



Subject: Multivariable Calculus
Class: BSCYS-IV
Section(s): A,B
Course Code: MA-105

Time Allowed: 180 Minutes
Max Marks: 100
FM's Name: Dr. Tanzeela Shaheen
FM's Signature: *[Signature]*

INSTRUCTIONS

- Nothing is to be written on the question paper.
- Calculators are allowed.

[Signature]
27/5/25.

CLO-1

Q1. (a) Let $f(x, y) = -\frac{xy}{x^2 + y^2}$. Find the limit along (10)

- i. The x-axis ii. The y-axis iii. The line $y = x$
iv. the line $y = -x$ v. the parabola $y = x^2$

(b) Use Lagrange multipliers to find the maximum and minimum values of f subject to the given constraint. Also, find the points at which these extreme values occur. (10)

$$f(x, y, z) = 3x + 6y + 2z; 2x^2 + 4y^2 + z^2 = 70$$

CLO-5

Q2. (a) Sketch the level curve $z = k$ for the $k = 0, 1, 2, 3, 4$, where $z = x^2 + 9y^2$. (10)

(b) Risk assessment tools are sometimes used in cybersecurity, helping organizations decide which threats to prioritize based on how likely they are and how damaging they could be. The following table describes risk level as a function of threat likelihood and impact level, that is,

$z = f(x, y)$ where z = risk level, x = threat likelihood, y = impact level.

	y					
	(1)	(2)	(3)	(4)	(5)	
x	10%	1	1	2	2	3
	30%	1	2	3	3	4
	50%	2	3	4	4	5
	70%	2	4	4	5	5

From the given table, find the risk level when

- i. The impact level is 2 and the threat likelihood is 13%
- ii. The impact level is 3.5 and the threat likelihood is 90%
- iii. The impact level is 5 and the threat likelihood is 80%
- iv. The impact level is 1.5 and the threat likelihood is 70%

CLO-4

Q3. (a) Sketch the domain of $f(x, y) = \sqrt{x^2 + y^2 - 4}$. (10)

(b) Evaluate the line integral along the curve C : (10)

$$\int_C (x^2 - y^2)dx + xdy$$

$$C: x = t^{2/3}, y = t \quad (-1 \leq t \leq 1)$$

CLO-2

Q4. (a) The temperature at a point (x, y) on a metal plate in the xy -plane is $T(x, y) = x^3 + 2y^2 + x$ degrees Celsius. Assume that distance is measured in centimeters and find the rate at which temperature changes with respect to distance if we start at the point $(1, 2)$ and move (10)

(a) to the right and parallel to the x -axis

(b) upward and parallel to the y -axis.

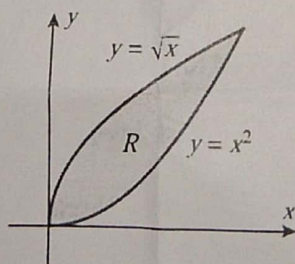
(b) Let a and b denote two sides of a triangle and let θ denote the included angle. (10)
Suppose that a , b , and θ vary with time in such a way that the area of the triangle remains constant. At a certain instant $a = 5\text{cm}$, $b = 4\text{cm}$, and $\theta = \pi/6$ radians, and at that instant both a and b are increasing at a rate of 3cm/s . Estimate the rate at which θ is changing at that instant.

CLO-3

Q5. (a) Find the absolute extrema of the given function on the indicated closed and bounded set R . (10)

$f(x, y) = xy - x - 3y$; R is the triangular region with vertices $(0, 0)$, $(0, 4)$, and $(5, 0)$.

(b) Evaluate $\iint_R (x + y)dA$, R is the region given below: (10)



((Good luck, students! May your brain be faster than your Wi-Fi))