Calculus-Unit 1

Applied Computer Science for AI

Written exam- Birindelli

Final grade
30 lode

Esercizes	Grade
1	4.+
2	3
3	4.+
4	6+
Mult. Ans.	15
Totale	32-

Family name: SEYEDMOHAMMADI

Name: MOBIN

Matricola:

Es. 1 [1+2+1 +0 Points] Given the sequence a_n defined in the following way

$$\begin{cases} a_0 = 10 \\ a_{n+1} = \frac{a_n}{4} + 1 \end{cases}$$

a) Compute a_1 and a_2.
b) Prove by induction that the sequence is monotone decreasing.
c) Determine the only possible value of the limit.
d) (Optional) Determine for which different value of a_o the sequence is increasing.

a) $a_1 = \frac{a_0}{4} + 1 = \frac{5}{2} + 1 = \frac{7}{2}$ $\quad a_2 = \frac{a_1}{4} + 1 = \frac{7}{8} + 1 = \frac{15}{8}$ X

b) $a_{n+1} < a_n \quad \frac{a_n}{4} + 1 < a_n \quad \frac{4}{3} < a_n$ as long as, $a_n > \frac{4}{3}$ it is decreasing o

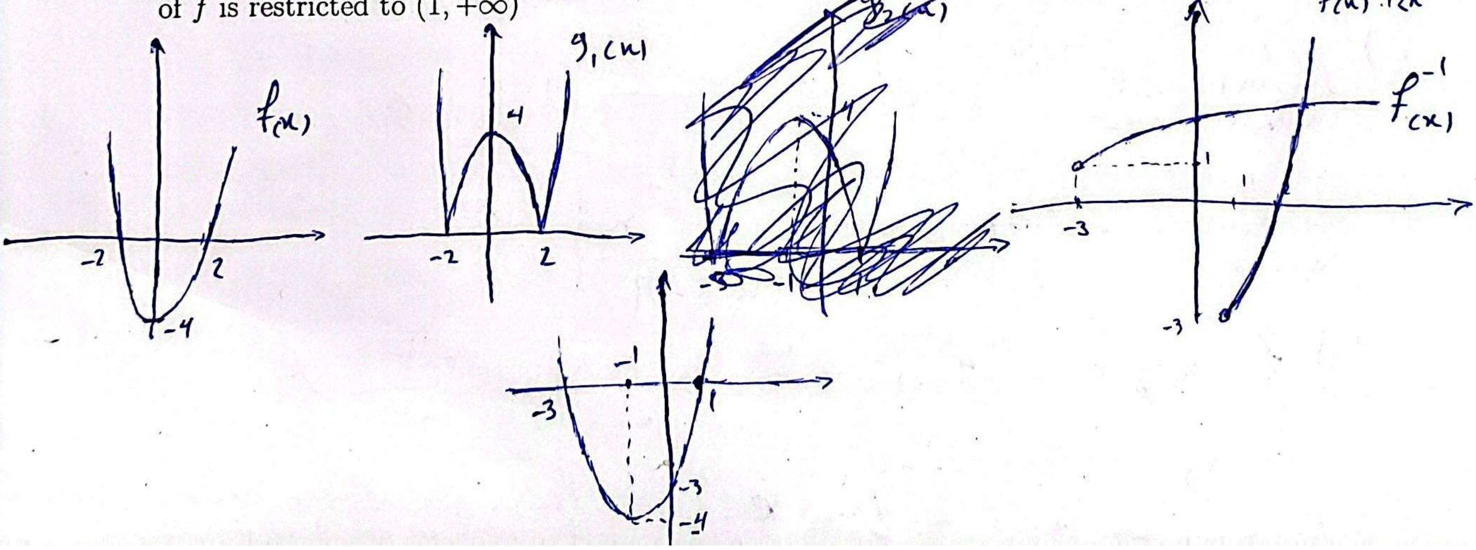
$\frac{4}{3} < a_n \Rightarrow \frac{4}{3} < \frac{a_n}{4} + 1 \Rightarrow \frac{4}{3} < a_{n+1} \Rightarrow$ as firs a_n is bigger than $\frac{4}{3}$ then all a are bigger than $\frac{4}{3}$. so it is allways decreasing x.

c) $a_n = \frac{a_n}{4} + 1 \quad \frac{3}{4} a_n = 1 \quad \boxed{a_n = \frac{4}{3}}$

D) as it is calculated in part-b- so if $a_0 < \frac{4}{3}$, the sequence was increa

$a_{n+1} > a_n \Rightarrow \boxed{\frac{4}{3} > a_n} \longrightarrow a_n < \frac{4}{3} \Rightarrow a_{n+1} < \frac{4}{3} \Rightarrow$ it is allways increasing

Es 2 [3 Points] Let $f(x) = x^2 - 4$. Draw the graph of the following functions: $g_1(x) := |f(x)|$, $g_2(x) := f(x+1)$, $g_3(x) = f^{-1}(x)$ (f^{-1} is the inverse function not $1/f$) provided that the domain of f is restricted to $(1, +\infty)$



Es 3 [4 points] Compute the following limit (justify your answer) $\lim_{x\to 0} \dfrac{2\cos(3x)-1}{xe^{x^2}\sin(2x)}$

~~$\lim_{x\to 0} \dfrac{2\cos(3x)-1}{xe^{x^2}\sin(2x)} = \dfrac{1-1}{0^+\times 1\times 0^+} = \dfrac{0}{0}$~~

~~$\lim_{x\to 0^-} \dfrac{2\cos(3x)-1}{xe^{x^2}\sin(2x)} = \dfrac{2-1}{0\times 1\times 0} = \dfrac{1}{0} = +\infty$~~

~~$\lim_{x\to\infty} \dfrac{2\cos(3x)-1}{xe^{x^2}\sin(2x)} = +\infty$~~

$\lim_{x\to 0} \dfrac{\cos(3x)-1}{xe^{x^2}\sin 2x} = \lim_{x\to 0} \dfrac{\left(1-\frac{9x^2}{2}\right)-1}{x\times 1\times 2x} = \dfrac{-\frac{9x^2}{2}}{2x^2} = \boxed{-\dfrac{9}{4}}$ ✗

Es 4 [1+2+1+2+1 points] Given the function

$$f(x) = \begin{cases} e^{\frac{x-1}{x}} & x \geq -1 \\ -(e^2)x & x < -1 \end{cases}$$

Determine:

a) Domain:

b) The limits at the boundary of the domains

c) The asymptotes

d) The derivative

e) The intervals of monotonicity

e) for $x<-1 \Rightarrow f'(x) = -e^2 < 0$ for all $\boxed{x<-1}$

for $x>-1$ $f'(x) = \dfrac{e^{\frac{x-1}{x}}}{x^2}$

$f'(x) > 0 \Rightarrow -1<x<0 \cup 0<x$ $(-1,0)\cup(0,+\infty)$

a) $x \neq 0 \Rightarrow D = \mathbb{R} - \{0\} \Rightarrow (-\infty, 0)\cup(0,+\infty)$ ✗

b) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} e^{\frac{x-1}{x}} = e^{-1/0^-} = e^{+\infty} = +\infty$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} e^{\frac{x-1}{x}} = e^{-1/0^+} = e^{-\infty} = 0$

} there is no limits defined for $f(x)$ at $x\to 0$ ✗

c) $\lim_{x\to\infty^+} f(x) = \lim_{x\to\infty^+} e^{\frac{x-1}{x}} = e^1 = e \Rightarrow \boxed{y = e}$ ✗

$\lim_{x\to\infty} f(x) = -(e^2)x \Rightarrow \boxed{y = -e^2 x}$

$\boxed{x = 0}$

D) for $x \geq -1 \Rightarrow f'(x) = e^{\frac{x-1}{x}} \times \dfrac{x-(x-1)}{x^2} = \dfrac{e^{\frac{x-1}{x}}}{x^2}$ ✗

for $x \leq -1 \Rightarrow f'(x) = -e^2$

for $x=-1 \Rightarrow$ $x=-1^+ \Rightarrow f(x) = e^2 \Rightarrow f'(x) = e^2$ $\Rightarrow$ so there is no derivatives for $f(x)$ in $x=-1$

$x = -1^- \Rightarrow f(x) = e^2 \Rightarrow f'(x) = -e^2$

Surname SEYED MOHAMMADI
name MOBIN
matricola

Es 5 [2 o -1 points] The function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \sqrt{1-x^2}$

1. Has a minimum but no maximum
2. Doesn't have a maximum or a minimum
3. Has a maximum but no minimum
4. Has a minimum and a maximum

Es 6 [2 o -1 punti] The derivative of $f(x) = \sin(2x)\log(\cos(2x))$ is:

1. $f'(x) = 2\left(\cos(2x)\log(\cos(2x)) - \frac{1}{\cos(2x)}\right)$
2. $f'(x) = -2\sin(2x)$
3. $f'(x) = 2\left(\cos(2x)\log(\cos(2x)) + \frac{\cos^2(2x)}{\sin(2x)}\right)$
4. $f'(x) = 2\left(\cos(2x)\log(\cos(2x)) - \frac{\sin^2(2x)}{\cos(2x)}\right)$
5. None of the other answers is correct

Es 7 [1/2 each answer] Let $f : [-1,1] \to \mathbb{R}$ be a continuous function. Then

1. The graph of the function f is symmetric since the domain is symmetric T F
2. If $f(-1) = f(1)$ then there exists x_o in the open interval such that $f(x_o) = f(1)$ T F
3. If f is invertible then it is monotone T F
4. The function reaches only the values between $\max f(x)$ and $\min f(x)$ T F
5. The function reaches all the values between $f(-1)$ and $f(0)$ T F

Es 8 [1/2 each answer] Given the value $z_1 = 1 - 3i$ in $\mathbb{C}$

1. $\frac{1}{z_1} = \frac{1}{10}(1+3i)$ T F True
2. $(3+i)z_1 = 6 - 8i$ T F
3. $(z_1)^3 = 1 - 9i$ T F
4. $|z_1| = 4$ T F

Es 9 [3 o -1 punti] Let a_n be a bounded sequence. Then necessarily

1. There exists a converging subsequence T F
2. The sequence is monotone T F
3. All subsequences converge T F
4. The sequence has a limit T F

Es 10 Let $f(x) = x^2\cos(3x)$. Then $T_5(x) = a_o + a_1x + a_2x^+a_3x^3 + a_4x^4 + a_5x^5$, the Taylor's polynomial of order 5 centered in zero is:

$a_o =$ 0 , $a_1 =$ 0 , $a_2 =$ 1 , $a_3 =$ 0 , $a_4 =$ -4.5 , $a_5 =$ 0