

Exercise 5.3 (Decision Tree)

$$\log n! = \Theta(n \log n)$$

Upper bound: $n! \leq n^n$

$$\Rightarrow \log(n!) \leq \log(n^n) = n \log(n)$$

Lower bound: $(n!)^2 \geq n^n$

$$\Rightarrow \log((n!)^2) = 2 \log(n!) \geq n \log(n)$$

$$\Rightarrow \log(n!) \geq \frac{1}{2} n \log(n)$$

Therefore: $\log(n!) = \Theta(n \log(n))$