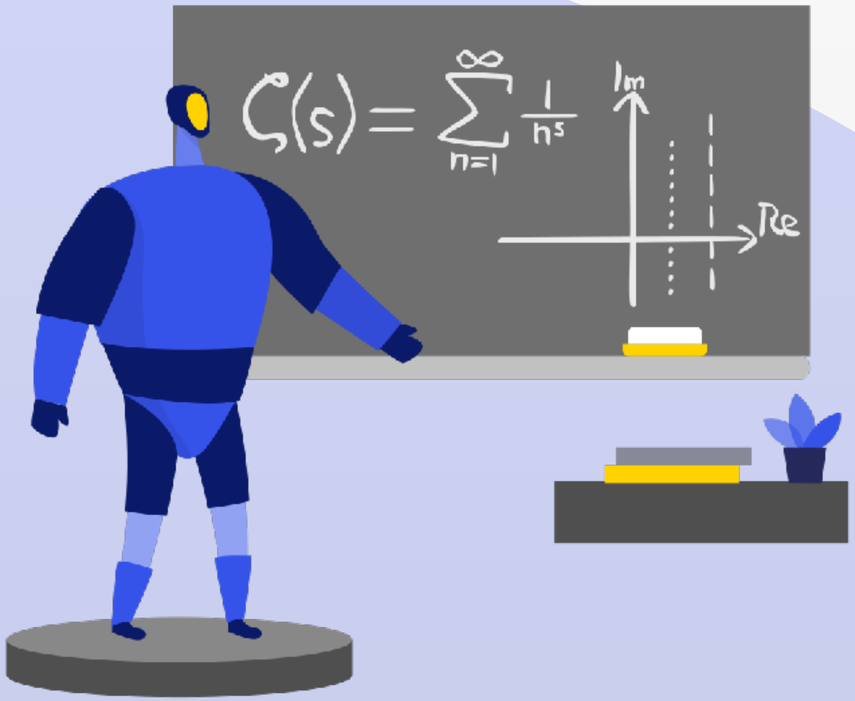
Superposition for Higher-Order Logic

My PhD Thesis in Simple Terms

Alexander Bentkamp

“At some point in the future, mathematicians will be replaced by computers.”

—allegedly predicted by Paul Cohen



The Language of Mathematics

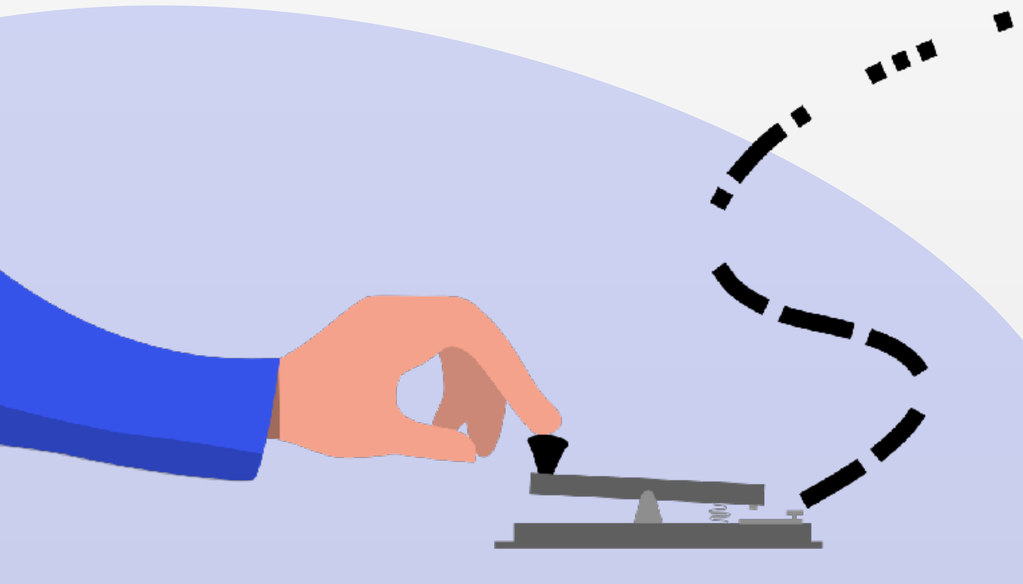
First-Order Logic

```
integral(app(
 app(
  app(S,app(app(S,app(K,plus)),
  app(app(S,mul),I)))
 ),
 app(K,one)
))
```

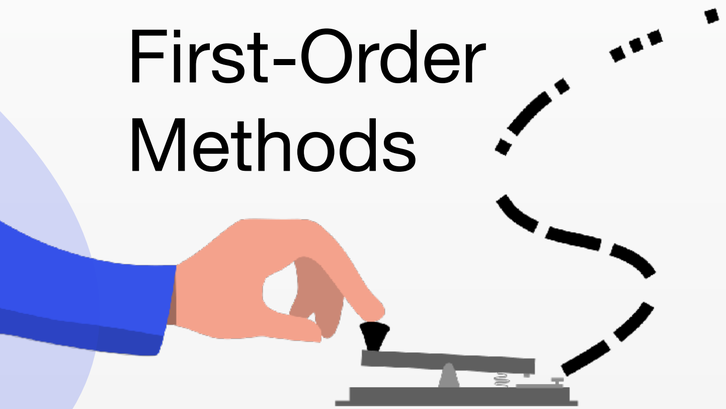
$$\int x^2 + 1 \, dx$$

```
integral (λx. plus (mul x x) one)
```

Higher-Order Logic

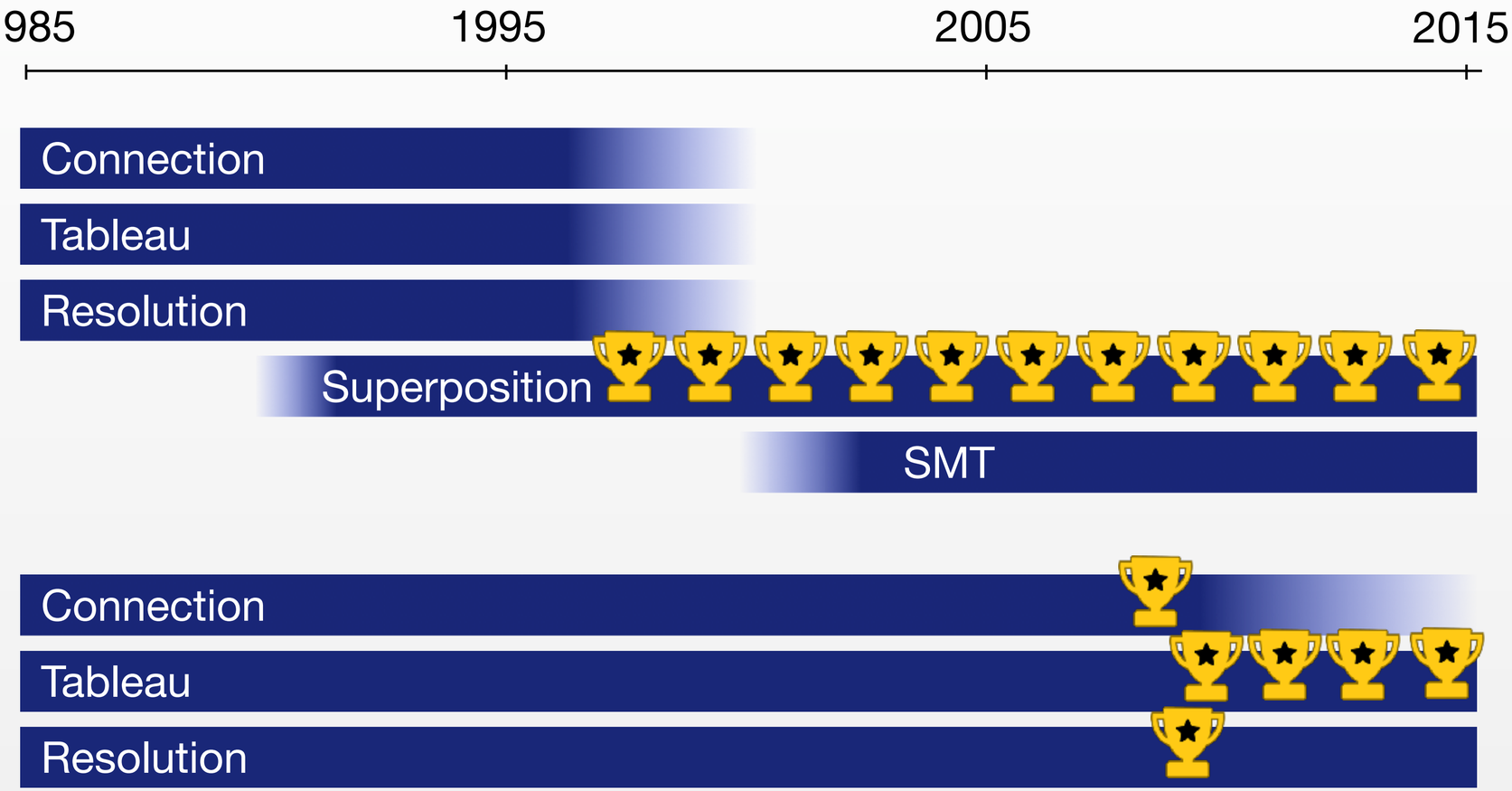


Proving Methods



First-Order Methods

Higher-Order Methods



??

Our Way to Higher-Order Logic

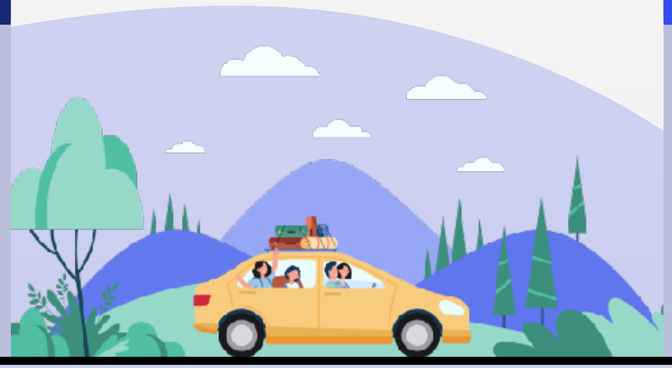
$$\sum_{\substack{p \text{ prime} \\ p \neq 2}} \frac{1}{p}$$

```
sum(
 app(app(S,
  app(app(S,
   app(app(S,app(K,ite)),
    app(app(S,
     app(app(S,app(K,and)),prime)),
     app(app(S,neq),app(K,2))))),
   app(app(S,app(div,1)),I))),
  app(K,0)))
```

1990

Superposition

Bachmair & Ganzinger



$$\sum_{\substack{p \text{ prime} \\ p \neq 2}} \frac{1}{p}$$

```
sum (S (S (S (K ite) (S
    (S (K and) prime)
    (S neq (K 2))))
  (S (div 1) I))
 (K 0))
```

2018

Superposition with Currying

Bentkamp et al.

$$\sum_{\substack{p \text{ prime} \\ p \neq 2}} \frac{1}{p}$$

```
sum (λp. ite
  (and (prime p)
     (neq p 2))
  (div 1 p) 0)
```

2019

Superposition with Lambdas

Bentkamp et al.

$$\sum_{\substack{p \text{ prime} \\ p \neq 2}} \frac{1}{p}$$

```
sum (λp.
 ite (prime p ∧ p ≠ 2)
   (div 1 p) 0)
```

2021

Higher-Order Superposition

Bentkamp et al.

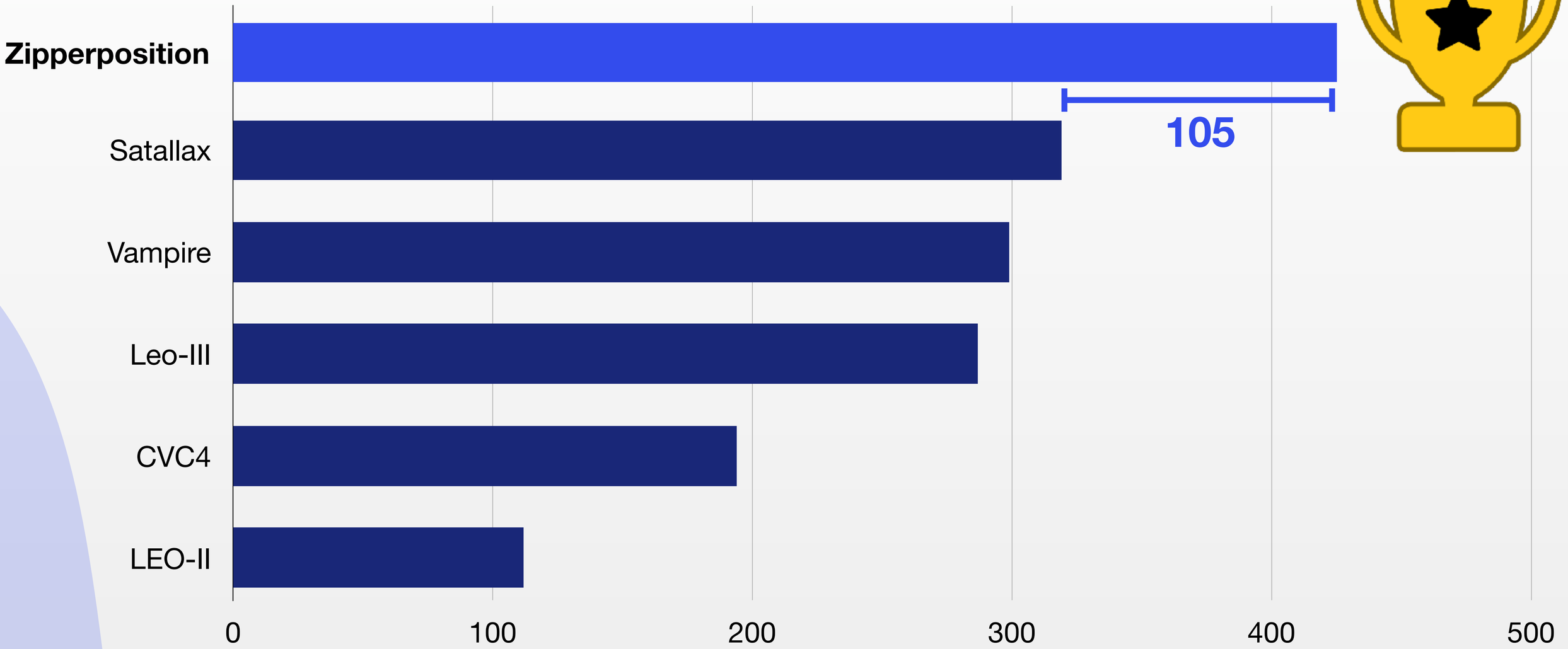
Illustration by pch.vector / Freepik

Guiding Principles

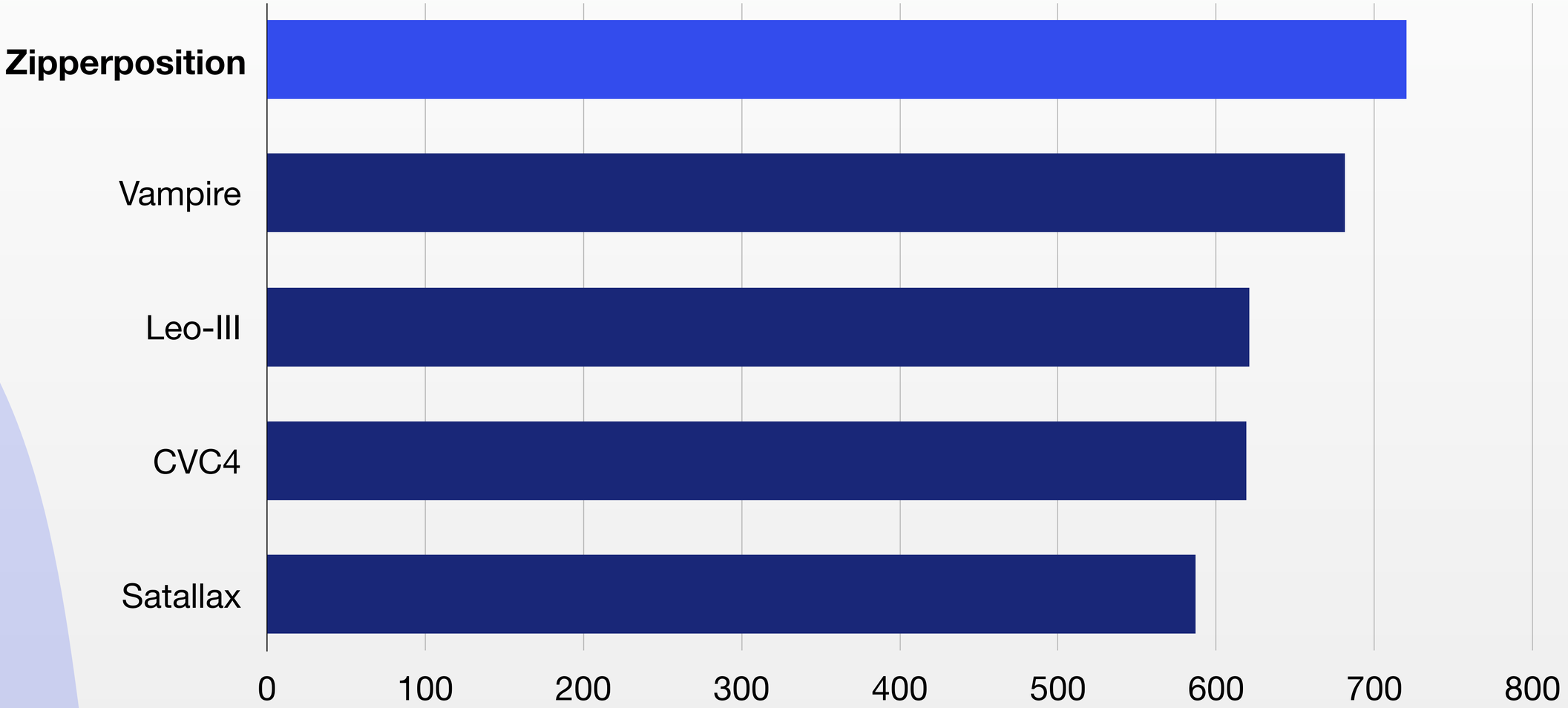
1. Don’t pay for what you don’t use!

2. Proof finding guarantee!

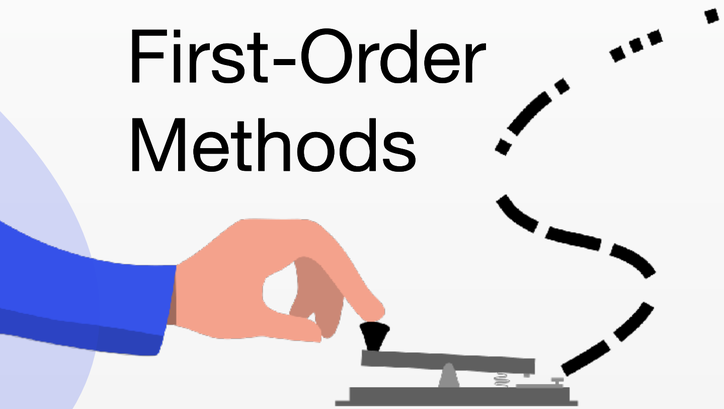
CASC Prover Competition 2020
Higher-Order Division



Our Evaluation

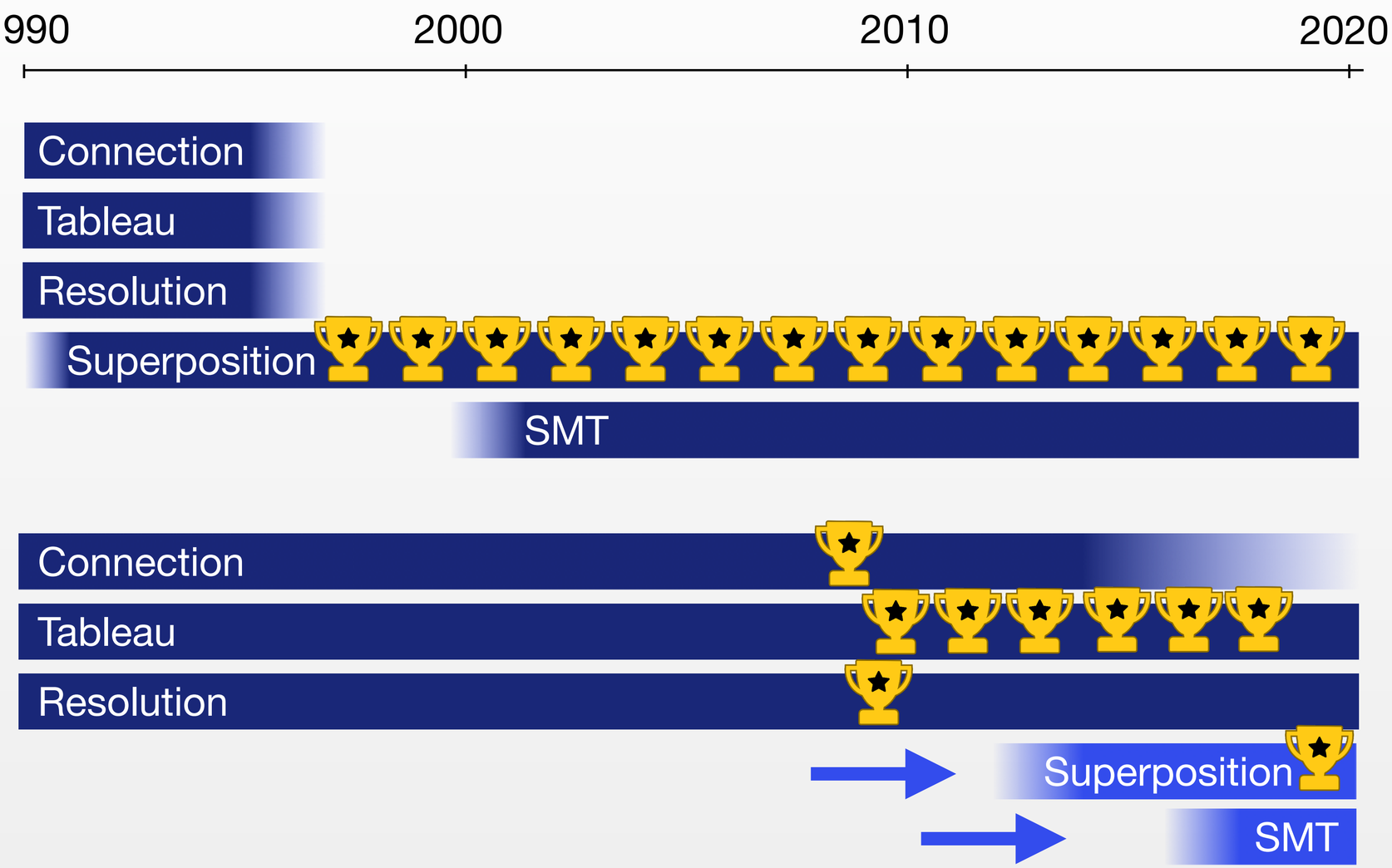


Five Years Later

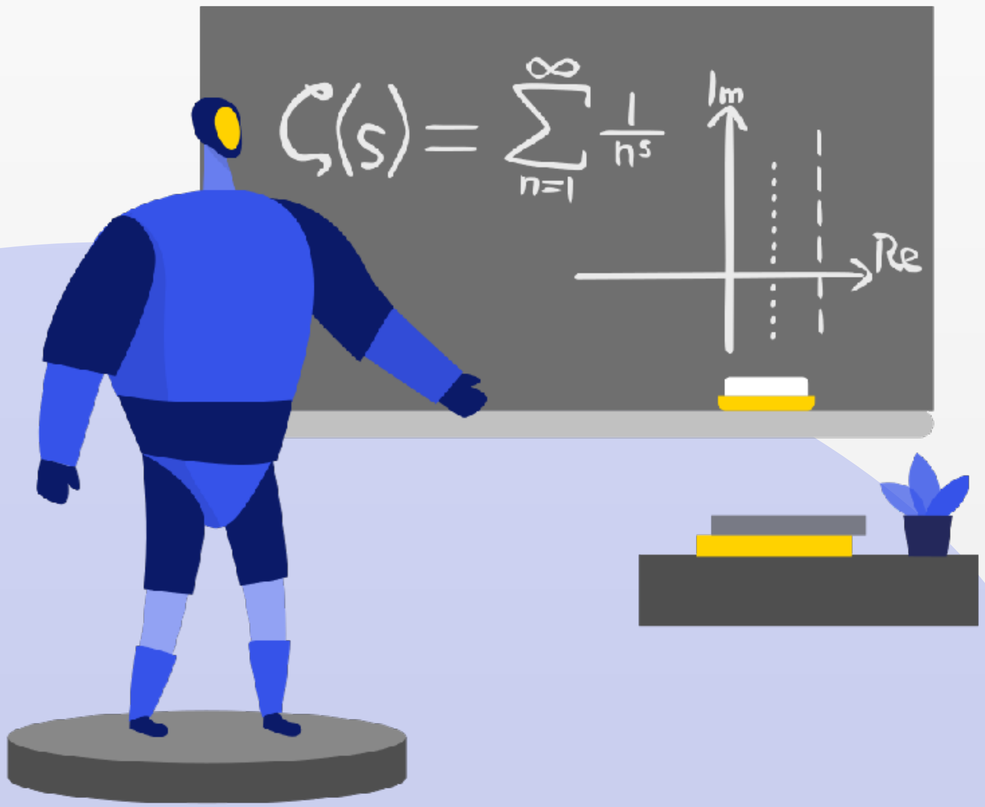


First-Order Methods

Higher-Order Methods



Are we there yet?
No.



Software and hardware engineering

Computer science

Some mathematics

Teaching of mathematics

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