INTRODUCTION

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-Francesca Cinelli

PHYSICS

20 February

Physics deterministic

- initial conditions known
- equations on how the system evolves

=> product

Deterministic laws

↳ system is reversible => conservation of information

↓ dynamical if it changes with time

↓ 1 single incoming arrow
1 single outgoing arrow

- not everything can be determined, the predictability is limited by resolution power

ex. p. 1

Scientific method

experience ⟶ hypothesis $\xrightarrow{\text{predict}}$ experimental verification

logical fallacies

$X \to C \to H \to P$

- if H, then C

1) not C -> H is false

2) C -> H is true ✗

C could depend on other things

└ no observation consistent with the hypothesis can confirm it

⟶ hypothesis needs to be falsifiable

Laws in physics

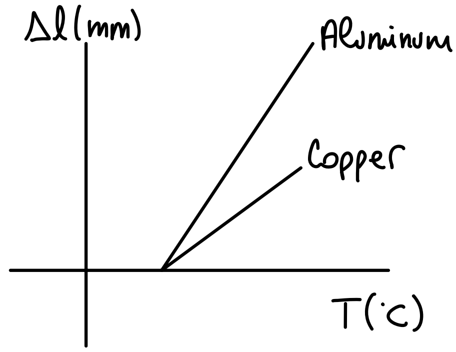
experiments → measurements of physical quantities
- associates a number to " "
- we have a reliability error
- key element = reproducibility → consistent results n times

laws → quantitative relation among physical quantities
└ validity → valid in region where the law was tested with general law

1) Thermal expansion

$\boxed{l = l_0(1 + \alpha T)}$ → linear relation

- the limit is melting



ex.

$\boxed{s = vt}$ - limit: v must be constant ↪ but also $v \ll c$

$c = 3 \cdot 10^8$ m/s
↳ special relativity
↳ ex. p.2 (muons)

International system of units → SI
↳ length (m)
↳ mass (kg)
↳ time (s)

Time (s)

def. A second is the time taken by 9192631770 oscillations of the light (of a specified wavelength) emitted by a cesium-133 atom

Length (m)

def. the meter is the length of the path traveled by light in a vacum during a time interval of $\frac{1}{\underbrace{3\cdot10^8}_{c}}$ of a second

Mass (Kg)

{see CMB}

def. A cylinder of platinum and iridium

Dimensional analysis

[ ] -> dimensions of a physical quantity

$[x] = L \quad [t] = T \qquad x = \frac{1}{2}at^2$

↳ derive the dimensions of "a" just by consistency $\Rightarrow [a] = \frac{L}{T^2}$

ex. p.3 for Plank's time

↳ time it takes for a photon to travel a distance equal to plank length

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Errors

- A measurement is the experimental result of a series of operations
- key element is to assess the reliability

$l = (52.5 \pm 0.2)\,cm$ absolute error $\rightarrow \varepsilon = 0.2\ cm$

if a true value exists:

error = | measured value - true value |

Error originates from:

- instrumental limitations
- accidental causes

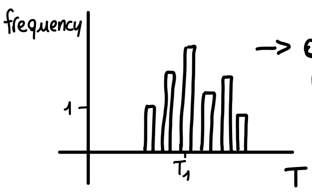
$\longrightarrow$

Errors are classified in:

- systematic $\rightarrow$ can be suppressed
- statistical $\rightarrow$ cannot

ex. p.3

Result of a measurement



$\rightarrow$ distribution of results

$$\langle T \rangle = \frac{1}{N}\sum_{i=0}^{N} T_i$$

$\downarrow$ average

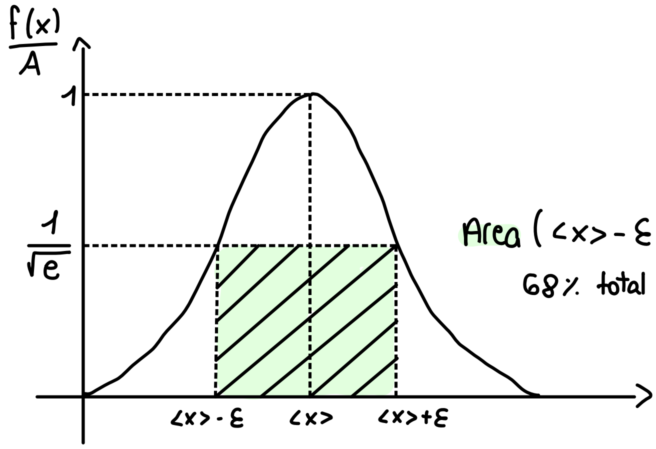
$$\varepsilon_T = \sqrt{\frac{1}{N-1}\sum_{i=0}^{N}(T_i - \langle T \rangle}$$

↳ standard deviation estimates the error you make when reporting $\langle T \rangle$ as a result

Gaussian distribution

$$f(x) = A e^{-\frac{(x-\langle x \rangle)^2}{2\varepsilon^2}}$$

if $x - \langle x \rangle = \varepsilon \longrightarrow f(x) = \frac{A}{\sqrt{e}}$



Area $(\langle x \rangle - \varepsilon, \langle x \rangle + \varepsilon) \simeq$ 68% total area

Propagation of error

$l_1, l_2 \rightarrow \varepsilon_1, \varepsilon_2$

1. $L = l_1 + l_2 \rightarrow \varepsilon_L = \varepsilon_1 + \varepsilon_2$

2. $L = l_1 - l_2 \rightarrow \varepsilon_L = \varepsilon_1 + \varepsilon_2$

3. ex. p. 3

In general:

$$a = f(x, y, \dots)$$

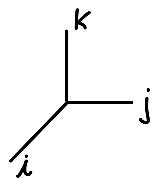
$$\varepsilon_a^2 = \left(\frac{df}{dx}\right)^2 \varepsilon_x^2 + \left(\frac{df}{dy}\right)^2 \varepsilon_y^2 + \dots$$

↳ we compute the derivative to take into account the variation from different PoV, depending on what error you are computing, since based on different parameters one might incide more than the other in computing total error

Math recap → p. 4-6

↳ important:

- $\vec{r} \cdot \vec{p} = |\vec{r}||\vec{p}| \cos\theta$
- $\vec{r} \times \vec{p} = |\vec{r}||\vec{p}| \sin\theta$



$$\vec{p} \times \vec{r} = (p_x \hat{\imath}, p_y \hat{\jmath}, p_z \hat{k}) \times (r_x \hat{\imath}, r_y \hat{\jmath}, r_z \hat{k})$$

$$= p_x r_y + p_x r_z + p_y r_x + p_y r_z + p_z r_x + p_z r_y$$

- Derivative = rate of change of a function

$$\vec{v} = \frac{d\vec{r}}{dt} \; ; \; \vec{a} = \frac{d\vec{v}}{dt}$$

→ is the quantity that describes the way v changes with t

- Integral = sequential sum → $A = \int_a^b f(t)dt = \lim_{\Delta t \to 0} \sum_i f(t_i)\Delta t$

Free fall in 1D

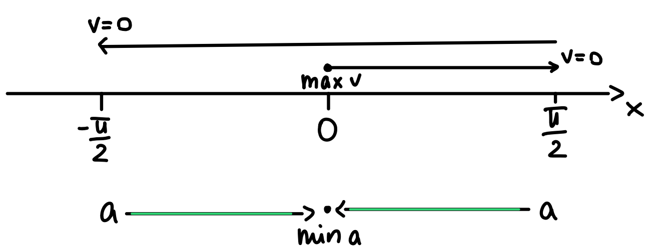
$$z(t) = z(0) + v_0 t - \frac{1}{2} g t^2 \underset{\frac{dz(t)}{dt}}{\overset{\int}{\rightleftarrows}} v_z(t) = v_0 - gt \underset{\frac{dv_z(t)}{dt}}{\overset{\int}{\rightleftarrows}} a_z = -g$$

ex. p. 7

Spring motion

$$x(t) = \sin(\omega t) \xrightarrow{d} v(t) = \omega \cos(\omega t) \xrightarrow{d} a(t) = \underbrace{-\omega^2 \sin(\omega t)}$$

same as x but opposite



ex. p. 7-8

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Components of vectors

$P_x = P\cos\theta \qquad P_y = P\sin\theta \qquad P = \sqrt{P_x^2 + P_y^2} \qquad \tan\theta = \frac{P_y}{P_x}$

$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \qquad \Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

Bullet motion - with no air

$= V_0\cos\theta \qquad = V_0\sin\theta$

- no horizontal acceleration
  - $V_{0x} = V_x \longrightarrow x = x_0 + V_{0x}t = x_0 + (V_0\cos\theta)t$
- vertical acceleration is free fall
  - $y = y_0 + V_{0y}t - \frac{1}{2}gt^2 = y_0 + (V_0\sin\theta)t - \frac{1}{2}gt^2$
  - $V_y = V_{0y} - gt = V_0\sin\theta - gt$

$\hookrightarrow V_y^2 = V_0^2\sin^2\theta + g^2t^2 - 2V_0\sin\theta\, g t$

ex. p. 8

$= (V_0\sin\theta)^2 + 2g\left(\frac{1}{2}gt^2 - V_0\sin\theta\, t\right)$

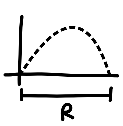
$= (V_0\sin\theta)^2 - 2g(y - y_0)$

Is the path parabolic?

$x - x_0 = V_0\cos\theta\, t \longrightarrow t = \frac{x - x_0}{V_0\cos\theta}$

$y - y_0 = V_0\sin\theta\, t - \frac{1}{2}gt^2 = \frac{\sin\theta}{\cos\theta}(x - x_0) - \frac{1}{2}g\frac{(x - x_0)^2}{(V_0\cos\theta)^2} = \underbrace{\tan\theta\,(x - x_0) - \frac{g}{2}\frac{(x - x_0)^2}{(V_0\cos\theta)^2}}_{y = ax + bx^2 \ \checkmark}$

Horizontal range - distance travelled



$x = x_0 + V_0\cos\theta\, t$

$R = x - x_0$

Note: if $\theta = 45^\circ$ R is max

$R = V_0\cos\theta\, t_R \longrightarrow t_R = \frac{R}{V_0\cos\theta}$

$y - y_0 = 0 = V_0\sin\theta\, t_R - \frac{1}{2}gt_R^2 = \frac{V_0\sin\theta}{V_0\cos\theta}R - \frac{1}{2}g\frac{R^2}{(V_0\cos\theta)^2} = \frac{V_0^2\sin\theta\cos\theta\,\cancel{R} - \frac{1}{2}gR^{\cancel{2}}}{(V_0\cos\theta)^2} = 0$

$\Longleftrightarrow V_0^2\sin\theta\cos\theta - \frac{1}{2}gR = 0 \longrightarrow R = \frac{2V_0^2}{g}\underbrace{\sin\theta\cos\theta}_{\sin(2\theta)} = \frac{V_0^2}{g}\sin(2\theta)$

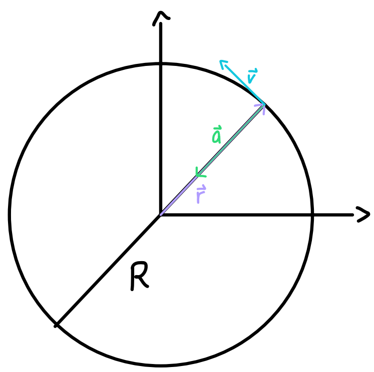
Maximum height

$$v_y = v_0 \sin\theta - g t_m \longrightarrow t_m = \frac{v_0 \sin\theta}{g}$$

$$\underbrace{y - y_0}_{h} = v_0 \sin\theta\, t_m - \frac{1}{2} g t_m^2$$

$$h = \frac{(v_0 \sin\theta)^2}{g} - \frac{1}{2}\frac{v_0^2}{g}\sin^2\theta \longrightarrow h = \frac{v_0^2 \sin^2\theta}{2g}$$

Circular motion



$$\vec{r} = R\cos\omega t\,\hat{\imath} + R\sin\omega t\,\hat{\jmath}$$

d

$$\vec{v} = -R\omega\sin\omega t\,\hat{\imath} + R\omega\cos\omega t\,\hat{\jmath}$$

d

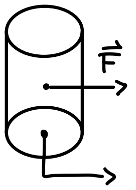
$$\vec{a} = -R\omega^2\cos\omega t\,\hat{\imath} - R\omega^2\sin\omega t\,\hat{\jmath}$$

1. $\vec{a} = -\omega^2 \vec{r}$
2. $\vec{r} \cdot \vec{v} = 0$
3. ω = angular velocity $= \frac{2\pi}{T}$
4. T = time needed to complete an oscillation $= \frac{2\pi R}{|v|}$
5. $|\vec{a}| = R\omega^2 = \frac{|v|^2}{R}$
6. $F = \frac{mv^2}{R}$

$$\left.\begin{array}{l} \text{if } t = T \rightarrow \cos(2\pi) \\ \text{if } t = \frac{T}{2} \rightarrow \cos(\pi) \end{array}\right\} \cos\omega t = \cos\left(\frac{2\pi}{T} t\right)$$

Forces

ex. p. 8-9



$m = 1kg$

ideal case = no friction

- we pull the object of 1kg to the right and we give it $a = 1 m/s^2$, the force that acts on the object has an intensity of 1 Newton -> $1N = 1 kg\ m/s^2$

Newton's laws - valid in inertial reference frames

1^{st}: in the absence of external forces, the object remains at rest or in uniform motion
↓
$\vec{F}_{net} = 0$

2^{nd}: the acceleration depends on the mass and on the amount of force $\longrightarrow \vec{F}_{net} = m\vec{a}$

↳ $F_{net,x} = ma_x$, $F_{net,y} = ma_y$, $F_{net,z} = ma_z$

3^{rd}: if an object exerts a force on another object, the second object exerts an equal and opposite force on the first

-> Forces are vectorial quantities

↳ 2 or more forces acting on a body should be summed vectorially

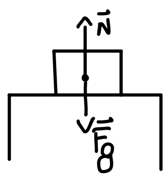
2 Mar

4 types of forces

1. Gravitational force $\vec{F}_g$

   $|F_g| = mg$ = weight

2. Normal force N



ground reaction force prevents solid objects from passing through each other

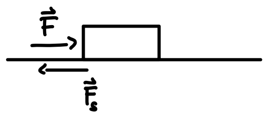
3. Friction force

resists the sliding or rolling of a solid object over another.

its direction is parallel to the surface and opposes motion.

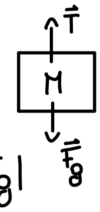
$|\vec{F}_s| = \mu |\vec{N}|$

↳ friction coefficient, depends on surface



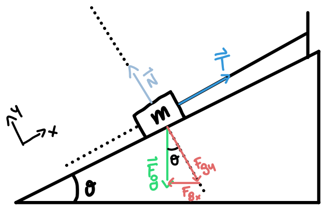
4. Tension

pulling force transmitted axially by the means of a string/rope



$|T| = |F_g|$

exercise



$\vec{T} + \vec{N} + \vec{F}_g = 0$

x: $T - mg\sin\theta = 0$

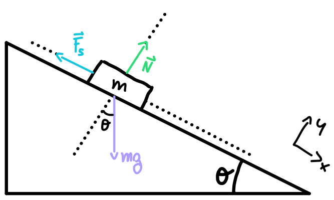
y: $N - mg\cos\theta = 0$

cut the rope → $T = 0$

→ $F_x = ma_x = -mg\sin\theta$

$a_x = -g\sin\theta$

Static friction coefficient



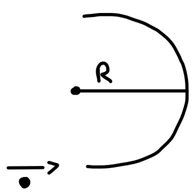
x: $mg\sin\theta - F_s = 0$ → $N\tan\theta - F_s = 0$ → $N\tan\theta = \mu_s N$

↓

$\tan\theta = \mu_s$

y: $N - mg\cos\theta = 0$ → $mg = \frac{N}{\cos\theta}$

Centripetal force



$\vec{F}_c = m\vec{a} = \frac{mv^2}{R} = \vec{F}_s = \mu_s N$

↳ friction between tire and road

$\left.\begin{array}{l} x: \frac{mv^2}{R} = \mu N \\ y: N = mg \end{array}\right\} \mu mg = \frac{mv^2}{R}$

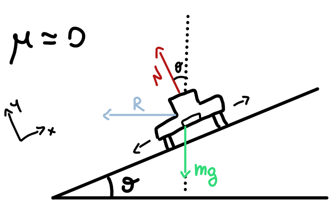
$v_{max} = \sqrt{\mu g R}$

↓

there is no mass!!

$\mu <<$ if ice

ice, $\mu \simeq 0$



x: $N\sin\theta = \frac{mv^2}{R}$ → $\tan\theta = \frac{v^2}{gR}$

y: $N\cos\theta - mg = 0$ → $N = \frac{mg}{\cos\theta}$

Energy conservation

- Kinetic energy $= \frac{1}{2} m v^2$
- potential energy

a particle with 0 velocity has 0 K energy -> $+V = +K$

$[K] = ML^2 T^2 = \text{Joules} \left(kg (m/s)^2 \right)$

$F(x) = -\frac{dU}{dx}$ -> derivative of potential energy -> $U(x) = -\int F(x)\,dx$

means that the force has always the direction that lowers p. energy

$\Delta U = -W$

$E = K + U$

$E = \frac{1}{2} m v^2 + U(x)$

Elastic potential energy

$U(x) = \frac{1}{2} K x^2$

Proof

if $E = \text{constant}$ -> $\frac{dE}{dt} = 0$

-> $\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2} m v^2\right) = \frac{1}{2} m 2 v \frac{dv}{dt} = mva$

-> $\frac{dU}{dt} = \frac{dU}{dx} \cdot \frac{dx}{dt} = \frac{dU}{dx} \cdot v$

$\frac{dE}{dt} = mva \frac{dU}{dx} v = v\left(ma + \frac{dU}{dx}\right)$

if E is constant -> $ma + \frac{dU}{dx} = 0$

how to derive eqn. of motion from p.energy

$ma - F(x) = 0$ -> Newton's law

$U = \frac{1}{2} A (x^2 + y^2)$ = potential energy

$F = -\frac{dU}{dx}$

x: $F_x = -\frac{dU}{dx} = -Ax$ → $ma_x = -Ax$ → $a_x = -\frac{A}{m} x$

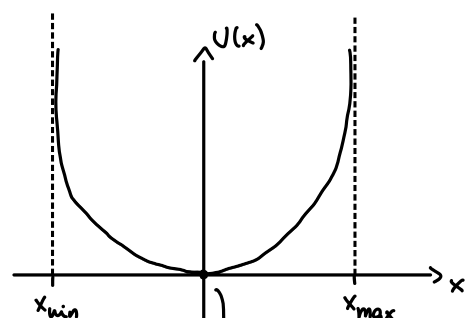
y: $F_y = -\frac{dU}{dy} = -Ay$ → $ma_y = -Ay$ → $a_y = -\frac{A}{m} y$

$\vec{a} = -\frac{A}{m} \vec{r}$ $\quad \vec{r}(x,y)$

Stable equilibrium

Hook's law for springs -> $F = -Ax$ (A -> constant; x -> displacement relative to natural length of the spring)

$F = -\frac{dU}{dx}$ -> $U = -\int F(x)\,dx$ -> $U(x) = \frac{1}{2} A x^2$ -> doesn't matter if x is + or − ↓ symmetric distribution



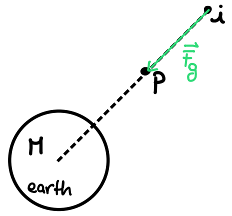
↳ x=0 natural length ↳ stable eq. -> will always return there $U(x) = \min$

ex. p. 10 - p. energy atoms in a molecule and distance
- unstable equilibrium

Potential energy of gravity

particle mass m

that moves from i to p



$U(y) = mgy$

Newton's gravity constant

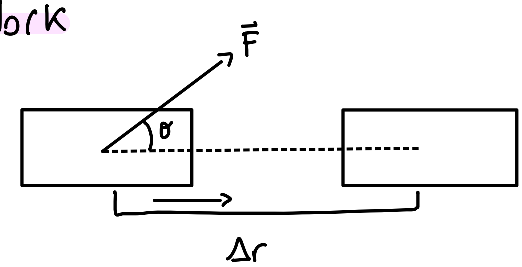
$\vec{F}_g = -\frac{GMm}{r^2}\hat{r}$

U_g = potential energy in point p $= -\int_{r_i}^{r_p} F(r)\cdot dr + U_i = GMm\int_{r_i}^{r_p}\frac{dr}{r^2} + U_i = -GMm\left(\frac{1}{r_p} - \underbrace{\frac{1}{r_i}}_{0}\right) + U_i$

choose i far away so that $U_i = 0 \rightarrow r_i = \infty$

$U_p = -\frac{GMm}{r}$

Work



→ energy transferred $\underbrace{\text{to}}_{+W}$ or $\underbrace{\text{from}}_{-W}$ an object by means of a force acting on the object

$[W] = Nm$ = Joules

Newton

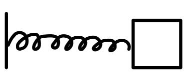
dW is a scalar quantity $= \vec{F}\cdot d\vec{r}$ $= Fr\cos\theta$

external force

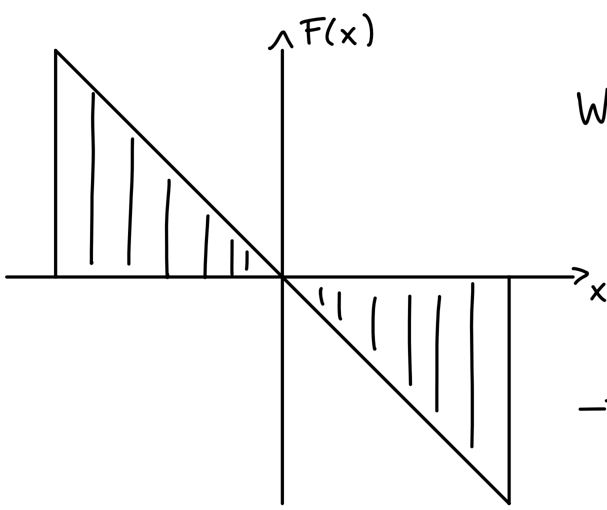
displacement

Note: $\Delta K = K_f - K_i = W$

Work of a spring



$F = -Ax$



$W = \int \vec{F}\cdot d\vec{r}$

$\{W_s = \frac{1}{2}Ax_i^2 - \frac{1}{2}Ax_f^2\}$

$\rightarrow W = 0$

Work of gravitational force

$W_g = mgd\cos\theta$

Conservative and non-conservative forces

-> Consider an object and surrounding system

-> the object and the rest of the ~ interact via a force

-> force acts on objects and does work W_1

-> things are inverted, the force does work W_2

} if force is conservative : $W_1 = -W_2$
↓
net work is going to be 0

• non-cons. = friction (process cannot be reverted, no potential)
 └ dissipative forces
  ↳ mechanical energy is not conserved

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Power

$\langle P \rangle = \dfrac{W}{\Delta t}$ -> average power of a force $[P] = ML^2T^{-3}$ (Joules/s)
↓
Watt

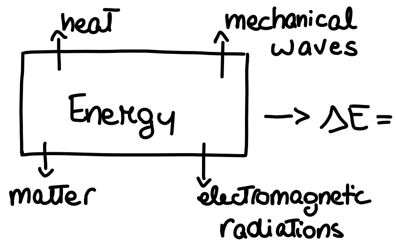
$P = \dfrac{dW}{dt} = \dfrac{\vec{F}d\vec{r}}{dt} = \vec{F}\cdot\vec{v}$ -> istantaneous power (scalar)

ex. p. 11

Isolated and non-isolated system

• isolated -> $\Delta E = 0$
 └ does not exchange energy with outside -> $E_{mec} = K + U$
  └ $\Delta E_{mec} = \Delta K + \Delta U = 0$

• non-isolated



-> $\Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$

ΔE_{mec} ↳ $\Delta K + \Delta U$

ΔE_{th} ↗ linked to the temperature of the system

ΔE_{int} ↳ dissipated

ex. p. 10-11

Center of mass

L its the point that moves as though all of the system's mass were concentrated there and all external forces were applied there

- 2 particles

  L $x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

  if $m_1 = m_2$

  $x_{cm} = \frac{m}{2m}(x_1 + x_2) = \langle x \rangle$

- many particles - useful when finite #

  L $\frac{1}{M}\sum_{i=1}^{n} m_i x_i \rightarrow \vec{r}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$

  $\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\, dm$

  differential mass element

ex. p. 12-13

p. 13-14 -> why CM is useful, problem of rigid bar and 2 masses

Linear momentum

$\boxed{\vec{p} = m\vec{v}}$ $[\vec{p}] = \frac{ML}{T}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt}$

$\Delta\vec{p} = \int_{t_i}^{t_f} \Sigma\vec{F}\, dt$

Impulse

$\vec{I}$ = impulse $= \Delta\vec{p}$ -> vectorial quantity

L> quantity of momentum that traverses the borders of the system due to applied force

L no applied force, $\Sigma\vec{F} = 0$

L> $\Delta\vec{p} = 0 \rightarrow \vec{p}$ is constant

L> conservation of momentum

ex. p. 11 decay of a pion

Collisions

L elastic -> kinetic energy is conserved

L inelastic -> " " " not "

} event during which 2 particles interact via a force

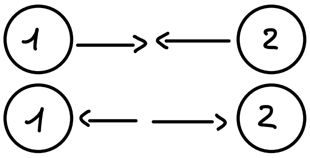
L completely inelastic - 2 bodies stick together after collision

L $v_f = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{m_1 + m_2}$

ex. p. 12

9 Mar

Elastic scattering (collisions)



$$\left.\begin{matrix} K \\ \vec{p} \end{matrix}\right\} \text{both Conserved} \quad \begin{cases} \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 & (2) \\ m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} & (1) \end{cases}$$

(2) $m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2)$

-> factorize $(a-b)^2 = (a-b)(a+b)$ -> $m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i})$ (2b)

(1) $m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i})$ (1b)

Divide $\frac{2b}{1b}$ -> $\frac{\cancel{m_1(v_{1i} - v_{1f})}(v_{1i} + v_{1f})}{\cancel{m_1(v_{1i} - v_{1f})}} = \frac{\cancel{m_2(v_{2f} - v_{2i})}(v_{2f} + v_{2i})}{\cancel{m_2(v_{2f} - v_{2i})}}$ -> $v_{1i} + v_{1f} = v_{2f} + v_{2i}$

$= v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$ (2c)

(1), (2c) $\longrightarrow$
$$\begin{cases} v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \\ v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i} \end{cases}$$

Case 1 -> $m_1 = m_2$

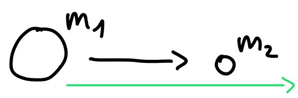
$$\begin{cases} v_{1f} = v_{2i} \\ v_{2f} = v_{1i} \end{cases}$$

Case 2 -> $v_{2i} = 0$ (at rest)

$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$

$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$

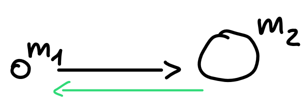
2.1 $m_1 >> m_2$



$v_{1f} = v_{1i}$

$v_{2f} = 2v_{1i}$

2.2 $m_2 >> m_1$



$v_{1f} = -v_{1i}$

$v_{2f} \sim 0$

Moment of inertia

$I = m \cdot r^2$ -> distance to rotation axis

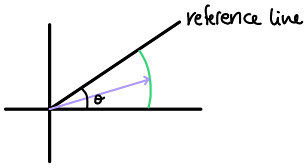
$= \sum m_i r_i^2$

$I = \int r^2 dm$

ex.p. 16

Rotation

$\theta = \frac{s}{r}$



Angular velocity

$\langle \omega \rangle = \frac{\Delta\theta}{\Delta t}$ $\quad \omega = \frac{d\theta}{dt}$

Angular acceleration

$\langle \alpha \rangle = \frac{\Delta\omega}{\Delta t}$ $\quad \alpha = \frac{d\omega}{dt}$

Velocity

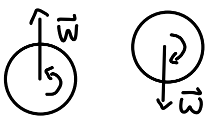
$\frac{dr}{dt} = \frac{d\theta}{dt} r \rightarrow v = \omega r$

Acceleration

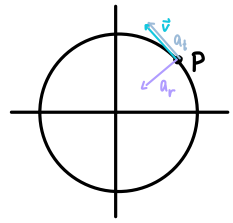
$a_t = \alpha r$

$a_r = \frac{v^2}{r} = \omega^2 r$

Rotating vector representation



└ in order to rotate a system you need a tangential force



Kinetic energy

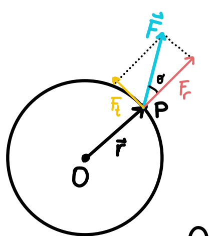
not = for all particles in rigid body

$K = \sum \frac{1}{2} m_i v_i^2$

$\hookrightarrow K = \sum \frac{1}{2} m_i (\omega r_i)^2 = \frac{1}{2}\Big(\underbrace{\sum m_i r_i^2}_{I}\Big)\omega^2$

$K = \frac{1}{2} I \omega^2$

Torque ($\vec{\tau}$)



radial component

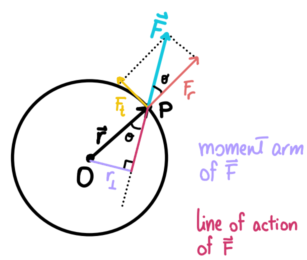
tangential component $\rightarrow F_t = F\sin\theta$

O = rotation axis

r → point where force acts

$\vec{\tau} = \vec{r} \times \vec{F} \rightarrow \tau = r(F\sin\theta) = rF_t$

$\rightarrow \tau = (r\sin\theta)F = r_\perp F$



Note:

$F_{net} = ma \rightarrow \tau_{net} = I\alpha \rightarrow \vec{\tau} = I\frac{d\vec{\omega}}{dt} = \frac{d}{dt}(I\vec{\omega}) = \frac{d\vec{L}}{dt}$

↳ Newtons 2nd law

↳ Newton's 2nd law in angular form

Work / Power

$W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$ $\quad P = \frac{dW}{dt} = \tau\omega$

13 March

Angular momentum

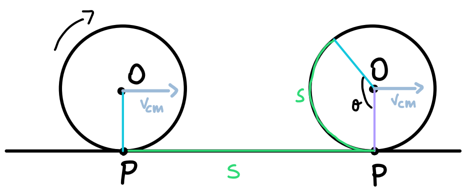
$\vec{L} = I\vec{\omega}$ → rigid body
↳ angular velocity

$\vec{\ell} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v}) = rmv\sin\theta$ → single particle

if net forces are zero, $\vec{L}$ is constant
$\vec{\tau} = 0 \rightarrow \vec{L} = \text{constant}$
} conservation law
↳ $\vec{L}_i = \vec{L}_f$

ex. p. 15

Rolling as a traslation and rotation



$s = \theta R$

linear speed → $v = \frac{ds}{dt}$
angular speed → $\omega = \frac{d\theta}{dt}$
} $v_{cm} = \omega R$

Kinetic energy of rolling

$$K = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

Smooth rolling motion

$a_{cm} = \alpha R$
↳ angular acceleration

Equilibrium (and elasticity)

$\vec{p} = \text{constant}$ $\vec{L} = \text{constant}$

- translational equilibrium
  $\vec{F}_{net} = 0$
- rotational equilibrium
  $\vec{\tau}_{net} = 0$

Gravitation

$F = \frac{GMm}{r^2}$ $G = 6.67 \times 10^{-11}\ m^3/kg \cdot s^2$

- earth is denser in the core $\rho \propto \frac{1}{r}$ (density, proportional)
- earth is flatter in poles → a_g higher there
- earth rotates

$a_g = \frac{GM}{r^2}$

$g = \underbrace{a_g - \omega^2 R}_{\text{effective acceleration}}$

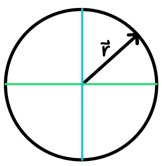
Kepler's laws

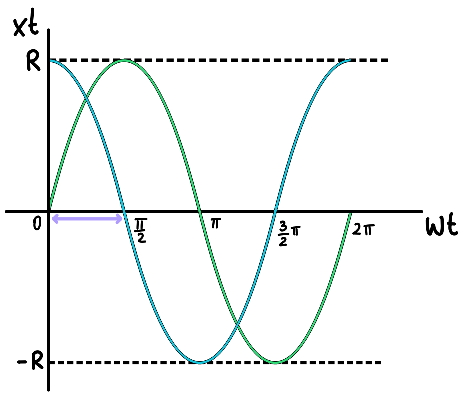
1. law of orbits: all planets move in elliptical orbits, with sun at one focus
2. law of areas: a line that connects a planet to the sun sweeps out equal areas in the plane of the planet's orbit in equal time intervals; that is, the rate $\frac{dA}{dt}$ at which it sweeps out area A is constant
3. law of periods: the square of the period of any planet is proportional to the cube of the semimajor axis of its orbit

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Oscillations and waves

$$\vec{r}(t) = R\cos\omega t\, \hat{\imath} + R\sin\omega t\, \hat{\jmath}$$





$x(t) = R\cos(\omega t)$

$y(t) = R\sin(\omega t)$

- phase is $\frac{\pi}{2}$

  ↳ shift

$\sin(\omega t) = \cos(\omega t - \frac{\pi}{2})$

$\to \omega t = 0 \Rightarrow \sin(0) = \cos(0 - \frac{\pi}{2}) = 1$

$\sin(\omega t) = \cos(\omega t + \frac{\pi}{2})$ ×

$[\cos(\omega t + \phi) = \cos\phi\cos\omega t - \sin\phi\sin\omega t]$

$x(t) = A\cos(\omega t + \phi)$

amplitude ↳ phase

$v(t) = -\omega A\sin(\omega t + \phi)$

$a(t) = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x(t)$

ex.p.15

16 March

Oscillations

$T = \text{period} = \frac{1}{\underset{\hookrightarrow \text{frequency}}{f}}$

harmonic motion - motion that repeats at regular intervals

$$x(t) = \underbrace{A}_{\text{amplitude}}\cos(\overbrace{\omega}^{\text{angular frequency}} t + \underbrace{\phi}_{\text{phase constant}}) \rightarrow \text{displacement} \qquad \omega = \frac{2\pi}{T}$$

$$v(t) = \frac{dx(t)}{dt} = \underbrace{-\omega A}_{\text{velocity amplitude}}\sin(\omega t + \phi)$$

$$a(t) = \frac{dv(t)}{dt} = -\underbrace{\omega^2}_{\text{acceleration amplitude}}\overbrace{A\cos(\omega t + \phi)}^{x(t)} = -\omega^2 x(t)$$

$$F = ma = m(-\omega^2 x) = -(m\omega^2)x$$

$$F = -Kx \rightarrow K = m\omega^2 \longrightarrow \omega = \sqrt{\frac{K}{m}} \rightarrow T = 2\pi\sqrt{\frac{m}{K}}$$

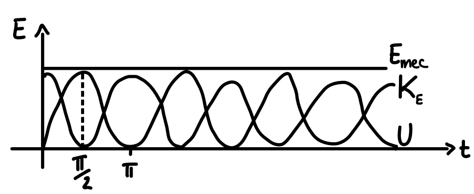
Energy

$$U(t) = \frac{1}{2}Kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

$$K_E(t) = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\sin^2(\omega t + \phi) = \frac{1}{2}KA^2\sin^2(\omega t + \phi)$$

$$E_{mec} = U + K_E = \frac{1}{2}kA^2\underline{\cos^2(\omega t + \phi)} + \frac{1}{2}kA^2\underline{\sin^2(\omega t + \phi)} \qquad \sin^2 + \cos^2 = 1$$

$$= \underbrace{\frac{1}{2}KA^2}_{\text{constant}}$$

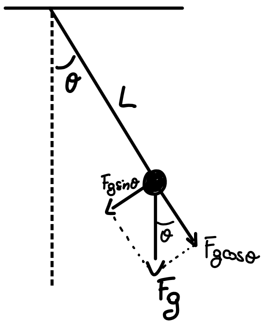


Angular harmonic motion

$\tau = -\kappa\theta$
↳ torsion constant (kappa)

$T = 2\pi\sqrt{\frac{I}{\kappa}}$ ⇝ inertia → Torsion pendulum

Pendulum - simple, small amplitude



$\tau = -L(F_g \sin\theta)$ $[\tau = r_\perp F]$

$-L(mg\sin\theta) = I\alpha$ $[\tau = I\alpha]$

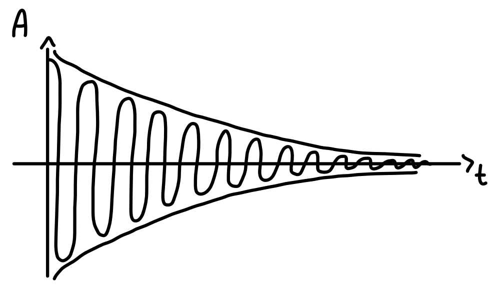
$\alpha = -\frac{mgL}{I}\theta$

$\omega = \sqrt{\frac{mgL}{I}} \rightarrow T = 2\pi\sqrt{\frac{I}{mgL}}$ $[I = mr^2]$

$T = 2\pi\sqrt{\frac{L}{g}}$

- physical, small amplitude → $T = 2\pi\sqrt{\frac{I}{mgh}}$

Damped harmonic motion (quenched)



↳ motion of oscillator reduced by external force

$F_d = -bv$
↳ damping constant

$-bv - kx = ma$

$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \rightarrow$ solution: $x(t) = \underbrace{Ae^{-\frac{bt}{2m}}}_{A'}\cos(\omega' t + \phi) \rightarrow \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

↳ angular freq. of the damped oscillator

↳ amplitude decreases with time exponentially

$E_{mec} = \frac{1}{2}kA^2 e^{-\frac{bt}{m}}$

↳ decreases exponentially with time (dissipation)

Resonance

$\underbrace{F_0\sin(\omega t)}_{\text{external force}} - \underbrace{\frac{b\,dx}{dt} - kx}_{\text{damped oscillation}} = ma \rightarrow x = A\cos(\omega t + \phi)$

⇝ friction

$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$

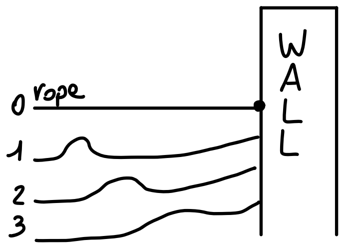
freq. ext. force ↲ ↳ freq. of system

if $b \rightarrow 0$
$\omega \rightarrow \omega_0$
A diverges!

Waves

↳ mechanical → need a medium to propagate (ex. sound)
↳ electromagnetic → propagate vacuum (ex. light)

- source of perturbation
- medium to be perturbated
- physical mechanism



if we flick the extreme of a rope, every segment of the rope moves in the direction perpendicular to the direction of propagation

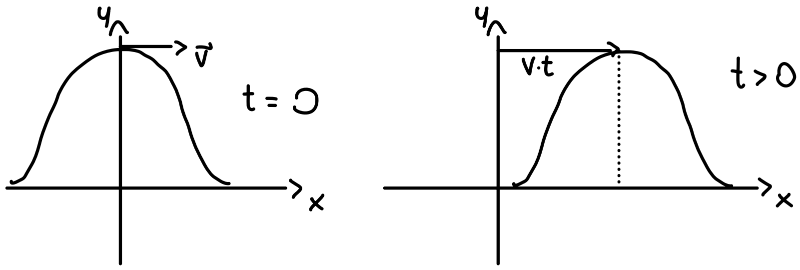
⇓

transverse waves

≠

longitudinal waves (ex. sound)

Mathematical representation of a wave



the wave shifts by distance v·t

→ assumption: the shape of the perturbation does not change with time

$y(x,t)$ → wave function

$y(x,0) = f(x)$ → shape of the impulse

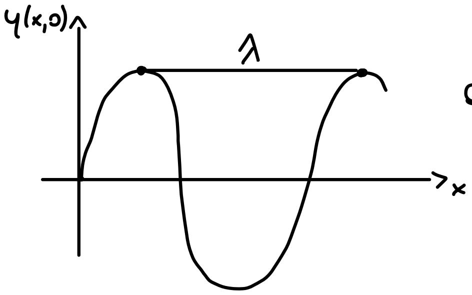
$y(x,t) = y(x-vt, 0)$

↳ shift

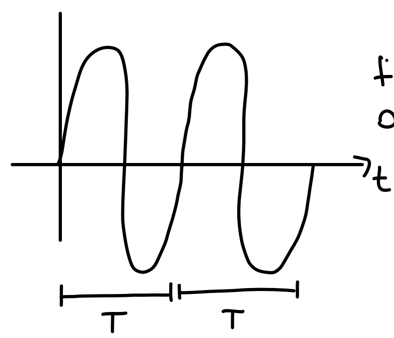
↳ $y(x,t) = f(x-vt)$ → impulse travelling to right

↳ $y(x,t) = f(x+vt)$ → " " " left

Wave length



distance between 2 identical points



time to do a complete oscillation

→ angular wave number

$y(x,0) = A\sin(ax)$ = wave shape

$y(x,0) = A\sin\left(\frac{2\pi}{\lambda}x\right)$

→ shift

$y(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x-vt)\right]$

↳ wave length

$v_w = \frac{\lambda}{T} = \frac{\Delta x}{\Delta t}$

Superposition of waves

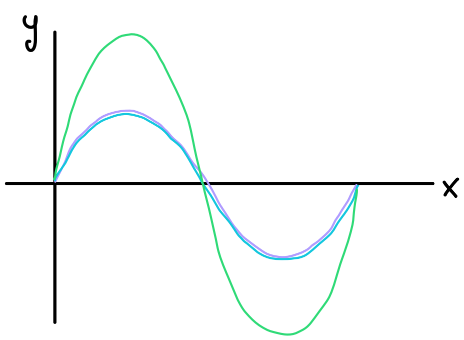
$$y'(x,t) = y_1(x,t) + y_2(x,t) = A\sin(kx-\omega t) + A\sin(kx-\omega t+\phi) = \underbrace{\left[2A\cos\left(\frac{\phi}{2}\right)\right]}_{\text{different amplitude}} \underbrace{\sin\left(kx-\omega t+\frac{\phi}{2}\right)}_{\text{sinusoidal function}}$$

sum of the sines with 2 angles α and β:

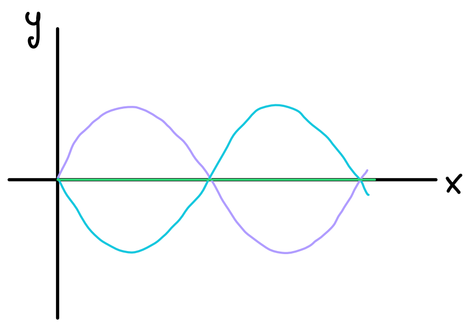
$$\sin\alpha + \sin\beta = 2\sin\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha+\beta)$$

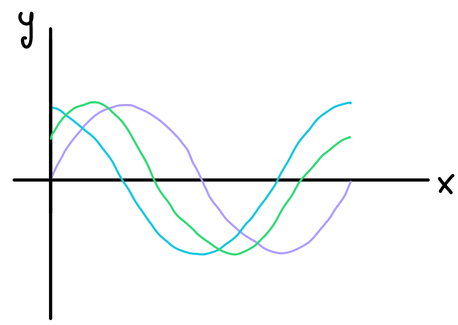
$$A' = 2A\cos\left(\frac{\phi}{2}\right)$$

- if $\phi = 0 \rightarrow A' = 2A \rightarrow$ constructive interference
- if $\phi = \pi \rightarrow A' = 0 \rightarrow$ destructive "

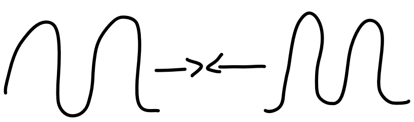


$y' = y_1 + y_2$





Standing waves



$$y'(x,t) = [2A\sin kx]\cos\omega t$$

amplitude varies with x

$A = 0$

$kx = n\pi$; $n = 0, 1, 2 \ldots$ $\quad k = \frac{2\pi}{\lambda}$

$x = \frac{n\lambda}{2}$

↳ nodes (points where amplitude is 0)

Doppler effect

$$f' = f\,\frac{v \pm v_D}{v \pm v_S}$$

- perceived frequency
- natural frequency
- speed of wave (ex. sound thru air)
- speed of detector / observer
- speed of source

rule:

when the motion of observer or source is toward the other, the sign on its speed must give an upward shift in frequency. When the motion of observer or source is away from the other, the sign on its speed must give a downward shift in frequency

23 March

- general notions on relativity → ex. p. 17

Block 2

Fluids and thermodynamics

→ fluids flow

→ fluids don't have a shape of its own → take shape of container

→ don't have ordered molecular structure

→ cannot maintain a shear stress at rest

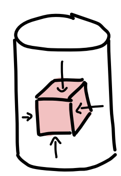
→ viscosity : friction between adjacent layers of the fluid

Density

$\rho = \frac{m}{V}$ → volume

Pressure

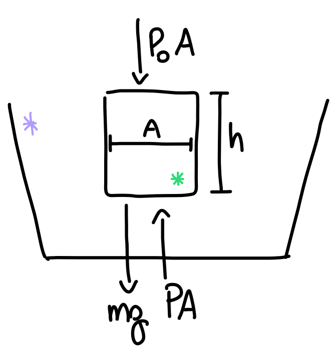
$P = \frac{F}{A}$ → area $[P] = \frac{N}{m^2} = Pa$



differential volume

ex. p. 18

Pressure in a fluid grows lineary with depth :



* fluid 2 = sample fluid

* fluid 1

Equilibrium:

$\Sigma F_y = PA - P_0 A - mg = 0$

$m = \rho V = \rho A h$ → $P = P_0 + \rho g h$

* just depends on depth → not on shape of container

‖

Pascal law

General formula:

$P_2 = P_1 + \rho g (y_1 - y_2)$ if $= 0$ then $P_1 = P_0$, $y_2 = h$

→ $P_0 = \rho g h$ density of the mercury in barometer → measurement of Pressure: - quicksilver column

$P = P_{ext} + \rho g h$

Archimede's law

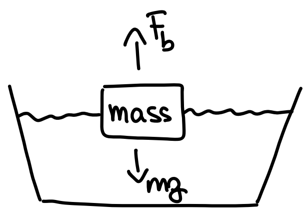
-> when a body is fully or partially submerged in a fluid, a buoyant force $\vec{F}_b$ from the surrounding fluid acts on the body. The force is directed upward and has a magnitude equal to the weight $m_f g$ of the fluid that has been displaced by the body

$$F_b = m_f g$$

↳ mass of the fluid displaced by body

-> when body floats

$$F_b = F_g \rightarrow F_g = m_f g \quad \Longrightarrow$$



$m = \rho_f V$

because if equilibrium:

$$-mg + F_b = 0$$

$$-m\cancel{g} + \rho_f V_m \cancel{g} = 0$$

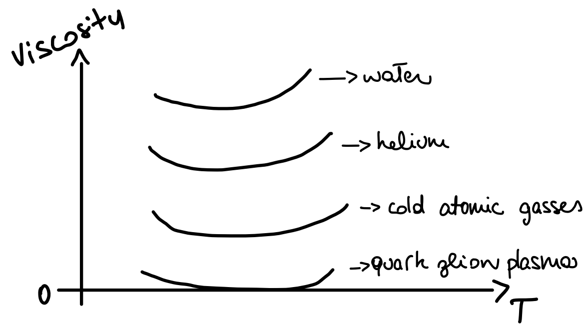
-> apparent weight

$$weight_{app} = weight - F_b$$

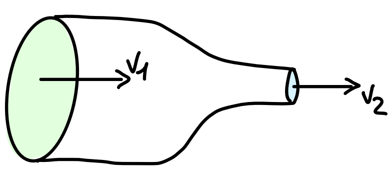
27 March

Ideal fluid

- Incompressible: density is constant
- Viscosity is zero
- stationary flow: velocity of each element of the fluid does not vary with time
- irrotational: angular momentum ($\vec{L}$) is zero at each point



Equation of continuity



$A_2 < A_1$

In a time Δt, what is ΔV (volume of fluid) that enters through A_1 (ideal fluid, density is constant) -> $\Delta V_1 = \Delta V_2$

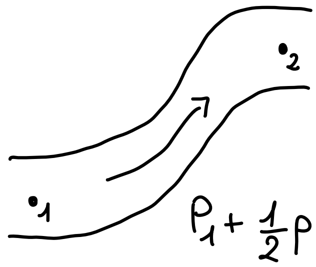
$$\Delta V_1 = A_1 \Delta x = A_1 v_1 \Delta t$$

$$\hookrightarrow A_1 v_1 \cancel{\Delta t} = A_2 v_2 \cancel{\Delta t}$$

$$A_1 v_1 = A_2 v_2 \qquad (Av = \text{volume flow})$$

Equation of Bernoulli

$$\underbrace{P}_{\text{pressure}} + \underbrace{\frac{1}{2}\rho v^2}_{\text{kinetic energy}} + \underbrace{\rho g h}_{\text{potential energy}} = \text{a constant}$$



$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

if $v = 0 \rightarrow P_1 - P_2 = \rho g h$ (Pascal)

ex. p. 18-19

3 April

Thermodynamics → energy transfers

→ explains the macroscopic properties of matter and their correlation to the mechanics of the atoms and molecules

→ in an isolated system, 2 objects that are at different temperature can transfer energy:
- ↳ heat
- ↳ electromagnetic radiation

energy is exchanged until thermal equilibrium is reached
↳ 2 bodies are at same temperature

Principle zero

→ if 2 bodies A and B are each in thermal equilibrium with body C, then A and B are in thermal equilibrium

Gas thermometer

→ defines the temperature as a function of internal energy

Kelvin scale

$T = -273.15\,^\circ C \rightarrow 0K = -273.15\,^\circ C$

Thermal expansion

→ linear expansion: $\Delta L = L\alpha\Delta T$ — coefficient of linear expansion

→ volume expansion: $\Delta V = V\beta\Delta T$ — coefficient of volume expansion
↳ $= 3\alpha$

Heat

-> energy transferred between a system and its environment because of a temperature difference that exists between them

$[Q]$ = calories
↳ heat

$1 \text{ cal} = 4.186 \text{ J}$
↳ necessary heat to increase the temperature of 1g of water by 1°C

Heat capacity

$$Q = C\Delta T = C(T_f - T_i)$$

$$C = \frac{Q}{\Delta T}$$

Specific heat -> change in heat

$$Q = cm\Delta T = cm(T_f - T_i)$$

$$c = \frac{Q}{m\Delta T} \qquad [c] = \text{J/kg°C}$$

Latent heat -> produce a change of phase

$L = \frac{Q}{\Delta m}$ (depends on nature of the change and substance) ex. p. 21

Macroscopic definition : Perfect gas

-> low density

-> short range forces

Mole

-> mass of substance containing $N_A = 6.022 \times 10^{23}$ molecules
N_A -> Avogadro's number

-> m_0 = mass of molecule $= \frac{M}{N_A}$
M -> molar mass

-> $n = \frac{N}{N_A}$
N -> number of molecules
n ↳ number of moles

Ideal Gasses

ex. p. 20

$pV = nRT$

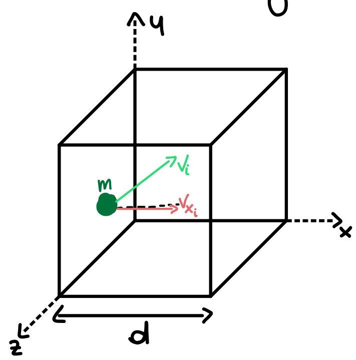
pressure (p), volume (V), gas constant (R) $= 8.31\ \mathrm{J/mol\cdot K}$

$= pV = \frac{N}{N_A} RT \longrightarrow pV = N k_B T$

Boltzmann constant

$k_B = \frac{R}{N_A} = 1.38 \times 10^{-23}\ \mathrm{J/K}$

Model for our gas



$\Delta p_{x_i} = -2 m v_{x_i}$

↳ momentum

$F_i \Delta t = \Delta p_{x_i} = -2 m v_{x_i}$

↳ average force exerted by wall on molecule i during collision time Δt

$\Delta t = \frac{2d}{v_{x_i}}$

↳ time between 2 collisions

$F_i = \frac{-m v_{x_i}^2}{d}$

↳ force exerted by wall

By Newton's third law $\rightarrow$ $F_{\text{particle}} = -F_{\text{wall}}$ (particle exerts on wall; wall exerts on particle)

$F = \frac{m v_{x_i}^2}{d}$

↳ force 1 particle exerts on wall

$F_N = \sum_{i=1}^{N} \frac{m v_{x_i}^2}{d} = \frac{m}{d} \sum_{i=1}^{N} v_{x_i}^2 = \frac{m}{d} N \overline{v_{x_i}^2}$ average

↳ force by N molecules on the wall

$\rightarrow$ particles move isotropically $(\overline{v^2} = 3\overline{v_x^2})$

↳ $F = \frac{m_0}{d} N\left(\frac{1}{3}\overline{v^2}\right) = \frac{N}{3} \cdot \frac{m_0 \overline{v^2}}{d}$

Pressure

$P = \frac{F}{A} = \frac{F}{d^2} = \frac{1}{3}\frac{N}{d^2}(m\overline{v^2}) = \frac{1}{3}\frac{N}{V}(m\overline{v^2})$

$P = \frac{2}{3}\frac{N}{V}\left(\frac{1}{2}m\overline{v^2}\right)$

$\frac{N}{V}$ ↳ density of molecules

$\left.\begin{array}{l} pV = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right) \\ pV = Nk_BT \end{array}\right\} \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right) = Nk_BT \Rightarrow T = \frac{2}{3}k_B\underbrace{\left(\frac{1}{2}m\overline{v^2}\right)}$

T: macroscopic temperature

↳ kinetic energy of molecules

↳ translational → $K = \frac{3}{2}k_BT$

↳ if we have only this: monoatomic molecules

ex. p. 20

$E^{kin} = E^{tot} = \frac{3}{2}nk_BT$ → only depends on T

Distribution of molecular speeds

→ Maxwell-Boltzmann

$$P(v) = 4\pi\left(\frac{m}{2\pi k_BT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2RT}}$$

13 April

First law of thermodynamics

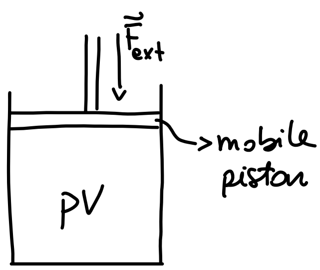
$\Delta E^{int} = Q + W$ → work done

ΔE^{int} ↳ change of internal energy

Q → transferred energy in form of heat

$\Delta E_{int} = Q - W$

Work in a thermodynamic transformation



$dW = \vec{F}_{ext} \cdot d\vec{r} = -pA\underbrace{d\vec{r}}_{dV} = -pdV$

$\vec{F}_{ext}$ ↓ $-F_{gas} = -PA$

↳ if gas compressed → dW

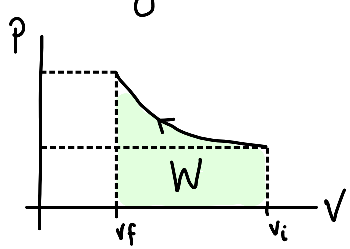
↳ " " expands → $-dW$

slow compression: the system is in equilibrium at every instant t

$$W = -\int_{V_i}^{V_f} p\, dV$$

ex. p. 22

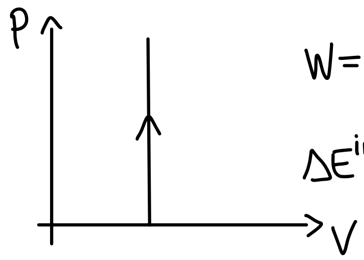
Path diagram



Types of transformations

ex. p. 21-23

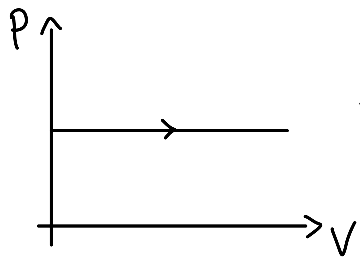
- transformation with constant volume (isochoric)



$W = 0 \quad (dV = 0)$

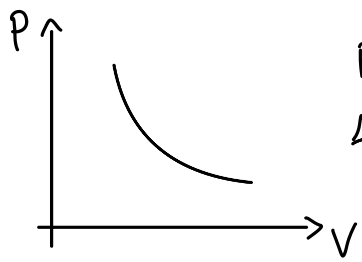
$\Delta E^{int} = Q$

- transformation with constant pressure (isobaric)



$\Delta E^{int} = Q + W$

- transformation with constant temperature (isothermal)



$pV = nRT = \text{constant}$

$\Delta E^{int} = 0 \;\rightarrow\; Q = -W$

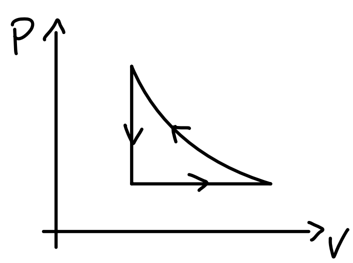
- adiabatic transformations (better at p. 21)

$Q = 0$

$\Delta E^{int} = W$ (thermically isolated systems)

$$\begin{cases} W = -p\,dV \qquad pV = nRT \\ p\,dV + V\,dp = nR\,dT \end{cases} \longrightarrow \frac{dV}{V} + \frac{dP}{P} = -\frac{(c_p - c_v)}{c_v}\,\frac{dV}{V} \qquad \begin{cases} PV^{\gamma} = \text{constant} \\ TV^{\gamma-1} = \text{constant} \end{cases} \quad \gamma > 0$$

- cyclic transformations



$\Delta E^{int} = 0$

$Q = -W$

Specific molar heat

$Q = nC_v \Delta T$ (isochoric)

$Q = nC_p \Delta T$ (isobaric)

monoatomic / simplified case

$E^{int} = \frac{2}{3} nRT$

isochoric $W = 0$

$Q = \frac{2}{3} nRT$ $\longrightarrow nC_v \Delta T = \frac{3}{2} nR\Delta T$

$C_v = \frac{3}{2} R \simeq 12.5 \ J/_{mol \cdot K}$

isobaric $Q = nC_p \Delta T$

$\Delta E^{int} = Q + W = nC_p \Delta T - p\Delta V \longrightarrow p$ is constant $\longrightarrow nC_v \Delta T = nC_p \Delta T - nR\Delta T$

$p\Delta V = nR\Delta T$

$C_p - C_v = R$

↳ molar specific heat in isobaric is larger than in isochoric

ex. p. 22

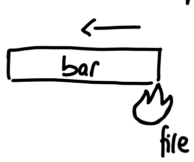
Mechanisms of energy transfer

⟶ **conduction**: transfer between objects in direct contact at different T

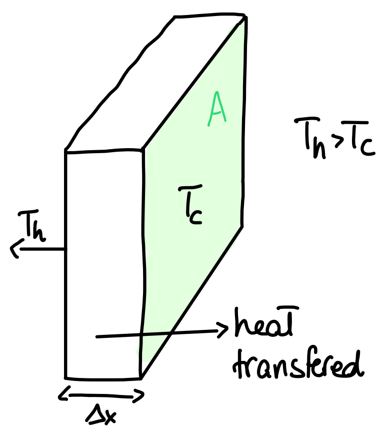
⟶ **convection**: heat transfer takes within the fluid

⟶ **radiation**: heat transfer via electromagnetic radiations

Conduction



heat transfered to the colder end of the bar



$T_h > T_c$

ex. p. 22

$$Power = \frac{Q}{\Delta t} \propto \underbrace{\frac{A\Delta T}{\Delta x}}_{general}$$

$$P = K \frac{A\Delta T}{\Delta x}$$

↳ constant: depends on material

Radiation

⟶ every object radiates electromagnetic waves due to the thermal motion of its molecules (thermal radiation)

$$P = \sigma \varepsilon A T^4$$

↳ Stefan-Boltzmann constant

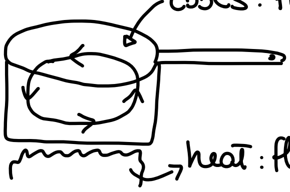
$[\sigma] = 5.669 \times 10^{-8} \ \frac{W}{m^2 \cdot K^4}$

• if = 0K ⟶ no radiation

• emissivity of object's surface

17 April

Convection



cools: fluid becomes more dense
-> fluid goes down

heat: fluid becomes less dense
-> fluid goes up

Reversible transformation

↳ system can return to the initial conditions via the same path

Irreversible transformation

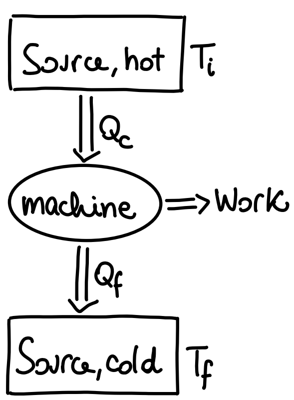
↳ happen in just one direction

↳ if occurs in a closed system, the entropy always increases, it never decreases

P. 585 - to see

Thermal machine

↳ a machine that operates in a cyclic mode, accumulating energy in the form of heat and releasing a fraction of it in the form of work



source = thermostat = constant temperature

$\Delta E_{int} = 0$

$Q = -W = W_{machine}$

$\begin{cases} W_m = |Q_c| - |Q_f| \\ \varepsilon = \text{machine efficiency} = \dfrac{\overset{\text{work}}{W_m}}{\underset{\text{input}}{|Q_c|}} = \dfrac{|Q_c| - |Q_f|}{|Q_c|} = 1 - \dfrac{|Q_f|}{|Q_c|} \end{cases}$

$\varepsilon = 100\%$ when $Q_f = 0$

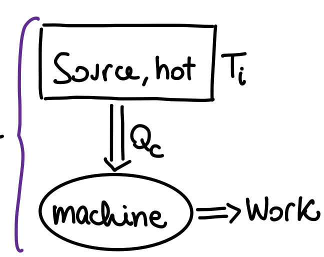
Second law of thermodynamics

↳ it is not possible to build a thermal machine with $\varepsilon = 100\%$

↳ can't absorb heat from a cold source spontaneously
↳ without adding work

↳ $\Delta S > 0$
↳ variation of entropy

impossible



20 April

Carnot engine (ideal engine)

$$\frac{|Q_f|}{|Q_c|} = \frac{T_f}{T_c} \Rightarrow \frac{|Q_f|}{T_f} = \frac{|Q_c|}{T_c}$$

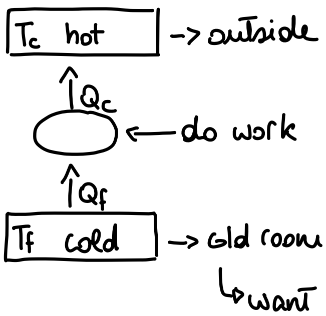
ex. p. 24-25

$\varepsilon_c = 1 - \frac{T_f}{T_c}$ → cannot be 0
↳ can never be 1

↓ uncertainty principle

→ upper limit for a thermal machine operating between 2 temperatures

Refrigerator machine



P_c = performance coefficient $= \frac{|Q_f|}{W_m} = \frac{|Q_f|}{|Q_c| - |Q_f|}$ → using that $\frac{|Q_f|}{T_f} = \frac{|Q_c|}{T_c}$ → $P_c = \frac{T_f}{T_c - T_f}$

$W_m = |Q_c| - |Q_f|$

in a reversible cycle:

$$\frac{-Q_f}{T_f} = \frac{Q_c}{T_c} \quad \Big| \quad \sum \frac{Q}{T} = 0$$

Entropy - macroscopic definition

$$\Delta S = \int_i^f \frac{dQ_r}{T}$$

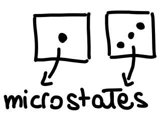
ΔS ↓ variation of entropy

dQ_r → variation of heat in reversible transformations

T → temperature

ex. p. 24-25

Entropy - microscopic definition



→ 4 macrostates

2 → ⊡⊡

6 → 5 possiblities of combinations

macrostates with more available microstates are more probable
↓
states that are more disordered are (chosen by nature) favored

$\underbrace{S = k_b \ln W}$ → number of associated microstates

↳ function of state: just depends on the values

Electromagnetism

Coulomb's law

$$\vec{F}_e = \frac{K_e \, q_1 q_2}{r^2} \hat{r}$$

* Coulomb constant $= 8.9876 \times 10^9 \, \frac{Nm^2}{C^2}$

$$K_e = \frac{1}{4\pi \varepsilon_0}$$

→ permittivity constant $= 8.85 \times 10^{-12} \, \frac{C^2}{N \cdot m^2}$
dielectric constant of vacuum

Electrical charges

$q_e = -e = -1.6 \times 10^{-19}$ C

$q_p = e = 1.6 \times 10^{-19}$ C

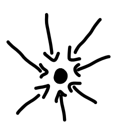
e: elementary charge

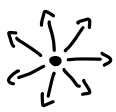
Electric field

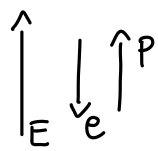
$\vec{E} = \frac{\vec{F}_e}{q_0}$ $\frac{N}{C}$ → property of the source

$\vec{F}_e = q\vec{E}$

direction

 electron

 proton



Note: $F_e \gg F_g$, we have differences and analogies between them (ex.p.26)

24 April

Types of electric field

- point charge

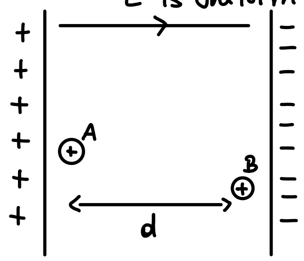
$$\vec{E} = \frac{q}{4\pi \varepsilon_0 r^2}$$

- charged ring   ex.p.26

$$E^{tot} = \int dE_x = \frac{k_e \, x}{(a^2+x^2)^{3/2}} Q \, \hat{x}$$

Motion of a charged particle in an uniform electric field

$\vec{E}$ is uniform along $\hat{x}$ in a certain region



• we place a proton in A, at rest

$V_B = ?$
- $\to$ energy conservation $\to \Delta K + \Delta U = 0$
- $\to$ equation of motion $\to \vec{F} = m\vec{a} = \vec{E}q$

$- \vec{a} = \frac{\vec{E}q}{m}$

$V_B = V_0 + at$

$V_B = at$

$t = \frac{V_B}{a}$

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$d = \frac{1}{2}at^2$

$d = \frac{1}{2}\cancel{a}\frac{V_B^2}{a^{\cancel{2}}} = \frac{1}{2}\frac{V_B^2}{a}$

$V_B = \sqrt{2da} = \sqrt{\frac{2dEq}{m}}$

Dipole in an electric field

qd = electric dipole moment $= \vec{p}$ Cm

$\vec{\tau} = \vec{p} \times \vec{E} = pE\sin\theta$

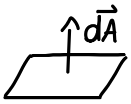
$U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$

$U = 0$ if $\vec{p} \perp \vec{E}$; $U = -pE$ if $p /\!/ E$ same direction ; $U = pE$ if $p /\!/ E$ opposite direction

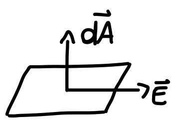
Electric Flux

$\phi_E = \oint \vec{E}\cdot\vec{dA}$

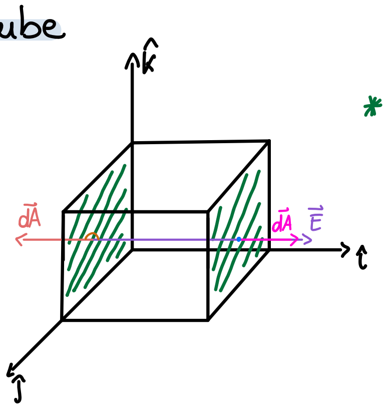
closed integral



if $\vec{E} \perp \vec{dA} \Rightarrow \phi_E = 0$



• cube

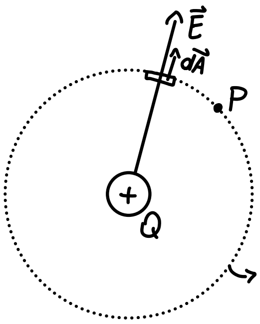


* for these 2 sides of the cube, $\vec{dA} /\!/ \vec{E} \to \vec{E}\cdot\vec{dA} \neq 0$

$\oint \vec{E}\cdot\vec{dA} = \int E\,dA\cos\pi + \int E\,dA\cos 0 = -El^2 + El^2 = 0$

Gauss theorem

↳ relation between the electric flux and the charge enclosed by a surface



↳ gaussian surface (sphere)
↳ $\vec{E} \parallel d\vec{A}$

$$\phi_E = \oint \vec{E}\cdot d\vec{A} = \int E \cdot dA = E\int dA = E\,4\pi r^2 = \frac{k_e Q}{\cancel{r^2}}\,4\pi \cancel{r^2} = k_e Q\, 4\pi = \frac{Q}{\varepsilon_0}$$

↳ E is constant in the surface of a sphere

↳ Coulomb

↳ $\varepsilon_0 = \frac{1}{4\pi k_e}$

→ $k_e = \frac{1}{4\pi\varepsilon_0}$

$$\phi_E = \frac{Q}{\varepsilon_0}$$

- cylinder / line of charge
  ↳ $E = \frac{\lambda}{2\pi\varepsilon_0 r}$
- plane
  ↳ $E = \frac{\sigma}{2\varepsilon_0}$
- sphere $(r \geq R)$
  ↳ $E = \frac{q}{4\pi\varepsilon_0 r^2}$
- sphere $(r < R)$
  ↳ $E = 0$

} shell

- uniform sphere of charge
  ↳ $E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r$

ex. p. 26

27 April

Electric potential → $V = \frac{U}{q}$ → $U = Vq$ ex. p. 27

$1\,\text{Volt} = \frac{J}{C} = \frac{Nm}{C}$

$\Delta U = q\Delta V = q(V_f - V_i)$

$W = -\Delta U = -q\Delta V = -q(V_f - V_i)$

Conservation of energy → $\Delta K + \Delta U = 0$

$\Delta K = -q\Delta V = -q(V_f - V_i)$

The electron-volt (eV)

↳ the work required to move a single elementary charge through a potential difference of 1 Volt

↳ $1\,eV = 1.602 \times 10^{-19}\,J$

$\Delta U = -q_0 \int_i^f \vec{E}\cdot d\vec{s}$ → variation of potential energy due to movement of a point like charge q

$\Delta V = -\int_i^f \vec{E}\cdot d\vec{s}$ → if E is uniform: $\Delta V = -E\Delta x$ → $E = -\frac{\Delta V}{\Delta x}$ (the electric field vector points from higher to lower potential)

→ else: $E = -\frac{dV}{ds}$

Potential of a charged particle → $V = k_e \frac{q}{r}$ (scalar)

" " N " " → $V = k_e \sum_{i=1}^{N} \frac{q_i}{r_i}$

" " an electric dipole → $V = k_e \frac{p\cos\theta}{r^2}$ ($p = qd$ - dipole moment)

} proof ex. p. 27-28

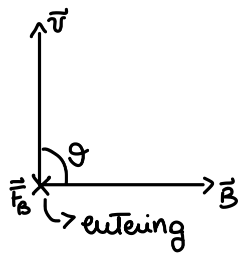
Magnetic fields

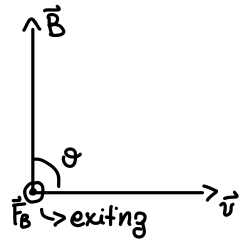
→ charge in motion produces a magnetic field

→ magnets

- ↳ natural magnets
- ↳ magnets with electric current

ex. p. 28





$B = \frac{F_B}{|q|v} \rightarrow \vec{F}_B = q\vec{v} \times \vec{B} \qquad F_B = |q| v B \sin\theta$

$[B] = \text{Tesla} = \frac{Ns}{Cm}$

$dW = \vec{F}_B \cdot d\vec{s} \;, \quad F_B \perp ds \rightarrow dW = 0$

Experiment: ex. p. 28

→ $v = \frac{E}{B}$

4 May

Current

↳ rapidity of charge flowing through a surface

$i = \frac{dq}{dt} \qquad [i] = \text{Ampère} = C/s$

- direction of current is that of the flow of positive charge
- current density $\vec{J}$ (same direction of velocity of moving + or opposite if −)

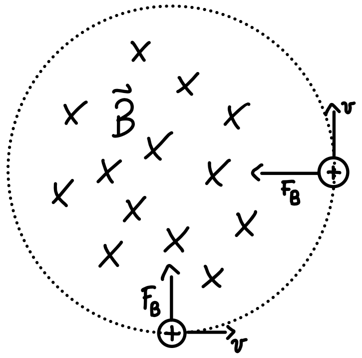
$i = \int \vec{J} \cdot d\vec{A} \qquad J = \frac{i}{A}$

$\Delta x = v_d \Delta t \qquad v_d = \frac{J}{ne} \qquad \vec{J} = (ne)\vec{v}_d \qquad i = nAv_d q$

↳ drift speed

see ex. p. 29 for formulas

Particle in a uniform magnetic field



=> circular motion

ex. p. 28

↳ particle moves in a circle

in a plane $\perp \vec{B}$ ($\vec{v} \perp \vec{B}$ initially)

$F_B = qvB$

→ 2nd law Newton $\quad ma = F_B = \frac{mv^2}{r}$

$r = \frac{mv}{qB}$

↳ useful particle identification method

=> helyx if $\vec{v}$ not $\perp$ to $\vec{B}$

→ angular frequency : $\omega = \frac{v}{r} = \frac{|q|B}{m}$

Magnetic force on a current-carrying wire (Lorentz force)

$\vec{F}_B = i\vec{L} \times \vec{B}$ → uniform magnetic field

i → current

$\vec{L}$ → length

↳ in direction of i

$F_B = iLB\sin\theta$

Torque on a current loop

$\vec{\tau} = i\vec{A} \times \vec{B}$

↳ vector perpendicular to plane of coil

Moment of a magnetic dipole

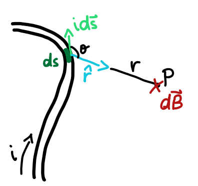
$\mu = NiA$ → area enclosed by each turn of the coil

direction of A

↳ number of turns in the coil

$\vec{\tau} = \vec{\mu} \times \vec{B}$

Magnetic field due to a current



- the magnitude of the field $d\vec{B}$ produced at point P at distance r by a current-length element $i d\vec{s}$:

$$dB = \frac{\mu_0}{4\pi} \frac{i\,ds\sin\theta}{r^2}$$

$4\pi \times 10^{-7}\ T\cdot m/A$

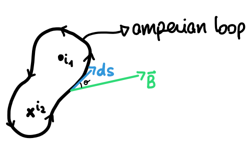
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,d\vec{s} \times \hat{r}}{r^2}$$ → Biot-Savart's law

K_m = magnetic permeability of vacuum

- magnetic field due to a current in a long straight wire → $B = \frac{\mu_0 i}{2\pi R}$

Ampere's law

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$$



Magnetic flux

$$\phi_B = \int \vec{B}\cdot d\vec{A} = BA\cos\theta$$

$[\phi_B] = Tm^2 = \text{Weber (Wb)}$

Faraday's law

Electromotive force $\to \mathcal{E}$

$$\mathcal{E} = -\frac{d\phi_B}{dt} = \oint \vec{E}\cdot d\vec{s}$$

$\hookrightarrow \Delta V = q\mathcal{E}$

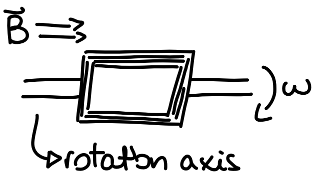
if B is constant $\to \mathcal{E}=0$

Lenz's law

- direction of the induced current
- the induced current is such that it tends to compensate the change of magnetic flux.

8 May

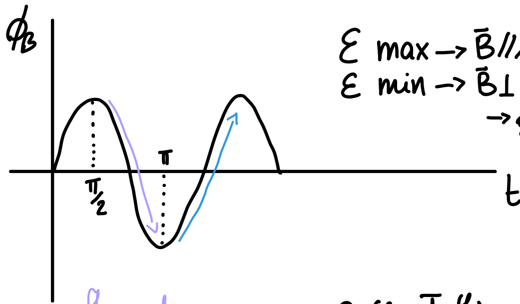
Alternate current



$$\phi_B = BA\cos\theta = BA\cos\omega t$$

$$\mathcal{E} = -\frac{d\phi_B N}{dt} = NBA\sin\omega t$$

N $\hookrightarrow$ elements of the coil



$\mathcal{E}$ max $\to \vec{B} /\!/ \vec{A}$

$\mathcal{E}$ min $\to \vec{B} \perp \vec{A}$

$\to \phi_B = 0$

flux decreases

flux increases

<u>current flips</u>

Maxwell's equations

1. $\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$ | Gauss (stationary charge can produce $\vec{E}$)

2. $\oint \vec{E}\cdot d\vec{s} = -\frac{d\phi_B}{dt}$ Faraday

3. $\oint \vec{B}\cdot d\vec{A} = 0$ Gauss (stationary charge cannot produce $\vec{B}$)

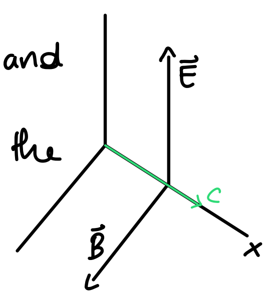
4. $\underbrace{\oint \vec{B}\cdot d\vec{s} = \mu_0 i}_{\text{Ampere}} + \underbrace{\varepsilon_0\mu_0 \frac{d\phi_E}{dt}}_{\text{Maxwell} \to \text{charge conservation}}$ Ampere

5. $F = q(\vec{E} + \vec{v}\times\vec{B})$

with no charges, vacuum, this = 0

Solution:

perpendicular and oscillating fields propagating with the speed of light



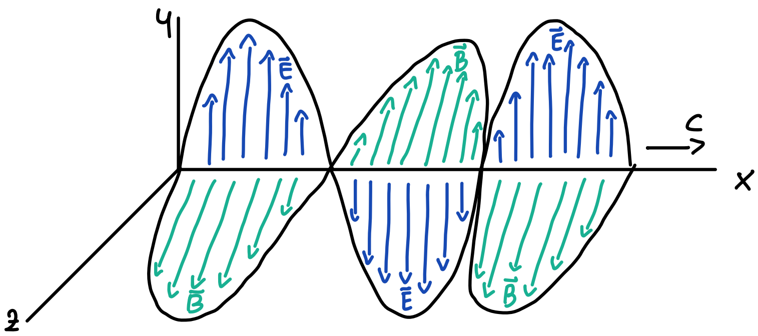
Electromagnetic waves

$E = E_{max} \sin(kx - \omega t)$

$B = B_{max} \sin(kx - \omega t)$

- wave speed $\rightarrow$ $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$ $\quad \dfrac{E}{B} = c$

p. 997



Poynting vector

ex. p. 30

$\rightarrow$ rate of energy transport per unit area in a wave

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \qquad [S] = \frac{W}{m^2}$$

$\rightarrow$ direction of $\vec{S}$ of an electromagnetic wave at any point gives the wave's direction of travel and the direction of energy transport at that point

$\rightarrow$ $E \perp B$ in electromagnetic wave

$$S = \frac{EB}{\mu_0} \quad \left[c = \frac{E}{B}\right] \quad S = \frac{E^2}{c\mu_0} \text{ or } \frac{cB^2}{\mu_0}$$

$\rightarrow$ Intensity of the wave

$$I = \langle S \rangle = \frac{\langle E^2 \rangle}{c\mu_0} = \frac{1}{c\mu_0} \langle E_{max}^2 \sin^2(kx - \omega t) \rangle$$

$\langle S \rangle$ $\hookrightarrow$ temporal average

isotropic point source:

$$I = \frac{\text{source power}}{\text{area}} = \frac{P_s}{4\pi r^2}$$

$4\pi r^2$ $\hookrightarrow$ area of sphere

$\rightarrow$ u_E – energy density associated to electric field $u_E = \frac{1}{2} \varepsilon_0 E^2$

$$- \; u_E = \frac{1}{2} \varepsilon_0 (cB)^2 = \frac{1}{2} \varepsilon_0 \frac{B^2}{\varepsilon_0 \mu_0} = \frac{B^2}{2\mu_0} = u_B$$

$\left.\right\}$ $u_B = u_E$ electromagnetic wave

$$u = u_E + u_B = \varepsilon_0 E^2$$

$\rightarrow$ momentum: $-\; \Delta p = \dfrac{\Delta U}{c}$ absorbed radiation

$-\; \Delta p = \dfrac{2\Delta U}{c}$ reflected radiation

$$\begin{cases} F = \dfrac{IA}{c} \\ F = \dfrac{2IA}{c} \end{cases} \qquad \begin{cases} P_r = \dfrac{I}{c} \\ P_r = \dfrac{2I}{c} \end{cases}$$

$\rightarrow$ force: $F = \dfrac{\Delta p}{\Delta t}$

$\Delta U = IA\Delta t$

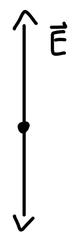
$\rightarrow$ pressure: $\dfrac{F}{A}$ $\qquad$ $\rightarrow$ power: $P_w = IA$

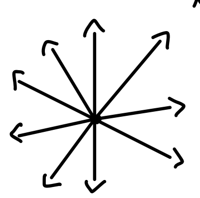
11 May

Optics

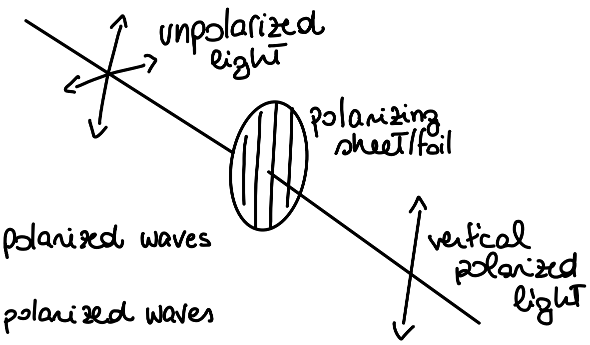
- polarization
- interference
- diffraction
- reflection
- refraction

Polarization





frontal view of polarized wave



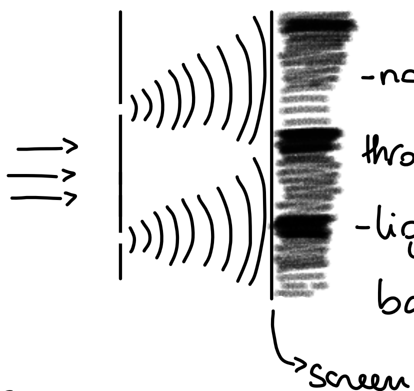
$I = \frac{1}{2} I_0$ if unpolarized waves

$I = I_0 \cos^2\theta$ if polarized waves

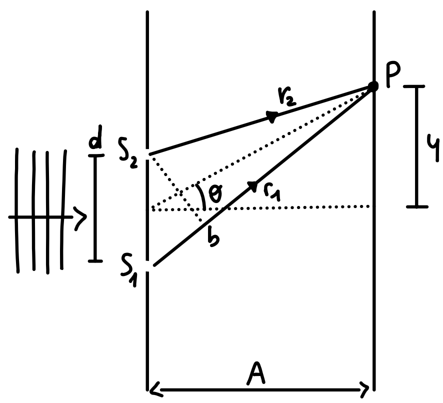
p.29

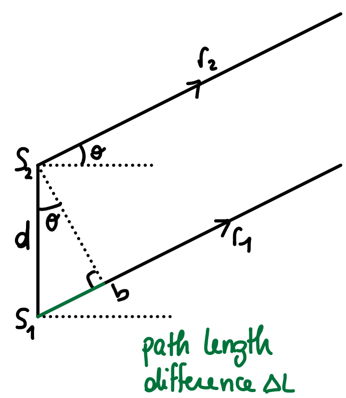
Interference

Young's double-slit experiment



- no direct path through the slit
- light and dark band patterns





$$\begin{cases} \tan\theta = \frac{y}{A} \\ y_L = A\tan\theta_L \\ y_D = A\tan\theta_D \\ y_L = \frac{Am\lambda}{d} \quad y_D = \frac{A(m+\frac{1}{2})\lambda}{d} \end{cases}$$

when θ is small: $\tan\theta \sim \sin\theta$

maxima: constructive interference - light • $\Delta L = d\sin\theta = m\lambda$

destructive interference - dark • $\Delta L = d\sin\theta = (m+\frac{1}{2})\lambda$

λ = wave length in vacuum ; $m = 0, 1, 2, \ldots$

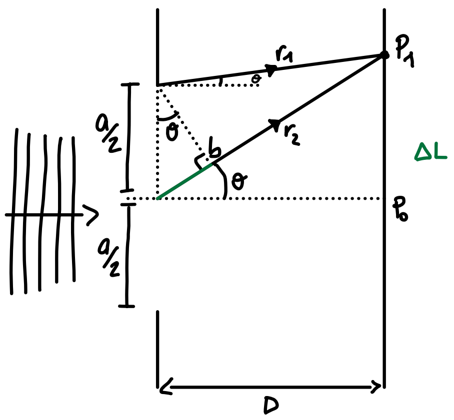
ex. p. 30

→ frequency: $f = \frac{c}{\lambda}$

experiment with electrons: p. 31

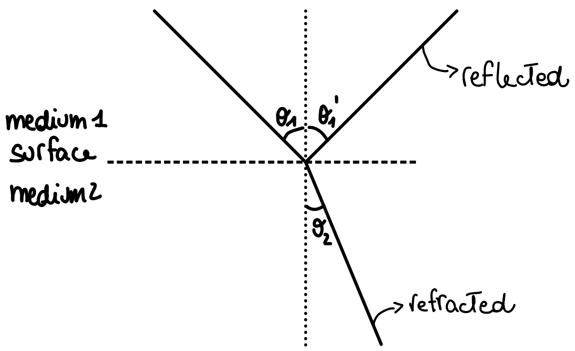
Diffraction

wide slit means much larger λ



$a \sin\theta = m\lambda$ • dark (minima)

Reflection and Refraction



-> $\theta_1' = \theta_1$ (reflection)

-> $n_1 \sin\theta_1 = n_2 \sin\theta_2$ (refraction)

-> n_1, n_2 are refraction indexes, depend on medium

-> $n \propto \frac{c}{v}$

↳ velocity of the light in the material

· if $n_1 = n_2 \Rightarrow \theta_1 = \theta_2'$

· if $n_2 > n_1 \Rightarrow \theta_1 > \theta_2$

15 May

Quantum physics

Photons

-> energy : $E = hf$ planck constant $= 6.63 \times 10^{-34}$ Js

-> light is made of photons or quanta

↳ plural of quantum

↳ elementary amount associated with a quantity that is quantized

-> momentum: $\frac{h}{\lambda}$

Black body radiator

↳ idealised object that absorbs all radiation and emits radiation

↳ something that emits thermal radiation

Wien's law

$\lambda_{max} T = \text{constant} = 2898\ \mu m \cdot K$ (at max radiancy)

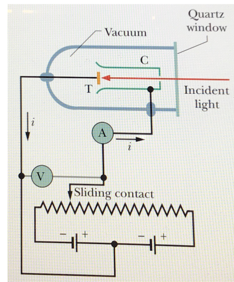
Radiated power / radiation intensity

$P_w = \sigma \varepsilon A T^4$

$\sigma = 5.6704 \times 10^{-8}\ W/m^2 \cdot K^4$ Stefan-Boltzmann constant

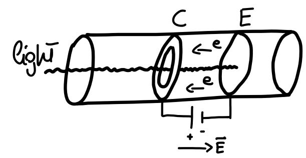
ε emissivity of radiating surface ($\varepsilon = 1$ for ideal b. body radiator)

Photoelectric effect



↳ Einstein's photoelectric experiments (1905)

1. beam hits foil (T)
2. electrons are produced (in E)
3. electrons are collected in C with the help of a potential difference
   ↳ photoelectrons
   ↳ when photon interacts with electron it gives it all its energy



4. production of photoelectric current i measured with meter A

Equation:

$hf = K_{max} + \Phi$ → work function

photon energy (hf), kinetic energy (K_{max}), ↳ extraction energy, energy that links electron to material, ~ few eV

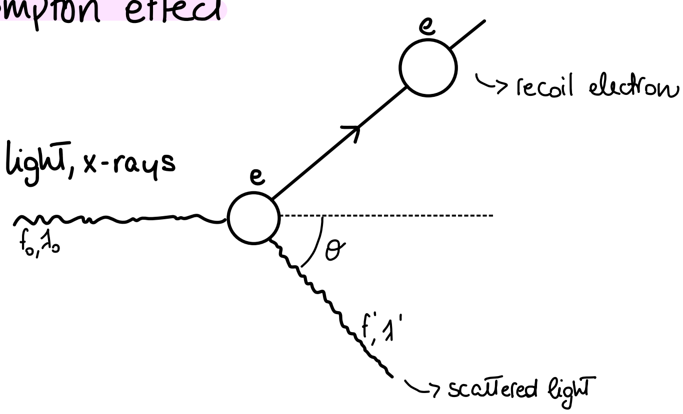
p. 30+33

$K_{max} = e \Delta V_s$

if $hf < \Phi_{max}$ → electrons are not emitted

→ $f_{threshold} = \dfrac{\Phi_{max}}{h}$ (if the frequency of the light is smaller than some threshold freq. some electrons are not emitted)

Compton effect



$$\lambda' - \lambda_0 = \Delta\lambda = \frac{h}{mc}(1 - \cos\theta)$$

ex. p. 31

De Broglie (1923)

→ all forms of matter exhibit wave-like propagation (not only light)

$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \rightarrow \lambda = \frac{h}{p}$ (de Broglie wavelength)

ex. p. 31

For any particle: $\left.\begin{array}{l} \lambda = \frac{\hbar}{p} = \frac{\hbar}{mv} \\ f = \frac{E}{\hbar} \end{array}\right\}$ general → Davidson-Gamer experiment

↳ measurement of the wave function of the electron

Heisenberg's Uncertainty Principle

$$\begin{cases} \Delta x \Delta p_x \geq \hbar \\ \Delta y \Delta p_y \geq \hbar \\ \Delta z \Delta p_z \geq \hbar \end{cases} \qquad \hbar = \frac{h}{2\pi}$$

→ can be written in terms of mass

$$\Delta m = \frac{\hbar}{\Delta r \cdot c}$$

(schwarzshild radius)

$\Delta r = \frac{2G\Delta m}{c^2} \rightarrow \Delta r = \sqrt{\frac{2G\hbar}{c^3}}$

· minimal space distance below which dragons may be

Wave function

$$\psi(x,y,z,t) = \psi(x,y,z)e^{-i\omega t}$$

$\omega = 2\pi f$, angular freq. of the matter wave

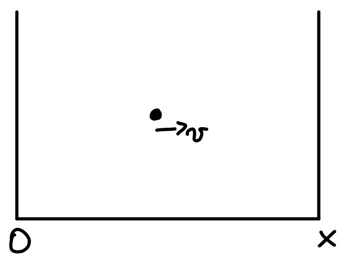
• $|\psi|^2$ must be 1

→ of a trapped electron

$\psi_n(x) = A\sin\left(\frac{n\pi x}{L}\right)$ $n = 1,2,3\ldots$

$P = \frac{hn}{2L}$

$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2 n^2}{8mL^2}$



↳ $n = 0$ not allowed, particle cannot be fixed

more at p.32 +34

22 May

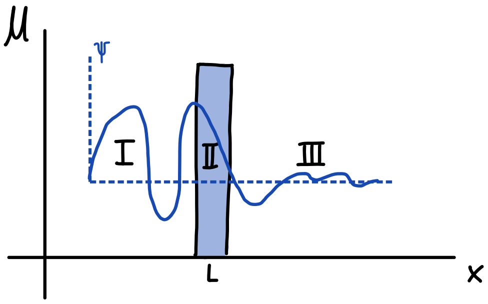
Schrödinger's equation

$$\frac{\hbar^2}{2m} \cdot \frac{d^2\psi}{dx^2} + U\psi = E\psi$$

$U(x)$ encodes the interaction of the particles with the environment

ex. p.33

Tunneling effect



- $U > 0$ in II
- $U = 0$ in I and III

barrier → normally not passable
→ in Quantum Mechanics all regions are accessible to the particle

$\Delta x \Delta p \geq \hbar/2 \quad \Delta E \cdot \Delta t \geq \hbar/2$

→ energy conservation can be violated by ΔE

⇒ Schrödinger's equation has a non-zero solution in II and III

To model this

T(transmission)
R(reflection) } $T + R = 1$ → under some approximations → $T \sim e^{-2cL}$ $C = \sqrt{\frac{2m(U-E)}{\hbar}} \left[\frac{1}{m}\right]$

L: length barrier

ex. p. 34

Nuclear Physics

nuclei

↳ protons (atomic number) Z
↳ neutrons (neutron number) N } mass number A   ⇒ $A = Z + N$

$^{A}_{Z}X$ → atom

→ if element is not stable → radioactive → undergo decay

↳ spontaneous nuclear transformations

emit α particle (^4He)
↳ alpha

Binding energy

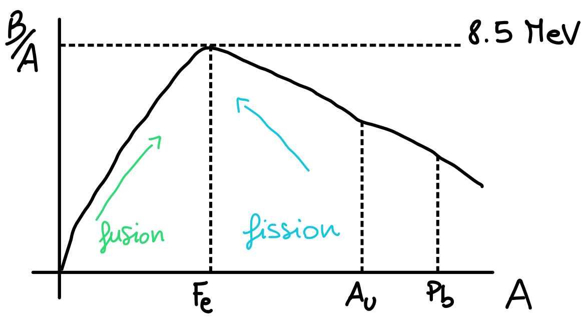
$$B(^{A}_{Z}X) = Nm_n + Zm_p - M(^{A}_{Z}X)$$

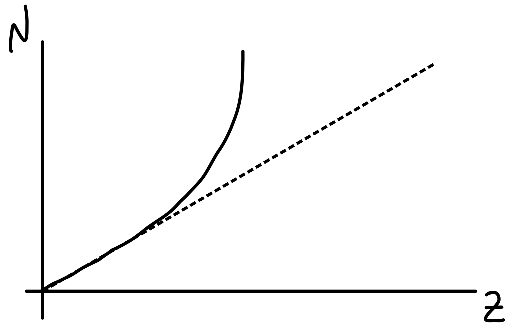
To bind nucleus together → strong force between prot. and neut. vs repulsive Coulomb force among prot.

in atom there are other constraints, more quantum numbers involved:

- n, l, ml, spin

29 May





$A \leq 40 \quad Z \sim N$

$A > 40 \quad N > Z$

↳ strong force is short range

Radioactivity

→ via different processes

α - decay

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha$$

→ stopped by a paper foil

β - decay

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e + \bar{\nu} \qquad \beta^- \text{- decays}$$

neutrinos → very small mass

→ stopped by a thin metal foil

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+ + \nu \qquad \beta^+ \text{- decays}$$

- charge needs to be conserved
- conservation of leptonic number

γ - rays

$${}^{A}_{Z}X^* \rightarrow {}^{A}_{Z}X + \gamma$$

exited neuclide ↳ high energy photon

→ Stopped by walls of concrete
L dangerous, can modify your cells

Radioactive decay law

$$\frac{dN}{dt} = -\lambda N$$

↳ number of neuclides

decay constant

$N(t) = N_0 e^{-\lambda t}$

can define the half life

→ time it takes for N to become half

↳ when $N(t) = \frac{N_0}{2}$; $\frac{1}{2} = e^{-\lambda t}$

$\ln \frac{1}{2} = -\lambda t \rightarrow t_{1/2} = \frac{\ln 2}{\lambda}$

Activity / decay rate

ex. p.35

$$R = \frac{dN}{dt} = \lambda N_0 e^{-\lambda t} = \lambda N(t) \qquad R(t) = R_0 e^{-\lambda t}$$

→ basis of carbonium dating → method to determine the age of an object containing organic material

- while alive ^{14}C is constant and equal to the content in the atmosphere, when the organism dies then ^{14}C decays exponentially
- $t_{1/2}$ ^{14}C = 5700 years (extimate time from death)

exam example p.36-37