3,0)}{\partial x}}{\frac{\partial F(3,0)}{\partial y}} = -\frac{-1}{-3} = -\frac{1}{3}$$

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Questão 5

$$z = f(x,y), \begin{cases} x = r\cos\theta \to \dfrac{\partial x}{\partial r} = \cos\theta, \quad \dfrac{\partial x}{\partial \theta} = -r\,\text{sen}\,\theta \\ y = r\,\text{sen}\,\theta \to \dfrac{\partial y}{\partial r} = \text{sen}\,\theta, \quad \dfrac{\partial y}{\partial \theta} = r\cos\theta \end{cases}$$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r}$$

$$= \frac{\partial z}{\partial x}\cos\theta + \frac{\partial z}{\partial y}\,\text{sen}\,\theta$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta}$$

$$= \frac{\partial z}{\partial x}(-r\,\text{sen}\,\theta) + \frac{\partial z}{\partial y}\, r\cos\theta$$

Dado $\left(\dfrac{\partial z}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial z}{\partial \theta}\right)^2 =$

$$= \left(\frac{\partial z}{\partial x}\cos\theta + \frac{\partial z}{\partial y}\,\text{sen}\,\theta\right)^2 + \left(\frac{\partial z}{\partial x}(-r\,\text{sen}\,\theta) + \frac{\partial z}{\partial y}\, r\cos\theta\right)^2$$

$$= \left(\frac{\partial z}{\partial x}\right)^2 (\cos^2\theta) + 2\,\frac{\partial z}{\partial x}(\cos\theta)\,\frac{\partial z}{\partial y}(\text{sen}\,\theta) + \left(\frac{\partial z}{\partial y}\right)^2 (\text{sen}^2\theta)$$

$$+ \frac{1}{r^2}\left[\left(\frac{\partial z}{\partial x}\right)^2 r^2\,\text{sen}^2\theta - 2\,\frac{\partial z}{\partial x}\,\frac{\partial z}{\partial y}\, r^2\,\text{sen}\,\theta\cos\theta + \left(\frac{\partial z}{\partial y}\right)^2 r^2\cos^2\theta\right]$$

$$= \left(\frac{\partial z}{\partial x}\right)^2 \cos^2\theta + \cancel{2\,\frac{\partial z}{\partial x}\cos\theta\,\frac{\partial z}{\partial y}\,\text{sen}\,\theta} + \left(\frac{\partial z}{\partial y}\right)^2 (\text{sen}^2\theta)$$

$$+ \left(\frac{\partial z}{\partial x}\right)^2 \text{sen}^2\theta \; \cancel{- 2\,\frac{\partial z}{\partial x}\,\frac{\partial z}{\partial y}\,\text{sen}\,\theta\cos\theta} + \left(\frac{\partial z}{\partial y}\right)^2 (\cos^2\theta)$$

$$= \left(\frac{\partial z}{\partial x}\right)^2 \cos^2\theta + \left(\frac{\partial z}{\partial x}\right)^2 \text{sen}^2\theta + \left(\frac{\partial z}{\partial y}\right)^2 \text{sen}^2\theta + \left(\frac{\partial z}{\partial y}\right)^2 (\cos^2\theta)$$

$$= \left(\frac{\partial z}{\partial x}\right)^2 (\cos^2\theta + \text{sen}^2\theta) + \left(\frac{\partial z}{\partial y}\right)^2 (\text{sen}^2\theta + \cos^2\theta)$$

$$= \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

cqd

Questão 6

a) No máximo crescimento,

$$D_u f = \|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2}$$

$$= \sqrt{(y+z)^2 + (x+z)^2 + (x+y)^2}$$

b) $P(1,-1,6)$

Considere $\vec{v}=(a,b,c)$ um vetor unitário.

$$D_u f(1,-1,6) = \nabla f(1,-1,6)\cdot(a,b,c)$$

$$=(5,7,0)(a,b,c) = 5a+7b=0$$

Para que a derivada direcional em $P(1,-1,6)$ seja nula, temos que satisfazer a condição $5a+7b=0$ e c é livre.