7 - Boolean Algebra

Wednesday, May 15, 2024 6:45 PM

Remark

- SOP and POS expressions might not lead to the simplest set of logic gates
- Fewer expression terms lead to fewer gates, that means we have a circuit with smaller area, and lower delay, cost, and power consumption

* To do so we use Axioms and theorems

- Duality in axioms and theorems:
  - ✓ ANDs and ORs, 0’s and 1’s interchanged

Boolean Axioms

Number	Axiom	Dual	Name
A1	$B = 0$ if $B \neq 1$	$B = 1$ if $B \neq 0$	Binary Field
A2	$\overline{0} = 1$	$\overline{1} = 0$	NOT
A3	$0 \cdot 0 = 0$	$1 + 1 = 1$	AND/OR
A4	$1 \cdot 1 = 1$	$0 + 0 = 0$	AND/OR
A5	$0 \cdot 1 = 1 \cdot 0 = 0$	$1 + 0 = 0 + 1 = 1$	AND/OR

A1’

To get the dual, replace: · with +
0 with 1

Boolean Theorems of One Variable

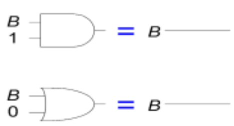
Number	Theorem	Dual	Name
T1	$B \cdot 1 = B$	$B + 0 = B$	Identity
T2	$B \cdot 0 = 0$	$B + 1 = 1$	Null Element
T3	$B \cdot B = B$	$B + B = B$	Idempotency
T4	$\overline{\overline{B}} = B$		Involution
T5	$B \cdot \overline{B} = 0$	$B + \overline{B} = 1$	Complements

Dual: Replace: · with +
0 with 1

- Identity theorem

$B \cdot 1 = B$

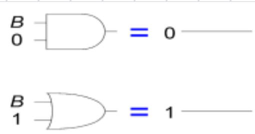
$B + 0 = B$



- Null theorem

$B \cdot 0 = 0$

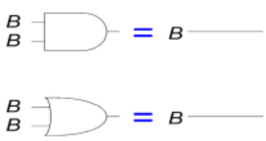
$B + 1 = 1$



- Idempotency theorem

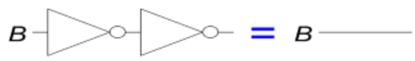
$B \cdot B = B$

$B + B = B$



- Involution theorem

$\overline{\overline{B}} = B$



- Complements theorem

$B \cdot \overline{B} = 0$

$B + \overline{B} = 1$