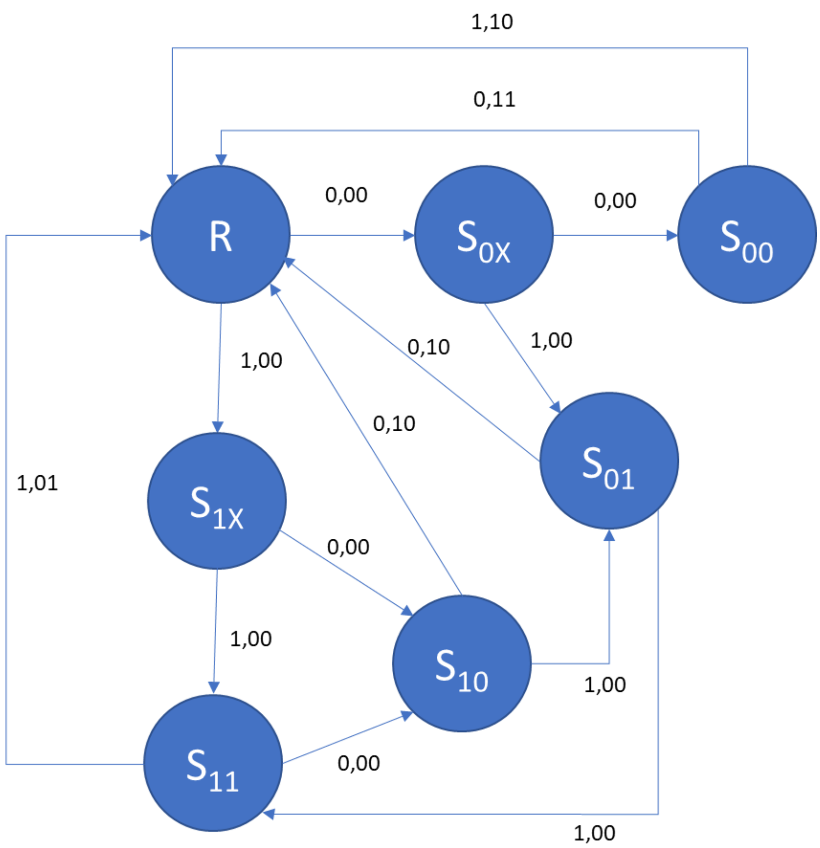
Exam - Computer Architecture Unit I [18/01/2023] (A) - Solution

Exercise 1 (8 points) Design a sequential circuit with an input x and two outputs z1 and z0. The output z1 must be equal to 1 if the last three bits on input contain at least two 0, whereas z0 must be equal to 1 if the last three bits are the same (i.e., 3 zeros or 3 ones). Do not consider overlaps. Draw the sequential circuit (use a ROM for the combinational part).

Example x 10100000111
z1 00010010000
z0 00000010001



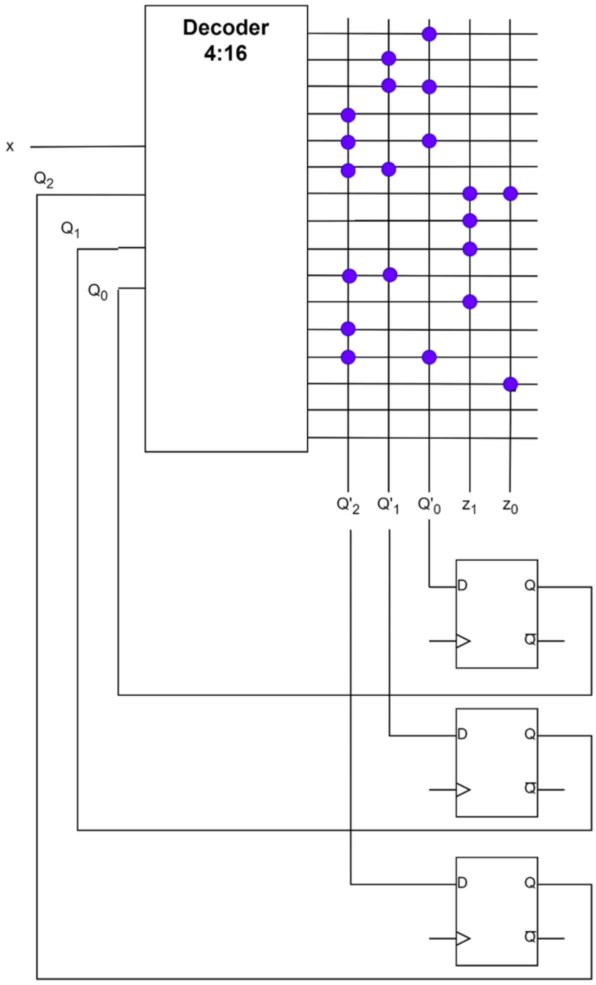
States encoding:

R	**000**
S0X	**001**
S1X	**010**
S00	**011**
S01	**100**
S10	**101**
S11	**110**

Outputs and next state table:

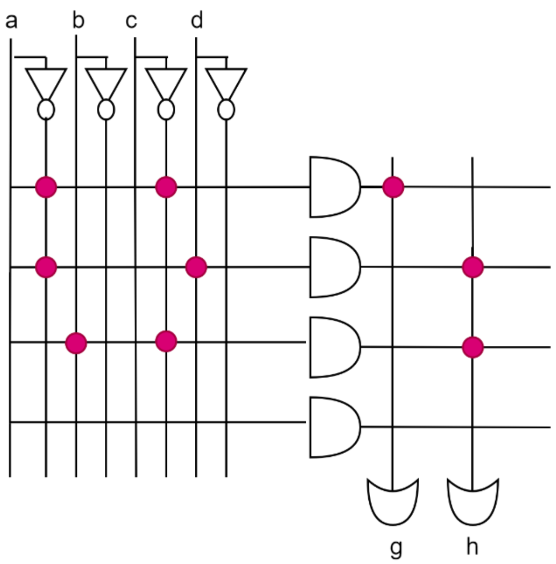
CS	S_2	S_1	S_0	x	NS	S_2'	S_1'	S_0'	z1	z0
R	0	0	0	0	S0X	0	0	1	0	0
R	0	0	0	1	S1X	0	1	0	0	0
S0X	0	0	1	0	S00	0	1	1	0	0
S0X	0	0	1	1	S01	1	0	0	0	0
S1X	0	1	0	0	S10	1	0	1	0	0
S1X	0	1	0	1	S11	1	1	0	0	0
S00	0	1	1	0	R	0	0	0	1	1
S00	0	1	1	1	R	0	0	0	1	0
S01	1	0	0	0	R	0	0	0	1	0
S01	1	0	0	1	S11	1	1	0	0	0
S10	1	0	1	0	R	0	0	0	1	0
S10	1	0	1	1	S01	1	0	0	0	0
S11	1	1	0	0	S10	1	0	1	0	0
S11	1	1	0	1	R	0	0	0	0	1

Circuit:



Exercise 2 (1+2+1+2 points) Consider the PLA shown below.

- Write the boolean expressions for functions *g* ed *h*
- Transform the boolean expression *f=g* ⊕ *h*, using boolean algebra's axiom, rules, and theorems, in canonical SOP form
- Write down the truth table for *f*
- Write down the minimal SOP and POS expressions for *f*



$g = \bar{a}\bar{c}$
$h = \bar{a}d + b\bar{c}$

Canonical SOP form:

$$f = \bar{a}\bar{c} \oplus (\bar{a}d + b\bar{c}) = ab\bar{c} + \bar{a}cd + \bar{a}\bar{b}\bar{c}\bar{d} = ab\bar{c}d + ab\bar{c}\bar{d} + \bar{a}bcd + \bar{a}\bar{b}cd + \bar{a}\bar{b}\bar{c}\bar{d}$$

Truth table for f:

a	b	c	d	f
0	0	0	0	**1**
0	0	0	1	**0**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**0**
0	1	0	1	**0**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**0**
1	0	1	1	**0**
1	1	0	0	**1**
1	1	0	1	**1**
1	1	1	0	**0**
1	1	1	1	**0**

Minimal SOP:

cd \ ab	00	01	11	10
00	1	0	1	0
01	0	0	1	0
11	1	1	0	0
10	0	0	0	0

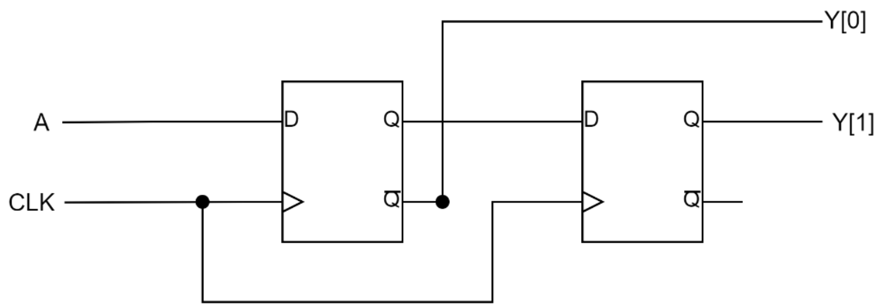
$$f = \bar{a}\bar{b}\bar{c}\bar{d} + ab\bar{c} + \bar{a}cd$$

Minimal POS:

cd \ ab	00	01	11	10
00	1	0	1	0
01	0	0	1	0
11	1	1	0	0
10	0	0	0	0

$$f = (\bar{a} + b)(\bar{c} + d)(\bar{a} + \bar{c})\left(a + c + \bar{d}\right)\left(a + \bar{b} + c\right)$$

Exercise 3 (4 points) Describe the following circuit using SystemVerilog:




```
module esercizio5(input  logic clk,
                  input  logic A,
                  output logic[1:0] Y);

  logic net;
        always_ff @(posedge clk)
         begin
          Y[0] <= ~A;
          net <= A;
          Y[1] <= net;
         end
endmodule
```

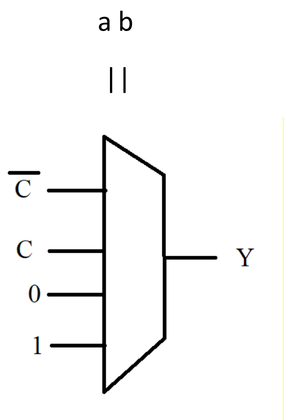
Exercise 4 (3 points)

A circuit receives the boolean inputs a, b, c, d and produces an output y such that:

y=1 if $a \cdot b = 1$ or $\bar{b} + \bar{c} = 0$ or $\bar{a}\bar{b}\bar{c} = 1$

- Write down the truth table
- Implement y with a 4-to-1 MUX using inputs a e b as control variables
- Draw the circuit corresponding to the NAND-NAND equation for the given circuit

a	b	c	d	f
0	0	0	0	**1**
0	0	0	1	**1**
0	0	1	0	**0**
0	0	1	1	**0**
0	1	0	0	**0**
0	1	0	1	**0**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**0**
1	0	1	1	**0**
1	1	0	0	**1**
1	1	0	1	**1**
1	1	1	0	**1**
1	1	1	1	**1**

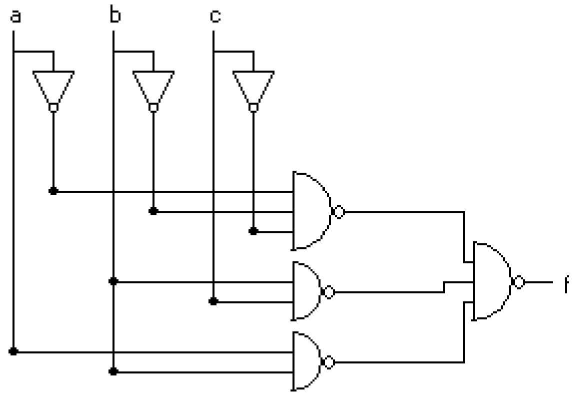


NAND-NAND:

Minimal POS: A’B’C’ + BC + AB

NAND-NAND = ((A’B’C’)’ (BC)’ (AB)’)’

Circuit:



Exercise 5 (1+2+1 points)
Represent A= -3.25 using the IEEE half-precision floating point standard. Sum A and B (using the algorithm for summing IEEE floating point numbers), with B = 0100_0110_0100_0000 and represent the result as a IEEE half-precision floating point number. Last, represent the 16-bits of the result in hexadecimal format.

A = -3.25 → $-11.01_2 = -1*2^1*1.101_2$

Sign = 1 (negative)

Exponent = 1

Exponent+bias = 1+15 = 16 = 10000_2

Mantissa = 1010000000

A = 1100_0010_1000_0000

B:

Sign = 0 (positive)

Exponent+bias = 10001_2 = 17

Exponent = 17-15 = 2

Mantissa = 1001000000_2

B = $1*2^2*1.1001_2 = 110.01_2 \rightarrow 6.25$

A+B – Shift and sum mantissas

11.0011 + *(x 2^2)*

01.1001 = *(x 2^2)*

00.1100 (x 2^2)

= $11.00_2 = 3_{10} = 1.100_2 \times 2^1$

Sign = 0 (positive)

Exponent = 1

Exponent+bias=1+15=16=10000_2

Mantissa=1000000000

IEEE Representation = 0100_0010_0000_0000 = 0x4200

Exercise 6 (5 points) Given the function $f = \bar{a}d \oplus \left(a\bar{b} + bc\right)$

Represent it in POS form using Boolean algebra axiom, rules, and theorems.

$$f = (d\bar{a}) \oplus \left(a\bar{b} + bc\right) =$$

$$d\bar{a}\overline{\left(a\bar{b} + bc\right)} + \overline{d\bar{a}}\left(a\bar{b} + bc\right) = d\bar{a} \cdot \left(\overline{a\bar{b}}\right) \cdot \left(\overline{bc}\right) + \left(\bar{d} + a\right)\left(a\bar{b} + bc\right) =$$

$$d \cdot \bar{a} \cdot (\bar{a} + b) \cdot \left(\bar{b} + \bar{c}\right) + \left(\bar{d} + a\right)\left(a\bar{b} + bc\right) =$$

$$d \cdot \bar{a} \cdot \left(\bar{b} + \bar{c}\right) + \left(\bar{d} + a\right)(a + b)(a + c)\left(\bar{b} + c\right) =$$

$$= (a + b + d)(a + c + d)\left(\bar{b} + c + d\right)\left(\bar{a} + \bar{b} + c\right)\left(a + \bar{b} + \bar{c} + \bar{d}\right)$$