Lab Report 5: Projectile Motion

PHY121

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Purpose

The purpose of this lab was to measure the distance of a metal ball shot from a spring loaded horizontally aligned "gun" before it hit the ground. We then used that distance to the calculate the ball's initial velocity and predict its distance when shot from the gun tilted at an angle.

Theory

Acceleration due to gravity only impacts the vertical component of motion. This means that a projectile shot horizontally and one dropped a the same height, will have the same amount of time in the air.

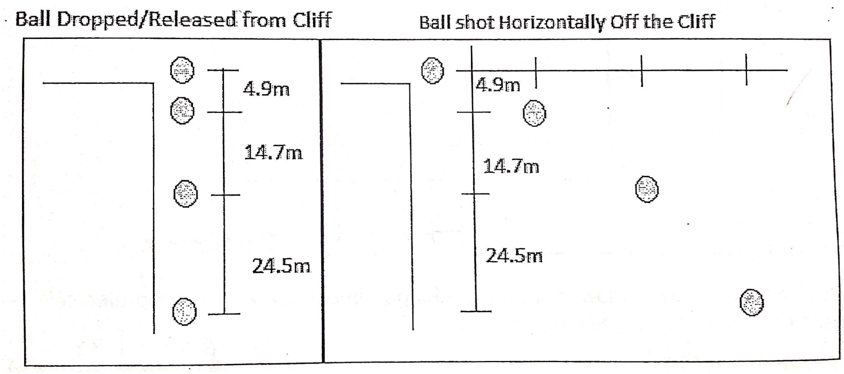

Figure 1: A ball dropped from a height will hit the ground at the same time as a ball that is launched horizontally, since gravity only impacts the vertical component of motion.

Since gravity only affects the vertical component of motion, then, assuming no air resistance, the initial horizontal velocity of an object will be the same throughout its motion. More specifically, the acceleration due to gravity of an object in the horizontal direction is $0\,m/s^2$, while in the vertical direction its $-9.8\,m/s^2$. This difference in accelerations in the x and y directions slightly changes the equations of kinematics used. All the equations of kinematics used for 1D motion still apply in the y direction, however, since acceleration due to gravity in the x direction is $0\,m/s^2$, the horizontal distance an object travels can be easily determined using the following equation:

$$x = Vo_x t$$

When dealing with objects fired at angles, we must use trigonometric functions to seperate the x and y directions (the same way one would break up a vector into x and y components). The following equations can be used to determine the initial velocity of an object in the x and y directions:

$$Vo_x = Vosin(\theta)$$
$$Vo_y = Vocos(\theta)$$

Procedure

Horizontal Gun

1. Measured height of the gun above ground using a **meter stick**.
2. Fired gun, then measured distance travelled.
3. Marked where the ball landed on the ground with tape.
4. Repeaated steps 1-3, five times to calculate average distance.
5. With the average distance and height of the gun above the ground, used equations a & b to calculate the ball's time in the air and initial velocity.

Inclined Gun

6. Guessed where the ball would land, marking our expectations with tape.
7. Inclined gun to 40°.
8. Measured new height of the gun above ground.
9. Calculated expected time in the air and horizontal distance using the initial velocity found in step 5.
10. Marked expected distance on the ground with tape.
11. Shot ball from the angled gun five times.
12. Calculated average distance and standard deviation of trials in step 4.
13. Repeated steps 6-12 at a different angle.

Calculations & Graphs

Equations of Kinematics For Vertical & Horizontal Motion

$$y = Vo_y t + \frac{1}{2} a_y t^2 \tag{1}$$
$$x = Vo_x t \tag{2}$$

Sample Calculation
hang time and inital velocity of horizontal launch

Knowns:

$$\begin{aligned} y &= \boldsymbol{-0.276\, m} \\ Vo_y &= \boldsymbol{0\, m/s} \\ a_y &= \boldsymbol{-9.8\, m/s^2} \\ x &= \boldsymbol{1.231\, m} \end{aligned}$$

Calculating Hang Time:

$$\begin{aligned} y &= Vo_y t + \frac{1}{2} a_y t^2 \\ -0.276 &= (0)(t) + \frac{1}{2}(-9.8)t^2 \\ -0.276 &= -4.9t^2 \\ 0.05\overline{632} &= t^2 \\ \sqrt{0.05\overline{632}} &= t \\ t &= \boxed{0.2373\, s} \end{aligned}$$

Calculating Initial Velocity:

$$\begin{aligned} x &= Vo_x t \\ 1.231 &= (Vo_x)(.2373) \\ Vo_x &= \boxed{5.187\, m/s} \end{aligned}$$

Equations for Vertical & Horizontal Components of Initial Velocity

$$Vo_y = Vosin(\theta) \tag{3}$$
$$Vo_x = Vocos(\theta) \tag{4}$$

Sample Calculation
inclined gun at 40 degrees

Knowns:

$$Vo = \mathbf{5.187\, m/s}$$

Calculating Vertical Initial Velocity:

$$\begin{aligned} Vo_y &= Vosin(\theta) \\ &= (5.187)(sin(40^\circ)) \\ Vo_y &= \boxed{3.334\, m/s} \end{aligned}$$

Calculating Horizontal Initial Velocity:

$$\begin{aligned} Vo_x &= Vocos(\theta) \\ &= (5.187)(cos(40^\circ)) \\ Vo_x &= \boxed{3.973\, m/s} \end{aligned}$$

Average Value Formula

$$\overline{a} = \frac{sum\, of\, values}{total\, \#\, of\, values}$$

Sample Calculation
average distance of horizontal ball launch

$$\begin{aligned} \overline{a} &= \frac{1.331 + 1.279 + 1.228 + 1.2 + 1.118}{5} \\ &= \boxed{1.231\, m} \end{aligned}$$

Standard Deviation Formula

$$\sigma = \sqrt{\frac{\Sigma(x_i - \overline{a})^2}{N}} = \sqrt{\frac{SS}{N}}$$

$\mathbf{N}$: Total number of values
$\overline{\mathbf{a}}$: Average value
$\mathbf{x_i}$: Each value from the data set
$\mathbf{SS}$: Sum of squares

Sample Calculation
std horizontal ball launches

$$\sigma = \sqrt{\frac{(1.331 - \overline{a})^2 + ... + (1.118 - \overline{a})^2}{5}} = \sqrt{\frac{0.0260428}{5}} = \boxed{0.07217\, m}$$

Relative Error Formula

$$RE = \left| \frac{V_A - V_E}{V_E} \right| \text{ x } 100\%$$

$\boldsymbol{V_A}$: Actual value observed
$\boldsymbol{V_E}$: Expected value

Sample Calculation
inclined launch at 40 degrees - measured vs calculations

$$RE = \left| \frac{2.925 - 2.704}{2.704} \right| \text{ x } 100\% = \boxed{8.157\%}$$

Horizontal Launch

Trials

Trials	Distances (m)
1	1.331
2	1.279
3	1.228
4	1.2
5	1.118
Average (m)	1.231
Standard Deviation (m)	0.07217

Table 1: Horizontal Launch Trials

Calculations

X	$x^{!}$	Vo_x	V_x	a_x	t
	$1.231\,m$	$5.18\,m/s$	$5.187\,m/s$	$0\,m/s^2$	$0.237\,s$

Y	$y^{!!}$	Vo_y	V_y	a_y
	$-0.276\,m$	$0\,m/s$	$-2.322\,m/s$	$-9.80\,m/s^2$

! Average distance of horizontal launch trials
!! Height of gun to ground

Table 2: Horizontal Launch Calculations

Inclined Launch at 40°

Calculations

$V^{!}$	θ
$5.187\,m/s$	40°

X	x	Vo_x	V_x	a_x	t
	$2.704\,m$	$3.973\,m/s$	$3.973\,m/s$	$0\,m/s^2$	$0.680\,s$

Y	y	Vo_y	V_y	a_y
	$-1.05\,m$	$3.334\,m/s$	$0\,m/s$	$-9.80\,m/s^2$

! Taken from horizontal launch trials since gun compression was the same.

Table 3: Inclined Launch at 40° Calculations

Trials

Trials	Distances (m)	Relative Error (%)
1	2.925	8.157
2	3.054	12.92
3	2.87	6.123
4	2.927	10.81
5	2.734	3.683
Average (m)	2.902	
Standard Deviation (m)	0.1034	

Table 4: Inclined Launch at 40° Trials

Inclined Launch at 55°

Calculations

$\boldsymbol{V}^!$	$\boldsymbol{\theta}$				
$5.187\,m/s$	$55°$				
X	$\boldsymbol{x}$	$\boldsymbol{Vo_x}$	$\boldsymbol{V_x}$	$\boldsymbol{a_x}$	$\boldsymbol{t}$
	$2.580\,m$	$2.975\,m/s$	$3.975\,m/s$	$0\,m/s^2$	$0.867\,s$
Y	$\boldsymbol{y}$	$\boldsymbol{Vo_y}$	$\boldsymbol{V_y}$	$\boldsymbol{a_y}$	
	$-1.05\,m$	$4.249\,m/s$	$0\,m/s$	$-9.80\,m/s^2$	

! Taken from horizontal launch trials since gun compression was the same.

Table 5: Inclined Launch at 55° Calculations

Trials

Trials	Distances (m)	Relative Error (%)
1	2.748	6.5
2	2.52	2.324
3	2.723	5.523
4	2.542	1.475
5	2.733	5.914
Average (m)	2.653	
Standard Deviation (m)	0.1001	

Table 6: Inclined Launch at 55° Trials

Conclusion