A Genetic Algorithm for Crop Rotation

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Summary

The Crop Rotation Problem

The Crop Rotation Problem (CRP) consists in developing a planting schedule, alternating different plant families within the same agricultural area to prevent excessive wear of the soil and reduce the use of chemicals to control pests

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- is a model for sustainable crop production
- the practice improves the physical, chemical and biological characteristics of soil
- helps in the biological control of pests and diseases
- provides a diversified production of food and other products

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(b) *continuity for same-family crops* - plants of the same family must not be planted consecutively on the same lot

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(a) *sowing season* - the time for planting and the life cycle of each culture must be respected

(b) *continuity for same-family crops* - plants of the same family must not be planted consecutively on the same lot

(c) *neighboring for same family crops* - plants belonging to the same family must not be planted in adjacent parcels of land or lots

Constraints:

(d) *green fertilization* - in the planning horizon each lot must have a plant of the leguminosae family and be subject to the above mentioned conditions (a) and (b), also each of these must be planted only once

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(f) *demand* - each culture has a pre-stablished market demand that must be satisfied

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1. C: trade crops
2. A: set of crops for green fertilization
3. F_p: set of plants of family p, $p = 1..N_f$
4. t_i: planting cycle of crop i, including soil preparation and harvesting
5. l_{ij}: profitability of crop i in the period j per unit of area

Parameters:

6. $I_i = [C_i, T_i]$: crop planting interval i, in which C_i is the earlier period and T_i is the later period
7. p_{ij}: production of crop i in the period j per unit of area
8. D_i: demand for crop i
9. $I_i^D = [C_i^D, T_i^D]$: demand interval of crop i, in which C_i^D is the earlier period and T_i^D is the later period of the demand
10. S_k: set of lots adjacent to lot k
11. $area_k$: area of lot k

Mathematical model

The decision variable is

$$x_{ijk} = \begin{cases} 1, & \text{if crop } i \text{ is planted in period } j \text{ in lot } k \\ 0, & \text{otherwise} \end{cases}$$

Mathematical model - objective function

The objective function

$$\text{maximize } z = \sum_{i \in C} \sum_{j \in I_i} \sum_{k=1}^{L} area_k \, l_{ij} \, x_{ijk} \tag{1}$$

sets out to maximizes the profitability of the rotation

Mathematical model - neighbors constraints

The constraints

$$\sum_{i \in F_p} \sum_{r=0}^{t_i - 1} \sum_{v \in S_k} x_{i(j-r)v} \leq L \left(1 - \sum_{i \in F_p} \sum_{r=0}^{t_i - 1} x_{ijk} \right), \; p = 1..N_f, j = 1..M, k = 1..L \tag{2}$$

prevent the plants of the same family from being neighbors

Mathematical model - planting consecutive constraints

The constraints

$$\sum_{i \in F_p} \sum_{r=0}^{t_i} x_{i(j-r)k} \leq 1, p = 1..N_f, j = 1..M, k = 1..L \qquad (3)$$

forbid plants of the same family from being consecutively planted on the same lot k

Mathematical model - not overlap constraints

The constraints

$$\sum_{i=1}^{N+1} \sum_{r=0}^{t_i - 1} x_{i(j-r)k} \leq 1, \; j = 1..M, \; k = 1..L \tag{4}$$

prevent two plants from occupying the same lot in the same time interval

Mathematical model - green fertilization and set aside constraints

The constraints (5) and (6)

$$\sum_{i \in A} \sum_{j=1}^{M} x_{ijk} \geq 1, \ \ k = 1..L \tag{5}$$

$$\sum_{j=1}^{M} x_{njk} \geq 1, \ \ k = 1..L \tag{6}$$

ensure that each lot has at least one green fertilization application and a set-aside period, respectively.

The set-aside period is represented by crop $n = (N + 1)$

Mathematical model - demand constraints

The constraints (7)

$$\sum_{j \in I_i^D} \sum_{k=1}^{L} area_k \, p_{ij} \, x_{ijk} \geq D_i, \;\; i \in C \tag{7}$$

$$x_{ijk} \in \{0,1\}, \;\; i = 1..N+1, \;\; j \in I_i, \;\; k = 1..L. \tag{8}$$

Note: if $j - r \leq 0$, replace $j - r$ by $j - r + M$

impose satisfaction of demand for crops during the respective period

Mathematical model

$$\text{maximize } z = \sum_{i \in C} \sum_{j \in I_i} \sum_{k=1}^{L} area_k \, l_{ij} \, x_{ijk}$$

Subject to

$$\sum_{i \in F_p} \sum_{r=0}^{t_i - 1} \sum_{v \in S_k} x_{i(j-r)v} \leq L \left(1 - \sum_{i \in F_p} \sum_{r=0}^{t_i - 1} x_{ijk} \right), \; p = 1..N_f, j = 1..M, k = 1..L$$

$$\sum_{i \in F_p} \sum_{r=0}^{t_i} x_{i(j-r)k} \leq 1, p = 1..N_f, j = 1..M, k = 1..L$$

$$\sum_{i=1}^{N+1} \sum_{r=0}^{t_i - 1} x_{i(j-r)k} \leq 1, \; j = 1..M, \; k = 1..L$$

$$\sum_{i \in A} \sum_{j=1}^{M} x_{ijk} \geq 1, \; k = 1..L$$

$$\sum_{j=1}^{M} x_{njk} \geq 1, \; k = 1..L$$

$$\sum_{j \in I_i^D} \sum_{k=1}^{L} area_k \, p_{ij} \, x_{ijk} \geq D_i, \; i \in C$$

$$x_{ijk} \in \{0, 1\}, \; i = 1..N+1, \; j \in I_i, \; k = 1..L.$$

Methodology - Solving strategies

1. We changed the decision-making variables
2. We developed a constructive heuristic that provides the initial population for the Genetic Algorithm (GA) for this problem
3. We developed and implemented a GA

Decision-making variables

- It is more convenient for this algorithm to be based on integer decision variables taking values in the interval $[1; N+1]$
- One solution to the problem is associated with an individual identified by a single chromosome, which is encoded through an $L \times M$ integer matrix
- Its element (k, j) belongs to $[1, N+1]$ and identifies which crop is being planted on lot k and in period j, for all $k = 1..L$ and $j = 1..M$

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This matrix is the timetable or schedule for the given planting area

Constructive heuristic

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Each initial solution/individual is built lot by lot thus imposing

- the correct planting period for each crop (condition (a))
- the planting sequence constraint (condition (b))

Penalization for inadmissible solutions

The fitness of an individual was initially set equal to the sum of the profitabilities on all lots. An exponential penalization process was used to punish the infeasible solutions due to violation of the remaining rotation conditions.

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- The penalizations of the neighborhood, set-aside period, green fertilization and demand violations were added
- Considering that their sum is p, the fitness of the individual is multiplied by

  $$e^{-p/K},$$

  where K is a positive constant equal to 10

Strategies performed in GA

The GA was applied with $G = 120$ generations, $P = 451$ individuals (solutions) in the population of all the generations and using Selection, Crossover, Elitism, Mutation and Migration operators.

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1. **Selection**: Biased Roulette Wheel
2. **Crossover**: a uniform crossover with a rate of 80%. The chromosome break points for this genetic operator were horizontally and randomly selected (to keep feasibility in the line).

Strategies performed in GA

3. **Elitism**: consists of saving the best solution before the action of the operators in each generation and inserting it in the population for the next generation

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5. **Migration**: 50% of the population in the generations 120γ is replaced by randomly generated individuals (with γ, a control parameter taking the value 0.80)

Strategies performed in GA

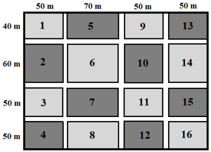
The probability $m(g)$ of **Mutation** or **Migration** in the generation $g (g \in [1..G])$ is given as:

$$m(g) = \frac{0.01}{0.01 + e^{-g/10}} \tag{9}$$

e.g., $m(g)$ increases with the generation g

The data for the real application

The computational experiment took into account a real instance of CRP with a planting area with 16 non-parallel lots, as shown in the figure



Planting area

The data for the real application

- A two-year planting program was considered and each period fixed at one month
- Crops 23 to 29 were selected for green fertilization and crop 30 for the set-aside period
- The plants represented by 1 to 25 correspond to marketing purposes

Computational results

The GA ran 100 times for this real life problem.
We measured:

- the average CPU time per GA run (t in seconds)
- the number of times the algorithm provided a feasible best solution out of 100 (α)
- the average profitability of the best solution found per run ($\bar{z}$ in R$)

Computational results

- the average penalization for the fitness of the best solution at the initial population per run ($\overline{p}_{initial}$) and the same for the best solution at the final population per run ($\overline{p}_{end}$)
- The value $\overline{\Delta} z$ representing the average relative deviation between the profitability of the best first and best final solutions found at each run of the GA

Computational results

Tabela: Average of computational results

t	$\overline{z}$	α	$p_{initial}$	p_{end}	$\overline{\Delta} z$
1,017	1.53×10^6	72	44.0	0.43	0.23

Computational results

The best feasible solution determined by the GA following the 100 runs, a rotation for the planting area studied, is given in the figure

	Year 1												Year 2											
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	28	28	23	23	23	23	30	1	1	26	26	26	15	15	15	15	15	15	15	12	12	12	28	28
2	13	13	13	13	13	13	30	9	9	9	9	1	1	23	23	23	23	2	2	2	2	20	20	20
3	10	19	19	19	19	19	30	2	2	2	2	27	27	27	13	13	13	13	13	13	10	10	10	10
4	15	15	15	15	15	15	30	9	9	9	9	25	25	25	22	22	22	22	22	22	16	16	16	15
5	13	13	13	13	13	13	30	23	23	23	23	18	18	18	18	18	30	21	21	21	21	20	20	20
6	23	1	1	16	16	16	30	2	2	2	2	28	28	28	28	8	8	6	6	6	6	23	23	23
7	27	27	7	7	3	3	3	23	23	23	23	2	2	2	2	19	19	19	19	19	30	1	1	27
8	28	28	28	2	2	2	2	1	1	18	18	18	18	18	16	16	16	7	7	12	12	12	30	28
9	22	22	22	22	22	22	30	15	15	15	15	15	15	15	4	4	4	4	4	4	4	27	27	27
10	15	15	15	24	24	24	30	9	9	9	9	1	1	11	11	11	11	11	8	8	15	15	15	15
11	19	19	19	19	19	8	8	2	2	2	2	25	25	25	5	5	5	5	11	11	11	11	11	30
12	16	16	16	5	5	5	5	30	12	12	12	2	2	2	2	23	23	23	23	1	1	25	25	25
13	23	23	23	14	14	14	14	25	25	25	20	20	20	22	22	22	22	22	22	16	16	16	30	23
14	28	28	21	21	21	21	30	30	15	15	15	15	15	15	15	5	5	5	5	12	12	12	28	28
15	26	2	2	2	2	30	30	12	12	12	27	27	27	1	1	8	8	19	19	19	19	19	26	26
16	28	28	28	8	8	1	1	15	15	15	15	15	15	15	3	3	3	30	11	11	11	11	11	28

The best soluction for the real instance for 16 lots

Conclusions

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1. The model the CRP possesses real presented for applicability insofar as it takes into account both technical as well as economic considerations
2. The method GA provides good quality feasible solutions in short computing time, thus proving to be a viable, simple and efficient approach to tackle a real instance of this problem of a highly complex combinatorial nature
3. Is a promising tool to help farmers in decision-making processes

References

- ALTIERI, M.A. *Agroecologia: bases científicas para uma agricultura sustentável.* Editora Agropecuária. Guaíba.(2002).
- EIBEN, A.E. *Coloring with Adaptive Evolutionary Algorithms.*Operations Research, v.4. 25-46 (1998).
- GOLDBERG, D. E. *Genetic Algorithms and Machine Learning. Kluwer Academic Publishers, v.3, 1989.*

References

- MICHALEWICZ, Z.; FOGEL, B. D. *How to solve it: Modern Heuristics. USA: , 2000. p.115-183.*
- RODRIGUES, L.M.S *Programação de rotação de Culturas: modelos e métodos de solução Tese de Doutorado, ICMC - USP, São Carlos. (2009).*
- SANTOS, L. M. R.; SANTOS, R. H.; ARENALES, M. N.; RAGGI, L. A. *Crop rotation scheduling with adjacency constraint.* Annals of Operations Research 190, p. 165-180, (2011).

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The average computational results for SA, AG+SA and M are given in the table:

Tabela: Mean of computational results for SA, AG+SA and Memetic

Method	t	$\overline{z}$	α	$p_{initial}$	p_{end}	$\overline{\Delta} z$
SA	158.4	1.86×10^6	98	52.0	0.02	0.25
AG+SA	189.6	2.20×10^6	99	57.0	0.01	0.32
M	568.2	1.98×10^6	99	44,3	0.01	0.24