Exam - Computer Architecture Unit I [01/02/2024] (A)

Surname: ______________________________ **Name:** __________________________

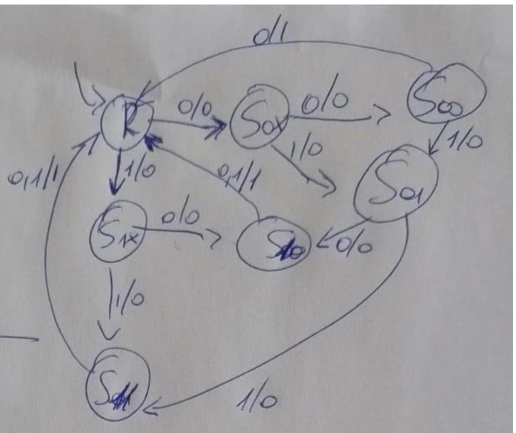
Student ID Number (Matricola): ____

DSA Students should solve only the first 4 exercises (grade will be scaled accordingly)

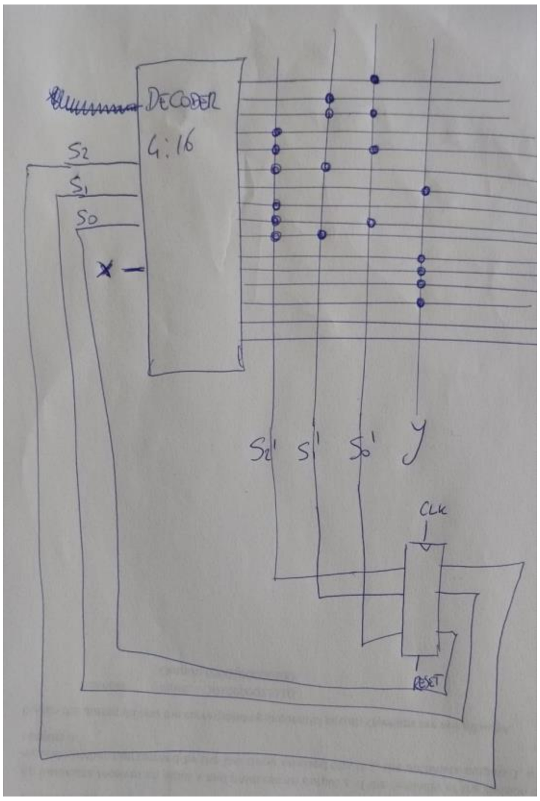
Exercise 1 (7 points) Design an FSM with an input x and an output y. The FSM outputs y when the last 3 bits, interpreted as a two's complement number, represent a negative number, or a number that is a multiple of 4. Do not consider overlaps. If the FSM didn't receive yet 3 bits, it outputs 0. Write down the circuit using a ROM for the combinational part.

Solution:

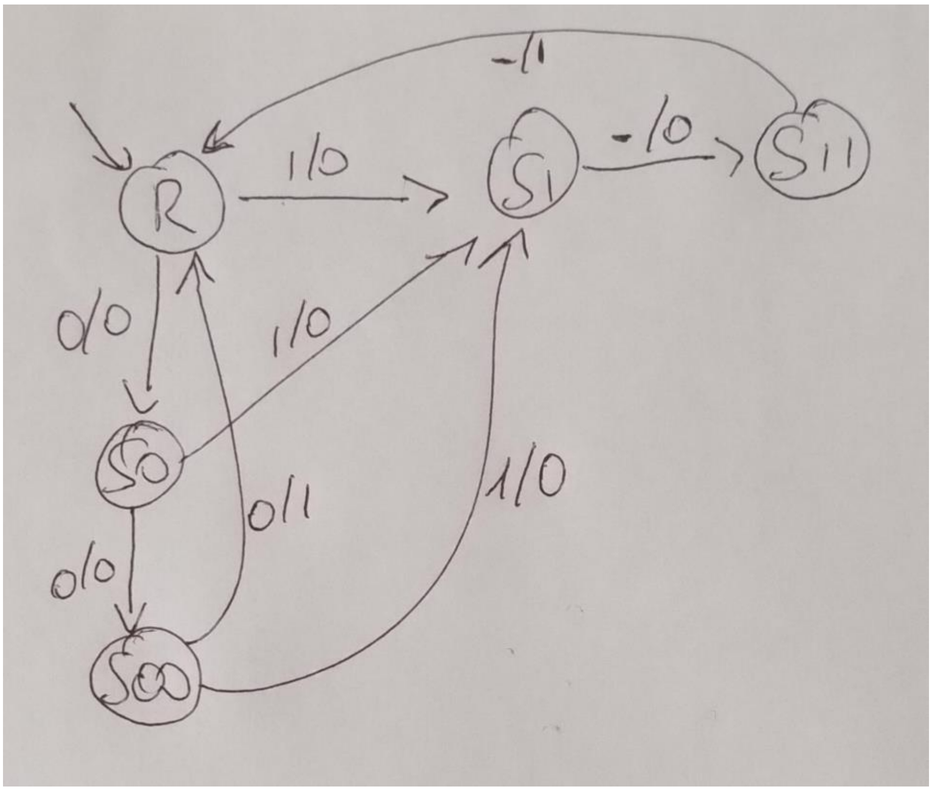
	x	y
(0)	000	1
(1)	001	0
(2)	010	0
(3)	011	0
(-4)	100	1
(-3)	101	1
(-2)	110	1
(-1)	111	1



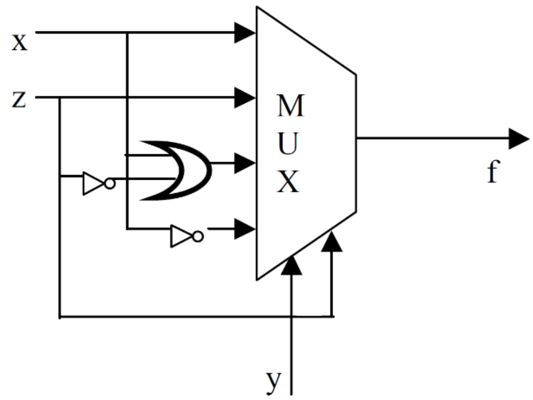
CS	S_2	S_1	S_0	x	NS	S_2'	S_1'	S_0'	Y
R	0	0	0	0	S_{0x}	0	0	1	0
R	0	0	0	1	S_{1x}	0	1	0	0
S_{0x}	0	0	1	0	S_{00}	0	1	1	0
S_{0x}	0	0	1	1	S_{01}	1	0	0	0
S_{1x}	0	1	0	0	S_{10}	1	0	1	0
S_{1x}	0	1	0	1	S_{11}	1	1	0	0
S_{00}	0	1	1	0	R	0	0	0	1
S_{00}	0	1	1	1	S_{01}	1	0	0	0
S_{01}	1	0	0	0	S_{10}	1	0	1	0
S_{01}	1	0	0	1	S_{11}	1	1	0	0
S_{10}	1	0	1	0	R	0	0	0	1
S_{10}	1	0	1	1	R	0	0	0	1
S_{11}	1	1	0	0	R	0	0	0	1
S_{11}	1	1	0	1	R	0	0	0	1



Note: Can also be solved using 5 states only (diagram shown below)



Exercise 2 (6 points) Write the function *f* described by the following circuit; then derive the minimal SOP and POS forms using K-maps; write down the all-NAND and all-NOR forms.



$$f = (\bar{y}\bar{z})x + (\bar{y}z)z + (y\bar{z})(x+\bar{z}) + (yz)\bar{x} =$$

$$= x\bar{y}\bar{z} + \bar{y}z + xy\bar{z} + y\bar{z} + \bar{x}yz$$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

z \ xy	00	01	11	10
0	0	1	1	1
1	1	1	0	1

$$POS = (x+y+z)(\bar{x}+\bar{y}+\bar{z})$$

$$SOP = \bar{x}z + y\bar{z} + x\bar{y}$$

$$\text{ALL-NAND} = \overline{\overline{\bar{x}z + y\bar{z} + x\bar{y}}} = \overline{\overline{\bar{x}z} \cdot \overline{y\bar{z}} \cdot \overline{x\bar{y}}}$$

$$\text{ALL-NOR} = \overline{\overline{(x+y+z)(\bar{x}+\bar{y}+\bar{z})}} = \overline{\overline{(x+y+z)} + \overline{(\bar{x}+\bar{y}+\bar{z})}}$$

Exercise 3 (4 points) Encode X = -129,125 as an IEEE 754 half-precision number. Then compute Z=X+Y, where Y=<0;00110;1110001100> in IEEE 754 half-precision format. Then convert the bits of Z to a base-16 number.

$X = -129{,}125 = 10000001.001 = 1.0000001001 \times 2^7$

$\boxed{\begin{array}{l} \text{Sign} = 1 \\ \text{Exponent} = 7 + 15 = 22 \\ \text{Mantissa} = 0000001001 \end{array}}$

$Y = +1.1110001100 \times 2^{-9} = 0.\underbrace{00 \cdots 0}_{16\ \text{bits}}1110001100 \times 2^7$

We only have 10 bits for the mantisse, thus $Y = 0$, and $Z = X + Y = X$

$Z = X = \underbrace{1\ 101}_{D}\underbrace{10\ 00}_{8}\underbrace{0\ 000}_{0}\underbrace{1001}_{9} = 0x\,D809$

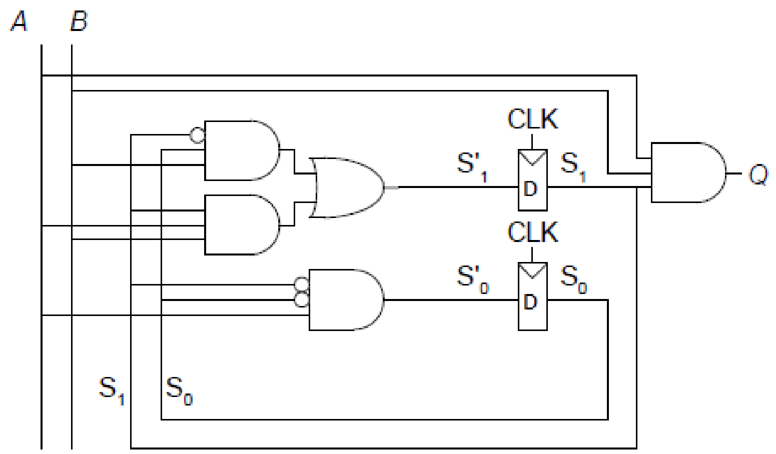
Exercise 4 (5 points) Describe in SystemVerilog an 8-bits shift register with synchronous reset and parallel load.

Solution

```
module shiftreg(input logic clk,
                input logic reset, load,
                input logic sin,
                input logic [7:0] d,
                output logic [7:0] q,
                output logic sout);
    always_ff @(posedge clk)
        if (reset) q <= 0;
        else if (load) q <= d;
        else q <= {q[N-2:0], sin};
    assign sout = q[N-1];
endmodule
```

Exercise 5 (5 points)

Analyze the following sequential circuit and draw the corresponding FSM. Then write down a sequence of input bits that, for the last bit, would produce an output equal to 1.



Solution:

$$S'_1 = \overline{S_1} S_0 B + S_1 A B$$

$$S'_0 = \overline{S_1}\,\overline{S_0} A$$

$$Q' = S_1 A B$$

Transition table

current state		inputs		next state		output
s_1	s_0	a	b	s'_1	s'_0	q
0	0	0	X	0	0	0
0	0	1	X	0	1	0
0	1	X	0	0	0	0
0	1	X	1	1	0	0
1	0	1	1	1	0	1
1	0	0	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	0	0	0

State encoding

state	encoding $s_{1:0}$
S0	00
S1	01
S2	10

State transition diagram:

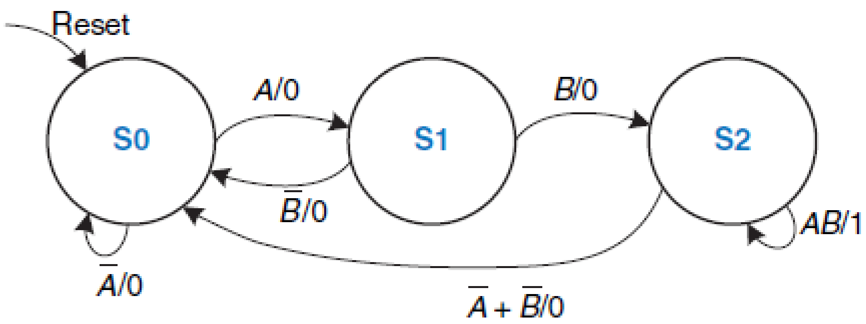


Figure 3.70 State transition diagram

Exercise 6 (3 points)

Using Boolean’s algebra theorems and axioms, simplify the following equation:

$$\left(a\bar{b} + \overline{\bar{b}c + c(\bar{a}+b)}\right) \oplus ac$$

Solution:
(You could also solve it in fewer steps by simplifying first (B’C + C(A’+B))’ = (B’C + A’C + BC)’ = (C + A’C)’ = C’)

$$\left(A\bar{B} + \overline{\bar{B}C + C(\bar{A}+B)}\right) \oplus AC =$$

$$= \left(A\bar{B} + \overline{\bar{B}C} \cdot \overline{C(\bar{A}+B)}\right) \oplus AC =$$

$$= \left(A\bar{B} + (B+\bar{C}) \cdot (\bar{C} + \overline{\bar{A}+B})\right) \oplus AC =$$

$$= \left(A\bar{B} + (B+\bar{C})(\bar{C} + A\bar{B})\right) \oplus AC =$$

$$= \left(A\bar{B} + \cancel{B\bar{C}} + \bar{C} + \cancel{A\bar{B}\bar{C}}\right) \oplus AC =$$

$$= (A\bar{B} + \bar{C}) \oplus AC =$$

$$= \overline{(A\bar{B} + \bar{C})}\,AC + (A\bar{B} + \bar{C})\,\overline{AC} =$$

$$= (\overline{A\bar{B}} \cdot C)\,AC + (A\bar{B} + \bar{C})(\bar{A} + \bar{C}) =$$

$$= (\bar{A} + B)\,AC + \cancel{A\bar{B}\bar{C}} + \cancel{\bar{A}\bar{C}} + \bar{C} =$$

$$= ABC + \bar{C} =$$

$$= AB + \bar{C}$$