Name: _______________ Math 40, Section _____

HW 05; Subspace Basis Dimension Rank Invertible Matrix Theorem Determinants

February 9, 2017

Section 3.5 Numbers; 8, 17, 42, 48, 58, 66(b)

Section 3.6 Numbers; 4 , 8, 12, 16, 32, 44, 50.

Section 4.2 Number; 8

3.5.8 *Let S be the collection of vectors* $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ *in* $\mathbb{R}^3$ *that satisfy the given property;*

$$|x - y| = |y - z|$$

Either prove that S forms a subspace of $\mathbb{R}^2$ *or give a counterexample to show that it does not.*

■

3.5.17 *Give bases for row(A), col(A), and null(A).*

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}.$$

■

$\boxed{\textbf{3.5.42}\ \ \textit{If } A \textit{ is a } 4 \times 2 \textit{ matrix, what are the possible values of nullity}(A)\textit{?}}$

■

3.5.48 *Do* $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$, *and* $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ *form a basis for* $\mathbb{R}^4$*?*

$\blacksquare$

3.5.58 *If A and B are $n \times n$ matricies of rank n, prove that AB has rank n.*

$\blacksquare$

3.5.66(b) *Let A be a skew-symmetric $n \times n$ matrix (see page 162). Prove that $I + A$ is invertible. [Hint: Show that $null(I + A) = \{0\}$].*

■

3.6.4 *Prove that the transformation is linear, using the definition (or the Remark following Example 3.55):*

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x + 2y \\ 3x - 4y \end{bmatrix}$$

.

■

3.6.8 *Give a counterexample to show that the transformation is not linear:*

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} |x| \\ |y| \end{bmatrix}$$

.

■

3.6.12 *Find the standard matrix of the linear transformation in Exercise 4.*

■

3.6.16 *Show that the transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ is linear by showing that it is a matrix transformation: R rotates a vector 45° counterclockwise about the origin.*

■

3.6.32 *Verify Theorem 3.32 by finding the matrix of $S \circ T$ (a) by direct substitution and (b) by matrix multiplication of $[S][T]$:*

$$T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -x_1 \end{bmatrix}$$

,

$$S \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_1 + 3y_2 \\ y_1 - y_2 \end{bmatrix}$$

■

3.6.44 *Let T be a linear transformation from* $\mathbb{R}^2$ *to* $\mathbb{R}^2$ *(or* $\mathbb{R}^3$ *to* $\mathbb{R}^3$*). Prove that T maps a straight line to a straight line or a point. [Hint: use the vector form of the equation of a line.]*

■

3.6.50 *Let ABCD be the square with vertices* $(-1,1)$, $(1,1)$, $(1,-1)$, *and* $(-1,-1)$. *Use the results in Exercises 44 and 45 to find and draw the image of ABCD under the given transformation:*

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ -3x_1 + x_2 \end{bmatrix}$$

.

■

4.2.8 *Compute the determinant using cofactor expansion along any row or column that seems convenient:*

$$\begin{vmatrix} 1 & 1 & -1 \\ 2 & 0 & 1 \\ 3 & -2 & 1 \end{vmatrix}$$

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■