Lab Report 5: Projectile Motion

PHY121

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Purpose

The purpose of this lab was to measure the distance of a metal ball shot from a spring loaded horizontally aligned "gun" before it hit the ground. We then used that distance to the calculate the ball's initial velocity and predict its distance when shot from the gun tilted at an angle.

Theory

Acceleration due to gravity only impacts the vertical component of motion. This means that a projectile shot horizontally and one dropped a the same height, will have the same amount of time in the air.
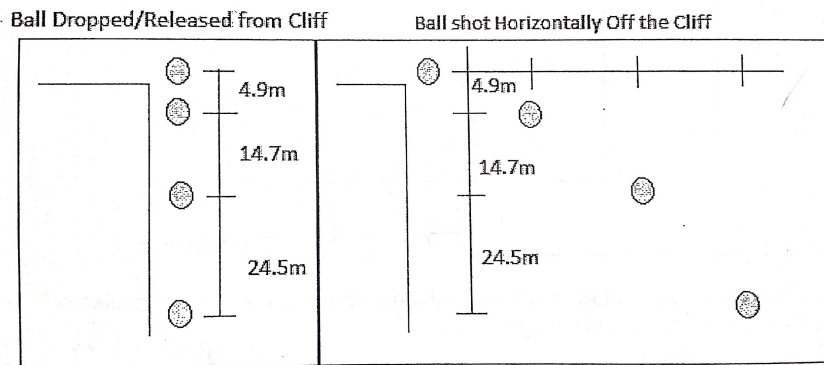
Figure 1: A ball dropped from a height will hit the ground at the same time as a ball that is launched horizontally, since gravity only impacts the vertical component of motion.

Since gravity only affects the vertical component of motion, then, assuming no air resistance, the initial horizontal velocity of an object will be the same throughout its motion. More specifically, the acceleration due to gravity of an object in the horizontal direction is $0\,m/s^2$, while in the vertical direction its $-9.8\,m/s^2$. This difference in accelerations in the x and y directions slightly changes the equations of kinematics used. All the equations of kinematics used for 1D motion still apply in the y direction, however, since acceleration due to gravity in the x direction is $0\,m/s^2$, the horizontal distance an object travels can be easily determined using the following equation:

$$x = Vo_x t$$

When dealing with objects fired at angles, we must use trigonometric functions to seperate the x and y directions (the same way one would break up a vector into x and y components). The following equations can be used to determine the initial velocity of an object in the x and y directions:

$$Vo_x = Vosin(\theta)$$
$$Vo_y = Vocos(\theta)$$

Procedure

Horizontal Gun

1. Measured height of the gun above ground using a **meter stick**.

2. Fired gun, then measured distance travelled.

3. Marked where the ball landed on the ground with tape.

4. Repeaated steps 1-3, five times to calculate average distance.

5. With the average distance and height of the gun above the ground, used equations of kinematics to calculate the ball's time in the air and initial velocity.

Inclined Gun

6. Guessed where the ball would land, marking our expectations with tape.

7. Inclined gun to 40°.

8. Measured new height of the gun above ground.

9. Calculated expected time in the air and horizontal distance using the initial velocity found in step 5.

10. Marked expected distance on the ground with tape.

11. Shot ball from the angled gun five times.

12. Calculated average distance and standard deviation of trials in step 4.

13. Repeated steps 6-12 at a different angle.

Calculations & Graphs

Equations of Kinematics For Vertical & Horizontal Motion

$$y = Vo_y t + \frac{1}{2}a_y t^2 \tag{1}$$

$$x = Vo_x t \tag{2}$$

Sample Calculation
hang time and inital velocity of horizontal launch

Knowns:

$$y = -0.276\,m$$
$$Vo_y = 0\,m/s$$
$$a_y = -9.8\,m/s^2$$
$$x = 1.231\,m$$

Calculating Hang Time:

$$y = Vo_y t + \frac{1}{2}a_y t^2$$
$$-0.276 = (0)(t) + \frac{1}{2}(-9.8)t^2$$
$$-0.276 = -4.9t^2$$
$$0.05\overline{632} = t^2$$
$$\sqrt{0.05\overline{632}} = t$$
$$t = \boxed{0.2373\,s}$$

Calculating Initial Velocity:

$$x = Vo_x t$$
$$1.231 = (Vo_x)(.2373)$$
$$Vo_x = \boxed{5.187\,m/s}$$

Equations for Vertical & Horizontal Components of Initial Velocity

$$Vo_y = Vosin(\theta) \tag{3}$$
$$Vo_x = Vocos(\theta) \tag{4}$$

Sample Calculation
inclined gun at 40 degrees

Knowns:

$$Vo = \textbf{5.187}\,\textbf{\textit{m/s}}$$

Calculating Vertical Initial Velocity:

$$Vo_y = Vosin(\theta)$$
$$= (5.187)(sin(40°))$$
$$Vo_y = \boxed{3.334\,m/s}$$

Calculating Horizontal Initial Velocity:

$$Vo_x = Vocos(\theta)$$
$$= (5.187)(cos(40°))$$
$$Vo_x = \boxed{3.973\,m/s}$$

Average Value Formula

$$\overline{a} = \frac{sum\,of\,values}{total\,\#\,of\,values}$$

Sample Calculation
average distance of horizontal ball launch

$$\overline{a} = \frac{1.331 + 1.279 + 1.228 + 1.2 + 1.118}{5}$$
$$= \boxed{1.231\,m}$$

Standard Deviation Formula

$$\sigma = \sqrt{\frac{\Sigma(x_i - \overline{a})^2}{N}}$$

$$= \sqrt{\frac{SS}{N}}$$

N : Total number of values

$\overline{\mathbf{a}}$: Average value

$\mathbf{x_i}$: Each value from the data set

SS : Sum of squares

Sample Calculation
std horizontal ball launches

$$\sigma = \sqrt{\frac{(1.331 - \overline{a})^2 + ... + (1.118 - \overline{a})^2}{5}}$$

$$= \sqrt{\frac{0.0260428}{5}}$$

$$= \boxed{0.07217 \, m}$$

Relative Error Formula

$$RE = \left| \frac{V_A - V_E}{V_E} \right| \text{ x } 100\%$$

$\boldsymbol{V_A}$: Actual value observed

$\boldsymbol{V_E}$: Expected value

Sample Calculation
inclined launch at 40 degrees - measured vs calculations

$$RE = \left| \frac{2.925 - 2.704}{2.704} \right| \text{ x } 100\%$$

$$= \boxed{8.157\%}$$

Horizontal Launch

Trials

Trials	Distances (m)
1	1.331
2	1.279
3	1.228
4	1.2
5	1.118
Average (m)	1.231
Standard Deviation (m)	0.07217

Table 1: Horizontal Launch Trials

Calculations

X	$x^{!}$	Vo_x	V_x	a_x	t
	$1.231\,m$	$5.18\,m/s$	$5.187\,m/s$	$0\,m/s^2$	$0.237\,s$

Y	$y^{!!}$	Vo_y	V_y	a_y
	$-0.276\,m$	$0\,m/s$	$-2.322\,m/s$	$-9.80\,m/s^2$

[!] Average distance of horizontal launch trials
[!!] Height of gun to ground

Table 2: Horizontal Launch Calculations

Inclined Launch at 40°

Calculations

$V^{!}$	θ
$5.187\,m/s$	$40°$

X	x	Vo_x	V_x	a_x	t
	$2.704\,m$	$3.973\,m/s$	$3.973\,m/s$	$0\,m/s^2$	$0.680\,s$

Y	y	Vo_y	V_y	a_y
	$-1.05\,m$	$3.334\,m/s$	$0\,m/s$	$-9.80\,m/s^2$

[!] Taken from horizontal launch trials since gun compression was the same.

Table 3: Inclined Launch at 40° Calculations

Trials

Trials	Distances (m)	Relative Error (%)
1	2.925	8.157
2	3.054	12.92
3	2.87	6.123
4	2.927	10.81
5	2.734	3.683
Average (m)	2.902	
Standard Deviation (m)	0.1034	

Table 4: Inclined Launch at 40° Trials

Inclined Launch at 55°

Calculations

$V^!$	θ
$5.187\,m/s$	$55°$

X	x	Vo_x	V_x	a_x	t
	$2.580\,m$	$2.975\,m/s$	$3.975\,m/s$	$0\,m/s^2$	$0.867\,s$

Y	y	Vo_y	V_y	a_y
	$-1.05\,m$	$4.249\,m/s$	$0\,m/s$	$-9.80\,m/s^2$

$^!$ Taken from horizontal launch trials since gun compression was the same.

Table 5: Inclined Launch at 55° Calculations

Trials

Trials	Distances (m)	Relative Error (%)
1	2.748	6.5
2	2.52	2.324
3	2.723	5.523
4	2.542	1.475
5	2.733	5.914
Average (m)	2.653	
Standard Deviation (m)	0.1001	

Table 6: Inclined Launch at 55° Trials

Conclusion

The purpose of this lab was to use the measurements from a ball launched horizontally to determine the distance that ball would travel when launched at an angle. By applying the equations of kinematics and recognizing the independence of a motion's horizontal and vertical components, we were able to make our calculations. First we measured the average distance of the ball, when launched horizontally five times. We measured an average distance of 1.231 meters (see Table 1). With a standard deviation of $7.2127\text{x}10^{-2}$ meters (Table 1), our measurements were precise. We measured the height of the gun to be 0.276 meters (Table 1) and, by applying equation 1, determined the ball's time in the air to be 0.2373 seconds (Table 1). Using equation 2, we determined the initial velocity of the ball to be $5.187\,m/s$ (Table 1).

Our calculations for the distance the ball would travel when launched at an incline of 40°, required us to split the initial velocity into horizontal and vertical components. Since we did not change the compression of the gun, only the angle, we used the initial velocity for x that we calculated in Table 1. Applying equations 1 and 2, we calculated the horizontal distance the ball would travel to be 2.704 meters (Table 3). The average distance of our inclined launch trials was 2.902 meters with a standard deviation of 0.1034 meters so our measurements were precise (see Table 4). While most our trials were within 10% error of our calculations, trials 2 and 4 had relative errors of 12.92% and 10.81% error respectively (Table 4). The higher relative error observed in the data can be attributed to the less than ideal conditions in which the experiment was conducted. External environmental factors such as vibrations of the table on which the gun was placed caused by the presence and activity of other people in the room, could have impacted the launch. To reduce this impact as much as possible, a repeat of this experiment could have the gun on an isolated platform, such that vibrations or similar environmental disturbances cannot affect the gun launch.

In contrast to the inclined launch at 40°, our calculations and subsequent trials for the inclined launch at 55° were much closer. We calculated the horizontal distance the gun would travel to be 2.580 meters (Table 5). The average distance of our trials at 55° was 2.653 meters with a standard deviation of 0.1001 meters (Table 6). All of our measurements were within 10% error, so not only were our measurements precise, they were also accurate. These trials were conducted near the end of class time when less people were around, so it's possible there was less environmental interference on our launches.