Exam - Computer Architecture Unit I [27/06/2023] (A)

Surname: ______________________________ Name: __________________________

Student ID Number (Matricola): ____

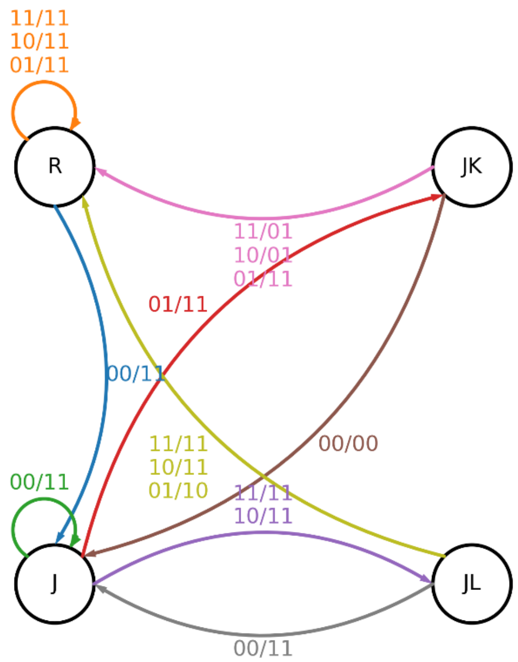
DSA Students should solve only the first 3 exercises (grade will be scaled accordingly)

Exercise 1 (8 points)

Design a sequential circuit with two inputs `a,b` that encode the characters J, K, L as follows:

`a,b`	character
00	J
01	K
1-	L

The circuit has 2 outputs `z` and `w`. The state machine outputs 00 when it finds the string JKJ, 01 when it finds the string JKL, 10 when it finds the string JLK, and 11 in all the other cases. Overlaps are allowed. Show the table and diagram describing the automata.



CS	S_0	S_1	a	b	NS	$\overline{S_0}$	$\overline{S_1}$	z	w
R	0	0	0	0	J	0	1	1	1
R	0	0	0	1	R	0	0	1	1
R	0	0	1	0	R	0	0	1	1
R	0	0	1	1	R	0	0	1	1
J	0	1	0	0	J	0	1	1	1
J	0	1	0	1	JK	1	0	1	1
J	0	1	1	0	JL	1	1	1	1
J	0	1	1	1	JL	1	1	1	1
JK	1	0	0	0	J	0	1	0	0
JK	1	0	0	1	R	0	0	1	1
JK	1	0	1	0	R	0	0	0	1
JK	1	0	1	1	R	0	0	0	1
JL	1	1	0	0	J	0	1	1	1
JL	1	1	0	1	R	0	0	1	0
JL	1	1	1	0	R	0	0	1	1
JL	1	1	1	1	R	0	0	1	1

Exercise 2 (7 points) A combinational circuit has a 4 bit input **abcd** and outputs Y=A+B in two's complement on 4 bits, where A is the unsigned number **ab** and B is the two's complement number **bcd**.

1. Write down the truth table
2. Implement Y with a PLA with the minimum number of rows.
3. We denote the bits of Y as Y= $y_3y_2y_1y_0$. Implement y_3 with at most three 4-to-1 multiplexers
4. We denote the bits of Y as Y= $y_3y_2y_1y_0$. Implement y_2 with NAND ports only

Solution

1.

a	b	c	d	y3	y2	y1	y0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	1
0	1	0	0	1	1	0	1
0	1	0	1	1	1	1	0
0	1	1	0	1	1	1	1
0	1	1	1	0	0	0	0
1	0	0	0	0	0	1	0
1	0	0	1	0	0	1	1
1	0	1	0	0	1	0	0
1	0	1	1	0	1	0	1
1	1	0	0	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	0	0	0	0	1
1	1	1	1	0	0	1	0

2.

y0 SOP kmap:

cd \ ab	00	01	11	10
00	0	1	1	0
01	1	0	0	1
11	1	0	0	1
10	0	1	1	0

y1 SOP kmap:

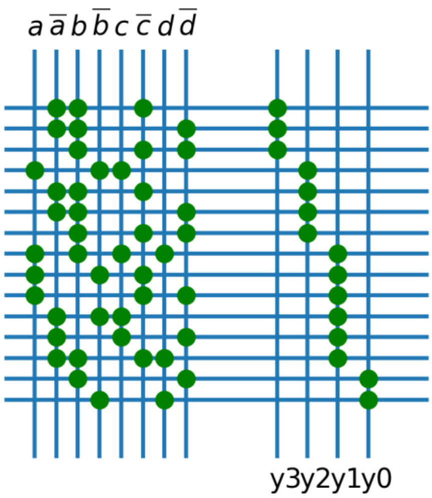
cd \ ab	00	01	11	10
00	0	0	1	1
01	0	1	0	1
11	1	0	1	0
10	1	1	0	0

y2 SOP kmap:

cd \ ab	00	01	11	10
00	0	1	1	0
01	0	1	0	0
11	0	0	0	1
10	0	1	0	1

y3 SOP kmap:

cd \ ab	00	01	11	10
00	0	1	1	0
01	0	1	0	0
11	0	0	0	0
10	0	1	0	0



3.

4.
SOP: (a and c and ~b) or (b and ~a and ~c) or (b and ~a and ~d) or (b and ~c and ~d)
ALL-NAND: ~(~(a and c and ~b) and ~(b and ~a and ~c) and ~(b and ~a and ~d) and ~(b and ~c and ~d)

Exercise 3 (5 points)
Write an HDL module for a 2:4 decoder.

Solution:

```
module decoder2_4(input logic [1:0] a,
                  output logic [3:0] y);
  always_comb
    case (a)
      2'b00: y = 4'b0001;
      2'b01: y = 4'b0010;
      2'b10: y = 4'b0100;
      2'b11: y = 4'b1000;
    endcase
endmodule
```

Exercise 4 (6 points)
a) Convert the number X=29,625 into IEEE half-precision format.
b) Consider the hexadecimal number **CBD0**, convert it to a 16-bit binary sequence, and interpret such sequence as a number Y in IEEE half-precision format.
c) Compute X+Y and X-Y and write the results into IEEE half-precision format.

Solution

a) $X=29{,}625_{(10)}=11101{,}101_{(2)}= 1{,}1101101_{(2)} \times 2^4$

In IEEE 754 half-precision format:
sign=0
exp=4+15=19=10011

X=<0;10011;1101101000>

b) $CBD0_{(16)}=1100101111010000_{(2)}$

In IEEE 754 half-precision format:

Y=<1;100010;1111010000>

c) To compute X+Y and X-Y I change Y's exponent. Since difference between exponents is 1, Y becomes: 0,1111101000 x 2^4

Since Y is negative, X+Y is a subtraction:

1,1101101 –
0,1111101

0,1110000 x 2^4 = 1,110000 x 2^3

After normalizing and recomputing the exponent, the result in IEEE format becomes:

<0;10010;1100000000>

Since Y is negative, X-Y is a sum:

1,1101101 +
0,1111101

10,1101010 x 2^4 = 1,01101010 x 2^5

After normalizing and recomputing the exponent, the IEEE format result is:

<0;10100;0110101000>

Exercise 5 (4 points)
Given the following Boolean expression *f,* simplify it using Boolean's algebra theorems and axioms and bring it to minimal POS form.

$$f = (\overline{(a+\bar{b})} \oplus \bar{b}c) + d$$

Solution:

$$\begin{aligned} f = (\overline{(a+\bar{b})} \oplus \bar{b}c) + d &= (\bar{a}b \oplus \bar{b}c) + d = \overline{\bar{a}b}\,\bar{b}c + \bar{a}b\,\overline{\bar{b}c} + d = (a+\bar{b})\bar{b}c + \bar{a}b(b+\bar{c}) + d \\ &= \bar{b}c + \bar{a}b + d = (\bar{b}+\bar{a})(\bar{b}+b)(c+\bar{a})(c+b) + d \\ &= (\bar{b}+\bar{a}+d)(c+\bar{a}+d)(c+b+d) \end{aligned}$$