Comparison between A star and Dijkstra Algorithm with different Kernel and Obstacle Sizes and Heuristics using Occupancy Grid in an Autonomous Vehicle

by

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With my signature, I certify that this thesis has been written by me using only the indicates resources and materials. Where I have presented data and results, the data and results are complete, genuine, and have been obtained by me unless otherwise acknowledged; where my results derive from computer programs, these computer programs have been written by me unless otherwise acknowledged. I further confirm that this thesis has not been submitted, either in part or as a whole, for any other academic degree at this or another institution.

Abrish Sabri Bremen,Germany, Date

Abstract

Consider this a separate document, although it is submitted together with the rest. The abstract aims at another audience than the rest of the proposal. It is directed at the final decision maker or generalist, who typically is not an expert at all in your field, but more a manager kind of person. Thus, don't go into any technical description in the abstract, but use it to motivate the work and to highlight the importance of your project.

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1 Introduction

Spatial model of a robot environment is a map and the process to build a map is called mapping. Occupancy grid mapping refers to a group of computer algorithms in probabilistic robotic autonomy for mobile / versatile robots which addresses the complication of generating maps from noisy and uncertain sensor measurement data with the assumption that the robot pose (position and orientation) is known. Data about the environment can be gathered from sensors progressively or be loaded from prior knowledge. Laser range finders, bump sensors, cameras and depth sensors are commonly used to discover obstacles in the robots environment.

Occupancy grid map is an array with occupancy variables. The term occupancy is characterized as a random variable. A random variable is a function that maps the sample space to the real numbers. Each element of an occupancy grid is represented with a corresponding occupancy variable which is an evenly spaced field of binary random variables each representing the presence of an obstacle at that location in the environment. Occupancy grid mapping requires bayesian filtering algorithm to maintain an occupancy grid map. Bayesian filtering applies recursive update to the map. A robot can never be certain about the world so the probabilistic idea of the occupancy rather than occupancy itself is utilized.

Method that is using occupancy grid divides area into cells (e.g. map pixels) and assign them as occupied or free. One of the grid cells is marked as robot position and another as a destination. All the grid cells are independent of each other. The occupancy probability if the grid cell is occupied is: $p(m_i) = 1$, if the grid cell is not occupied: $p(m_i) = 0$, and if there is no given knowledge of the grid cell: $p(m_i) = 0.5$. The state of each grid cell is assumed to be static.

Finding the trajectory is based on finding shortest line that do not cross any of the occupied cells. This is a difficult issue in developing autonomous frameworks in light of the fact that limiting the danger of impacts removes the productivity of most navigation strategies. This problem is liable to vulnerability and incomplete data with respect to the condition of the vehicle, the obstacles, and the responses of the vehicle to inputs. Robust methodologies for safe and efficient navigation require replanning to compensate for uncertainty and changes in the environment. The success of such methodologies is dependent on the nature of detection, planning and control, and on the transient interactions between those tasks [1]. This paper involves comparing two algorithms with additional assistance of kernel sizes, different heuristics and obstacles size and tested in a simulation that incorporates these sensors to find the most efficient trajectory plan. It focuses on finding a continuous path that can be followed from an initial configuration (or state) to a goal configuration. A safe path is one that prevents collisions with obstacles, and an efficient path is one which minimises cost.

For my thesis, I compare Dijkstra and A star with different kernel sizes and heuristics in the simulation. Dijkstra is a classic Greedy algorithm due to the process of choosing locally optimal decision and restart the operation from there. A star algorithm is a classic breadth-first-search (BFS) algorithm for finding the shortest path between two points due to its optimisation capability. It processes vertices in increasing order of their distance from the source, which are also called root vertices. The shortest path between two vertices is a path with the shortest length (i.e. least number of edges), also called link-distance.

- Let $G = (u, v)$ be a weighted undirected graph, with weight function $w : E \mapsto R$ mapping edges to real-valued weight. If $e(u, v)$, then we write $w(u, v)$ for $w(e)$.
- The length of a path $p =< v_0, v_1, v_2, v_3 ... v_k >$ would be the total of the weights of its constituent edges as in:

$$length(p) = \sum_{1}^{k} w(v_{i-1}, v_i)$$

- The distance from u to v, denoted by $\delta(u, v)$ is the length of the minimum path if there is a path from u to v and ∞ is otherwise.

The general idea of Dijkstra algorithm is to report vertices in increasing order of their distances from the source vertex while constructing the shortest path tree edge by edge; at each step adding one new edge, corresponding to the construction of the shortest path to the current new vertex. This is accomplished in the following steps:

- Maintain an estimate $d[v]$ of the $\delta(s, v)$ of the shortest path for each vertex v.
- Always $d[v] \geq \delta(s, v)$ and $d[v]$ equals the length of a known path ($d[v] = \infty$ if we have no path so far).
- Initially, $d[s] = 0$ and all other $d[v]$ values are set ∞. The algorithm will then process the vertices one by one in some order. The processed vertex's estimate will be validated as being the real shortest distance; i.e. $d[v] = \delta(s, v)$.

The term processing a vertex u means finding new paths and updating $d[v]$ for all $v \epsilon adj[u]$ if necessary. The process by which an estimate is updated is called relaxation. When all vertices have been processed, $d[v] = \delta(s, v)$ for all v [2]. The path computed using the classic Dijkstra algorithm is the shortest; however, it may not be the most feasible.

The A star algorithm follows a technique that makes use of well informed search procedure. It computes the shortest path between two nodes in a graph. While Dijkstra algorithm indiscriminately picks the next node accessible, the A star algorithm uses heuristic that estimates the distance from any node to the target to choose the best node leading to the target. This estimate acts as an archetype for the algorithm and speeds up the calculation.

During the A star algorithm, each vertex has one of the following three states:

- The vertex is not known and thus has not been processed therefore there is no path from the beginning to this vertex.
- The vertex is in the priority queue. Some route prompting this vertex is known, yet there might be a shorter way.
- The vertex is completely processed. The shortest path from the starting vertex to the current one is known.

The algorithm first adds the starting vertex to the empty priority queue. Each vertex in the priority queue has an f-value. This value is the sum of the distance from the starting vertex to this vertex and the estimate of its distance to the target vertex. The vertex with the smallest f-value drives the priority queue and will be processed next. The algorithm now takes the vertex with the minimum f-value from the priority queue until the queue is empty or a path to the target vertex has been found. In the event that the vertex taken from the queue is the target vertex, at that point the algorithm has found the shortest path and the program terminates. If the priority queue becomes empty, then no path from the start to the target is possible and the algorithm terminates. Subsequent to processing a vertex from the priority queue, its neighbors are examined. The algorithm recognizes three cases:

- The neighbor has already been processed at that point the algorithm does nothing
- The neighbor is already in the priority queue, If the current path is an alternate shortcut, update its f-value.
- The neighbor is not in the priority queue. Process the f-value of the vertex and add it to the priority queue.

Each time when updating the cost of some vertex, the algorithm saves the edge that was used for the update as the predecessor of the node. Towards the end of the algorithm, the shortest path to each vertex can be constructed by going backwards using the predecessor edges until the starting vertex is reached. In the event that a vertex cannot be reached from the starting vertex, at that point its cost remains infinite. [3].

Dijkstra Algorithm is ensured to locate the shortest path given the input graph. A star algorithm is ensured to find the shortest path if the heuristic is never larger than the true distance. As the heuristic decreases, A star transforms into Dijkstra Algorithm. As the heuristic increases, A star transforms into Greedy Best First Search.

The major disadvantage of Dijkstra algorithm is the fact that it runs a blind search there by consuming a lot of time and in addition a waste of necessary resources and as a result it generally makes it slower than A star. Another disadvantage is that it cannot handle negative edges. This leads to acyclic graphs and most often cannot obtain the right shortest path. Dijkstra algorithm has an order of n^2 so it is efficient enough to use for relatively large problems.

2 Statement and Motivation of Research

In recent years, a whole lot of conclusive research results have been concluded using Dijkstra Algorithm and A star (which is an optimized version of Dijkstra Algorithm) to be the most fitting method for path planning in autonomous vehicles. [2] proposes a robotic path planning using a multilayer dictionary which provides more comprehensive data structure for Dijkstra algorithm in an indoor environment application where GPS coordinates and compass orientation are unreliable. Another group of researchers in [4] analyzed node-based optimal algorithms based on 3D path planning. Node based optimal algorithms deal with nodes and arcs weight information. Their task is to find the optimal path through calculating the cost by traversing through the nodes. Essentially, two main examples of node-based optimal algorithms are Dijkstra and A star. Their survey concluded these type of algorithms cannot be further optimize the result beyond the decomposition of the environment. The results of these kind of algorithms rely much on the preconstructed graph and can be combined with other methods to achieve global optimal. [5] developed an interpolation based method for ideal cost-to-go function based on Dijkstra and A star. It produced an effective method for estimating feedback of a plan and determined the shortest path for motion over a simplicial complex of an arbitrary dimension. The paper also demonstrated that the computational cost is significantly reduced by implementing an A star like heuristic.

There also has been some extensive research comparing the two algorithms A star and Dijkstra in terms of their computational cost, efficiency and simplicity. After considerable amount of analysis of the two algorithms [6] Dijkstra is significantly similar to A star, except there is no heuristic; H is always 0 because of which the algorithm expands out equally in all directions. In other cases, A star scans the area only in the direction of the destination. Thus, Dijkstra ends up exploring a sizeable area before the target is found making it slower than the A star. However, both of the algorithms have their own importance; Dijkstra is mostly used when the destination of the target is unknown while A star is mainly applied when both the source and the destination is known. As an illustration, there is a delivery service unit that needs to pick up an accessory from a branded store. It may know where several stores of that brand are but it wants to go to the closest one. Here, Dijkstras is better than A star because we dont know which one is closest. The only alternative is to repeatedly use A star to find the distance to each one and then choose that path. There are probably countless similar situations where the location whereabouts might be known but not know where it might be or which one might be closest. Therefore, A star is better when we both know the initial point and final point. A star is both thorough (finds a path if one exists) and optimal (always finds the shortest path) if the admissible heuristic function is used.

In the field of path planning for motion in virtual environments and artificial intelligence such as games, there have been some group of researchers focusing on navigation to find the optimized path [[7],[8],[9]]. The authors in [8] compared various path finding algorithms in unmanned aerial vehicles for detecting targets and keeping them in its sensor range in various environments and their performance compared to establish and monitor a path for communication. K. Khantanapoka and K.Chinnasarn [9] compared the path finding algorithms in intelligent environments in order to find the contrast in their time and space complexity. Sathyaraj et al. [7] developed a path planning strategy which let the randomly deployed autonomous robots in the environment move forward till an obstacle is met. For each agent (robot), an estimation of the relative location of the obstacle nearby according to the measurements of infrared sensors was deduced which let the common agent diverge before collision. Since common agents are used to imitate real people, this moving strategy satisfies the real situation that people walk around. For pursuer, it needs to navigate to specific position when evader is located therefore the researchers applied Dijkstra algorithm for path planning.

On the contrary, instead of focusing only on finding the optimized path using the Dijkstra and A star, the proposed method in this thesis uses the sensors implemented in the simulation. This thesis is based on a simulation in the loop (SIL) procedure that incorporates real observations into the simulation in an effortless manner by synchronization of simulated conditions with real-world data. The simulation helps in analyzing and optimizing critical components like robot localization considering the components behavior in deep sea robotic operations and shows the benefit of the presented simulation in the loop approach in the context of DexROV research project. The SIL framework synchronizes simulated and real-world data by incorporating environmental and spatial feedback collected from field-trials which provides an augmented virtual environment reflecting environmental/spatial conditions from real-world missions to test, benchmark and compare behaviors of system modules, preserves the benefits of continuous system integration to perform such benchmarks using real or simulated components or a combination of both and allows to perform tests on distributed deployment, interfaces/pipelines, data regressions/degradation, and fault recovery/safety [Appendix A]. This simulation was created by the Robotics research group of Prof. Andreas Birk. It is provided by Jacobs University Bremen under the supervision of Dr. Francesco Maureli, Dr. Szymon Krupinski and Arturo Gomez Chavez.

The proposed method takes advantage of the prior knowledge that the sensors provide such as camera view, position, resolution, height and width of the grid. Since the simulation illustrates an underwater vehicles environment with a vessel nearby, the physics of the vehicle is disabled in order to get a more precise measurement of the mapping for simplicity reasons. Subsequently, making this measurement off-center than the real world calculations. For simplicity, it is assumed that the prior knowledge of the simulation/map services is believable. The occupancy grid map is based on real-time information and it updates with the detection based on camera image. The contribution focuses on planning the global path for autonomous navigation, however, in this thesis, the focus will be in checking which grid cell is occupied or free in the occupancy grid and the time it takes to find the optimal path from starting position to the goal.

3 Representation of the Data

As mentioned above in this paper, the two algorithms implemented are Dijkstra and A star for finding the optimized path for the robot used in the simulation. A generative implementation of these two algorithms could have been constructed by explicitly coding basic rules in some logic or formal grammar. However, due to the utility of the simulation extended expertise was required. It is of crucial importance how data set is represented and what information is retrieved from the data set. Subsections 3.1, 3.2 and 3.3 describe the representation of simulation performed, data collected, and data utilized, respectively. Subsection 3.4 describes the procedures of selecting and manipulating the data obtained through the simulation.

3.1 Representation of Simulation performed & ROS

Since the simulation is part of the DexRov research project, evidently it is run on the ROS. ROS (Robot Operating System) provides tools and libraries to help software developers simplify the task of creating complex and robust robot behavior across a variety of robot applications. ROS greatly aids in displaying the valuable data needed to perform the task required such as width and height between the travelled distance, resolution, the initial position of the robot and the goal it needs to reach. By running the command rostopic echo /projected_map the data is presented in the form of pose (position and orientation) [1]. The simulation is initiated by launching a vehicle, vessel and octomap launch file; followed by starting Rviz (ROS visualization), a 3D visualizer for displaying sensor data and state information from ROS. It displays live representations of sensor values coming over ROS Topics including camera data, infrared distance measurements, sonar data, and more. Rviz is an essential part of this section. The robot moves through a joystick and the software displays the live representation of the trajectory mapped by the robots movement [2].

```
#INITIAL POSITION IN METERS (START)
pose:
  position:
    x: 10.6115179532
    y: -12.484507967
    z: 2.94719725414
  orientation:
    x: -0.00106181641047
    y: 0.00037097801211
    z: -0.943184935889
    w: -0.332266326362
---
```

```
#FINAL POSITION IN METERS (GOAL)
pose:
  position:
    x: 1.21908376738
    y: 4.85488249354
    z: 2.88283420207
  orientation:
    x: -0.000842838538483
    y: -0.000929650401635
    z: -0.67355026858
    w: 0.739140352754
---
```

Figure 1: Data set displayed for the source and destination of the robot through Rviz

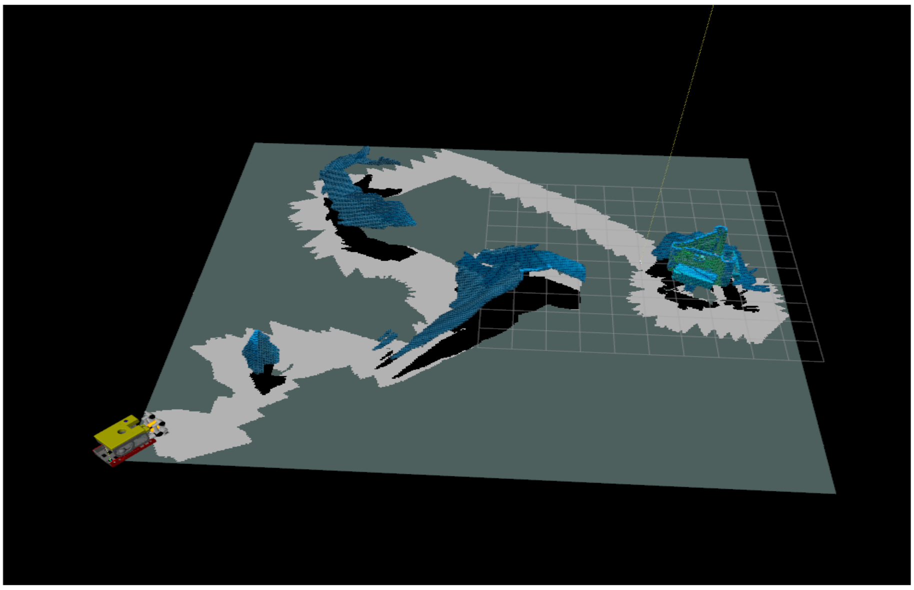


Figure 2: Projected map rendered via Rviz

Gazebo is used to simulate the robots route. This software produces the actual simulation of the underwater environment where the robot, vessel is shown by a third body. This part of the software involves disabling the physics of the environment. The software acts as an eye while Rviz collects all the data and utilizes this information to produce an occupancy grid and an octomap [3].

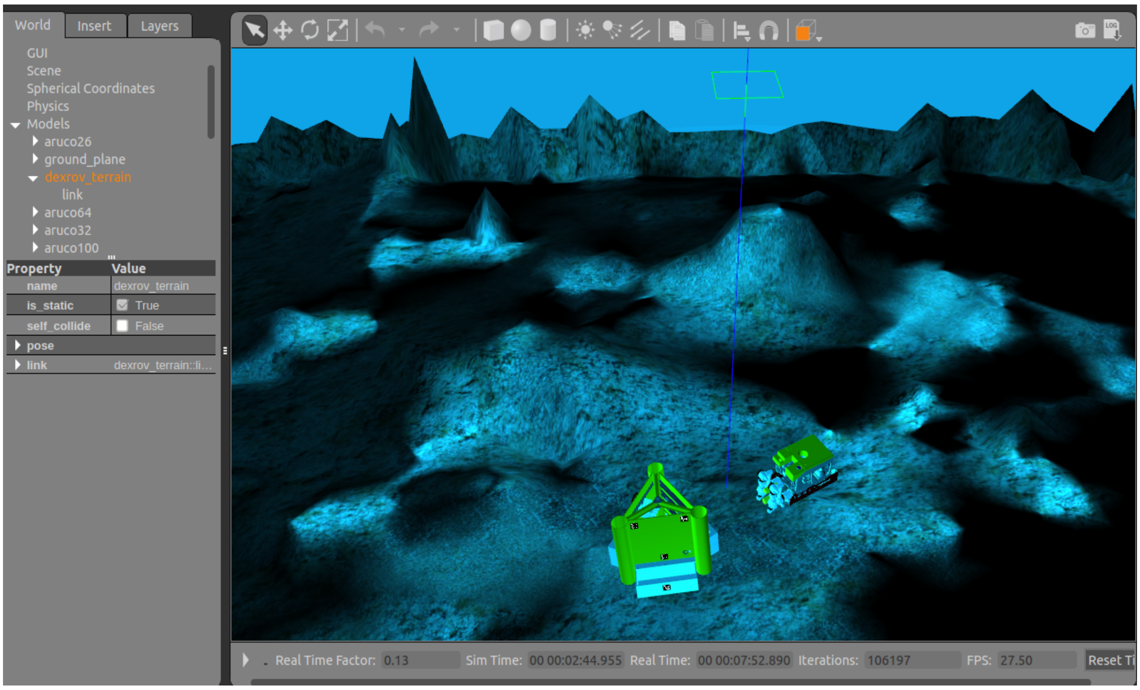


Figure 3: Simulation of the environment via Gazebo

3.2 Representation of Data

The data is displayed in left tab of Rviz. It exhibits vital things about the ROS Topics that have been enabled and are currently running. For this simulation, there are at least 150 ROS topics active varying from stereo, depth, and underwater camera / left and right , each having their numerous parameters such as image_raw, state, rotation, etc. Topics containing information about rov, gazebo, octomap, projected map are also active. The data collected for this research exists of stereo camera, ROV model, projected_map, pose, grid, and octomap. Rviz also generates a text file that displays the map as a 1D array. The text file is indeed applied in the usage of dijkstra and A star algorithm. The data obtained in the text file is displayed as an array of 100, 0 and -1. 100 implies that the cell in the map is occupied/blocked, 0 implies that the cell is free/empty while -1 implies that the cell is unexplored. Based on this information the data is further probed on.

3.3 Representation of Data Set/Text file

The data set is displayed in a 1D array. The data is converted into a 2D array by using the width and height values given. This data is then converted to be represented as a graph to be used in the path planning algorithms by traversing through each cell one by one and checking the cells neighbours. Logically, each cell would have 8 neighbours at most. The graph is formulated by comparing the values between the neighbour cell and the current cell. If the value of the cell in the map is 100, 0 , -1, in the graph it would have a value of 100,1,50, respectively. After creating the graph, the shortest path and the best path from source to the destination is found respectively through the two path planning algorithms. The path (display of various cells) is extracted from the cells in the map and multiplied by the resolution. These values are stored as X and Y values into a YAML file to be used in the launch file.

3.4 Representation of Images

In a spatial transformation each point (x, y) of an input image coordinates is mapped to a point (u, v) in a new coordinate system. After algorithms read data files and compute the path, the path is iterated through and each coordinate value is multiplied by the given resolution in order to convert the coordinates into meters. Before converting the coordinates, a rotation operator performs a geometric transform which maps the coordinates (x, y) in an input image onto a position of (u, v) in an output image by rotating it through a specified angle about an origin. Rotation is mostly used to improve the visual appreance of an image.

$$\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xcos\theta & -ysin\theta \\ xsin\theta & ycos\theta \end{bmatrix}$$

$$\theta = 90$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$x' = -y$$
$$y = x$$

The coordinates in meters are multipled by the resolution of the defined kernel size to scale the image coordinates properly. This method is called scaling.

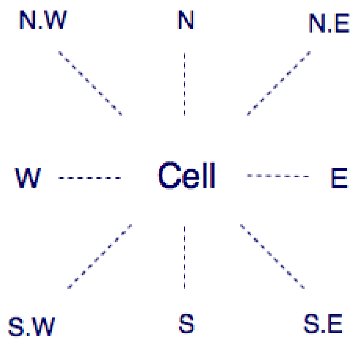
```
x = (vecTemp[i].second)*-1
x = x*resolution*KERNEL_SIZE
y = (vecTemp[i].first)
y = y*resolution*KERNEL_SIZE
```

The x and y values are stored in another vector that stores the path in resolution in a pair form.

4 Method of Implementation

4.1 Implementation of Dijkstra:

For dijkstra, the implementation idea was taken by Ms Chitra Nayals implementation. Before computing the dijkstra, the data read by the text file is mapped and then converted into a graph of nodes based on each neighbour of each cell in the map. The data of graph is stored by using vector of vectors. The row of the graph is constructed by multiplying the row of the cell in the map to the max width of the map and adding the column number. The column of the graph is based on the current cells neighbours position in the map. If the north cell above the current cell exists the value of the column will be computed by subtracting 1 from the row, multiplied by the width and since its in the same column, the column value will remain the same. Ideally, if we consider north east neighbour of the current cell, the row will be subtracted by 1 and the column will be increased by 1. Each cell has 8 neighbours at most and they are computed as such:



$$Cell - PoppedCell(x, y)$$
$$N - North(x - 1, y)$$
$$S - South(x + 1, y)$$
$$E - East(x, y + 1)$$
$$W - West(x, y - 1)$$
$$N.E - NorthEast(x - 1, y + 1)$$
$$N.W - NorthWest(x - 1, y - 1)$$
$$S.E - SouthEast(x + 1, y + 1)$$
$$S.W - SouthWest(x + 1, y - 1)$$

Considering i as row and j as column for the graph; to compute i and j the formulas followed are:

i = (row of the current cell*width of the map)+column of the current cell

For north neighbour:

j = (row of the neighbour cell * width of the map)+column of the neighbour cell

Written in the program as:

$$i = (row * W) + col$$
$$j = (rneigh * W) + cneigh$$

Note: The formula of i remains the same , j changes according to the neighbour. Evidently, the values of rneigh and cneigh are found out by the position of the neighbour.

Since the real data set consists of $366 * 362$ by $362 * 366$ value set, to test the concept and the working of the algorithm, the test data set (map) taken included $3 * 3$ value set.

0	-1	0
100	0	-1
-1	0	100

As previously mentioned, 0,-1 and 100 depicted unique values in the graph. The graph computed is:

0	50	0	100	1	0	0	0	0
50	0	50	100	50	50	0	0	0
0	50	0	0	1	50	0	0	0
100	100	0	0	100	0	100	100	0
1	50	1	100	0	50	50	1	100
0	50	50	0	50	0	0	50	100
0	0	0	100	50	0	0	50	0
0	0	0	100	1	50	50	0	100
0	0	0	0	100	100	0	100	0

While the graph is filled by converting the map into 2D by using the logic of rows and columns, the graph itself is read by an entirely different logic. The first row represents each cell methodically and the values in the column represents each cells relation to their neighbour cell.

For example: Reviewing the first column: the first cell of the map relation with all other cells; the first value is 0 because it has no connection with itself of course, the second value is 50 because the next neighbour is -1 and the third value is 0 because there is no connection between the first cell and the third cell in the map and so on.

0	$\rightarrow$	-1	=	50
0	$\rightarrow$	0	=	1
0	$\rightarrow$	100	=	100
0	$\rightarrow$	0	=	1
0	$\rightarrow$	-1	=	0
0	$\rightarrow$	-1	=	0
0	$\rightarrow$	0	=	0
0	$\rightarrow$	100	=	0

Using this logic, the graph will always have the size of $H * W$. H and W being the height and width of the map respectively. In this case, if the map is 3 by 3, the graph will have the size of 9 by 9. The graph will also always be a left diagonal matrix. Dijkstra is applied on the graph and the shortest path from the source to the destination is computed. A separate function for Dijkstra is written in which a set is created to keep track of shortest path tree vertices meaning vertices for which minimum distance is calculated from the source and confirmed. An array $dist[i]$ having the size of $H * W$ is created to hold the shortest distance from the source to the vertex for each vertex. Initially, the shortest path tree set is empty and values of distances for all the vertices are maximum number of the integer (INT_MAX) or INFINITE. For the source vertex, the distance value is allotted to be 0 so that the source acts as the starting point. The program processes a loop to find the shortest path for all the vertices. It firsts select a vertex l that does not exist in the set but has a minimum distance from the source and adds it to the set. It iterates through all the adjoining vertices and detects if the sum of $l's$ distance from the source and weight of the next adjoining edge is less than the distance value of iterated vertex, then it updates the value of that vertex.

```
 for (int i = 0; i < H*W - 1; i++)
{
   int l = mDist(dist, spt);
   spt[l] = true;
   for (int v = 0; v < H*W; v++)
   {
      if (!spt[v] && graph[l][v] &&
         dist[l] != INT_MAX && dist[l]+ graph[l][v] < dist[v])
      {
         dist[v] = dist[l] + graph[l][v];
      }
   }
}
```

Listing 1: Code implemented to find the shortest path for all vertices

Since the path is in 2D and it represents x and y coordinates, the path is stored in a pair of doubles of vectors of vector. The path is extracted from the map instead of the graph because map depicts the real number of cells. After extracting the path, the x and y values for the simulation are computed by dividing and finding the modulus of the cells position number respectively and multiplied by the resolution value to be portrayed appropriately for the robots orientation.

```
void Path2D(vector<vector<int> >&v1,vector<vector<pair<double,double> > >&v)
{
   double x,y;
   for(int i=0; i<v1.size();i++)
   {
      vector<pair<double,double> > tmp;
   for(int j=0; j<v1[i].size();j++)
   {
      x = (v1[i][j])*resolution*KERNEL_SIZE;
      y = (v1[i][j])*resolution*KERNEL_SIZE;
      x = x-ORIGIN_MAP;
      y = y-ORIGIN_MAP;
      tmp.push_back(make_pair(x,y));
   }
   v.push_back(tmp);
   }
}
```

Listing 2: Code implemented to print the path in resolution form.

The first vector of vector of integers v_1 is the vector that stores the path in cell(map) values. It has to be an integer because the modulus operator, % (remainder operator) is a binary operator meaning it only takes two operands at a time. It also only manages integer types such as int, short, and long, etc. Thus, another vector of vector of pairs of doubles have to be initialized to store the path in resolution format because the resolution has a value of 0.0500000007451 which is a double.

The libraries used to get the complete functionality of the code are the following:

#include $<$stdio.h$>$
#include $<$limits.h$>$
#include $<$iostream$>$
#include $<$fstream$>$
#include $<$sstream$>$
#include $<$vector$>$
#include $<$iomanip$>$ #include ”yaml-cpp/yaml.h”
#include ”opencv2/imgproc/imgproc.hpp”
#include ”opencv2/highgui/highgui.hpp”

stdio.h header in this research paper is used for the variable type: FILE. An object type to store information for a file stream. fstream and sstream are classes to read and operate on files. Through the combination of these various libraries functions the data from the file is read and stored in the vector of vector of integers which is called map in this paper. iostream uses the objects cout, cin, etc for sending data to and from the standard streams input and output. These objects are also a part of namespace std. Including the vector header file helps construct the vector and provides with vector functions which makes adding and deleting elements from the vector easier. Vectors are sequence containers that changes size dynamically. Logically they are the same as arrays that can be resized. Internally, vectors also uses dynamically allocated array to store their elements. Vectors are very efficient accessing its elements and relatively efficient adding or removing elements from its end comparing to other dynamic sequence containers. It is important to state that operations that involve inserting or removing elements at positions other than the end, vectors usually perform worse than the other dynamic sequence containers. The header <limits.h> provides various properties such as macros defined in the header, and value limit for several variable types. INT_MAX is used several times during implementing the algorithms. This means an integer can store up to a maximum value of 2147483647. The iomanip header file is used for the setprecision() function. It is used to affect the state of iostream objects. Yaml-cpp header file has a YAML parser and emitter in C++ for the purpose to make and write yaml files. The last two opencv header files are used for image processing and HighGUI module in opencv is responsible for quick and simple GUIs.

Some of the globally defined variables and macros are the following:

```
using namespace std;
using namespace cv;
#define KERNEL_SIZE 7
#define ORIGIN_MAP 15
#define resolution 0.0500000007451;
int H ,W;
vector<vector<int> > map;
vector<vector<int> > graph;
vector<vector<int> >v1;
vector<vector<pair<double,double> > >v2;
```

These vectors were chosen to be defined globally because the vectors map, graph, v_1 and v_2 and the integers H and W are accessed by multiple functions. The advantages of global variable are that they can be accessed from all the functions defined in the program and the variables are only needed to be declared only once. Global variables general perception is that they are unsafe and provide a high risk to a programs computability. If too many variables are declared as global they will remain in the memory till program is executed and terminated. The data stored as a global variable is also vulnerable to other functions that can easily modify it. However, in this case, map, graph, v_1 and v_2 are not changed anywhere in the function, the value is the same and with each function the values of these variables are updated which are utilized somewhere else. Similarly, H and W are initialized in the main function but used in a lot of functions that are needed in the computation of the map and the graph. The main problem claimed against global variables are that they are changeable. One can allot the variables as a watchpoint while debugging. It would be impossible to follow this procedure with a local variable as watchpoints are canceled when a variable goes out of scope but with a global variable, the variable can be queried at any time. Lastly, resolution, KERNEL_SIZE and ORIGIN_MAP (starting position of the robot) are manoeuvred as a macro. One significant difference between a macro and a global variable is that macro is not stored in the memory, in this case resolution is just a substitute for the value 0.0500000007451 which is what we need to multiply the values in the code above to get the final result. In this research, three different kernel sizes are used; 3,5 and 7. The chosen kernel size is defined initially, similarly origin map states the initial coordinates of the robot to match the simulation. Macros are also preprocessor directives, their values cannot be changed like variables. Therefore, this method is memory efficient for the defined macros but no so much for the other variables as the variables such as map, graph, etc. need to have memory allocated in order for the data to be accessed, modified and computed.

The run time of Dijkstra algorithm depends on how the Priority Queue is implemented. In this paper, we use an array thus we have a time complexity of O(n^2) and space complexity of O(n^2) as well. N being the number of nodes. The bonus of this algorithm is that it often does not have to investigate all edges. If edges are relatively expensive to compute, the Dijkstra algorithm might be faster. Since Dijkstra explores more map, it also uses more memory which was the biggest complication faced in this research.

4.2 Implementation of A star

The implementation of A star imitates the same idea as the Dijkstra. The main difference lies in some additional functions. As previously mentioned, A star search algorithm selects a cell based on its f-value which is a variable equivalent to the sum of two other variables; g and h. f is defined as the total cost of the path via the current cell. With each iteration, it selects the cell that holds the minimum f value and process that cell. g is defined as the cost to move from the initial point to the stated cell on the graph, backing the path generated from the starting cell to the given cell. h is defined as the estimated cost to move from the given cell on the graph to the destination cell. This variable is also called heuristic. The heuristic is a simple estimate of the distance between each cell and the destination cell, therefore it is especially important that the computation of H is simple and easy as the value will be determined at least once for each cell before reaching the destination cell. Therefore the implementation of this H value varies depending on the properties of the graph being processed. This variable is found through various methods of calculations, three common heuristic functions are mentioned below.

1. **Manhattan Distance**
2. **Diagonal Distance**
3. **Euclidean Distance**

These are the approximation methods to compute the value of H, there are methods to compute the exact value of H, however, they are not efficiently utilized because they are generally very time consuming.

The Manhattan distance is computed by summing the absolute values of differences in destination cells x and y coordinates and the current cells x and y coordinates respectively i.e.,

$$h(n) = |n.x - Z.x| + |n.y - Z.x|$$

n = current cell
Z = destination cell

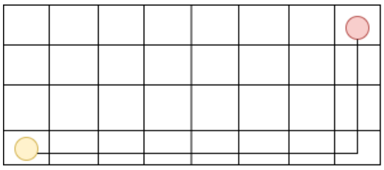


Figure 4: The Manhattan Distance Heuristics.

Assume orange point as the starting node/cell and red point as the destination cell

This is the most simple heuristic method and is ideal for graphs/grids/maps that allow movements in four directions (up,down,left,right)

There are two types of diagonal distance. One being the uniform cost of diagonal planning where the cost of diagonal planning is equivalent to the cost of non-diagonal. It is computed by finding the maximum of absolute values of differences in the destination cells x and y coordinates and the current cells x and y coordinates respectively i.e.,

$$h(n) = c.max(|n.x - Z.x| + |n.y - Z.x|)$$

n = current cell
Z = destination cell
c = cost of movement

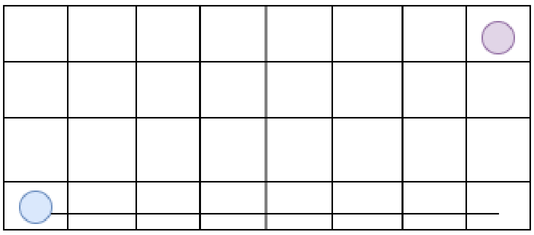


Figure 5: The Uniform Diagonal Distance Heuristics.

Assume blue point as the starting node/cell and purple point as the destination cell

The other diagonal distance occurs when the cost of diagonal planning varies from the cost of non-diagonal.

$$h(n) = c\_d\_min + c\_n(d\_max - d\_min)$$

$$d\_max = max(|n.x - Z.x|, |n.y - Z.y|)$$

$$d\_min = min(|n.x - Z.x|, |n.y - Z.y|)$$

n = current cell

Z = destination cell
c = cost of movement
c_n = cost of non-diagonal movement
c_d = cost of diagonal movement
c_d = c_n x $\sqrt{2}$ *= c_n x 1.414*

The non-uniform diagonal distance cost is simply computed by summing the minimum cost value of diagonal movement and the differences in maximum and minimum value of cost of non-diagonal movement in the destination cells x and y coordinates and the current cells x and y coordinates respectively. Both of the diagonal distances is used for graphs/grids/maps that allow movement in eight directions.

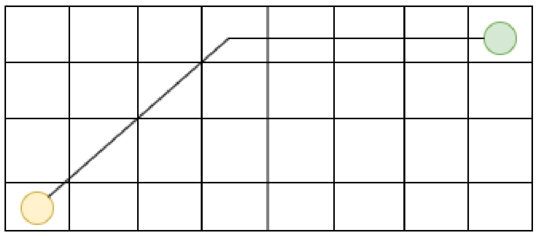


Figure 6: The Diagonal Distance Heuristics.

Assume yellow point as the starting node/cell and green point as the destination cell

The euclidean distance is computed by finding the distance between the current cell and the destination cell using the distance formula. Sum of x and y coordinates in the differences of destination cells x and y coordinates and the current cells x and y coordinates respectively. This distance is used for graphs/grids/maps that allow movement at any angle.

$$h(n) = \sqrt{(n.x - Z.x)^2 + (n.y - Z.y)^2}$$

n = current cell
Z = destination cell

Assume orange point as the starting node/cell and purple point as the destination cell

The data was tested on different heuristic functions through the test data set. However, this time along with a 3x3 value set, the functions were tested on different data set to have a better estimate such as: 3x6, 4x6, 10x10, 10x16, 16x6.

The results of the path are following:

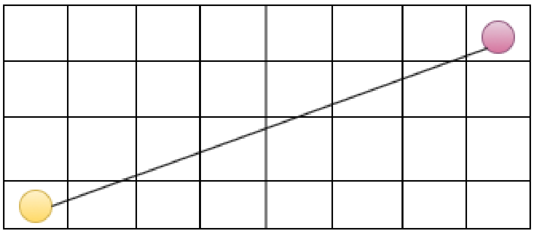


Figure 7: The Euclidean Distance Heuristics.

For 3x3; source: (0,1) destination (3,2)
Manhattan:
(0,1) → (1,2) → (2,3) → (3,2)
Diagonal:
(0,1) → (1,1) → (2,1) → (3,2)
Euclidean:
(0,1) → (1,1) → (2,2) → (3,2)

For 3x6; source: (0,5) destination (3,2)
Manhattan:
(0,5) → (1,4) → (2,3) → (3,2)
Diagonal:
(0,5) → (1,4) → (2,3) → (3,2)
Euclidean:
(0,5) → (1,4) → (2,3) → (3,2)

For 4x6; source: (0,5) destination (3,0)
Manhattan:
(0,5) → (0,4) → (0,3) → (1,2) → (2,1) → (3,0)
Diagonal:
(0,5) → (0,4) → (0,3) → (1,2) → (2,1) → (3,0)
Euclidean:
(0,5) → (1,4) → (2,3) → (2,2) → (3,1) → (3,0)

For 10x10; source: (0,9) destination (2,0)
Manhattan:
(0,9) → (0,8) → (0,7) → (0,6) → (0,5) → (0,4) → (0,3) → (0,2) → (1,1) → (2,0)
Diagonal:
(0,9) → (0,8) → (0,7) → (0,6) → (0,5) → (0,4) → (0,3) → (0,2) → (1,1) → (2,0)
Euclidean:
(0,9) → (0,8) → (0,7) →(0,6) → (0,5) → (0,4) → (1,3) → (1,2) → (2,1) → (2,0)

For 10x16; source: (0,15) destination (9,0)
Manhattan:
(0,15) → (0,14) → (0,13) → (0,12) → (0,11) → (0,10) → (0,9) → (1,8) → (2,7) → (3,6) → (4,5) → (5,4) → (6,3) → (7,2) → (8,1) → (9,0)
Diagonal:
(0,15) → (0,14) → (0,13) → (0,12) → (0,11) → (0,10) → (0,9) → (1,8) → (2,7) → (3,6) →

(4,5) → (5,4) → (6,3) → (7,2) → (8,1) → (9,0)
Euclidean:
(0,15) → (1,14) → (2,13) → (3,12) → (4,11) → (4,10) → (5,9) → (5,8) → (6,7) → (6,6) → (7,5) → (7,4) → (8,3) → (8,2) → (9,1) → (9,0)

For 16x6; source: (0,15) destination (5,0)
Manhattan:
(0,15) → (0,14) → (0,13) → (0,12) → (0,11) → (0,10) → (0,9) → (0,8) → (0,7) → (0,6) → (0,5) → (1,4) → (2,3) → (3,2) → (4,1) → (5,0)
Diagonal:
(0,15) → (0,14) → (0,13) → (0,12) → (0,11) → (0,10) → (0,9) → (0,8) → (0,7) → (0,6) → (0,5) → (1,4) → (2,3) → (3,2) → (4,1) → (5,0)
Euclidean:
(0,15) → (0,14) → (0,13) → (0,12) → (0,11) → (0,10) → (1,9) → (1,8) → (2,7) → (2,6) → (3,5) → (3,4) → (4,3) → (4,2) → (5,1) → (5,0)

5 out of 6 trials demonstrate similar results for manhattan and diagonal heuristics while 1 out of 6 trails show similarity between euclidean and the others. Euclidean distance is shorter than manhattan or diagonal distance and gives the shortest paths but on hindsight A* takes longer to run. Looking closely, it is evident that euclidean turns at any angle and tries to find the shortest path as compare to manhattan and diagonal which only checks their adjacent cells (nodes) and moves closer to the destination point accordingly.

```
double EuclideanHVal(int row, int col, Pair destination)
{
   //W = row, H = col.
   return ((double)sqrt((row - destination.first) * (row - destination.first) +
       (col - destination.second) * (col - destination.second)));
}
```

Listing 3: Code implemented to find the Euclidean Heuristic function

Note: Usage of Pair makes it easier to extract the first and second elements rather than using a loop or an array. Explained further down below.

Besides the additional heuristic function in the implementation of a star, two more functions were added, one to determine if the source or the destination is blocked and the other to check if the destination has been found. The added libraries to complete the functionality of a star are the following:

#include <cfloat>
#include <cmath>
#include <set>
#include <stack>
#include<string>
#include <utility>

cfloat header defines the characteristics of floating types with their minimal or maximal values FLT_MAX is used several times during implementing the algorithms. This means a float can store up to a maximum value of 1e+37. It is used in storing the values of f,g and h. String header file contains the operations on string such as getline(). The set and stack headers define the set and stack container classes. Utility is a header file which contains params/objects of two different types of values. It is used in make_pair function

The altered global defined variables and macros are the following:

```
#define KERNEL_SIZE 3
#define ORIGIN_MAP 5
#define OH 366 //original height
#define OW 362 //original width
#define OSx 320 //original source x
#define OSy 40 //original source y
#define ODx 320 //original destination x
#define ODy 300 //original destination y
//resolution values
double res3 = 0.15;
double res5 = 0.25;
double res7 = 0.35;
typedef pair<int, int> Pair;
typedef pair<int,int> pathTemp;
typedef pair<double,double> pathStore;
typedef pair<int, pair<int, int> > pPair;
vector<pathStore> vecPath;
```

In our implementation, typedef pair $<int, int> Pair$ is initialized twice, one with data type of stack and the other with vector. To make things easier, stack$< Pair > Path$ is utilized for storing the row and column of the path in 1D. vector$< Pair > vecTemp$ stores the same data in 2D, since the data required is in resolution which is a double value, we initialize typedef pair$< double, double > pathStore$ once; equivalent to vector$< pathStore >$ $vecPath$. $vecPath$ stores the path in 2D with the resolution. vector$< pathStore >$ is also used as a function that returns the trace of the path from source to the destination. Defining typedef pair$< int, int > Pair$ and typedef pair$< int, pair < int, int >> pPair$ is globally appropriate for our implementation because $Pair$ itself is used five times in different functions as $Pair\ dest$ and $Pair\ src$. $pPair$ is used for openlist. Most importantly, the major difference between dijkstra and astar implementation is the added usage of $Pair$ that is because in a star applications the destination and the source is usually known. Keeping this in mind, the source and the destination cells are given in 2D in the main function as x and y coordinates as one Pair which makes it simpler to extract the first and second components when needed. $std :: stack$ is only initialized once, because stack is not flexible enough to provide several operations as compare to $std :: vector$. In case of $std :: stack$, operations are only performed in a calculated manner, where the elements are only needed to $push()$ above the last element or $pop()$ the last element. While $std :: vector$ has several accessibility and modification operations, elements can be inserted in between or be erased in between. Lastly, vector class underlying data structure is a dynamic array, it represents the list of objects under a resizable array. Vector is also the best choice in retrieval cases.

For open list we use a set data structure of C++ STL ($pPair$) which is implemented as Red-Black Tree. and for closed list we used a boolean hash table for best performance. Also to lessen the time taken to calculate g, dynamic programming is used.

Red-Black Tree is a self balancing binary search tree where every node (cell in our case) pursues specific rules such as the root of tree is always black, a node is either red or black, a red node cannot have a red parent or red child and all leaves are black. Most of this tree operations take O(h) time where h is the height of the tree. If after every insertion and deletion of a node in open list it is guaranteed that the height of the tree remains $O(log\ n)$ then for all other operations such as e.g., search, max, min, insert, delete.. etc it is also $O(log\ n)$. The height of the tree is always $O(log\ n)$ when n is the number of nodes (cells).

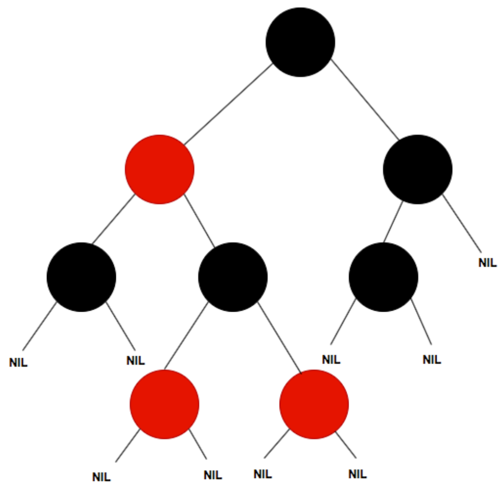


Figure 8: Displays an example of a red-black tree

A struct is also defined to carry some essential parameters

```
struct cell
{
// Width and Height index of its parent
// Note that 0 <= i <= W-1 & 0 <= j <= H-1
int parent_i, parent_j;
// f = g + h
// g = movement cost
// h = heuristic
double f, g, h;
};
```

This struct is declared as a 2D array to hold the details of a cell in a function that traces the path from the source to the destination cell and in a function that finds the shortest path between the source and the destination cell

$$cell\ cellInfo[W][H];$$

```
vector<pathStore> trace(cell **cellInfo, Pair destination)
{
   printf("\nThe Path is ");
   int RW = destination.first;
   int RH = destination.second;
   double x,y;
   int m,n;
   stack<Pair> Path;

   while (!(cellInfo[RW][RH].parent_i == RW && cellInfo[RW][RH].parent_j ==
       RH))
   {
      Path.push(make_pair(RW, RH));
      int temp_RW = cellInfo[RW][RH].parent_i;
      int temp_RH = cellInfo[RW][RH].parent_j;
      RW = temp_RW;
      RH = temp_RH;
   }

   vector<pathTemp> vecTemp;

   Path.push(make_pair(RW, RH));
   while (!Path.empty())
   {
      pair<int, int> p = Path.top();

      Path.pop();
      vecTemp.push_back(make_pair(p.first,p.second));
      cout << "-> (" << p.first << "," << p.second << ") ";
   }

   cout << endl;

   for(int i=0; i<vecTemp.size(); i++)
   {

      x = (vecTemp[i].second)*-1;
      x = x*resolution*KERNEL_SIZE;
      y = (vecTemp[i].first)*resolution*KERNEL_SIZE;
      x = x+ORIGIN_MAP;
      y = y-ORIGIN_MAP;
      vecPath.push_back(make_pair(x,y));
   }
   cout << "\nPath in resolution" << endl;
   for(int i =0; i<vecPath.size();i++)
   {
      cout << fixed << setprecision(20)<< vecPath[i].first << " " <<fixed <<
          setprecision(20)<< vecPath[i].second << " " << endl;
   }
   cout << endl;
```

```
    Mat imgmat = imread("../images/map_image.jpg");
    Vec3b color;
    color[0] = 0;
    color[1] = 255;
    color[2] = 0;

    for(int i =0; i< vecTemp.size();i++)
    {
       m = (vecTemp[i].second)*KERNEL_SIZE;
       n = (vecTemp[i].first)*KERNEL_SIZE;
       imgmat.at<Vec3b>(Point(m,n)) = color;
    }

    imshow("window",imgmat);
    imwrite("../images/manhattan.jpg",imgmat);
    waitKey(0);

    return vecPath;
}
```

Listing 4: Code implemented to trace the path from the source to the destination cell. Converts the path from 1D into 2D using resolution [10]

The first part of this code contains changing the coordinates of the path into resolution, multiplying those values by the given kernel_size [5.2] and applying mathematical operations relative to the robots origin position to x and y respectively. Before multiplying the x value with the resolution, x and y values are rotated by a $90°$ angle in order to display the tracked path from the map to an image. The openCV operations are discussed in detail in Evaluation section of this paper.

A star is implemented by initializing two lists; open list and closed list. Open list is a pair of pair of int while closed list uses a boolean hash table. A star function takes three parameters; the graph, source cell coordinates and the destination cell coordinates. First it checks if the source and the destination coordinates are valid, and whether the source and the destination is accessible and not blocked. The closed list is initialized as a boolean 2D array and initialized to false; stating it is empty.

$$bool\ closedList[H][W];$$
$$memset(closedList, false, sizeof(closedList));$$

The struct cell is declared as:

$$cellInfo[i][j].f = FLT\_MAX;$$
$$cellInfo[i][j].g = FLT\_MAX;$$
$$cellInfo[i][j].h = FLT\_MAX;$$
$$cellInfo[i][j].parent\_i = -1;$$
$$cellInfo[i][j].parent\_j = -1;$$

And then the parameters are initialized as:

$$i = source.first;$$
$$j = source.second;$$
$$cellInfo[i][j].f = 0.0;$$
$$cellInfo[i][j].g = 0.0;$$
$$cellInfo[i][j].h = 0.0;$$
$$cellInfo[i][j].parent\_i = i;$$
$$cellInfo[i][j].parent\_j = j;$$

Open list is created by using typedef pair $< int, pair < int, int >> pPair$ structure that stores the information of f,i and j; where $f = g + h$ and i and j are the row and column index of the cell.

$$set < pPair > openList;$$

Initially open list holds the source cell and its f is given a value of zero. Open list iterates through all the cells inside and finds the cell with the least f. It pops that cell off the open list and generates that cells 8 neighbours and sets the parents of that cell equal to that cell. For each neighbour it checks whether the neighbour is the destination:

neighbour.g = cell.g + distance between the neighbour and cell
neighbour.h = distance from destination cell to the neighbour cell
neighbour.f = neighbour.g + neighbour.h

A star also checks If a cell is in the same position as its neighbour in the open list and has a lower f than the neighbour then it skips that neighbour. If a cell is in the same position as its neighbour in the closed list and holds a lower f value than the neighbour or if it is blocked, the neighbour is skipped, the cell is added to the open list and the parameters of the cell are updated and f,g and h are stored.

```
// To store the 'g', 'h' and 'f' of the 8 successors
double gNew, hNew, fNew;
int rneigh, cneigh, gi, gj;
gi = (i*W)+j;
// North neighbour
if (isValid(i - 1, j,row_max,col_max) == true)
{
   if (isDestination(i - 1, j, destination) == true)
   {
      cellInfo[i - 1][j].parent_i = i;
      cellInfo[i - 1][j].parent_j = j;
      cout << "The destination cell is found" << endl;
      vecPath = trace(cellInfo, destination);
```

```
            ifFound = true;
            return;
        }
            else if (closedList[i - 1][j] == false &&
            isUnBlocked(graph, i - 1, j) == true)
        {
            rneigh = i-1;
            cneigh = j;
            gj = (rneigh*W)+cneigh;
            gNew = cellDetails[i][j].g + graph[gi][gj];
            hNew = EuclideanHVal(i - 1, j, destination);
            fNew = gNew + hNew;
            if (cellInfo[i - 1][j].f == FLT_MAX ||
            cellInfo[i - 1][j].f > fNew)
            {
                openList.insert(make_pair(fNew,make_pair(i - 1, j)));
                cellInfo[i - 1][j].parent_i = i;
                cellInfo[i - 1][j].parent_j = j;
                cellInfo[i - 1][j].f = fNew;
                cellInfo[i - 1][j].g = gNew;
                cellInfo[i - 1][j].h = hNew;
            }
        }
    }
```

The iteration does not stop until all the elements inside open list have been processed. After the loop, the open list is empty and the function concludes whether the destination cell has been found or not. The struct is deallocated and the function ends.

5 Evaluation of the Investigation

5.1 Tie Breaking Mechanism

Despite the improvements in search algorithms for cost optimal path planning, the exponential growth of the extent of the search frontier in A star is unavoidable. Tie-breaking significantly affects the performance of search algorithms when there are zero cost operators that incite large level areas in the search space. A star needs to apply tie-breaking techniques in order to choose which vertex to extend when multiple vertices have a similar evaluation score.

The A star implementation is integrated with a tie-breaking mechanism after the simple implementation fails to find the path in our real graph i.e., 366x362. The algorithm terminates with a Killed by 9 error message code, a SIGKILL. This error is caused by OutOfMemory. In order to resolve this complication, tie-breaking mechanism was implemented in the algorithm. One approach was to find a way to break ties by preferring paths that were along the straight line from the source cell to the destination cell.

```
double TieBreakHValue(int W, int H, Pair source, Pair destination)
{
   double dx1;
   double dy1;
   double dx2;
   double dy2;
   double cross;
   double heuristic = 0;
   if(map1[row][col] == 3) {
      heuristic = 1000;
      return heuristic;
   }
   dx1 = row - dest.first;
   dy1 = col - dest.second;
   dx2 = src.first - dest.first;
   dy2 = src.second - dest.second;
   cross = dx1*dy2 - dx2*dy1;
   heuristic += cross*0.001;
   return heuristic;
}
```

This chunk of code calculates the vector cross product between the source and the destination vector and the current cell to destination vector. When these vectors do not queue up, the cross product is larger. The code gives more value to a path that lies along the straight line from the source cell to the destination cell. However, this tie-breaking technique fails drastically as there are several obstacles in our predefined graph.

Tested with our previous 3x3 map:
Source: (0,0) , Destination: (2,0)

0, -1, 0,
100, 0,-1,

-1, 0,100

The path is (0,0) $\rightarrow$ (1,0) $\rightarrow$ (2,0)

Evidently, this is incorrect as (1,0) is an obstacle and the robot can not go through this cell. To fix this problem an if statement was added in case the cell has a value of 100, update the value of heuristic to 1000.

However, tie-breaking mechanism did not solve the OutOfMemory error. This problem occurs in Dijkstra as well. With the original map data set requiring a vector of 132492 (366x362) by 132492 to be allocated, the process allocates too much memory putting pressures on the OS. The OS kills such processes for the sake of system stability. When the process is killed, its action is logged in at /var/log/messages. Each process has a limited free storage associated with it for dynamic memory allocation. When the application devours all the memory from load, at that point the process crashes with OutOfMemory error. After 1 minute and 5 seconds, the process is terminated after CMPRS reaches 59G+. CMPRS shows the measure of bytes of compressed data in memory that belongs to the process.

5.2 Convolution with filter2D

Subsequently, filter2D method was utilized through downscaling the real map by three times. It is an opencv function that uses a method known as convolution by using a kernel. Convolution is a procedure that takes place between all the parts of images and the kernel. The kernel is an operator such as a fixed array of integers consisting of an anchor point located at the center of the array.

Essentially this filter calculates the value of convolution by placing the kernel anchor on top of the intent pixel. The kernel coefficients are multiplied by the corresponding image pixel values and summed together, the result is placed at the anchor and the process is repeated for the rest of the pixels.

$$dst(x,y) = \sum_{i=0}^{M_i-1} \sum_{j=0}^{M_j-1} kernel(i,j) * src(x+i-anchor_i, y+j-anchor_j)$$

This equation manifests all the existing procedures described above in filter2D. [11]

Function: void **filter2D**(InputArray **src**, OutputArray **dst**, int **ddepth**, InputArray **kernel**, Point **anchor**=Point(-1,-1), double **delta**=0, int **borderType**=BORDER_DEFAULT)

The Parameter denotes:

src: *input image*
dst: *output image*
ddepth: *depth of the destination image*
When ddepth = -1 the output image has the same depth as the source
kernel: *convolution kernel*
anchor: *indicates the relative position of the determined pixel.*

Default value (-1,-1) indicates the kernel center
delta: *value added to filtered pixels. Default value = 0*
borderType: *pixel extrapolation method*

Owing to the fact that convolution method functions by multiplying the kernel coefficients, the values of the original map are changed from 100 to 3 such that extreme bias is avoided, in any other way if there is even one 100 the result would be so high that the whole block will be perceived as an obstacle.

These values are stored as a vector and later converted into Mat float. In order to extract the reduced map, the filter2D is applied on Mat float to an integer 8UC1. The reason being that an image cannot allocate floating values. It only keeps integers 0 to 255. If the image is of floating type then any value greater than 1.0 is shown as a white pixel and value less than 0.0 is shown as a black pixel. While converting, the matrix stores values from the source matrix and then rounds them to the nearest possible value of the destination data type. If the value is out of range, it picks up either minimum or maximum values. In this example, all the values which are negatives will become 0 in the destination matrix as the destination type is CV_8U1 and the minimum possible value is 0. All the floating point values will be floored. No automatic mapping is done. Therefore, to convert the float matrix to int in the scale from 0 to 255, float values are rescaled so they fit in the destination matrix scale. The minimum and maximum values are found through the minMaxLoc opencv function and applied in scaling.

To properly scale the values the following function is applied:

```
void displaymat(Mat& matname, Mat &dst)
{
   double minVal;
   double maxVal;
   Point minLoc;
   Point maxLoc;
   minMaxLoc(matname,&minVal,&maxVal,&minLoc,&maxLoc);
   cout << "min: " << minVal << endl;
   cout << "max: " << maxVal << endl;
   if (minVal!=maxVal){
      matname.convertTo(dst,CV_8U,255.0/(maxVal-minVal),-255.0*minVal/(maxVal-minVal));
   }
}
```

```
int main ( int argc, char** argv )
{
   ifstream inFile;
   ofstream outFile;
   string strFileName = "num2.txt";
   string strOutFile = "downsample7.txt";
   int r = 366, c = 362;
   float V[r*c];

   readFile(strFileName, V, inFile);
```

```
//Copying vector to Mat
Mat dst1;

Mat M(r,c,CV_32FC1,V);
printvals(M);

// Declare variables
Mat intMat;
Mat dst;
Mat kernel;
Point anchor;
double delta;
int ddepth;
int kernel_size;
double minVal;
double maxVal;
Point minLoc;
Point maxLoc;
Mat downsample(dr,dc,CV_32FC1);
Mat intdownsample;
int newrow = 0, newcol = 0;

// Initialize arguments for the filter
anchor = Point( -1, -1 );
delta = 0;
ddepth = -1;

// Update kernel size for a normalized box filter
kernel_size = 7;
kernel = Mat::ones( kernel_size, kernel_size, CV_32FC1 )/
    (float)(kernel_size*kernel_size);
// Apply filter
cv::filter2D(M, dst, ddepth , kernel, anchor, delta, BORDER_DEFAULT );

// Min and Max value of Mat

displaymat(M,intMat);
imshow("Filter Map",intMat);
imwrite("Filter_Map.jpg",intMat);

int dr = ceil(float(M.rows)/kernel_size);
int dc = ceil(float(M.cols)/kernel_size);

cout << dr << " " << dc << endl;
//downsampling - kernel operation
for(int i = kernel_size/2; i<dst.rows; i+=kernel_size)
{
   newcol=0;
   for(int j = kernel_size/2; j<dst.cols; j+=kernel_size)
   {
      if(dst.at<float>(i,j) > 0)
      {
         downsample.at<float>(newrow,newcol) = 3;
      }
      else if(dst.at<float>(i,j) < 0)
```

```
            {
               downsample.at<float>(newrow,newcol) = -1;
            }
            else
            {
               downsample.at<float>(newrow,newcol) = 0;
            }
            newcol++;
         }
            cout << endl;
            newrow++;
         }

   displaymat(downsample,intdownsample);
   printvals(downsample);

   imshow("downscaled7",intdownsample);
   imwrite("downscaled7.jpg",intdownsample);
   writeFile(strOutFile,downsample,outFile);
   waitKey(0);

   return 0;

}
```

Listing 5: C++ Program to implement filter2D for reducing the map

This function not only reduces the original map by the given kernel_size but prints out the coordinates of the map in reduced map and the images. The a star then use the reduced data as shown in [4.2] and computes the path on it. This code is also used for different kernel_sizes such as 3, 5 and 7 and tested on a star with different heuristics.

The path printed by a star after applying convolution is reduced to 122 x 121 which is 3 times less of the original map. Before computing the path, the x and y coordinates are multiplied with the applied kernel_size. The source is 10,10 and the destination is 120,120.

5.3 Path Outline

Path outline for coordinates:
source = 30,30 ; destination = 360,360

5.3.1 Shortest Path in A star

With Kernel Size 3

The path computed:

(10,10) → (10,11) → (10,12) → (10,13) → (11,14) → (12,15) → (13,16) → (14,17) → (15,18) → (16,19) → (17,20) → (18,21) → (19,22) → (20,23) → (20,24) → (20,25) → (20,26) → (21,27) → (22,28) → (23,29) → (24,30) → (25,31) → (26,32) → (27,33) →

(28,34) → (29,35) → (30,36) → (31,37) → (32,38) → (33,39) → (34,40) → (35,41) → (36,42) → (37,41) → (38,42) → (39,43) → (40,44) → (41,45) → (42,46) → (43,45) → (43,44) → (43,43) → (44,42) → (44,41) → (44,40) → (44,39) → (44,38) → (44,37) → (44,36) → (44,35) → (44,34) → (45,33) → (46,32) → (47,31) → (48,30) → (49,29) → (50,30) → (50,31) → (50,32) → (50,33) → (50,34) → (50,35) → (51,36) → (51,37) → (51,38) → (51,39) → (51,40) → (52,41) → (52,42) → (52,43) → (52,44) → (52,45) → (52,46) → (53,47) → (53,48) → (53,49) → (53,50) → (53,51) → (53,52) → (53,53) → (53,54) → (53,55) → (53,56) → (53,57) → (53,58) → (54,59) → (54,60) → (54,61) → (54,62) → (54,63) → (54,64) → (55,65) → (55,66) → (55,67) → (55,68) → (56,69) → (57,70) → (58,71) → (59,72) → (60,73) → (61,74) → (62,75) → (63,76) → (64,77) → (65,78) → (66,79) → (67,80) → (68,81) → (69,82) → (70,83) → (71,84) → (72,85) → (73,86) → (74,87) → (75,88) → (76,89) → (77,90) → (78,91) → (79,90) → (79,89) → (79,88) → (79,87) → (79,86) → (79,85) → (79,84) → (79,83) → (79,82) → (79,81) → (79,80) → (79,79) → (79,78) → (79,77) → (79,76) → (79,75) → (79,74) → (79,73) → (79,72) → (79,71) → (79,70) → (79,69) → (80,68) → (81,67) → (82,66) → (83,65) → (84,64) → (85,63) → (86,62) → (87,61) → (88,60) → (89,59) → (90,58) → (91,57) → (92,56) → (93,55) → (94,54) → (95,53) → (96,52) → (97,53) → (97,54) → (97,55) → (97,56) → (98,57) → (98,58) → (98,59) → (98,60) → (99,61) → (99,62) → (99,63) → (99,64) → (100,65) → (100,66) → (100,67) → (101,68) → (101,69) → (101,70) → (101,71) → (102,72) → (102,73) → (102,74) → (102,75) → (102,76) → (102,77) → (103,78) → (103,79) → (103,80) → (103,81) → (103,82) → (104,83) → (104,84) → (104,85) → (104,86) → (105,87) → (105,88) → (105,89) → (105,90) → (105,91) → (105,92) → (105,93) → (106,94) → (106,95) → (106,96) → (106,97) → (107,98) → (107,99) → (107,100) → (107,101) → (107,102) → (108,103) → (108,104) → (108,105) → (108,106) → (108,107) → (108,108) → (109,109) → (110,110) → (111,111) → (112,112) → (113,113) → (114,114) → (115,115) → (116,116) → (117,117) → (118,118) → (119,119) → (120,120)

Total vertices/cells explored 108

With Kernel Size 5:

(2,2) → (3,2) → (4,2) → (5,2) → (6,2) → (7,2) → (8,2) → (9,2) → (10,2) → (11,2) → (12,2) → (13,2) → (14,3) → (15,4) → (16,5) → (17,5) → (18,6) → (19,7) → (20,8) → (21,9) → (22,10) → (23,11) → (24,12) → (24,13) → (24,14) → (24,15) → (24,16) → (24,17) → (24,18) → (24,19) → (24,20) → (24,21) → (24,22) → (24,23) → (24,24)

Total vertices/cells explored 36

With Kernel Size 7:

(1,1) → (2,1) → (3,1) → (4,1) → (5,1) → (6,1) → (7,1) → (8,1) → (9,1) → (10,2) → (11,3) → (12,4) → (13,5) → (14,6) → (15,7) → (16,8) → (17,9) → (17,10) → (17,11) → (17,12) → (17,13) → (17,14) → (17,15) → (17,16) → (17,17)

Total vertices/cells explored 25

5.3.2 Shortest Path in Dijkstra

With Kernel Size 3:

Path Computed in terms of vertices:

10→122→244→366→488→610→732→854→976→1098→1220→1342→1464→1586→1708→1830→1952→2074→2196→2318→2440→2562→2684→2806→2928→3050→3172→3294→3416→3538→3660→3782→3904→4026→4148→4149→4150→4272→4273→4274→4275→4276→4398→4518→4640→4762→4884→5006→5128→5248→5368→5488→5608→5728→5848→5968→5967→5966→5844→5843→5842→5841→5840→5839→5838→5958→6080→6202→6324→6446→6568→6690→6812→6934→7056→7178→7300→7422→7544→7666→7788→7910→8032→8154→8276→8398→8399→8400→8522→8523→8524→8646→8647→8648→8649→8650→8772→8773→8774→8775→8897→8898→8899→8900→8901→9023→9024→9025→9026→9148→9149→9029→9030→9031→9153→9154→9155→9156→9278→9279→9280→9281→9403→9404→9405→9406→9407→9529→9649→9769→9889→10009→10129→10249→10369→10489→10609→10729→10849→10969→11089→11209→11329→11449→11569→11689→11688→11687→11686→11685→11684→11562→11561→11560→11559→11558→11557→11556→11434→11433→11432→11431→11430→11429→11549→11548→11668→11790→11912→12034→12156→12278→12400→12522→12644→12766→12888→13010→13132→13254→13376→13498→13620→13742→13864→13986→14108→14230→14352→14474→14596→14718→14719→14720→14721→14722→14723→14724→14725→14726→14727→14728→14729→14730→14731→14732→14733→14734→14735→14736→14737→14738→14739→14740→14741→14742→14743→14744→14745→14746→14747→14748→14749→14750→14751→14752→14753→14754→14755→14756→14757→14758→14759→14760→14761

Total vertices explored in kernel size 3 are 231

With Kernel Size 5:

0→74→148→222→296→370→444→518→592→666→740→814→888→962→890→818→892→893→894→895→896→897→971→1045→1046→1120→1121→1122→1196→1197→1271→1345→1419→1493→1567→1640→1713→1786→1860→1933→2007→2081→2155→2229→2303→2304→2378→2379→2307→2308→2309→2310→2238→2166→2094→2095→2023→2024→1952→1880→1807→1733→1732→1731→1803

Vertices ecplored in Kernel size 5 are 65

With Kernel Size 7:

0→53→106→159→212→265→318→371→424→477→426→479→428→429→430→431→484→537→590→591→592→593→646→699→751→804→856→908→960→1013→1066→1119→1172→1173→1174→1175→1176→1177→1126→1075→1024→972→920→869→817→766→714→663→610→557→506→454→402

Vertices explored in Kernel size 7 are 53

Dijkstra's path is shown in vertices to show that dijkstra explores more map than A star. Since Dijkstra has no heuristic, the path travels from one vertex to another and does not pick another vertex in any direction but one that is connected to the current vertex. The path computed in resolution is stored in a YAML file and configured on the simulator for the robot to move accordingly. The computation in Dijkstra depends on the distance of previous vertices before the current vertex therefore Dijkstra also prints all the paths from the source uptill the last vertex before the destination. Appropriately, only the path between the source and the destination (last path) is printed and mentioned in this thesis. The distance is also calculated and printed for the same reason.

The results proves Dijkstra explores more area than A star. The vertices explored in different kernel sizes are more than twice the vertices or cells explored in A star.

Note: This conclusion was drawn by comparing the results of 10 different coordinate sets with various kernel sizes, Dijkstra almost always explores double the amount of vertices explored in A star

5.4 YAML Configuration

After these coordinates are converted into resolution; they are stored in a YAML file in order for the simulation to run. The robot moves accordingly and computes the same path displayed in the images below 10b.

```
YAML::Emitter yaml_out;
yaml_out << YAML::BeginMap;
yaml_out << YAML::Key << "waypoints";
yaml_out << YAML::Value << YAML::BeginSeq ;
for(int i =0; i<vecPath.size();i++)
{
   yaml_out << YAML::BeginMap;
   yaml_out << YAML::Key <<"position";
   yaml_out << YAML::Value << YAML::BeginMap;
   yaml_out << YAML::Key << "x";
   yaml_out << YAML::Value << vecPath[i].first;
   yaml_out << YAML::Key << "y";
   yaml_out << YAML::Value << vecPath[i].second;
   yaml_out << YAML::Key << "z";
   yaml_out << YAML::Value << "4.5";
   yaml_out << YAML::EndMap;
   yaml_out << YAML::Key << "orientation";
   yaml_out << YAML::Value << YAML::BeginMap;
   yaml_out << YAML::Key << "x";
   yaml_out << YAML::Value << "0.0444210774910485";
   yaml_out << YAML::Key << "y";
   yaml_out << YAML::Value << "-0.039973645527O3113";
   yaml_out << YAML::Key << "z";
   yaml_out << YAML::Value << "0.7459565426241741";
   yaml_out << YAML::Key << "w";
   yaml_out << YAML::Value << "0.663308157686 91";
   yaml_out << YAML::EndMap;
   yaml_out << YAML::EndMap;
}
yaml_out << YAML::EndSeq;
yaml_out << YAML::EndMap;
cout << "Here's the output YAML:\n" << yaml_out.c_str();

cout << endl;

ofstream inFile;
inFile.open("../config/yamlastardata.yaml");
inFile << yaml_out.c_str();
```

```
inFile.close();
```

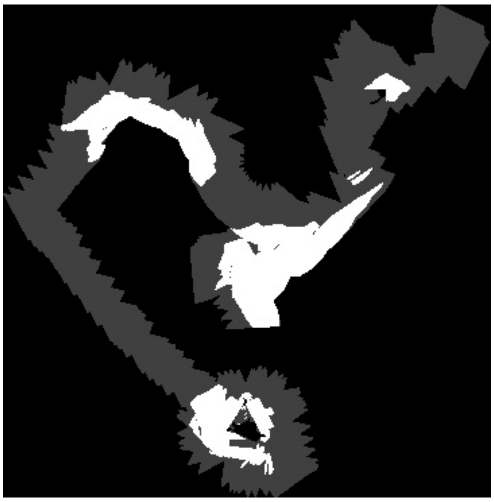
Listing 6: Code to store the coordinates in a YAML file


```yaml
waypoints:
  - position:
      x: 13.499999977647001
      y: -13.499999977647001
      z: 4.5
    orientation:
      x: 0.0444210774910485
      y: -0.03997364552703113
      z: 0.7459565426241741
      w: 0.66330815768691
  - position:
      x: 13.3499999754117
      y: -13.499999977647001
      z: 4.5
    orientation:
      x: 0.0444210774910485
      y: -0.03997364552703113
      z: 0.7459565426241741
      w: 0.66330815768691
  - position:
      x: 13.1999999731764
      y: -13.499999977647001
      z: 4.5
    orientation:
      x: 0.0444210774910485
      y: -0.03997364552703113
      z: 0.7459565426241741
      w: 0.66330815768691
  - position:
      x: 13.0499999709411
      y: -13.499999977647001
      z: 4.5
```

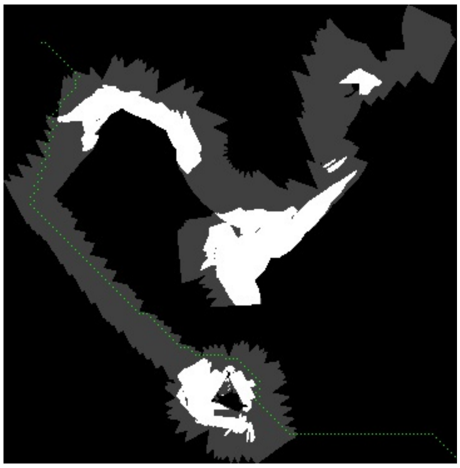

Figure 9: Output of YAML file: File name: yamlastardata.yaml

5.5 Path displayed as seen on simulator

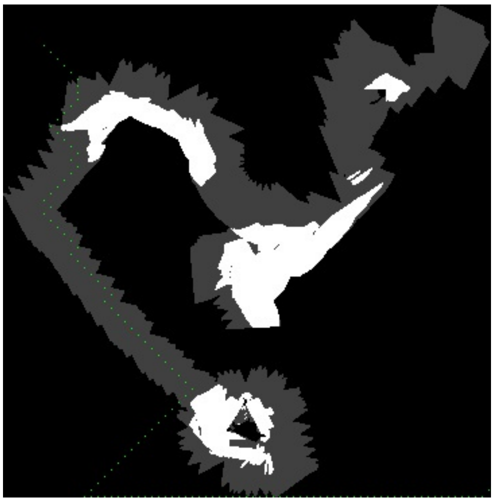
The following images display the result of the path computed:
Original coordinates: source = 30,30 ; destination = 360,360



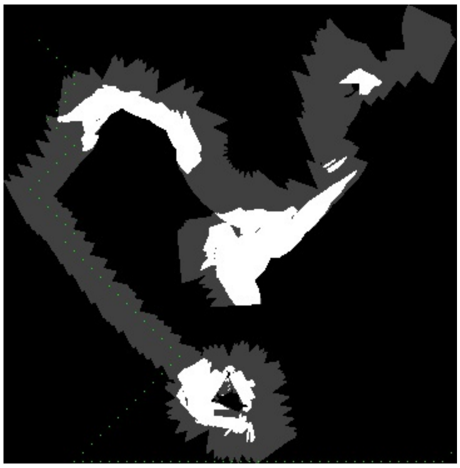
(a) Original map



(b) Path computed using Euclidean Heuristic on A star with Kernel Size 3



(c) Path computed using Euclidean Heuristic on A star with Kernel Size 5



(d) Path computed using Euclidean Heuristic on A star with Kernel Size 7

Figure 10: Path computed on original map with downscaled data

5.6 Downsampled Images

These images are produced by 5.2. 12 displays the same path as in 10. 10 uses the original filtered map with dimensions 366 by 362 to display the result of varied kernel size data. Kernel Size 3 has a dimension of 122 by 121, kernel size 5 has a dimension of 74 by 73 and kernel size 7 has a dimension of 53 by 52. The dimension variation is the cause of the difference between 10b, 10c and 12. The path in the latter is more clear and concised as compare to the path computed in the orignal map because of less resolution. The difference in path formation is also visible from kernel size 5, A star distinguishes the white block as a bigger obstacle than it actually is and changes its route.

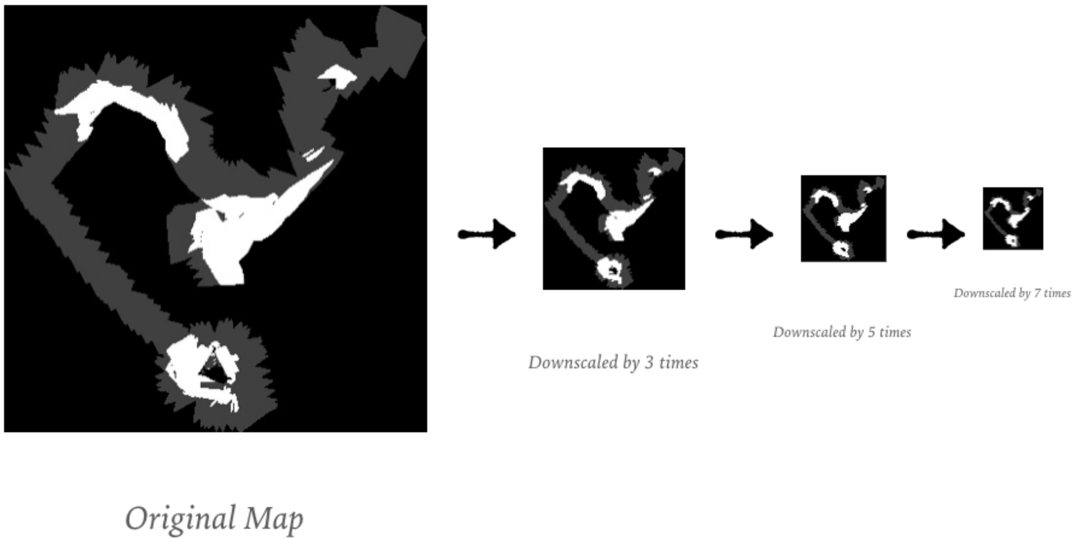


Figure 11: Set of downscaled images of the map

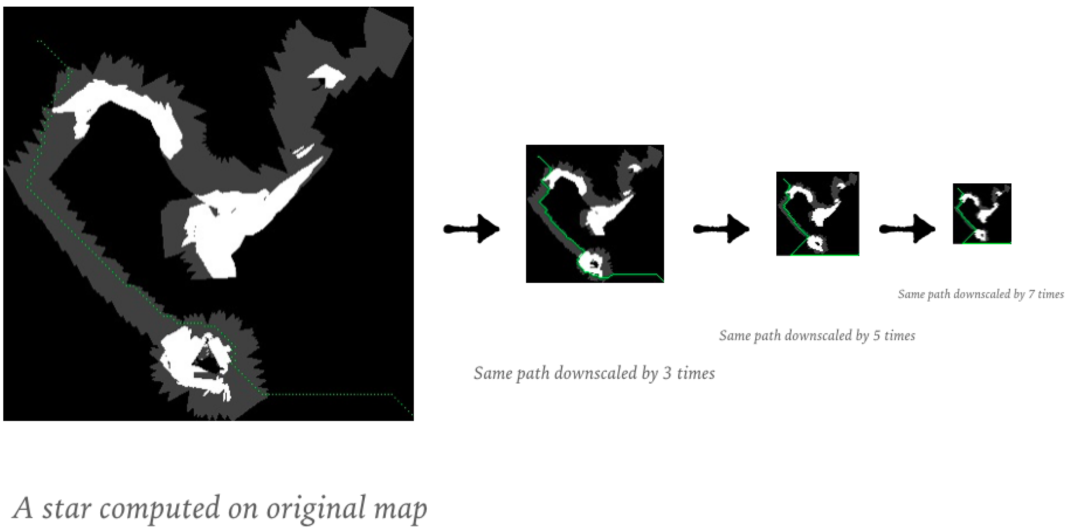


Figure 12: Set of downscaled images with the path computed

5.7 Time & Space Complexity

The Dijkstra algorithm is implemented as a depth first search algorithm with adjacent arrays being the main data structure so the time complexity will be O(V^2). However, the way this algorithm was written to conduct this research was to have a two-dimensional graph of $H * W$ by $H * W$ and to access each vertex, the number of vertices are given, then the time complexity will be proportional to $O(H * W \times H * W)^2$

The length of longest path = x. For each vertex, it's siblings are stored so when it's children have been visited and the parent vertex is explored, the next sibling to visit should be known. For x vertices throughout the path, extra s vertices are stored for each of the x vertices. So the most memory it can take up is the longest possible path. Thus, the space complexity of Dijkstra function is $O(x * s)$. However, in order to store the vertices in an array, it takes $O(V)$ and to store the shortest distance for every vertex it takes another $O(V)$ which equals to $O(V)$ since 2 is constant.

A star function itself has the time complexity of $O(E)$ where E is the number of edges in the graph as the graph is connected by a series of edges from the source cell to the destination cell. The total number of real edges results in $O(H * W \times H * W)$. Altogether it has the same worse time as the Dijkstra $O(H * W \times H * W)^2$.

A star uses more memory but it is the absolute algorithm meaning if the vertex is in the lowest depth possible, it will give the optimal solution. The space complexity of A star is $O(H * W)$.

Algorithm	Kernel Size	Heuristic	Time
Dijkstra	3	-	5.8 s
Dijkstra	5	-	4.2 s
Dijkstra	7	-	2.9 s
A star	3	euclidean	1 s
A star	3	manhattan	93ms
A star	3	diagonal	83ms
A star	5	euclidean	30 ms
A star	5	manhattan	26 ms
A star	5	diagonal	26 ms
A star	7	euclidean	26 ms
A star	7	manhattan	20 ms
A star	7	diagonal	19 ms

Table 1: Total time taken to compute paths with different heuristics and kernel sizes

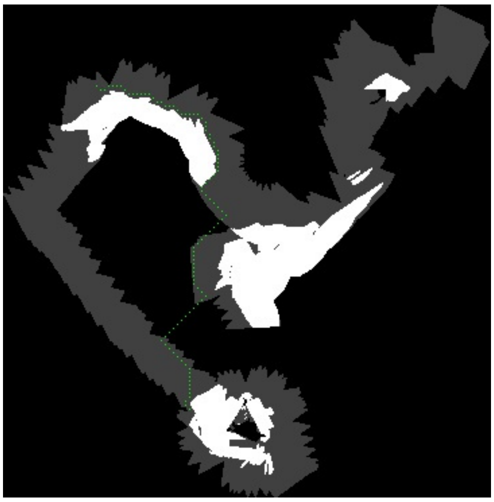
6 Comparision with Heuristics & Obstacle Sizes

6.1 Path Computation and Comparison with different Heuristics

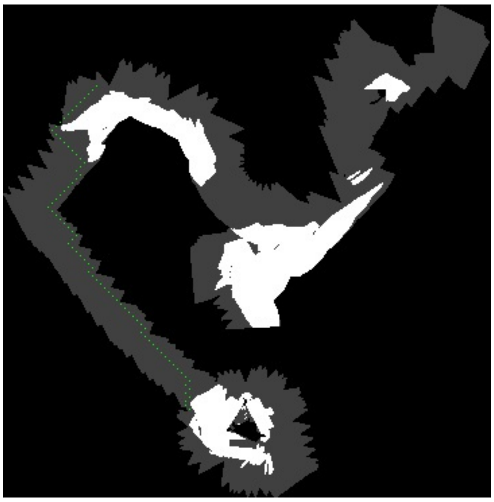
A star is computed with three different heuristics; Manhattan, Diagonal and Euclidean. Moreover, these three heuristics are compared with 10 different coordinate sets.

1. Source: 30,30 Destination: 360,360
2. Source: 60,70 Destination: 60,340
3. Source: 60,70 Destination: 140,300
4. Source: 60,90 Destination: 180,210
5. Source: 60,60 Destination: 180,315
6. Source: 300,70 Destination: 300,210
7. Source: 300,70 Destination: 180,315
8. Source: 180,180 Destination: 315,315
9. Source: 40,320 Destination: 340,60
10. Source: 40,320 Destination: 300,320

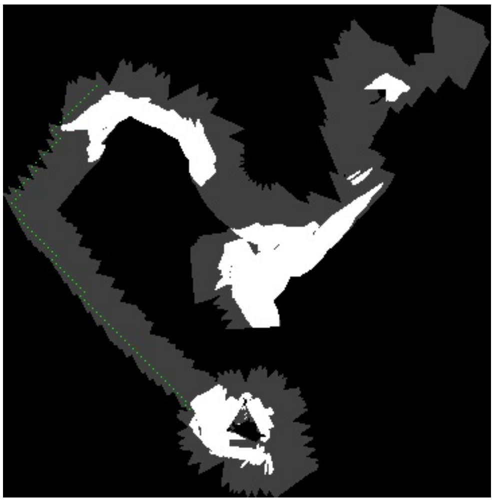
The paths are computed in detail in B with the three heuristics mentioned. The images below display the different paths with three heuristics. The difference in path is visible. As the kernel size increases, the path with different heuristics might compute a different path but might have the same length or number of cells explored. In some cases, two heuristics give the shortest path such as in coordinates 3,5,6,7,8 and 9. While with kernel size 7 all three heuristics have the same length in coordinates 2,3,5 and 6. After comparing the results, Euclidean computes the shortest path 86% of the time , Manhattan comes second with 63% and Diagonal with 53%. When acknowledged singly, Euclidean gives the shortest path 50% of the time while Manhattan and Diagonal gives the shortest path 33% and 16% of the time respectively.



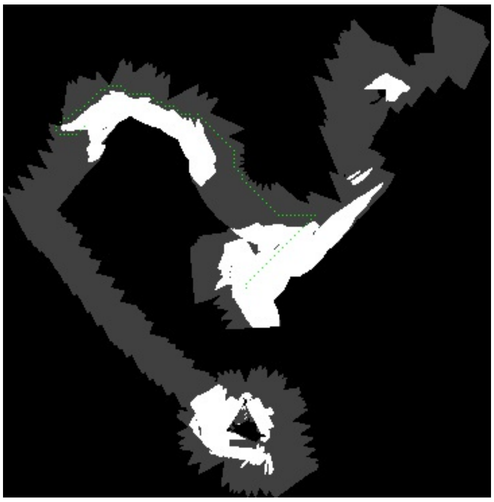
(a) Diagonal Heuristic with coordinates of 3



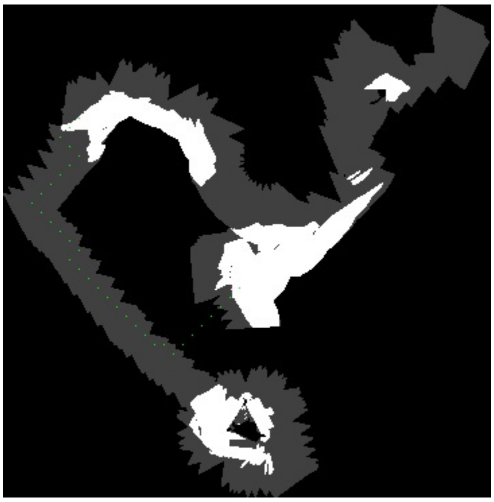
(b) Manhattan Heuristic with coordinates of 3



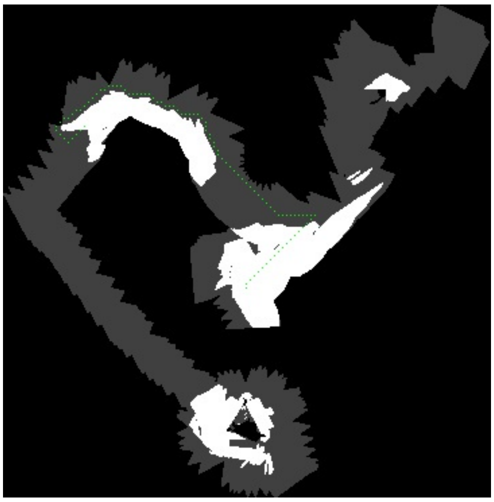
(c) Euclidean Heuristic with coordinates of 3



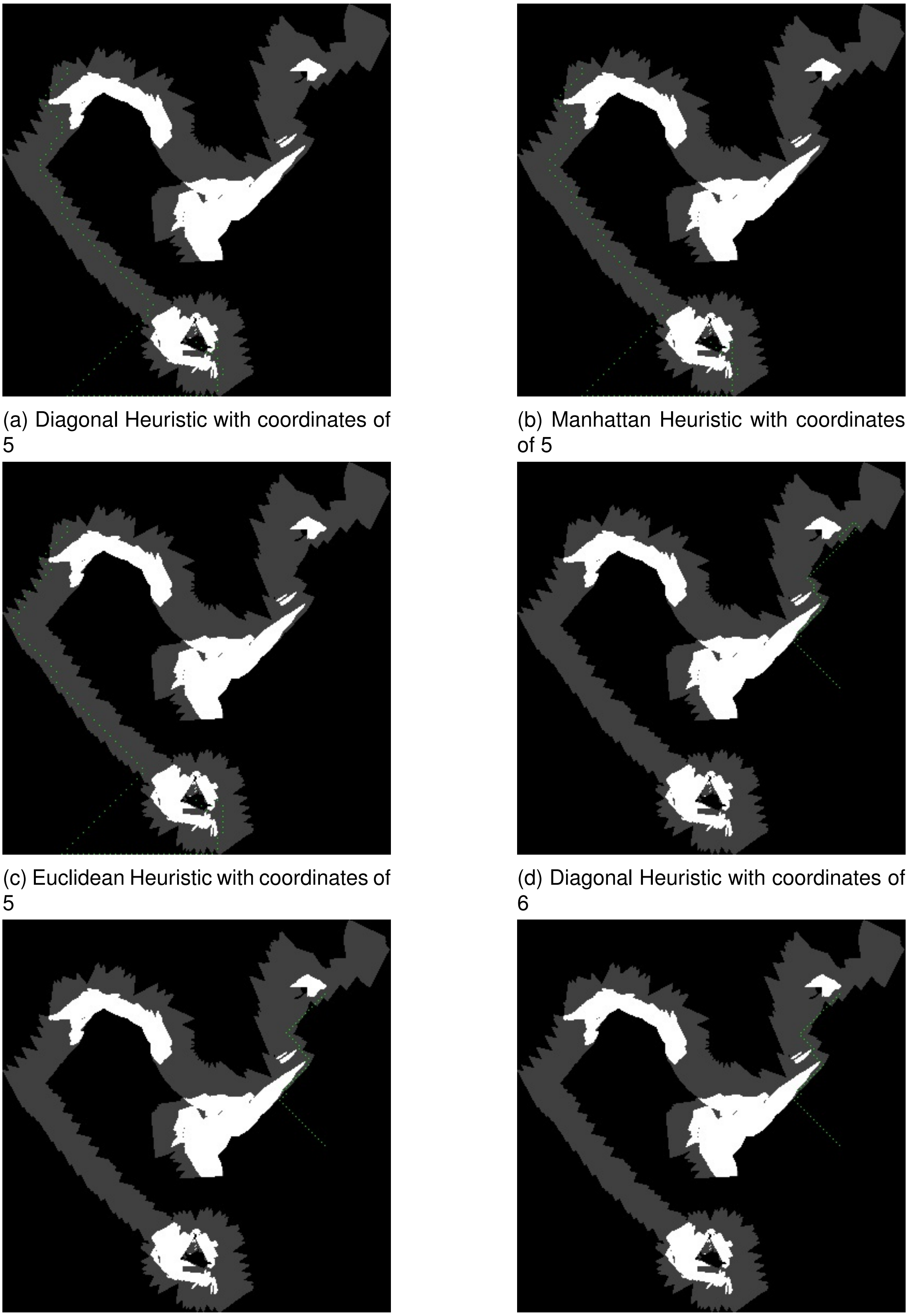
(d) Diagonal Heuristic with coordinates of 4



(e) Manhattan Heuristic with coordinates of 4



(f) Euclidean Heuristic with coordinates of 4



(a) Diagonal Heuristic with coordinates of 5

(b) Manhattan Heuristic with coordinates of 5

(c) Euclidean Heuristic with coordinates of 5

(d) Diagonal Heuristic with coordinates of 6

(e) Manhattan Heuristic with coordinates of 6

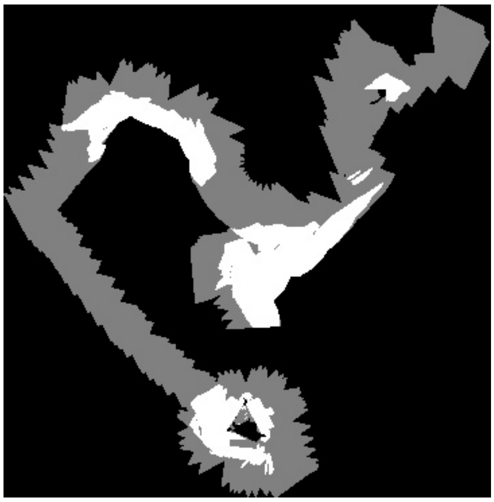
(f) Euclidean Heuristic with coordinates of 6

Figure 14: Paths with different heuristics

6.2 Path Computation and Comparison with different Obstacle Sizes

In order to emphasize on the obstacle, free and unknown cell values. The values were switched from 100, -1 and 0 to 100 , 2 and 1 and to 60,50 and 40. 100 and 60 are for obstacles. -1 and 50 for unseen cells and 1 and 40 for free cells.

(a) Filtered Map of 100, 2 & 1



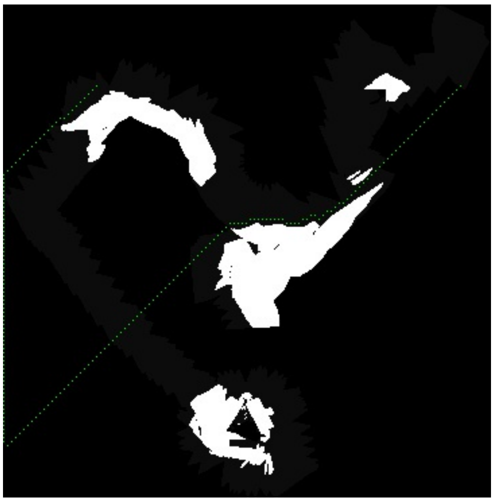
(b) Filtered Map of 60,50 & 40

As evident by the images, the free cells in 15a is barely visible, the grey area is closer to black since the values are close. In 15b the free cells have a lighter grey color closer to white.

By changing the values of the depicted values of the graph, the path changes completely.When the value from 50 to 2 is altered, there is almost no recognization of free cells becuase weighted value of 2 is much closer to 1 where 1 depicts the value of a free cell. Thus the same path in 10b is very different in 16a by using the same heuristic. The path difference is also visible between 16c and 14f. In both images, the path does not follow the free and known route and rather hits the boundaries first, seems like the path is computed blindly and follows the boundaries until it comes closer to the boundary that is closest to the destination point. This similarity is presented in the rest of the images as well.

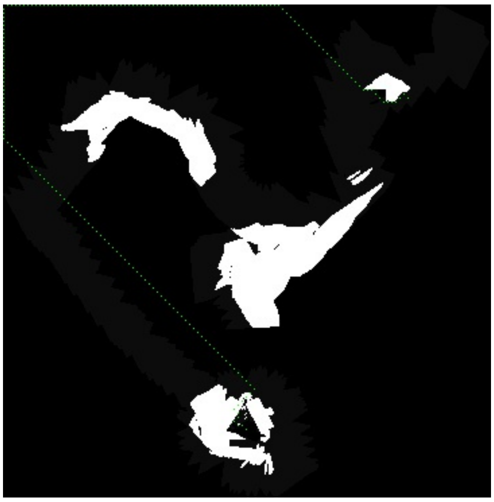
When the values are changed to 60,50 & 40. The path somewhat follows the known free cells but because of values being so close, in some cases, it pases through the obstacles while in other cases the unknown cells are mistaken with free cells. The path in 17a and 17c is still similar to the original path computed. 17b and 17e completely disregards the free and unknown cell notion and computes the shortest path by making a straight line and passes through the obstacles respectively.

(a) Coordinates with no. 1

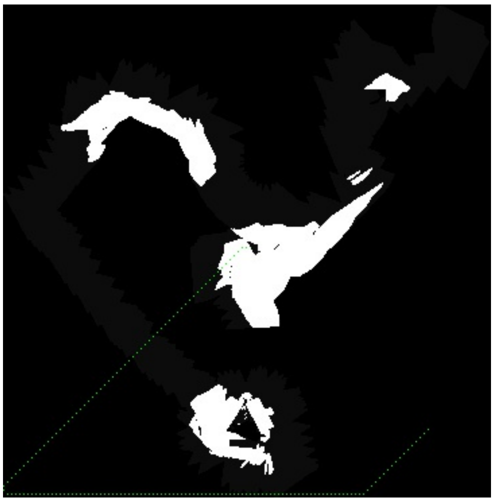


(b) Coordinates with no. 2

(c) Coordinates with no. 6

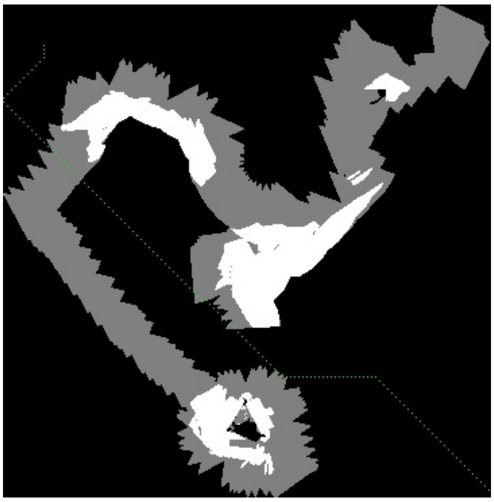


(d) Coordinates with no. 7

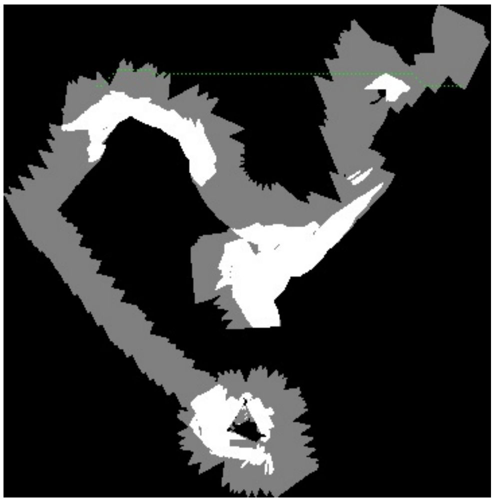


(e) Coordinates with no. 8

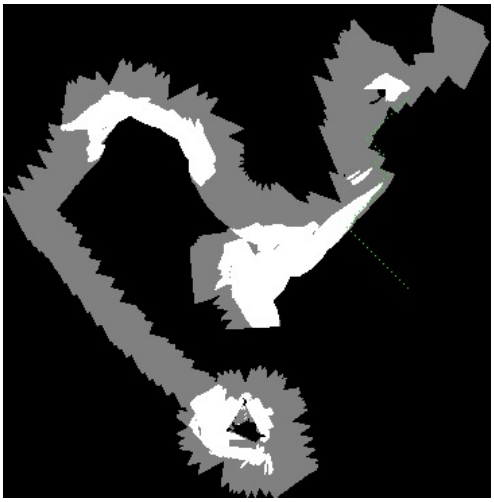
Figure 16: Paths computed on image 15a



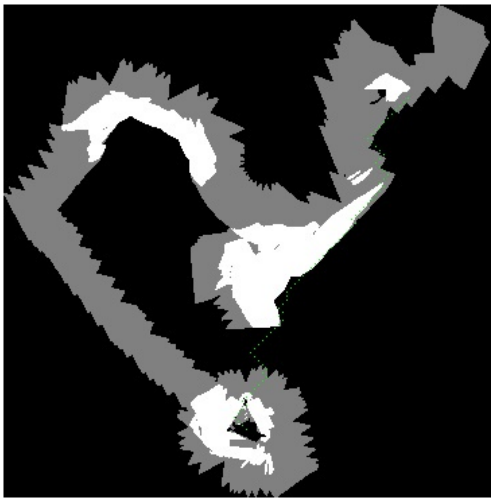
(a) Coordinates with no. 1



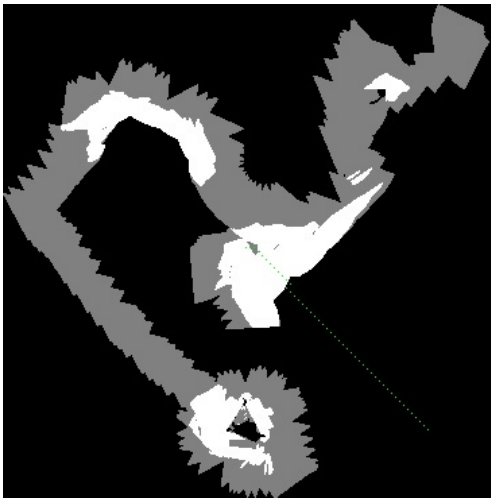
(b) Coordinates with no. 2



(c) Coordinates with no. 6



(d) Coordinates with no. 7



(e) Coordinates with no. 8

Figure 17: Paths computed on image 15b

7 Compilation & Simulation Start-up Process

7.1 Gazebo Simulator

To run the simulation on Gazebo the following steps are proceeded:

1. **roslaunch dexrov_meta startup_simulartion.launch ;** starts up the simulation
2. **roslaunch dexrov_meta startup_vehicle_sim.launch ;** vehicle is imported in the simulation
3. **roslaunch dexrov_meta startup_vessel_sim.launch ;** vessel is imported in the simulation
4. **roslaunch dexrov_meta rviz.launch ;** for launching rviz and visualization purpose to keep track of all the topics and nodes
5. **roslaunch dexrov_meta perception_octomap.launch ;** for launching the octomap
6. **rosrun joy joy_node_dev:=/dev/input/js1 ;** in case the joystick does not work
7. **roslaunch abrish navigate_path.launch ;** This command makes the robot move through the YAML file. YAML file is stowed in navigate_path.launch file <arg name="config_filename" default="$(find abrish)/config/yamlastardata.yaml"$/>

7.2 Compilation of Algorithms

To run the algorithms the following steps are followed:

- g++ filter2D.cpp -o filter2D 'pkg-config –cflags –libs opencv'
- g++ -std=c++11 astar.cpp -o astar -lyaml-cpp 'pkg-config –cflags –libs opencv'
- g++ -std=c++11 dij.cpp -o dij -lyaml-cpp 'pkg-config –cflags –libs opencv'

8 Drawbacks & Potential Improvements

Before unfolding an algorithm the right data structure has to be adopted for the optimization of the algorithm. For the open list, the best choice is a list that has both a fast insert and extract minimum operation. It is more important for the insert operation to have better performance of the insert operation than the extract minimum operation since the algorithm on average does not extract all the cells (nodes) inserted into the open list. These functionalities exist in the priority queues such as a binary heap and a Fibonacci heap. Binary heap gives a time complexity of Θ(log n) for both insert and extract minimum operations. The fibonacci heap further optimizes the insert operation to a linear time Θ(1). Another less recurrent operation that occurs is decrease key, when the g cost of a cell (node) in the open list needs an update. The fibonacci heap beats the rest of the data structure in this regard with a linear decrease key time complexity Θ(1).

Both algorithms can be implemented with an adjacency list which will improve the time complexity by $O(N + E)$ where N is the number of nodes/vertices and E is the number of edges.

Although A star produces the shortest optimal path but it is not always consistent as it relies heavily on the heuristics

9 Conclusion

Compared to A star, Dijkstra has a much better space complexity however it may not find the optimal solution. Dijkstra does not compute the most optimal shortest path but stops when it finds a minimal path from one vertex to another. Although the time complexity is the same for both algorithms, A star calculates the shortest path much faster than Dijkstra. A star computes the optimal path with kernel size 3 with the euclidean heuristic function. This research paper also deduce the importance of values assigned to various objects to indicate their characteristics. The values should be optimum otherwise a confused path will be evaluated. In addition, the kernel sizes are crucial, less resolution and data on the image and the graph returns altered path as less resolution disturbs the significance of mixed objects defined.

10 Further Research

11 Acknowledgements

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In addition, I would like to thank my friends Murt and Zazzle for helping me survive the stress and not letting me give up.

A iros Simulation

B Paths with different Heuristics

SOURCE 30,30 DESTINATION 360,360

Euclidean:
(10,10) →(10,11) →(11,12) →(12,13) →(13,14) →(14,15) →(15,16) →(16,17) → (17,18) →(18,19) →(19,20) →(20,19) →(21,19) →(22,19) →(23,18) →(24,17) → (25,16) →(26,15) →(27,15) →(28,14) →(29,13) →(30,12) →(31,13) →(32,14) → (33,13) →(34,12) →(35,13) →(36,12) →(37,11) →(38,12) →(39,11) →(40,12) → (41,11) →(42,10) →(43,11) →(44,10) →(45,9) →(46,10) →(47,9) →(48,8) → (49,9) → (50,8) →(51,7) →(52,8) →(53,7) →(54,8) →(55,9) →(56,10) → (57,11) →(58,12) → (59,13) →(60,14) →(61,15) →(62,16) →(63,17) →(64,18) → (65,19) →(66,20) → (67,21) →(68,22) →(69,23) →(70,24) →(71,25) →(72,26) → (73,27) →(74,28) → (75,29) →(76,30) →(77,31) →(78,32) →(79,33) →(80,34) → (81,35) →(82,36) → (82,37) →(82,38) →(83,39) →(84,40) →(85,41) →(86,42) → (87,43) →(88,44) → (89,45) →(90,46) →(91,47) →(91,48) →(91,49) →(92,50) → (93,51) →(93,52) → (93,53) →(93,54) →(93,55) →(93,56) →(93,57) →(93,58) → (94,59) →(94,60) → (94,61) →(94,62) →(95,63) →(96,64) →(97,65) →(98,66) → (99,67)→(100,68)→ (101,67)→(102,68)→(103,67)→(104,68)→(105,67)→(106,68)→ (107,69)→ (108,70) →(109,71)→(110,72) →(111,73) →(112,74) →(113,75) →(114,76) → (114,77) →(114,78) →(114,79) →(114,80) →(114,81) →(114,82) →(114,83) → (114,84) →(114,85) →(114,86) →(114,87) →(114,88) →(114,89) →(114,90) → (114,91) →(114,92) →(114,93) →(114,94) →(114,95) →(114,96) →(114,97) → (114,98) →(114,99) →(114,100) →(114,101) →(114,102) →(114,103) →(114,104)→ (114,105)→(114,106)→(114,107)→(114,108) →(114,109)→(114,110)→(114,111) → (114,112)→(114,113)→(114,114)→(115,115) →(116,116) →(117,117) →(118,118) → (119,119) →(120,120) = 164

Diagonal:
(10,10) →(10,11) →(11,12) →(12,13) →(13,14) →(14,15) →(15,16) →(16,17) → (17,18) →(18,19) →(19,20) →(20,21) →(21,22) →(22,23) →(21,24) →(21,25) → (20,26) →(20,27) →(20,28) →(20,29) →(21,30) →(22,31) →(22,32) →(22,33) → (22,34) →(22,35) →(22,36) →(23,37) →(24,38) →(24,39) →(25,40) →(25,41) → (25,42) →(26,43) →(27,44) →(27,45) →(27,46) →(28,47) →(27,48) →(28,49) → (29,50) →(30,51) →(31,52) →(32,53) →(33,54) →(34,55) →(35,56) →(36,57) → (37,57) →(38,57) →(39,57) →(40,57) →(41,58) →(42,59) →(43,59) →(44,60) → (45,61) →(46,62) →(47,63) →(48,62) →(49,61) →(50,60) →(51,59) →(52,58) → (53,57) →(54,56) →(55,55) →(56,56) →(57,55) →(58,55) →(58,54) →(59,53) → (60,52) →(61,53) →(62,52) →(63,51) →(64,52) →(65,51) →(66,52) →(67,53) → (68,52) →(69,51) →(70,50) →(71,51) →(72,50) →(73,49) →(74,48) →(75,47) → (76,46) →(77,45) →(78,44) →(79,43) →(80,42) →(81,41) →(82,40) →(83,39) → (84,40) →(85,41) →(86,42) →(87,43) →(88,44) →(89,45) →(90,46) →(91,47) → (92,48) →(93,49) →(94,50) →(95,49) →(95,48) →(96,47) →(97,46) →(98,45) → (99,44) →(100,43) →(101,44) →(102,45) →(103,45) →(104,44) →(105,43) →(106,44) →(107,45) →(108,46) →(109,47) →(110,48) →(111,49) →(112,50) →(112,51) →(112,52) →(112,53) →(112,54) →(113,55) →(114,56) →(115,57) →(116,58) →(117,59) →(117,60) →(117,61) →(117,62) →(117,63) →(117,64) →(117,65) →(117,66) →(117,67) →(117,68) →(117,69) →(117,70) →(117,71) →(117,72) →(116,73) →(115,74) →(115,75) →(114,76) →(114,77) →(114,78) →(114,79) →(114,80) →(114,81)

→(114,82) →(114,83) →(114,84) →(114,85) →(114,86) →(114,87) →(114,88) →(114,89) →(114,90) →(114,91) →(114,92) →(114,93) →(114,94) →(114,95) →(114,96) →(114,97) →(114,98) →(114,99) →(114,100) →(114,101)→(114,102)→(114,103)→(114,104)→(114,105) →(114,106) →(114,107) →(114,108) →(114,109)→(114,110)→(114,111)→(114,112)→(114,113)→ (114,114) →(115,115) →(116,116) →(117,117) →(118,118) →(119,119) →(120,120) = 196

Manhattan:

(10,10) →(10,11) →(11,12) →(12,13) →(13,14) →(14,15) →(15,16) →(16,17) → (17,18) →(18,19) →(19,20) →(20,21) →(21,20) →(22,20) →(23,20) →(24,19) → (25,18) →(26,17) →(27,16) →(28,15) →(29,14) →(30,13) →(31,12) →(32,13) → (33,14) →(34,15) →(35,16) →(36,17) →(37,18) →(38,19) →(39,20) →(40,19) → (41,18) →(42,19) →(43,18) →(44,17) →(45,16) →(46,15) →(47,14) →(48,13) → (49,12) →(50,11) →(51,12) →(52,13) →(53,14) →(54,15) →(55,16) →(56,17) → (57,18) →(58,17) →(59,16) →(60,17) →(61,18) →(62,19) →(63,20) →(64,21) → (65,22) →(66,21) →(67,22) →(68,23) →(69,24) →(70,25) →(71,26) →(72,27) → (73,28) →(74,29) →(75,30) →(76,31) →(77,32) →(78,33) →(79,34) →(80,35) → (81,34) →(82,35) →(83,36) →(84,37) →(85,38) →(86,39) →(87,40) →(88,41) → (89,42) →(90,43) →(91,44) →(92,45) →(93,46) →(93,47) →(93,48) →(93,49) → (93,50) →(93,51) →(93,52) →(93,53) →(93,54) →(94,55) →(95,56) →(95,57) → (95,58) →(95,59) →(95,60) →(95,61) →(96,62) →(96,63) →(97,64) →(98,65) → (98,66) →(99,67) →(100,68) →(101,69) →(102,70) →(103,69) →(104,70) →(105,71) →(106,72) →(107,73) →(108,72) →(109,73) →(110,74) →(111,73) →(112,74) →(113,75) →(114,76) →(114,77) →(114,78) →(114,79) →(114,80) →(114,81) →(114,82) →(114,83) →(114,84) →(114,85) →(114,86) →(114,87) →(114,88) →(114,89) →(114,90) →(114,91) →(114,92) →(114,93) →(114,94) →(114,95) →(114,96) →(114,97) →(114,98) →(114,99) →(114,100)→(114,101)→(114,102)→(114,103)→ (114,104)→(114,105) →(114,106) →(114,107) →(114,108)→(114,109)→(114,110)→(114,111) →(114,112)→(114,113) →(114,114) →(115,115) →(116,116) →(117,117) →(118,118) →(119,119) →(120,120) = 165

Kernel Size 5:

Euclidean:

(6,6) →(7,7) →(8,8) →(9,9) →(10,10) →(11,11) →(12,11) →(13,11) → (14,11) →(15,10) →(16,9) →(17,8) →(18,7) →(19,8) →(20,9) →(21,10) → (22,11) →(23,10) →(24,11) →(25,10) →(26,9) →(27,8) →(28,7) →(29,6) → (30,7) →(31,6) →(32,7) →(33,8) →(34,9) →(35,10) →(36,11) →(37,10) → (38,11) →(39,12) →(40,13) →(41,14) →(42,15) →(43,16) →(44,15) →(45,16) → (46,17) →(47,18) →(48,19) →(49,20) →(50,21) →(51,22) →(52,23) →(53,24) → (54,25) →(55,26) →(56,27) →(57,28) →(58,27) →(59,26) →(60,25) →(61,24) → (62,23) →(63,22) →(64,21) →(65,20) →(66,19) →(67,18) →(68,17) →(69,16) → (70,15) →(71,14) →(72,13) →(73,12) →(73,13) →(73,14) →(73,15) →(73,16) → (73,17) →(73,18) →(73,19) →(73,20) →(73,21) →(73,22) →(73,23) →(73,24) → (73,25) →(73,26) →(73,27) →(73,28) →(73,29) →(73,30) →(73,31) →(73,32) → (73,33) →(73,34) →(73,35) →(73,36) →(73,37) →(73,38) →(73,39) →(73,40) → (73,41) →(73,42) →(73,43) →(73,44) →(73,45) →(73,46) →(73,47) →(73,48) → (73,49) →(73,50) →(73,51) →(73,52) →(73,53) →(73,54) →(73,55) →(73,56) → (73,57) →(73,58) →(73,59) →(73,60) →(73,61) →(73,62) →(73,63) →(73,64) → (73,65) →(73,66) →(73,67) →(73,68) →(73,69) →(73,70) →(73,71) →(72,72) = 128

Diagonal:

(6,6) →(7,7) →(8,8) →(9,9) →(10,10) →(11,11) →(12,12) →(13,13) → (14,12) →(14,11) →(15,10) →(16,9) →(17,8) →(18,7) →(19,8) →(20,9) → (21,10) →(22,11) →(23,10) →(24,11) →(25,10) →(26,9) →(27,8) →(28,7) → (29,6) →(30,7) →(31,6) →(32,7) →(33,8) →(34,9) →(35,10) →(36,11) → (37,10) →(38,11) →(39,12) →(40,13) →(41,14) →(42,15)

→(43,16) →(44,15) → (45,16) →(46,17) →(47,18) →(48,19) →(49,20) →(50,21) →(51,22) →(52,23) → (53,24) →(54,25) →(55,26) →(56,27) →(57,28) →(58,27) →(59,26) →(60,25) → (61,24) →(62,23) →(63,22) →(64,21) →(65,20) →(66,19) →(67,18) →(68,17) → (69,16) →(70,15) →(71,14) →(72,13) →(73,12) →(73,13) →(73,14) →(73,15) → (73,16) →(73,17) →(73,18) →(73,19) →(73,20) →(73,21) →(73,22) →(73,23) → (73,24) →(73,25) →(73,26) →(73,27) →(73,28) →(73,29) →(73,30) →(73,31) → (73,32) →(73,33) →(73,34) →(73,35) →(73,36) →(73,37) →(73,38) →(73,39) → (73,40) →(73,41) →(73,42) →(73,43) →(73,44) →(73,45) →(73,46) →(73,47) → (73,48) →(73,49) →(73,50) →(73,51) →(73,52) →(73,53) →(73,54) →(73,55) → (73,56) →(73,57) →(73,58) →(73,59) →(73,60) →(73,61) →(73,62) →(73,63) → (73,64) →(73,65) →(73,66) →(73,67) →(73,68) →(73,69) →(73,70) →(73,71) → (72,72) = 129

Manhattan:
(6,6) →(7,7) →(8,8) →(9,9) →(10,10) →(11,11) →(12,11) →(13,11) → (14,11) →(15,10) →(16,9) →(17,8) →(18,7) →(19,8) →(20,9) →(21,10) → (22,11) →(23,10) →(24,11) →(25,10) →(26,9) →(27,8) →(28,7) →(29,6) → (30,7) →(31,6) →(32,7) →(33,8) →(34,9) →(35,10) →(36,11) →(37,10) → (38,11) →(39,12) →(40,13) →(41,14) →(42,15) →(43,16) →(44,15) →(45,16) → (46,17) →(47,18) →(48,19) →(49,20) →(50,21) →(51,22) →(52,23) →(53,24) → (54,25) →(55,26) →(56,27) →(57,28) →(58,27) →(59,26) →(60,25) →(61,24) → (62,23) →(63,22) →(64,21) →(65,20) →(66,19) →(67,18) →(68,17) →(69,16) → (70,15) →(71,14) →(72,13) →(73,12) →(73,13) →(73,14) →(73,15) →(73,16) → (73,17) →(73,18) →(73,19) →(73,20) →(73,21) →(73,22) →(73,23) →(73,24) → (73,25) →(73,26) →(73,27) →(73,28) →(73,29) →(73,30) →(73,31) →(73,32) → (73,33) →(73,34) →(73,35) →(73,36) →(73,37) →(73,38) →(73,39) →(73,40) → (73,41) →(73,42) →(73,43) →(73,44) →(73,45) →(73,46) →(73,47) →(73,48) → (73,49) →(73,50) →(73,51) →(73,52) →(73,53) →(73,54) →(73,55) →(73,56) → (73,57) →(73,58) →(73,59) →(73,60) →(73,61) →(73,62) →(73,63) →(73,64) → (73,65) →(73,66) →(73,67) →(73,68) →(73,69) →(73,70) →(73,71) →(72,72) = 128

Kernel size 7

Euclidean:
(4,4) →(5,5) →(6,6) →(7,7) →(8,8) →(9,8) →(10,7) →(11,7) → (12,6) →(13,5) →(14,6) →(15,7) →(16,8) →(17,7) →(18,6) →(19,5) → (20,4) →(21,3) →(22,4) →(23,5) →(24,6) →(25,7) →(26,8) →(27,7) → (28,8) →(29,9) →(30,10) →(31,11) →(32,12) →(33,13) →(34,14) →(35,15) → (36,16) →(37,17) →(38,18) →(39,19) →(40,20) →(41,19) →(42,18) →(43,17) → (44,16) →(45,15) →(46,14) →(47,13) →(48,12) →(49,11) →(50,10) →(51,9) → (52,8) →(52,9) →(52,10) →(52,11) →(52,12) →(52,13) →(52,14) →(52,15) → (52,16) →(52,17) →(52,18) →(52,19) →(52,20) →(52,21) →(52,22) →(52,23) → (52,24) →(52,25) →(52,26) →(52,27) →(52,28) →(52,29) →(52,30) →(52,31) → (52,32) →(52,33) →(52,34) →(52,35) →(52,36) →(52,37) →(52,38) →(52,39) → (52,40) →(52,41) →(52,42) →(52,43) →(52,44) →(52,45) →(52,46) →(52,47) → (52,48) →(52,49) →(52,50) →(51,51) = 92

Diagonal:
(4,4) →(5,5) →(6,6) →(7,7) →(8,8) →(9,9) →(10,8) →(10,7) → (11,6) →(12,5) →(13,4) →(14,3) →(15,2) →(16,1) →(17,0) →(18,1) → (19,0) →(20,1) →(21,2) →(22,3) →(23,4) →(24,5) →(25,6) →(26,7) → (27,8) →(28,7) →(29,8) →(30,9) →(31,10) →(32,11) →(33,12) →(34,13) → (35,14) →(36,15) →(37,16) →(38,17) →(39,18) →(40,19) →(41,18) →(42,17) → (43,16) →(44,15) →(45,14) →(46,13) →(47,12) →(48,11) →(49,10) →(50,9) → (51,8)

→(52,7) →(52,8) →(52,9) →(52,10) →(52,11) →(52,12) →(52,13) → (52,14) →(52,15) →(52,16) →(52,17) →(52,18) →(52,19) →(52,20) →(52,21) → (52,22) →(52,23) →(52,24) →(52,25) →(52,26) →(52,27) →(52,28) →(52,29) → (52,30) →(52,31) →(52,32) →(52,33) →(52,34) →(52,35) →(52,36) →(52,37) → (52,38) →(52,39) →(52,40) →(52,41) →(52,42) →(52,43) →(52,44) →(52,45) → (52,46) →(52,47) →(52,48) →(52,49) →(52,50) →(51,51) = 94

Manhattan:
(4,4) →(5,5) →(6,6) →(7,7) →(8,8) →(9,8) →(10,7) →(11,6) → (12,5) →(13,5) →(14,6) →(15,7) →(16,8) →(17,7) →(18,6) →(19,5) → (20,4) →(21,3) →(22,4) →(23,5) →(24,6) →(25,7) →(26,8) →(27,7) → (28,8) →(29,9) →(30,10) →(31,11) →(32,12) →(33,13) →(34,14) →(35,15) → (36,16) →(37,17) →(38,18) →(39,19) →(40,20) →(41,19) →(42,18) →(43,17) → (44,16) →(45,15) →(46,14) →(47,13) →(48,12) →(49,11) →(50,10) →(51,9) → (52,8) →(52,9) →(52,10) →(52,11) →(52,12) →(52,13) →(52,14) →(52,15) → (52,16) →(52,17) →(52,18) →(52,19) →(52,20) →(52,21) →(52,22) →(52,23) → (52,24) →(52,25) →(52,26) →(52,27) →(52,28) →(52,29) →(52,30) →(52,31) → (52,32) →(52,33) →(52,34) →(52,35) →(52,36) →(52,37) →(52,38) →(52,39) → (52,40) →(52,41) →(52,42) →(52,43) →(52,44) →(52,45) →(52,46) →(52,47) → (52,48) →(52,49) →(52,50) →(51,51) = 92

SOURCE 180,180 DESTINATION 315,315

Kernel Size 3

Euclidean:
(60,60) →(60,59) →(61,58) →(62,57) →(63,56) →(64,55) →(65,54) →(66,53) → (67,52) →(68,51) →(69,50) →(70,49) →(71,48) →(72,47) →(73,46) →(74,45) → (75,44) →(76,43) →(77,42) →(78,41) →(79,40) →(80,39) →(81,38) →(82,37) → (82,38) →(83,39) →(84,40) →(85,41) →(86,42) →(87,43) →(88,44) →(89,45) → (90,46) →(91,47) →(91,48) →(91,49) →(92,50) →(93,51) →(93,52) →(93,53) → (93,54) →(93,55) →(93,56) →(93,57) →(93,58) →(94,59) →(94,60) →(94,61) → (94,62) →(95,63) →(96,64) →(97,65) →(98,66) →(99,67) →(100,68) →(101,67) →(102,68) →(103,69) →(104,70) →(105,71) →(106,72) →(107,73) →(107,74) →(107,75) →(107,76) →(107,77) →(107,78) →(107,79) →(107,80) →(107,81) →(107,82) →(107,83) →(107,84) →(107,85) →(107,86) →(107,87) →(106,88) →(106,89) →(106,90) →(106,91) →(106,92) →(106,93) →(106,94) →(106,95) →(106,96) →(106,97) →(106,98) →(106,99) →(106,100) →(106,101) →(106,102) →(106,103) →(106,104) →(105,105) = 94

Diagonal:
(60,60) →(60,59) →(61,58) →(62,57) →(63,56) →(64,55) →(65,54) →(66,53) → (67,52) →(68,51) →(69,50) →(70,51) →(71,52) →(72,51) →(73,50) →(74,49) → (75,48) →(76,47) →(77,46) →(78,45) →(79,44) →(80,43) →(81,42) →(82,41) → (83,40) →(84,39) →(85,40) →(86,41) →(87,42) →(88,43) →(89,44) →(90,45) → (91,46) →(92,47) →(93,48) →(93,49) →(93,50) →(93,51) →(93,52) →(93,53) → (93,54) →(94,55) →(95,56) →(95,57) →(95,58) →(95,59) →(95,60) →(95,61) → (96,62) →(96,63) →(97,64) →(98,65) →(98,66) →(99,67) →(100,68) →(101,69) →(102,70) →(103,69) →(104,70) →(105,71) →(106,72) →(107,73) →(107,74) →(107,75) →(107,76) →(107,77) →(106,78) →(106,79) →(106,80) →(106,81) →(106,82) →(106,83) →(106,84) →(106,85) →(106,86) →(106,87) →(106,88) →(106,89) →(106,90) →(106,91) →(106,92) →(106,93) →(106,94) →(106,95) →(106,96) →(106,97)

→(106,98) →(106,99) →(106,100) →(106,101) →(106,102) →(106,103) →(106,104) →(105,105) = 94

Manhattan:

(60,60) →(60,59) →(61,58) →(62,57) →(63,56) →(64,55) →(65,54) →(66,53) → (67,52) →(68,51) →(69,50) →(70,51) →(71,52) →(71,53) →(72,54) →(73,55) → (74,56) →(75,57) →(76,58) →(77,57) →(78,58) →(79,59) →(80,58) →(81,57) → (82,56) →(83,55) →(84,54) →(85,53) →(86,52) →(87,51) →(88,50) →(89,49) → (90,48) →(91,47) →(92,48) →(93,49) →(93,50) →(93,51) →(93,52) →(93,53) → (93,54) →(94,55) →(95,56) →(95,57) →(95,58) →(95,59) →(95,60) →(95,61) → (96,62) →(96,63) →(97,64) →(98,65) →(98,66) →(99,67) →(100,68) →(101,69) →(102,70) →(103,69) →(104,70) →(105,71) →(106,72) →(107,73) →(107,74) →(107,75) →(107,76) →(107,77) →(106,78) →(106,79) →(106,80) →(106,81) →(106,82) →(106,83) →(106,84) →(106,85) →(106,86) →(106,87) →(106,88) →(106,89) →(106,90) →(106,91) →(106,92) →(106,93) →(106,94) →(106,95) →(106,96) →(106,97) →(106,98) →(106,99) →(106,100) →(106,101) →(106,102) →(106,103) →(106,104) →(105,105) = 94

Kernel Size 5

Euclidean:

(36,36) →(37,36) →(38,35) →(39,34) →(40,33) →(41,32) →(42,31) →(43,30) → (44,29) →(45,28) →(46,27) →(47,26) →(48,25) →(49,24) →(50,23) →(51,24) → (52,25) →(53,26) →(54,27) →(55,28) →(55,29) →(56,30) →(57,29) →(58,28) → (59,27) →(60,26) →(61,25) →(62,24) →(63,23) →(64,22) →(65,21) →(66,20) → (67,19) →(68,18) →(69,17) →(70,16) →(71,15) →(72,14) →(73,13) →(73,14) → (73,15) →(73,16) →(73,17) →(73,18) →(73,19) →(73,20) →(73,21) →(73,22) → (73,23) →(73,24) →(73,25) →(73,26) →(73,27) →(73,28) →(73,29) →(73,30) → (73,31) →(73,32) →(73,33) →(73,34) →(73,35) →(73,36) →(73,37) →(73,38) → (73,39) →(73,40) →(73,41) →(73,42) →(73,43) →(73,44) →(73,45) →(73,46) → (73,47) →(73,48) →(73,49) →(73,50) →(73,51) →(73,52) →(73,53) →(73,54) → (73,55) →(73,56) →(73,57) →(73,58) →(73,59) →(73,60) →(73,61) →(73,62) → (73,63) →(73,64) →(73,65) →(73,66) →(73,67) →(73,68) →(73,69) →(73,70) → (73,71) →(72,72) →(71,72) →(70,72) →(69,72) →(68,72) →(67,72) →(66,72) → (65,72) →(64,72) →(63,72) →(62,72) →(61,72) →(60,72) →(59,72) →(58,72) → (57,72) →(56,72) →(57,71) →(58,70) →(59,69) →(60,68) →(61,67) →(62,66) → (63,65) →(64,64) →(63,63) = 123

Diagonal:

(36,36) →(36,35) →(35,34) →(34,33) →(34,32) →(34,31) →(35,30) →(36,31) → (37,30) →(38,31) →(39,30) →(40,31) →(41,30) →(42,29) →(43,28) →(44,27) → (45,26) →(46,25) →(47,24) →(48,23) →(49,22) →(50,21) →(51,22) →(52,23) → (53,24) →(54,25) →(55,26) →(56,27) →(57,28) →(58,27) →(59,26) →(60,25) → (61,24) →(62,23) →(63,22) →(64,21) →(65,20) →(66,19) →(67,18) →(68,17) → (69,16) →(70,15) →(71,14) →(72,13) →(73,12) →(73,13) →(73,14) →(73,15) → (73,16) →(73,17) →(73,18) →(73,19) →(73,20) →(73,21) →(73,22) →(73,23) → (73,24) →(73,25) →(73,26) →(73,27) →(73,28) →(73,29) →(73,30) →(73,31) → (73,32) →(73,33) →(73,34) →(73,35) →(73,36) →(73,37) →(73,38) →(73,39) → (73,40) →(73,41) →(73,42) →(73,43) →(73,44) →(73,45) →(73,46) →(73,47) → (73,48) →(73,49) →(73,50) →(73,51) →(73,52) →(73,53) →(73,54) →(73,55) → (73,56) →(73,57) →(73,58) →(73,59) →(73,60) →(73,61) →(73,62) →(73,63) → (73,64) →(73,65) →(73,66) →(73,67) →(73,68) →(73,69) →(73,70) →(73,71) → (72,72) →(71,72) →(70,72) →(69,72) →(68,72) →(67,72) →(66,72) →(65,72) → (64,72) →(63,72) →(62,72) →(61,72) →(60,72) →(59,72) →(58,72) →(57,72) → (58,71) →(59,70) →(60,69) →(61,68) →(62,67) →(63,66)

→(64,65) →(64,64) → (63,63) = 129
Manhattan:
(36,36) →(36,35) →(35,34) →(34,33) →(34,32) →(34,31) →(35,30) →(36,31) → (37,30) →(38,31) →(39,30) →(40,31) →(41,30) →(42,29) →(43,28) →(44,27) → (45,26) →(46,25) →(47,24) →(48,23) →(49,22) →(50,21) →(51,22) →(52,23) → (53,24) →(54,25) →(55,26) →(56,27) →(57,28) →(58,27) →(59,26) →(60,25) → (61,24) →(62,23) →(63,22) →(64,21) →(65,20) →(66,19) →(67,18) →(68,17) → (69,16) →(70,15) →(71,14) →(72,13) →(73,12) →(73,13) →(73,14) →(73,15) → (73,16) →(73,17) →(73,18) →(73,19) →(73,20) →(73,21) →(73,22) →(73,23) → (73,24) →(73,25) →(73,26) →(73,27) →(73,28) →(73,29) →(73,30) →(73,31) → (73,32) →(73,33) →(73,34) →(73,35) →(73,36) →(73,37) →(73,38) →(73,39) → (73,40) →(73,41) →(73,42) →(73,43) →(73,44) →(73,45) →(73,46) →(73,47) → (73,48) →(73,49) →(73,50) →(73,51) →(73,52) →(73,53) →(73,54) →(73,55) → (73,56) →(73,57) →(73,58) →(73,59) →(73,60) →(73,61) →(73,62) →(73,63) → (73,64) →(73,65) →(73,66) →(73,67) →(73,68) →(73,69) →(73,70) →(73,71) → (72,72) →(71,72) →(70,72) →(69,72) →(68,72) →(67,72) →(66,72) →(65,72) → (64,72) →(63,72) →(62,72) →(61,72) →(60,72) →(59,72) →(58,72) →(57,72) → (56,72) →(57,71) →(58,70) →(59,69) →(60,68) →(61,67) →(62,66) →(63,65) → (64,64) →(63,63) = 130

Kernel Size 7

Euclidean:
(25,25) →(26,25) →(27,24) →(28,23) →(29,22) →(30,21) →(31,20) →(32,19) → (33,18) →(34,17) →(35,16) →(36,15) →(37,16) →(38,17) →(39,18) →(40,19) → (41,19) →(42,18) →(43,17) →(44,16) →(45,15) →(46,14) →(47,13) →(48,12) → (49,11) →(50,10) →(51,9) →(52,8) →(52,9) →(52,10) →(52,11) →(52,12) → (52,13) →(52,14) →(52,15) →(52,16) →(52,17) →(52,18) →(52,19) →(52,20) → (52,21) →(52,22) →(52,23) →(52,24) →(52,25) →(52,26) →(52,27) →(52,28) → (52,29) →(52,30) →(52,31) →(52,32) →(52,33) →(52,34) →(52,35) →(52,36) → (52,37) →(52,38) →(52,39) →(52,40) →(52,41) →(52,42) →(52,43) →(52,44) → (51,44) →(50,44) →(49,44) →(48,44) →(47,44) →(46,44) →(45,45) = 71
Diagonal:
(25,25) →(25,24) →(24,23) →(24,22) →(25,22) →(26,21) →(27,20) →(28,21) → (29,20) →(30,19) →(31,18) →(32,17) →(33,16) →(34,15) →(35,14) →(36,15) → (37,16) →(38,17) →(39,18) →(40,19) →(41,19) →(42,18) →(43,17) →(44,16) → (45,15) →(46,14) →(47,13) →(48,12) →(49,11) →(50,10) →(51,9) →(52,8) → (52,9) →(52,10) →(52,11) →(52,12) →(52,13) →(52,14) →(52,15) →(52,16) → (52,17) →(52,18) →(52,19) →(52,20) →(52,21) →(52,22) →(52,23) →(52,24) → (52,25) →(52,26) →(52,27) →(52,28) →(52,29) →(52,30) →(52,31) →(52,32) → (52,33) →(52,34) →(52,35) →(52,36) →(52,37) →(52,38) →(52,39) →(52,40) → (52,41) →(52,42) →(52,43) →(52,44) →(52,45) →(51,45) →(50,45) →(49,45) → (48,45) →(47,45) →(46,45) →(45,45) = 76
Manhattan:
(25,25) →(25,24) →(24,23) →(24,22) →(25,22) →(26,21) →(27,20) →(28,21) → (29,20) →(30,19) →(31,18) →(32,17) →(33,16) →(34,15) →(35,14) →(36,15) → (37,16) →(38,17) →(39,18) →(40,19) →(41,19) →(42,18) →(43,17) →(44,16) → (45,15) →(46,14) →(47,13) →(48,12) →(49,11) →(50,10) →(51,9) →(52,8) → (52,9) →(52,10) →(52,11) →(52,12) →(52,13) →(52,14) →(52,15) →(52,16) → (52,17) →(52,18) →(52,19) →(52,20) →(52,21) →(52,22) →(52,23) →(52,24) → (52,25) →(52,26) →(52,27) →(52,28) →(52,29) →(52,30) →(52,31) →(52,32) → (52,33) →(52,34) →(52,35) →(52,36) →(52,37) →(52,38) →(52,39) →(52,40) → (52,41) →(52,42) →(52,43) →(52,44) →(51,44) →(50,44) →(49,44) →(48,44)

→ (47,44) →(46,44) →(45,45) = 75

SOURCE 60,60 DESTINATION 180,315

Kernel Size 3
Euclidean:
(20,20) →(21,20) →(22,20) →(23,20) →(24,19) →(25,18) →(26,17) →(27,16) → (28,15) →(29,14) →(30,13) →(31,12) →(32,13) →(33,14) →(34,13) →(35,12) → (36,11) →(37,10) →(38,9) →(39,8) →(40,7) →(41,8) →(42,7) →(43,6) → (44,5) →(45,4) →(46,3) →(47,2) →(48,3) →(49,4) →(50,5) →(51,6) → (52,7) →(53,8) →(54,9) →(55,10) →(56,11) →(57,12) →(58,13) →(59,14) → (60,15) →(61,16) →(62,17) →(63,18) →(64,19) →(65,20) →(66,21) →(67,22) → (68,23) →(69,24) →(70,25) →(71,26) →(72,27) →(73,28) →(74,29) →(75,30) → (76,31) →(77,32) →(78,33) →(79,34) →(80,35) →(81,34) →(82,35) →(83,36) → (84,37) →(85,38) →(86,39) →(87,40) →(88,41) →(89,42) →(90,43) →(91,44) → (92,45) →(93,46) →(93,47) →(93,48) →(93,49) →(93,50) →(93,51) →(93,52) → (93,53) →(93,54) →(94,55) →(95,56) →(95,57) →(95,58) →(95,59) →(95,60) → (95,61) →(96,62) →(96,63) →(97,64) →(98,65) →(99,64) →(100,63) →(101,62) →(102,61) →(103,60) →(104,59) →(105,60) = 100
Manhattan:
(20,20) →(21,20) →(22,20) →(23,20) →(24,19) →(25,18) →(26,17) →(27,16) → (28,15) →(29,14) →(30,13) →(31,12) →(32,13) →(33,14) →(34,15) →(35,16) → (36,17) →(37,18) →(38,19) →(39,20) →(40,19) →(41,18) →(42,19) →(43,18) → (44,17) →(45,16) →(46,15) →(47,14) →(48,13) →(49,12) →(50,11) →(51,12) → (52,13) →(53,14) →(54,15) →(55,16) →(56,17) →(57,18) →(58,17) →(59,16) → (60,17) →(61,18) →(62,19) →(63,20) →(64,21) →(65,22) →(66,21) →(67,22) → (68,23) →(69,24) →(70,25) →(71,26) →(72,27) →(73,28) →(74,29) →(75,30) → (76,31) →(77,32) →(78,33) →(79,34) →(80,35) →(81,34) →(82,35) →(83,36) → (84,37) →(85,38) →(86,39) →(87,40) →(88,41) →(89,42) →(90,43) →(91,44) → (92,45) →(93,46) →(93,47) →(93,48) →(93,49) →(93,50) →(93,51) →(93,52) → (93,53) →(93,54) →(94,55) →(95,56) →(95,57) →(95,58) →(95,59) →(95,60) → (95,61) →(95,62) →(96,61) →(97,60) →(98,59) →(99,58) →(100,57) →(101,58) →(102,58) →(103,59) →(104,59) →(105,60) = 100
Diagonal:
(20,20) →(21,21) →(22,22) →(22,23) →(21,24) →(21,25) →(20,26) →(20,27) → (20,28) →(20,29) →(21,30) →(22,31) →(22,32) →(22,33) →(22,34) →(22,35) → (22,36) →(23,37) →(24,38) →(24,39) →(25,40) →(25,41) →(25,42) →(26,43) → (27,44) →(27,45) →(27,46) →(28,47) →(27,48) →(28,49) →(29,50) →(30,51) → (31,52) →(32,53) →(33,54) →(34,55) →(35,56) →(36,57) →(37,57) →(38,57) → (39,57) →(40,57) →(41,58) →(42,59) →(43,59) →(44,60) →(45,60) →(46,60) → (47,60) →(48,60) →(49,60) →(50,59) →(51,58) →(52,57) →(53,56) →(54,55) → (55,54) →(56,53) →(57,52) →(58,52) →(59,52) →(60,52) →(61,52) →(62,52) → (63,52) →(64,52) →(65,52) →(66,52) →(67,53) →(68,52) →(69,51) →(70,51) → (71,52) →(71,53) →(72,54) →(73,55) →(74,56) →(75,57) →(76,58) →(77,58) → (78,59) →(79,60) →(80,59) →(81,58) →(82,57) →(83,56) →(84,55) →(85,54) → (86,53) →(87,52) →(88,51) →(89,50) →(90,49) →(91,48) →(92,49) →(93,50) → (93,51) →(93,52) →(93,53) →(93,54) →(94,55) →(95,56) →(96,57) →(96,58) → (95,59) →(95,60) →(95,61) →(95,62) →(96,61) →(97,60) →(98,59) →(99,58) → (100,57) →(101,58) →(102,58) →(103,59) →(104,59) →(105,60) = 118

Kernel Size 5

Euclidean:
(12,12) →(13,12) →(14,11) →(15,10) →(16,9) →(17,8) →(18,7) →(19,8) → (20,9) →(21,8) →(22,7) →(23,6) →(24,5) →(25,4) →(26,3) →(27,4) → (28,3) →(29,2) →(30,3) →(31,2) →(32,3) →(33,4) →(34,5) →(35,6) → (36,7) →(37,8) →(38,9) →(39,10) →(40,9) →(41,10) →(42,11) →(43,12) → (44,13) →(45,14) →(46,15) →(47,16) →(48,17) →(49,18) →(50,19) →(51,20) → (52,21) →(53,22) →(54,23) →(55,24) →(56,25) →(57,26) →(58,26) →(59,25) → (60,24) →(61,23) →(62,22) →(63,21) →(64,20) →(65,19) →(66,18) →(67,17) → (68,16) →(69,15) →(70,14) →(71,13) →(72,12) →(73,11) →(73,12) →(73,13) → (73,14) →(73,15) →(73,16) →(73,17) →(73,18) →(73,19) →(73,20) →(73,21) → (73,22) →(73,23) →(73,24) →(73,25) →(73,26) →(73,27) →(73,28) →(73,29) → (73,30) →(73,31) →(73,32) →(73,33) →(73,34) →(73,35) →(73,36) →(73,37) → (73,38) →(73,39) →(72,40) →(71,40) →(70,40) →(69,41) →(68,41) →(67,41) → (66,41) →(65,41) →(64,41) →(63,40) →(64,39) →(65,38) →(64,37) →(63,36) = 104

Manhattan:
(12,12) →(13,12) →(14,11) →(15,10) →(16,9) →(17,8) →(18,7) →(19,8) →(20,9) →(21,10) →(22,11) →(23,10) →(24,11) →(25,10) →(26,9) →(27,8) →(28,7) →(29,6) →(30,7) →(31,6) →(32,7) →(33,8) →(34,9) →(35,10) →(36,11) →(37,10) →(38,11) →(39,12) →(40,13) →(41,14) →(42,15) →(43,16) →(44,15) →(45,16) →(46,17) →(47,18) →(48,19) →(49,20) →(50,21) →(51,22) →(52,23) →(53,24) →(54,25) →(55,26) →(56,27) →(57,28) →(58,27) →(59,26) →(60,25) →(61,24) →(62,23) →(63,22) →(64,21) →(65,20) →(66,19) →(67,18) →(68,17) →(69,16) →(70,15) →(71,14) →(72,13) →(73,12) →(73,13) →(73,14) →(73,15) →(73,16) →(73,17) →(73,18) →(73,19) →(73,20) →(73,21) →(73,22) →(73,23) →(73,24) →(73,25) →(73,26) →(73,27) →(73,28) →(73,29) →(73,30) →(73,31) →(73,32) →(73,33) →(73,34) →(73,35) →(73,36) →(73,37) →(73,38) →(73,39) →(72,40) →(71,40) →(70,40) →(69,41) →(68,41) →(67,40) →(66,40) →(65,40) →(64,40) →(63,40) →(64,39) →(65,38) →(64,37) →(63,36) = 92

Diagonal:
(12,12) →(13,12) →(14,11) →(15,10) →(16,9) →(17,8) →(18,7) →(19,8) → (20,9) →(21,10) →(22,11) →(23,11) →(24,11) →(25,10) →(26,9) →(27,9) → (28,8) →(29,7) →(30,7) →(31,7) →(32,8) →(33,9) →(34,10) →(35,10) → (36,11) →(37,11) →(38,12) →(39,11) →(40,12) →(41,13) →(42,14) →(43,15) → (44,16) →(45,17) →(46,18) →(47,19) →(48,20) →(49,21) →(50,22) →(51,23) → (52,24) →(53,25) →(54,26) →(55,27) →(56,28) →(57,27) →(58,27) →(59,26) → (60,25) →(61,24) →(62,23) →(63,22) →(64,21) →(65,20) →(66,19) →(67,18) → (68,17) →(69,16) →(70,15) →(71,14) →(72,13) →(73,12) →(73,13) →(73,14) → (73,15) →(73,16) →(73,17) →(73,18) →(73,19) →(73,20) →(73,21) →(73,22) → (73,23) →(73,24) →(73,25) →(73,26) →(73,27) →(73,28) →(73,29) →(73,30) → (73,31) →(73,32) →(73,33) →(73,34) →(73,35) →(73,36) →(73,37) →(73,38) → (73,39) →(72,40) →(71,40) →(70,40) →(69,41) →(68,41) →(67,40) →(66,40) → (65,39) →(64,40) →(63,40) →(64,39) →(65,38) →(64,37) →(63,36) = 103

Kernel Size 7

Euclidean:
(8,8) →(9,8) →(10,7) →(11,6) →(12,5) →(13,5) →(14,6) →(15,7) → (16,6) →(17,5) →(18,4) →(19,3) →(20,2) →(21,1) →(22,2) →(23,3) → (24,4) →(25,5) →(26,6) →(27,7) →(28,8) →(29,7) →(30,8) →(31,9) → (32,10) →(33,11) →(34,12) →(35,13) →(36,14) →(37,15) →(38,16) →(39,17) → (40,18) →(41,19) →(42,18) →(43,17) →(44,16) →(45,15) →(46,14) →(47,13) → (48,12) →(49,11) →(50,10) →(51,9) →(52,8) →(52,9) →(52,10) →(52,11) → (52,12) →(52,13) →(52,14) →(52,15) →(52,16) →(52,17) →(52,18) →(52,19) → (52,20) →(52,21) →(52,22) →(52,23) →(52,24) →(52,25) →(52,26) →(52,27) → (52,28) →(51,28)

→(50,28) →(49,29) →(48,29) →(47,29) →(46,29) →(45,29) → (46,28) →(47,27) →(46,26) →(45,25) = 76

Manhattan:

(8,8) →(9,8) →(10,7) →(11,6) →(12,5) →(13,5) →(14,6) →(15,7) → (16,8) →(17,7) →(18,6) →(19,5) →(20,4) →(21,3) →(22,4) →(23,5) → (24,6) →(25,7) →(26,8) →(27,7) →(28,8) →(29,9) →(30,10) →(31,11) → (32,12) →(33,13) →(34,14) →(35,15) →(36,16) →(37,17) →(38,18) →(39,19) → (40,20) →(41,19) →(42,18) →(43,17) →(44,16) →(45,15) →(46,14) →(47,13) → (48,12) →(49,11) →(50,10) →(51,9) →(52,8) →(52,9) →(52,10) →(52,11) → (52,12) →(52,13) →(52,14) →(52,15) →(52,16) →(52,17) →(52,18) →(52,19) → (52,20) →(52,21) →(52,22) →(52,23) →(52,24) →(52,25) →(52,26) →(52,27) → (52,28) →(51,28) →(50,28) →(49,29) →(48,29) →(47,29) →(46,29) →(45,29) → (46,28) →(47,27) →(46,26) →(45,25) = 76

Diagonal:

(8,8) →(9,8) →(10,7) →(11,6) →(12,5) →(13,5) →(14,6) →(15,7) → (16,8) →(17,7) →(18,7) →(19,6) →(20,5) →(21,4) →(22,5) →(23,5) → (24,6) →(25,7) →(26,8) →(27,8) →(28,7) →(29,8) →(30,9) →(31,10) → (32,11) →(33,12) →(34,13) →(35,14) →(36,15) →(37,16) →(38,17) →(39,18) → (40,19) →(41,19) →(42,18) →(43,17) →(44,16) →(45,15) →(46,14) →(47,13) → (48,12) →(49,11) →(50,10) →(51,9) →(52,8) →(52,9) →(52,10) →(52,11) → (52,12) →(52,13) →(52,14) →(52,15) →(52,16) →(52,17) →(52,18) →(52,19) → (52,20) →(52,21) →(52,22) →(52,23) →(52,24) →(52,25) →(52,26) →(52,27) → (52,28) →(51,28) →(50,28) →(49,29) →(48,29) →(47,29) →(46,29) →(45,29) → (46,28) →(47,27) →(46,26) →(45,25) = 76

SOURCE 60,90 DESTINATION 180,210

Kernel 3

Euclidean:

(30,20) →(31,19) →(32,18) →(33,17) →(34,16) →(33,15) →(32,14) →(31,13) → (30,13) →(29,14) →(28,15) →(28,16) →(27,17) →(26,18) →(26,19) →(25,20) → (24,21) →(23,22) →(22,23) →(21,24) →(21,25) →(20,26) →(20,27) →(20,28) → (20,29) →(21,30) →(22,31) →(22,32) →(22,33) →(22,34) →(22,35) →(22,36) → (23,37) →(24,38) →(24,39) →(25,40) →(25,41) →(25,42) →(26,43) →(27,44) → (27,45) →(27,46) →(27,47) →(27,48) →(28,49) →(29,50) →(30,50) →(31,50) → (32,51) →(33,51) →(34,52) →(35,52) →(36,53) →(37,53) →(38,54) →(39,55) → (40,56) →(41,57) →(42,58) →(43,59) →(44,60) →(45,61) →(46,62) →(47,63) → (48,64) →(49,65) →(50,66) →(51,67) →(52,68) →(52,69) →(52,70) →(52,71) → (52,72) →(52,73) →(52,74) →(52,75) →(52,76) →(52,77) →(53,76) →(54,75) → (55,74) →(56,73) →(57,72) →(58,71) →(59,70) →(60,69) →(61,68) →(62,67) → (63,66) →(64,65) →(65,64) →(66,63) →(67,62) →(68,61) →(69,60) →(70,60) = 96

Manhattan:

(30,20) →(31,19) →(32,18) →(33,17) →(34,16) →(33,15) →(32,14) →(31,13) → (30,13) →(29,14) →(28,15) →(28,16) →(27,17) →(26,18) →(26,19) →(25,20) → (24,21) →(23,22) →(22,23) →(21,24) →(21,25) →(20,26) →(20,27) →(20,28) → (20,29) →(21,30) →(22,31) →(22,32) →(22,33) →(22,34) →(22,35) →(22,36) → (23,37) →(24,38) →(24,39) →(25,40) →(25,41) →(25,42) →(26,43) →(27,44) → (27,45) →(27,46) →(27,47) →(27,48) →(28,49) →(29,50) →(30,51) →(31,52) → (32,53) →(33,54) →(34,55) →(35,56) →(36,57) →(37,56) →(38,57) →(39,56) → (40,57) →(41,58) →(42,59) →(43,58) →(44,59) →(45,60) →(46,61) →(47,62) → (48,63) →(49,64) →(50,65) →(51,66) →(52,67) →(52,68) →(52,69) →(52,70) → (52,71) →(52,72) →(52,73) →(52,74) →(52,75) →(52,76) →(52,77) →(53,76) → (54,75)

→(55,74) →(56,73) →(57,72) →(58,71) →(59,70) →(60,69) →(61,68) → (62,67) →(63,66)
→(64,65) →(65,64) →(66,63) →(67,62) →(68,61) →(69,60) → (70,60) = 97
Diagonal:
(30,20) →(31,19) →(32,18) →(33,17) →(34,16) →(33,15) →(32,14) →(31,13) → (30,13)
→(29,14) →(28,15) →(28,16) →(27,17) →(26,18) →(26,19) →(25,20) → (24,21) →(23,22)
→(22,23) →(21,24) →(21,25) →(20,26) →(20,27) →(20,28) → (20,29) →(21,30) →(22,31)
→(22,32) →(22,33) →(22,34) →(22,35) →(22,36) → (23,37) →(24,38) →(24,39) →(25,40)
→(25,41) →(25,42) →(26,43) →(27,44) → (27,45) →(27,46) →(27,47) →(27,48) →(28,49)
→(29,50) →(30,51) →(31,52) → (32,53) →(33,54) →(34,55) →(35,56) →(36,57) →(37,56)
→(38,57) →(39,56) → (40,57) →(41,58) →(42,59) →(43,58) →(44,59) →(45,60) →(46,61)
→(47,62) → (48,63) →(49,64) →(50,65) →(51,66) →(52,67) →(52,68) →(52,69) →(52,70)
→ (52,71) →(52,72) →(52,73) →(52,74) →(52,75) →(52,76) →(52,77) →(53,76) → (54,75)
→(55,74) →(56,73) →(57,72) →(58,71) →(59,70) →(60,69) →(61,68) → (62,67) →(63,66)
→(64,65) →(65,64) →(66,63) →(67,62) →(68,61) →(69,60) → (70,60) = 97

Kernel 5
Euclidean:
(18,12) →(19,11) →(20,10) →(21,9) →(22,8) →(23,7) →(24,6) →(25,5) -¿ (26,4) →(27,3)
→(28,2) →(29,1) →(30,2) →(31,3) →(32,4) →(33,5) → (34,4) →(35,5) →(36,6) →(37,7)
→(38,8) →(39,9) →(40,10) →(41,11) → (42,12) →(43,13) →(44,14) →(45,15) →(46,16)
→(47,17) →(48,18) →(49,19) → (50,20) →(51,21) →(52,22) →(53,23) →(54,24) →(55,25)
→(56,26) →(57,27) → (58,26) →(59,25) →(60,24) →(61,23) →(62,22) →(63,21) →(64,20)
→(65,19) → (66,18) →(67,17) →(68,16) →(69,15) →(70,14) →(71,13) →(72,12) →(73,11)
→ (73,12) →(73,13) →(73,14) →(73,15) →(73,16) →(73,17) →(73,18) →(73,19) → (73,20)
→(73,21) →(73,22) →(73,23) →(73,24) →(73,25) →(73,26) →(73,27) → (73,28) →(73,29)
→(73,30) →(73,31) →(73,32) →(73,33) →(73,34) →(73,35) → (73,36) →(73,37) →(73,38)
→(73,39) →(73,40) →(73,41) →(73,42) →(73,43) → (73,44) →(73,45) →(73,46) →(73,47)
→(73,48) →(73,49) →(73,50) →(73,51) → (73,52) →(73,53) →(73,54) →(73,55) →(73,56)
→(73,57) →(73,58) →(73,59) → (73,60) →(73,61) →(73,62) →(73,63) →(73,64) →(73,65)
→(73,66) →(73,67) → (73,68) →(73,69) →(73,70) →(73,71) →(72,72) →(71,72) →(70,72)
→(69,72) → (68,72) →(67,72) →(66,72) →(65,72) →(64,72) →(63,72) →(62,72) →(61,72)
→ (60,72) →(59,72) →(58,72) →(57,72) →(56,72) →(55,72) →(54,72) →(53,72) → (52,72)
→(51,72) →(50,72) →(49,72) →(48,72) →(47,72) →(46,72) →(45,72) → (44,72) →(43,72)
→(42,72) →(41,72) →(40,72) →(39,72) →(38,72) →(37,72) → (36,72) →(35,72) →(34,72)
→(33,72) →(32,72) →(31,72) →(30,72) →(29,72) → (28,72) →(27,72) →(26,72) →(25,72)
→(24,72) →(23,72) →(22,72) →(21,72) → (20,72) →(19,72) →(18,72) →(17,72) →(16,72)
→(15,72) →(14,72) →(13,72) → (12,72) →(11,72) →(10,72) →(9,72) →(10,71) →(11,70)
→(12,69) →(13,68) → (14,67) →(15,66) →(16,65) →(17,64) →(18,63) →(19,62) →(20,61)
→(21,60) → (22,59) →(23,58) →(24,57) →(25,56) →(25,55) →(26,54) →(27,53) →(27,52)
→ (28,51) →(29,50) →(30,49) →(31,48) →(32,47) →(33,46) →(34,45) →(35,44) → (36,43)
→(37,42) →(38,41) →(39,40) →(40,39) →(41,38) →(42,37) →(42,36) = 216
Manhattan:
(18,12) →(19,11) →(20,10) →(21,11) →(22,10) →(23,11) →(24,10) →(25,9) → (26,8)
→(27,9) →(28,8) →(29,7) →(30,6) →(31,7) →(32,8) →(33,9) → (34,10) →(35,9) →(36,10)
→(37,11) →(38,12) →(39,11) →(40,12) →(41,13) → (42,14) →(43,15) →(44,16) →(45,17)
→(46,18) →(47,19) →(48,20) →(49,21) → (50,22) →(51,23) →(52,24) →(53,25) →(54,26)
→(55,27) →(56,28) →(57,27) → (58,27) →(59,26) →(60,25) →(61,24) →(62,23) →(63,22)
→(64,21) →(65,20) → (66,19) →(67,18) →(68,17) →(69,16) →(70,15) →(71,14) →(72,13)
→(73,12) → (73,13) →(73,14) →(73,15) →(73,16) →(73,17) →(73,18) →(73,19) →(73,20)

→ (73,21) →(73,22) →(73,23) →(73,24) →(73,25) →(73,26) →(73,27) →(73,28) → (73,29) →(73,30) →(73,31) →(73,32) →(73,33) →(73,34) →(73,35) →(73,36) → (73,37) →(73,38) →(73,39) →(73,40) →(73,41) →(73,42) →(73,43) →(73,44) → (73,45) →(73,46) →(73,47) →(73,48) →(73,49) →(73,50) →(73,51) →(73,52) → (73,53) →(73,54) →(73,55) →(73,56) →(73,57) →(73,58) →(73,59) →(73,60) → (73,61) →(73,62) →(73,63) →(73,64) →(73,65) →(73,66) →(73,67) →(73,68) → (73,69) →(73,70) →(73,71) →(72,72) →(71,72) →(70,72) →(69,72) →(68,72) → (67,72) →(66,72) →(65,72) →(64,72) →(63,72) →(62,72) →(61,72) →(60,72) → (59,72) →(58,72) →(57,72) →(56,72) →(55,72) →(54,72) →(53,72) →(52,72) → (51,72) →(50,72) →(49,72) →(48,72) →(47,72) →(46,72) →(45,72) →(44,72) → (43,72) →(42,72) →(41,72) →(40,72) →(39,72) →(38,72) →(37,72) →(36,72) → (35,72) →(34,72) →(33,72) →(32,72) →(31,72) →(30,72) →(29,72) →(28,72) → (27,72) →(26,72) →(25,72) →(24,72) →(23,72) →(22,72) →(21,72) →(20,72) → (19,72) →(18,72) →(17,72) →(16,72) →(15,72) →(14,72) →(13,72) →(12,72) → (11,72) →(10,72) →(9,72) →(10,71) →(11,70) →(12,69) →(13,68) →(14,67) → (15,66) →(16,65) →(17,64) →(18,63) →(19,62) →(20,61) →(21,60) →(22,59) → (23,58) →(24,57) →(25,56) →(25,55) →(26,54) →(27,53) →(27,52) →(28,51) → (29,50) →(30,49) →(31,48) →(32,47) →(33,46) →(34,45) →(35,44) →(36,43) → (37,42) →(38,41) →(39,40) →(40,39) →(41,38) →(42,37) →(42,36) = 215

Diagonal:

(18,12) →(19,11) →(20,10) →(21,11) →(22,10) →(23,11) →(24,10) →(25,9) → (26,8) →(27,9) →(28,8) →(29,7) →(30,6) →(31,7) →(32,8) →(33,9) → (34,10) →(35,9) →(36,10) →(37,11) →(38,12) →(39,11) →(40,12) →(41,13) → (42,14) →(43,15) →(44,16) →(45,17) →(46,18) →(47,19) →(48,20) →(49,21) → (50,22) →(50,23) →(51,24) →(52,25) →(53,26) →(54,27) →(55,28) →(56,29) → (57,28) →(58,27) →(59,26) →(60,25) →(61,24) →(62,23) →(63,22) →(64,21) → (65,20) →(66,19) →(67,18) →(68,17) →(69,16) →(70,15) →(71,14) →(72,13) → (73,12) →(73,13) →(73,14) →(73,15) →(73,16) →(73,17) →(73,18) →(73,19) → (73,20) →(73,21) →(73,22) →(73,23) →(73,24) →(73,25) →(73,26) →(73,27) → (73,28) →(73,29) →(73,30) →(73,31) →(73,32) →(73,33) →(73,34) →(73,35) → (73,36) →(73,37) →(73,38) →(73,39) →(73,40) →(73,41) →(73,42) →(73,43) → (73,44) →(73,45) →(73,46) →(73,47) →(73,48) →(73,49) →(73,50) →(73,51) → (73,52) →(73,53) →(73,54) →(73,55) →(73,56) →(73,57) →(73,58) →(73,59) → (73,60) →(73,61) →(73,62) →(73,63) →(73,64) →(73,65) →(73,66) →(73,67) → (73,68) →(73,69) →(73,70) →(73,71) →(72,72) →(71,72) →(70,72) →(69,72) → (68,72) →(67,72) →(66,72) →(65,72) →(64,72) →(63,72) →(62,72) →(61,72) → (60,72) →(59,72) →(58,72) →(57,72) →(56,72) →(55,72) →(54,72) →(53,72) → (52,72) →(51,72) →(50,72) →(49,72) →(48,72) →(47,72) →(46,72) →(45,72) → (44,72) →(43,72) →(42,72) →(41,72) →(40,72) →(39,72) →(38,72) →(37,72) → (36,72) →(35,72) →(34,72) →(33,72) →(32,72) →(31,72) →(30,72) →(29,72) → (28,72) →(27,72) →(26,72) →(25,72) →(24,72) →(23,72) →(22,72) →(21,72) → (20,72) →(19,72) →(18,72) →(17,72) →(16,72) →(15,72) →(14,72) →(13,72) → (12,72) →(11,72) →(12,71) →(13,70) →(14,69) →(15,68) →(16,67) →(17,66) → (17,65) →(18,64) →(19,63) →(20,62) →(21,61) →(22,60) →(23,59) →(23,58) → (24,57) →(25,56) →(25,55) →(26,54) →(27,53) →(27,52) →(28,51) →(29,50) → (30,49) →(31,48) →(32,47) →(33,46) →(34,45) →(35,44) →(36,43) →(37,42) → (38,41) →(39,40) →(40,39) →(41,38) →(42,37) →(42,36) = 214

Kernel 7

Euclidean:

(12,8) →(13,7) →(14,6) →(15,7) →(16,8) →(17,7) →(18,6) →(19,5) → (20,4) →(21,3) →(22,4) →(23,5) →(24,6) →(25,7) →(26,8) →(27,7) → (28,8) →(29,9) →(30,10) →(31,11) →(32,12) →(33,13) →(34,14) →(35,15) → (36,16) →(36,17) →(37,18) →(36,19) →(35,20) →(34,21) →(33,22) →(32,23) → (31,24) →(30,25) = 34

Manhattan:
(12,8) →(11,7) →(10,8) →(9,9) →(8,9) →(8,10) →(8,11) →(8,12) → (8,13) →(8,14) →(8,15) →(9,16) →(9,17) →(10,18) →(11,19) →(11,20) → (11,21) →(12,22) →(13,23) →(14,22) →(15,23) →(16,24) →(17,23) →(18,24) → (19,25) →(20,26) →(21,27) →(22,28) →(22,29) →(22,30) →(22,31) →(23,30) → (24,29) →(25,28) →(26,27) →(27,26) →(28,25) →(29,24) →(30,25) = 39
Diagonal:
(12,8) →(11,7) →(10,8) →(9,9) →(8,9) →(8,10) →(8,11) →(8,12) -¿ (8,13) →(8,14) →(8,15) →(9,16) →(9,17) →(10,18) →(11,19) →(11,20) → (11,21) →(12,22) →(13,23) →(14,23) →(15,24) →(16,24) →(17,24) →(18,24) → (19,25) →(20,26) →(21,27) →(22,28) →(22,29) →(22,30) →(22,31) →(23,30) → (24,29) →(25,28) →(26,27) →(27,26) →(28,25) →(29,24) →(30,25) = 39

SOURCE 70,300 DESTINATION 315,180

Kernel 3
Euclidean:
(23,100) →(24,99) →(25,98) →(26,97) →(27,96) →(28,95) →(29,94) →(30,93) → (31,92) →(32,91) →(33,90) →(34,89) →(35,88) →(36,87) →(37,86) →(38,85) → (39,84) →(40,83) →(41,82) →(42,81) →(43,80) →(44,79) →(45,78) →(46,77) → (47,76) →(48,75) →(49,74) →(50,73) →(51,72) →(52,71) →(53,70) →(52,69) → (53,68) →(52,67) →(53,66) →(52,65) →(53,64) →(52,63) →(53,62) →(52,61) → (53,60) →(54,59) →(53,58) →(54,57) →(53,56) →(54,55) →(55,54) →(56,53) → (57,52) →(58,51) →(59,50) →(60,49) →(61,48) →(62,47) →(63,48) →(64,47) → (65,48) →(66,47) →(67,48) →(68,47) →(69,48) →(70,47) →(71,48) →(72,47) → (73,46) →(74,45) →(75,44) →(76,43) →(77,42) →(78,41) →(79,40) →(80,39) → (81,38) →(82,37) →(82,38) →(83,39) →(84,40) →(85,41) →(86,42) →(87,43) → (88,44) →(89,45) →(90,46) →(91,47) →(91,48) →(91,49) →(92,50) →(93,51) → (93,52) →(93,53) →(93,54) →(93,55) →(93,56) →(93,57) →(93,58) →(94,59) → (94,60) →(94,61) →(95,62) →(96,61) →(97,60) →(98,59) →(99,58) →(100,57) → (101,58) →(102,58) →(103,57) →(104,58) →(104,59) →(105,60) = 110
Euclidean:
(23,100) →(24,99) →(25,98) →(26,97) →(27,96) →(28,95) →(29,94) →(30,93) → (31,92) →(32,91) →(33,90) →(34,89) →(35,88) →(36,87) →(37,86) →(38,85) → (39,84) →(40,83) →(41,82) →(42,81) →(43,80) →(44,79) →(45,78) →(46,77) → (47,76) →(48,75) →(49,74) →(50,73) →(51,72) →(52,71) →(53,70) →(53,69) → (53,68) →(53,67) →(53,66) →(53,65) →(53,64) →(53,63) →(53,62) →(53,61) → (54,60) →(55,59) →(56,58) →(57,57) →(57,56) →(57,55) →(58,55) →(59,54) → (60,53) →(61,52) →(62,51) →(63,50) →(64,49) →(65,48) →(66,47) →(67,46) → (68,45) →(69,44) →(70,43) →(71,42) →(72,41) →(73,40) →(74,39) →(75,38) → (76,37) →(77,36) →(78,35) →(79,34) →(80,35) →(81,34) →(82,35) →(83,36) → (84,37) →(85,38) →(86,39) →(87,40) →(88,41) →(89,42) →(90,43) →(91,44) → (92,45) →(92,46) →(92,47) →(92,48) →(93,49) →(93,50) →(93,51) →(93,52) → (93,53) →(93,54) →(94,55) →(95,56) →(95,57) →(95,58) →(95,59) →(95,60) → (95,61) →(96,62) →(96,63) →(97,64) →(98,65) →(99,64) →(100,63) →(101,62) →(102,61) →(103,60) →(104,59) →(105,60) = 108
Manhattan:
(23,100) →(24,99) →(25,98) →(26,97) →(27,96) →(28,95) →(29,94) →(30,93) → (31,92) →(32,91) →(33,90) →(34,89) →(35,88) →(36,87) →(37,86) →(38,85) → (39,84) →(40,83) →(41,82) →(42,81) →(43,80) →(44,79) →(45,78) →(46,77) → (47,76) →(48,75) →(49,74)

→(50,73) →(51,72) →(52,71) →(53,70) →(53,69) → (53,68) →(53,67) →(53,66) →(53,65) →(53,64) →(52,63) →(51,62) →(50,61) → (49,60) →(50,59) →(51,58) →(52,57) →(53,56) →(54,55) →(55,54) →(56,53) → (57,52) →(58,53) →(59,52) →(60,53) →(61,52) →(62,53) →(63,52) →(64,51) → (65,52) →(66,51) →(67,52) →(68,51) →(69,50) →(70,51) →(71,52) →(71,53) → (72,54) →(73,55) →(74,56) →(75,57) →(76,58) →(77,57) →(78,58) →(79,59) → (80,58) →(81,57) →(82,56) →(83,55) →(84,54) →(85,53) →(86,52) →(87,51) → (88,50) →(89,49) →(90,48) →(91,47) →(92,48) →(93,49) →(93,50) →(93,51) → (93,52) →(93,53) →(93,54) →(94,55) →(95,56) →(95,57) →(95,58) →(95,59) → (95,60) →(95,61) →(95,62) →(96,61) →(97,60) →(98,59) →(99,58) →(100,57) → (101,58) →(102,58) →(103,59) →(104,59) →(105,60) = 109

Diagonal:

(23,100) →(24,99) →(25,98) →(26,97) →(27,96) →(28,95) →(29,94) →(30,93) → (31,92) →(32,91) →(33,90) →(34,89) →(35,88) →(36,87) →(37,86) →(38,85) → (39,84) →(40,83) →(41,82) →(42,81) →(43,80) →(44,79) →(45,78) →(46,77) → (47,76) →(48,75) →(49,74) →(50,73) →(51,72) →(52,71) →(53,70) →(53,69) → (53,68) →(53,67) →(53,66) →(53,65) →(53,64) →(53,63) →(53,62) →(53,61) → (54,60) →(53,59) →(54,58) →(53,57) →(54,56) →(55,55) →(56,56) →(57,55) → (58,55) →(58,54) →(59,53) →(60,53) →(61,53) →(62,53) →(63,52) →(64,52) → (65,52) →(66,52) →(67,53) →(68,52) →(69,51) →(70,51) →(71,52) →(71,53) → (72,54) →(73,55) →(74,56) →(75,57) →(76,58) →(77,58) →(78,59) →(79,60) → (80,59) →(81,58) →(82,57) →(83,56) →(84,55) →(85,54) →(86,53) →(87,52) → (88,51) →(89,50) →(90,49) →(91,48) →(92,49) →(93,50) →(93,51) →(93,52) → (93,53) →(93,54) →(94,55) →(95,56) →(96,57) →(96,58) →(95,59) →(95,60) → (95,61) →(95,62) →(96,61) →(97,60) →(98,59) →(99,58) →(100,57) →(101,58) →(102,58) →(103,59) →(104,59) →(105,60) = 108

Kernel Size 5

Euclidean:

(14,60) →(15,59) →(16,58) →(17,57) →(18,56) →(19,55) →(20,54) →(21,53) → (22,52) →(23,51) →(24,50) →(25,49) →(26,48) →(27,47) →(28,46) →(29,45) -¿ (30,44) →(31,43) →(32,42) →(31,41) →(30,40) →(31,39) →(30,38) →(31,37) → (31,36) →(32,35) →(31,34) →(32,33) →(33,32) →(34,32) →(34,31) →(35,30) → (36,29) →(37,28) →(38,27) →(39,26) →(40,25) →(41,24) →(42,23) →(43,22) → (44,21) →(45,20) →(46,19) →(47,18) →(48,17) →(49,18) →(50,19) →(51,20) → (52,21) →(53,22) →(54,23) →(55,24) →(56,25) →(57,26) →(58,27) →(59,26) → (60,25) →(61,24) →(62,23) →(63,22) →(64,21) →(65,20) →(66,19) →(67,18) → (68,17) →(69,16) →(70,15) →(71,14) →(72,13) →(73,12) →(73,13) →(73,14) → (73,15) →(73,16) →(73,17) →(73,18) →(73,19) →(73,20) →(73,21) →(73,22) → (73,23) →(73,24) →(73,25) →(73,26) →(73,27) →(73,28) →(73,29) →(73,30) → (73,31) →(73,32) →(73,33) →(73,34) →(73,35) →(73,36) →(73,37) →(73,38) → (73,39) →(72,40) →(71,40) →(70,40) →(69,41) →(68,41) →(67,41) →(66,41) → (65,41) →(64,41) →(63,40) →(64,39) →(65,38) →(64,37) →(63,36) = 111

Manhattan:

(14,60) →(15,59) →(16,58) →(17,57) →(18,56) →(19,55) →(20,54) →(21,53) → (22,52) →(23,51) →(24,50) →(25,49) →(26,48) →(27,47) →(28,46) →(29,45) → (30,44) →(31,43) →(32,42) →(31,41) →(31,40) →(31,39) →(31,38) →(30,37) → (29,36) →(30,35) →(31,34) →(32,33) →(33,32) →(34,32) →(34,31) →(35,30) → (36,31) →(37,30) →(38,31) →(39,30) →(40,31) →(41,30) →(42,29) →(43,28) → (44,27) →(45,26) →(46,25) →(47,24) →(48,23) →(49,22) →(50,21) →(51,22) → (52,23) →(53,24) →(54,25) →(55,26) →(56,27) →(57,28) →(58,27) →(59,26) → (60,25) →(61,24) →(62,23) →(63,22) →(64,21) →(65,20) →(66,19) →(67,18) → (68,17) →(69,16) →(70,15) →(71,14) →(72,13) →(73,12) →(73,13) →(73,14)

→ (73,15) →(73,16) →(73,17) →(73,18) →(73,19) →(73,20) →(73,21) →(73,22) → (73,23) →(73,24) →(73,25) →(73,26) →(73,27) →(73,28) →(73,29) →(73,30) → (73,31) →(73,32) →(73,33) →(73,34) →(73,35) →(73,36) →(73,37) →(73,38) → (73,39) →(72,40) →(71,40) →(70,40) →(69,41) →(68,41) →(67,40) →(66,40) → (65,40) →(64,40) →(63,40) →(64,39) →(65,38) →(64,37) →(63,36) = 111

Diagonal:

(14,60) →(15,59) →(16,58) →(17,57) →(18,56) →(19,55) →(20,54) →(21,53) → (22,52) →(23,51) →(24,50) →(25,49) →(26,48) →(27,47) →(28,46) →(29,45) → (30,44) →(31,43) →(32,42) →(31,41) →(31,40) →(31,39) →(31,38) →(31,37) → (31,36) →(32,35) →(31,34) →(32,33) →(33,32) →(34,32) →(34,31) →(35,31) → (36,31) →(37,31) →(38,31) →(39,30) →(40,31) →(41,30) →(42,30) →(43,29) → (44,28) →(45,27) →(46,26) →(47,25) →(48,24) →(49,23) →(50,22) →(51,23) → (52,24) →(53,25) →(54,26) →(55,27) →(56,28) →(57,27) →(58,27) →(59,26) → (60,25) →(61,24) →(62,23) →(63,22) →(64,21) →(65,20) →(66,19) →(67,18) → (68,17) →(69,16) →(70,15) →(71,14) →(72,13) →(73,12) →(73,13) →(73,14) → (73,15) →(73,16) →(73,17) →(73,18) →(73,19) →(73,20) →(73,21) →(73,22) → (73,23) →(73,24) →(73,25) →(73,26) →(73,27) →(73,28) →(73,29) →(73,30) → (73,31) →(73,32) →(73,33) →(73,34) →(73,35) →(73,36) →(73,37) →(73,38) → (73,39) →(72,40) →(71,40) →(70,40) →(69,41) →(68,41) →(67,40) →(66,40) → (65,39) →(64,40) →(63,40) →(64,39) →(65,38) →(64,37) →(63,36) = 111

Kernel Size 7

Euclidean:

(10,42) →(11,41) →(12,40) →(13,39) →(14,38) →(15,37) →(16,36) →(17,35) → (18,34) →(19,33) →(20,32) →(21,31) →(22,30) →(21,29) →(22,28) →(21,27) → (22,26) →(22,25) →(21,24) →(22,23) →(23,22) →(24,21) →(25,20) →(26,19) → (27,18) →(28,17) →(29,16) →(30,15) →(31,14) →(32,13) →(33,12) →(34,13) → (35,14) →(36,15) →(37,16) →(38,17) →(39,18) →(40,19) →(41,18) →(42,17) → (43,16) →(44,15) →(45,14) →(46,13) →(47,12) →(48,11) →(49,10) →(50,9) → (51,8) →(52,7) →(52,8) →(52,9) →(52,10) →(52,11) →(52,12) →(52,13) → (52,14) →(52,15) →(52,16) →(52,17) →(52,18) →(52,19) →(52,20) →(52,21) → (52,22) →(52,23) →(52,24) →(52,25) →(52,26) →(52,27) →(52,28) →(51,28) → (50,28) →(49,29) →(48,29) →(47,29) →(46,29) →(45,29) →(46,28) →(47,27) → (46,26) →(45,25) = 82

Manhattan:

(10,42) →(11,41) →(12,40) →(13,39) →(14,38) →(15,37) →(16,36) →(17,35) → (18,34) →(19,33) →(20,32) →(21,31) →(22,30) →(22,29) →(22,28) →(22,27) → (21,26) →(20,25) →(21,24) →(22,23) →(23,22) →(24,21) →(25,20) →(26,19) → (27,18) →(28,17) →(29,16) →(30,15) →(31,14) →(32,13) →(33,14) →(34,13) → (35,14) →(36,15) →(37,16) →(38,17) →(39,18) →(40,19) →(41,19) →(42,18) → (43,17) →(44,16) →(45,15) →(46,14) →(47,13) →(48,12) →(49,11) →(50,10) → (51,9) →(52,8) →(52,9) →(52,10) →(52,11) →(52,12) →(52,13) →(52,14) → (52,15) →(52,16) →(52,17) →(52,18) →(52,19) →(52,20) →(52,21) →(52,22) → (52,23) →(52,24) →(52,25) →(52,26) →(52,27) →(52,28) →(51,28) →(50,28) → (49,29) →(48,29) →(47,29) →(46,29) →(45,29) →(46,28) →(47,27) →(46,26) → (45,25) = 81

Diagonal:

(10,42) →(11,41) →(12,40) →(13,39) →(14,38) →(15,37) →(16,36) →(17,35) → (18,34) →(19,33) →(20,32) →(21,31) →(22,30) →(22,29) →(22,28) →(22,27) → (22,26) →(22,25) →(21,24) →(22,23) →(23,22) →(24,21) →(25,20) →(26,19) → (27,18) →(28,17) →(29,16) →(30,15) →(31,14) →(32,13) →(33,14) →(34,13) → (35,14) →(36,15) →(37,16) →(38,17) →(39,18) →(40,19) →(41,19) →(42,18) → (43,17) →(44,16) →(45,15) →(46,14) →(47,13)

→(48,12) →(49,11) →(50,10) → (51,9) →(52,8) →(52,9) →(52,10) →(52,11) →(52,12) →(52,13) →(52,14) → (52,15) →(52,16) →(52,17) →(52,18) →(52,19) →(52,20) →(52,21) →(52,22) → (52,23) →(52,24) →(52,25) →(52,26) →(52,27) →(52,28) →(51,28) →(50,28) → (49,29) →(48,29) →(47,29) →(46,29) →(45,29) →(46,28) →(47,27) →(46,26) → (45,25) = 81

SOURCE 70,300 DESTINATION 210,300

Kernel 3
Euclidean:
(23,100) →(24,99) →(25,98) →(26,97) →(27,96) →(28,95) →(29,94) →(30,93) → (31,92) →(32,91) →(33,90) →(34,89) →(35,88) →(36,88) →(37,89) →(38,90) → (39,91) →(40,92) →(41,91) →(42,92) →(43,93) →(44,94) →(45,95) →(46,94) → (47,93) →(48,92) →(49,91) →(50,90) →(51,89) →(52,88) →(53,87) →(54,88) → (55,87) →(56,86) →(57,87) →(58,88) →(59,89) →(60,90) →(61,91) →(62,92) → (63,93) →(64,94) →(65,95) →(66,96) →(67,97) →(68,98) →(69,99) →(70,100) = 48
Manhattan:
(23,100) →(24,99) →(25,98) →(26,97) →(27,96) →(28,95) →(29,94) →(30,93) → (31,92) →(32,91) →(33,90) →(34,89) →(35,88) →(36,89) →(37,90) →(38,91) → (39,92) →(40,92) →(41,93) →(42,94) →(43,95) →(44,94) →(45,95) →(46,94) → (47,93) →(48,92) →(49,91) →(50,90) →(51,89) →(52,88) →(53,87) →(54,88) → (55,87) →(56,86) →(57,87) →(58,88) →(59,89) →(60,90) →(61,91) →(62,92) → (63,93) →(64,94) →(65,95) →(66,96) →(67,97) →(68,98) →(69,99) →(70,100) = 48
Diagonal:
(23,100) →(22,101) →(21,102) →(20,103) →(19,104) →(19,105) →(20,106) →(21,105) →(22,104) →(23,103) →(24,102) →(25,101) →(26,100) →(27,99) →(28,98) →(29,97) →(30,96) →(31,95) →(32,94) →(33,93) →(34,92) →(35,91) →(36,90) →(37,91) → (38,92) →(39,92) →(40,92) →(41,93) →(42,94) →(43,95) →(44,94) →(45,95) → (46,94) →(47,93) →(48,92) →(49,91) →(50,90) →(51,89) →(52,88) →(53,87) → (54,88) →(55,87) →(56,86) →(57,87) →(58,88) →(59,89) →(60,90) →(61,91) → (62,92) →(63,93) →(64,94) →(65,95) →(66,96) →(67,97) →(68,98) →(69,99) → (70,100) = 49

Kernel Size 5
Euclidean:
(14,60) →(15,59) →(16,58) →(17,57) →(18,56) →(19,55) →(20,54) →(21,53) → (20,53) →(19,53) →(18,53) →(17,53) →(16,53) →(15,53) →(14,53) →(13,53) → (12,52) →(11,53) →(10,54) →(9,55) →(9,56) →(9,57) →(9,58) →(9,59) → (10,60) →(10,61) →(10,62) →(10,63) →(10,64) →(9,65) →(9,66) →(9,67) → (9,68) →(8,69) →(7,70) →(7,71) →(7,72) →(8,72) →(9,72) →(10,72) → (11,72) →(12,72) →(13,72) →(14,72) →(15,72) →(16,72) →(17,72) →(18,72) → (19,72) →(20,72) →(21,72) →(22,72) →(23,72) →(24,72) →(25,72) →(26,72) → (27,72) →(28,72) →(29,71) →(30,70) →(31,69) →(32,68) →(33,67) →(34,66) → (35,65) →(36,64) →(37,63) →(38,62) →(39,61) →(40,60) →(41,59) →(42,60) = 72
Manhattan:
(14,60) →(15,59) →(16,58) →(17,57) →(18,56) →(19,55) →(20,54) →(21,53) → (20,53) →(19,53) →(18,53) →(17,53) →(16,53) →(15,53) →(14,53) →(13,53) → (12,52) →(11,53) →(10,54) →(9,55) →(9,56) →(9,57) →(9,58) →(10,59) → (10,60) →(10,61) →(10,62) →(10,63) →(10,64) →(9,65) →(9,66) →(9,67) → (9,68) →(8,69) →(7,70) →(7,71) →(7,72) →(8,72) →(9,72) →(10,72) → (11,72) →(12,72) →(13,72) →(14,72) →(15,72) →(16,72)

→(17,72) →(18,72) → (19,72) →(20,72) →(21,72) →(22,72) →(23,72) →(24,72) →(25,72) →(26,72) → (27,72) →(28,72) →(29,71) →(30,70) →(31,69) →(32,68) →(33,67) →(34,66) → (35,65) →(36,64) →(37,63) →(38,62) →(39,61) →(40,60) →(41,59) →(42,60) = 72
Diagonal:
(14,60) →(13,60) →(12,61) →(11,62) →(11,63) →(10,64) →(9,65) →(9,66) → (9,67) →(9,68) →(8,69) →(7,70) →(7,71) →(7,72) →(8,72) →(9,72) → (10,72) →(11,72) →(12,72) →(13,72) →(14,72) →(15,72) →(16,72) →(17,72) → (18,72) →(19,72) →(20,72) →(21,72) →(22,72) →(23,72) →(24,72) →(25,72) → (26,72) →(27,72) →(28,72) →(29,71) →(30,70) →(31,69) →(32,68) →(33,67) → (34,66) →(35,65) →(36,64) →(37,63) →(38,62) →(39,61) →(40,60) →(41,59) → (42,60) = 49

Kernel Size 7
Euclidean:
(10,42) →(11,41) →(12,40) →(13,39) →(14,38) →(15,37) →(16,38) →(16,39) → (17,40) →(18,39) →(19,40) →(20,40) →(21,39) →(22,38) →(23,37) →(24,36) → (25,37) →(26,38) →(27,39) →(28,40) →(29,41) →(30,42) = 22
Manhattan:
(10,42) →(11,41) →(12,40) →(13,39) →(14,38) →(15,37) →(16,38) →(16,39) → (17,40) →(18,39) →(19,40) →(20,40) →(21,39) →(22,38) →(23,37) →(24,36) → (25,37) →(26,38) →(27,39) →(28,40) →(29,41) →(30,42) = 22
Diagonal:
(10,42) →(11,41) →(12,40) →(13,39) →(14,38) →(15,37) →(16,38) →(16,39) → (17,40) →(18,39) →(19,40) →(20,40) →(21,39) →(22,38) →(23,37) →(24,36) → (25,37) →(26,38) →(27,39) →(28,40) →(29,41) →(30,42) = 22

SOURCE 60,70 DESTINATION 300,140

Kernel 3
Euclidean:
(20,23) →(21,22) →(22,21) →(23,20) →(24,19) →(25,18) →(26,17) →(27,16) → (28,15) →(29,14) →(30,13) →(31,12) →(32,13) →(33,14) →(34,13) →(35,12) -¿ (36,11) →(37,10) →(38,9) →(39,8) →(40,7) →(41,8) →(42,7) →(43,6) → (44,5) →(45,4) →(46,3) →(47,4) →(48,3) →(49,2) →(50,3) →(51,4) → (52,5) →(53,4) →(54,5) →(55,6) →(56,7) →(57,8) →(58,9) →(59,10) → (60,11) →(61,12) →(62,13) →(63,12) →(64,13) →(65,14) →(66,15) →(67,16) → (68,17) →(69,18) →(70,19) →(71,20) →(72,19) →(73,20) →(74,21) →(75,22) → (76,23) →(77,24) →(78,25) →(79,26) →(80,27) →(81,28) →(82,29) →(83,30) → (84,31) →(85,32) →(86,33) →(87,34) →(88,35) →(89,36) →(90,37) →(91,38) → (92,39) →(93,40) →(94,41) →(95,42) →(96,43) →(97,44) →(98,45) →(99,45) → (100,46) = 81
Manhattan:
(20,23) →(21,22) →(22,21) →(23,20) →(24,19) →(25,18) →(26,17) →(27,16) → (28,15) →(29,14) →(30,13) →(31,12) →(32,13) →(33,14) →(34,15) →(35,16) → (36,17) →(37,18) →(38,19) →(39,20) →(40,19) →(41,18) →(42,19) →(43,18) → (44,17) →(45,16) →(46,15) →(47,14) →(48,13) →(49,12) →(50,11) →(51,12) → (52,13) →(53,14) →(54,15) →(55,16) →(56,17) →(57,18) →(58,17) →(59,16) → (60,17) →(61,18) →(62,19) →(63,20) →(64,21) →(65,22) →(66,21) →(67,22) → (68,23) →(69,24) →(70,25) →(71,26) →(72,27) →(73,28) →(74,29) →(75,30) → (76,31) →(77,32) →(78,33) →(79,34) →(80,35) →(81,34) →(82,35) →(83,36) → (84,37) →(85,38) →(86,39) →(87,40) →(88,41) →(89,42) →(90,43) →(91,44) → (92,45) →(93,46) →(94,45) →(95,46) →(96,45) →(97,46) →(98,45) →(99,45) → (100,46)

= 81
Diagonal:
(20,23) →(21,24) →(21,25) →(20,26) →(20,27) →(20,28) →(20,29) →(21,30) → (22,31) →(22,32) →(22,33) →(22,34) →(22,35) →(22,36) →(23,37) →(24,38) → (24,39) →(25,40) →(25,41) →(25,42) →(26,43) →(27,44) →(27,45) →(27,46) → (27,47) →(27,48) →(28,49) →(29,50) →(30,50) →(31,50) →(32,51) →(33,51) → (34,52) →(35,52) →(36,53) →(37,53) →(38,53) →(39,53) →(40,53) →(41,53) → (42,52) →(43,51) →(44,50) →(45,49) →(46,49) →(47,50) →(48,51) →(49,52) → (50,53) →(51,54) →(52,55) →(53,54) →(54,53) →(55,52) →(56,51) →(57,50) → (58,49) →(59,48) →(60,47) →(61,47) →(62,47) →(63,47) →(64,47) →(65,47) → (66,47) →(67,47) →(68,47) →(69,47) →(70,48) →(71,49) →(72,50) →(73,49) → (74,48) →(75,47) →(76,46) →(77,45) →(78,44) →(79,43) →(80,42) →(81,41) → (82,40) →(83,39) →(84,40) →(85,41) →(86,42) →(87,43) →(88,44) →(89,45) → (90,46) →(91,46) →(92,46) →(93,46) →(94,46) →(95,46) →(96,46) →(97,46) → (98,45) →(99,45) →(100,46)
= 99

Kernel Size 5
Euclidean:
(12,14) →(13,13) →(13,12) →(14,11) →(15,10) →(16,9) →(17,8) →(18,7) → (19,8) →(20,9) →(21,8) →(22,7) →(23,6) →(24,5) →(25,4) →(26,3) → (27,4) →(28,3) →(29,2) →(30,3) →(31,2) →(32,3) →(33,4) →(34,5) → (35,6) →(36,7) →(37,8) →(38,9) →(39,10) →(40,9) →(41,10) →(42,11) → (43,12) →(44,13) →(45,14) →(46,15) →(47,16) →(48,17) →(49,18) →(50,19) → (51,20) →(52,21) →(53,22) →(54,23) →(55,24) →(56,25) →(57,26) →(58,27) → (59,26) →(60,27) →(60,28) = 51
Manhattan:
(12,14) →(13,13) →(14,12) →(14,11) →(15,10) →(16,9) →(17,8) →(18,7) → (19,8) →(20,9) →(21,10) →(22,11) →(23,10) →(24,11) →(25,10) →(26,9) → (27,8) →(28,7) →(29,6) →(30,7) →(31,6) →(32,7) →(33,8) →(34,9) → (35,10) →(36,11) →(37,10) →(38,11) →(39,12) →(40,13) →(41,14) →(42,15) → (43,16) →(44,15) →(45,16) →(46,17) →(47,18) →(48,19) →(49,20) →(50,21) → (51,22) →(52,23) →(53,24) →(54,25) →(55,26) →(56,27) →(57,28) →(58,27) → (59,26) →(60,27) →(60,28) = 51
Diagonal:
(12,14) →(13,13) →(14,12) →(14,11) →(15,10) →(16,9) →(17,8) →(18,7) → (19,8) →(20,9) →(21,10) →(22,11) →(23,11) →(24,11) →(25,10) →(26,9) → (27,9) →(28,8) →(29,7) →(30,7) →(31,7) →(32,8) →(33,9) →(34,10) → (35,10) →(36,11) →(37,11) →(38,12) →(39,12) →(40,13) →(41,14) →(42,15) → (43,16) →(44,16) →(45,17) →(46,18) →(47,19) →(48,20) →(49,21) →(50,22) → (51,23) →(52,24) →(53,25) →(54,26) →(55,27) →(56,28) →(57,28) →(58,27) → (59,26) →(60,27) →(60,28) = 51
Kernel Size 7
Euclidean:
(8,10) →(9,9) →(9,8) →(10,7) →(11,6) →(12,5) →(13,5) →(14,6) → (15,7) →(16,6) →(17,5) →(18,4) →(19,3) →(20,2) →(21,1) →(22,2) → (23,3) →(24,4) →(25,5) →(26,6) →(27,7) →(28,8) →(29,7) →(30,8) → (31,9) →(32,10) →(33,11) →(34,12) →(35,13) →(36,14) →(37,15) →(38,16) → (39,17) →(40,18) →(41,19) →(42,20) = 36
Manhattan:
(8,10) →(9,9) →(10,8) →(10,7) →(11,6) →(12,5) →(13,5) →(14,6) → (15,7) →(16,8) →(17,7) →(18,6) →(19,5) →(20,4) →(21,3) →(22,4) → (23,5) →(24,6) →(25,7) →(26,8) →(27,7) →(28,8) →(29,9) →(30,10) → (31,11) →(32,12) →(33,13) →(34,14) →(35,15) →(36,16) →(37,17) →(38,18) → (39,19) →(40,20) →(41,19) →(42,20) = 36
Diagonal:

(8,10) →(9,9) →(10,8) →(10,7) →(11,6) →(12,5) →(13,5) →(14,6) → (15,7) →(16,8) →(17,7) →(18,7) →(19,6) →(20,5) →(21,4) →(22,5) → (23,5) →(24,6) →(25,7) →(26,8) →(27,8) →(28,8) →(29,9) →(30,10) → (31,11) →(32,12) →(33,13) →(34,14) →(35,15) →(36,16) →(37,17) →(38,18) → (39,19) →(40,20) →(41,19) →(42,20) = 36
SOURCE 60,70 DESTINATION 60,340

Kernel Size 3
Euclidean:
(20,23) →(20,24) →(20,25) →(20,26) →(20,27) →(20,28) →(20,29) →(20,30) → (20,31) →(20,32) →(20,33) →(20,34) →(20,35) →(20,36) →(21,37) →(22,38) → (23,39) →(24,40) →(24,41) →(24,42) →(24,43) →(24,44) →(24,45) →(25,46) → (26,47) →(27,48) →(28,49) →(29,50) →(30,51) →(31,52) →(32,53) →(33,54) → (34,55) →(35,56) →(36,57) →(37,56) →(38,57) →(39,56) →(40,57) →(41,58) → (42,59) →(43,59) →(44,60) →(44,61) →(45,62) →(45,63) →(46,64) →(46,65) → (46,66) →(46,67) →(46,68) →(46,69) →(47,70) →(48,71) →(49,72) →(49,73) → (49,74) →(50,75) →(49,76) →(49,77) →(49,78) →(49,79) →(49,80) →(49,81) → (49,82) →(49,83) →(48,84) →(47,84) →(46,84) →(45,84) →(44,84) →(43,84) → (42,83) →(41,84) →(40,85) →(39,86) →(38,87) →(37,88) →(36,89) →(35,89) → (34,90) →(33,90) →(32,90) →(31,91) →(30,92) →(29,92) →(28,93) →(27,93) → (26,94) →(25,94) →(24,95) →(24,96) →(24,97) →(24,98) →(23,99) →(23,100) → (22,101) →(21,102) →(20,103) →(19,104) →(19,105) →(19,106) →(19,107) →(18,108) →(17,109) →(16,110) →(17,111) →(18,112) →(19,113) →(20,113) = 110
Manhattan:
(20,23) →(20,24) →(20,25) →(20,26) →(20,27) →(20,28) →(20,29) →(20,30) → (20,31) →(20,32) →(20,33) →(20,34) →(21,35) →(22,36) →(23,37) →(23,38) → (23,39) →(24,40) →(24,41) →(24,42) →(24,43) →(24,44) →(24,45) →(25,46) → (26,47) →(27,48) →(28,49) →(29,50) →(30,51) →(31,52) →(32,53) →(33,54) → (34,55) →(35,56) →(36,57) →(37,56) →(38,57) →(39,56) →(40,57) →(41,58) → (42,59) →(43,59) →(44,60) →(44,61) →(45,62) →(45,63) →(46,64) →(46,65) → (46,66) →(46,67) →(46,68) →(46,69) →(47,70) →(48,71) →(49,72) →(49,73) → (50,74) →(50,75) →(49,76) →(49,77) →(49,78) →(49,79) →(49,80) →(49,81) → (49,82) →(49,83) →(48,84) →(47,85) →(46,86) →(45,87) →(44,88) →(44,89) → (43,90) →(42,91) →(41,92) →(40,92) →(39,92) →(38,92) →(37,91) →(36,90) → (35,89) →(34,90) →(33,90) →(32,90) →(31,91) →(30,92) →(29,92) →(28,93) → (27,93) →(26,94) →(25,94) →(24,95) →(24,96) →(24,97) →(23,98) →(22,99) → (22,100) →(21,101) →(20,102) →(20,103) →(19,104) →(19,105) →(19,106) →(19,107) →(18,108) →(17,109) →(16,110) →(17,111) →(18,112) →(19,113) →(20,113) = 111
Diagonal:
(20,23) →(20,24) →(20,25) →(20,26) →(20,27) →(20,28) →(20,29) →(20,30) → (20,31) →(20,32) →(20,33) →(20,34) →(21,35) →(22,36) →(23,37) →(23,38) → (23,39) →(24,40) →(24,41) →(24,42) →(24,43) →(24,44) →(24,45) →(25,46) → (26,47) →(27,48) →(28,49) →(29,50) →(30,51) →(31,52) →(32,53) →(33,54) → (34,55) →(35,56) →(36,57) →(37,56) →(38,57) →(39,56) →(40,57) →(41,58) → (42,59) →(43,59) →(44,60) →(44,61) →(45,62) →(45,63) →(46,64) →(46,65) → (46,66) →(46,67) →(46,68) →(46,69) →(47,70) →(48,71) →(49,72) →(49,73) → (50,74) →(50,75) →(49,76) →(49,77) →(49,78) →(49,79) →(49,80) →(49,81) → (49,82) →(49,83) →(48,84) →(47,85) →(46,86) →(45,87) →(44,88) →(44,89) → (43,90) →(42,91) →(41,92) →(40,92) →(39,92) →(38,93) →(37,92) →(36,91) → (36,90) →(35,89) →(34,90) →(33,90) →(32,90) →(31,91) →(30,92) →(29,92) → (28,93) →(27,93) →(26,94) →(25,94) →(24,95) →(24,96) →(24,97) →(23,98) → (22,99) →(22,100) →(21,101) →(20,102) →(20,103) →(19,104) →(19,105) →(19,106) →(19,107) →(18,108) →(17,109) →(16,110) →(17,111) →(18,112) →(19,113) →(20,113) = 112

Kernel Size 5
Euclidean:
(12,14) →(13,13) →(13,12) →(14,11) →(15,10) →(16,9) →(17,8) →(18,7) → (19,8) →(20,9) →(21,8) →(22,7) →(23,6) →(24,5) →(25,4) →(26,3) → (27,4) →(28,3) →(29,2) →(30,3) →(31,2) →(32,3) →(33,4) →(34,5) → (35,6) →(36,7) →(37,8) →(38,9) →(39,10) →(40,9) →(41,10) →(42,11) → (43,12) →(44,13) →(45,14) →(46,15) →(47,16) →(48,17) →(49,18) →(50,19) → (51,20) →(52,21) →(53,22) →(54,23) →(55,24) →(56,25) →(57,26) →(58,27) → (59,26) →(60,25) →(61,24) →(62,23) →(63,22) →(64,21) →(65,20) →(66,19) → (67,18) →(68,17) →(69,16) →(70,15) →(71,14) →(72,13) →(73,12) →(73,13) → (73,14) →(73,15) →(73,16) →(73,17) →(73,18) →(73,19) →(73,20) →(73,21) → (73,22) →(73,23) →(73,24) →(73,25) →(73,26) →(73,27) →(73,28) →(73,29) → (73,30) →(73,31) →(73,32) →(73,33) →(73,34) →(73,35) →(73,36) →(73,37) → (73,38) →(73,39) →(73,40) →(73,41) →(73,42) →(73,43) →(73,44) →(73,45) → (73,46) →(73,47) →(73,48) →(73,49) →(73,50) →(73,51) →(73,52) →(73,53) → (73,54) →(73,55) →(73,56) →(73,57) →(73,58) →(73,59) →(73,60) →(73,61) → (73,62) →(73,63) →(73,64) →(73,65) →(73,66) →(73,67) →(73,68) →(73,69) → (73,70) →(73,71) →(72,72) →(71,72) →(70,72) →(69,72) →(68,72) →(67,72) → (66,72) →(65,72) →(64,72) →(63,72) →(62,72) →(61,72) →(60,72) →(59,72) → (58,72) →(57,72) →(56,72) →(55,72) →(54,72) →(53,72) →(52,72) →(51,72) → (50,72) →(49,72) →(48,72) →(47,72) →(46,72) →(45,72) →(44,72) →(43,72) → (42,72) →(41,72) →(40,72) →(39,72) →(38,72) →(37,72) →(36,72) →(35,72) → (34,72) →(33,72) →(32,72) →(31,72) →(30,72) →(29,72) →(28,72) →(27,72) → (26,72) →(25,72) →(24,72) →(23,72) →(22,72) →(21,72) →(20,72) →(19,72) → (18,72) →(17,72) →(16,72) →(15,72) →(14,72) →(13,72) →(12,72) →(11,72) → (10,72) →(11,71) →(12,70) →(13,69) →(12,68) = 189
Manhattan:
(12,14) →(12,13) →(13,12) →(14,11) →(15,10) →(16,9) →(17,8) →(18,7) → (19,8) →(20,9) →(21,10) →(22,11) →(23,10) →(24,11) →(25,10) →(26,9) → (27,8) →(28,7) →(29,6) →(30,7) →(31,6) →(32,7) →(33,8) →(34,9) → (35,10) →(36,11) →(37,10) →(38,11) →(39,12) →(40,13) →(41,14) →(42,15) → (43,16) →(44,15) →(45,16) →(46,17) →(47,18) →(48,19) →(49,20) →(50,21) → (51,22) →(52,23) →(53,24) →(54,25) →(55,26) →(56,27) →(57,28) →(58,27) → (59,26) →(60,25) →(61,24) →(62,23) →(63,22) →(64,21) →(65,20) →(66,19) → (67,18) →(68,17) →(69,16) →(70,15) →(71,14) →(72,13) →(73,12) →(73,13) → (73,14) →(73,15) →(73,16) →(73,17) →(73,18) →(73,19) →(73,20) →(73,21) → (73,22) →(73,23) →(73,24) →(73,25) →(73,26) →(73,27) →(73,28) →(73,29) → (73,30) →(73,31) →(73,32) →(73,33) →(73,34) →(73,35) →(73,36) →(73,37) → (73,38) →(73,39) →(73,40) →(73,41) →(73,42) →(73,43) →(73,44) →(73,45) → (73,46) →(73,47) →(73,48) →(73,49) →(73,50) →(73,51) →(73,52) →(73,53) → (73,54) →(73,55) →(73,56) →(73,57) →(73,58) →(73,59) →(73,60) →(73,61) → (73,62) →(73,63) →(73,64) →(73,65) →(73,66) →(73,67) →(73,68) →(73,69) → (73,70) →(73,71) →(72,72) →(71,72) →(70,72) →(69,72) →(68,72) →(67,72) → (66,72) →(65,72) →(64,72) →(63,72) →(62,72) →(61,72) →(60,72) →(59,72) → (58,72) →(57,72) →(56,72) →(55,72) →(54,72) →(53,72) →(52,72) →(51,72) → (50,72) →(49,72) →(48,72) →(47,72) →(46,72) →(45,72) →(44,72) →(43,72) → (42,72) →(41,72) →(40,72) →(39,72) →(38,72) →(37,72) →(36,72) →(35,72) → (34,72) →(33,72) →(32,72) →(31,72) →(30,72) →(29,72) →(28,72) →(27,72) → (26,72) →(25,72) →(24,72) →(23,72) →(22,72) →(21,72) →(20,72) →(19,72) → (18,72) →(17,72) →(16,72) →(15,72) →(14,72) →(13,72) →(12,72) →(11,72) → (10,72) →(11,71) →(12,70) →(13,69) →(12,68) = 189
Diagonal:
(12,14) →(12,13) →(13,12) →(14,11) →(15,10) →(16,9) →(17,8) →(18,7) → (19,8) →(20,9) →(21,10) →(22,11) →(23,10) →(24,11) →(25,10) →(26,9) → (27,8) →(28,7) →(29,6)

→(30,7) →(31,6) →(32,7) →(33,8) →(34,9) → (35,10) →(36,11) →(37,10) →(38,11) →(39,12) →(40,13) →(41,14) →(42,15) → (43,16) →(44,16) →(45,17) →(46,18) →(47,19) →(48,20) →(49,21) →(50,22) → (50,23) →(51,24) →(52,25) →(53,26) →(54,27) →(55,28) →(55,29) →(56,30) → (55,31) →(55,32) →(55,33) →(55,34) →(56,35) →(56,36) →(55,37) →(55,38) → (56,39) →(57,40) →(58,41) →(58,42) →(59,43) →(60,43) →(61,42) →(62,43) → (63,43) →(64,44) →(65,43) →(66,44) →(67,44) →(68,44) →(69,43) →(70,42) → (71,41) →(72,40) →(73,41) →(73,42) →(73,43) →(73,44) →(73,45) →(73,46) → (73,47) →(73,48) →(73,49) →(73,50) →(73,51) →(73,52) →(73,53) →(73,54) → (73,55) →(73,56) →(73,57) →(73,58) →(73,59) →(73,60) →(73,61) →(73,62) → (73,63) →(73,64) →(73,65) →(73,66) →(73,67) →(73,68) →(73,69) →(73,70) → (73,71) →(72,72) →(71,72) →(70,72) →(69,72) →(68,72) →(67,72) →(66,72) → (65,72) →(64,72) →(63,72) →(62,72) →(61,72) →(60,72) →(59,72) →(58,72) → (57,72) →(56,72) →(55,72) →(54,72) →(53,72) →(52,72) →(51,72) →(50,72) → (49,72) →(48,72) →(47,72) →(46,72) →(45,72) →(44,72) →(43,72) →(42,72) → (41,72) →(40,72) →(39,72) →(38,72) →(37,72) →(36,72) →(35,72) →(34,72) → (33,72) →(32,72) →(31,72) →(30,72) →(29,72) →(28,72) →(27,72) →(26,72) → (25,72) →(24,72) →(23,72) →(22,72) →(21,72) →(20,72) →(19,72) →(18,72) → (17,72) →(16,72) →(15,72) →(14,72) →(13,72) →(12,72) →(11,72) →(10,72) → (11,71) →(12,70) →(13,69) →(12,68) = 172

Kernel Size 7
Euclidean:
(8,10) →(8,11) →(8,12) →(8,13) →(8,14) →(8,15) →(9,16) →(9,17) → (10,18) →(10,19) →(10,20) →(11,21) →(12,22) →(13,23) →(14,22) →(15,23) → (16,24) →(17,23) →(18,24) →(19,25) →(20,26) →(20,27) →(20,28) →(20,29) → (21,30) →(21,31) →(22,32) →(21,33) →(21,34) →(20,35) →(20,36) →(19,36) → (18,35) →(17,36) →(16,37) →(15,37) →(14,37) →(13,37) →(12,37) →(11,37) → (10,37) →(9,37) →(8,37) →(7,38) →(6,39) →(6,40) →(6,41) →(7,42) → (7,43) →(7,44) →(7,45) →(6,46) →(6,47) →(7,48) →(8,48) = 55
Manhattan:
(8,10) →(8,11) →(8,12) →(8,13) →(8,14) →(8,15) →(9,16) →(9,17) → (10,18) →(10,19) →(10,20) →(11,21) →(12,22) →(13,23) →(14,22) →(15,23) → (16,24) →(17,23) →(18,24) →(19,25) →(20,26) →(20,27) →(20,28) →(20,29) → (21,30) →(21,31) →(22,32) →(21,33) →(21,34) →(20,35) →(20,36) →(19,36) → (18,35) →(17,36) →(16,37) →(15,37) →(14,37) →(13,37) →(12,37) →(11,37) → (10,37) →(9,37) →(8,37) →(7,38) →(6,39) →(6,40) →(6,41) →(7,42) → (7,43) →(7,44) →(7,45) →(6,46) →(6,47) →(7,48) →(8,48) = 55
Diagonal:
(8,10) →(8,11) →(8,12) →(8,13) →(8,14) →(8,15) →(9,16) →(9,17) → (10,18) →(10,19) →(10,20) →(11,21) →(12,22) →(13,23) →(14,22) →(15,23) → (16,24) →(17,23) →(18,24) →(19,25) →(20,26) →(20,27) →(20,28) →(20,29) → (21,30) →(21,31) →(22,32) →(21,33) →(21,34) →(20,35) →(20,36) →(19,36) → (18,35) →(17,36) →(16,37) →(15,37) →(14,38) →(13,38) →(12,39) →(11,38) → (10,37) →(9,38) →(8,37) →(7,38) →(6,39) →(6,40) →(6,41) →(7,42) → (7,43) →(7,44) →(7,45) →(6,46) →(6,47) →(7,48) →(8,48) = 55

SOURCE 320,40 DESTINATION 60,340

Kernel Size 3
Euclidean:
(106,13) →(106,14) →(106,15) →(105,16) →(104,17) →(103,18) →(102,19) →(101,20) →(100,21) →(99,22) →(98,23) →(97,24) →(96,25) →(95,26) →(94,27) →(93,28) → (92,29)

→(91,30) →(90,31) →(89,32) →(88,33) →(87,33) →(86,33) →(85,33) → (84,33) →(83,33) →(82,33) →(81,33) →(80,33) →(79,33) →(78,33) →(77,32) → (76,31) →(75,30) →(74,29) →(73,28) →(72,28) →(71,27) →(70,26) →(69,25) → (68,24) →(67,23) →(66,22) →(65,22) →(64,22) →(63,21) →(62,20) →(61,19) → (60,18) →(59,17) →(58,17) →(57,17) →(56,17) →(55,16) →(54,15) →(53,14) → (52,13) →(51,12) →(50,12) →(49,12) →(48,12) →(47,12) →(46,12) →(45,12) → (44,12) →(43,12) →(42,12) →(41,12) →(40,12) →(39,12) →(38,12) →(37,12) → (36,13) →(35,13) →(34,13) →(33,13) →(32,13) →(31,13) →(30,13) →(29,14) → (28,15) →(28,16) →(27,17) →(26,18) →(26,19) →(25,20) →(24,21) →(23,22) → (22,23) →(21,24) →(21,25) →(20,26) →(20,27) →(20,28) →(20,29) →(20,30) → (20,31) →(20,32) →(20,33) →(20,34) →(20,35) →(20,36) →(21,37) →(22,38) → (23,39) →(24,40) →(24,41) →(24,42) →(24,43) →(24,44) →(24,45) →(25,46) → (26,47) →(27,48) →(28,49) →(29,50) →(30,51) →(31,50) →(32,51) →(33,52) → (34,53) →(35,52) →(36,53) →(37,53) →(38,54) →(39,55) →(40,56) →(41,57) → (42,58) →(43,59) →(44,60) →(44,61) →(45,62) →(45,63) →(46,64) →(46,65) → (46,66) →(46,67) →(46,68) →(46,69) →(47,70) →(48,71) →(49,72) →(49,73) → (49,74) →(50,75) →(50,76) →(50,77) →(50,78) →(50,79) →(50,80) →(50,81) → (50,82) →(49,83) →(48,84) →(47,84) →(46,84) →(45,84) →(44,84) →(43,84) → (42,83) →(41,83) →(40,83) →(39,83) →(38,83) →(37,83) →(36,83) →(35,84) → (34,85) →(33,86) →(32,87) →(31,88) →(30,89) →(29,90) →(28,91) →(27,92) → (26,93) →(25,94) →(24,95) →(24,96) →(24,97) →(24,98) →(23,99) →(23,100) → (22,101) →(21,102) →(20,103) →(19,104) →(19,105) →(19,106) →(19,107) →(18,108) →(17,109) →(16,110) →(16,111) →(17,112) →(18,113) →(19,112) →(20,113) = 199

Manhattan:

(106,13) →(106,14) →(106,15) →(105,16) →(104,17) →(103,18) →(102,19) →(101,20) →(100,21) →(99,22) →(98,23) →(97,24) →(96,25) →(95,26) →(94,27) →(93,28) → (92,29) →(91,30) →(90,31) →(89,32) →(88,33) →(87,33) →(86,33) →(85,33) → (84,33) →(83,33) →(82,33) →(81,33) →(80,33) →(79,33) →(78,33) →(77,32) → (76,31) →(75,30) →(74,29) →(73,28) →(72,28) →(71,27) →(70,26) →(69,25) → (68,24) →(67,23) →(66,22) →(65,22) →(64,22) →(63,21) →(62,20) →(61,19) → (60,18) →(59,17) →(58,17) →(57,17) →(56,17) →(55,16) →(54,15) →(53,14) → (52,13) →(51,12) →(50,12) →(49,13) →(48,13) →(47,13) →(46,13) →(45,13) → (44,13) →(43,13) →(42,13) →(41,13) →(40,13) →(39,13) →(38,13) →(37,13) → (36,13) →(35,13) →(34,13) →(33,13) →(32,13) →(31,13) →(30,13) →(29,14) → (28,15) →(27,16) →(26,17) →(25,18) →(24,19) →(23,20) →(22,21) →(21,22) → (20,23) →(20,24) →(20,25) →(20,26) →(20,27) →(20,28) →(20,29) →(20,30) → (20,31) →(20,32) →(20,33) →(20,34) →(20,35) →(20,36) →(21,37) →(22,38) → (23,39) →(24,40) →(24,41) →(24,42) →(24,43) →(24,44) →(24,45) →(25,46) → (26,47) →(27,48) →(28,49) →(29,50) →(30,51) →(31,52) →(32,53) →(33,54) → (34,55) →(35,56) →(36,57) →(37,56) →(38,57) →(39,56) →(40,57) →(41,58) → (42,59) →(43,59) →(44,60) →(44,61) →(45,62) →(45,63) →(46,64) →(46,65) → (46,66) →(46,67) →(46,68) →(46,69) →(47,70) →(48,71) →(49,72) →(49,73) → (49,74) →(50,75) →(49,76) →(49,77) →(49,78) →(49,79) →(49,80) →(49,81) → (49,82) →(49,83) →(48,84) →(47,85) →(46,86) →(45,87) →(44,88) →(44,89) → (43,90) →(42,91) →(41,92) →(40,92) →(39,92) →(38,92) →(37,91) →(36,90) → (35,89) →(34,90) →(33,90) →(32,90) →(31,91) →(30,92) →(29,92) →(28,93) → (27,93) →(26,94) →(25,94) →(24,95) →(24,96) →(24,97) →(23,98) →(22,99) → (22,100) →(21,101) →(20,102) →(20,103) →(19,104) →(19,105) →(19,106) →(19,107) →(18,108) →(17,109) →(16,110) →(17,111) →(18,112) →(19,113) →(20,113) = 199

Diagonal:

(106,13) →(106,14) →(106,15) →(105,16) →(104,17) →(103,18) →(102,19) →(101,20) →(100,21) →(99,22) →(98,23) →(97,24) →(96,25) →(95,26) →(94,27) →(93,28) → (92,29) →(91,30) →(90,31) →(89,32) →(88,33) →(87,34) →(86,35) →(85,36) → (84,37) →(83,38)

→(82,37) →(82,36) →(81,35) →(80,35) →(79,35) →(78,34) → (77,34) →(76,33) →(75,32) →(75,31) →(74,30) →(74,29) →(73,28) →(72,28) → (71,27) →(70,26) →(69,25) →(68,25) →(67,24) →(67,23) →(66,22) →(65,22) → (64,22) →(63,21) →(62,20) →(61,20) →(60,19) →(60,18) →(59,17) →(58,18) → (57,18) →(56,17) →(55,17) →(54,16) →(54,15) →(53,14) →(52,13) →(51,12) → (50,12) →(49,13) →(48,14) →(47,15) →(46,15) →(45,16) →(44,17) →(43,18) → (42,19) →(41,19) →(40,20) →(39,21) →(38,20) →(37,20) →(36,19) →(35,19) → (34,20) →(33,20) →(32,20) →(31,19) →(31,18) →(32,17) →(32,16) →(32,15) → (32,14) →(31,13) →(30,13) →(29,14) →(28,15) →(27,16) →(26,17) →(25,18) → (24,19) →(23,20) →(22,21) →(21,22) →(20,23) →(20,24) →(20,25) →(20,26) → (20,27) →(20,28) →(20,29) →(20,30) →(20,31) →(20,32) →(20,33) →(20,34) → (20,35) →(20,36) →(21,37) →(22,38) →(23,39) →(24,40) →(24,41) →(24,42) → (24,43) →(24,44) →(24,45) →(25,46) →(26,47) →(27,48) →(28,49) →(29,50) → (30,51) →(31,52) →(32,53) →(33,54) →(34,55) →(35,56) →(36,57) →(37,56) → (38,57) →(39,56) →(40,57) →(41,58) →(42,59) →(43,59) →(44,60) →(44,61) → (45,62) →(45,63) →(46,64) →(46,65) →(46,66) →(46,67) →(46,68) →(46,69) → (47,70) →(48,71) →(49,72) →(49,73) →(49,74) →(50,75) →(49,76) →(49,77) → (49,78) →(49,79) →(49,80) →(49,81) →(49,82) →(49,83) →(48,84) →(47,85) → (46,86) →(45,87) →(44,88) →(44,89) →(43,90) →(42,91) →(41,92) →(40,92) → (39,92) →(38,93) →(37,92) →(36,91) →(36,90) →(35,89) →(34,90) →(33,90) → (32,90) →(31,91) →(30,92) →(29,92) →(28,93) →(27,93) →(26,94) →(25,94) → (24,95) →(24,96) →(24,97) →(23,98) →(22,99) →(22,100) →(21,101) →(20,102) →(20,103) →(19,104) →(19,105) →(19,106) →(19,107) →(18,108) →(17,109) →(16,110) →(17,111) →(18,112) →(19,113) →(20,113) = 212

Kernel Size 5

Euclidean:

(60,14) →(61,13) →(62,12) →(63,11) →(64,10) →(65,9) →(66,8) →(67,7) → (68,6) →(69,5) →(70,4) →(71,3) →(72,2) →(73,1) →(73,2) →(73,3) → (73,4) →(73,5) →(73,6) →(73,7) →(73,8) →(73,9) →(73,10) →(73,11) → (73,12) →(73,13) →(73,14) →(73,15) →(73,16) →(73,17) →(73,18) →(73,19) → (73,20) →(73,21) →(73,22) →(73,23) →(73,24) →(73,25) →(73,26) →(73,27) → (73,28) →(73,29) →(73,30) →(73,31) →(73,32) →(73,33) →(73,34) →(73,35) → (73,36) →(73,37) →(73,38) →(73,39) →(73,40) →(73,41) →(73,42) →(73,43) → (73,44) →(73,45) →(73,46) →(73,47) →(73,48) →(73,49) →(73,50) →(73,51) → (73,52) →(73,53) →(73,54) →(73,55) →(73,56) →(73,57) →(73,58) →(73,59) → (73,60) →(73,61) →(73,62) →(73,63) →(73,64) →(73,65) →(73,66) →(73,67) → (73,68) →(73,69) →(73,70) →(73,71) →(72,72) →(71,72) →(70,72) →(69,72) → (68,72) →(67,72) →(66,72) →(65,72) →(64,72) →(63,72) →(62,72) →(61,72) → (60,72) →(59,72) →(58,72) →(57,72) →(56,72) →(55,72) →(54,72) →(53,72) → (52,72) →(51,72) →(50,72) →(49,72) →(48,72) →(47,72) →(46,72) →(45,72) → (44,72) →(43,72) →(42,72) →(41,72) →(40,72) →(39,72) →(38,72) →(37,72) → (36,72) →(35,72) →(34,72) →(33,72) →(32,72) →(31,72) →(30,72) →(29,72) → (28,72) →(27,72) →(26,72) →(25,72) →(24,72) →(23,72) →(22,72) →(21,72) → (20,72) →(19,72) →(18,72) →(17,72) →(16,72) →(15,72) →(14,72) →(13,72) → (12,72) →(11,72) →(10,72) →(11,71) →(12,70) →(13,69) →(12,68) = 151

Manhattan:

(60,14) →(61,13) →(62,12) →(63,11) →(64,10) →(65,9) →(66,8) →(67,7) → (68,6) →(69,5) →(70,4) →(71,3) →(72,2) →(73,1) →(73,2) →(73,3) → (73,4) →(73,5) →(73,6) →(73,7) →(73,8) →(73,9) →(73,10) →(73,11) → (73,12) →(73,13) →(73,14) →(73,15) →(73,16) →(73,17) →(73,18) →(73,19) → (73,20) →(73,21) →(73,22) →(73,23) →(73,24) →(73,25) →(73,26) →(73,27) → (73,28) →(73,29) →(73,30) →(73,31) →(73,32) →(73,33) →(73,34) →(73,35) → (73,36) →(73,37) →(73,38) →(73,39) →(73,40) →(73,41) →(73,42) →(73,43) → (73,44) →(73,45) →(73,46) →(73,47) →(73,48) →(73,49) →(73,50) →(73,51) → (73,52)

→(73,53) →(73,54) →(73,55) →(73,56) →(73,57) →(73,58) →(73,59) → (73,60) →(73,61) →(73,62) →(73,63) →(73,64) →(73,65) →(73,66) →(73,67) → (73,68) →(73,69) →(73,70) →(73,71) →(72,72) →(71,72) →(70,72) →(69,72) → (68,72) →(67,72) →(66,72) →(65,72) →(64,72) →(63,72) →(62,72) →(61,72) → (60,72) →(59,72) →(58,72) →(57,72) →(56,72) →(55,72) →(54,72) →(53,72) → (52,72) →(51,72) →(50,72) →(49,72) →(48,72) →(47,72) →(46,72) →(45,72) → (44,72) →(43,72) →(42,72) →(41,72) →(40,72) →(39,72) →(38,72) →(37,72) → (36,72) →(35,72) →(34,72) →(33,72) →(32,72) →(31,72) →(30,72) →(29,72) → (28,72) →(27,72) →(26,72) →(25,72) →(24,72) →(23,72) →(22,72) →(21,72) → (20,72) →(19,72) →(18,72) →(17,72) →(16,72) →(15,72) →(14,72) →(13,72) → (12,72) →(11,72) →(10,72) →(11,71) →(12,70) →(13,69) →(12,68) = 151

Diagonal:

(60,14) →(61,13) →(62,12) →(63,11) →(64,10) →(65,9) →(66,8) →(67,7) → (68,6) →(69,5) →(70,4) →(71,3) →(72,2) →(73,1) →(73,2) →(73,3) → (73,4) →(73,5) →(73,6) →(73,7) →(73,8) →(73,9) →(73,10) →(73,11) → (73,12) →(73,13) →(73,14) →(73,15) →(73,16) →(73,17) →(73,18) →(73,19) → (73,20) →(73,21) →(73,22) →(73,23) →(73,24) →(73,25) →(73,26) →(73,27) → (73,28) →(73,29) →(73,30) →(73,31) →(73,32) →(73,33) →(73,34) →(73,35) → (73,36) →(73,37) →(73,38) →(73,39) →(72,40) →(73,41) →(73,42) →(73,43) → (73,44) →(73,45) →(73,46) →(73,47) →(73,48) →(73,49) →(73,50) →(73,51) → (73,52) →(73,53) →(73,54) →(73,55) →(73,56) →(73,57) →(73,58) →(73,59) → (73,60) →(73,61) →(73,62) →(73,63) →(73,64) →(73,65) →(73,66) →(73,67) → (73,68) →(73,69) →(73,70) →(73,71) →(72,72) →(71,72) →(70,72) →(69,72) → (68,72) →(67,72) →(66,72) →(65,72) →(64,72) →(63,72) →(62,72) →(61,72) → (60,72) →(59,72) →(58,72) →(57,72) →(56,72) →(55,72) →(54,72) →(53,72) → (52,72) →(51,72) →(50,72) →(49,72) →(48,72) →(47,72) →(46,72) →(45,72) → (44,72) →(43,72) →(42,72) →(41,72) →(40,72) →(39,72) →(38,72) →(37,72) → (36,72) →(35,72) →(34,72) →(33,72) →(32,72) →(31,72) →(30,72) →(29,72) → (28,72) →(27,72) →(26,72) →(25,72) →(24,72) →(23,72) →(22,72) →(21,72) → (20,72) →(19,72) →(18,72) →(17,72) →(16,72) →(15,72) →(14,72) →(13,72) → (12,72) →(11,72) →(10,72) →(11,71) →(12,70) →(13,69) →(12,68) = 151

Kernel Size 7

Euclidean:

(42,10) →(43,9) →(44,8) →(45,7) →(46,6) →(47,5) →(48,4) →(49,3) → (50,2) →(51,1) →(52,0) →(52,1) →(52,2) →(52,3) →(52,4) →(52,5) → (52,6) →(52,7) →(52,8) →(52,9) →(52,10) →(52,11) →(52,12) →(52,13) → (52,14) →(52,15) →(52,16) →(52,17) →(52,18) →(52,19) →(52,20) →(52,21) → (52,22) →(52,23) →(52,24) →(52,25) →(52,26) →(52,27) →(52,28) →(51,28) → (50,28) →(49,29) →(48,29) →(47,29) →(46,29) →(45,29) →(44,29) →(43,29) → (42,29) →(41,28) →(40,27) →(40,26) →(40,25) →(40,24) →(39,24) →(38,24) → (37,24) →(36,24) →(35,24) →(34,24) →(33,24) →(32,24) →(31,23) →(30,23) → (31,22) →(30,21) →(29,21) →(28,21) →(27,21) →(26,22) →(25,22) →(24,22) → (23,23) →(22,24) →(22,25) →(22,26) →(22,27) →(22,28) →(22,29) →(22,30) → (22,31) →(22,32) →(21,33) →(21,34) →(20,35) →(20,36) →(19,36) →(18,35) → (17,36) →(16,37) →(15,37) →(14,37) →(13,37) →(12,37) →(11,37) →(10,37) → (9,37) →(8,37) →(7,38) →(6,39) →(6,40) →(6,41) →(7,42) →(7,43) → (7,44) →(7,45) →(6,46) →(6,47) →(7,48) →(8,48) = 110

Manhattan:

(42,10) →(43,9) →(44,8) →(45,7) →(46,6) →(47,5) →(48,4) →(49,3) → (50,2) →(51,1) →(52,0) →(52,1) →(52,2) →(52,3) →(52,4) →(52,5) → (52,6) →(52,7) →(52,8) →(52,9) →(52,10) →(52,11) →(52,12) →(52,13) → (52,14) →(52,15) →(52,16) →(52,17) →(52,18) →(52,19) →(52,20) →(52,21) → (52,22) →(52,23) →(52,24) →(52,25) →(52,26) →(52,27) →(52,28) →(51,28) → (50,28) →(49,29) →(48,29) →(47,29) →(46,29) →(45,29) →(44,29)

→(43,29) → (42,29) →(41,29) →(40,28) →(39,27) →(40,26) →(40,25) →(40,24) →(39,24) → (38,24) →(37,24) →(36,24) →(35,24) →(34,24) →(33,24) →(32,24) →(31,23) → (30,23) →(31,22) →(30,21) →(29,21) →(28,21) →(27,21) →(26,22) →(25,22) → (24,22) →(23,23) →(22,24) →(22,25) →(22,26) →(22,27) →(22,28) →(22,29) → (22,30) →(22,31) →(22,32) →(21,33) →(21,34) →(20,35) →(20,36) →(19,36) → (18,35) →(17,36) →(16,37) →(15,37) →(14,37) →(13,37) →(12,37) →(11,37) → (10,37) →(9,37) →(8,37) →(7,38) →(6,39) -¿(6,40) →(6,41) →(7,42) → (7,43) →(7,44) →(7,45) →(6,46) →(6,47) →(7,48) →(8,48) = 111

Diagonal:

(42,10) →(43,9) →(44,8) →(45,7) →(46,6) →(47,5) →(48,4) →(49,3) → (50,2) →(51,1) →(52,0) →(52,1) →(52,2) →(52,3) →(52,4) →(52,5) → (52,6) →(52,7) →(52,8) →(52,9) →(52,10) →(52,11) →(52,12) →(52,13) → (52,14) →(52,15) →(52,16) →(52,17) →(52,18) →(52,19) →(52,20) →(52,21) → (52,22) →(52,23) →(52,24) →(52,25) →(52,26) →(52,27) →(52,28) →(52,29) → (51,29) →(50,29) →(49,29) →(48,29) →(47,29) →(46,29) →(45,29) →(44,29) → (43,29) →(42,29) →(41,29) →(40,28) →(39,27) →(40,26) →(40,25) →(40,24) → (39,24) →(38,24) →(37,24) →(36,24) →(35,24) →(34,24) →(33,24) →(32,24) → (31,23) →(30,23) →(31,22) →(30,21) →(29,21) →(28,21) →(27,21) →(26,22) → (25,22) →(24,22) →(23,23) →(22,24) →(22,25) →(22,26) →(22,27) →(22,28) → (22,29) →(22,30) →(22,31) →(22,32) →(21,33) →(21,34) →(20,35) →(20,36) → (19,36) →(18,35) →(17,36) →(16,37) →(15,37) →(14,38) →(13,38) →(12,39) → (11,38) →(10,37) →(9,38) →(8,37) →(7,38) →(6,39) →(6,40) →(6,41) → (7,42) →(7,43) →(7,44) →(7,45) →(6,46) →(6,47) →(7,48) →(8,48) = 112

SOURCE 320,40 DESTINATION 320,300

Kernel Size 3

Euclidean:

(106,13) →(106,14) →(106,15) →(105,16) →(104,17) →(103,18) →(102,19) →(101,20) →(100,21) →(99,22) →(98,23) →(97,24) →(96,25) →(95,26) →(94,27) →(93,28) → (92,29) →(91,30) →(90,31) →(89,32) →(88,33) →(88,34) →(88,35) →(88,36) → (88,37) →(88,38) →(88,39) →(88,40) →(88,41) →(88,42) →(88,43) →(88,44) → (89,45) →(90,46) →(91,47) →(91,48) →(91,49) →(92,50) →(93,51) →(93,52) → (93,53) →(93,54) →(93,55) →(93,56) →(93,57) →(93,58) →(94,59) →(94,60) → (94,61) →(94,62) →(95,63) →(96,64) →(97,65) →(98,66) →(99,67) →(100,68) → (101,67) →(102,68) →(103,69) →(104,70) →(105,71) →(106,72) →(107,73) →(107,74) →(107,75) →(107,76) →(107,77) →(107,78) →(107,79) →(107,80) →(107,81) →(107,82) →(107,83) →(107,84) →(107,85) →(107,86) →(107,87) →(107,88) →(107,89) →(107,90) →(107,91) →(107,92) →(107,93) →(107,94) →(107,95) →(107,96) →(107,97) →(107,98) →(107,99) →(106,100) = 90

Manhattan:

(106,13) →(106,14) →(106,15) →(105,16) →(104,17) →(103,18) →(102,19) →(101,20) →(100,21) →(99,22) →(98,23) →(97,24) →(96,25) →(95,26) →(94,27) →(93,28) → (92,29) →(91,30) →(90,31) →(89,32) →(88,33) →(89,34) →(90,35) →(91,36) → (91,37) →(92,38) →(93,39) →(93,40) →(93,41) →(93,42) →(93,43) →(93,44) → (93,45) →(93,46) →(93,47) →(93,48) →(93,49) →(93,50) →(93,51) →(93,52) → (93,53) →(93,54) →(94,55) →(95,56) →(95,57) →(95,58) →(95,59) →(95,60) → (95,61) →(96,62) →(96,63) →(97,64) →(98,65) →(98,66) →(99,67) →(100,68) → (101,69) →(102,70) →(103,69) →(104,70) →(105,71) →(106,72) →(107,73) →(107,74) →(107,75) →(107,76) →(107,77) →(107,78) →(107,79) →(107,80) →(107,81) →(107,82) →(107,83) →(107,84) →(107,85) →(107,86) →(107,87)

→(107,88) →(107,89) →(107,90) →(107,91) →(107,92) →(107,93) →(107,94) →(107,95) →(107,96) →(107,97) →(107,98) →(107,99) →(106,100) = 90

Diagonal:

(106,13) →(106,14) →(106,15) →(106,16) →(105,17) →(104,18) →(103,19) →(102,20) →(101,21) →(100,22) →(100,23) →(100,24) →(99,25) →(98,26) →(97,27) →(96,28) →(95,29) →(94,30) →(93,31) →(92,32) →(91,33) →(90,34) →(90,35) →(91,36) → (91,37) →(92,38) →(93,39) →(93,40) →(93,41) →(93,42) →(93,43) →(93,44) → (93,45) →(93,46) →(93,47) →(93,48) →(93,49) →(93,50) →(93,51) →(93,52) → (93,53) →(93,54) →(94,55) →(95,56) →(95,57) →(95,58) →(95,59) →(95,60) → (95,61) →(96,62) →(96,63) →(97,64) →(98,65) →(98,66) →(99,67) →(100,68) → (101,69) →(102,70) →(103,69) →(104,70) →(105,71) →(106,72) →(107,73) →(107,74) →(107,75) →(107,76) →(107,77) →(107,78) →(107,79) →(107,80) →(107,81) →(107,82) →(107,83) →(107,84) →(107,85) →(107,86) →(107,87) →(107,88) →(107,89) →(107,90) →(107,91) →(107,92) →(107,93) →(107,94) →(107,95) →(107,96) →(107,97) →(107,98) →(107,99) →(106,100) = 90

Kernel Size 5

Euclidean:

(64,8) →(65,7) →(66,6) →(67,5) →(68,4) →(69,3) →(70,2) →(71,1) → (72,0) →(73,0) →(73,1) →(73,2) →(73,3) →(73,4) →(73,5) →(73,6) → (73,7) →(73,8) →(73,9) →(73,10) →(73,11) →(73,12) →(73,13) →(73,14) → (73,15) →(73,16) →(73,17) →(73,18) →(73,19) →(73,20) →(73,21) →(73,22) → (73,23) →(73,24) →(73,25) →(73,26) →(73,27) →(73,28) →(73,29) →(73,30) → (73,31) →(73,32) →(73,33) →(73,34) →(73,35) →(73,36) →(73,37) →(73,38) → (73,39) →(73,40) →(73,41) →(73,42) →(73,43) →(73,44) →(73,45) →(73,46) → (73,47) →(73,48) →(73,49) →(73,50) →(73,51) →(73,52) →(73,53) →(73,54) → (73,55) →(73,56) →(73,57) →(73,58) →(73,59) →(73,60) →(73,61) →(73,62) → (73,63) →(73,64) →(73,65) →(73,66) →(73,67) →(73,68) →(73,69) →(73,70) → (73,71) →(72,72) →(71,72) →(70,72) →(69,72) →(68,72) →(67,72) →(66,72) → (65,72) →(64,72) →(63,72) →(62,72) →(61,72) →(60,72) →(59,72) →(58,72) → (57,72) →(56,72) →(55,72) →(54,72) →(53,72) →(54,71) →(55,70) →(56,69) → (57,68) →(58,67) →(59,66) →(60,65) →(61,64) →(62,63) →(63,62) →(63,61) → (64,60) = 113

Manhattan:

(64,8) →(65,7) →(66,6) →(67,5) →(68,4) →(69,3) →(70,2) →(71,1) → (72,0) →(73,0) →(73,1) →(73,2) →(73,3) →(73,4) →(73,5) →(73,6) → (73,7) →(73,8) →(73,9) →(73,10) →(73,11) →(73,12) →(73,13) →(73,14) → (73,15) →(73,16) →(73,17) →(73,18) →(73,19) →(73,20) →(73,21) →(73,22) → (73,23) →(73,24) →(73,25) →(73,26) →(73,27) →(73,28) →(73,29) →(73,30) → (73,31) →(73,32) →(73,33) →(73,34) →(73,35) →(73,36) →(73,37) →(73,38) → (73,39) →(73,40) →(73,41) →(73,42) →(73,43) →(73,44) →(73,45) →(73,46) → (73,47) →(73,48) →(73,49) →(73,50) →(73,51) →(73,52) →(73,53) →(73,54) → (73,55) →(73,56) →(73,57) →(73,58) →(73,59) →(73,60) →(73,61) →(73,62) → (73,63) →(73,64) →(73,65) →(73,66) →(73,67) →(73,68) →(73,69) →(73,70) → (73,71) →(72,72) →(71,72) →(70,72) →(69,72) →(68,72) →(67,72) →(66,72) → (65,72) →(64,72) →(63,72) →(62,72) →(61,72) →(60,72) →(59,72) →(58,72) → (57,72) →(56,72) →(55,72) →(54,72) →(53,72) →(54,71) →(55,70) →(56,69) → (57,68) →(58,67) →(59,66) →(60,65) →(61,64) →(62,63) →(63,62) →(63,61) → (64,60) = 113

Diagonal:

(64,8) →(65,7) →(66,6) →(67,5) →(68,4) →(69,3) →(70,2) →(71,1) → (72,0) →(73,0) →(73,1) →(73,2) →(73,3) →(73,4) →(73,5) →(73,6) → (73,7) →(73,8) →(73,9) →(73,10) →(73,11) →(73,12) →(73,13) →(73,14) → (73,15) →(73,16) →(73,17) →(73,18) →(73,19) →(73,20) →(73,21) →(73,22) → (73,23) →(73,24) →(73,25) →(73,26) →(73,27) →(73,28)

→(73,29) →(73,30) → (73,31) →(73,32) →(73,33) →(73,34) →(73,35) →(73,36) →(73,37) →(73,38) → (73,39) →(73,40) →(73,41) →(73,42) →(73,43) →(73,44) →(73,45) →(73,46) → (73,47) →(73,48) →(73,49) →(73,50) →(73,51) →(73,52) →(73,53) →(73,54) → (73,55) →(73,56) →(73,57) →(73,58) →(73,59) →(73,60) →(73,61) →(73,62) → (73,63) →(73,64) →(73,65) →(73,66) →(73,67) →(73,68) →(73,69) →(73,70) → (73,71) →(72,72) →(71,72) →(70,72) →(69,72) →(68,72) →(67,72) →(66,72) → (65,72) →(64,72) →(63,72) →(62,72) →(61,72) →(60,72) →(59,72) →(58,72) → (57,72) →(56,72) →(55,72) →(54,72) →(55,71) →(56,70) →(57,69) →(58,68) → (59,67) →(60,66) →(60,65) →(61,64) →(62,63) →(63,62) →(63,61) →(64,60) = 113

Kernel Size 7

Euclidean:

(45,5) →(46,4) →(47,3) →(48,2) →(49,1) →(50,0) →(51,1) →(52,0) → (52,1) →(52,2) →(52,3) →(52,4) →(52,5) →(52,6) →(52,7) →(52,8) → (52,9) →(52,10) →(52,11) →(52,12) →(52,13) →(52,14) →(52,15) →(52,16) → (52,17) →(52,18) →(52,19) →(52,20) →(52,21) →(52,22) →(52,23) →(52,24) → (52,25) →(52,26) →(52,27) →(52,28) →(52,29) →(52,30) →(52,31) →(52,32) → (52,33) →(52,34) →(52,35) →(52,36) →(52,37) →(52,38) →(52,39) →(52,40) → (52,41) →(51,41) →(50,41) →(49,41) →(48,41) →(47,41) →(46,41) →(45,42) = 56

Manhattan:

(45,5) →(46,4) →(47,3) →(48,2) →(49,1) →(50,0) →(51,1) →(52,0) → (52,1) →(52,2) →(52,3) →(52,4) →(52,5) →(52,6) →(52,7) →(52,8) → (52,9) →(52,10) →(52,11) →(52,12) →(52,13) →(52,14) →(52,15) →(52,16) → (52,17) →(52,18) →(52,19) →(52,20) →(52,21) →(52,22) →(52,23) →(52,24) → (52,25) →(52,26) →(52,27) →(52,28) →(52,29) →(52,30) →(52,31) →(52,32) → (52,33) →(52,34) →(52,35) →(52,36) →(52,37) →(52,38) →(52,39) →(52,40) → (52,41) →(51,41) →(50,41) →(49,41) →(48,41) →(47,41) →(46,41) →(45,42) = 56

Diagonal:

(45,5) →(46,4) →(47,3) →(48,2) →(49,1) →(50,0) →(51,1) →(52,0) → (52,1) →(52,2) →(52,3) →(52,4) →(52,5) →(52,6) →(52,7) →(52,8) → (52,9) →(52,10) →(52,11) →(52,12) →(52,13) →(52,14) →(52,15) →(52,16) → (52,17) →(52,18) →(52,19) →(52,20) →(52,21) →(52,22) →(52,23) →(52,24) → (52,25) →(52,26) →(52,27) →(52,28) →(52,29) →(52,30) →(52,31) →(52,32) → (52,33) →(52,34) →(52,35) →(52,36) →(52,37) →(52,38) →(52,39) →(52,40) → (52,41) →(52,42) →(51,42) →(50,42) →(49,42) →(48,42) →(47,42) →(46,42) → (45,42) = 57

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