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Exercise 1 1. Using the following expansions at $x = 0$

$$\begin{aligned}
\sqrt{1+y} &= 1 + \frac{1}{2}y - \frac{1}{8}y^2 + \frac{1}{16}y^3 + o(x^3)\,, \\
\cos y &= 1 - \frac{x^2}{2} + \frac{x^4}{4!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1})\,, \\
\sin y &= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})\,,
\end{aligned}$$

write the Taylor expansion around $x = 0$ up to order 5 of the function

$$f(x) = \sqrt{1+x^2} - \sin x$$

with Peano's remainder;

2. using the expansion of f compute the limit

$$\lim_{x\to 0} \frac{\sqrt{1+x^2} - \sin x - 1}{x \cos x^2}\,.$$

Exercise 2 (a) Compute the indefinite integral

$$\int \frac{2 - \sin x}{2x + \cos x}\, dx\,;$$

(b) Find the primitive $G(x)$ of

$$f(x) = \frac{2 - \sin x}{2x + \cos x}$$

such that $G(0) = 1$.

Exercise 3 Given the function

$$f(x) = \frac{x+2}{(|x|+3)(x-3)}$$

1. compute the definite integral
$$\int_{-3}^{0} f(x)\,dx;$$
2. compute the improper integral
$$\int_{-3}^{+\infty} f(x)\,dx\,.$$