EX: COMPUTE ALL THE $p[j]$'s OF THE WIS ALGORITHM IN $O(n)$ TIME.

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THE HEART OF A DP (DYNAMIC PROGRAMMING) APPROACH IS ITS TABLE M.

A DP APPROACH SOLVES A PROBLEM BY MEANS OF ITS SUBPROBLEMS.

FOR A DP APPROACH TO GO THROUGH IT IS USEFUL THAT:

(i) THERE IS ONLY A POLYNOMIAL NUMBER OF SUBPROBLEMS TO CONSIDER;

(ii) THE VALUE OF AN OPTIMAL SOLUTION FOR A SUBPROBLEM CAN BE COMPUTED EFFICIENTLY GIVEN THE VALUES OF THE OPTIMAL SOLUTIONS TO ITS SUBPROBLEMS;

(iii) THE SUBPROBLEMS SHOULD HAVE AN "ORDERING", SO THAT YOU CAN SOLVE THE RECURRENCE.

WIS: $OPT_j = \max\left(w_j + OPT_{p(j)},\ OPT_{j-1}\right)$.

(i) $n+1$ SUBPROBLEMS $(0 \le j \le n)$

(ii) $O(1)$ TIME (1 SUM, AND 1 MAX)

(iii) CONSIDER THE PREFIXES ORDERED BY THEIR SIZES $(p(j) < j$ AND $j-1 < j)$.

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SEGMENTATION

"THEYOUTHEVENT" → THE YOUTH EVENT

"THEYME" → THEY OUT HE VENT

→ THE YOU THE VENT

A PREFIX CODE IS GIVEN BY A SET $S$ OF BINARY STRINGS (CODEWORDS) SUCH THAT $\forall x, y \in S$, WITH $x \neq y$, $x$ IS NOT A PREFIX OF $y$.

PREFIX CODE:
A → 0
B → 10
C → 11

"ABC" → "$\underbrace{0}_{A}\underbrace{10}_{B}\underbrace{11}_{C}$"

---

NOT A PREFIX CODE:
A → 0
B → 01
C → 11

"ABC" → "$\underbrace{0}_{A}\underbrace{01}_{B}\underbrace{11}_{C}$"

$0^n 01^m$

IF $m$ IS ODD THEN "$A^n B C^{\frac{m-1}{2}}$"

IF $m$ IS EVEN THEN "$A^{n+1} C^{\frac{m}{2}}$"

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SEGMENTATION

WE ARE GIVEN A SEQUENCE OF TOKENS $w_1, w_2, \dots, w_n$, AND WE WANT TO SEGMENT IT — SELECT $1 = i_1 < i_2 < \dots < i_k = n-1$, FOR SOME $k \ge 1$ — THAT IS, TO CUT IT INTO THE SEGMENTS $(w_{i_1}, \dots, w_{i_2-1}), (w_{i_2}, \dots, w_{i_3-1}) \cdots (w_{i_{k-1}}, \dots, w_{i_k-1})$.

GIVEN THAT THE COST OF THE SEGMENT $(w_i, \dots, w_j)$ IS $c_{ij}$ $(\forall i \le j)$, FIND A SEGMENTATION OF MINIMUM TOTAL COST.

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(E.G., SET $c_{ij} = 1$ IF THE SLICE $s[i:j+1]$ DOES NOT CORRESPOND TO A WORD, AND SET $c_{ij} = 0$ OTHERWISE).

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LET $OPT(j)$ BE THE COST OF THE MIN-COST SEGMENTATION OF $w_1, w_2, \dots, w_j$:

L: $OPT(j) = \min_{i=1,\dots,j}\left(c_{ij} + OPT(i-1)\right)$ $\forall j \ge 1$, AND $OPT(0) = 0$.

```
DEF SEGMENTATION(C):        //C IS A n×n ARRAY
    M = [None] * (n+1)
    M[0] = 0
    FOR j = 1..n:
        m = C1j
        FOR i = 2,...,j:
            IF Cij + M[i-1] < m:
                m = Cij + M[i-1]
        M[j] = m
```

EX: FIND AN OPTIMAL SEGMENTATION USING M.
("ROLL BACK THE COMPUTATION STORED IN M")