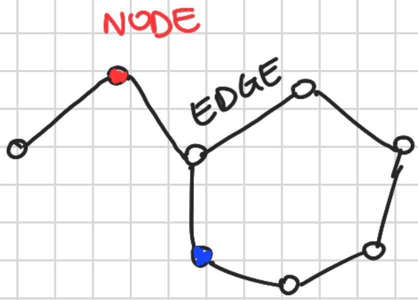
Algorithms

DATE: 22/2/22
LOL



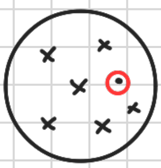
ALGORITHMS
↓
mathematical procedure that allows us to tell what's the shortest path from the red point to the blue point.

- **OPTIMISATION PROBLEMS**
  - → we will use optimized algoriths.
  - → we will be looking for the most optimal solutions.

DETERMINISTIC ALGORITHM → doesn't make random decisions.

- **OPTIM. PROBLEM (shortest path, minimum spanning tree)**

SPANNING TREE → graph with no cycles.

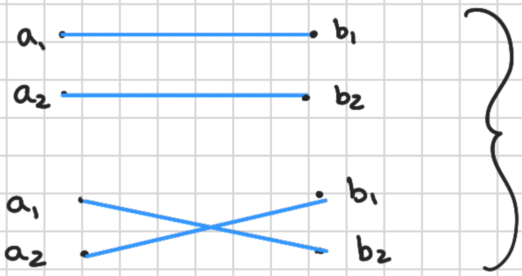

→ shortest path

- **ALGORITHM TECHNIQUES**

  - greedy algorithms
  - dinamic programming
  - divide - et - impera (divide - and - conquer)

"STABLE MATCHING" PROBLEM

COMPANIES (a_n) APPLICANTS (b_n)



POSSIBLE MATCHES BETWEEN COMPANIES AND APPLICANTS

b_1 PREFERS a_1 to a_2
b_2 " a_2 to a_1
a_1 " b_1 to b_2
a_2 " b_2 to b_1

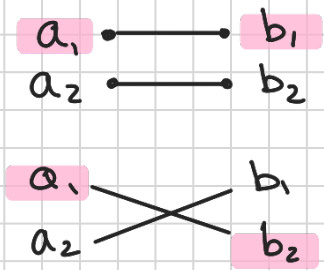
IF WE CONSIDER THESE CASES, EVERYONE WILL BE SATISFIED

prefers

$a_1 : b_1 > b_2$
$a_2 : b_1 > b_2$

$b_1 : a_1 > a_2$
$b_2 : a_1 > a_2$

$\Rightarrow$



WORKS ONLY FOR a_1 & b_1

WORKS ONLY FOR A_1 & b_2

UNSTABLE MATCHING

Def: Let A and B be two sets of cardinality $|A| = |B| = n$, with $A \cap B = \emptyset$:

A perfect matching between A and B is a pairing of the elements of A with the elements of B. That is, $M \subseteq [\{a,b\} \mid a \in A \wedge b \in B]$

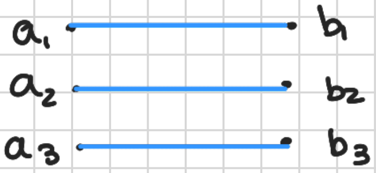
M is a perfect matching if and only if (IFF) $\forall a \in A$ there exists exactly one $b \in B$ such that $\{a, b\} \in M$ (AND viceversa)

example

One company will have only one applicant and viceversa.

$A = \{a_1, a_2, a_3\}$

$B = \{b_1, b_2, b_3\}$



$M = \{\{a_1,b_1\},\{a_2,b_2\},\{a_3,b_3\}\}$

M is a matching IFF $\exists$ bijective function

$f: A \to B$ $\quad \{\{a, f(a)\} \mid a \in A\} = M$

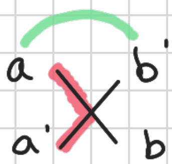
NOT A MATCHING (b_1 left alone, b_2 has 2 matches)

STABLE MATCHING

Let A, B be two sets, $|A| = |B| = n$, $A \cap B = \emptyset$ suppose that each $a \in A$ has a preference order on B and, likewise, each $b \in B$ has a preference order on A.

Given a perfect matching M of A and B, we say that M is unstable if it contains two pairs $\{a, b\}$, $\{a', b'\} \in M$, such that:

- a PREFERS b' to b AND
- b' 〃 a to a'



A MATCHING M is STABLE, IF ITS NOT UNSTABLE (bruh...)

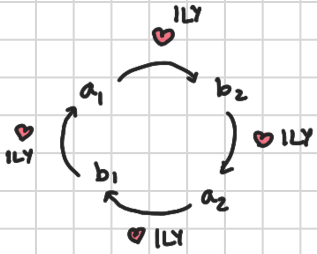
Questions:

① Does a stable matching always exist?
② How can we find a stable matching (if it exists)?

ANOTHER CASE :

$a_1 : b_1 > b_2$
$a_2 : b_2 > b_1$

$b_1 : a_2 > a_1$
$b_2 : a_1 > a_2$



INTUITIVE ALGORITHM

- suppose <u>a</u> is unmatched, let <u>a</u> propose an engagement to his most preferred <u>b</u> $\in B$
    - says "YES" IF b is unmatched,
    - says "NO" if b is matched with a' that she likes better than "a"
    - says "YES" if b is matched with "a'" and she likes "a" more than "a'"
- <u>a</u> NEVER ASKS the same b more than once.

(slowly dying .....)