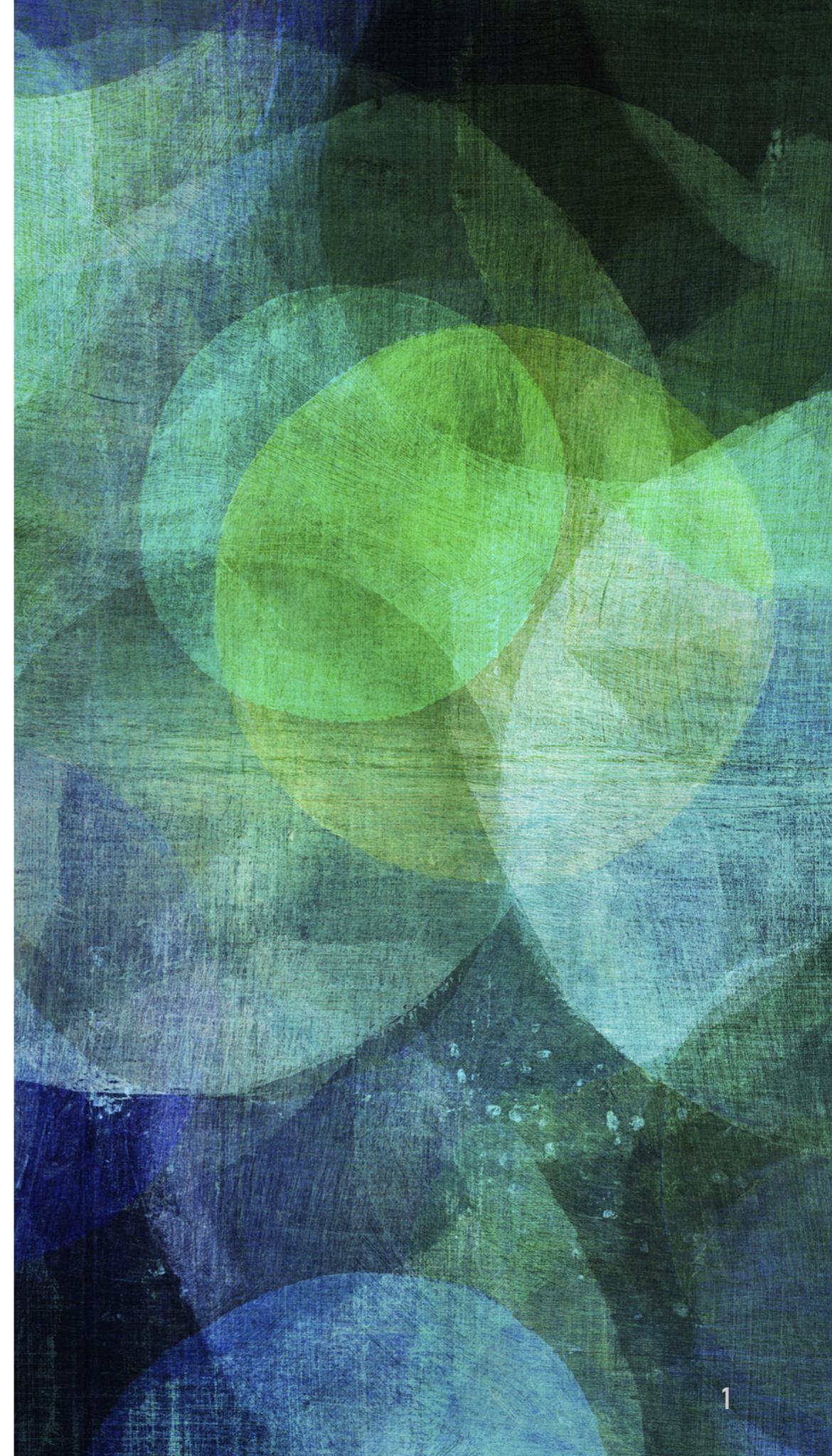


# COMPARISON BETWEEN A STAR AND DIJKSTRA ALGORITHM WITH DIFFERENT KERNEL AND OBSTACLE SIZES AND HEURISTICS USING OCCUPANCY GRID IN AN AUTONOMOUS VEHICLE

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# Project Goal

- The aim of this research is to find the shortest, smartest path between a source and a destination
  - by comparing the two algorithms
    - A star and Dijkstra
    - heuristics
  - within different kernel and obstacle sizes
  - Overall processing time taken
  - Space and Time Complexity

# Motivation

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- Finds the shortest path between a source and the destination through obstacles
- How does A star and Dijkstra output differ?
- Does different kernel sizes affect the path?
- Is there a significant difference when different heuristics are used?
- Is there a significant difference in path when obstacles sizes are increased?

# Importance

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- Algorithms to find a shortest path are important not only in robotics, but also in video games, network routing and gene sequencing [3]

## Related Work on this Task

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1. Introduction to A\* [4]
2. On the heuristics of A\* in ITS and robot path planning [6]
3. Syed Abdullah Fadzli, Sani Iyal Abdulkadir, Mokhairi Makhtar, and Azrul Amri Jamal. Robotic indoor path planning using Dijkstra's algorithm with multi-layer dictionaries. In 2015 2nd International Conference on Information Science and Security (ICISS), pages 1-4. IEEE, 2015. [7]

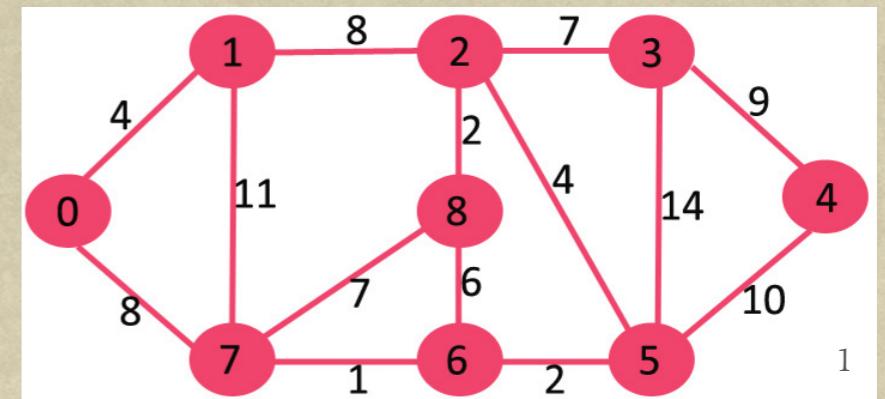
# OCCUPANCY GRID

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- An array with occupancy variables
- This method divides area into cells and categorizes as occupied, free or unknown.
- Occupancy probability
  - if the grid cell is occupied:  $p(m_i) = 1$
  - if the grid cell is not occupied:  $p(m_i) = 0$
  - no knowledge of the grid cell:  $p(m_i) = 0.5$
- Trajectory is found by computing the shortest path that do not cross any of the occupied cells

# Dijkstra Algorithm

- Algorithm to calculate the shortest path from the source to all vertices in the given graph (grid)
- Invented by Edsger W. Dijkstra in 1956
- Greedy Algorithm
- Weighted Graph  $G = (E, V)$
- Graphs must be connected
- Directed and undirected graph
- Edges must have non-negative weights
- Time Complexity; worst-case:  $O(V)^2$
- Space Complexity; worst-case:  $O(V)$
- Applications:
  - Geographical Maps
  - IP routing
  - Telephone network

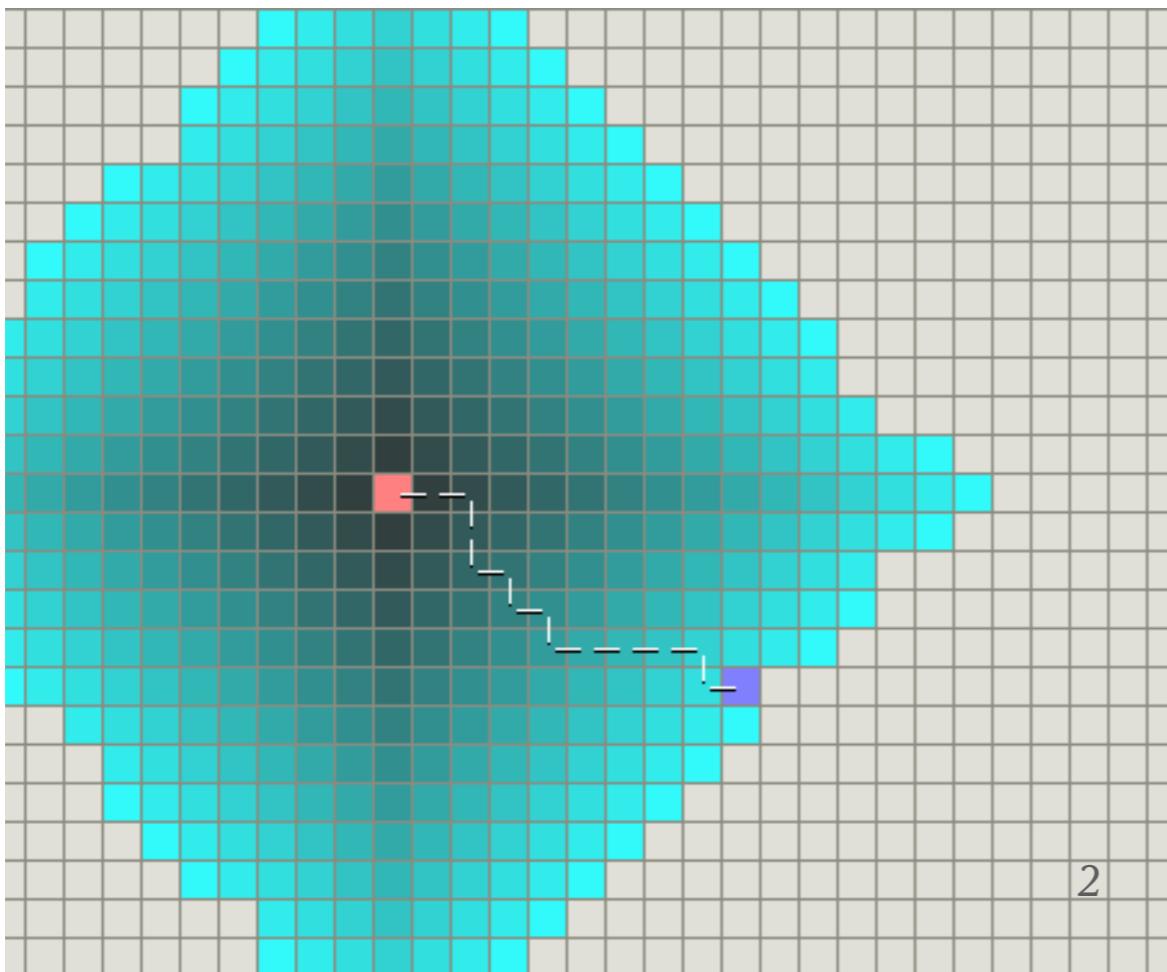


# A star Algorithm

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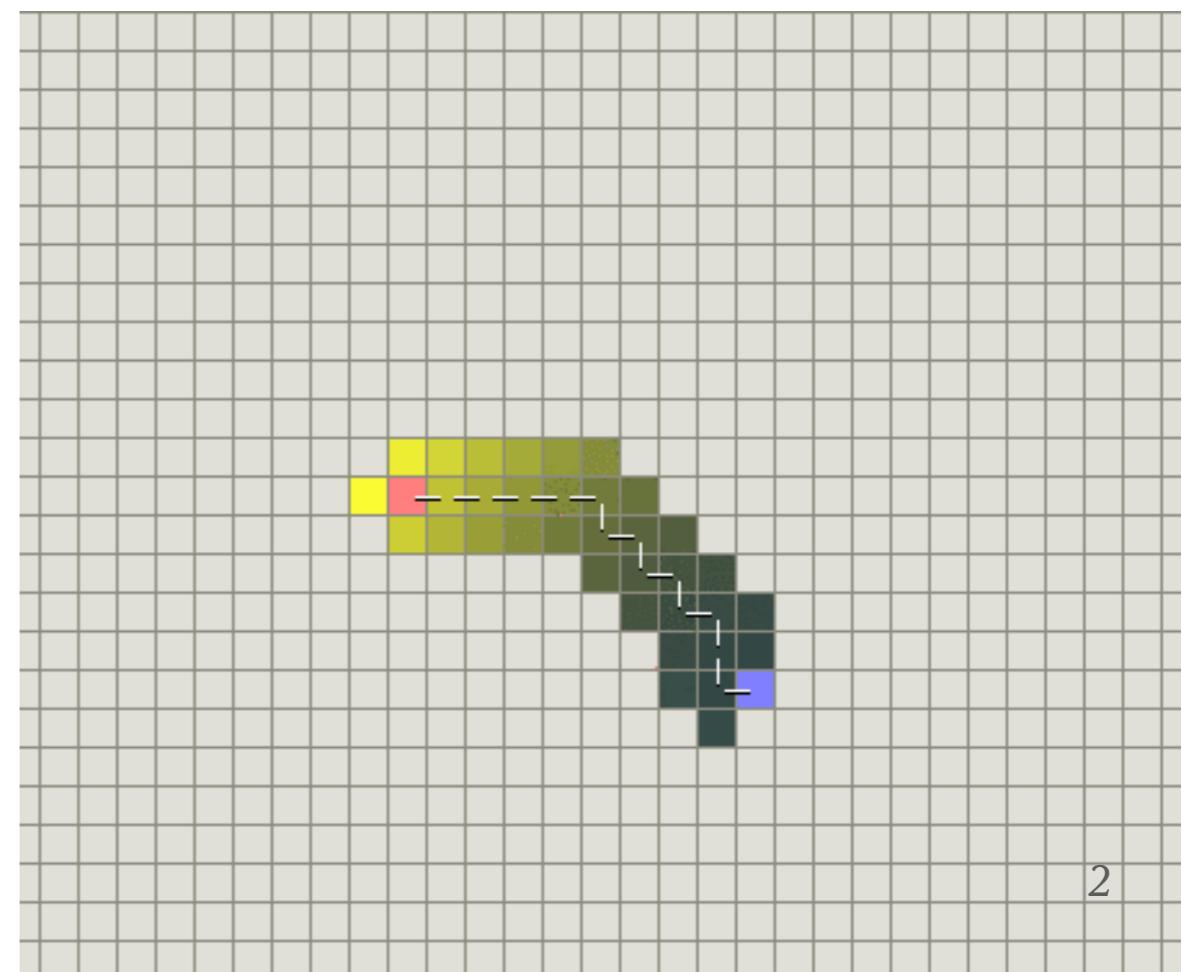
- Algorithm to calculate the shortest path from the source to a given destination.
- Extension of Dijkstra by using heuristics
- Published in 1968
- Time Complexity; worst-case:  $O(|E|) = O(b^d)$
- Space Complexity; worst-case:  $O(|V|) = O(b^d)$
- A star uses a function that finds an estimate of the total cost of a path for each node.
  - $f(n) = g(n)+h(n)$ 
    - $f(n)$ ; total cost of path through node
    - $g(n)$ ; current cost to reach node
    - $h(n)$ ; estimated cost from current node to destination node
- Three types of heuristics: Manhattan, Diagonal & Euclidean

# Dijkstra



Blue color shows the number of nodes visited  
by Dijkstra's algorithm

# A Star



Yellow color means nodes closer to source node  
and green means nodes closer to the  
destination

# Differences in number of vertices explore to compute the path

Dijkstra

```
0->122->244->366->488->610->732->854->976->1098->1220->1342->1464->1586->1708->1830->1952->2074->2196->2318->2440->2562->2684->2806->2928->3050->3172->3294->3416->3538->3660->3782->3904->4026->4148->4149->4150->4272->4273->4274->4275->4276->4398->4518->4640->4762->4884->5006->5128->5248->5368->5488->5608->5728->5848->5968->5966->5844->5843->5842->5841->5840->5839->5838->5958->6080->6202->6324->6446->6568->6690->6812->6934->7056->7178->7300->7422->7544->7666->7788->7910->8032->8154->8276->8398->8399->8400->8522->8523->8524->8646->8647->8648->8649->8650->8772->8773->8774->8775->8897->8898->8899->8900->8901->9023->9024->9025->9026->9148->9149->9029->9030->9031->9153->9154->9155->9156->9278->9279->9280->9281->9403->9404->9405->9406->9407->9529->9649->9769->9889->10009->10129->10249->10369->10489->10609->10729->10849->10969->11089->11209->11329->11449->11569->11689->11688->11687->11686->11684->11562->11561->11560->11559->11558->11557->11556->11434->11433->11432->11431->11430->11429->11549->11548->11668->11790->11912->12034->12156->12278->12400->12522->12644->12766->12888->13010->13132->13254->13376->13498->13620->13742->13864->13986->14108->14230->14352->14474->14596->14718->14719->14720->14721->14722->14723->14724->14725->14726->14727->14728->14729->14730->14731->14732->14733->14734->14735->14736->14737->14738->14739->14740->14741->14742->14743->14744->14745->14746->14747->14748->14749->14750->14751->14752->14753->14754->14755->14756->14757->14758->14759->14760->14761
```

231

```
0->74->148->222->296->370->444->518->592->666->740->814->888->962->890->818->892->893->894->895->896->897->971->1045->1046->1120->1121->1122->1196->1197->1271->1345->1419->1493->1567->1640->1713->1786->1860->1933->2007->2081->2155->2229->2303->2304->2378->2379->2307->2308->2309->2310->2238->2166->2094->2095->2023->2024->1952->1880->1807->1733->1732->1731->1803
```

65

```
0->53->106->159->212->265->318->371->424->477->426->479->428->429->430->431->484->537->590->591->592->593->646->699->751->804->856->908->960->1013->1066->1119->1172->1173->1174->1175->1176->1177->1126->1075->1024->972->920->869->817->766->714->663->610->557->506->454->402
```

53

A Star

```
(10,10) -> (11,10) -> (12,10) -> (13,10) -> (14,10) -> (15,10) -> (16,10) -> (17,10) -> (18,10) -> (19,10) -> (20,10) -> (21,10) -> (22,11) -> (23,12) -> (24,13) -> (25,13) -> (26,13) -> (27,13) -> (28,13) -> (29,13) -> (30,13) -> (31,13) -> (32,13) -> (33,14) -> (34,15) -> (35,16) -> (36,17) -> (37,18) -> (38,19) -> (39,20) -> (40,21) -> (41,22) -> (42,23) -> (43,24) -> (44,25) -> (45,26) -> (46,27) -> (47,28) -> (48,29) -> (49,30) -> (50,31) -> (51,32) -> (52,33) -> (53,34) -> (54,35) -> (55,36) -> (56,37) -> (57,38) -> (58,39) -> (59,40) -> (60,41) -> (61,42) -> (62,43) -> (63,44) -> (64,45) -> (65,46) -> (66,47) -> (67,48) -> (68,49) -> (69,50) -> (70,51) -> (71,52) -> (72,53) -> (73,54) -> (74,55) -> (75,56) -> (76,57) -> (77,58) -> (78,59) -> (79,60) -> (80,61) -> (80,62) -> (80,63) -> (80,64) -> (80,65) -> (80,66) -> (80,67) -> (80,68) -> (80,69) -> (80,70) -> (80,71) -> (80,72) -> (80,73) -> (80,74) -> (80,75) -> (80,76) -> (80,77) -> (80,78) -> (80,79) -> (80,80) -> (81,81) -> (82,82) -> (83,83) -> (84,84) -> (85,85) -> (86,86) -> (87,87) -> (88,88) -> (89,89) -> (90,90) -> (91,91) -> (92,92) -> (93,93) -> (94,94) -> (95,95) -> (96,96) -> (97,97) -> (98,98) -> (99,99) -> (100,100) -> (101,101) -> (102,102) -> (103,103) -> (104,104) -> (105,105) -> (106,106) -> (107,107) -> (108,108) -> (109,109) -> (110,110) -> (111,111) -> (112,112) -> (113,113) -> (114,114) -> (115,115) -> (116,116) -> (117,117) -> (118,118) -> (119,119) -> (120,120)
```

108

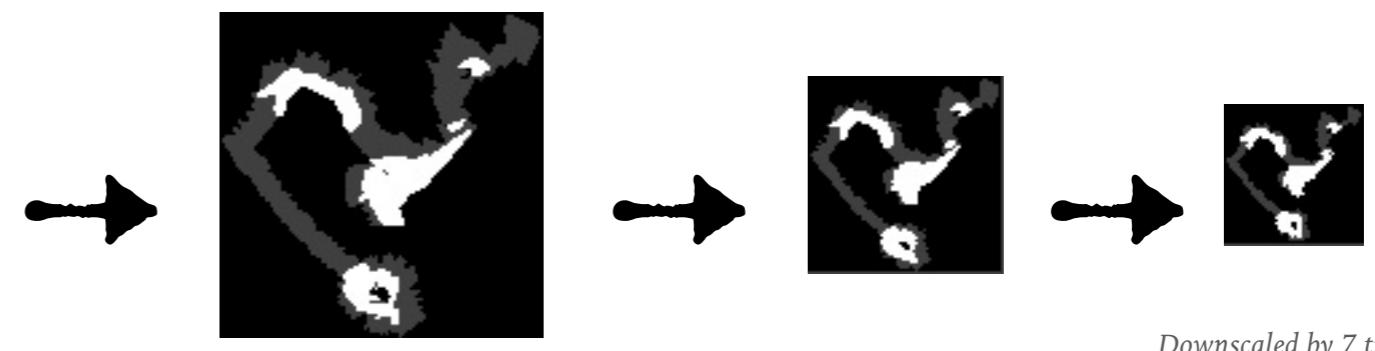
```
(2,2) -> (3,2) -> (4,2) -> (5,2) -> (6,2) -> (7,2) -> (8,2) -> (9,2) -> (10,2) -> (11,2) -> (12,2) -> (13,2) -> (14,3) -> (15,4) -> (16,5) -> (17,5) -> (18,6) -> (19,7) -> (20,8) -> (21,9) -> (22,10) -> (23,11) -> (24,12) -> (24,13) -> (24,14) -> (24,15) -> (24,16) -> (24,17) -> (24,18) -> (24,19) -> (24,20) -> (24,21) -> (24,22) -> (24,23) -> (24,24)
```

36

```
(1,1) -> (2,1) -> (3,1) -> (4,1) -> (5,1) -> (6,1) -> (7,1) -> (8,1) -> (9,1) -> (10,2) -> (11,3) -> (12,4) -> (13,5) -> (14,6) -> (15,7) -> (16,8) -> (17,9) -> (17,10) -> (17,11) -> (17,12) -> (17,13) -> (17,14) -> (17,15) -> (17,16) -> (17,17)
```

25

# Downsampled Images

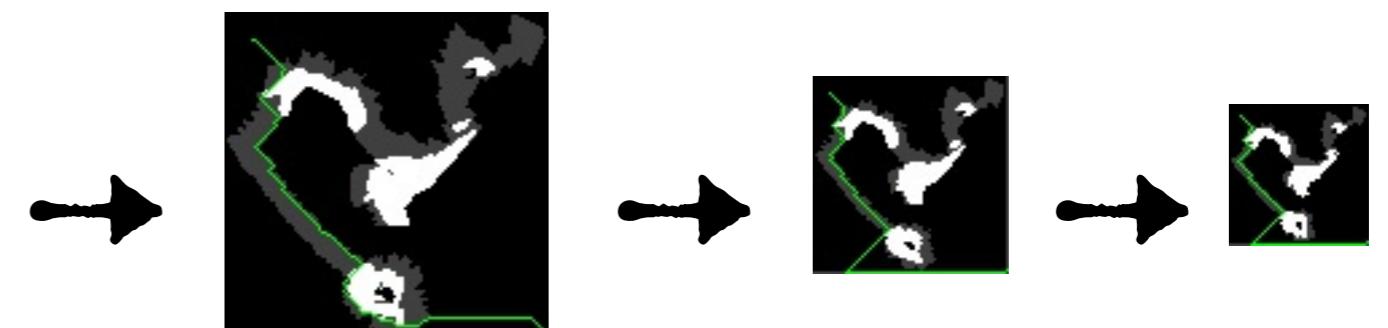
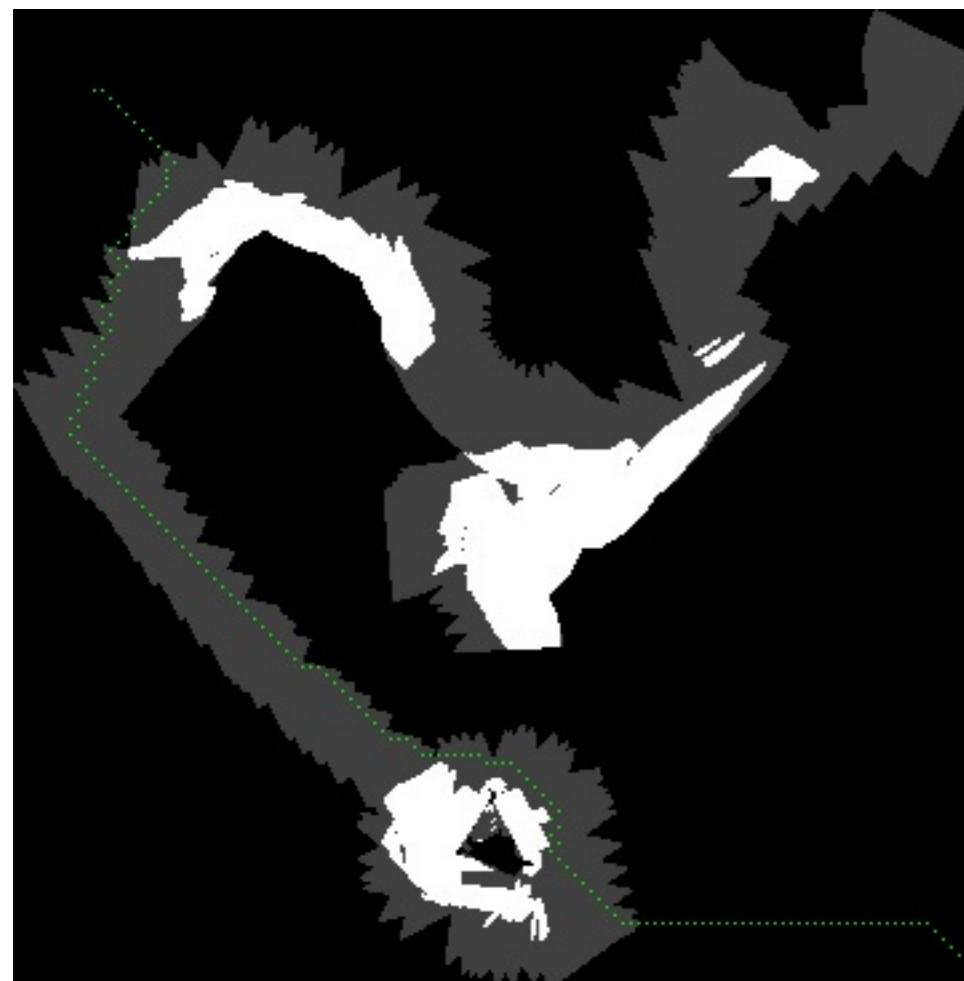


Downscaled by 7 times

Downscaled by 5 times

Downscaled by 3 times

*Original Map*



Same path downscaled by 7 times

Same path downscaled by 5 times

Same path downscaled by 3 times

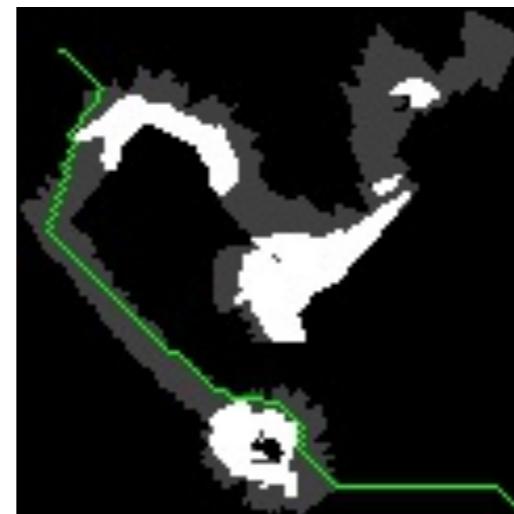
*A star computed on original map*

## DIFFERENCES IN PATH WITH DIFFERENT KERNEL SIZES IN A STAR



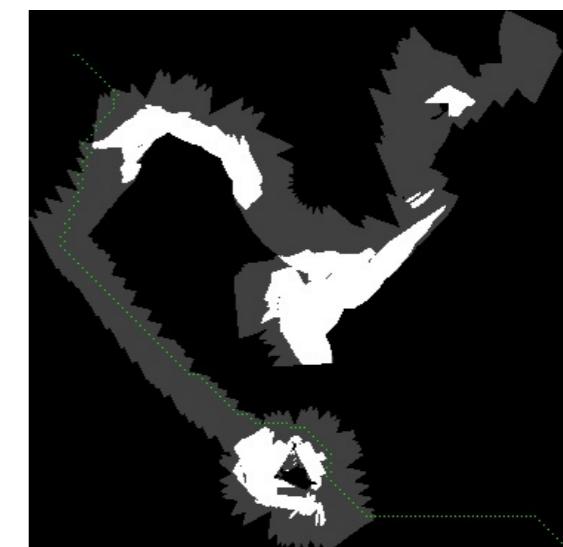
Downscaled image

Kernel size : 3



Path Computed by A star

Source: (30,30) Destination: (360,360)



Correct Path after computing the correct

source and destination points within the right

resolution : Source: (10,10) Destination: (120,120)



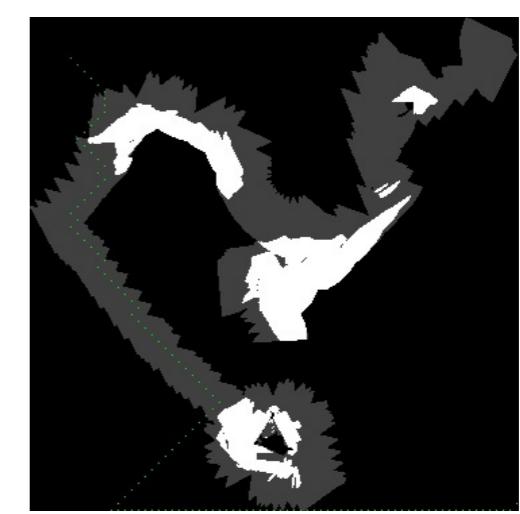
Downscaled image

Kernel size : 5



Source: (6,6) Destination: (72,72)

Path differs due to grid size differences



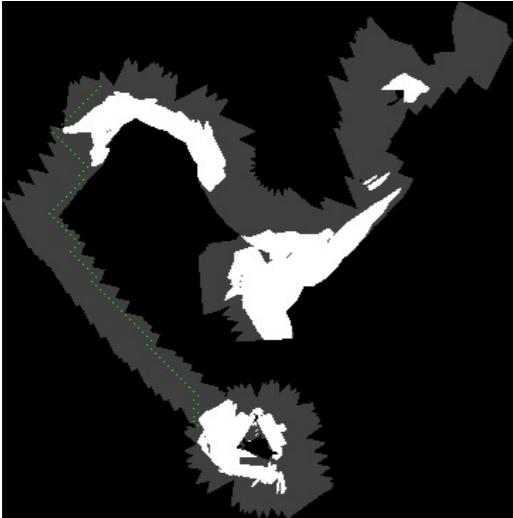
Correct Path:

Source (0,0) -> (6,6)

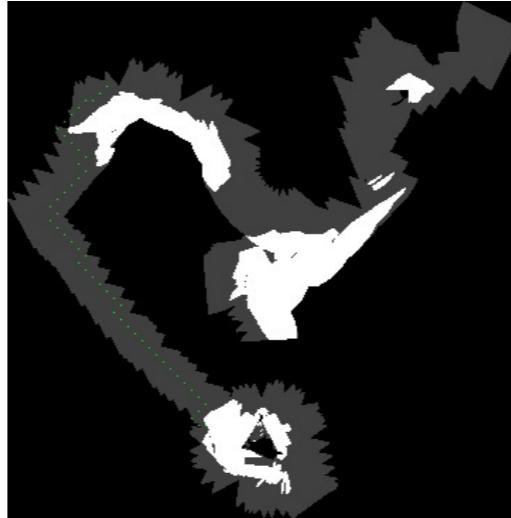
Destination: (120,120) -> (72,72)

# Differences in Path with different heuristics in A star

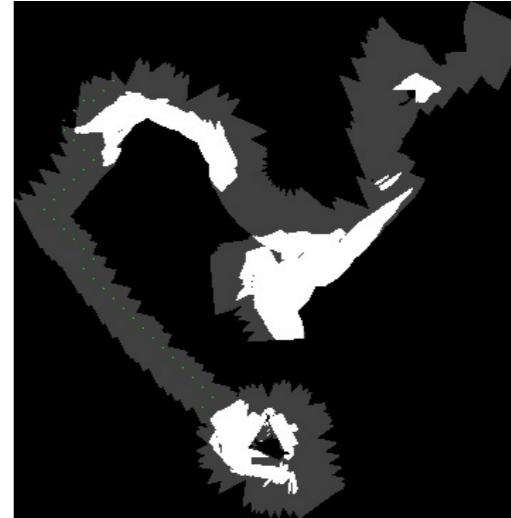
Manhattan:



Kernel Size 3

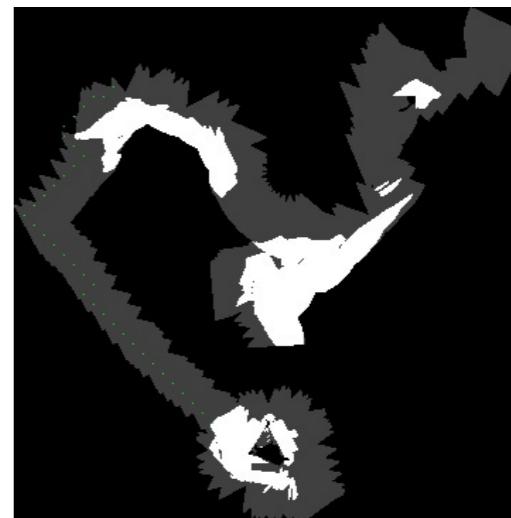
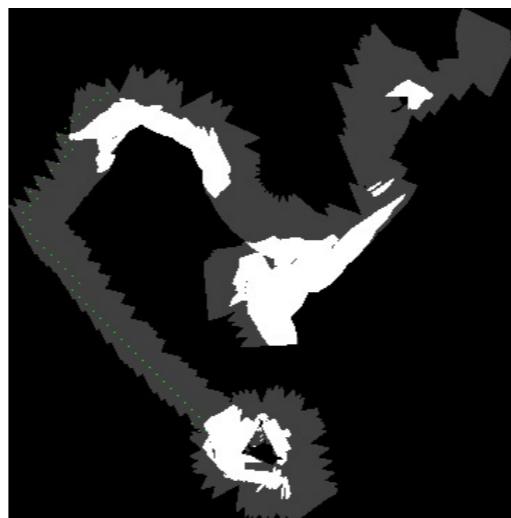
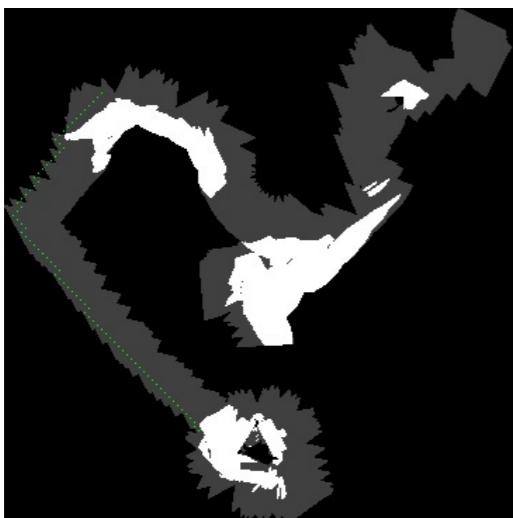


Kernel Size 5

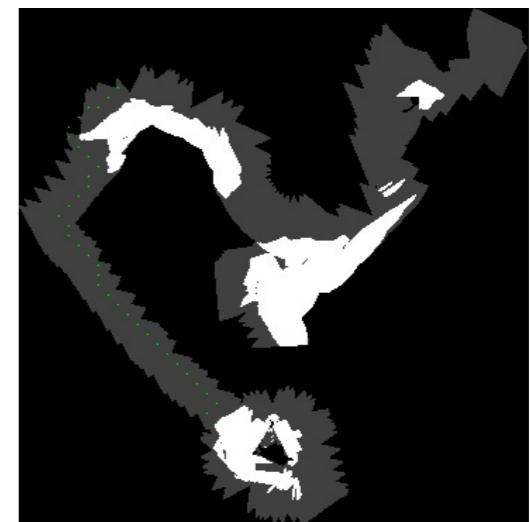
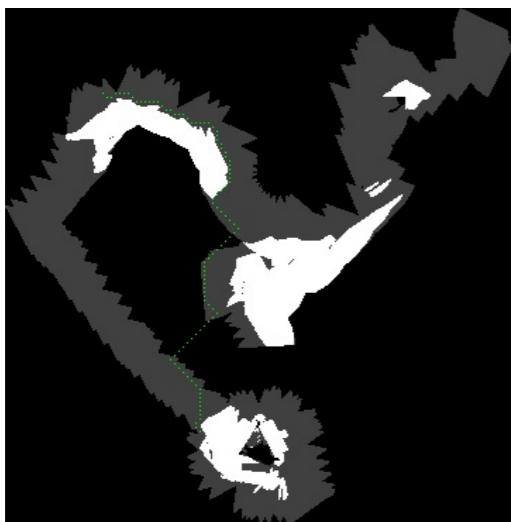


Kernel Size 7

Euclidean:



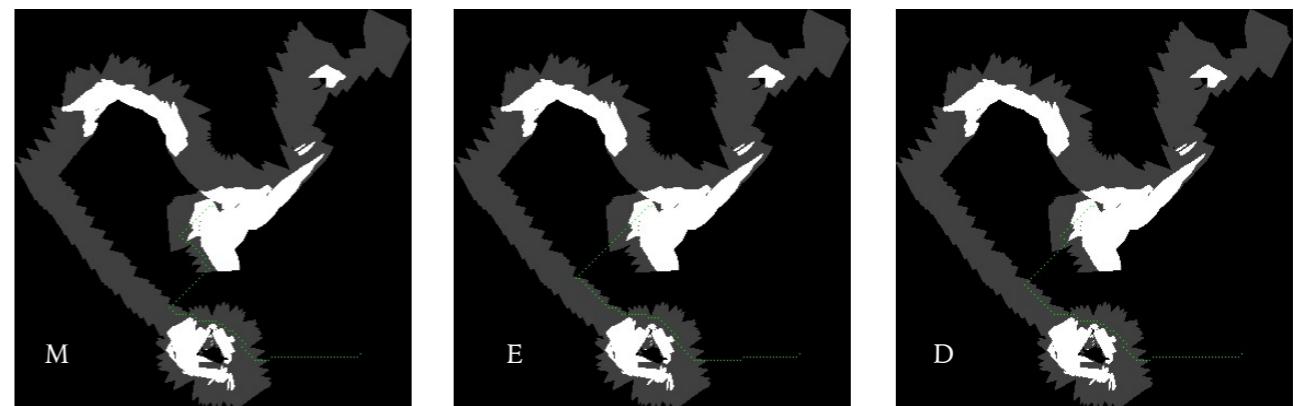
Diagonal:



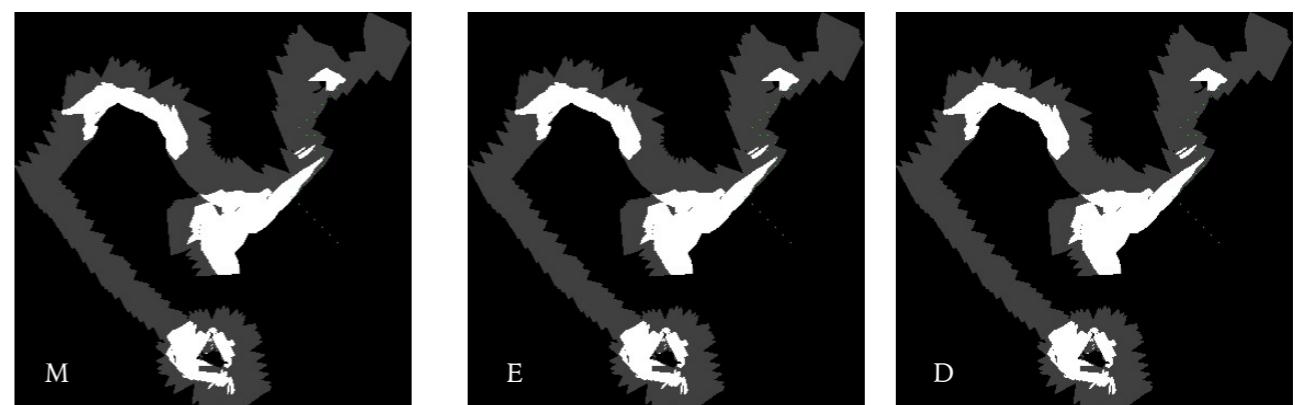
## COMPARISON WITH ALL THREE HEURISTICS

Compared these heuristics with 10 sample coordinates in different kernel sizes

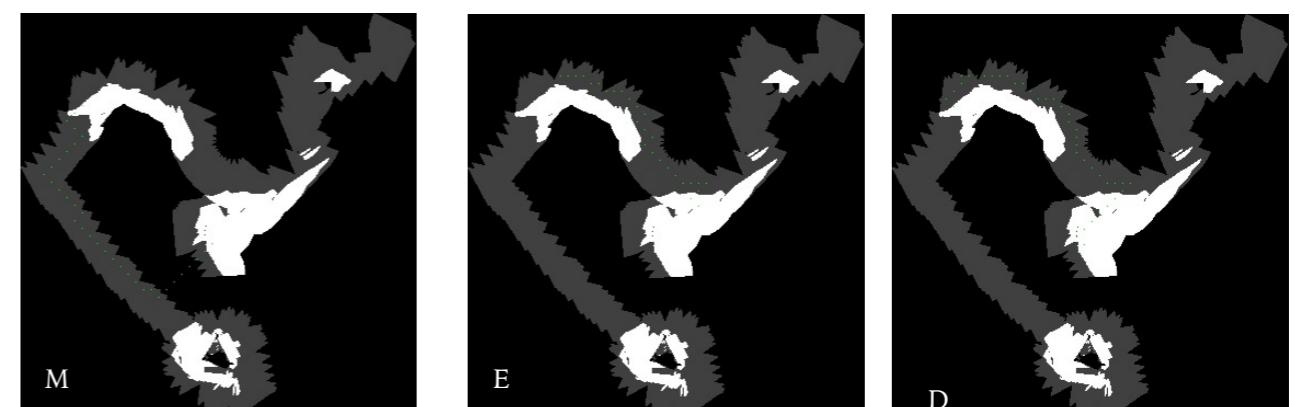
1. Source: 30,30 Destination: 360,360
2. Source: 60,70 Destination: 60,340
3. Source: 60,70 Destination: 140,300
4. Source: 60,90 Destination: 180,210
5. Source: 60,60 Destination: 180,315
6. Source: 300,70 Destination: 300,210
7. Source: 300,70 Destination: 180,315
8. Source: 180,180 Destination: 315,315
9. Source: 40,320 Destination: 340,60
10. Source: 40,320 Destination: 300,320



Path computed with coordinates no 8 in kernel size 3



Path computed with coordinates no 6 in kernel size 7



Path computed with coordinates no 4 in kernel size 5

## Comparison with different Obstacle Sizes I

.....

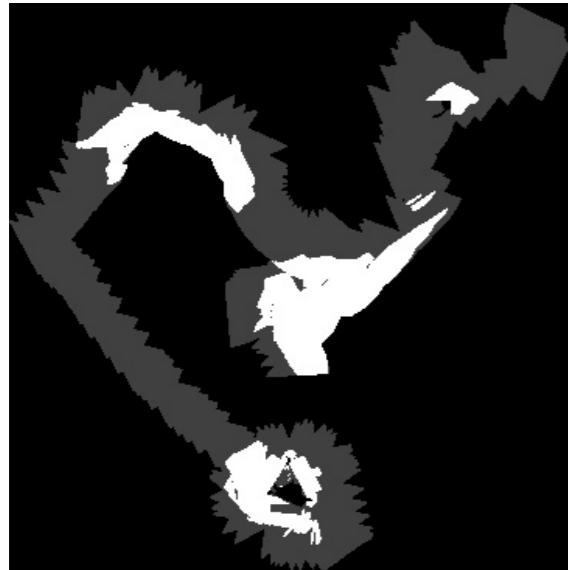


Fig:1 100,50,1



Fig:2 100,2,1

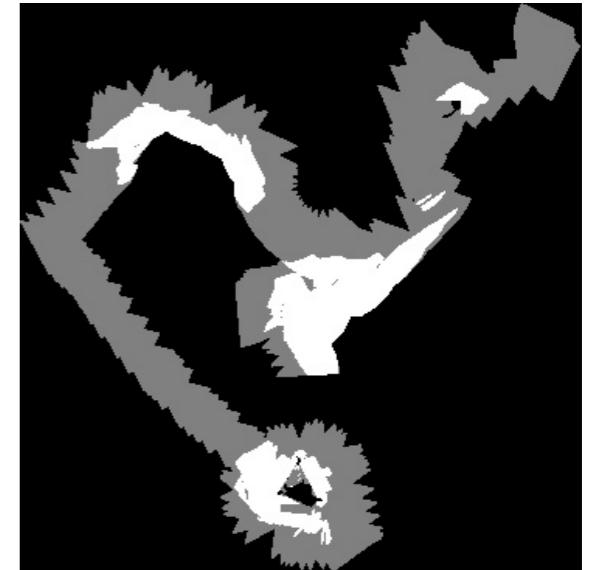


Fig:3 60,50,40

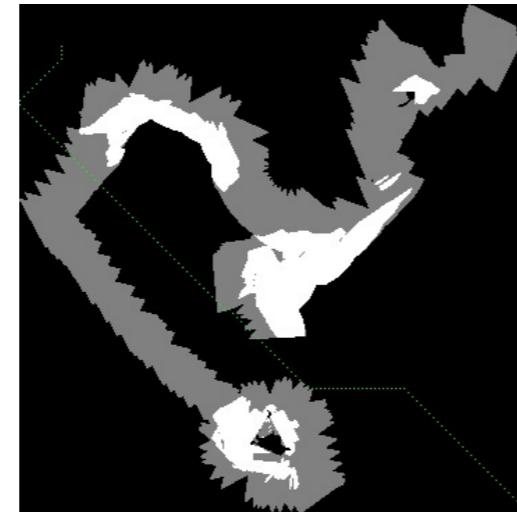
The free cells in fig 2 is barely visible

In fig 3, the free cells have a lighter grey color closer to white.

## Comparison with different Obstacle Sizes II

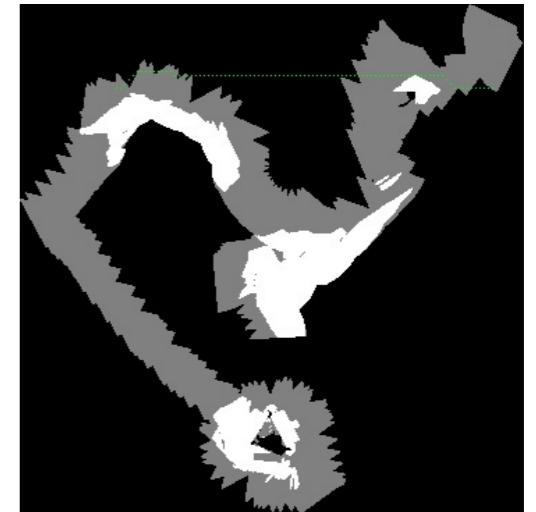
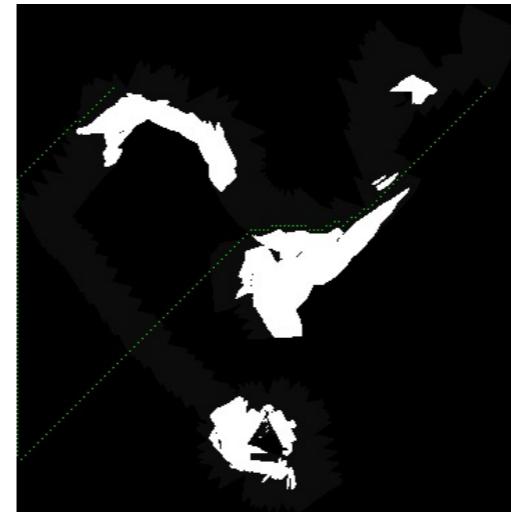
Source : 30,30

Destination: 360,360



Source: 60,70

Destination: 60,340



- By changing the values, the path changes completely
- There is almost no recognition of free cells in first set
- In second set, the unknown cells are mistaken with free cells and computes the shortest path by making a straight line and passes through the obstacles respectively.

# PATH COMPARISON

## Kernel Size 3

### Euclidean

$(30,20) \rightarrow (31,19) \rightarrow (32,18) \rightarrow (33,17) \rightarrow (34,16) \rightarrow (33,15) \rightarrow (32,14) \rightarrow (31,13) \rightarrow$   
 $(30,13) \rightarrow (29,14) \rightarrow (28,15) \rightarrow (28,16) \rightarrow (27,17) \rightarrow (26,18) \rightarrow (26,19) \rightarrow (25,20) \rightarrow$   
 $(24,21) \rightarrow (23,22) \rightarrow (22,23) \rightarrow (21,24) \rightarrow (21,25) \rightarrow (20,26) \rightarrow (20,27) \rightarrow (20,28) \rightarrow$   
 $(20,29) \rightarrow (21,30) \rightarrow (22,31) \rightarrow (22,32) \rightarrow (22,33) \rightarrow (22,34) \rightarrow (22,35) \rightarrow (22,36) \rightarrow$   
 $(23,37) \rightarrow (24,38) \rightarrow (24,39) \rightarrow (25,40) \rightarrow (25,41) \rightarrow (25,42) \rightarrow (26,43) \rightarrow (27,44) \rightarrow$   
 $(27,45) \rightarrow (27,46) \rightarrow (27,47) \rightarrow (27,48) \rightarrow (28,49) \rightarrow (29,50) \rightarrow (30,50) \rightarrow (31,50) \rightarrow$   
 $(32,51) \rightarrow (33,51) \rightarrow (34,52) \rightarrow (35,52) \rightarrow (36,53) \rightarrow (37,53) \rightarrow (38,54) \rightarrow (39,55) \rightarrow$   
 $(40,56) \rightarrow (41,57) \rightarrow (42,58) \rightarrow (43,59) \rightarrow (44,60) \rightarrow (45,61) \rightarrow (46,62) \rightarrow (47,63) \rightarrow$   
 $(48,64) \rightarrow (49,65) \rightarrow (50,66) \rightarrow (51,67) \rightarrow (52,68) \rightarrow (52,69) \rightarrow (52,70) \rightarrow (52,71) \rightarrow$   
 $(52,72) \rightarrow (52,73) \rightarrow (52,74) \rightarrow (52,75) \rightarrow (52,76) \rightarrow (52,77) \rightarrow (53,76) \rightarrow (54,75) \rightarrow$   
 $(55,74) \rightarrow (56,73) \rightarrow (57,72) \rightarrow (58,71) \rightarrow (59,70) \rightarrow (60,69) \rightarrow (61,68) \rightarrow (62,67) \rightarrow$   
 $(63,66) \rightarrow (64,65) \rightarrow (65,64) \rightarrow (66,63) \rightarrow (67,62) \rightarrow (68,61) \rightarrow (69,60) \rightarrow (70,60) = 9$

Manhattan

(30,20) -> (31,19) -> (32,18) -> (33,17) -> (34,16) -> (33,15) -> (32,14) -> (31,13) ->  
 (30,13) -> (29,14) -> (28,15) -> (28,16) -> (27,17) -> (26,18) -> (26,19) -> (25,20) ->  
 (24,21) -> (23,22) -> (22,23) -> (21,24) -> (21,25) -> (20,26) -> (20,27) -> (20,28) ->  
 (20,29) -> (21,30) -> (22,31) -> (22,32) -> (22,33) -> (22,34) -> (22,35) -> (22,36) ->  
 (23,37) -> (24,38) -> (24,39) -> (25,40) -> (25,41) -> (25,42) -> (26,43) -> (27,44) ->  
 (27,45) -> (27,46) -> (27,47) -> (27,48) -> (28,49) -> (29,50) -> (30,51) -> (31,52) ->  
 (32,53) -> (33,54) -> (34,55) -> (35,56) -> (36,57) -> (37,56) -> (38,57) -> (39,56) ->  
 (40,57) -> (41,58) -> (42,59) -> (43,58) -> (44,59) -> (45,60) -> (46,61) -> (47,62) ->  
 (48,63) -> (49,64) -> (50,65) -> (51,66) -> (52,67) -> (52,68) -> (52,69) -> (52,70) ->  
 (52,71) -> (52,72) -> (52,73) -> (52,74) -> (52,75) -> (52,76) -> (52,77) -> (53,76) ->  
 (54,75) -> (55,74) -> (56,73) -> (57,72) -> (58,71) -> (59,70) -> (60,69) -> (61,68) ->  
 (62,67) -> (63,66) -> (64,65) -> (65,64) -> (66,63) -> (67,62) -> (68,61) -> (69,60) ->  
 (70,60) = 97

### Diagonal

Diagonal  
 $(30,20) \rightarrow (31,19) \rightarrow (32,18) \rightarrow (33,17) \rightarrow (34,16) \rightarrow (33,15) \rightarrow (32,14) \rightarrow (31,13) \rightarrow$   
 $(30,13) \rightarrow (29,14) \rightarrow (28,15) \rightarrow (28,16) \rightarrow (27,17) \rightarrow (26,18) \rightarrow (26,19) \rightarrow (25,20) \rightarrow$   
 $(24,21) \rightarrow (23,22) \rightarrow (22,23) \rightarrow (21,24) \rightarrow (21,25) \rightarrow (20,26) \rightarrow (20,27) \rightarrow (20,28) \rightarrow$   
 $(20,29) \rightarrow (21,30) \rightarrow (22,31) \rightarrow (22,32) \rightarrow (22,33) \rightarrow (22,34) \rightarrow (22,35) \rightarrow (22,36) \rightarrow$   
 $(23,37) \rightarrow (24,38) \rightarrow (24,39) \rightarrow (25,40) \rightarrow (25,41) \rightarrow (25,42) \rightarrow (26,43) \rightarrow (27,44) \rightarrow$   
 $(27,45) \rightarrow (27,46) \rightarrow (27,47) \rightarrow (27,48) \rightarrow (28,49) \rightarrow (29,50) \rightarrow (30,51) \rightarrow (31,52) \rightarrow$   
 $(32,53) \rightarrow (33,54) \rightarrow (34,55) \rightarrow (35,56) \rightarrow (36,57) \rightarrow (37,56) \rightarrow (38,57) \rightarrow (39,56) \rightarrow$   
 $(40,57) \rightarrow (41,58) \rightarrow (42,59) \rightarrow (43,58) \rightarrow (44,59) \rightarrow (45,60) \rightarrow (46,61) \rightarrow (47,62) \rightarrow$   
 $(48,63) \rightarrow (49,64) \rightarrow (50,65) \rightarrow (51,66) \rightarrow (52,67) \rightarrow (52,68) \rightarrow (52,69) \rightarrow (52,70) \rightarrow$   
 $(52,71) \rightarrow (52,72) \rightarrow (52,73) \rightarrow (52,74) \rightarrow (52,75) \rightarrow (52,76) \rightarrow (52,77) \rightarrow (53,76) \rightarrow$   
 $(54,75) \rightarrow (55,74) \rightarrow (56,73) \rightarrow (57,72) \rightarrow (58,71) \rightarrow (59,70) \rightarrow (60,69) \rightarrow (61,68) \rightarrow$   
 $(62,67) \rightarrow (63,66) \rightarrow (64,65) \rightarrow (65,64) \rightarrow (66,63) \rightarrow (67,62) \rightarrow (68,61) \rightarrow (69,60) \rightarrow$   
 $(70,60) = 97$

## Kernel Size 7

## Euclidean

(42,10) -> (43,9) -> (44,8) -> (45,7) -> (46,6) -> (47,5) -> (48,4) -> (49,3) ->  
 (50,2) -> (51,1) -> (52,0) -> (52,1) -> (52,2) -> (52,3) -> (52,4) -> (52,5) ->  
 (52,6) -> (52,7) -> (52,8) -> (52,9) -> (52,10) -> (52,11) -> (52,12) -> (52,13) ->  
 (52,14) -> (52,15) -> (52,16) -> (52,17) -> (52,18) -> (52,19) -> (52,20) -> (52,2  
 (52,22) -> (52,23) -> (52,24) -> (52,25) -> (52,26) -> (52,27) -> (52,28) -> (51,2  
 (50,28) -> (49,29) -> (48,29) -> (47,29) -> (46,29) -> (45,29) -> (44,29) -> (43,2  
 (42,29) -> (41,28) -> (40,27) -> (40,26) -> (40,25) -> (40,24) -> (39,24) -> (38,2  
 (37,24) -> (36,24) -> (35,24) -> (34,24) -> (33,24) -> (32,24) -> (31,23) -> (30,2  
 (31,22) -> (30,21) -> (29,21) -> (28,21) -> (27,21) -> (26,22) -> (25,22) -> (24,2  
 (23,23) -> (22,24) -> (22,25) -> (22,26) -> (22,27) -> (22,28) -> (22,29) -> (22,3  
 (22,31) -> (22,32) -> (21,33) -> (21,34) -> (20,35) -> (20,36) -> (19,36) -> (18,3  
 (17,36) -> (16,37) -> (15,37) -> (14,37) -> (13,37) -> (12,37) -> (11,37) -> (10,3  
 (9,37) -> (8,37) -> (7,38) -> (6,39) -> (6,40) -> (6,41) -> (7,42) -> (7,43) ->  
 (7,44) -> (7,45) -> (6,46) -> (6,47) -> (7,48) -> (8,48) = 110

## Manhattan

(42,10) -> (43,9) -> (44,8) -> (45,7) -> (46,6) -> (47,5) -> (48,4) -> (49,3) ->  
 (50,2) -> (51,1) -> (52,0) -> (52,1) -> (52,2) -> (52,3) -> (52,4) -> (52,5) ->  
 (52,6) -> (52,7) -> (52,8) -> (52,9) -> (52,10) -> (52,11) -> (52,12) -> (52,13) ->  
 (52,14) -> (52,15) -> (52,16) -> (52,17) -> (52,18) -> (52,19) -> (52,20) -> (52,21)  
 (52,22) -> (52,23) -> (52,24) -> (52,25) -> (52,26) -> (52,27) -> (52,28) -> (51,29)  
 (50,28) -> (49,29) -> (48,29) -> (47,29) -> (46,29) -> (45,29) -> (44,29) -> (43,28)  
 (42,29) -> (41,29) -> (40,28) -> (39,27) -> (40,26) -> (40,25) -> (40,24) -> (39,23)  
 (38,24) -> (37,24) -> (36,24) -> (35,24) -> (34,24) -> (33,24) -> (32,24) -> (31,23)  
 (30,23) -> (31,22) -> (30,21) -> (29,21) -> (28,21) -> (27,21) -> (26,22) -> (25,22)  
 (24,22) -> (23,23) -> (22,24) -> (22,25) -> (22,26) -> (22,27) -> (22,28) -> (22,29)  
 (22,30) -> (22,31) -> (22,32) -> (21,33) -> (21,34) -> (20,35) -> (20,36) -> (19,35)  
 (18,35) -> (17,36) -> (16,37) -> (15,37) -> (14,37) -> (13,37) -> (12,37) -> (11,37)  
 (10,37) -> (9,37) -> (8,37) -> (7,38) -> (6,39) -> (6,40) -> (6,41) -> (7,42) ->  
 (7,43) -> (7,44) -> (7,45) -> (6,46) -> (6,47) -> (7,48) -> (8,48) = 111

#### Diagonal:

$(42,10) \rightarrow (43,9) \rightarrow (44,8) \rightarrow (45,7) \rightarrow (46,6) \rightarrow (47,5) \rightarrow (48,4) \rightarrow (49,3) \rightarrow$   
 $(50,2) \rightarrow (51,1) \rightarrow (52,0) \rightarrow (52,1) \rightarrow (52,2) \rightarrow (52,3) \rightarrow (52,4) \rightarrow (52,5) \rightarrow$   
 $(52,6) \rightarrow (52,7) \rightarrow (52,8) \rightarrow (52,9) \rightarrow (52,10) \rightarrow (52,11) \rightarrow (52,12) \rightarrow (52,13) \rightarrow$   
 $(52,14) \rightarrow (52,15) \rightarrow (52,16) \rightarrow (52,17) \rightarrow (52,18) \rightarrow (52,19) \rightarrow (52,20) \rightarrow (52,2)$   
 $(52,22) \rightarrow (52,23) \rightarrow (52,24) \rightarrow (52,25) \rightarrow (52,26) \rightarrow (52,27) \rightarrow (52,28) \rightarrow (52,2)$   
 $(51,29) \rightarrow (50,29) \rightarrow (49,29) \rightarrow (48,29) \rightarrow (47,29) \rightarrow (46,29) \rightarrow (45,29) \rightarrow (44,2)$   
 $(43,29) \rightarrow (42,29) \rightarrow (41,29) \rightarrow (40,28) \rightarrow (39,27) \rightarrow (40,26) \rightarrow (40,25) \rightarrow (40,2)$   
 $(39,24) \rightarrow (38,24) \rightarrow (37,24) \rightarrow (36,24) \rightarrow (35,24) \rightarrow (34,24) \rightarrow (33,24) \rightarrow (32,2)$   
 $(31,23) \rightarrow (30,23) \rightarrow (31,22) \rightarrow (30,21) \rightarrow (29,21) \rightarrow (28,21) \rightarrow (27,21) \rightarrow (26,2)$   
 $(25,22) \rightarrow (24,22) \rightarrow (23,23) \rightarrow (22,24) \rightarrow (22,25) \rightarrow (22,26) \rightarrow (22,27) \rightarrow (22,2)$   
 $(22,29) \rightarrow (22,30) \rightarrow (22,31) \rightarrow (22,32) \rightarrow (21,33) \rightarrow (21,34) \rightarrow (20,35) \rightarrow (20,3)$   
 $(19,36) \rightarrow (18,35) \rightarrow (17,36) \rightarrow (16,37) \rightarrow (15,37) \rightarrow (14,38) \rightarrow (13,38) \rightarrow (12,3)$   
 $(11,38) \rightarrow (10,37) \rightarrow (9,38) \rightarrow (8,37) \rightarrow (7,38) \rightarrow (6,39) \rightarrow (6,40) \rightarrow (6,41) \rightarrow$   
 $(7,42) \rightarrow (7,43) \rightarrow (7,44) \rightarrow (7,45) \rightarrow (6,46) \rightarrow (6,47) \rightarrow (7,48) \rightarrow (8,48) = 112$

Kernel Size 7

## Euclidean

$(12,8) \rightarrow (13,7) \rightarrow (14,6) \rightarrow$   
 $(15,7) \rightarrow (16,8) \rightarrow (17,7) \rightarrow$   
 $(18,6) \rightarrow (19,5) \rightarrow (20,4) \rightarrow$   
 $(21,3) \rightarrow (22,4) \rightarrow (23,5) \rightarrow$   
 $(24,6) \rightarrow (25,7) \rightarrow (26,8) \rightarrow$   
 $(27,7) \rightarrow (28,8) \rightarrow (29,9) \rightarrow$   
 $(30,10) \rightarrow (31,11) \rightarrow (32,12) -$   
 $> (33,13) \rightarrow (34,14) \rightarrow (35,15)$   
 $\rightarrow (36,16) \rightarrow (36,17) \rightarrow (37,18)$   
 $\rightarrow (36,19) \rightarrow (35,20) \rightarrow (34,21)$   
 $\rightarrow (33,22) \rightarrow (32,23) \rightarrow (31,24)$   
 $\rightarrow (30,25) = 34$

## Manhattan

Manhattan  
 (12,8) -> (11,7) -> (10,8) ->  
 (9,9) -> (8,9) -> (8,10) ->  
 (8,11) -> (8,12) -> (8,13) ->  
 (8,14) -> (8,15) -> (9,16) ->  
 (9,17) -> (10,18) -> (11,19) ->  
 (11,20) -> (11,21) -> (12,22) ->  
 (13,23) -> (14,22) -> (15,23) ->  
 (16,24) -> (17,23) -> (18,24)  
 -> (19,25) -> (20,26) -> (21,27)  
 -> (22,28) -> (22,29) -> 22,30  
 -> (22,31) -> (23,30) -> (24,29)  
 -> (25,28) -> (26,27) -> (27,26)  
 -> (28,25) -> (29,24) -> (30,25)  
 - 39 -

Diagonal

Diagonal  
 (12,8) -> (11,7) -> (10,8) ->  
 (9,9) -> (8,9) ->(8,10) ->(8,11)  
 ->(8,12) ->(8,13) -> (8,14) ->  
 (8,15) -> (9,16) -> (9,17) ->  
 (10,18) -> (11,19) -> (11,20) -  
 >(11,21) -> (12,22) -> (13,23)  
 -> (14,23) ->(15,24)-> (16,24)  
 -> (17,24) ->(18,24) ->(19,25)  
 -> (20,26)->(21,27) -> (22,28)  
 ->(22,29) ->(22,30) -> (22,31)  
 -> (23,30)->(24,29) -> (25,28)  
 -> 26,27) -> (27,26) -> (28,25)  
 -> (29,24) -> (30,25) = 39

- Different heuristics compute different paths.
- In some cases, the path is different as the kernel size increases
- In other cases, two heuristics give the shortest path such as in coordinates 3,5,6,7,8 and 9.
- Euclidean computes the shortest path 50% of the time.
- Manhattan 33%
- Diagonal 17%

# Time & Space Complexity

## Time Complexity

Dijkstra function :  $O(V^2)$

A star function:  $O(E)$

Dijkstra Program file:  $O(H*W \times H*W)^2$

A star Program file:  $O(H*W \times H*W)^2$

## Space Complexity

Dijkstra:  $O(H*W)$

A star:  $O(H*W)$

Algorithm	Kernel Size	Heuristic	Time
Dijkstra	3	-	5.8 s
Dijkstra	5	-	4.2 s
Dijkstra	7	-	2.9 s
A star	3	euclidean	1 s
A star	3	manhattan	93ms
A star	3	diagonal	83ms
A star	5	euclidean	30 ms
A star	5	manhattan	26 ms
A star	5	diagonal	26 ms
A star	7	euclidean	26 ms
A star	7	manhattan	20 ms
A star	7	diagonal	19 ms

Table 1: Total time taken to compute paths with different heuristics and kernel sizes

## Conclusion

---

- Dijkstra has better space complexity than A star but it may not find the optimal solution
- Although the time complexity is the same for both algorithms.
- A star calculates the shortest path much faster than Dijkstra.
- A star computes the optimal path with kernel size 3 with the euclidean heuristic function.
- The values assigned to various objects should be optimum otherwise a confused path will be evaluated.
- Kernel sizes are crucial, less resolution and data on the image and the graph returns altered path as less resolution disturbs the significance of mixed objects defined.

# Libraries & outside code used 1

```
#include <cfloat>
#include <cmath>
#include <fstream>
#include <iostream>
#include <set>
#include <sstream>
#include <stack>
#include <string>
#include <utility>
#include <vector>
#include <stdlib.h>
#include <stdio.h>
#include <iomanip>
#include <limits.h>
#include "yaml-cpp/yaml.h"
#include "opencv2/imgproc/imgproc.hpp"
#include "opencv2/highgui/highgui.hpp"
```

- roslaunch dexrov\_meta startup\_simulartion.launch ; starts up the simulation
- roslaunch dexrov\_meta startup\_vehicle\_sim.launch ; vehicle is imported in the simulation
- roslaunch dexrov\_meta startup\_vessel\_sim.launch ; vessel is imported in the simulation
- roslaunch dexrov\_meta rviz.launch ; for launching rviz and visualization purpose to keep track of all the topics and nodes
- roslaunch dexrov\_meta perception\_octomap.launch ; for launching the octomap
- rosrun joy joy\_node \_dev:=/dev/input/js1 ; in case the joystick does not work

```
g++ filter2D.cpp -o filter2D `pkg-config --cflags --libs opencv`
g++ -std=c++11 astar.cpp -o astar -lyaml-cpp `pkg-config --cflags --libs opencv`
g++ -std=c++11 dij.cpp -o dij -lyaml-cpp `pkg-config --cflags --libs opencv`
```

```
if (cellDetails[i - 1][j + 1].f == FLT_MAX || cellDetails[i - 1][j + 1].f > fNew)
{openList.insert(make_pair(fNew,
make_pair(i - 1, j + 1)));
cellDetails[i - 1][j + 1].f = fNew;
cellDetails[i - 1][j + 1].g = gNew;
cellDetails[i - 1][j + 1].h = hNew;
cellDetails[i - 1][j + 1].parent_i = i;
cellDetails[i - 1][j + 1].parent_j = j; }
```

code snippet from astar.cpp  
for updating the values of f and g.

```
kernel = Mat::ones( kernel_size,
kernel_size, CV_32FC1 )/ (float)
(kernel_size*kernel_size);
cv::filter2D(M, dst, ddepth , kernel,
anchor, delta, BORDER_DEFAULT );
```

code snippet from filter2D.cpp

## Boundaries of this thesis

.....

- SIGKILL 9 ; 132492 vector
- Out of Memory
- Tie Breaking Mechanism
- Filter2D convolution

$$dst(x, y) = \sum_{i=0}^{M_i-1} \sum_{j=0}^{M_j-1} kernel(i, j) * src(x + i - anchor_i, y + j - anchor_j)$$

## Possible Improvements

- Data Structure
- Heuristics
- Map Representations
- Variety of Map Images

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