Problem 1.1 Asymptotic Analysis - Albrit Bendo

a) $f(n) = 3n$; $g(n) = n^3$

$f \in O(g)$

$f(n) \leq g(n) \cdot c$

$3n \leq n^3 \cdot c$

True ✓ $O(n^3)$

as $c \geq 3 \rightarrow n^3 \geq 3n$

$f \in \Omega(g)$

$f(n) \geq g(n) \times c$

$3n \geq n^3 \times c$

False ✗

as $c \geq 3 \rightarrow n^3 \geq 3n$

$f \in \Theta(g)$

$c_1 g(n) \leq f(n) \leq c_2 g(n)$

$c_1 n^3 \leq 3n \leq c_2 n^3$

False ✗

as $c_1 n^3 \geq 3n$

$g \in O(f)$

$g(n) \leq f(n) \times c$

$n^3 \leq 3n \times c$

False ✗

as $n^3 \geq 3n \cdot c$

$g \in \Omega(f)$

$g(n) \geq f(n) \times c$

$n^3 \geq 3n \times c$

True ✓ $\Omega(n)$

as $n^3 \geq 3n \times c$

$g \in \Theta(f)$

$c_1 f(n) \leq g(n) \leq c_2 f(n)$

$c_1 \cdot 3n \leq n^3 \leq c_2 3n$

False ✗

as $n^3 \geq c_2 3n$

$f \in o(g)$

$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$

$\lim_{n \to \infty} \frac{3}{n^2} = 0$

True ✓ $o(n^3)$

as $g(n) > f(n)$

$f \in \omega(g)$

$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$

$\lim_{n \to \infty} \frac{3}{n^2} = \infty$

False ✗

as $g(n) > f(n)$

$g \in o(f)$

$\lim_{n \to \infty} \frac{g(n)}{f(n)} = 0$

$\lim_{n \to \infty} \frac{n^2}{3} = 0$

False ✗

as $g(n) > f(n)$

$g \in \omega(f)$

$\lim_{n \to \infty} \frac{g(n)}{f(n)} = \infty$

$\lim_{n \to \infty} \frac{n^2}{3} = \infty$

True ✓ $\omega(n)$

as $g(n) > f(n)$

b) $f(n) = 7n^{0.7} + 2n^{0.2} + 13 \log n$; $g(n) = \sqrt{n}$

$f \in O(g)$

$f(n) \leq g(n) \times c$

$7n^{0.7} + 2n^{0.2} + 13 \log n \leq \sqrt{n} \times c$

$7\sqrt[10]{n^7} + 2\sqrt[5]{n} + 13 \log n \leq \sqrt{n} \cdot c$

as $7n^{0.7} = 7n^{\frac{7}{10}}$ & $2n^{0.2} = 2n^{\frac{1}{5}}$

False ✗

as $f(n) \geq g(n) \times c$

$f \in \Omega(g)$

$f(n) \geq g(n) \times c$

$7\sqrt[10]{n^7} + 2\sqrt[5]{n} + 13 \log n \geq \sqrt{n} \cdot c$

True ✓ $\Omega(\sqrt{n})$

as $f(n) \geq g(n)$

$f \in \Theta(g)$

$c_1 g(n) \leq f(n) \leq c_2 g(n)$

$c_1\sqrt{n} \leq 7\sqrt[10]{n^7} + 2\sqrt[5]{n} + 13\log n \leq c_2\sqrt{n}$

False ✗

as $f(n) \geq g(n)$

$g \in O(f)$

$g(n) \leq f(n) \cdot c$

$\sqrt{n} \leq 7\sqrt[10]{n^7} + \sqrt[5]{n} + 13\log n \cdot c$

True ✓ $O(f)$

as $f(n) \geq g(n)$

$g \in \Omega(f)$

$g(n) \geq f(n) \cdot c$

$\sqrt{n} \geq 7\sqrt[10]{n^7} + 2\sqrt[5]{n} + 13\log n$

False ✗

as $g(n) < f(n)$

$g \in \Theta(f)$

$c_1 f(n) \leq g(n) \leq c_2 f(n)$

$c_1 * \left(7\sqrt[10]{n^7} + 2\sqrt[5]{n} + 13\log n\right) \leq \sqrt{n} \leq c_2 * \left(7\sqrt[10]{n^7} + 2\sqrt[5]{n} + 13\log n\right)$

False ✗

as $c_1 f(n) \geq g(n)$

$f \in o(g)$

$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$

$\lim_{n \to \infty} \frac{7\sqrt[10]{n^7} + 2\sqrt[5]{n} + 13\log n}{\sqrt{n}} = 0$

False ✗

as it tends to ∞ (infinity)
$f(n) > g(n)$

$f \in \omega(g)$

$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$

$\lim_{n \to \infty} \frac{7\sqrt[10]{n^7} + 2\sqrt[5]{n} + 13\log n}{\sqrt{n}} = \infty$

True ✓

as it tends to ∞ because
$f(n) > g(n)$

$g \in o(f)$

$\lim_{n \to \infty} \frac{g(n)}{f(n)} = 0$

$\lim_{n \to \infty} \frac{\sqrt{n}}{7\sqrt[10]{n^7} + 2\sqrt[5]{n} + 13\log n} = 0$

True ✓ $o(f)$

as it tends to 0

$g \in \omega(f)$

$\lim_{n \to \infty} \frac{g(n)}{f(n)} = \infty$

$\lim_{n \to \infty} \frac{\sqrt{n}}{7\sqrt[10]{n^7} + 2\sqrt[5]{n} + 13\log n} = \infty$.

False ✗

as it tends to 0

c) $f(n) = n^2 / \log(n)$; $g(n) = n \log n$

$f \in O(g)$

$f(n) \leq g(n) \times c_1$

$\frac{n^2}{\log n} \leq n \log n \cdot c_1$

$\div \log n$

$\frac{n^2}{(\log n)^2} \leq n \cdot c_1$

False ✗

as $f(n) \geq g(n)$

$f \in \Omega(g)$

$f(n) \geq g(n) \times c_1$

$\frac{n^2}{\log n} \geq n \log n \cdot c_1$

$\frac{n^2}{(\log n)^2} \geq n \times c_1$

True ✓ $\Omega(g)$

as $f(n) \geq g(n)$

$f \in \Theta(g)$

$c_1 g(n) \leq f(n) \leq c_2 g(n)$

$c_1 (n \log n) \leq \frac{n^2}{\log n} \leq c_2 (n \log n)$

$c_1 n \leq \frac{n^2}{(\log n)^2} \leq c_2 n$

False ✗

as $c_2 g(n) \leq f(n)$

$g \in O(f)$

$g(n) \leq f(n) \times c_1$

$n \log n \leq \frac{n^2}{\log(n)}$

$n \leq \frac{n^2}{(\log n)^2}$

True ✓ $O(f)$

as $g(n) < f(n)$

$g \in \Omega(f)$

$g(n) \geq f(n) \cdot c_1$

$n \log n \geq \frac{n^2}{\log(n)}$

$n \geq \frac{n^2}{(\log n)^2}$

False ✗ $\Omega(f)$

as $g(n) < f(n)$

$g \in \Theta(f)$

$c_1 f(n) \leq g(n) \leq c_2 f(n)$

$\frac{n^2}{(\log n)^2} \leq n \log n \leq \frac{n^2}{(\log n)^2}$

False ✗

as $c_1 f(n) > g(n)$

$f \in o(g)$

$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$

$\lim_{n \to \infty} \frac{\frac{n^2}{\log n}}{n \log n} = \frac{\cancel{n^2}}{\cancel{n}(\log n)^2} = 0$

$\frac{n}{(\log n)^2} = 0$

False ✗

as $f(n) > g(n)$

$f \in \omega(g)$

$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$

$\lim_{n \to \infty} = \frac{\frac{n^2}{\log n}}{n \log n} = \frac{n^2}{n(\log n)^2} = \infty$

$\frac{n}{(\log n)^2}$

True ✓ $\omega(g)$

as $f(n) > g(n)$

$g \in o(f)$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = 0$$

$$\lim_{n\to\infty} \frac{\frac{n\log n}{\frac{n^2}{\log n}}}{} = \frac{\frac{n(\log n)^2}{n^2}}{\frac{(\log n)^2}{n}} = 0$$

True ✓ $o(f)$

as $f(n) > g(n)$

$g \in \omega(f)$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \infty$$

$$\lim_{n\to\infty} \frac{n\log n}{\frac{n^2}{\log n}} = \frac{\frac{n(\log n)^2}{n^2}}{\frac{(\log n)^2}{n}} = \infty$$

False ✗

as $f(n) > g(n)$

d) $f(n) = (\log(3n))^3$; $g(n) = 9\log n$

$f \in O(g)$

$f(n) \le c\,g(n)$

$(\log(3n))^3 \le c(9\log n)$

True ✓ $O(f)$

as $(\log 3n)^3 < (9\log n)\cdot c$

$f \in \Omega(g)$

$f(n) \geq c\,g(n)$

$(\log 3n)^3 \geq c \cdot 9\log n$

False ✗

as $f(n) < g(n)$

$f \in \Theta(g)$

$c_1 g(n) \le f(n) \le c_2 g(n)$

$c_1(9\log n) \le (\log 3n)^3 \le c_2(9\log n)$

False ✗

as $c_1 g(n) > f(n)$

$g \in O(f)$

$g(n) \le c f(n)$

$9\log n \le c\,(\log 3n)^3$

False ✗

as $g(n) > f(n)$

$g \in \Omega(f)$

$g(n) \geq f(n)\cdot c$

$9\log n \geq (\log 3n)^3$

True ✓ $\Omega(f)$

as $f(n) > g(n)$

$g \in \Theta(f)$

$c_1 f(n) \le g(n) \le c_2 f(n)$

$c_1(\log 3n)^3 \le 9\log n \le c_2(\log 3n)^3$

False ✗

as $g(n) \geq c_2 f(n)$

$f \in o(g)$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$$

$$\lim_{n\to\infty} \frac{(\log 3n)^3}{8 \log n} = 0$$

True ✓ $o(g)$

as $g(n) > f(n)$

$f \in \omega(g)$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$$

$$\lim_{n\to\infty} \frac{(\log 3n)^3}{8 \log n} = \infty$$

False ✗

as $f(n) < g(n)$

$g \in o(f)$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = 0$$

$$\lim_{n\to\infty} \frac{8 \log n}{(\log 3n)^3} = 0$$

False ✗

as $g(n) \not> f(n)$

$g \in \omega(f)$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \infty$$

$$\lim_{n\to\infty} \frac{8 \log n}{(\log 3n)^3} = \infty$$

True ✓ $\omega(f)$

as $f(n) < g(n)$

Note: I got the results by building the graph for each of them but since there are 40 graphs to be written / drawn I did not include them here.

P.S.: Hope it won't be a problem.