First name: *Last name:* *Matricola:*

Exercise 1 Study the convergence of the following series and find the sum of the series

$$\sum_{n=0}^{+\infty} (\tan|x|)^n \, .$$

Exercise 2 Find the Taylor expansion at $x = 0$ of $\arctan x$ until order $n = 3$ and use it to study the convergence of the following series

$$\sum_{n=1}^{\infty} \left(n \arctan\left(\frac{1}{n}\right) - 1 \right)^2 .$$

Exercise 3 Solve the following Cauchy problem

$$\begin{cases} y''(x) + 9y = \cos 2x\,, \\ y(0) = 0 \\ y'(0) = 1\,. \end{cases}$$

Exercise 4 Solve the following improper integral

$$\int_0^2 x^2 \ln(8 - x^3)\, dx\,.$$

First name: *Last name:* *Matricola:*

Exercise 1 Study the convergence of the following series and find the sum of the series

$$\sum_{n=0}^{+\infty} (\arctan |x|)^n \, .$$

Exercise 2 Find the Taylor expansion at $x = 0$ of $\ln(1 + x)$ until order $n = 3$ and use it to study the convergence of the following series

$$\sum_{n=1}^{\infty} \left(n \ln\left(1 + \frac{1}{n}\right) - 1 \right)^2 .$$

Exercise 3 Solve the following Cauchy problem

$$\begin{cases} y'(x) = -2y + e^{-2x}\,, \\ y(0) = 1/2\,. \end{cases}$$

Exercise 4 Solve the following improper integral

$$\int_0^1 \frac{e^{\sqrt{x}} - \sqrt{x^3}}{\sqrt{x}}\, dx\,.$$