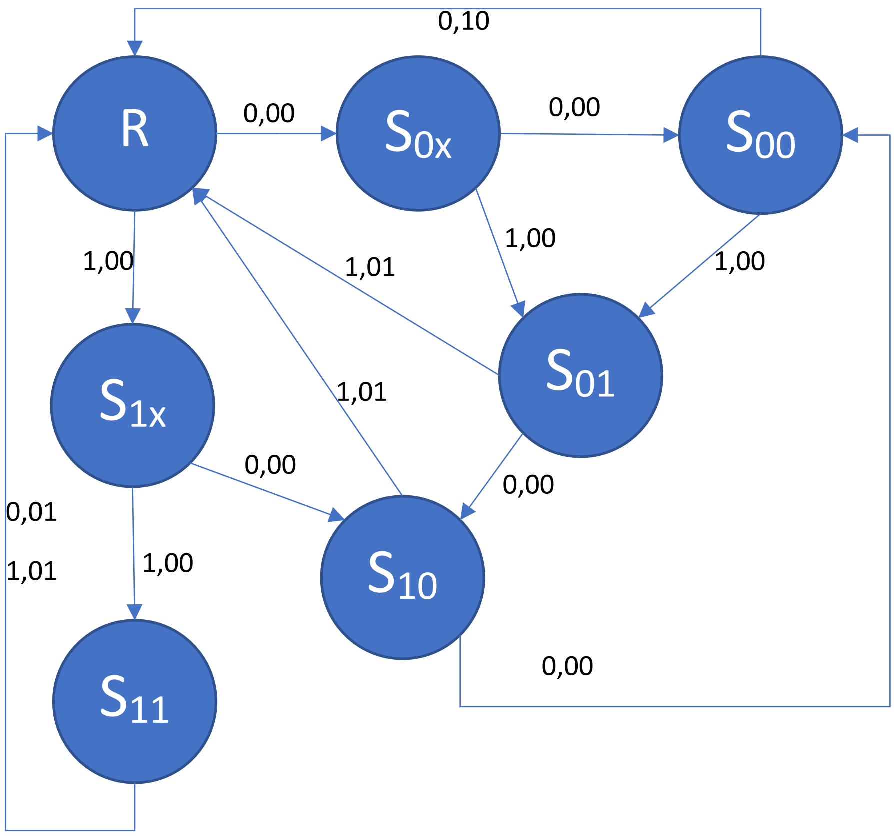
Exam - Computer Architecture Unit I [18/01/2023] (B) - Solution

Esercizio 1 (8 punti) Design a sequential circuit with an input x and two outputs z1 and z0. The output z1 must be equal to 1 if the last three bits on input are all equal to 0, whereas z0 must be equal to 1 if the last three bits contain at least two 1s. Do not consider overlaps. Draw the sequential circuit (use a ROM for the combinational part).

Example x 000001011100
z1 001000000000
z0 000000010010



States encoding:

R	**000**
S0X	**001**
S1X	**010**
S00	**011**
S01	**100**
S10	**101**
S11	**110**

Outputs and next state table:

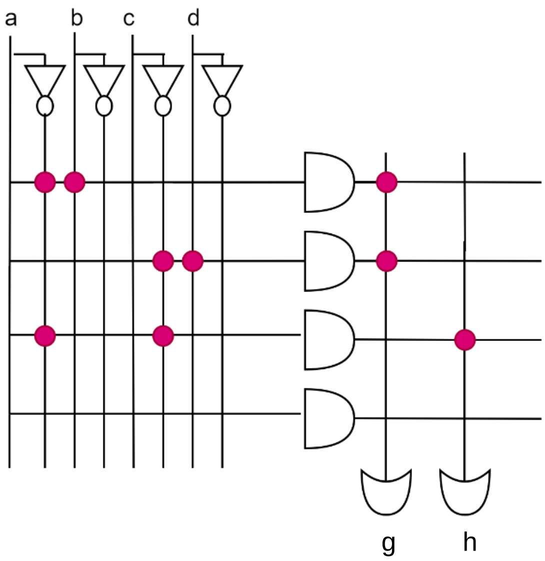
CS	S_2	S_1	S_0	x	NS	S_2'	S_1'	S_0'	z1	z0
R	0	0	0	0	S0X	0	0	1	0	0
R	0	0	0	1	S1X	0	1	0	0	0
S0X	0	0	1	0	S00	0	1	1	0	0
S0X	0	0	1	1	S01	1	0	0	0	0
S1X	0	1	0	0	S10	1	0	1	0	0
S1X	0	1	0	1	S11	1	1	0	0	0
S00	0	1	1	0	R	0	0	0	1	0
S00	0	1	1	1	S01	1	0	0	0	0
S01	1	0	0	0	S10	1	0	1	0	0
S01	1	0	0	1	R	0	0	0	0	1
S10	1	0	1	0	S00	0	1	1	0	0
S10	1	0	1	1	R	0	0	0	0	1
S11	1	1	0	0	R	0	0	0	0	1
S11	1	1	0	1	R	0	0	0	0	1

Circuit:

Same as track A, but with different dots on the ROM.

Exercise 2 (1+2+1+2 points) Consider the PLA shown below.

- Write the boolean expressions for functions *g* ed *h*
- Transform the boolean expression *f=g* ⊕ *h*, using boolean algebra's axiom, rules, and theorems, in canonical SOP form
- Write down the truth table for *f*
- Write down the minimal SOP and POS expressions for *f*



$h = \bar{a}\bar{c}$
$g = \bar{a}b + \bar{c}d$

Canonical SOP form

$$f = \bar{a}\bar{c} \oplus (\bar{a}b + \bar{c}d) = \overline{(\bar{a}\bar{c})}(\bar{a}b + \bar{c}d) + (\bar{a}\bar{c})\overline{(\bar{a}b + \bar{c}d)}$$

$$= (a + c)(\bar{a}b + \bar{c}d) + (\bar{a}\bar{c})\left(\overline{(\bar{a}b)} \cdot \overline{(\bar{c}d)}\right) = \bar{a}bc + a\bar{c}d + (\bar{a}\bar{c})\left((a + \bar{b})(c + \bar{d})\right)$$

$$= \bar{a}bc + a\bar{c}d + (\bar{a}\bar{c})\left(ac + \bar{b}c + a\bar{d} + \bar{b}\bar{d}\right) = a\bar{c}d + \bar{a}bc + \bar{a}\bar{b}\bar{c}\bar{d}$$

$$= ab\bar{c}d + a\bar{b}\bar{c}d + \bar{a}bcd + \bar{a}bc\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d}$$

Truth table for f:

a	b	c	d	f
0	0	0	0	**1**
0	0	0	1	**0**
0	0	1	0	**0**
0	0	1	1	**0**
0	1	0	0	**0**
0	1	0	1	**0**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**0**
1	0	1	1	**0**
1	1	0	0	**0**
1	1	0	1	**1**
1	1	1	0	**0**
1	1	1	1	**0**

Minimal SOP:

cd \ ab	00	01	11	10
00	1	0	0	0
01	0	0	1	1
11	0	1	0	0
10	0	1	0	0

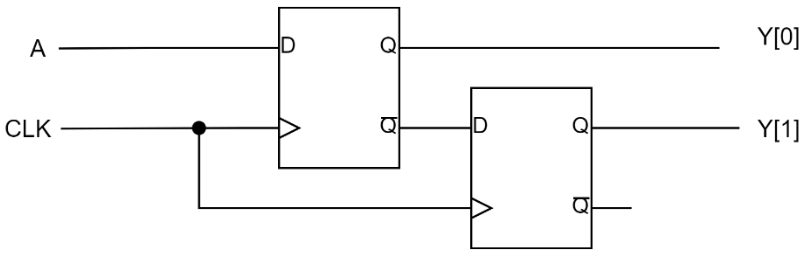
$$f = \mathbf{\bar{a}\bar{b}\bar{c}\bar{d} + a\bar{c}d + \bar{a}bc}$$

Minimal POS:

cd \ ab	00	01	11	10
00	1	0	0	0
01	0	0	1	1
11	0	1	0	0
10	0	1	0	0

$$f = (\bar{a} + d)(b + \bar{c})(\bar{a} + \bar{c})\left(a + \bar{b} + c\right)\left(a + c + \bar{d}\right)$$

Exercise 3 (4 points) Describe the following circuit using SystemVerilog:




```
module esercizio5(input  logic clk,
                  input  logic A,
                  output logic[1:0] Y);

 logic net;
        always_ff @(posedge clk)
         begin
          Y[0] <= A;
          Y[1] <= ~Y[0];
         end
endmodule
```

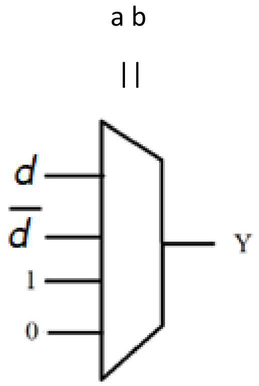
Exercise 4 (3 points)

A circuit receives the boolean inputs a, b, c, d and produces an output y such that:

y=1 if $a \cdot \bar{b} = 1$ or $b + \bar{d} = 0$ or $\bar{a}b\bar{d} = 1$

- Write down the truth table
- Implement y with a 4-to-1 MUX using inputs a e b as control variables
- Draw the circuit corresponding to the NAND-NAND equation for the given circuit

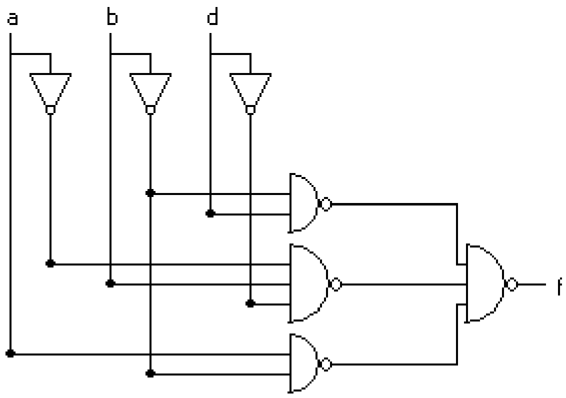
a	b	c	d	f
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**1**
0	1	0	1	**0**
0	1	1	0	**1**
0	1	1	1	**0**
1	0	0	0	**1**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**0**
1	1	1	1	**0**



NAND-NAND

Minimal POS: B'D + A'BD' + AB'

NAND-NAND: ((B'D)'(A'BD')'(AB')')'



Exercise 5 (1+2+1 points)

Represent A= 3.75 using the IEEE half-precision floating point standard. Sum A and B (using the algorithm for summing IEEE floating point numbers), with B = 1100_0110_1100_0000 and represent the result as a IEEE half-precision floating point number. Last, represent the 16-bits of the result in hexadecimal format.

A = 3.75 → $11.11_2 = 1*2^1*1.111_2$

Sign = 0 (positive)

Exponent = 1

Exponent+bias = 1+15 = 16 = 10000_2

Mantissa = 1110000000

A = 0100_0011_1000_0000

B:

Sign = 1 (negative)

Exponent+bias = 10001_2 = 17

Exponent = 17-15 = 2

Mantissa = 1011000000_2

$B = -1*2^2*1.1011_2 = 110.11_2 \rightarrow -6.75$

A+B – Shift and sum mantissas

00.1111 + *(x 2^2)*
10.0101 = *(x 2^2)*

11.0100 (x 2^2)

$= -00.11_2 \times 2^2 = -11.0_2 = -3_{10} = -1.100_2 \times 2^1$
Sign = 1 (negative)
Exponent = 1
Exponent+bias=1+15=16=10000_2
Mantissa=1000000000

IEEE representation = 1100_0010_0000_0000 = 0xC200

Exercise 6 (5 points) Given the function

$$f = \bar{b}c \oplus (\bar{a}b + ad)$$

Represent it in POS form using Boolean algebra axiom, rules, and theorems.

$$f = \left(\bar{b}c\right) \oplus (\bar{a}b + ad)=$$

$$\bar{b}c\overline{(\bar{a}b + ad)} + \overline{\bar{b}c}(\bar{a}b + ad)=\bar{b}c \cdot \left(\overline{\bar{a}b}\right) \cdot \left(\overline{ad}\right) + (b + \bar{c})(\bar{a}b + ad) =$$

$$c \cdot \bar{b} \cdot \left(a + \bar{b}\right) \cdot \left(\bar{a} + \bar{d}\right) + (b + \bar{c})(\bar{a}b + ad)=$$

$$c \cdot \bar{b} \cdot \left(\bar{a} + \bar{d}\right) + (b + \bar{c})(\bar{a} + d)(a + b)(b + d)=$$

$$=(\bar{a} + c + d)(a + b + c)(b + c + d)\left(\bar{a} + \bar{b} + d\right)\left(\bar{a} + b + \bar{c} + \bar{d}\right)$$