Algorithms

Solve the following exercises.

1. An undirected graph is connected if there is a path from any node, to any other node. A directed graph is strongly connected if there is a well-directed path from any node, to any other node.

   - What is the minimum number of edges of a connected undirected graph composed of $n \geq 2$ nodes?
   - What is the minimum number of arcs of a strongly connected directed graph composed of $n \geq 2$ nodes?

2. You want to hike a long trail (possibly, over multiple days). You start at km $x_0 = 0$ on the trail; the trail contains resting areas at kms $0 < x_1 < x_2 < \cdots < x_k$, and ends at km $x_{k+1} > x_k$. In a day, you can hike at most d kms; then, you have to stop at a resting area for the night. Assuming that $x_{i+1} - x_i \leq d$ for each $i \in \{0, 1, 2, \ldots, k\}$, prepare a schedule that minimizes the number of days of the hike. Prove that your algorithm is correct and bound its running time. More points will be awarded to faster solutions.

   Example 1: if $k = 2$ and $x_1 = 4$, $x_2 = 8$, $x_3 = 11$, and $d = 4$, then the optimal hike lasts 3 days — the hiker should stop at x_1 for the first night, and at x_2 for the second.

   Example 2: if $k = 4$ and $x_1 = 2$, $x_2 = 3$, $x_3 = 4$, $x_4 = 6$, $x_5 = 7$, and $d = 4$, then the optimal hike lasts 2 days — for instance, the hiker can stop at x_2 for the first and only night (alternatively, the hiker could stop at x_3 for the first and only night).