Algorithms 2023/2024
June Exam

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Solve the following exercises.

1. Consider the following statement: "Let $G(V, E, w)$ be a connected undirected graph with a positive weight $w(e)$ on each edge $e \in E$. Let e^* be an edge of minimum weight in E. Then, there exist two nodes x, y in $G(V, E, w)$, and a shortest path π connecting x and y, such that e^* is in π."

   Determine whether the statement is true or false: if it is true, prove it; if it is false, give a counterexample.

2. Let A and B be two arrays of n integers each, and let s be an integer. Do there exist two indices $0 \le i, j \le n - 1$ such that $A[i] + B[j] = s$?

   Write an efficient algorithm which, given A, B and s as input, returns True if the above question has a positive answer, and False otherwise. Prove that your algorithm is correct, and bound its running time.

   Larger scores will be awarded to faster solutions.

   Example 1: If $A = [23, 12, 40, 8]$, $B = [18, 4, 1, 10]$ and $s = 13$, then your algorithm should return True (since $A[1] + B[2] = s$).

   Example 2: If $A = [20, 12, 8]$, $B = [18, 4, 10]$ and $s = 13$, then your algorithm should return False.