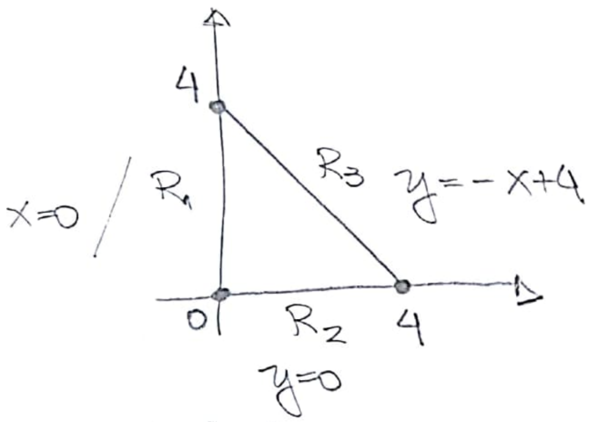
CM 202B - Gabarito Prova 3A

	d_5	d_6	d_7	d_8
GRR 9999	3	4	5	6

Questão 1:

$f(x,y) = 8x^3 + 2xy - 3x^2 + y^2 + 2$

Triang. de Vértices: $(0,0), (4,0), (0,4)$



No interior:

$\frac{\partial f}{\partial x} = 24x^2 + 2y - 6x = 0$

$\frac{\partial f}{\partial y} = 2x + 2y = 0 \Rightarrow x = -y$

$\Rightarrow 24y^2 + 2y + 6y = 0$

$24y^2 + 8y = 0$

$y(24y + 8) = 0$

$y = 0, -\frac{1}{3}$

$x = 0, \frac{1}{3}$

$\begin{cases} x = 0 \\ y = 0 \end{cases}$ ok!

$\cancel{\begin{cases} x = 1/3 \\ y = -1/3 \end{cases}}$

Descartado pq está fora do triângulo

$R_1: x=0$

$f(0,y) = y^2+2$

$f' = 2y = 0 \Rightarrow \begin{cases} x=0 \\ y=0 \end{cases} \quad \begin{cases} x=0 \\ y=4 \end{cases}$

$R_2: y=0$

$f(x,0) = 8x^3-3x^2+2$

$f' = 24x^2-6x = 0$

$x(24x-6)=0$

$\begin{cases} x=0 \\ y=0 \end{cases} \quad \begin{cases} x=\frac{1}{4} \\ y=0 \end{cases} \quad \begin{cases} x=4 \\ y=0 \end{cases}$

$R_3: y=-x+4$

$f(x,-x+4) = 8x^3+2x(-x+4)-3x^2+(-x+4)^2+2$

$f(x,-x+4) = 8x^3-2x^2+8x-3x^2+(-x+4)^2+2$

$f(x,-x+4) = 8x^3-5x^2+8x+(-x+4)^2+2$

$f' = 24x^2-10x+8+2(-x+4)(-1)$

$f' = 24x^2-10x+8-2(-x+4)$

$f' = 24x^2-10x+\cancel{8}+2x-\cancel{8}=0 \Rightarrow 24x^2-8x=0$

$x(24x-8)=0$

$$\begin{cases} x = 0 \\ y = 4 \end{cases} \qquad \begin{cases} x = \frac{1}{3} \\ y = \frac{11}{3} \end{cases} \qquad \begin{cases} x = 4 \\ y = 0 \end{cases}$$

Comparação:

Ptos Críticos Encontrados

$(0,0) \rightsquigarrow f(0,0) = 2$

$(0,4) \rightsquigarrow f(0,4) = 4^2 + 2 = 18$

$(1/4, 0) \rightsquigarrow f(1/4, 0) = 8 \cdot \left(\frac{1}{4}\right)^3 - 3\left(\frac{1}{4}\right)^2 + 2 = \frac{31}{16} \cong 2$

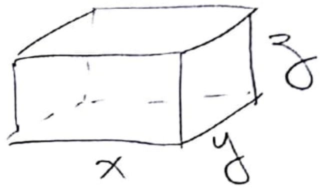
$(4,0) \rightsquigarrow f(4,0) = 8 \cdot 4^3 - 3 \cdot 4^2 + 2 = 466$

$(1/3, 11/3) \rightsquigarrow f(1/3, 11/3) = \frac{8}{3^3} + \frac{2 \cdot 11}{3^2} - \frac{3}{3^2} + \frac{121}{3^2} + 2 = \frac{482}{27} \cong 18$

$\therefore$ $(1/4, 0)$ é pto de mínimo local

$(4,0)$ é pto de máximo local

Questão 2



a) min $\boxed{2xy + xz + yz}$ função objetivo

s.a $\boxed{xyz = 8}$ restrição

Custo material da base: \$1

Custo material da lataral: \$1/2

b) $L(x,y,z) = 2xy + xz + yz - \lambda(xyz - 8)$

$\frac{\partial L}{\partial x} = 2y + z - \lambda(yz) = 0 \quad (\cdot x) \Rightarrow x(2y+z) = \lambda(xyz)$

$\frac{\partial L}{\partial y} = 2x + z - \lambda(xz) = 0 \quad (\cdot y) \Rightarrow y(2x+z) = \lambda(xyz)$

$\frac{\partial L}{\partial z} = x + y - \lambda(xy) = 0 \quad (\cdot z) \Rightarrow z(x+y) = \lambda(xyz)$

$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow xyz = 8$

$\frac{x(2y+z)}{y(2x+z)} = \frac{\lambda(xyz)}{\lambda(xyz)} \Rightarrow x(2y+z) = y(2x+z)$

$\cancel{2xy} + xz = \cancel{2xy} + yz$

$xz - yz = 0$

$z(x-y) = 0 \Rightarrow x = y$

$\frac{y(2x+z)}{z(x+y)} = \frac{\lambda(xyz)}{\lambda(xyz)} \Rightarrow y(2x+z) = z(x+y)$

$2xy + \cancel{yz} = xz + \cancel{yz}$

$x(2y - z) = 0$

$z = 2y$

$xyz = 8 \Rightarrow y \cdot y \cdot 2y = 8 \Rightarrow 2y^3 = 8 \Rightarrow y^3 = 4$

$\boxed{x = \sqrt[3]{4}, \quad y = \sqrt[3]{4}, \quad z = 2\sqrt[3]{4}}$

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c) $C = 2xy + xz + yz$

$$xyz = 8 \Rightarrow z = \frac{8}{xy}$$

$$C = 2xy + x\cdot\frac{8}{xy} + y\frac{8}{xy} = 2xy + \frac{8}{y} + \frac{8}{x}$$

$$\frac{\partial C}{\partial x} = 2y - \frac{8}{x^2} = 0 \Rightarrow 2y = \frac{8}{x^2} \Rightarrow ① \; y = \frac{4}{x^2}$$

$$\frac{\partial C}{\partial y} = 2x - \frac{8}{y^2} = 0 \Rightarrow 2x = \frac{8}{y^2} \Rightarrow ② \; y^2 = \frac{4}{x}$$

$$① \; y = \frac{4}{x^2} \Rightarrow y^2 = \frac{4^2}{x^4}$$

$$② \; y^2 = \frac{4}{x}$$

$$\frac{4^2}{x^4} = \frac{4}{x}$$

$$4x^4 = 4^2 x$$

$$x^4 - 4x = 0$$

$$x(x^3 - 4) = 0$$

$$x^3 - 4 = 0$$

$$\boxed{x = \sqrt[3]{4}}$$

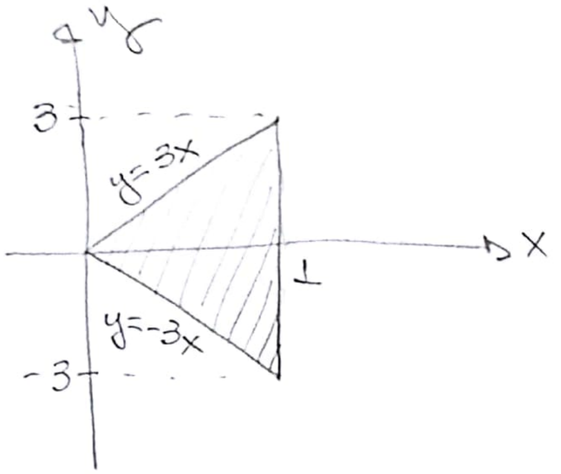
$$① \; y = \frac{4}{x^2} = \frac{4}{(\sqrt[3]{4})^2} = \frac{4\sqrt[3]{4}}{(\sqrt[3]{4})^2\sqrt[3]{4}} = \frac{\cancel{4}\sqrt[3]{4}}{\cancel{4}} = \sqrt[3]{4}$$

$$\boxed{y = \sqrt[3]{4}}$$

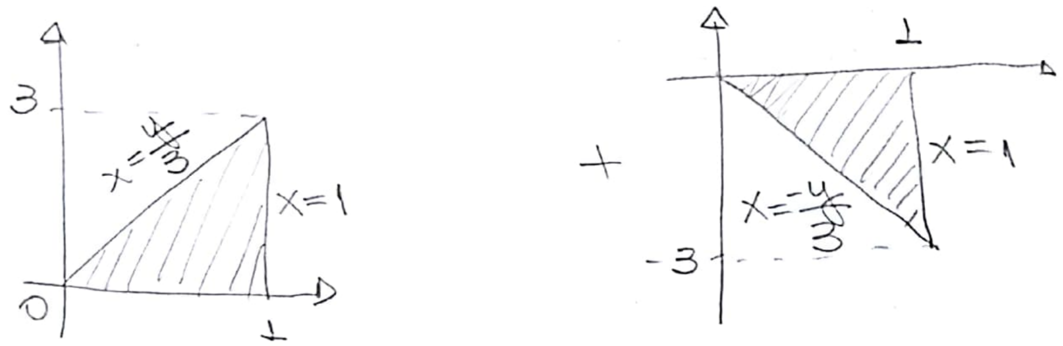
$$z = \frac{8}{xy} = \frac{8}{\sqrt[3]{4}\sqrt[3]{4}} = \frac{8}{(\sqrt[3]{4})^2}\frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{8\sqrt[3]{4}}{4} \Rightarrow \boxed{z = 2\sqrt[3]{4}}$$

Questão 3

a)



b)



$$\int_0^1\int_{-3x}^{3x} f(x,y)\,dy\,dx = \int_0^3\int_{y/3}^1 f(x,y)\,dx\,dy + \int_{-3}^0\int_{-y/3}^1 f(x,y)\,dx\,dy$$

c)

$$\int_0^1\int_{-3x}^{3x}(x+y)\,dy\,dx = \int_0^1 \left(xy+y^2\right)\Big|_{-3x}^{3x} dx = \int_0^1 3x^2+9x^2$$

$$\int_{-3x}^{3x}(x+y)\,dy = xy+\frac{y^2}{2}\Big|_{-3x}^{3x} = 3x^2+\frac{9x^2}{2}-\left(-3x^2+\frac{9x^2}{2}\right)$$

$$= 6x^2$$

$$\int_0^1 6x^2\,dx = 6\frac{x^3}{3}\Big|_0^1 = 2x^3\Big|_0^1 = 2$$

Questão 4

$$\int_0^{\pi/2}\int_0^{\pi/2} \text{sen}\, x\, \text{sen}\, y\, dx\, dy = \int_0^{\pi/2} \text{sen}\, x\, dx \int_0^{\pi/2} \text{sen}\, y\, dx$$

$$= \left(-\cos x\Big|_0^{\pi/2}\right)\left(-\cos y\Big|_0^{\pi/2}\right)$$

$$= \left(-\cos\frac{\pi}{2} + \cos 0\right)\left(-\cos\frac{\pi}{2} + \cos 0\right) = (0+1)(0+1) = 1$$

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