Math 55

Homework 4, Due Thursday October 4th, 2018

1. Use a combinatorial proof to prove that for $n \geq 2$,

$$\sum_{k=0}^{n} k^2 \binom{n}{k} = n2^{n-1} + n(n-1)2^{n-2}$$

2. Prove that for $n \geq 2$,

$$\sum_{k=0}^{n} k^2 \binom{n}{k} = n2^{n-1} + n(n-1)2^{n-2}$$

Using the binomial theorem.

3. Use the binomial theorem with Binet's formula to prove that for $n \geq 0$,

$$\sum_{k=0}^{n} \binom{n}{k} F_k = F_{2n}$$

Optional BONUS for problem 3 (turn in this solution to me, and do not work on this problem with others): Prove the above identity combinatorially.

4. Use the principle of Inclusion-Exclusion to determine how many numbers between 1 and 250 are not divisible by 2,3 or 5.

5. Delegations A, B, and C have respectively, 3, 3, and 4 distinct members. How many ways can the delegates be seated in a row such that:

a) All delegates from A are together.

b) All delegates are together from at least one delegation.

6. Prove that the number of ways to assign m distinct students to n distinct classrooms such that no classroom is empty is

$$\sum_{k=0}^{n} \binom{n}{k} (n-k)^m (-1)^k.$$

7. Now that you know what the sum in the previous exercise counts, determine what it evaluates to

a) when $m < n$,

b) when $m = n$,

c) when $m = n + 1$.

8. Use the binomial theorem to prove the identity

$$\sum_{k=0}^{n}(-1)^k\binom{n}{k}10^{n-k} = 9^n.$$

9. Give a combinatorial proof of the previous identity using P.I.E. (Hint: Notice that when $k = 0$, the summand is equal to 10^n.)