Algorithms 2021/2022

July Exam

Solve the following exercises.

1. Let $G(V, E, w)$ be a connected undirected weighted graph, that is, let V be the graph's set of nodes, let E be the graph's set of edges, and let $w : E \to \{0, 1\}$ be the function that assigns weights to edges. Observe that, for each edge $e \in E$, the weight of e is either 0 or 1. (For instance, $V = \{1, 2, 3\}$, $E = \{\{1, 2\}, \{2, 3\}\}$, and $w(\{1, 2\}) = 1, w(\{2, 3\}) = 0$.)

   Consider the following claim: *"If T is the set of edges of a minimum spanning tree of $G(V, E, w)$, then T is also the set of edges of a minimum spanning tree of $G(V, E, w')$, where $w'(e) = 1 - w(e)$ for each $e \in E$."*

   Determine whether the claim is true or false. That is, either prove the claim, or give a counterexample.

2. Let v be an array of n non-negative integers. We say that v is 3-increasing, if there exist three indices $0 \leq i < j < k \leq n - 1$ such that $v[i] < v[j] < v[k]$. Give an algorithm that returns `True` if v is 3-increasing, and `False` otherwise. *(Faster solutions will be awarded more points.)*

   Example 1: if $v = [5, 2, 1, 7, 0, 9]$, the Algorithm should return `True` ($i = 2, j = 3, k = 5$ is a triple of indices $0 \leq i < j < k \leq n - 1$ such that $v[i] < v[j] < v[k]$);

   Example 2: if $v = [5, 2, 1, 7, 0]$, the Algorithm should return `False` (there are no indices $0 \leq i < j < k \leq n - 1$ such that $v[i] < v[j] < v[k]$);

   Example 3: if $v = [1, 10, 20, 50, 100]$, the Algorithm should return `True` (indeed, in this case, each triple of indices $0 \leq i < j < k \leq n - 1$ satisfies $v[i] < v[j] < v[k]$).