Problem 2.2 Albrit Bendo

a) $T(n) = 36T(n/6) + 2n$

By Master Method we know $\left.\begin{array}{l} a = 36 \\ b = 6 \\ f(n) = 2n \end{array}\right\} T(n) = \Theta(n^{\log_b a})$

if $f(n)$ polynomially smaller than $n^{\log_b a}$

$\left.\begin{array}{l} \Rightarrow n^{\log_6 36} = n^2 \\ 2n = n^1 \end{array}\right\} \Rightarrow n^{\log_b a} > f(n) \Rightarrow \Theta(n^2)$

b) $T(n) = 5T(n/3) + 17n^{1.2}$

By Master Method we know $\left.\begin{array}{l} a = 5 \\ b = 3 \\ f(n) = 1.2 \end{array}\right\} T(n) = \Theta(n^{\log_b a})$

if $f(n)$ polynomially smaller than $n^{\log_b a}$

$\left.\begin{array}{l} \Rightarrow n^{\log_3 5} = n^{1.4} \\ 17n^{1.2} = n^{1.2} \end{array}\right\} \Rightarrow n^{\log_b a} > f(n) = \Theta(n^{1.4})$

c) $T(n) = 12T(n/2) + n^2 \lg n$

We get $a = 12$, $b = 2$, $f(n) = n^2 \lg n$ where $f(n) = n^2 \lg n$

$n^{\log_b a} = n^{\log_2 12} = n^{3.5}$

$\Rightarrow n^{3.5} > n \log_{10} n$

regularity condition

$af(n/b) \le cf(n) \quad c < 1$

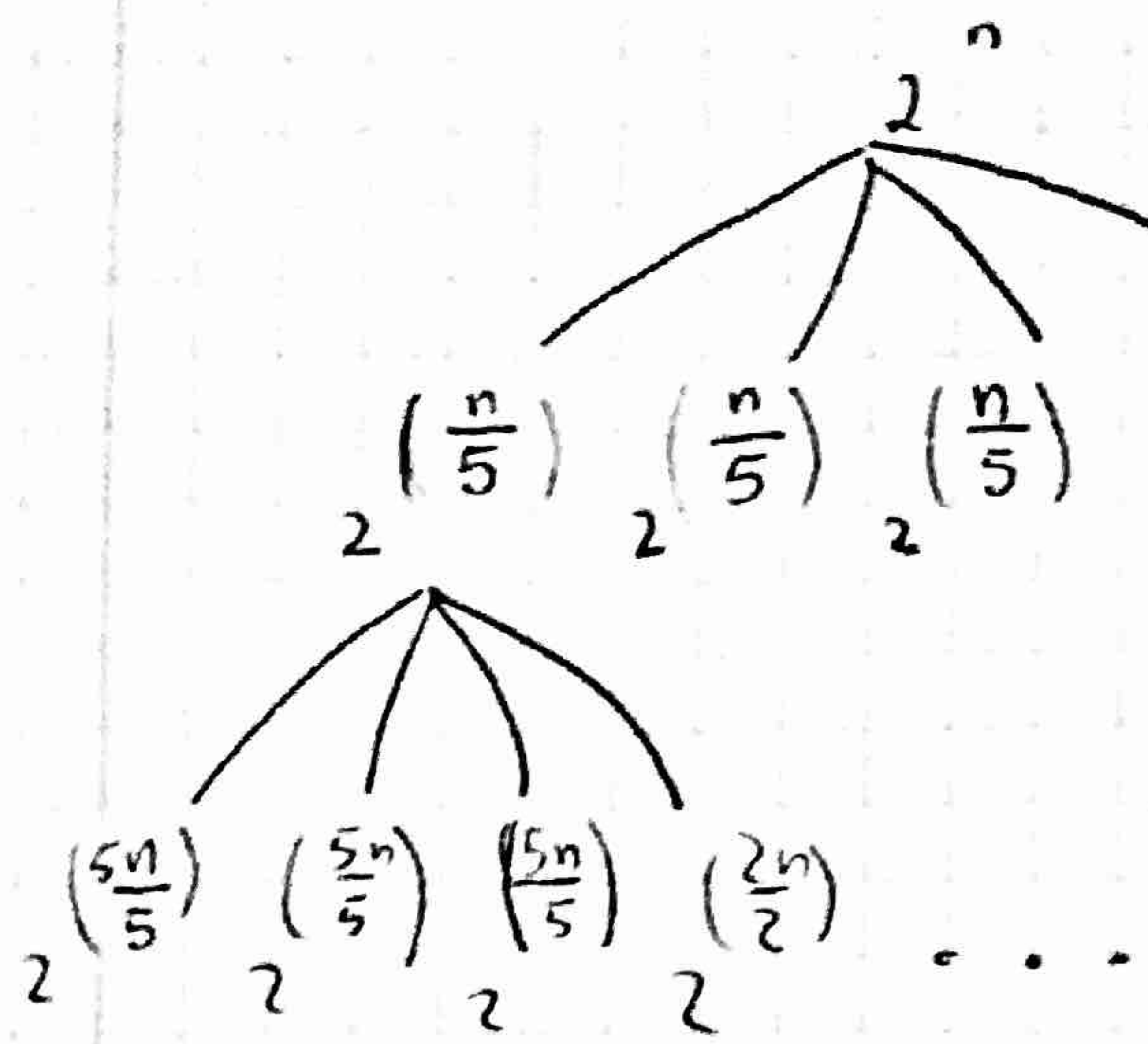
$a \cdot (n/b)^2 \log_{10}(n/b) \le cf(n)$

$12(n/2)^2 \log_{10}(n/2) \le cf(n)$

$\frac{12}{4} = 3$ True for $c = \frac{12}{4}$

So $T(n) = \Theta(f(n)) = \Theta(n^{\log_2 12})$

d) $T(n) = 3T(n/5) + T(n/2) + 2^n$ - Using Recursion Tree



We get:

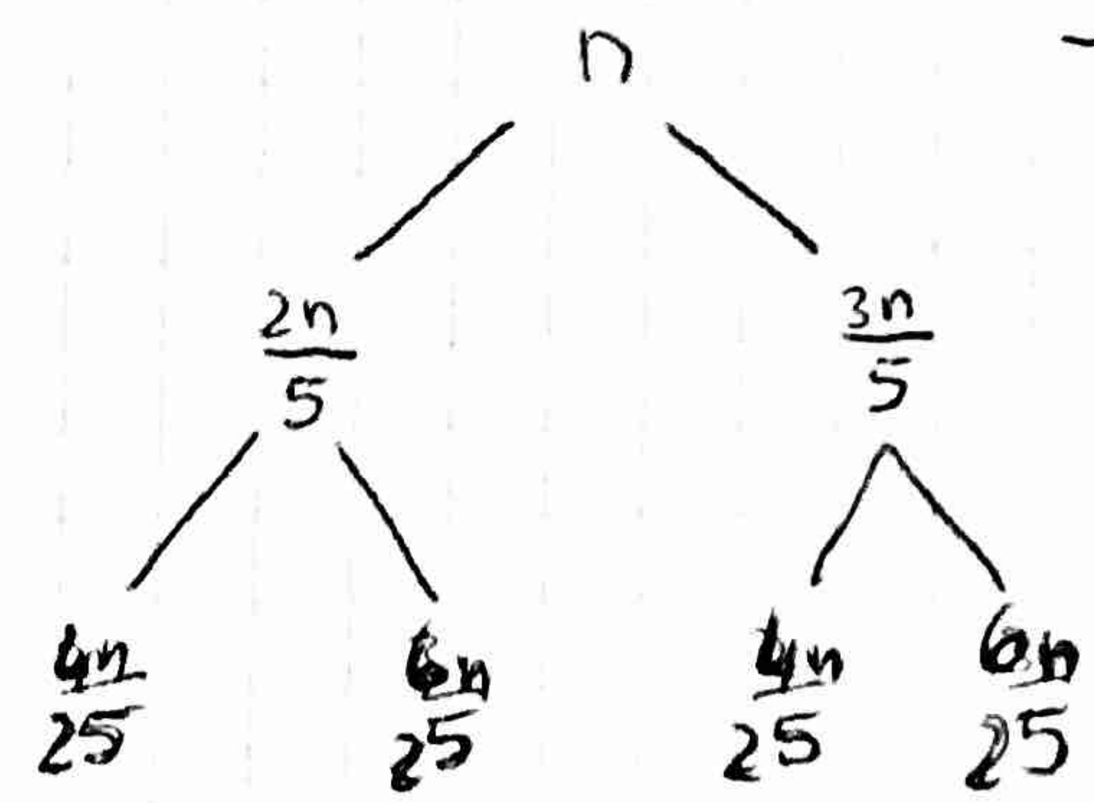
$2^n + 3 \times 2^{(\frac{n}{5})} + 2^{(\frac{n}{5})} + 2^{(\frac{5n}{5})} + \ldots$

we see that the common base is 2^n as:

$\underline{2^n}(1 + \ldots)$ geometric series

Therefore $\Theta(2^n)$

e) $T(n) = T(2n/5) + T(3n/5) + \Theta(n)$ - Using Recursion Tree



- n

$T(n) = n + n + \ldots$

So $T(n) \in \Theta(n \lg n)$
$= \Theta(n \lg n)$