Lab Report 11: Center of Mass and Torque

PHY121

November, 18th, 2024

Professor R. Lathrop — Professor M. Petralia

Abereni Opuiyo

Table of Contents

Purpose

To use static equilibrium situations to solve for the center of mass of rigid rods.

Theory

Torque is the measure of force needed to cause an object to rotate. Torque can be calculated by multiplying the force applied to an object, the radius of that force from the object's pivot point, and the angle between the component of the force that is perpendicular to the pivot axis. This relationship between the angle of the force applied, and an object's pivot point can be represented using the following equation:

$$T = \|\mathbf{r}\|\|\mathbf{F}\|\sin\theta \tag{1}$$

Since gravity is a force that acts on all parts of an object, it applies torque at multiple points at the same time. Instead of calculating the torque caused by gravity at multiple positions on an object, a single position, the object's *center of mass*, is used. Objects with a uniform distribution of mass, typically symmetrical with no varying mass, usually have their center of mass right at the center as shown below:

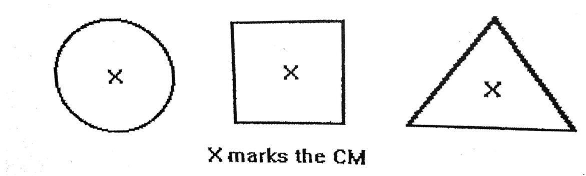

Figure 1: Symmetrical objects usually have their center of mass...at the center.

An object's center of mass can be calculated using the following equation:

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + ...}{M_T}$$

$$\begin{aligned} x_{\text{com}} &: \text{center of mass} \\ m_1 &: \text{mass of object 1} \\ m_2 &: \text{mass of object 2} \\ x_1 &: \text{position of object 1} \\ x_2 &: \text{position of object 2} \\ M_T &: \text{total mass of system} \end{aligned}$$

Procedure

1. Found the center of the **meter stick** by going to the halfway point (0.5 m).
2. Placed **knife edge clamp** on the **meter stick.**
3. Slid **knife edge clamp** on **meter stick** until **meter stick** was balanced on the **knife edge** support.
4. Recorded center of mass of the **meter stick**.
5. Attached 60 g of mass to the **meter stick** at 20 cm from one end.
6. Recorded distance of the 60 g to the pivot point.
7. Placed 100 g on the **meter stick** such that it remained balanced.
8. Solved for the actual placement position of the 100 g using center of mass equation.
9. Attached 20 g of mass to the **meter stick** at 10 cm from the end on the same side as the 60 g.
10. Adjusted placement of the 100 g so that the **meter stick** remained balanced.
11. Repeated step 8.
12. Removed all attached masses.
13. Placed 50 g of mass on the meter stick at 20 cm from one end.
14. Moved **knife edge** placement such that the **meter stick's** center of mass changed.
15. Solved for **meter stick's** center of mass using center of mass equation.

Calculations & Graphs

Center of Mass

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2 + ...}{M_T} \tag{2}$$

$$\begin{aligned}
x_{\text{com}} &: \text{center of mass} \\
m_1 &: \text{mass of object 1} \\
m_2 &: \text{mass of object 2} \\
x_1 &: \text{position of object 1} \\
x_2 &: \text{position of object 2} \\
M_T &: \text{total mass of system}
\end{aligned}$$

Sample Calculation

using values from Scenario 1

$$\begin{aligned}
x_{\text{com}} &= \frac{m_1 x_1 + m_2 x_2 + ...}{M_T} \\
&= \frac{(0.06\text{ kg})(0.8\text{ m}) + (0.1\text{ kg})(0.333\text{ m})}{(0.16\text{ kg})} \\
x_{\text{com}} &= \boxed{0.509\text{ m}}
\end{aligned}$$

Percent Error

$$\text{PD} = \left| \frac{\text{measured - actual}}{\text{actual}} \right| \text{ x } 100\% \tag{3}$$

Sample Calculation

percent error between measured and calculated position for 100 g in Scenario 1

$$\text{PD} = \left| \frac{\text{measured - actual}}{\text{actual}} \right| \text{ x } 100\%$$

$$\text{PD} = \left| \frac{0.333\text{ m - }0.334\text{ m}}{0.334\text{ m}} \right| \text{ x } 100\%$$

$$\text{PD} = \boxed{0.29\%}$$

Graphs & Tables

Figure 2: Scenario 1

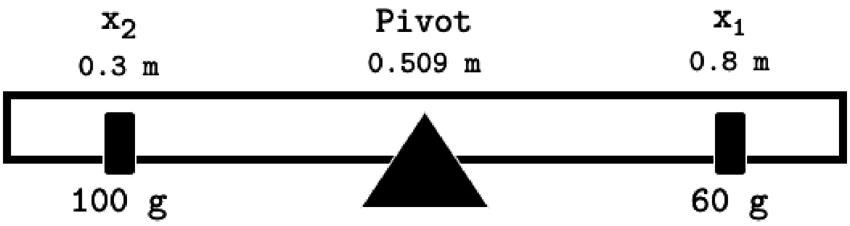

Table 1: Scenario 1 - Values

$\mathbf{x_{com}}$ **(m)**	0.509		
$\mathbf{x_1}$ **(m)**	0.8	$\mathbf{m_1}$ **(kg)**	0.06
$\mathbf{x_2}$ **Actual (m)**	0.333	$\mathbf{m_2}$ **(kg)**	0.1
$\mathbf{x_2}$ **Expected (m)**	0.3344	$\mathbf{M_T}$ **(kg)**	0.16
$\mathbf{x_2}$ **Error (%)**	0.4186		

Figure 3: Scenario 2

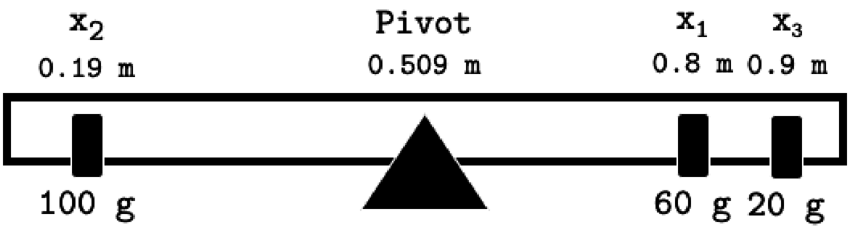

Table 2: Scenario 2 - Values

$\mathbf{x_{com}}$ **(m)**	0.509		
$\mathbf{x_1}$ **(m)**	0.83	$\mathbf{m_1}$ **(kg)**	0.06
$\mathbf{x_2}$ **Actual (m)**	0.19	$\mathbf{m_2}$ **(kg)**	0.1
$\mathbf{x_2}$ **Expected (m)**	0.2322	$\mathbf{m_3}$ **(kg)**	0.02
$\mathbf{x_3}$ **(m)**	0.93	$\mathbf{M_T}$ **(kg)**	0.18
$\mathbf{x_2}$ **Error (%)**	18.17		

Figure 4: Scenario 3

Table 3: Scenario 3 - Values

$\mathbf{x_{com}}$ **(m)**	0.6		
$\mathbf{x_1}$ **(m)**	0.7	$\mathbf{m_1}$ **(kg)**	0.05
$\mathbf{x_{com}}$ **Expected (m)**	0.5833	$\mathbf{M_T}$ **(kg)**	0.06
$\mathbf{x_{com}}$ **Error (%)**	2.857		

Questions

1. **Should the 100 grams be placed closer or further away from the pivot point when compared to the 60 grams?**

   The 100 g should be placed closer so that the torque it applies roughly equals the torque being applied by the 60 g on the other side.

2. **How does the calculated value compare to the experimental value you found? (Percent error)**

   The calculated value is extremely close to the experimental value we found, with an error of only 0.4186%.

3. **How does the calculated value compare to the experimental value you found? (Percent error)**

   The calculated value is 4 cm off what we found through experimentation. There error is 18.17%. This is most likely due to not taking into account the mass of the special mass holder used to hold up the 0.02 g in our calculations.

4. **Where do you need to place the knife edge such that the meter stick would be balanced?**

   We needed to place the knife edge closer to the 50 g so that the meter stick remained balanced.

5. **Explain in your own words why the meter stick needed to be shifted to find the new balance point. How can the system balance with one mass placed on it?**

Since there's no other mass on the other side of the meter stick to apply the necessary torque to balance out the torque of the 50 g, the meter stick needed to be shifted. By changing the meter stick's center of mass such that it's closer to the 50 g mass, we reduce the torque that it applies, balancing the rod again.

6. **Describe the procedure that you used to find the center of mass.**

   We unhooked the knife edge and kept sliding the meter stick until it stopped tipping over to the right.

7. **What can you deduce about the distribution of mass inside the non uniform rod? (Is one side heavier than the other? How does that affect finding the center of mass?)**

   There's an uneven distribution of mass inside a non uniform rod. One side IS heavier than the other, so the center of mass needs to be closer to the heavier side, reducing the torque on that end to make the net torque from left to right closer to 0.

Conclusion

The purpose of this lab was to use static equilibrium to solve for the center of mass of a rigid rod. By using a meter stick as our rigid rod, and balancing it on a knife edge, we were able to experimentally and through calculation, determine not only its center of mass, but also what positions to place masses on it and maintain static equilibrium. First, we experimentally found the meter stick's center of mass to be 0.509 m (see Table 1). Then we placed a 60 g mass 20 cm from one end of the meter stick, applying a torque on that side (see Figure 2). Next we experimentally determined the position to place a 100 g mass to the left of the meter stick's pivot such that the net torque on the meter stick was 0 (see Figure 2). Through experimentation, we determined the position of the 100 g mass to balance the meter stick, to be at 0.333 m on the meter stick (see Table 1). Since torque decreases the closer an object is to the pivot point, it makes sense that in order to balance out the torque of the 60 g mass on the other side, the 100 g mass was closer to it than the 60 g mass. Using equation 2 and rearranging it to solve for the expected position of the 100 g mass, we found there to be a 0.4186% difference with our measured value.

In the next scenario, we had results that differed a bit from our calculations. We added a 20 g mass to the same side as the 60 g mass, and through experimentation, found the position to place the 100 g mass to be at 0.19 m on the meter stick. Using equation 2, we found the expected position of the 100 g mass to be 0.2322 m, which is an 18.17% difference from our measured value (see Table 2). Considering that the expected value is closer to the pivot, it's possible that the 20 g mass, and its mass holder, were greater than we thought. Since torque increases the further an object is from the pivot (see 1), the 100 g we measured was applying a greater torque than the expected value (see Table 2 and equation 1). This suggests that either the 20 g mass and mass holder were more than 20 g, or the positions of them masses were farther away from the pivot than we recorded. Were I to repeat this experiment again, I would measure the weight of the mass holders as well and ensure that each mass is tightly attached to the meter stick to minimize as much potential sliding as possible.

In scenario 3 we removed all masses from the meter stick and attached only a 50 g mass, 30 cm from one end of the meter stick. This time we did not add a counter balancing mass to the other end, so we needed to change the meter stick's center of mass. By equation 1, the further an object is from the pivot point, the greater torque it applies. The uniform rod then becomes non uniform, and its distribution of mass unequal. Therefore, to reduce the torque of the 50 g mass, we needed to shift the meter stick's center of mass. Through experimentation we, found that shifting the pivot of the meter stick to 0.6 m, 10 cm behind the 50 g mass, balanced the it (see Figure 4). Using equation 2, we found the expected center of mass to be 0.5833 m, which is only a 2.857% from our measured center of mass (see Table 3).