$$\sum_{n=1}^{\infty} \frac{3^n(n^2+\cos(e^n))}{5^n}(x-1)^n$$

1) Find the radius of convergence of the following power series

2) Study the series at the extremes of the interval of convergence

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Using Taylor expansion of $f(x) = e^x$ and $g(x) = \log(1+x)$, solve the following limit

$$\lim_{x\to 0} \frac{7(e^x-1)+3\ln(1+x)-(3x^3+2x^2+5x)}{x^3}$$

Solve the following Cauchy problem and determine for what interval the proposed solution exists

$$\begin{cases} y' = \dfrac{2y}{x\ln y} \\ y(1) = \sqrt{e} \end{cases}$$

Determine if the following improper integral exists or not

$$\int_{-1}^{2} \frac{\sqrt{x+1}+1}{x^2-2x-3}\,dx$$