Name: ______________ Math 40, Section _____

HW 06; Determinants and Intro to EigenThings February 23, 2018

Section 4.2 Numbers; 14,24,52,54,56

Exploration Number 10

Section 4.1 Numbers; 6, 12, 14, 20, 26

Section 4.3 Numbers; 6a, 6b, 18

4.2.14 *Compute the determinant using cofactor expansion alon any row or column that seems convenient*

$$\begin{vmatrix} 2 & 0 & 3 & -1 \\ 1 & 0 & 2 & 2 \\ 0 & -1 & 1 & 4 \\ 2 & 0 & 1 & -3 \end{vmatrix}$$

■

4.2.24 *Evaluate the given determinant using elementary row and/or column operations and Theorem 4.3 to reduce the reduce the matrix to row echelon form.*

$$\begin{vmatrix} 1 & -1 & 0 & 3 \\ 2 & 5 & 2 & 6 \\ 0 & 1 & 0 & 0 \\ 1 & 4 & 2 & 1 \end{vmatrix}$$

■

4.2.52 *Assume that A and B are $n \times n$ matrices with $det A = 3$ and $det B = -2$.*

What is** $det(AA^T)$**?

$\blacksquare$

4.2.54 *A and B are $n \times n$ matrices;*

$$\text{If } B \text{ is invertible, prove that } det(B^{-1}AB) = det(A)$$

$\blacksquare$

4.2.56 *A and B are $n \times n$ matrices;*

*A square matric A is called **nilpotent** if $A^m = 0$ for some $m > 1$ (The word nilpotent comes from the Latin, nil, meaning "nothing" and potere, meaning "to have power".A nilpotent matrix is thus one that that becomes "nothing" - when raised to some power) Find all possible values of det(A) if A is nilpotent.*

■

Exploration 10

Let A be a 3×3 matric and let P be the parallelepiped determined by the vectors $\boldsymbol{u}$, $\boldsymbol{v}$, and $\boldsymbol{w}$. Let $T_A(P)$ denote the parallelpiped determined by $T_A(\boldsymbol{u}) = A\boldsymbol{u}$, $T_A(\boldsymbol{v}) = A\boldsymbol{v}$, and $T_A(\boldsymbol{w}) = A\boldsymbol{w}$. Prove that the volume of $T_A(P)$ is given by $|det A|$ (volume of P)

∎

4.1.6

Show that $\mathbf{v}$ *is an eigenvector of A and find the corresponding eigenvaluel*

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

■

4.1.12 *Show that λ is an eignevalue of A and find one eignevector corresponding to this eigenvector corresponding to this eigenvalue;*

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 4 & 2 & 0 \end{bmatrix}, \lambda = 2$$

■

4.1.14 *Find the eigenvalues and eigenvectors of A geometrically*

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ (\textit{reflection in the line} } y = x)$$

■

4.1.20 *The unit vectors* $\boldsymbol{x}$ *in* $\mathbb{R}^2$ *and their images* $A\boldsymbol{x}$ *under the action of a* 2×2 *matrix* A *are drawn head-to-tail as in Figure 4.7/ Estimate the eigenvectors and eigenvalues of* A *from each "eigenpicture"*

∎

4.1.26 *Using the method of Example 4.5 to find all of eigenvalues of the matrix A. Give bases for each of the corresponding eigenspaces. Illustrate the eigenspaces and the effect of multiplying eigenvectors by A as in Figure 4.8*

$$A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$$

■

4.3.6 *(a) Compute the characteristic polynomial of A and (b) the eigenvales of A;*

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 3 \\ 2 & 0 & 1 \end{bmatrix}$$

■

4.3.18

A is a 3×3 *matrix with eigenvectors;*

$$\mathbf{v}_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}\ \mathbf{v}_2 = \begin{bmatrix}1\\1\\0\end{bmatrix}\text{, and } \mathbf{v}_3 = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

with corresponding eigenvalues $\lambda_1 = -\frac{1}{3}$, $\lambda_2 = \frac{1}{3}$, *and* $\lambda_3 = 1$, *respectivley, and;*

$$\mathbf{x} = \begin{bmatrix}2\\1\\2\end{bmatrix}$$

Find $A^k\boldsymbol{x}$ *What happens as k becomes large(ie., $k \to \infty$)*

∎