W271 Lab 3 Spring 2016

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Below, we define some functions we will be using in the problem set:

```r
# Functions for Parts 2, 3, 4
get.best.arima <- function(x.ts, maxord = c(1, 1, 1)) {
    best.aic <- 1e+08
    all.aics <- vector()
    all.models <- vector()
    n <- length(x.ts)
    for (p in 0:maxord[1]) for (d in 0:maxord[2]) for (q in 0:maxord[3]) {
        fit <- arima(x.ts, order = c(p, d, q), method = "ML")
        fit.aic <- -2 * fit$loglik + (log(n) + 1) * length(fit$coef)
        if (fit.aic < best.aic) {
            best.aic <- fit.aic
            best.fit <- fit
            best.model <- c(p, d, q)
        }
        all.aics <- c(all.aics, fit.aic)
        all.models <- c(all.models, sprintf("(%d, %d, %d)", p,
            d, q))
    }
    list(best = list(best.aic, best.fit, best.model), others = data.frame(aics = all.aics,
        models = all.models))
}

get.best.sarima <- function(x.ts, maxord = c(1, 1, 1, 1, 1, 1),
    freq = frequency(x.ts)) {
    best.aic <- 1e+08
    all.aics <- vector()
    all.models <- vector()
    n <- length(x.ts)
    for (p in 0:maxord[1]) for (d in 0:maxord[2]) for (q in 0:maxord[3]) for (P in 0:maxord[3]) for (D i
        fit <- arima(x.ts, order = c(p, d, q), seasonal = list(order = c(P,
            D, Q), freq), method = "CSS", optim.control = list(maxit = 10000))
        fit.aic <- -2 * fit$loglik + (log(n) + 1) * length(fit$coef)
        if (fit.aic < best.aic) {
            best.aic <- fit.aic
            best.fit <- fit
            best.model <- c(p, d, q, P, D, Q)
        }
        all.aics <- c(all.aics, fit.aic)
        all.models <- c(all.models, sprintf("(%d, %d, %d, %d, %d, %d)",
            p, d, q, P, D, Q))
    }
    list(best = list(best.aic, best.fit, best.model), others = data.frame(aics = all.aics,
        models = all.models))
}
```

```r
plot.time.series <- function(x.ts, bins = 30, name, lag = 25,
    plot = "quad") {
    str(x.ts)
    par(mfrow = c(2, 2))
    if (plot == "single") {
        par(mfrow = c(1, 1))
    }
    hist(x.ts, bins, main = paste("Histogram of", name, sep = " "),
        xlab = "Values")
    plot(x.ts, main = paste("Plot of", name, sep = " "), ylab = "Values",
        xlab = "Time")
    acf(x.ts, lag = lag, main = paste("ACF of", name, sep = " "))
    pacf(x.ts, lag = lag, main = paste("PACF of", name, sep = " "))
}

plot.residuals.ts <- function(x.mod, model_name) {
    par(mfrow = c(1, 1))
    hist(x.mod$residuals, 30, main = paste("Histogram of", model_name,
        "Residuals", sep = " "), xlab = "Values")
    par(mfrow = c(2, 2))
    plot(x.mod$residuals, fitted(x.mod), main = paste(model_name,
        "Fitted vs. Residuals", sep = " "), ylab = "Fitted Values",
        xlab = "Residuals")
    plot(x.mod$residuals, main = paste(model_name, "Residuals",
        sep = " "), ylab = paste("Residuals", sep = " "))
    acf(x.mod$residuals, main = paste("ACF of", model_name, sep = " "))
    pacf(x.mod$residuals, main = paste("PACF of", model_name,
        sep = " "))
    Box.test(x.mod$residuals, type = "Ljung-Box")
}

estimate.ar <- function(x.ts) {
    x.ar = ar(x.ts)
    print("Difference in AICs")
    print(x.ar$aic)
    print("AR parameters")
    print(x.ar$ar)
    print("AR order")
    print(x.ar$order)
    return(x.ar)
}

plot.orig.model.resid <- function(x.ts, x.mod, orig_name, model_name,
    xlim, ylim) {
    df <- data.frame(cbind(x.ts, fitted(x.mod), x.mod$residuals))
    class(df)
    stargazer(df, type = "text", title = "Descriptive Stat",
        digits = 1)

    summary(x.ts)
    summary(x.mod$residuals)
    par(mfrow = c(1, 1))
    plot.ts(x.ts, col = "red", main = paste(orig_name, "Original vs Estimated",
```

```
        model_name, "Series with Residuals", sep = " "), ylab = paste(orig_name,
        "Original and Estimated Values", sep = " "), xlim = xlim,
        ylim = ylim, pch = 1, lty = 2)
    par(new = T)
    plot.ts(fitted(x.mod), col = "blue", axes = T, xlab = "",
        ylab = "", xlim = xlim, ylim = ylim, lty = 1)
    leg.txt <- c(paste(orig_name, "Original Series", sep = " "),
        "Estimated Series", "Residuals")
    legend("topleft", legend = leg.txt, lty = c(2, 1, 2), col = c("red",
        "blue", "green"), bty = "n", cex = 1)
    par(new = T)
    plot.ts(x.mod$residuals, axes = F, xlab = "", ylab = "",
        col = "green", xlim = xlim, ylim = ylim, lty = 2, pch = 1,
        col.axis = "green")
    axis(side = 4, col = "green")
    mtext("Residuals", side = 4, line = 2, col = "green")
}

plot.model.forecast <- function(x.mod, mod.fcast, orig_name,
    num_steps, x, y) {
    par(mfrow = c(1, 1))
    plot(mod.fcast, main = paste(num_steps, "-Step Ahead Forecast and",
        orig_name, "Original & Estimated Series", sep = " "),
        xlab = "Time", ylab = paste(orig_name, "Original, Estimated, and Forecasted Values",
            sep = " "), xlim = x, ylim = y, lty = 2, lwd = 1.5)
    par(new = T)
    plot.ts(fitted(x.mod), col = "blue", lty = 2, lwd = 2, xlab = "",
        ylab = "", xlim = x, ylim = y)
    leg.txt <- c(paste(orig_name, "Original Series", sep = " "),
        "Estimated Series", "Forecast")
    legend("topleft", legend = leg.txt, lty = c(2, 2, 1), lwd = c(1,
        2, 2), col = c("black", "blue", "blue"), bty = "n", cex = 1)
}
```

Part 1 (25 points): Modeling House Values

Step 1 - Univariate Analysis

1. **Crime Rate** - This variable is positively skewed, with 90% of datapoints having a crime rate below 11.2%, but outliers above that go up to 89%. We take the log to create a new variable before proceeding.
2. **nonRetailBusiness** - Has a suspiciously high mode at 0.18, which may indicate that a lot of the data points come from the same neighbourhood, which would explain the high number of occurences of a single value.
3. **withWater** - This is a categorical variable. 6.75% of homes in the given sample are in neighbourhoods within 5 miles of a water body.
4. **ageHouse** - This value is in percentage terms and not in strict proportion like other variables in the dataset. Over 50% of the houses in the dataset are in neighbourhoods with a proportion of houses older than 1950 that is greater than 78%.
5. **distanceToCity** - 75% of the houses are less than 15 miles away from a city, and 90% are less than 25 miles away. However, the variable has a large outlier, which is almost 55 miles away from a city. We take a log of the variable before proceeding, in order to make it more evenly distributed.
6. **distanceToHighway** - Definition of the variable is not provided in the dictionary. We see that there are 104 datapoints, exactly 24 miles away from the highway, so we assume this variable measures distance of a neighbourhood from the highway. This is exactly the same number of points for which nonRetailBusiness has a value of 0.18, so it further strengthens the argument that a lot of the datapoints seem to be for houses in the same or very close neighbourhoods.
7. **pupilTeacherRatio** - We find another variable with a high modal value of exactly 23.2 pupils per teacher. This furthers the above argument that a large part of the sample is taken from a single neighbourhood.
8. **pctLowIncome** - 90% of the homes come from neighbourhoods with less than 30% households being low-income, however we do have values going up to 49% in the dataset.
9. **homeValue** - The distribution has 95% of houses valued at well below $1 million, however, there are outliers above that value upto $1.125 million. We take the log of the variable to make it closer to a normal distribution.
10. **pollutionIndex** - The distribution is scattered with a median of 38.8, and a large outlier at 72.1.
11. **nBedRooms** - This variable has close to a normal distribution, with the mean and median around 4.25 bedrooms on average for a single family home, however there are small, as well as large outliers in the distribution.

```r
q1.dataset = read.csv("houseValueData.csv")

str(q1.dataset)
```

```
## 'data.frame':    400 obs. of  11 variables:
##  $ crimeRate_pc     : num  37.6619 0.5783 0.0429 22.5971 0.0664 ...
##  $ nonRetailBusiness: num  0.181 0.0397 0.1504 0.181 0.0405 ...
##  $ withWater        : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ ageHouse         : num  78.7 67 77.3 89.5 74.4 71.3 68.2 97.3 92.2 96.2 ...
##  $ distanceToCity   : num  2.71 4.12 7.82 1.95 5.54 ...
##  $ distanceToHighway: int  24 5 4 24 5 5 5 5 3 5 ...
##  $ pupilTeacherRatio: num  23.2 16 21.2 23.2 19.6 23.9 22.2 17.7 20.8 17.7 ...
##  $ pctLowIncome     : int  18 9 13 41 8 9 12 18 5 4 ...
##  $ homeValue        : int  245250 1125000 463500 166500 672750 596250 425250 483750 852750 1125000 .
##  $ pollutionIndex   : num  52.9 42.5 31.4 55 36 37 34.9 72.1 33.8 45.5 ...
##  $ nBedRooms        : num  4.2 6.3 4.25 3 4.86 ...
```

```r
summary(q1.dataset)
```

```
##   crimeRate_pc      nonRetailBusiness   withWater         ageHouse
##  Min.   : 0.00632   Min.   :0.0074    Min.   :0.0000   Min.   :  2.90
##  1st Qu.: 0.08260   1st Qu.:0.0513    1st Qu.:0.0000   1st Qu.: 45.67
##  Median : 0.26600   Median :0.0969    Median :0.0000   Median : 77.95
##  Mean   : 3.76256   Mean   :0.1115    Mean   :0.0675   Mean   : 68.93
##  3rd Qu.: 3.67481   3rd Qu.:0.1810    3rd Qu.:0.0000   3rd Qu.: 94.15
##  Max.   :88.97620   Max.   :0.2774    Max.   :1.0000   Max.   :100.00
##  distanceToCity   distanceToHighway pupilTeacherRatio  pctLowIncome
##  Min.   : 1.228   Min.   : 1.000    Min.   :15.60     Min.   : 2.00
##  1st Qu.: 3.240   1st Qu.: 4.000    1st Qu.:19.90     1st Qu.: 8.00
##  Median : 6.115   Median : 5.000    Median :21.90     Median :14.00
##  Mean   : 9.638   Mean   : 9.582    Mean   :21.39     Mean   :15.79
##  3rd Qu.:13.628   3rd Qu.:24.000    3rd Qu.:23.20     3rd Qu.:21.00
##  Max.   :54.197   Max.   :24.000    Max.   :25.00     Max.   :49.00
##    homeValue        pollutionIndex    nBedRooms
##  Min.   : 112500   Min.   :23.50   Min.   :1.561
##  1st Qu.: 384188   1st Qu.:29.88   1st Qu.:3.883
##  Median : 477000   Median :38.80   Median :4.193
##  Mean   : 499584   Mean   :40.61   Mean   :4.266
##  3rd Qu.: 558000   3rd Qu.:47.58   3rd Qu.:4.582
##  Max.   :1125000   Max.   :72.10   Max.   :6.780
```

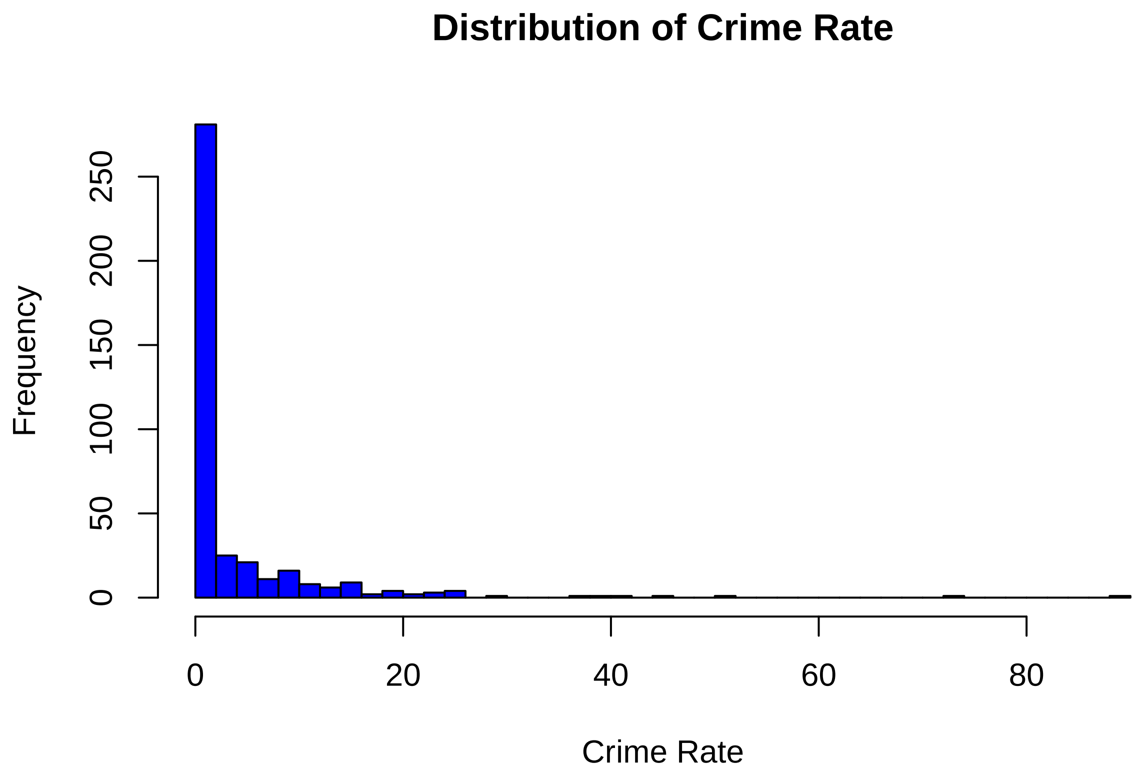
```r
# Performing univariate analysis Crime Rate
summary(q1.dataset$crimeRate_pc)
```

```
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
##  0.00632  0.08260  0.26600  3.76300  3.67500 88.98000
```

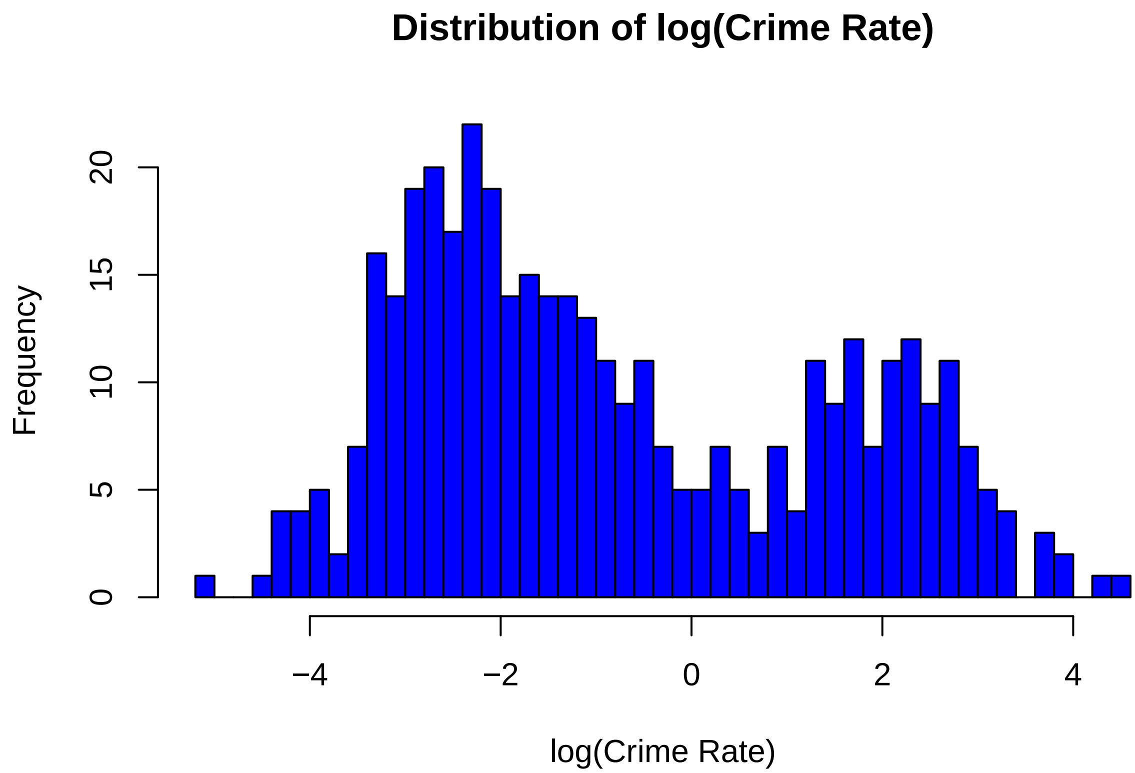
```r
quantile(q1.dataset$crimeRate_pc, probs = c(0.01, 0.05, 0.1, 0.25, 0.5,
    0.75, 0.9, 0.95, 0.99, 1))
```

```
##          1%          5%         10%         25%         50%         75%
##  0.0143128  0.0310980  0.0410280  0.0825975  0.2660050  3.6748075
##         90%         95%         99%        100%
## 11.2021500 18.1052800 41.5713690 88.9762000
```

```r
hist(q1.dataset$crimeRate_pc, breaks = 60, col = "blue", main = "Distribution of Crime Rate",
    xlab = "Crime Rate")
```




```r
# This variable is extremely +vely skewed, so we take the log
q1.dataset$logCrimeRate_pc = log(q1.dataset$crimeRate_pc)
hist(q1.dataset$logCrimeRate_pc, breaks = 60, col = "blue", main = "Distribution of log(Crime Rate)",
    xlab = "log(Crime Rate)")
```



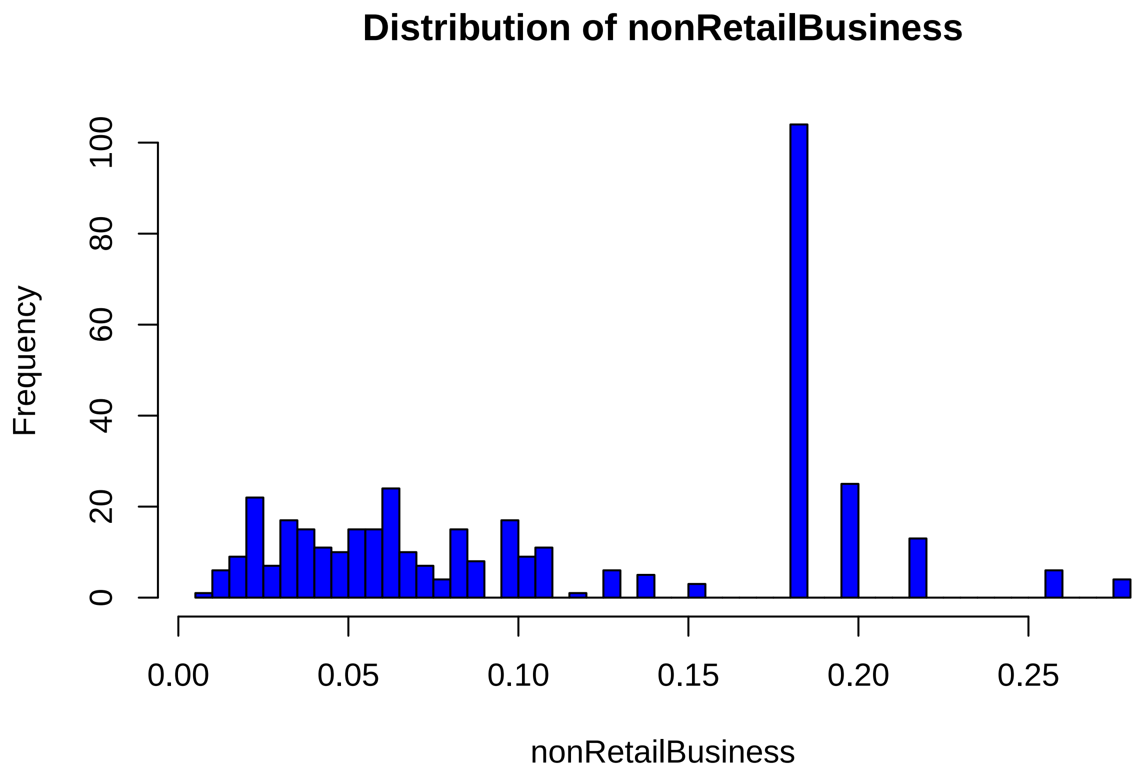

```r
# nonRetailBusiness
summary(q1.dataset$nonRetailBusiness)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.0074  0.0513  0.0969  0.1115  0.1810  0.2774
```

```r
quantile(q1.dataset$nonRetailBusiness, probs = c(0.01, 0.05, 0.1, 0.25,
    0.5, 0.75, 0.9, 0.95, 0.99, 1))
```

```
##       1%       5%      10%      25%      50%      75%      90%      95%
## 0.013794 0.021725 0.028900 0.051300 0.096900 0.181000 0.195800 0.218900
##      99%     100%
## 0.256709 0.277400
```

```r
hist(q1.dataset$nonRetailBusiness, breaks = 60, col = "blue", main = "Distribution of nonRetailBusiness
    xlab = "nonRetailBusiness")
```




```r
head(q1.dataset[order(q1.dataset$nonRetailBusiness, decreasing = TRUE),
    c("nonRetailBusiness")], n = 50)
```

```
##  [1] 0.2774 0.2774 0.2774 0.2774 0.2565 0.2565 0.2565 0.2565 0.2565 0.2565
## [11] 0.2189 0.2189 0.2189 0.2189 0.2189 0.2189 0.2189 0.2189 0.2189 0.2189
## [21] 0.2189 0.2189 0.2189 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958
## [31] 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958
## [41] 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1810 0.1810
```

```r
tail(sort(table(q1.dataset$nonRetailBusiness)), 5)
```

```
##
## 0.2189  0.062 0.0814 0.1958  0.181
##     13     14     15     25    104
```

```r
# suspicious that this has such a large modal value. May be some coded
# val

# withWater
summary(q1.dataset$withWater)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.0000  0.0000  0.0000  0.0675  0.0000  1.0000
```

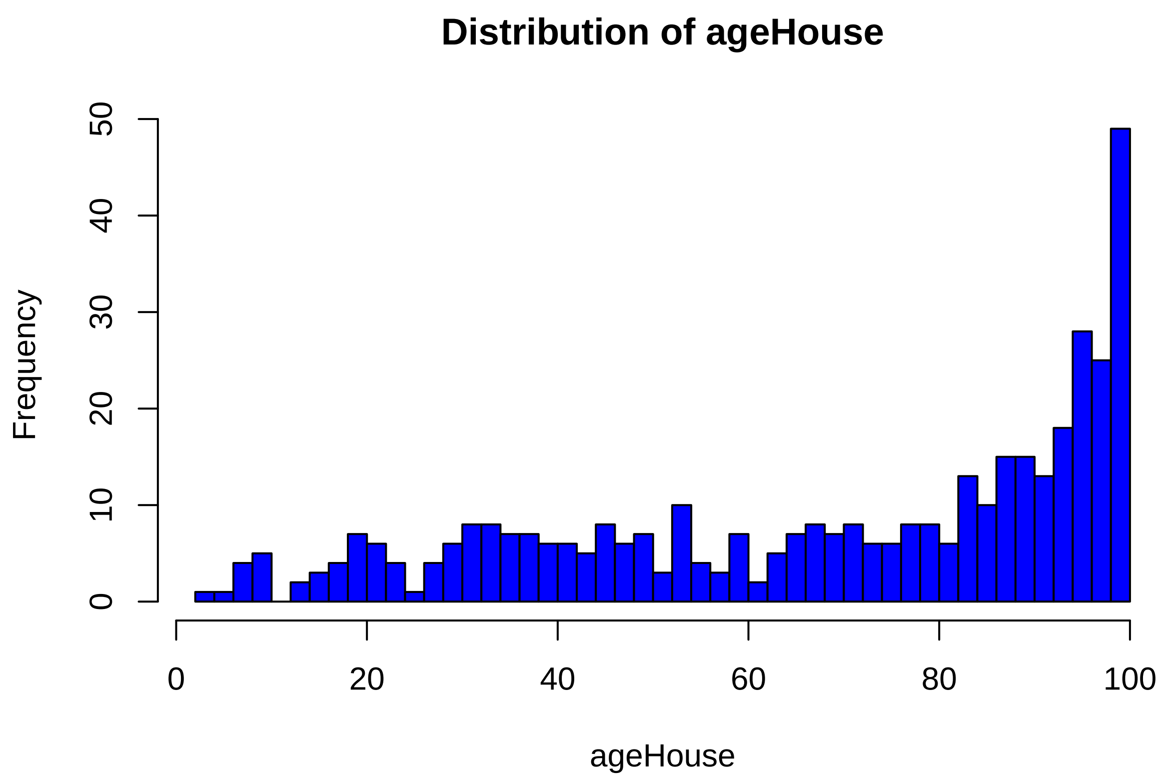
```r
q1.dataset$withWater = factor(q1.dataset$withWater)
# 7% have water

# ageHouse
summary(q1.dataset$ageHouse)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    2.90   45.68   77.95   68.93   94.15  100.00
```

```r
quantile(q1.dataset$ageHouse, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75,
    0.9, 0.95, 0.99, 1))
```

```
##      1%      5%     10%     25%     50%     75%     90%     95%     99%
##   7.788  18.370  27.690  45.675  77.950  94.150  98.410 100.000 100.000
##    100%
## 100.000
```

```r
hist(q1.dataset$ageHouse, breaks = 60, col = "blue", main = "Distribution of ageHouse",
    xlab = "ageHouse")
```



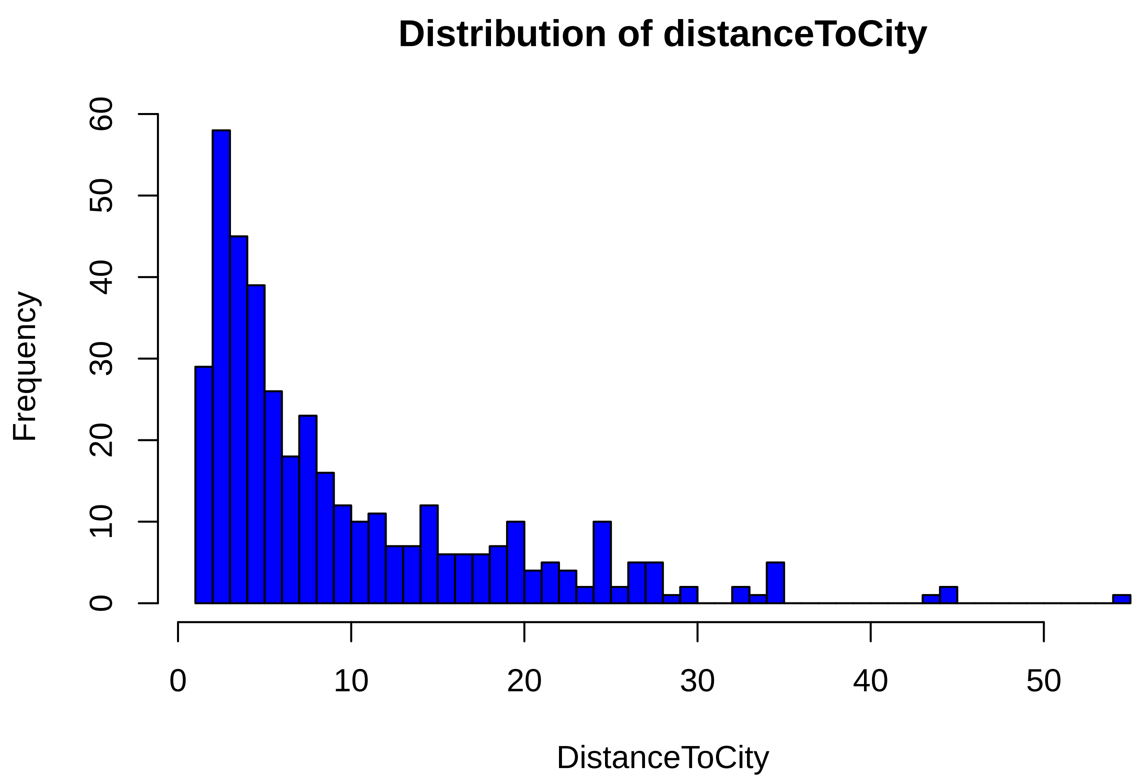

```r
# Looks like a % value.  May require a power transformation

# disttocity
summary(q1.dataset$distanceToCity)
```

```
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
##    1.228    3.240    6.115    9.638   13.630   54.200
```

```r
quantile(q1.dataset$distanceToCity, probs = c(0.01, 0.05, 0.1, 0.25, 0.5,
    0.75, 0.9, 0.95, 0.99, 1))
```

```
##         1%         5%        10%        25%        50%        75%        90%
##  1.342576  1.889692  2.158538  3.239878  6.114617 13.627873 22.682747
##        95%        99%       100%
## 26.939533 35.063729 54.197188
```

```r
hist(q1.dataset$distanceToCity, breaks = 60, col = "blue", main = "Distribution of distanceToCity",
    xlab = "DistanceToCity")
```



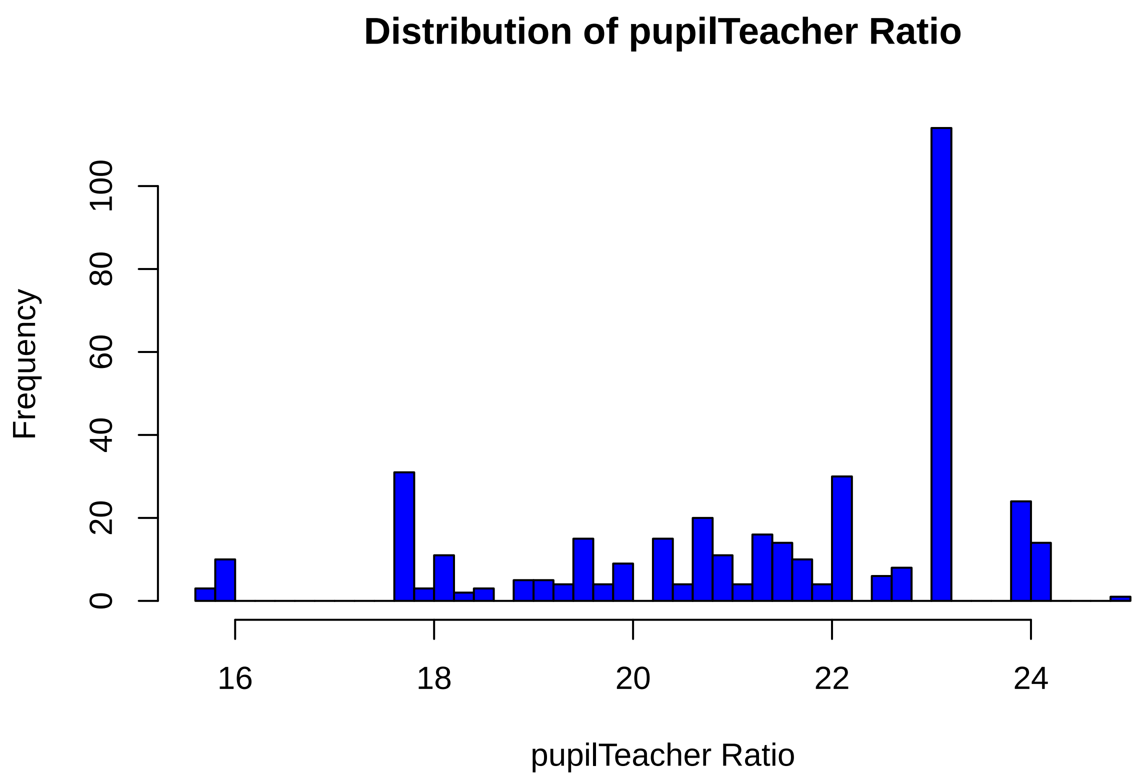

```r
q1.dataset$logDistanceToCity = log(q1.dataset$distanceToCity)
# skewed with a large outlier at the end. Keep in mind while running
# model

# pupilTeacher
summary(q1.dataset$pupilTeacherRatio)
```

```
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
##    15.60    19.90    21.90    21.39    23.20    25.00
```

```r
quantile(q1.dataset$pupilTeacherRatio, probs = c(0.01, 0.05, 0.1, 0.25,
    0.5, 0.75, 0.9, 0.95, 0.99, 1))
```

```
##   1%   5%  10%  25%  50%  75%  90%  95%  99% 100%
## 16.0 17.7 17.7 19.9 21.9 23.2 23.2 24.0 24.2 25.0
```

```r
hist(q1.dataset$pupilTeacherRatio, breaks = 60, col = "blue", main = "Distribution of pupilTeacher Rati
    xlab = "pupilTeacher Ratio")
```




```r
tail(sort(table(q1.dataset$pupilTeacherRatio)), 5)
```

```
##
##   24 22.2 20.8 17.7 23.2
##   16   17   20   28  110
```

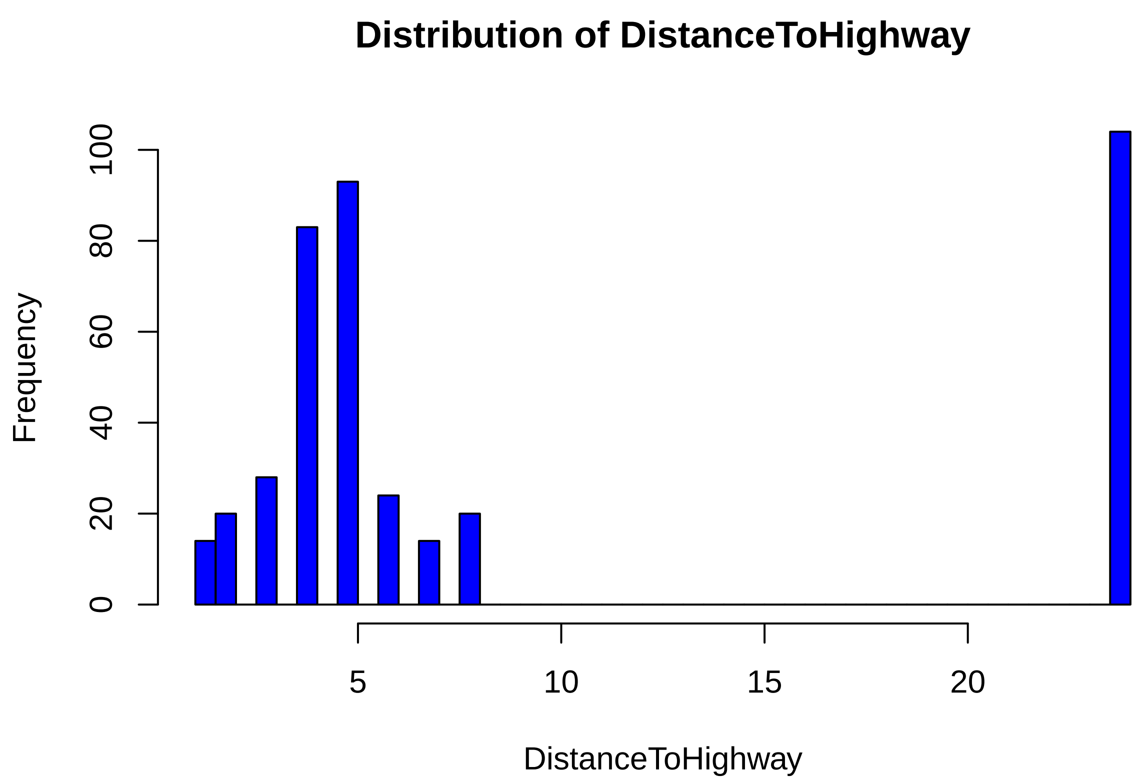
```r
# High mode at 23.2, suspicious

# dist to highway
summary(q1.dataset$distanceToHighway)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   1.000   4.000   5.000   9.582  24.000  24.000
```

```r
quantile(q1.dataset$distanceToHighway, probs = c(0.01, 0.05, 0.1, 0.25,
    0.5, 0.75, 0.9, 0.95, 0.99, 1))
```

```
##   1%   5%  10%  25%  50%  75%  90%  95%  99% 100%
##    1    2    3    4    5   24   24   24   24   24
```

```r
hist(q1.dataset$distanceToHighway, breaks = 60, col = "blue", main = "Distribution of DistanceToHighway
    xlab = "DistanceToHighway")
```




```r
tail(sort(table(q1.dataset$distanceToHighway)), 5)
```

```
##
##   6   3   4   5  24
##  24  28  83  93 104
```

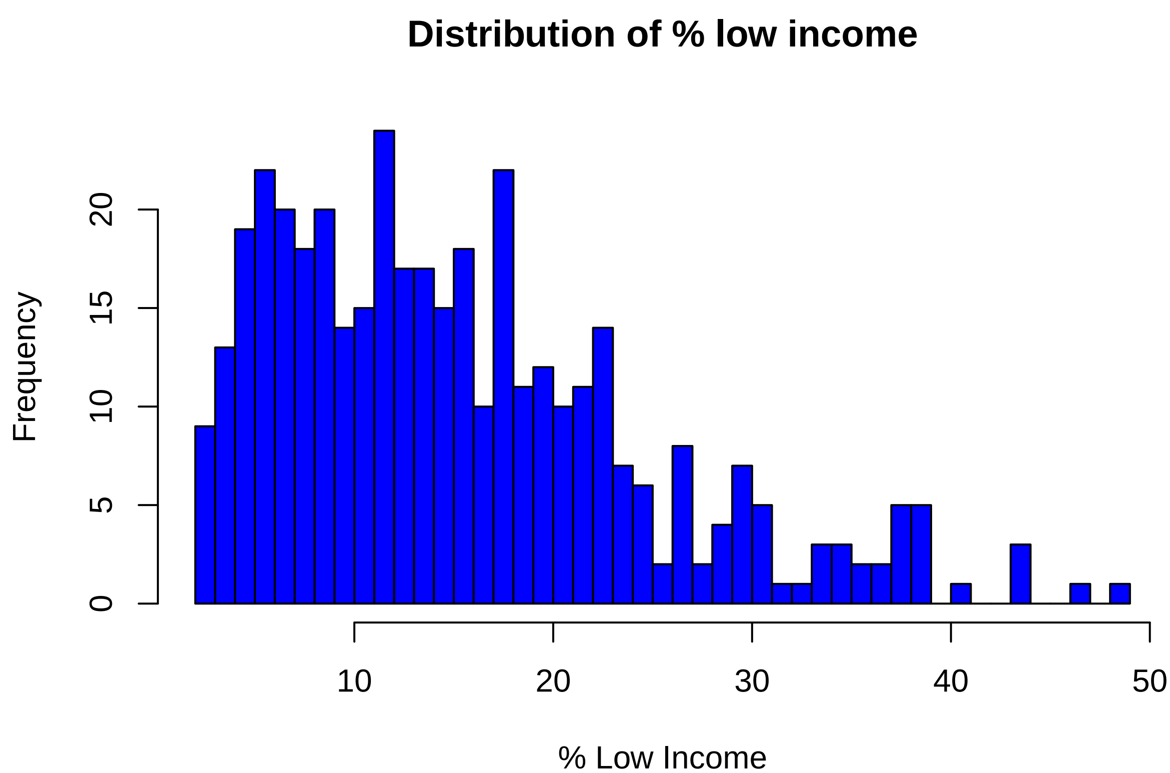
```r
# Very strange that so many values are exactly 24. May not be best
# thing for regression.

# pctlowincome
summary(q1.dataset$pctLowIncome)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##     2.0     8.0    14.0    15.8    21.0    49.0
```

```r
quantile(q1.dataset$pctLowIncome, probs = c(0.01, 0.05, 0.1, 0.25, 0.5,
    0.75, 0.9, 0.95, 0.99, 1))
```

```
##    1%    5%   10%   25%   50%   75%   90%   95%   99%  100%
##  3.00  4.00  5.00  8.00 14.00 21.00 29.10 35.05 44.00 49.00
```

```r
hist(q1.dataset$pctLowIncome, breaks = 60, col = "blue", main = "Distribution of % low income",
    xlab = "% Low Income")
```




```r
tail(sort(table(q1.dataset$pctLowIncome)), 5)
```

```
##
##  7  9  6 18 12
## 20 20 22 22 24
```

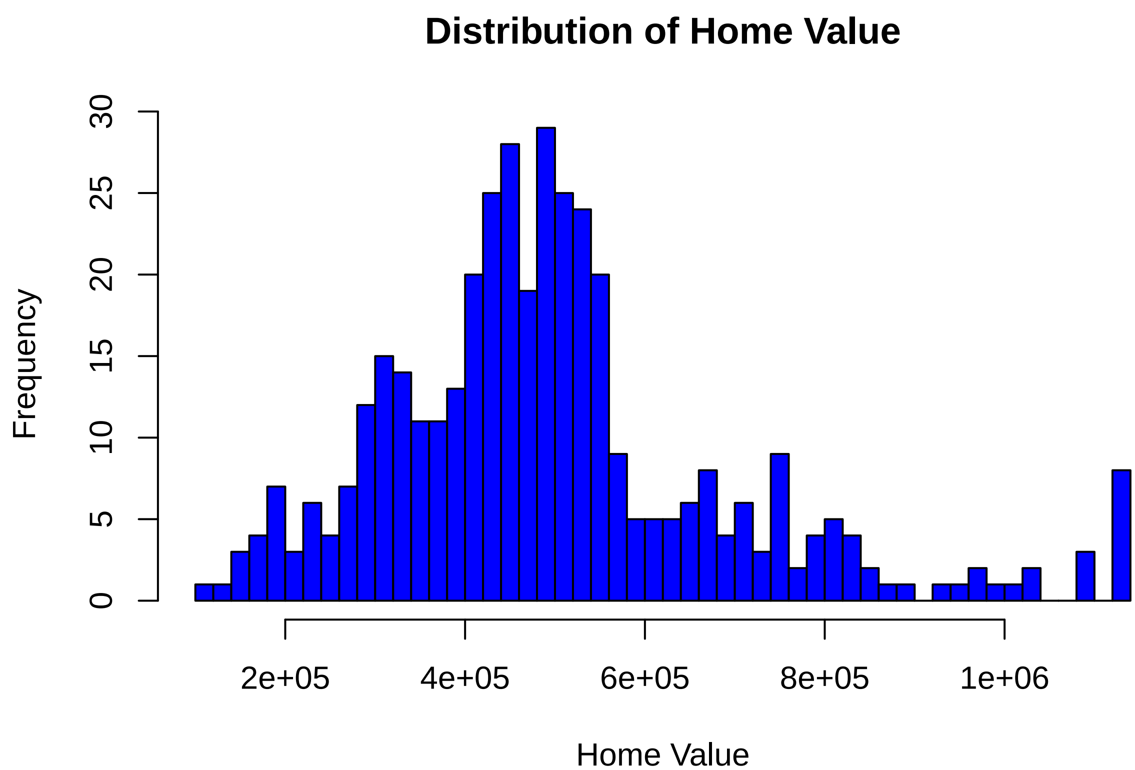
```r
# slight neg skew

# Home value
summary(q1.dataset$homeValue)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  112500  384200  477000  499600  558000 1125000
```

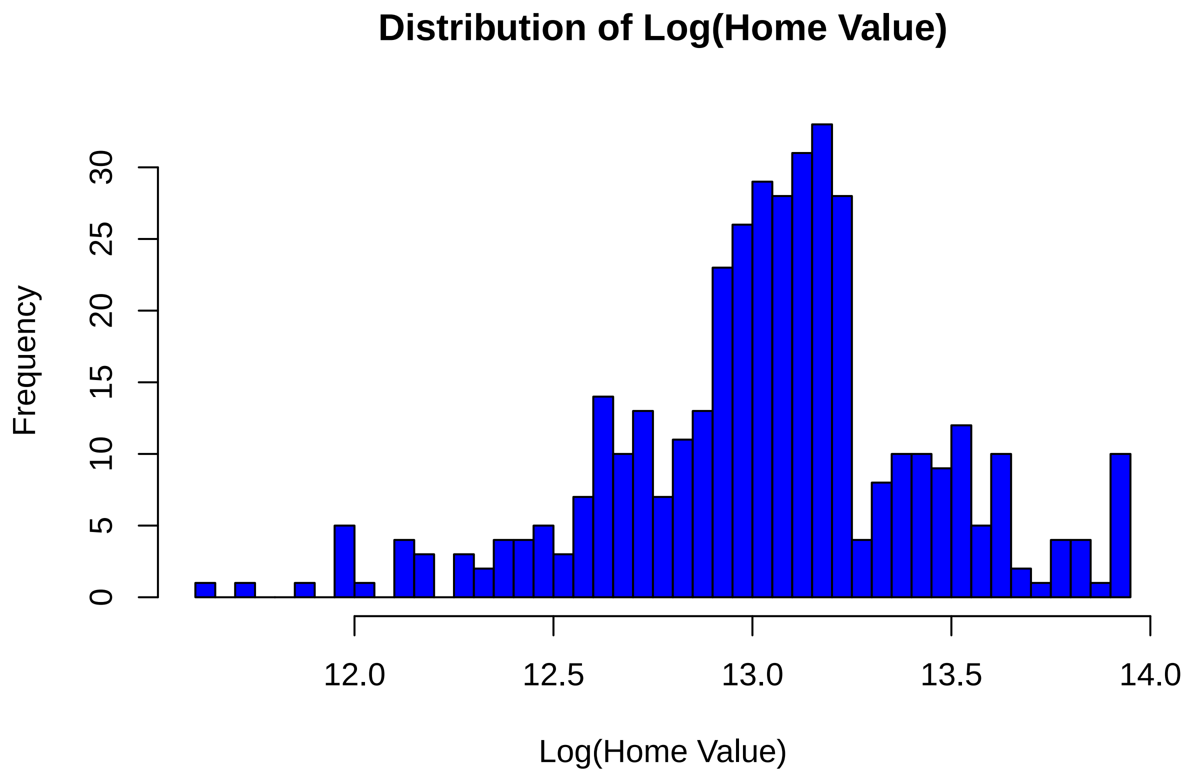
```r
quantile(q1.dataset$homeValue, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75,
    0.9, 0.95, 0.99, 1))
```

```
##        1%        5%       10%       25%       50%       75%       90%
##  157500.0  229500.0  291825.0  384187.5  477000.0  558000.0  749475.0
##       95%       99%      100%
##  871987.5 1125000.0 1125000.0
```

```r
hist(q1.dataset$homeValue, breaks = 60, col = "blue", main = "Distribution of Home Value",
    xlab = "Home Value")
```




```r
q1.dataset$logHomeValue = log(q1.dataset$homeValue)
hist(q1.dataset$logHomeValue, breaks = 60, col = "blue", main = "Distribution of Log(Home Value)",
    xlab = "Log(Home Value)")
```



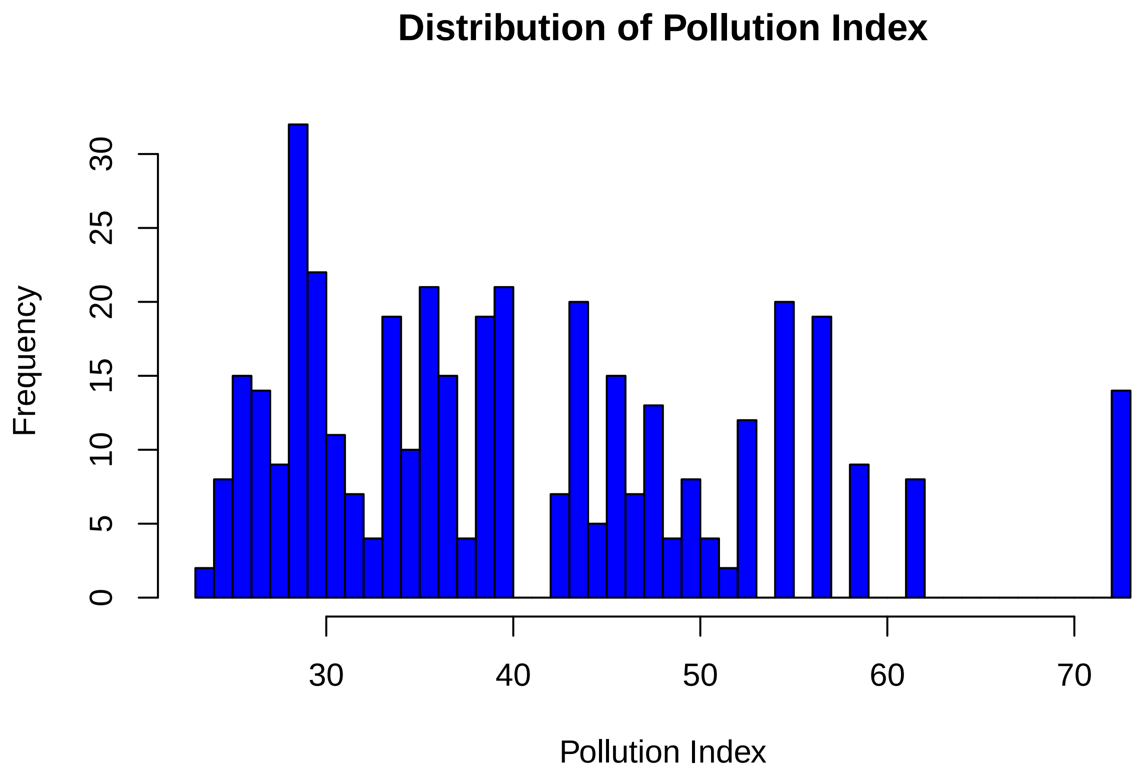

```
# Pretty normal

# pollution Index
summary(q1.dataset$pollutionIndex)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   23.50   29.87   38.80   40.61   47.58   72.10
```

```
quantile(q1.dataset$pollutionIndex, probs = c(0.01, 0.05, 0.1, 0.25, 0.5,
    0.75, 0.9, 0.95, 0.99, 1))
```

```
##     1%     5%    10%    25%    50%    75%    90%    95%    99%   100%
## 24.398 25.880 27.600 29.875 38.800 47.575 56.300 62.000 72.100 72.100
```

```
hist(q1.dataset$pollutionIndex, breaks = 60, col = "blue", main = "Distribution of Pollution Index",
    xlab = "Pollution Index")
```



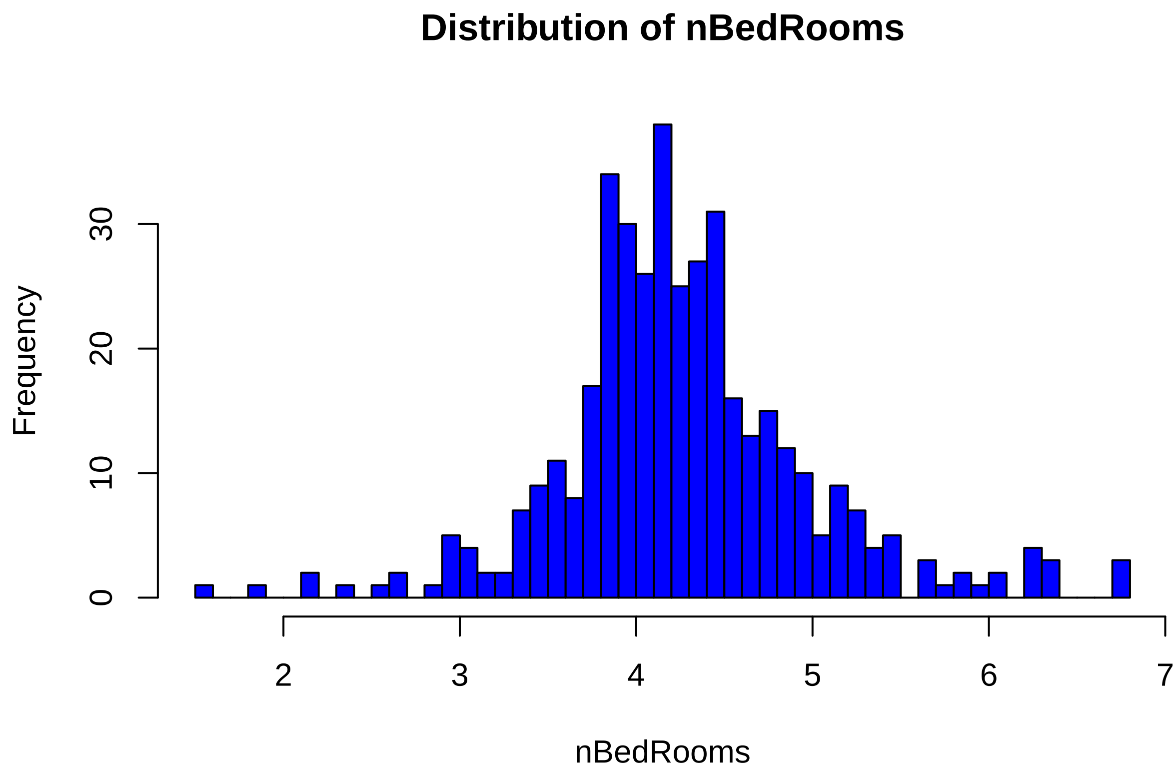

```r
# scattered dist, one high outlier at 72

# nbedrooms
summary(q1.dataset$nBedRooms)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   1.561   3.883   4.193   4.266   4.582   6.780
```

```r
quantile(q1.dataset$nBedRooms, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75,
    0.9, 0.95, 0.99, 1))
```

```
##      1%      5%     10%     25%     50%     75%     90%     95%     99%
## 2.36570 3.26770 3.53550 3.88300 4.19300 4.58175 5.14710 5.45480 6.37523
##    100%
## 6.78000
```

```r
hist(q1.dataset$nBedRooms, breaks = 60, col = "blue", main = "Distribution of nBedRooms",
    xlab = "nBedRooms")
```




```
# Pretty normal
```

Step 2 - Bivariate Analysis

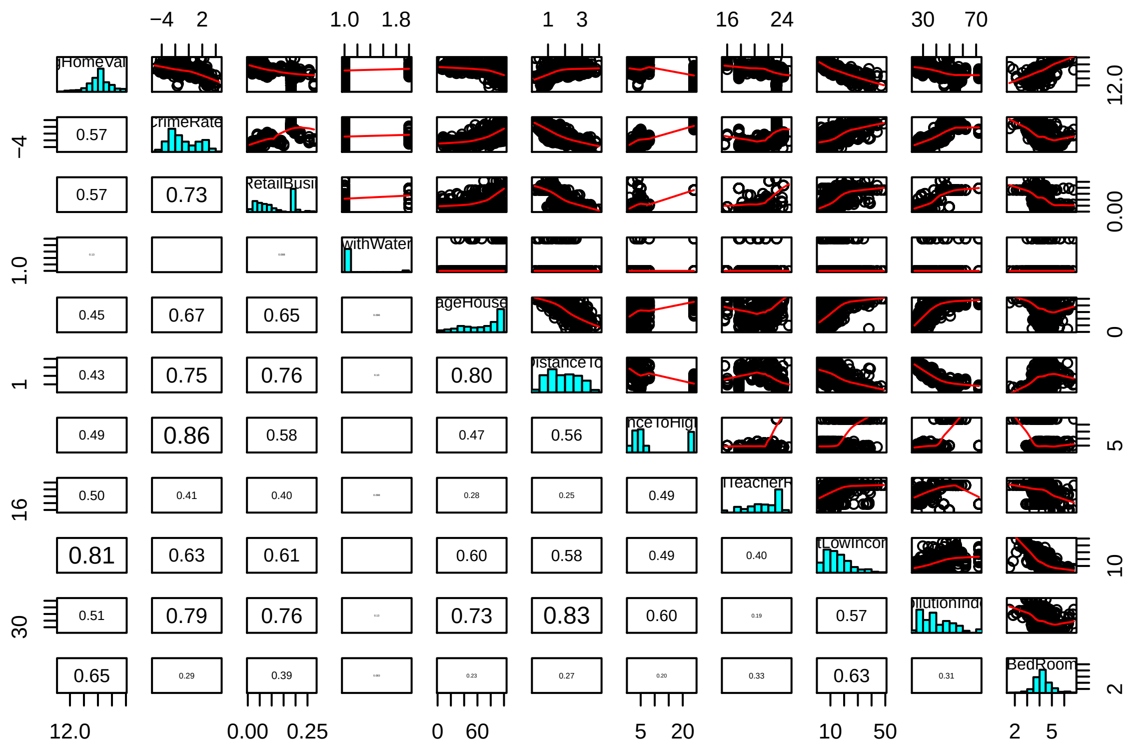
We examine bivariate correlations and scatterplots for all variables in the dataset. In this analysis if a variable was transformed, the transformed version is used.

Conclusions

1. A lot of variables show strong correlations with each other in the dataset (absolute val of correlation > 0.7). We must be wary of multicollinearity when including these variables together in our regression models which would make our model coefficients lose precision. At the same time, it is important to include necessary variables in order to prevent any omitted variable bias.

- logCrimeRate shows a strong correlation with logdistanceToCity, distanceToHighway, pollutionIndex and nonRetailBusiness
- nonRetailBusiness shows a strong correlation with logdistanceToCity, pollutionIndex and logCrimeRate
- ageHouse also shows a strong correlation with logdistanceToCity and pollutionIndex
- logDistancetoCity shows a strong correlation with pollutionIndex, logCrimeRate, nonRetailBusiness, ageHouse

```
panel.hist <- function(x, ...) {
    usr <- par("usr")
    on.exit(par(usr))
    par(usr = c(usr[1:2], 0, 1.5))
    h <- hist(x, plot = FALSE)
    breaks <- h$breaks
    nB <- length(breaks)
    y <- h$counts
    y <- y/max(y)
    rect(breaks[-nB], 0, breaks[-1], y, col = "cyan", ...)
}

panel.cor <- function(x, y, digits = 2, prefix = "", cex.cor, ...) {
    usr <- par("usr")
    on.exit(par(usr))
    par(usr = c(0, 1, 0, 1))
    r <- abs(cor(x, y))
    txt <- format(c(r, 0.123456789), digits = digits)[1]
    txt <- paste0(prefix, txt)
    if (missing(cex.cor))
        cex.cor <- 0.8/strwidth(txt)
    text(0.5, 0.5, txt, cex = cex.cor * r)
}

pairs(logHomeValue ~ logCrimeRate_pc + nonRetailBusiness + withWater +
    ageHouse + logDistanceToCity + distanceToHighway + pupilTeacherRatio +
    pctLowIncome + pollutionIndex + nBedRooms, data = q1.dataset, upper.panel = panel.smooth,
    lower.panel = panel.cor, diag.panel = panel.hist)
```


CrimeRate
RetailBusi
withWater
ageHouse
DistanceTo
nceToHigh
TeacherR
LowIncon
llutionInd
BedRoom
0.57
0.57
0.73
0.45
0.67
0.65
0.43
0.75
0.76
0.80
0.49
0.86
0.58
0.47
0.56
0.50
0.41
0.40
0.28
0.25
0.49
0.81
0.63
0.61
0.60
0.58
0.49
0.40
0.51
0.79
0.76
0.73
0.83
0.60
0.19
0.57
0.65
0.29
0.39
0.23
0.27
0.20
0.33
0.63
0.31

Step 3 - Model Estimation

Conclusion

We end up with the following model:

$$
\begin{aligned}
log(homeValue) = \\
& \beta_0 + \beta_1 withWater + \beta_2 pctLowIncome + \\
& \beta_3 pupilTeacherRatio + \beta_4 pollutionIndex + \\
& \beta_5 nBedRooms + \beta_6 withWater * pollutionIndex
\end{aligned} \tag{1}
$$

Procedure used to select the model, along with diagnostics detailed below. The practical conclusions of the model, and the final interpretation are detailed in the next section.

Procedure

We start off with a naive approach, including all variables in the regression to observe results.

```r
model.1 = lm(logHomeValue ~ logCrimeRate_pc + nonRetailBusiness + withWater +
    ageHouse + logDistanceToCity + distanceToHighway + pupilTeacherRatio +
    pctLowIncome + pollutionIndex + nBedRooms, data = q1.dataset)
summary(model.1)
```

```
##
## Call:
## lm(formula = logHomeValue ~ logCrimeRate_pc + nonRetailBusiness +
##     withWater + ageHouse + logDistanceToCity + distanceToHighway +
##     pupilTeacherRatio + pctLowIncome + pollutionIndex + nBedRooms,
##     data = q1.dataset)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.73040 -0.09641 -0.00502  0.09332  0.78653
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)       14.2334735  0.2107218  67.546  < 2e-16 ***
## logCrimeRate_pc   -0.0107344  0.0129755  -0.827 0.408585
## nonRetailBusiness -0.4128304  0.2645968  -1.560 0.119520
## withWater1         0.1411053  0.0409450   3.446 0.000631 ***
## ageHouse           0.0001650  0.0006556   0.252 0.801461
## logDistanceToCity -0.1288004  0.0254045  -5.070 6.17e-07 ***
## distanceToHighway -0.0010709  0.0025088  -0.427 0.669728
## pupilTeacherRatio -0.0303139  0.0060208  -5.035 7.33e-07 ***
## pctLowIncome      -0.0238385  0.0018412 -12.947  < 2e-16 ***
## pollutionIndex    -0.0081282  0.0018766  -4.331 1.89e-05 ***
## nBedRooms          0.1028429  0.0192037   5.355 1.46e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.1999 on 389 degrees of freedom
## Multiple R-squared:  0.7529, Adjusted R-squared:  0.7465
## F-statistic: 118.5 on 10 and 389 DF,  p-value: < 2.2e-16
```

```r
AIC(model.1)
```

```
## [1] -140.1337
```

```r
BIC(model.1)
```

```
## [1] -92.23611
```

In this model, we see that several variables do not have statistical significance. We see that the coefficients lack precision, having extremely high standard errors.

Before we move on to more parsimonious models, we will examine interaction variables to see if they add any explanatory power to our model. We have two categorical variables in our dataset: withWater and distanceToHighway (though a numerical variable, it has only nine distinct values, effectively functioning as a categorical). We add all possible interactions with these variables to see if we obtain any noteworthy result.

Although it is possible to include interaction terms of continuous variables, the results are complex to explain. For simplicity, these interactions are excluded from the analysis.

```r
model.2 = lm(logHomeValue ~ logCrimeRate_pc + nonRetailBusiness + withWater +
    ageHouse + logDistanceToCity + distanceToHighway + pupilTeacherRatio +
    pctLowIncome + pollutionIndex + nBedRooms + distanceToHighway:logCrimeRate_pc +
    distanceToHighway:nonRetailBusiness + distanceToHighway:ageHouse +
    distanceToHighway:pupilTeacherRatio + distanceToHighway:pctLowIncome +
    distanceToHighway:pollutionIndex + distanceToHighway:nBedRooms + withWater:pollutionIndex +
    withWater:logCrimeRate_pc + withWater:nonRetailBusiness + withWater:ageHouse +
    withWater:logDistanceToCity + withWater:pupilTeacherRatio + withWater:pctLowIncome +
    withWater:nBedRooms, data = q1.dataset)
summary(model.2)
```

```
##
## Call:
## lm(formula = logHomeValue ~ logCrimeRate_pc + nonRetailBusiness +
##     withWater + ageHouse + logDistanceToCity + distanceToHighway +
##     pupilTeacherRatio + pctLowIncome + pollutionIndex + nBedRooms +
##     distanceToHighway:logCrimeRate_pc + distanceToHighway:nonRetailBusiness +
##     distanceToHighway:ageHouse + distanceToHighway:pupilTeacherRatio +
##     distanceToHighway:pctLowIncome + distanceToHighway:pollutionIndex +
##     distanceToHighway:nBedRooms + withWater:pollutionIndex +
##     withWater:logCrimeRate_pc + withWater:nonRetailBusiness +
##     withWater:ageHouse + withWater:logDistanceToCity + withWater:pupilTeacherRatio +
##     withWater:pctLowIncome + withWater:nBedRooms, data = q1.dataset)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.66948 -0.07603 -0.00873  0.06591  0.68353
##
## Coefficients:
##                                        Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)                          1.298e+01  4.468e-01  29.055  < 2e-16
## logCrimeRate_pc                      4.846e-02  1.559e-02   3.108 0.002027
## nonRetailBusiness                   -1.489e+00  4.294e-01  -3.468 0.000585
## withWater1                           1.801e-01  8.372e-01   0.215 0.829771
## ageHouse                            -3.679e-03  7.789e-04  -4.723 3.30e-06
## logDistanceToCity                   -1.238e-01  2.274e-02  -5.445 9.40e-08
## distanceToHighway                    1.881e-02  7.910e-02   0.238 0.812165
## pupilTeacherRatio                   -3.658e-02  1.793e-02  -2.040 0.042057
## pctLowIncome                        -1.443e-03  2.692e-03  -0.536 0.592234
## pollutionIndex                      -3.252e-04  2.705e-03  -0.120 0.904352
## nBedRooms                            3.753e-01  2.812e-02  13.346  < 2e-16
## logCrimeRate_pc:distanceToHighway   -1.020e-02  1.447e-03  -7.052 8.58e-12
## nonRetailBusiness:distanceToHighway  2.281e-01  8.831e-02   2.583 0.010177
## ageHouse:distanceToHighway           2.572e-04  8.876e-05   2.898 0.003976
## distanceToHighway:pupilTeacherRatio  2.983e-03  3.840e-03   0.777 0.437728
## distanceToHighway:pctLowIncome      -1.189e-03  1.600e-04  -7.429 7.50e-13
## distanceToHighway:pollutionIndex    -7.424e-04  1.844e-04  -4.026 6.88e-05
## distanceToHighway:nBedRooms         -1.827e-02  1.668e-03 -10.949  < 2e-16
## withWater1:pollutionIndex           -1.191e-02  5.861e-03  -2.032 0.042887
## logCrimeRate_pc:withWater1           6.485e-02  6.100e-02   1.063 0.288479
## nonRetailBusiness:withWater1         1.332e+00  1.408e+00   0.946 0.344643
## withWater1:ageHouse                  1.852e-03  2.952e-03   0.627 0.530828
## withWater1:logDistanceToCity         8.978e-02  1.229e-01   0.730 0.465649
## withWater1:pupilTeacherRatio         2.894e-02  2.386e-02   1.213 0.225988
## withWater1:pctLowIncome             -6.662e-03  6.682e-03  -0.997 0.319416
## withWater1:nBedRooms                -1.047e-01  5.846e-02  -1.790 0.074245
##
## (Intercept)                         ***
## logCrimeRate_pc                     **
## nonRetailBusiness                   ***
## withWater1
## ageHouse                            ***
## logDistanceToCity                   ***
## distanceToHighway
## pupilTeacherRatio                   *
## pctLowIncome
## pollutionIndex
## nBedRooms                           ***
## logCrimeRate_pc:distanceToHighway   ***
## nonRetailBusiness:distanceToHighway *
## ageHouse:distanceToHighway          **
## distanceToHighway:pupilTeacherRatio
## distanceToHighway:pctLowIncome      ***
## distanceToHighway:pollutionIndex    ***
## distanceToHighway:nBedRooms         ***
## withWater1:pollutionIndex           *
## logCrimeRate_pc:withWater1
## nonRetailBusiness:withWater1
## withWater1:ageHouse
## withWater1:logDistanceToCity
## withWater1:pupilTeacherRatio
## withWater1:pctLowIncome
## withWater1:nBedRooms                .
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1584 on 374 degrees of freedom
## Multiple R-squared:  0.8508, Adjusted R-squared:  0.8408
## F-statistic: 85.29 on 25 and 374 DF,  p-value: < 2.2e-16
```

```
AIC(model.2)
```

```
## [1] -311.9024
```

```
BIC(model.2)
```

```
## [1] -204.1328
```

```
waldtest(model.1, model.2)
```

```
## Wald test
##
## Model 1: logHomeValue ~ logCrimeRate_pc + nonRetailBusiness + withWater +
##     ageHouse + logDistanceToCity + distanceToHighway + pupilTeacherRatio +
##     pctLowIncome + pollutionIndex + nBedRooms
## Model 2: logHomeValue ~ logCrimeRate_pc + nonRetailBusiness + withWater +
##     ageHouse + logDistanceToCity + distanceToHighway + pupilTeacherRatio +
##     pctLowIncome + pollutionIndex + nBedRooms + distanceToHighway:logCrimeRate_pc +
##     distanceToHighway:nonRetailBusiness + distanceToHighway:ageHouse +
##     distanceToHighway:pupilTeacherRatio + distanceToHighway:pctLowIncome +
##     distanceToHighway:pollutionIndex + distanceToHighway:nBedRooms +
##     withWater:pollutionIndex + withWater:logCrimeRate_pc + withWater:nonRetailBusiness +
##     withWater:ageHouse + withWater:logDistanceToCity + withWater:pupilTeacherRatio +
##     withWater:pctLowIncome + withWater:nBedRooms
##   Res.Df Df      F    Pr(>F)
## 1    389
## 2    374 15 16.357 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We do see some additional explanatory power through the addition of the interaction terms as we obtain a model with higher R square, lower AIC and lower BIC, as well as a significant Wald Test p value. However, apart from having a model which is extremely difficult to interpret, we also notice that most of the coefficient estimates are extremely small, having very little practical significance. Since we are not aiming for a predictive model, we remove most of these terms to get a model which can be easily interpreted.

Now, we want to narrow down our model to include only variables that really add to the explanatory power of the model, that reduce multicollinearity, that provide some valuable practical significance, while meeting the think-tank's specific ask of desirable neighbourhood features and environmental features' relation to home values.

We remove the following variables:

1. nonRetailBusiness: It has a high correlation with several of the variables in the dataset, and is not of direct interest to answering the question asked.
2. ageHouse: It is not of direct consequence to the question asked.

3. DistanceToCity has a high correlation with pollutionIndex, a variable we are definitely interested in, so we remove it to reduce the loss of precision that comes with multicollinearity
4. distanceToHighway interactions except the interaction with log crime: This is the only interaction with a variable still in the model which has statistical and practical significance.
5. withWater interactions except the interaction with pollutionIndex: This interaction seems to have some practical as well as statistical significance.
6. pupilToTeacherRatio: It is not of direct consequence to the question asked.

```
model.3 = lm(logHomeValue ~ logCrimeRate_pc + withWater + distanceToHighway +
    pctLowIncome + pollutionIndex + nBedRooms + distanceToHighway:logCrimeRate_pc +
    withWater:pollutionIndex, data = q1.dataset)
summary(model.3)
```

```
##
## Call:
## lm(formula = logHomeValue ~ logCrimeRate_pc + withWater + distanceToHighway +
##     pctLowIncome + pollutionIndex + nBedRooms + distanceToHighway:logCrimeRate_pc +
##     withWater:pollutionIndex, data = q1.dataset)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.6632 -0.1264 -0.0251  0.1023  0.9295
##
## Coefficients:
##                                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)                       12.948468   0.126530 102.335  < 2e-16
## logCrimeRate_pc                    0.039525   0.014945   2.645  0.00851
## withWater1                         0.385954   0.136740   2.823  0.00501
## distanceToHighway                  0.003694   0.003130   1.180  0.23860
## pctLowIncome                      -0.022256   0.001839 -12.104  < 2e-16
## pollutionIndex                    -0.002170   0.001618  -1.342  0.18050
## nBedRooms                          0.135016   0.018827   7.171 3.74e-12
## logCrimeRate_pc:distanceToHighway -0.005979   0.001154  -5.180 3.55e-07
## withWater1:pollutionIndex         -0.005643   0.002868  -1.968  0.04980
##
## (Intercept)                       ***
## logCrimeRate_pc                   **
## withWater1                        **
## distanceToHighway
## pctLowIncome                      ***
## pollutionIndex
## nBedRooms                         ***
## logCrimeRate_pc:distanceToHighway ***
## withWater1:pollutionIndex         *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.206 on 391 degrees of freedom
## Multiple R-squared:  0.736,  Adjusted R-squared:  0.7306
## F-statistic: 136.3 on 8 and 391 DF,  p-value: < 2.2e-16
```

```r
AIC(model.3)
```

```
## [1] -117.7322
```

```r
BIC(model.3)
```

```
## [1] -77.81753
```

We see that the distanceToHighway and pollutionIndex variables don't seem to have statistical significance. We will remove the distanceToHighway variable since it does not have direct consequence to the question asked. Further, we remove the logCrimeRate variable, since it has a high correlation with pollutionIndex, a variable of importance to us. We do this to observe if its removal increases the precision of the pollutonIndex variable. Note that this also means that we remove the interaction of distanceToHighway and logCrimeRate

```r
model.4 = lm(logHomeValue ~ withWater + pctLowIncome + nBedRooms + pollutionIndex +
    withWater:pollutionIndex, data = q1.dataset)
summary(model.4)
```

```
##
## Call:
## lm(formula = logHomeValue ~ withWater + pctLowIncome + nBedRooms +
##     pollutionIndex + withWater:pollutionIndex, data = q1.dataset)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.74284 -0.11845 -0.01227  0.10345  0.82889
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)              13.047261   0.103389 126.196  < 2e-16 ***
## withWater1                0.378494   0.143117   2.645   0.0085 **
## pctLowIncome             -0.025916   0.001785 -14.521  < 2e-16 ***
## nBedRooms                 0.122748   0.019624   6.255 1.04e-09 ***
## pollutionIndex           -0.003105   0.001235  -2.515   0.0123 *
## withWater1:pollutionIndex -0.004727   0.002997  -1.577   0.1155
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2171 on 394 degrees of freedom
## Multiple R-squared:  0.7047, Adjusted R-squared:  0.7009
## F-statistic:   188 on 5 and 394 DF,  p-value: < 2.2e-16
```

```r
AIC(model.4)
```

```
## [1] -78.87188
```

```r
BIC(model.4)
```

```
## [1] -50.93163
```

We fail to find statistical significance for our pollutionIndex variable, and are unable to make any confident claims about this variable's impact on homeValue. Perhaps this variable, which was significant at a lower level in earlier models with more variables, is losing precision due to an omitted variable bias.

We try adding back the pupilTeacherRatio variable to see if that makes any difference to the model.

```
model.5 = lm(logHomeValue ~ withWater + pctLowIncome + pupilTeacherRatio +
    pollutionIndex + nBedRooms + withWater:pollutionIndex, data = q1.dataset)
summary(model.5)
```

```
##
## Call:
## lm(formula = logHomeValue ~ withWater + pctLowIncome + pupilTeacherRatio +
##     pollutionIndex + nBedRooms + withWater:pollutionIndex, data = q1.dataset)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.73377 -0.11039 -0.01038  0.10830  0.90027
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)              13.795774   0.151009  91.357  < 2e-16 ***
## withWater1                0.405199   0.136166   2.976  0.00310 **
## pctLowIncome             -0.023177   0.001748 -13.256  < 2e-16 ***
## pupilTeacherRatio        -0.034392   0.005266  -6.531 2.03e-10 ***
## pollutionIndex           -0.003212   0.001174  -2.736  0.00651 **
## nBedRooms                 0.110955   0.018750   5.918 7.12e-09 ***
## withWater1:pollutionIndex -0.005762   0.002855  -2.019  0.04422 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2064 on 393 degrees of freedom
## Multiple R-squared:  0.7336, Adjusted R-squared:  0.7295
## F-statistic: 180.4 on 6 and 393 DF,  p-value: < 2.2e-16
```

```
AIC(model.5)
```

```
## [1] -118.0838
```

```
BIC(model.5)
```

```
## [1] -86.15208
```

We obtain statistical significance at the 5% level for all our coefficients, while maintaining a high R squared value. While AIC and BIC values are not the lowest, we prefer the parsimony of this model, and choose this as our final model to present to the think-tank.

We do not consider introducing instrument variables to the model for the following reasons:

1. The predictors have extremely low correlation with the error term, making them all effectively exogenous (shown below).

2. While some variables do meet the criteria for instrument relevance, they are correlated with multiple variables in the model, so they do not make good overall candidates for instruments as they would introduce multicollinearity to the prediction of the variable for which they would serve as instruments.

```
# Correlations of predictors with residuals (error term)
cor(q1.dataset$pctLowIncome, model.5$residuals)
```

```
## [1] -3.439642e-17
```

```
cor(q1.dataset$pollutionIndex, model.5$residuals)
```

```
## [1] 4.878514e-16
```

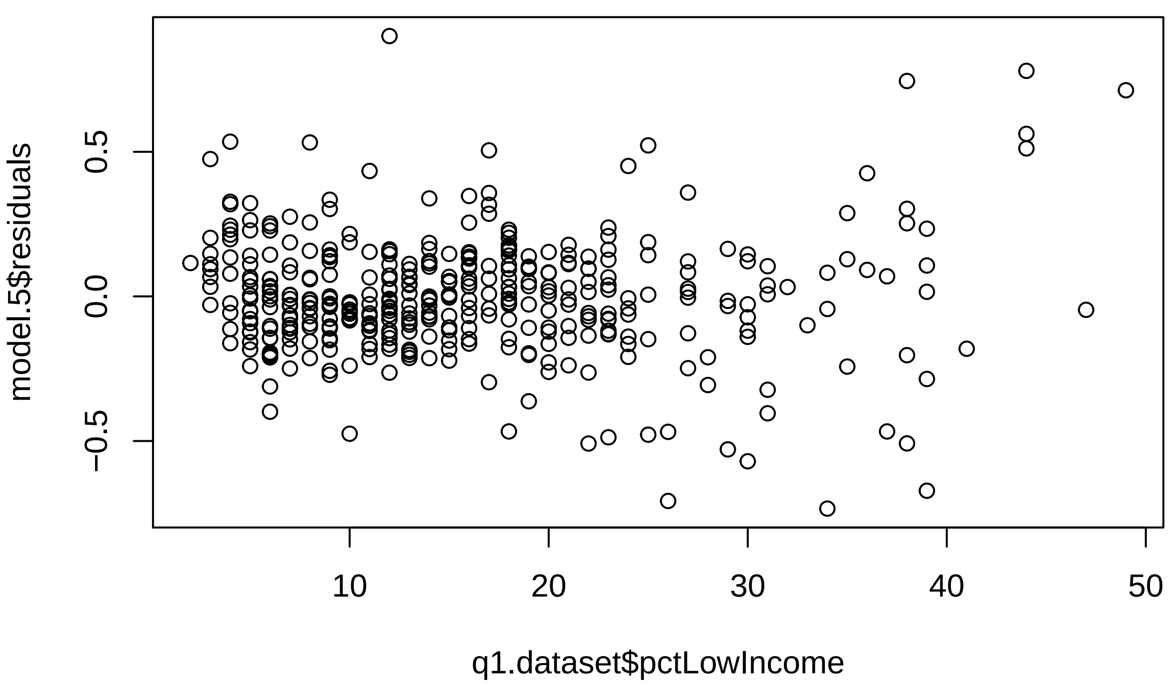
```
cor(q1.dataset$pupilTeacherRatio, model.5$residuals)
```

```
## [1] 2.94866e-15
```

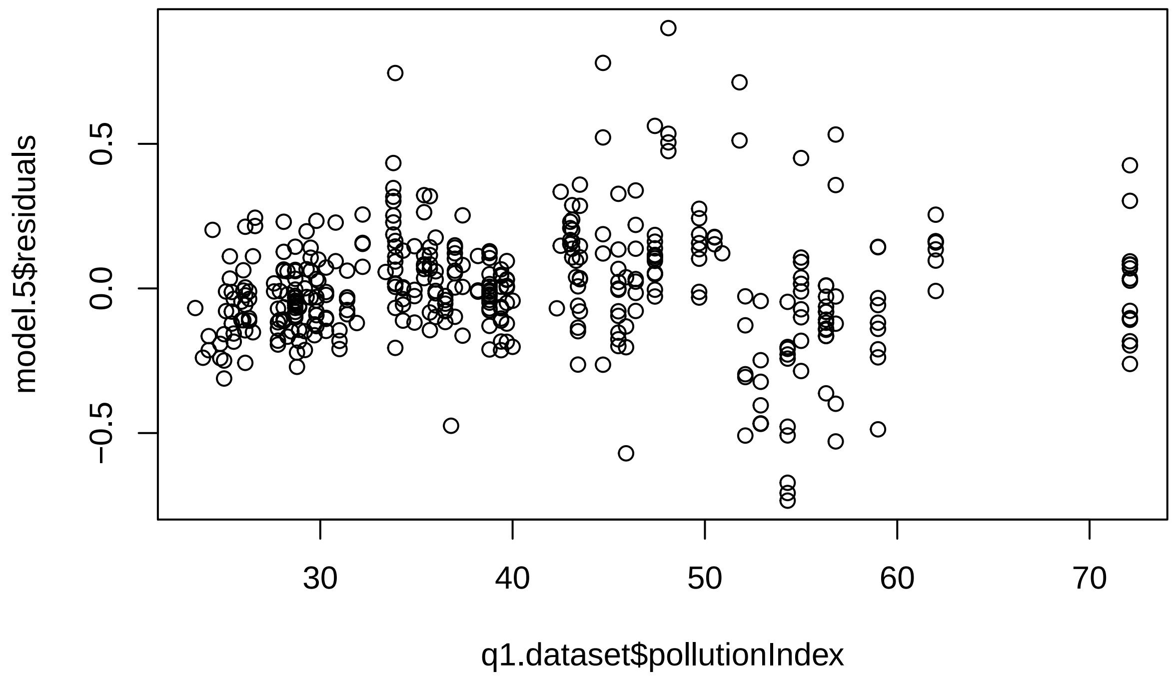
```
cor(q1.dataset$nBedRooms, model.5$residuals)
```

```
## [1] 4.091586e-16
```

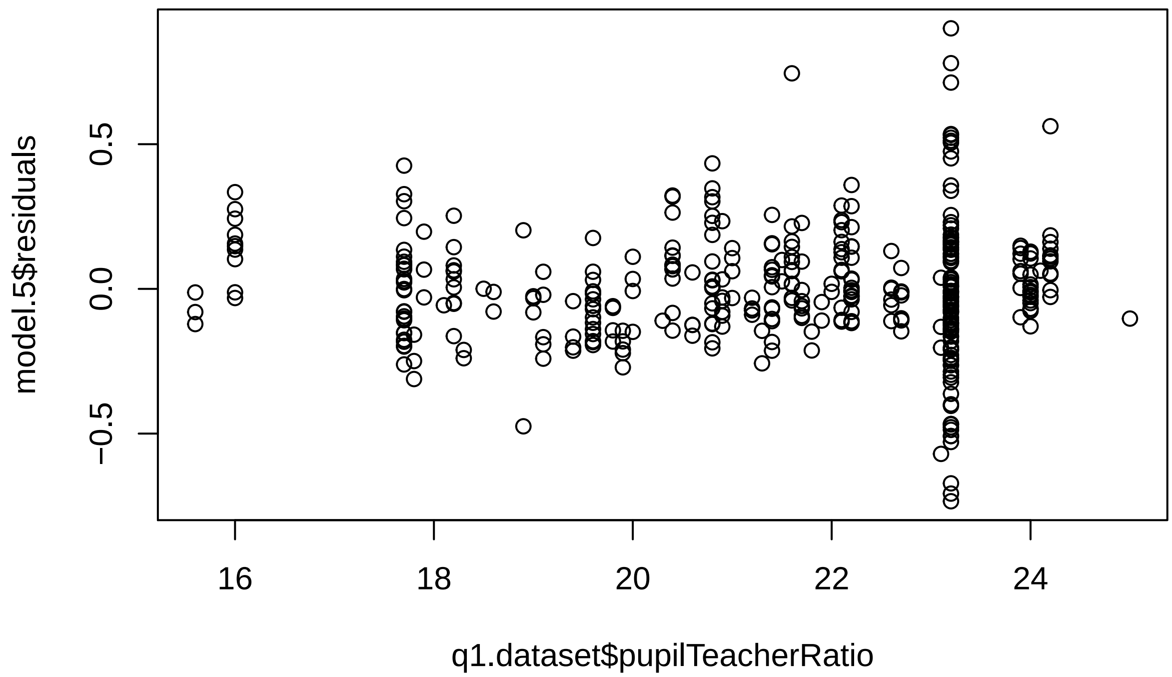
```
# Plots of predictors with residuals (error term)
plot(q1.dataset$pctLowIncome, model.5$residuals)
```



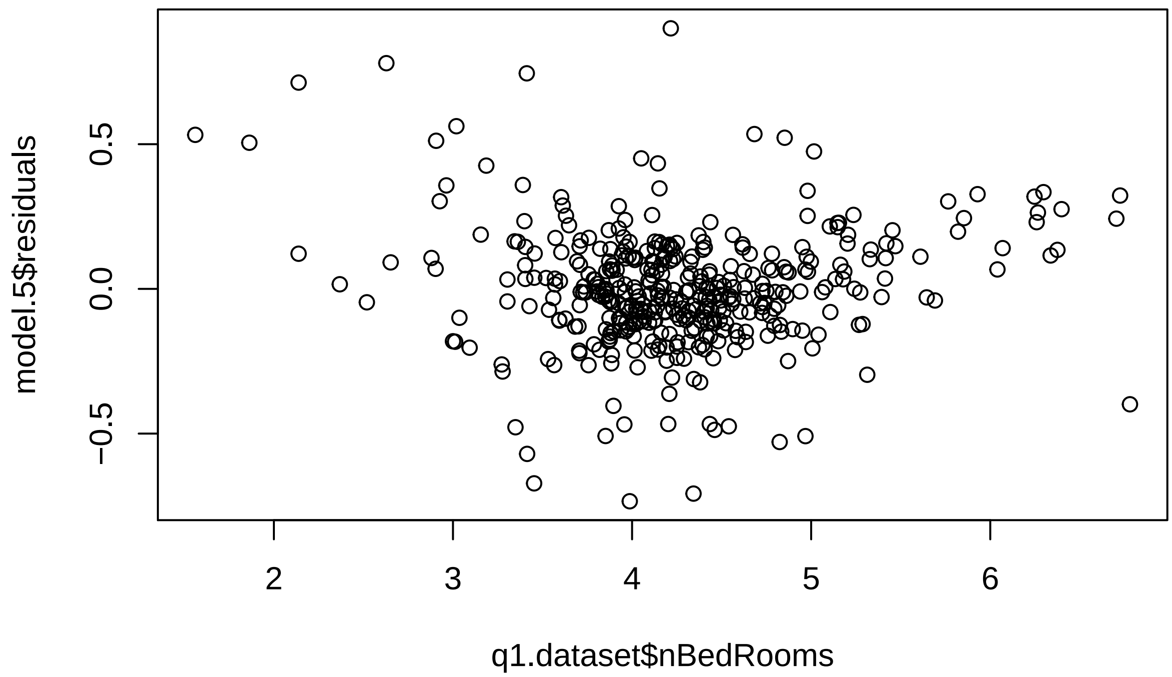
```r
plot(q1.dataset$pollutionIndex, model.5$residuals)
```




```r
plot(q1.dataset$pupilTeacherRatio, model.5$residuals)
```




```r
plot(q1.dataset$nBedRooms, model.5$residuals)
```



Now, we look at diagnostics for our final model.

1. The residuals vs fitted plot shows a very slight upward trend. The slope is negligible, so we assume zero conditional mean to hold.
2. Errors follow a close to normal distribution. In either case, we have 400 observations, enabling us to rely on OLS asymptotics.
3. The scale-location plot shows some trend which is not a significant cause for concern. It shows some heteroskedasticity, which we account for by taking robust standard errors below. Variables in our model remain significant. The Wald-Test shows that the model also remains significant.
4. The Residuals vs Leverage plot shows some outliers but no major cause for concern.

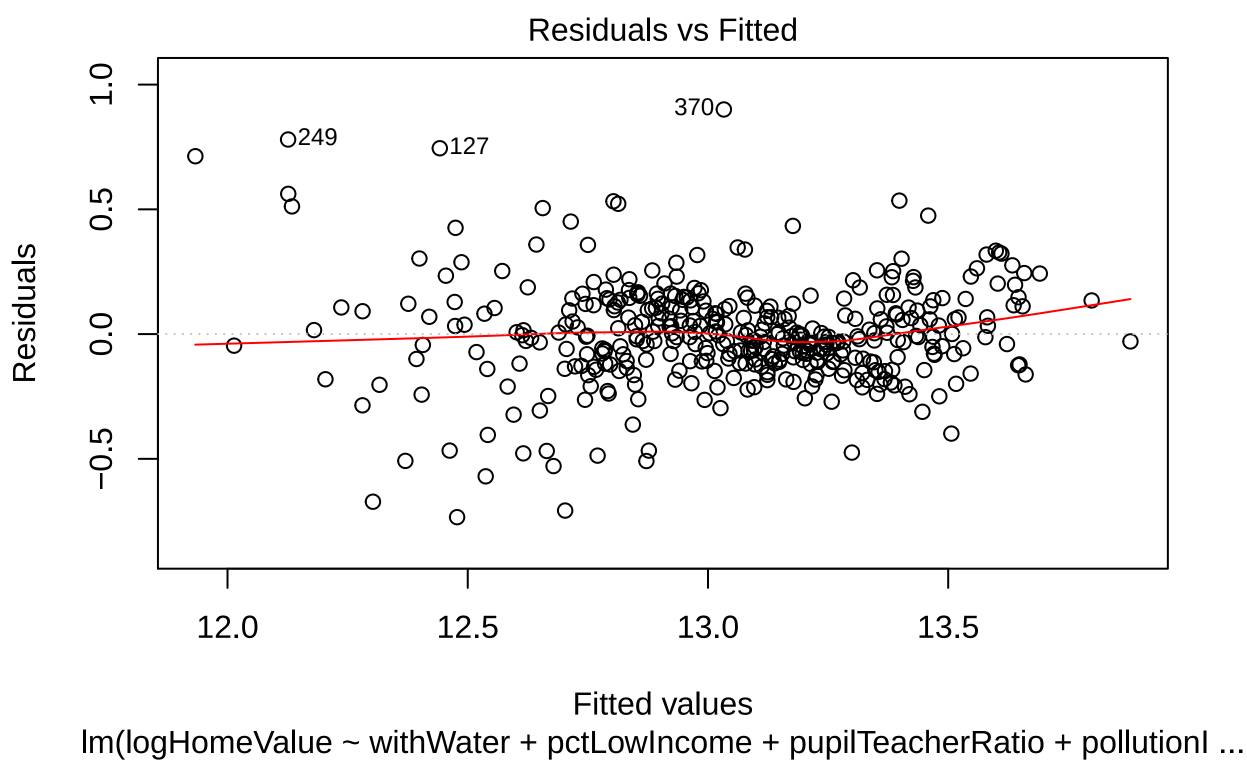
```r
coeftest(model.5)
```

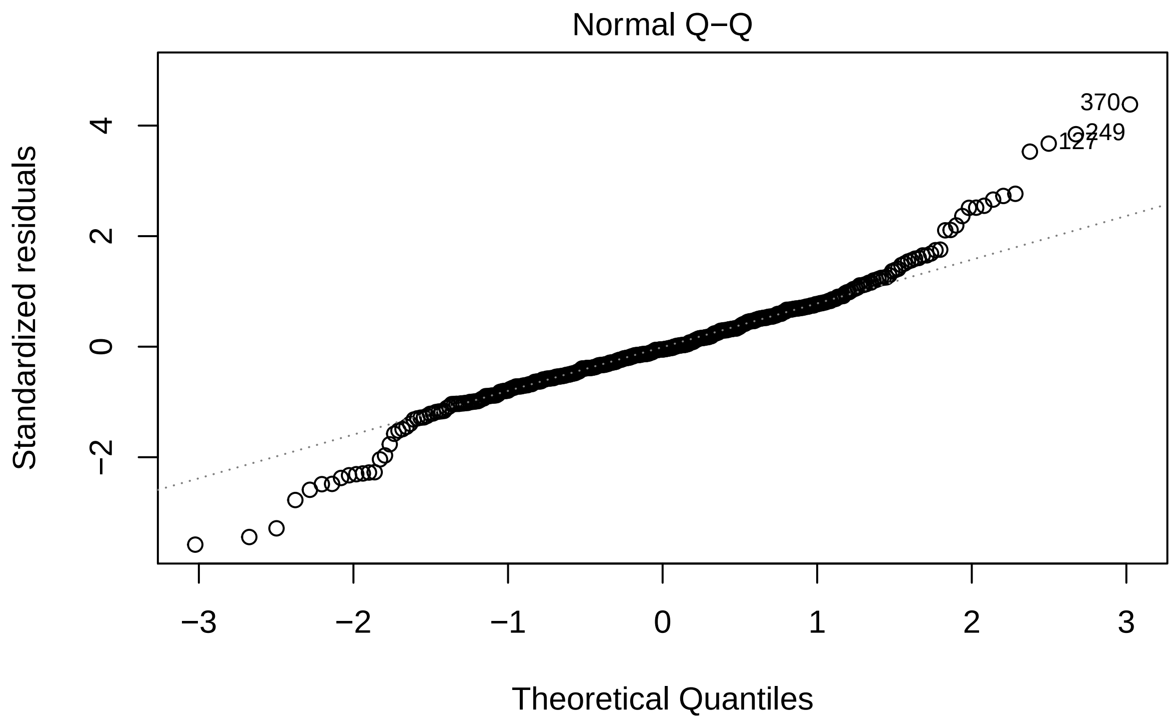
```
##
## t test of coefficients:
##
##                              Estimate Std. Error  t value  Pr(>|t|)
## (Intercept)                13.7957736  0.1510091  91.3573 < 2.2e-16 ***
## withWater1                  0.4051988  0.1361656   2.9758  0.003103 **
## pctLowIncome               -0.0231771  0.0017484 -13.2563 < 2.2e-16 ***
## pupilTeacherRatio          -0.0343924  0.0052663  -6.5307 2.028e-10 ***
## pollutionIndex             -0.0032122  0.0011742  -2.7357  0.006507 **
## nBedRooms                   0.1109547  0.0187498   5.9176 7.116e-09 ***
## withWater1:pollutionIndex -0.0057618  0.0028545  -2.0185  0.044217 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
waldtest(model.5, vcov = vcovHC)
```

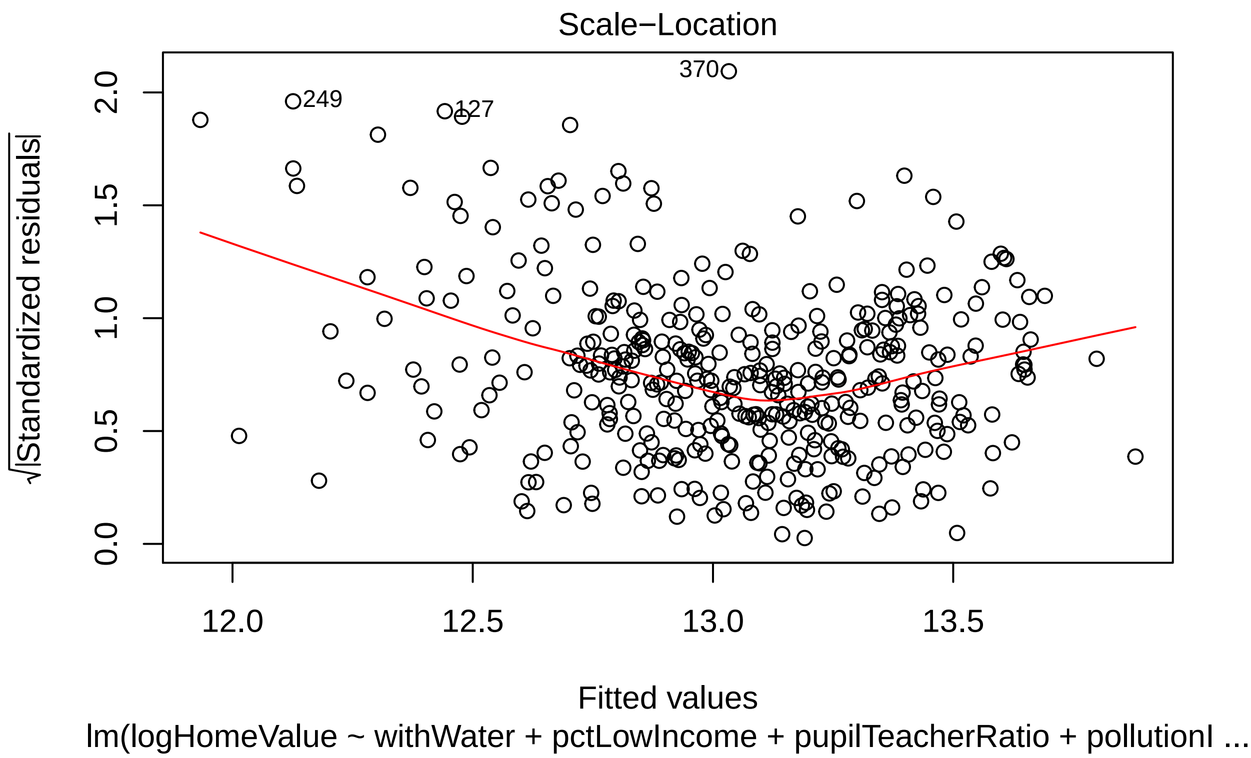
```
## Wald test
##
## Model 1: logHomeValue ~ withWater + pctLowIncome + pupilTeacherRatio +
##     pollutionIndex + nBedRooms + withWater:pollutionIndex
## Model 2: logHomeValue ~ 1
##   Res.Df Df      F    Pr(>F)
## 1    393
## 2    399 -6 108.23 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
plot(model.5)
```

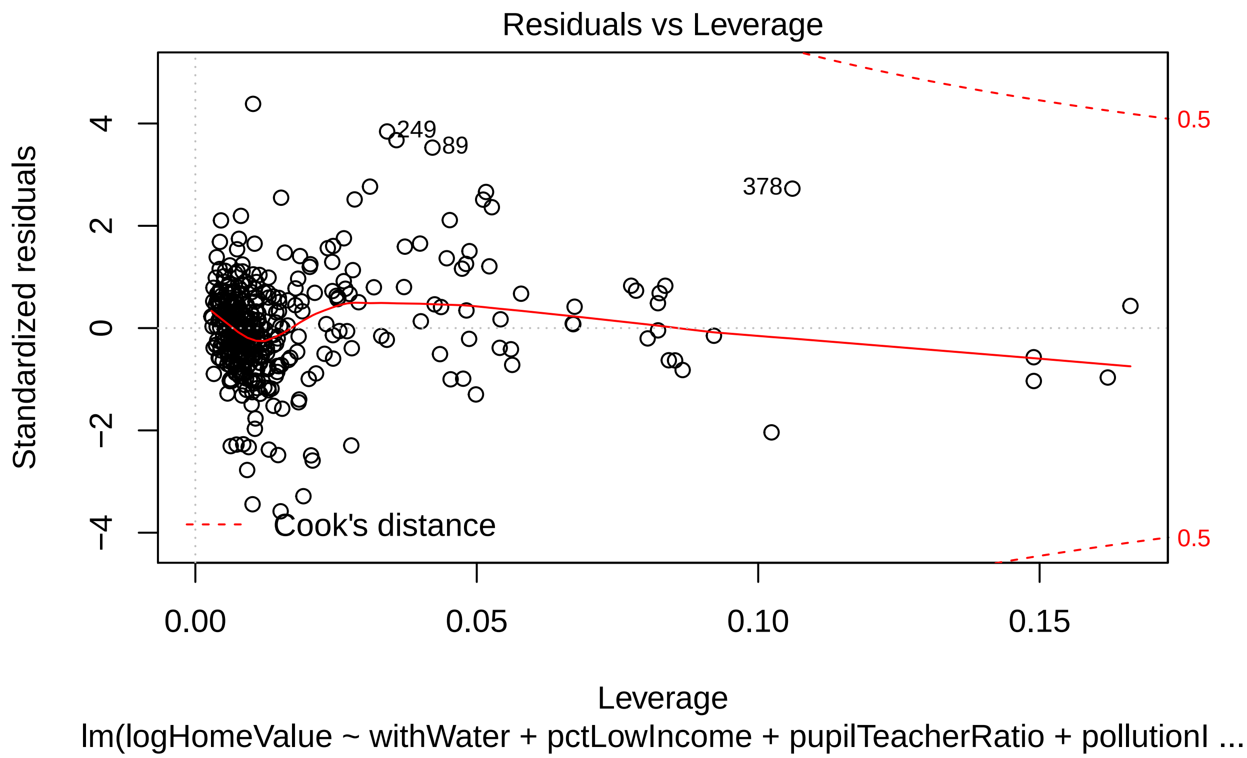




lm(logHomeValue ~ withWater + pctLowIncome + pupilTeacherRatio + pollutionI ...



Scale−Location
370
249
127
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
12.0
12.5
13.0
13.5
Fitted values
lm(logHomeValue ~ withWater + pctLowIncome + pupilTeacherRatio + pollutionI ...


Residuals vs Leverage
Standardized residuals
Leverage
lm(logHomeValue ~ withWater + pctLowIncome + pupilTeacherRatio + pollutionI ...
Cook's distance
0.5
0.5
249
89
378
4
2
0
−2
−4
0.00
0.05
0.10
0.15

Step 4 - Final Model Conclusions

Conclusion

Below we present a comparison of the initial and the final models run. We see that despite removing 5 variables, there is a reduction of only approximately 2% in R squared, implying that we have preserved most of the explanatory power of the model, while sticking with parsimony.

```
stargazer(model.1, model.5, type = "text")
```

```
##
## ============================================================================
##                                              Dependent variable:
##                              -------------------------------------------------
##                                                  logHomeValue
##                                         (1)                      (2)
## ----------------------------------------------------------------------------
## logCrimeRate_pc                        -0.011
##                                        (0.013)
##
## nonRetailBusiness                      -0.413
##                                        (0.265)
##
## withWater1                            0.141***                  0.405***
##                                        (0.041)                  (0.136)
##
## ageHouse                               0.0002
##                                        (0.001)
##
## logDistanceToCity                     -0.129***
##                                        (0.025)
##
## distanceToHighway                      -0.001
##                                        (0.003)
##
## pupilTeacherRatio                     -0.030***                -0.034***
##                                        (0.006)                  (0.005)
##
## pctLowIncome                          -0.024***                -0.023***
##                                        (0.002)                  (0.002)
##
## pollutionIndex                        -0.008***                -0.003***
##                                        (0.002)                  (0.001)
##
## nBedRooms                             0.103***                  0.111***
##                                        (0.019)                  (0.019)
##
## withWater1:pollutionIndex                                       -0.006**
##                                                                 (0.003)
##
## Constant                              14.233***                13.796***
##                                        (0.211)                  (0.151)
##
```

```
## ------------------------------------------------------------------------------
## Observations                            400                     400
## R2                                     0.753                   0.734
## Adjusted R2                            0.747                   0.730
## Residual Std. Error              0.200 (df = 389)        0.206 (df = 393)
## F Statistic                  118.510*** (df = 10; 389) 180.365*** (df = 6; 393)
## ==============================================================================
## Note:                                              *p<0.1; **p<0.05; ***p<0.01
```

Explanation of the Model

The final results from the model we present to the think-tank are as follows:

1. Home values are 41% greater for homes located within 5 miles of water. (Interaction effect of withWater explained below with pollutionIndex)
2. For every unit percentage increase in low-income households, home values are close to 2-3% lower.
3. For every one unit increase in the pupilTeacherRatio in a neighbourhood, home values are 3-4% lower.
4. For an additional unit on the pollutionIndex, we expect to see a decrease in home value of 0.3% if it is in a neighbourhood not within 5 miles of a water body. However, if it is within 5 miles of a water body, we expect an almost 1% decrease in home value.
5. If the number of bedrooms in a single-family house is one unit higher, we see 11% higher home values.

Part 2 (25 points): Modeling and Forecasting a Real-World Macroeconomic / Financial time series

Approach summary
We fit the series with an ARIMA(0, 1, 0) model, followed by fitting our residuals with GARCH and using this to make our forecast with accurate confidence intervals depicting the series. The next sections implement the details of our approach.

EDA on series
The series appears to be a time series of financial data, presumably one of the daily closing price of some financial instrumemt or index.

We observe that the time series is non-stationary in the mean. Therefore we can attempt to diff the time series to see if the resulting series is stationary in the mean. We also observe that the PACF of the time series indicates a correlation at lag 1 that ressembles an AR(1) series.

The plot of the original financial series does not indicate any amount of seasonality. The observation is confirmed by the PACF which doesn't display any significant amount of correlation at any lag other than the first. We will therefore not try to fit a model that includes a seasonal component to the data. We next proceed to analyze the first difference of the time series.

The 1st difference series is stationary in the mean. We also observe that after the first difference is taken, the ACF of the series, just like the PACF suggest white noise dynamics. But the first difference series does show clustered volatility in the variance which will require that we model the residuals of the fitted series using GARCH.

```r
# Load the data and describe it
data <- read.csv(file.path("lab3_series02.csv"))

# Describing Series
str(data)
```

```
## 'data.frame':    2332 obs. of  2 variables:
##  $ X         : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ DXCM.Close: num  9.88 9.79 9.68 9.64 9.42 9.47 9.16 8.99 8.6 8.81 ...
```

```r
summary(data)
```

```
##        X            DXCM.Close
##  Min.   :   1.0   Min.   :  1.390
##  1st Qu.: 583.8   1st Qu.:  8.188
##  Median :1166.5   Median : 12.355
##  Mean   :1166.5   Mean   : 23.210
##  3rd Qu.:1749.2   3rd Qu.: 32.565
##  Max.   :2332.0   Max.   :101.910
```

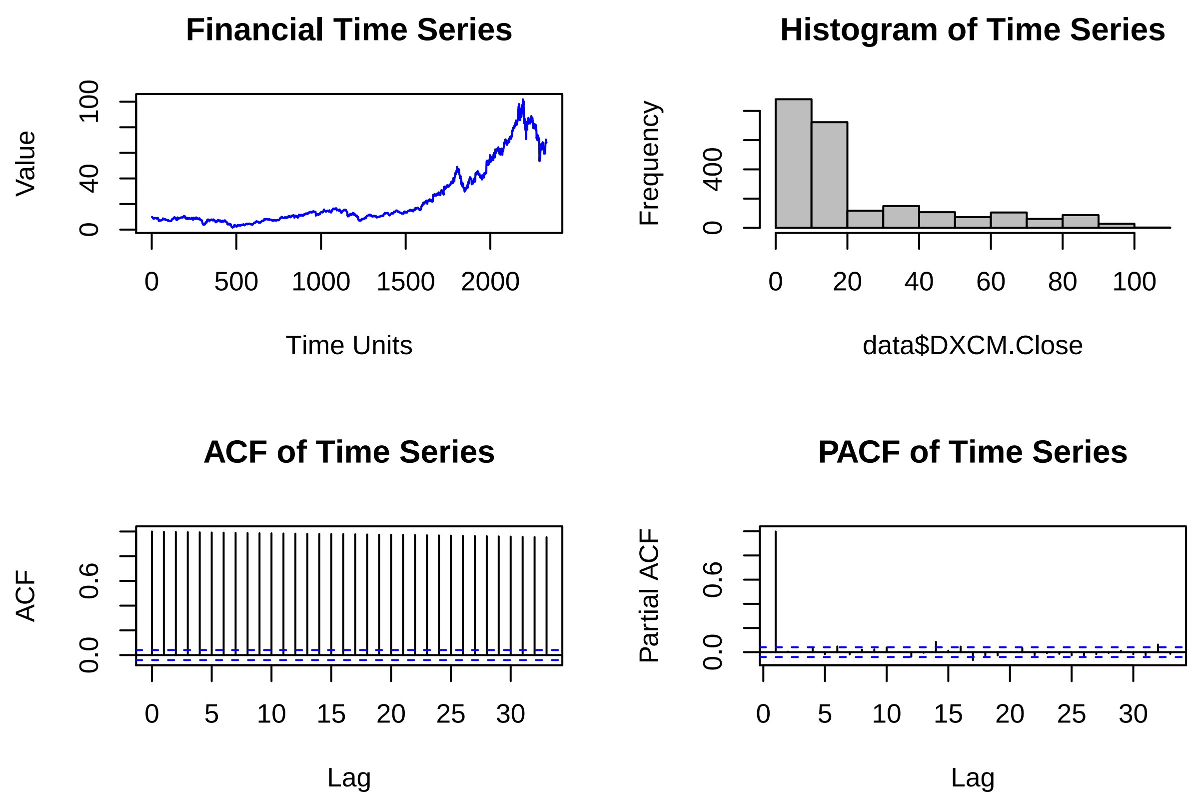
```r
cbind(head(data), tail(data))
```

```
##   X DXCM.Close    X DXCM.Close
## 1 1       9.88 2327      67.63
## 2 2       9.79 2328      70.49
## 3 3       9.68 2329      67.79
## 4 4       9.64 2330      68.72
## 5 5       9.42 2331      68.43
## 6 6       9.47 2332      68.08
```

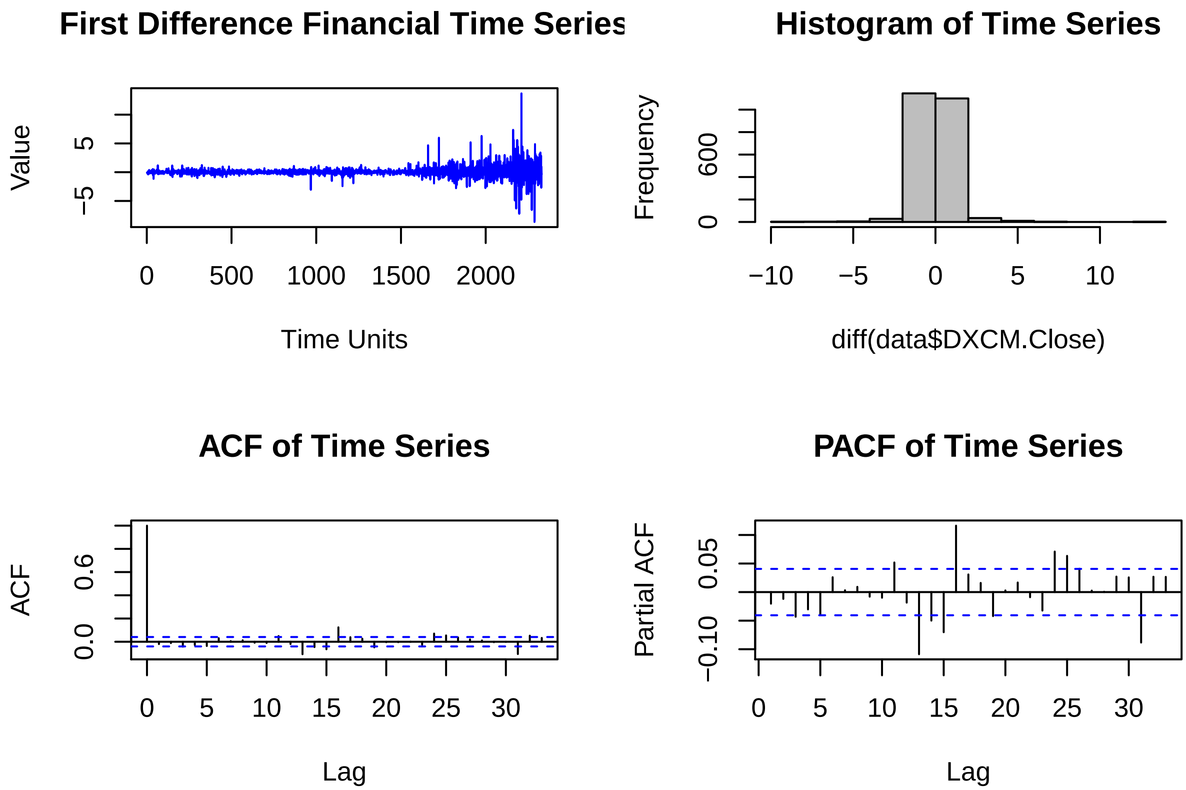
```r
quantile(as.numeric(data$DXCM.Close), c(0, 0.01, 0.05, 0.1, 0.25,
    0.5, 0.75, 0.9, 0.95, 0.99, 1))
```

```
##       0%       1%       5%      10%      25%      50%      75%      90%
##   1.3900   2.7531   4.1700   6.1630   8.1875  12.3550  32.5650  63.5210
##      95%      99%     100%
##  80.0430  91.6887 101.9100
```

```r
# EDA on series
par(mfrow = c(2, 2))
plot.ts(data$DXCM.Close, main = "Financial Time Series", ylab = "Value",
    xlab = "Time Units", col = "blue")
hist(data$DXCM.Close, col = "gray", main = "Histogram of Time Series")
acf(data$DXCM.Close, main = "ACF of Time Series")
pacf(data$DXCM.Close, main = "PACF of Time Series")
```




```r
# EDA on the first difference series
par(mfrow = c(2, 2))
plot.ts(diff(data$DXCM.Close), main = "First Difference Financial Time Series",
    ylab = "Value", xlab = "Time Units", col = "blue")
hist(diff(data$DXCM.Close), col = "gray", main = "Histogram of Time Series")
acf(diff(data$DXCM.Close), main = "ACF of Time Series")
pacf(diff(data$DXCM.Close), main = "PACF of Time Series")
```



Evaluating a best model to fit the series data

We now try to determine the best ARIMA model based on the best AIC for that series. Interestingly enough, the best model based on AIC is a seasonal model with (p,d,q,P,D,Q) of values (0, 0, 0, 0, 1, 0). Since we're assuming a frequency of one, that model is the same as the second best model, which has orders (p,d,q,P,D,Q) of (0, 1, 0, 0, 0, 0). We therefore decide to use (p,d,q,P,D,Q)=(0, 1, 0, 0, 0, 0) as our fitted model going forward.

```
data.best <- get.best.sarima(data$DXCM, maxord = rep(3, 6), 1)
data.best$best
```

```
## [[1]]
## [1] 6246.144
##
## [[2]]
##
## Call:
## arima(x = x.ts, order = c(p, d, q), seasonal = list(order = c(P, D, Q), freq),
##     method = "CSS", optim.control = list(maxit = 10000))
##
##
## sigma^2 estimated as 0.8536:  part log likelihood = -3123.07
##
## [[3]]
## [1] 0 0 0 0 1 0
```

```r
data.best$others[order(data.best$others$aics)[1:20], ]
```

```
##          aics             models
## 5    6246.144 (0, 0, 0, 0, 1, 0)
## 257  6246.144 (0, 1, 0, 0, 0, 0)
## 6    6253.987 (0, 0, 0, 0, 1, 1)
## 69   6253.987 (0, 0, 1, 0, 1, 0)
## 258  6253.987 (0, 1, 0, 0, 0, 1)
## 321  6253.987 (0, 1, 1, 0, 0, 0)
## 21   6254.999 (0, 0, 0, 1, 1, 0)
## 273  6254.999 (0, 1, 0, 1, 0, 0)
## 1029 6254.999 (1, 0, 0, 0, 1, 0)
## 1281 6254.999 (1, 1, 0, 0, 0, 0)
## 10   6256.568 (0, 0, 0, 0, 2, 1)
## 73   6256.568 (0, 0, 1, 0, 2, 0)
## 262  6256.568 (0, 1, 0, 0, 1, 1)
## 325  6256.568 (0, 1, 1, 0, 1, 0)
## 514  6256.568 (0, 2, 0, 0, 0, 1)
## 577  6256.568 (0, 2, 1, 0, 0, 0)
## 22   6259.117 (0, 0, 0, 1, 1, 1)
## 274  6259.117 (0, 1, 0, 1, 0, 1)
## 1030 6259.117 (1, 0, 0, 0, 1, 1)
## 1093 6259.117 (1, 0, 1, 0, 1, 0)
```

The fitted model is one with a differential component of value one, and has no other parameters for which we need to validate 95 confidence intervals.

```r
# Fitting a first difference series to our model
data.fit <- Arima(data$DXCM, order = c(0, 1, 0), seasonal = list(order = c(0,
    0, 0)), method = "CSS-ML")
data.res <- data.fit$resid
quantile(as.numeric(data.res), c(0, 0.01, 0.05, 0.1, 0.25, 0.5,
    0.75, 0.9, 0.95, 0.99, 1))
```

```
##      0%      1%      5%     10%     25%     50%     75%     90%     95%
## -8.6500 -2.4369 -1.1100 -0.5600 -0.2300  0.0000  0.2500  0.6400  1.1500
##     99%    100%
##  2.8538 13.7000
```

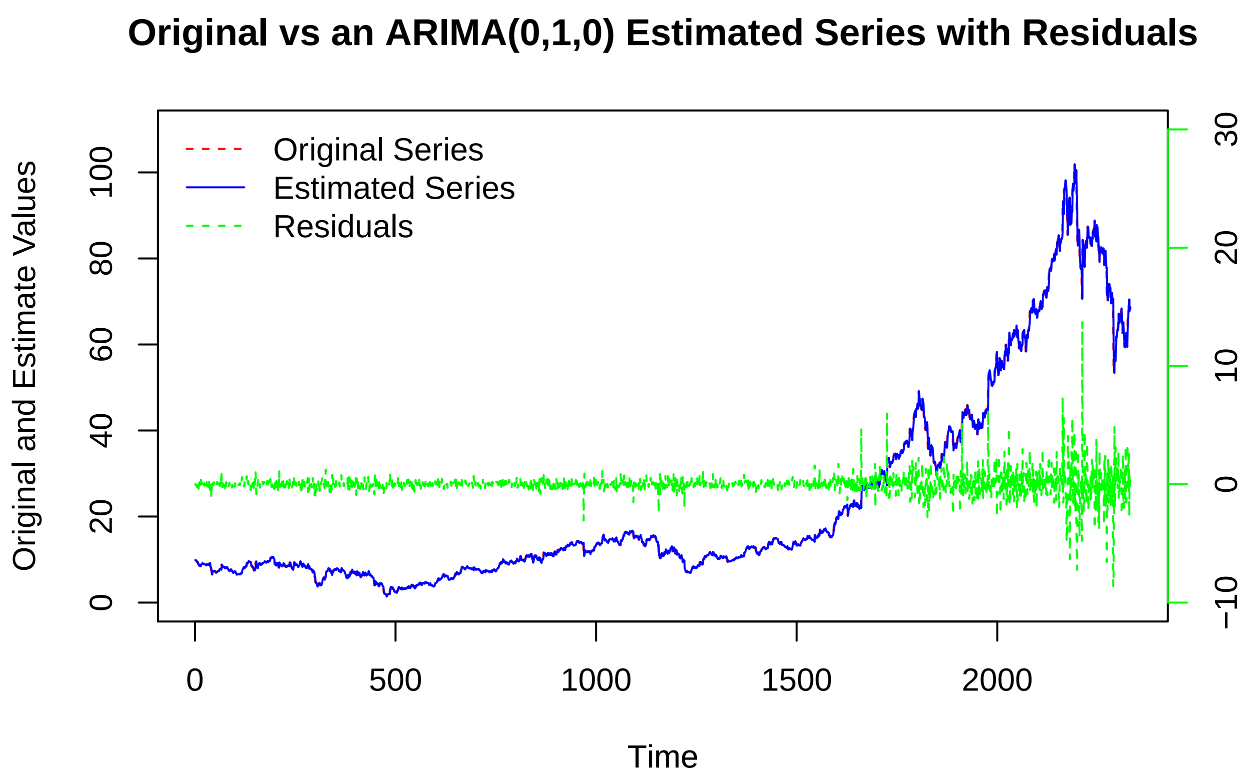
```r
t(confint(data.fit))
```

```
##
## 2.5 %
## 97.5 %
```

We now perform in-sample fit using the fitted series to assess our fitted model. The fitted series models the original series very well and the model selection seems appropriate based on in-sample fit.

```r
# Performing in-sample fit using our fitted series
par(mfrow = c(1, 1))
plot.ts(data$DXCM, lty = 2, col = "red", main = "Original vs an ARIMA(0,1,0) Estimated Series with Resi
```

```
    ylab = "Original and Estimate Values", ylim = c(0, 110))
par(new = T)
plot(fitted(data.fit), col = "blue", axes = F, ylab = "", ylim = c(0,
    110))
leg.txt <- c("Original Series", "Estimated Series", "Residuals")
legend("topleft", legend = leg.txt, lty = c(2, 1, 2), col = c("red",
    "blue", "green"), bty = "n", cex = 1)
par(new = T)
plot.ts(data.res, axes = F, xlab = "", ylab = "", col = "green",
    ylim = c(-10, 30), pch = 1, lty = 2)
axis(side = 4, col = "green")
mtext("Residuals", side = 4, line = 2, col = "green")
```



To further assess the fitted series, We now perform 36 steps backtesting. What the out of sample forecast shows is that the original series falls for the most part within the 80% confidence interval and almost totally within the 95% confidence interval of the prediction. However, we can see from the plot of the financial time series that the volatility of the series residuals seems to increase over time. Clustered volatility likely explains this dynamic. We next turn our eyes to the residuals of the fitted series and to the analysis of the dynamics of its variance.

```
# Performing 36 steps backtesting using our fitted series
data.fit.back <- Arima(data$DXCM[1:(length(data$DXCM) - 36)],
    order = c(0, 1, 0), seasonal = list(order = c(0, 0, 0)),
    method = "CSS-ML")
summary(data.fit.back)
```

```
## Series: data$DXCM[1:(length(data$DXCM) - 36)]
## ARIMA(0,1,0)
##
## sigma^2 estimated as 0.8223:  log likelihood=-3031.91
## AIC=6065.83   AICc=6065.83   BIC=6071.57
##
## Training set error measures:
##                      ME      RMSE       MAE         MPE     MAPE      MASE
## Training set 0.02262625 0.9065937 0.4574782 0.007157606 2.477381 0.9995739
##                     ACF1
## Training set -0.01924297
```

```r
length(fitted(data.fit.back))
```

```
## [1] 2296
```

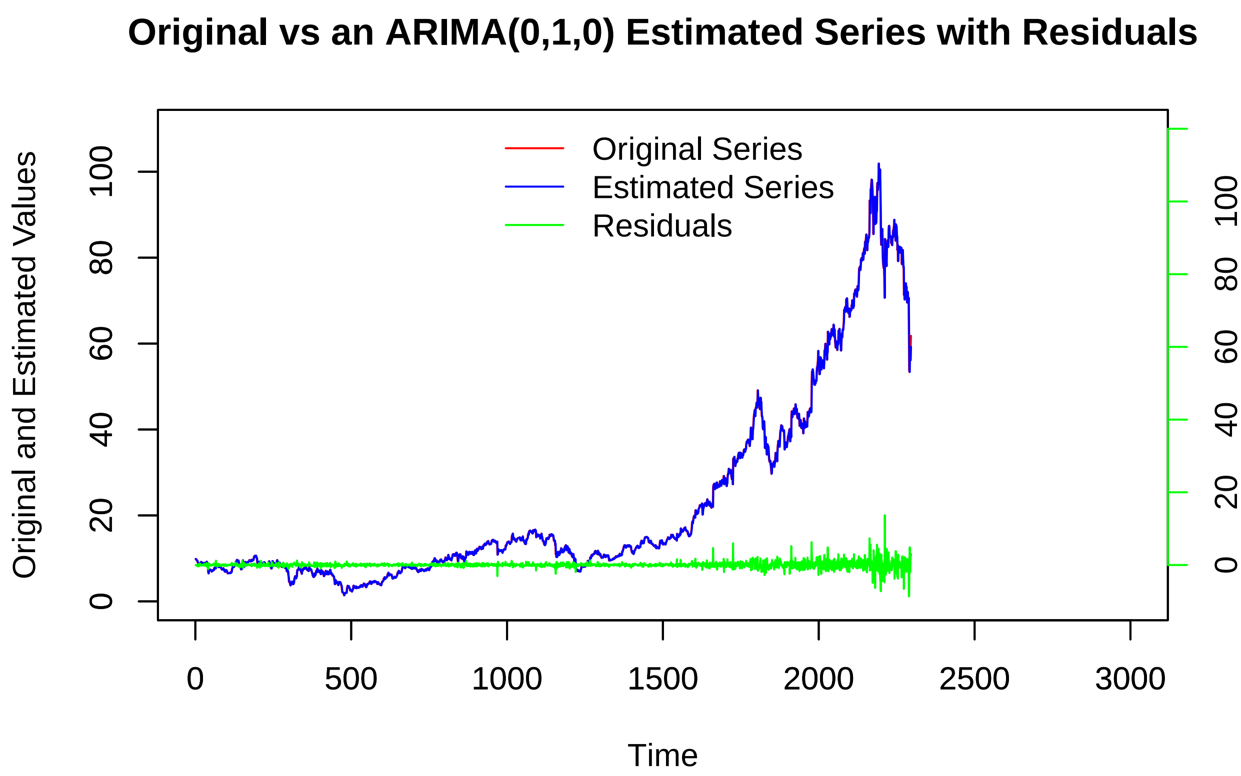
```r
length(data.fit.back$resid)
```

```
## [1] 2296
```

```r
df = cbind(data$DXCM[1:(length(data$DXCM) - 36)], fitted(data.fit.back),
    data.fit.back$resid)
colnames(df) = c("orig_series", "fitted_vals", "resid")
head(df)
```

```
##      orig_series fitted_vals        resid
## [1,]        9.88     9.87012  0.009879995
## [2,]        9.79     9.88000 -0.090000000
## [3,]        9.68     9.79000 -0.110000000
## [4,]        9.64     9.68000 -0.040000000
## [5,]        9.42     9.64000 -0.220000000
## [6,]        9.47     9.42000  0.050000000
```

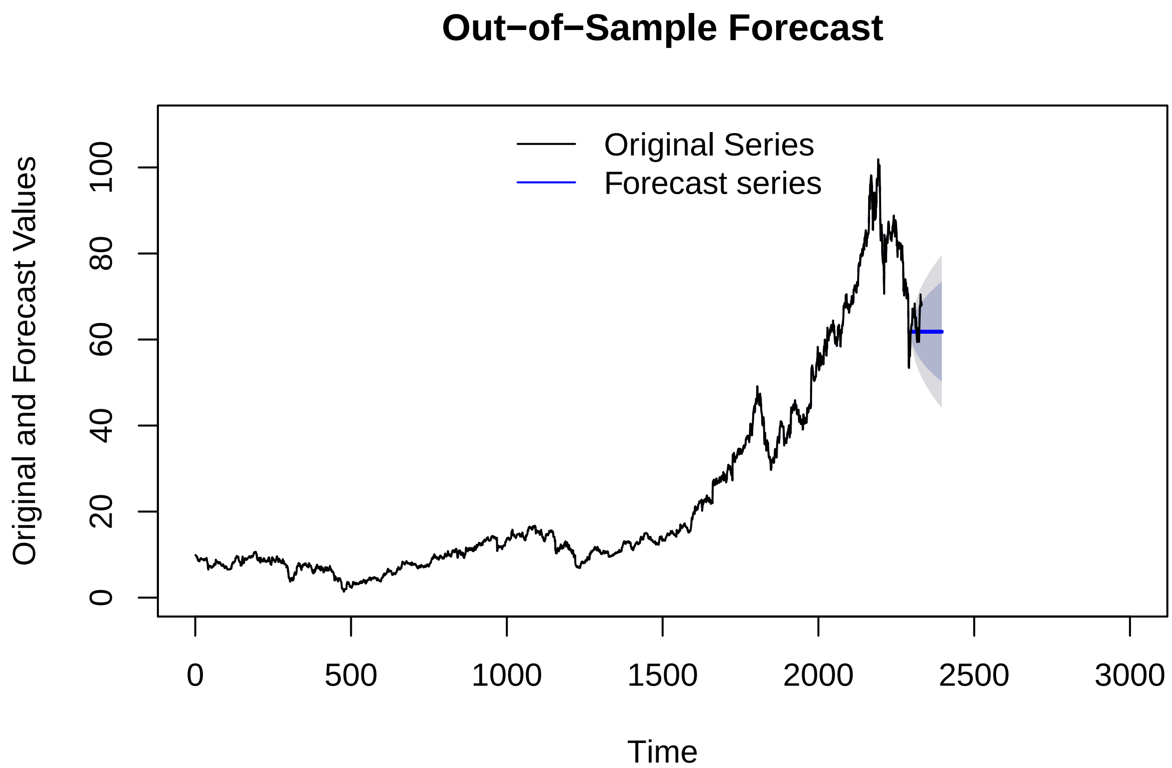
```r
# Step 1: Plot the original and estimate series
par(mfrow = c(1, 1))
plot.ts(df[, "orig_series"], col = "red", main = "Original vs an ARIMA(0,1,0) Estimated Series with Resi
    ylab = "Original and Estimated Values", xlim = c(0, 3000),
    ylim = c(0, 110))
par(new = T)
plot.ts(df[, "fitted_vals"], col = "blue", axes = T, xlab = "",
    ylab = "", xlim = c(0, 3000), ylim = c(0, 110))
leg.txt <- c("Original Series", "Estimated Series", "Residuals")
legend("top", legend = leg.txt, lty = 1, col = c("red", "blue",
    "green"), bty = "n", cex = 1)
par(new = T)
plot.ts(df[, "resid"], axes = F, xlab = "", ylab = "", col = "green",
    xlim = c(0, 3000), ylim = c(-10, 120), pch = 1)
axis(side = 4, col = "green")
mtext("Residuals", side = 4, line = 2, col = "green")
```




```r
# Step 2: Out of sample forecast
data.fit.back.fcast <- forecast.Arima(data.fit.back, h = 100)
length(data.fit.back.fcast$mean)
```

```
## [1] 100
```

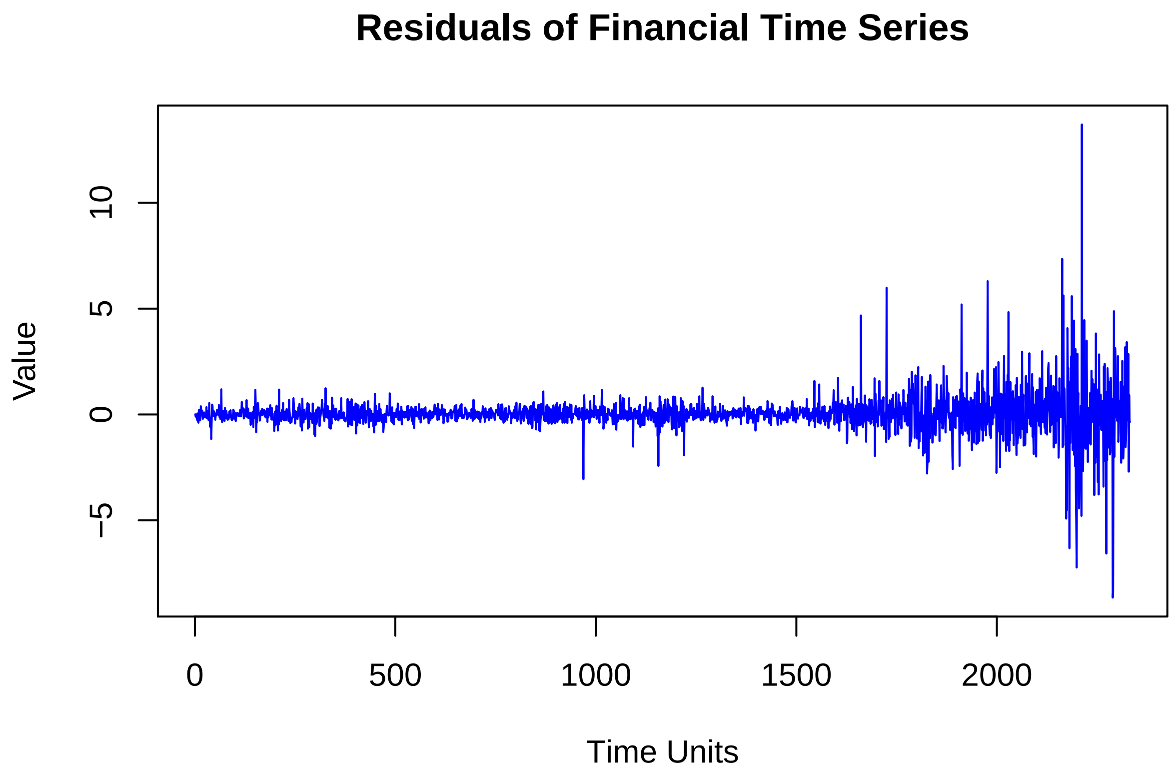
```r
par(mfrow = c(1, 1))
plot(data.fit.back.fcast, lty = 2, col = "navy", main = "Out-of-Sample Forecast",
    ylab = "Original and Forecast Values", xlim = c(0, 3000),
    ylim = c(0, 110))
par(new = T)
plot.ts(data$DXCM, axes = F, lty = 1, xlim = c(0, 3000), ylim = c(0,
    110), ylab = "")
leg.txt <- c("Original Series", "Forecast series")
legend("top", legend = leg.txt, lty = 1, col = c("black", "blue"),
    bty = "n", cex = 1)
```



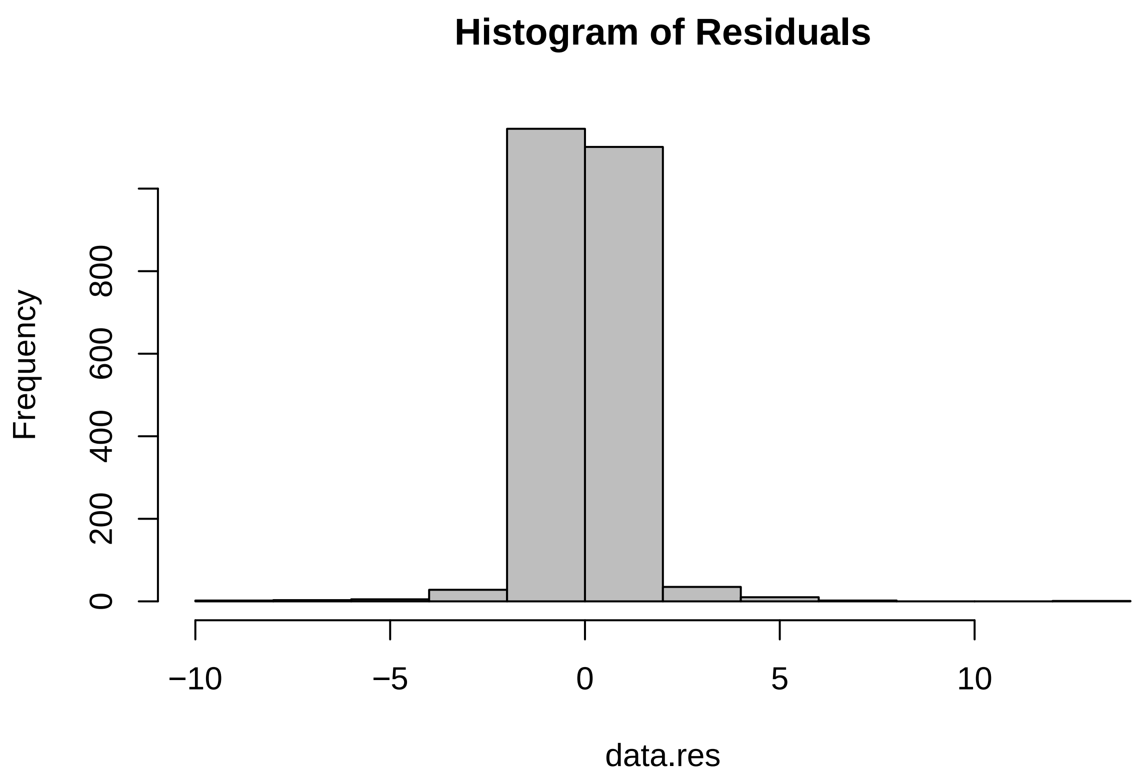
Residuals analysis

We observe from the residual time series that the variance of the series is non-stationary. The series exhibits volatility with a variance changing in a regular way. It exhibits conditional heteroskedasticity. An observation of the ACF of the squared residuals series provides confirmation of the variance dynamics. Therefore, we decide to model its residuals using GARCH

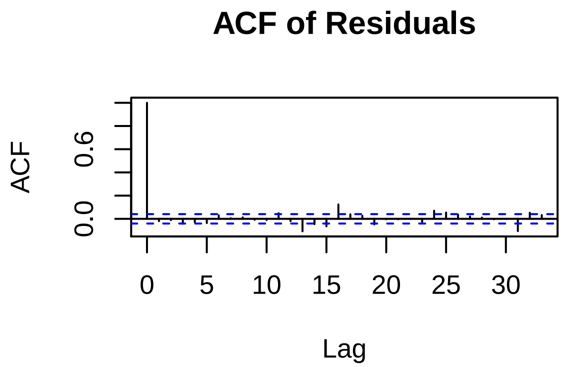
```
# Plot the residuals time series
par(mfrow = c(1, 1))
plot.ts(data.res, main = "Residuals of Financial Time Series",
    ylab = "Value", xlab = "Time Units", col = "blue")
```

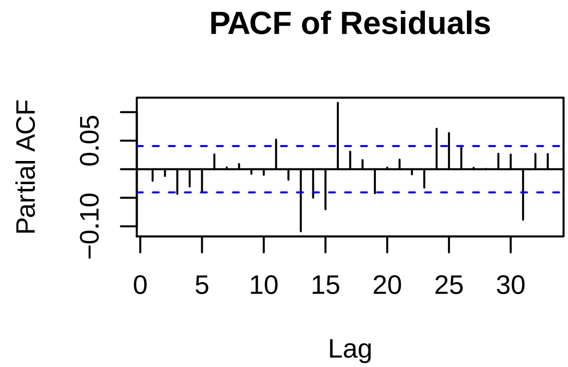


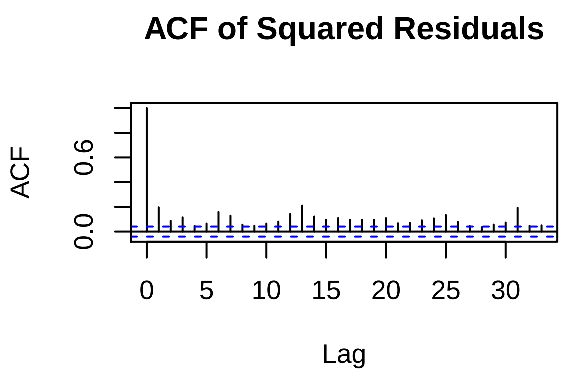

```r
par(mfrow = c(1, 1))
hist(data.res, col = "gray", main = "Histogram of Residuals")
```




```r
par(mfrow = c(2, 2))
acf(data.res, main = "ACF of Residuals")
pacf(data.res, main = "PACF of Residuals")
acf(data.res^2, main = "ACF of Squared Residuals")
```







We chose the default (p, q) = (1, 1) parameters of the function for our GARCH model. The parameters of the model are all significant based on a 95% confidence interval.

We observe from the ACF of the residuals of the GARCH fitted series that they have the charactersitics of white noise with mostly non-significant correlations at all lags of the ACF. What the GARCH model of the residuals tells is that we can expect more or less volatility through the forecast of the point series that invalidate the confidence intervals of our point predictions since those were made with the assumption of a stationary variance.

```
# Model the residuals of the financial time series using
# GARCH
data.garch <- garch(data.res, trace = F)
```

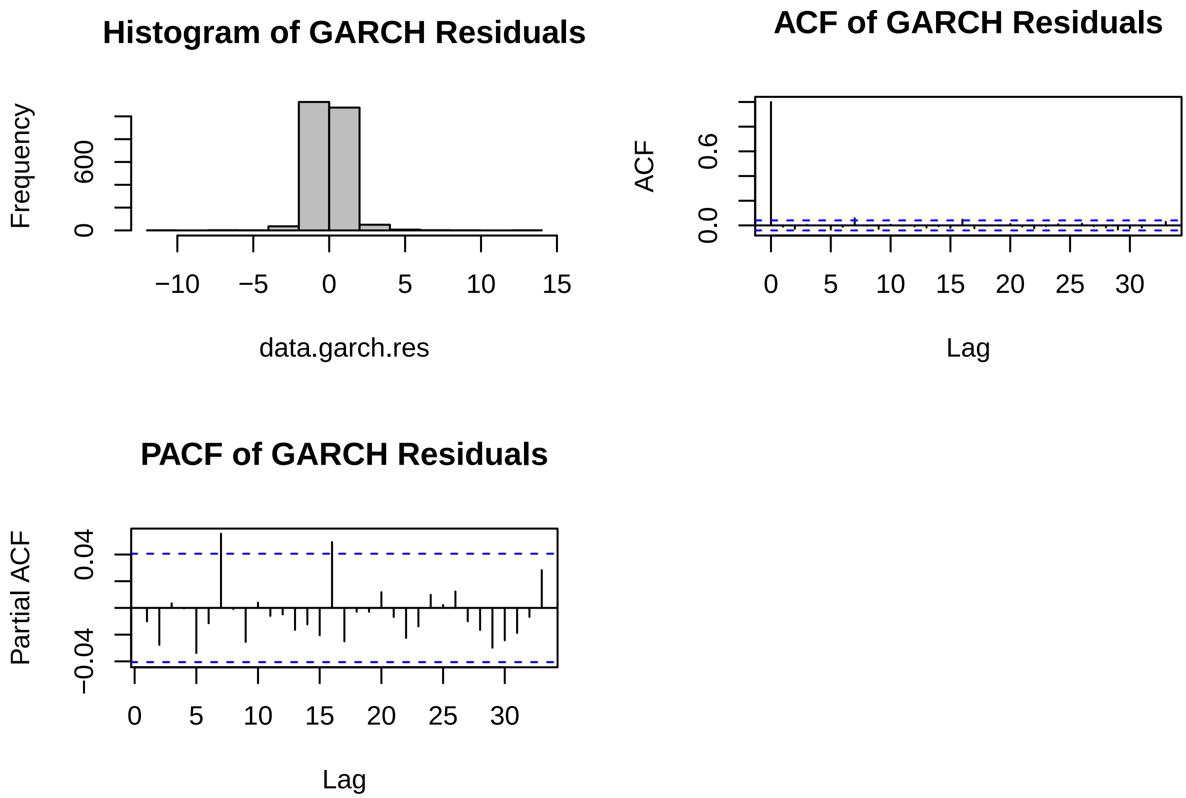
```
## Warning in sqrt(pred$e): NaNs produced
```

```
t(confint(data.garch))
```

```
##                  a0         a1        b1
## 2.5 %  5.480144e-05 0.03050862 0.9655033
## 97.5 % 4.229287e-04 0.03925722 0.9734176
```

```
data.garch.res <- resid(data.garch)[-1]

# Plot a histogram, ACF and PACF of the residuals after
# fitting a GARCH model
```

```r
par(mfrow = c(2, 2))
hist(data.garch.res, col = "gray", main = "Histogram of GARCH Residuals")
acf(data.garch.res, na.action = na.pass, main = "ACF of GARCH Residuals")
pacf(data.garch.res, na.action = na.pass, main = "PACF of GARCH Residuals")
```



36 steps ahead preditions
With the previous observations about volatility in mind, we still use the fitted series to predict 36 steps ahead. We will later adjust the confidence intervals of the predictions using our fitted GARCH model.

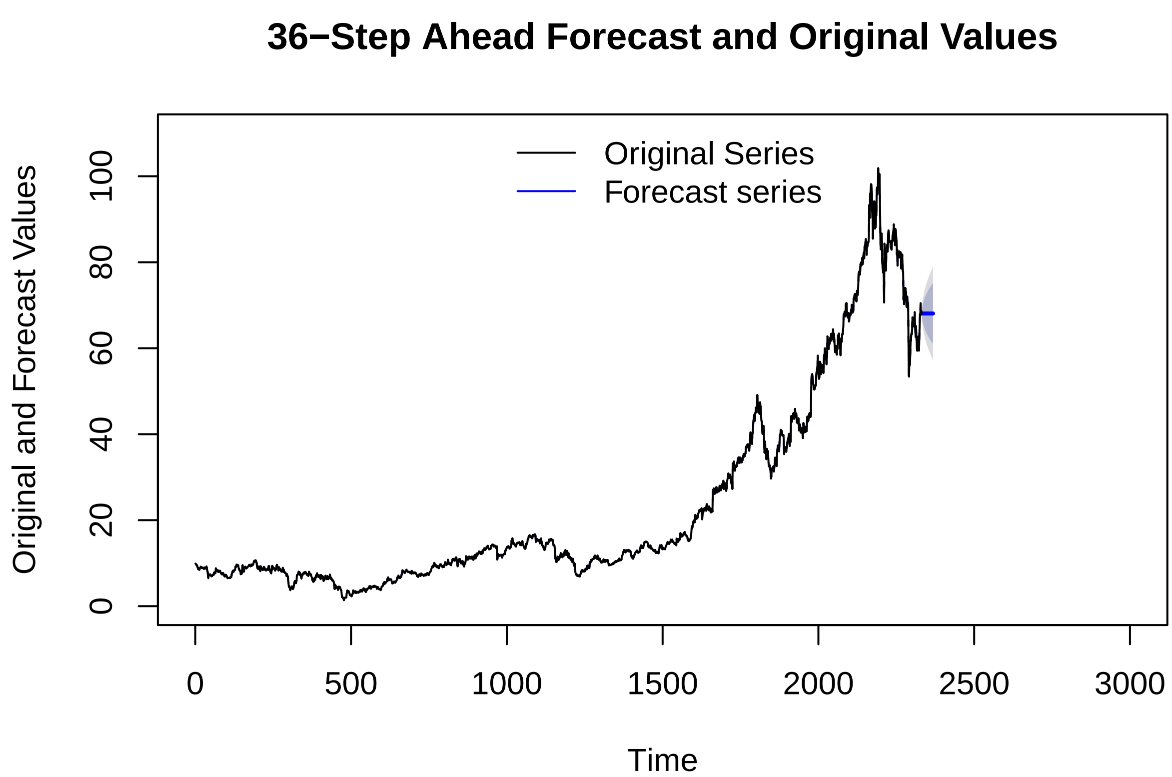
To perform a 36 steps ahead forecast of our series, we use the original point estimate of the series, that being the (0,1,0) ARIMA model initially estimated. The GARCH model of the residuals will additionally be used to forcast the variance of the series, and help us adjust the confidence interval of the prediction.

```r
# 36 steps ahead sample forecast of the financial time series
data.fit.ahead.fcast <- forecast.Arima(data.fit, h = 36)
length(data.fit.ahead.fcast$mean)
```

```
## [1] 36
```

```r
par(mfrow = c(1, 1))
plot(data.fit.ahead.fcast, lty = 2, col = "navy", main = "36-Step Ahead Forecast and Original Values",
    ylab = "Original and Forecast Values", xlim = c(0, 3000),
    ylim = c(0, 110))
par(new = T)
plot.ts(data$DXCM, axes = F, lty = 1, xlim = c(0, 3000), ylim = c(0,
```

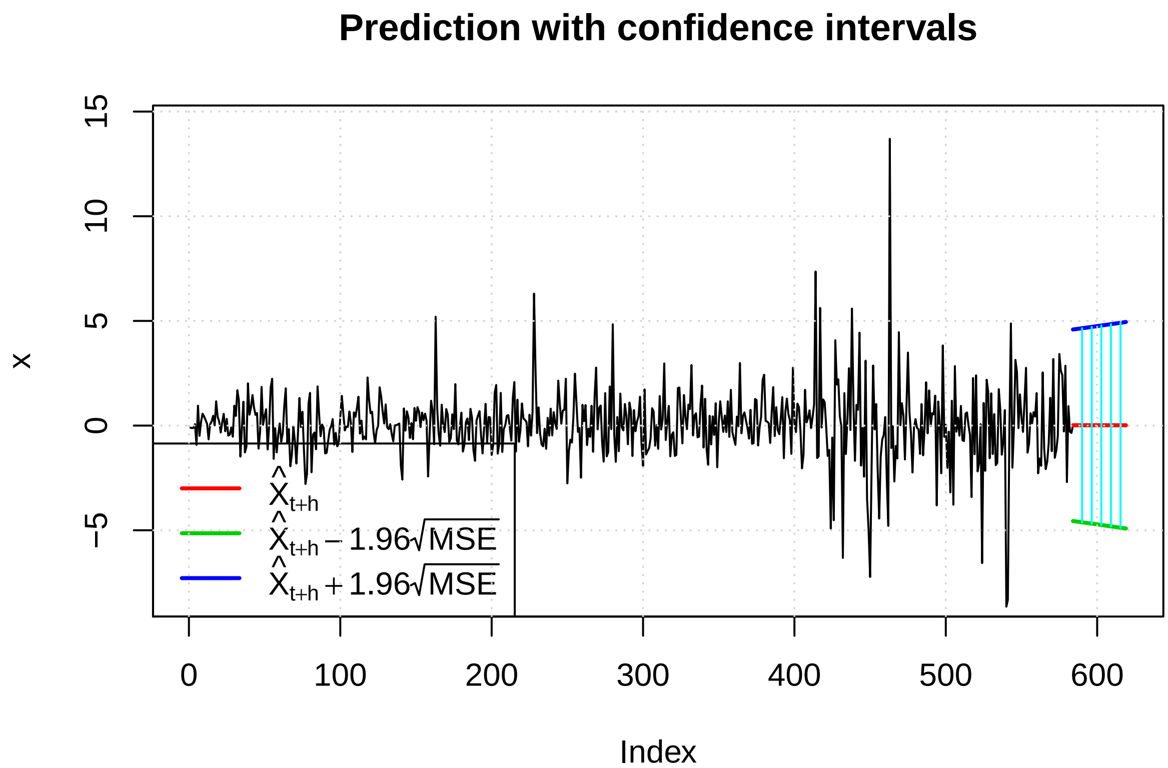
```r
    110), ylab = "")
leg.txt <- c("Original Series", "Forecast series")
legend("top", legend = leg.txt, lty = 1, col = c("black", "blue"),
    bty = "n", cex = 1)
```



Having acknowledged the confidence interval problem on the prediction caused by the non-stationary variance of the financial search time series, we want to use our fitted GARCH model to predict the mean and variance of the residuals of the series.

The results of this prediction are a better estimate of the 95% confidence interval of the residuals of the global warming time series after modeling with our ARIMA (0,1,0) model. We note that the volatility is predicted by the GARCH model to be in the range of -5 to 5, wider than the range predicted by the ARIMA model but consistent with the volatility observed towards the end of the original time series.

```r
data.garch.fit = garchFit(~garch(1, 1), data = data.res, trace = FALSE)
data.garch.pred <- predict(data.garch.fit, n.ahead = 36, plot = TRUE)
```



Using our GARCH model fitted on the residuals, we now plot the predicted confidence intervals obtained with GARCH modeling over the original fitted ARIMA(0,1,0) series.

The mean series of the 36 steps ahead predctions obtained from the fitted ARIMA(0,1,0) model and the corresponding lower and upper confidence intervals after substituting for the conditional variance obtained from GARCH model are:

```
# Reset the 95% confidence interval using the conditional
# variance from GARCH
data.fit.ahead.fcast$lower[, 2] <- -(((data.fit.ahead.fcast$mean -
    data.fit.ahead.fcast$lower[, 2])/(sqrt(data.fit.ahead.fcast$model$sigma2))) *
    data.garch.pred$standardDeviation) + data.fit.ahead.fcast$mean

data.fit.ahead.fcast$upper[, 2] <- (((data.fit.ahead.fcast$upper[,
    2] - data.fit.ahead.fcast$mean)/(sqrt(data.fit.ahead.fcast$model$sigma2))) *
    data.garch.pred$standardDeviation) + data.fit.ahead.fcast$mean

df <- data.frame(cbind(data.fit.ahead.fcast$mean, data.fit.ahead.fcast$lower[,
    2], data.fit.ahead.fcast$upper[, 2]))
names(df) <- c("Mean", "Lower 95% conf", "Upper 95% conf")
stargazer(df, type = "text", summary = FALSE)
```

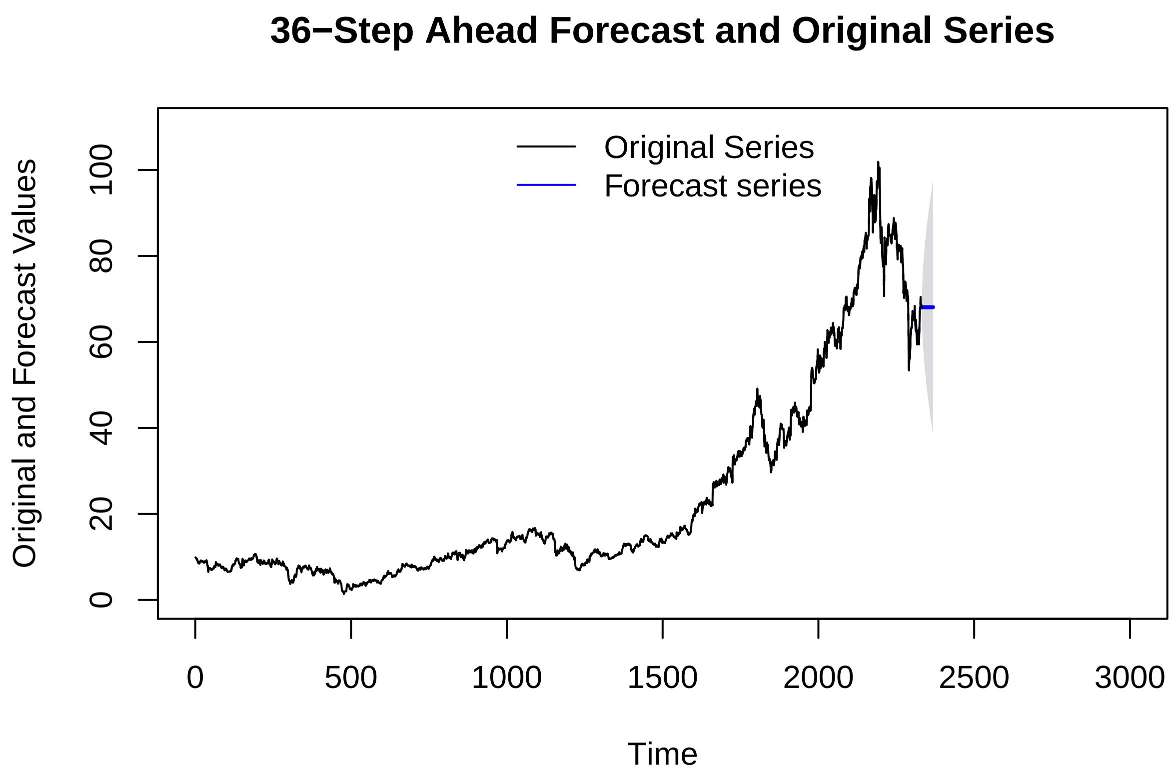
```
##
## =======================================
##     Mean  Lower 95% conf Upper 95% conf
## ---------------------------------------
```

```
## 1  68.080     63.506         72.654
## 2  68.080     61.597         74.563
## 3  68.080     60.122         76.038
## 4  68.080     58.872         77.288
## 5  68.080     57.762         78.398
## 6  68.080     56.753         79.407
## 7  68.080     55.819         80.341
## 8  68.080     54.944         81.216
## 9  68.080     54.117         82.043
## 10 68.080     53.330         82.830
## 11 68.080     52.577         83.583
## 12 68.080     51.852         84.308
## 13 68.080     51.153         85.007
## 14 68.080     50.476         85.684
## 15 68.080     49.819         86.341
## 16 68.080     49.179         86.981
## 17 68.080     48.555         87.605
## 18 68.080     47.946         88.214
## 19 68.080     47.349         88.811
## 20 68.080     46.765         89.395
## 21 68.080     46.191         89.969
## 22 68.080     45.628         90.532
## 23 68.080     45.073         91.087
## 24 68.080     44.528         91.632
## 25 68.080     43.990         92.170
## 26 68.080     43.460         92.700
## 27 68.080     42.937         93.223
## 28 68.080     42.420         93.740
## 29 68.080     41.909         94.251
## 30 68.080     41.404         94.756
## 31 68.080     40.905         95.255
## 32 68.080     40.410         95.750
## 33 68.080     39.921         96.239
## 34 68.080     39.435         96.725
## 35 68.080     38.954         97.206
## 36 68.080     38.477         97.683
## ---------------------------------------
```

Having computed 95% confidence intervals based on GARCH, we now plot the predicted point series with the 95% confidence interval updated.

```
# Clear the 80% confidence interval
data.fit.ahead.fcast$lower[, 1] <- data.fit.ahead.fcast$mean
data.fit.ahead.fcast$upper[, 1] <- data.fit.ahead.fcast$mean

# Plot the forecast with the updated 95% confidence inteval
par(mfrow = c(1, 1))
plot(data.fit.ahead.fcast, lty = 2, col = "navy", main = "36-Step Ahead Forecast and Original Series",
    ylab = "Original and Forecast Values", xlim = c(0, 3000),
    ylim = c(0, 110))
par(new = T)
plot.ts(data$DXCM, axes = F, lty = 1, xlim = c(0, 3000), ylim = c(0,
    110), ylab = "")
leg.txt <- c("Original Series", "Forecast series")
```

```
legend("top", legend = leg.txt, lty = 1, col = c("black", "blue"),
    bty = "n", cex = 1)
```



Part 3 (25 points): Forecast the Web Search Activity for Global Warming

Data Analysis - Original Series

1. The time series has weekly values (630 of them) starting at 1/4/04 and ending at 1/24/16. The minimum value is -0.551 and the maximum value is 4.104.
2. Time series plot shows that the series is very persistent, The series is basically flat from 2004 to 2012. After 2012, there is a sharp trend upward. There is more volatility after 2012. There are spikes and dips which could be seasonal with a yearly frequency. The series is not stationary in the mean.
3. Histogram shows is heavily positively skewed with most values between -0.551 and -0.3.
4. ACF of the series has a gradual decay toward 0. The plot of the ACF is for 104 lags (2 years). There is a gradual decay from correlation 1 at 0 lags to correlation 0.2 at 104 lags. This gradual decay is an indication that the series likely has an AR(p) component. There is also a slight dip in the correlation just after lag 0 and just after lag 52 (labeled 1.0 on the plot). This supports a theory that there is seasonality at frequency 52 lags (or 52 weeks or 1 year) in the time series.
5. PACF drops off immediately after first lag. This gives an indication that the series likely has an AR(1) component. There are 4 points that fall outside the 95% confidence interval (blue lines) at lags 3, 5, 11 and 14. The PACF shows some signs of seasonality.

Data Analysis - 1st Difference of Original Series

1. Plot. The plot of the 1st differenced time series shows that it is stationary in the mean. It does not look like white noise. It has a lot more volatility on the right hand side of the equation.
2. Histogram. This has a normal distribution like white noise.
3. ACF. Mostly drops off after 1 lag with several lags out of the 95% confidence interval.
4. PACF. Mostly drops off after 2 lags and slowly oscillates to zero.

Part A - Using Entire Dataset

Summary: Below, we try and fit a SARIMA model to the entire dataset. We find the fit unsuitable, and in Part B move on to analysis of a smaller part in order to produce a better forecast. We detail below the procedure used.

1. **Try AR models.** Since AR models are used to model time series that are stationary in the mean and the non-differenced version of this model is not stationary in the mean, then we will not estimate an AR model for this series.

2. **Try ARIMA models.** Since we have identified that there is a seasonal component in this series, we will skip estimating models that do not incorporate a seasonal component and move to SARIMA models.

3. **Try SARIMA models.** From the plot of the original series, it looks like this series has a seasonal component with a 52-week periodicity. Use the get.best.sarima() function to estimate a SARIMA(p,d,q,P,D,Q) model with maximum parameters c(2,2,2,2,2,2). The best AIC output is -1276.817 with a model of SARIMA(1,2,2,1,0,2). For parsimony try running get.best.sarima() with c(1,1,1,1,1,1). A parsimonious model from this output is SARIMA(0,1,1,1,0,1) with AIC -1246.412 which is very close to the AIC output from c(2,2,2,2,2,2). For parsimony we will choose SARIMA(0,1,1,1,0,1). The In-Sample fit of this estimated model matches the original model very well as evidenced in the plot. The coefficients: ma1=-0.4493386, sar1=1.2826553, sma1=-0.4615329 are all significant and fall in to the 95% confidence interval as shown below in the R code.
   The residuals do not look like white noise (see analysis below). This is probably because there is time-varying volatility in the model.

- Histogram: Yes. This looks like a normal distribution.
- Fitted vs. Residuals: No. The plot does not look like an evenly distributed cloud.
- Plot: No. The plot does not look random, there is a lot of volatility on the right hand side of the graph which looks like time-varying volatility.
- ACF: No. The ACF drops off after lag 0, but has only a few lags where the correlation comes out of the 95% confidence interval (CI)
- PACF: No. The PACF shows correlation with several values outside of the 95% CI.

  We have a decent fit for the model, but we notice that there are 2 distinct parts of this time series and we move to an investigation of whether to split the series or not.

```r
# Read in the time series data
glob.warm = read.csv("globalWarming.csv", header = TRUE)
glob.warm.ts = ts(glob.warm$data.science, start = 2004, frequency = 52)
### Data Analysis - Original Series Print descriptive
### statistics
str(glob.warm.ts)
```

```
##  Time-Series [1:630] from 2004 to 2016: -0.44 -0.474 -0.423 -0.551 -0.486 -0.551 -0.453 -0.462 -0.55
```

```r
summary(glob.warm.ts)
```

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## -0.551000 -0.506000 -0.485000  0.000038 -0.200000  4.104000
```

```r
cbind(head(glob.warm.ts), tail(glob.warm.ts))
```

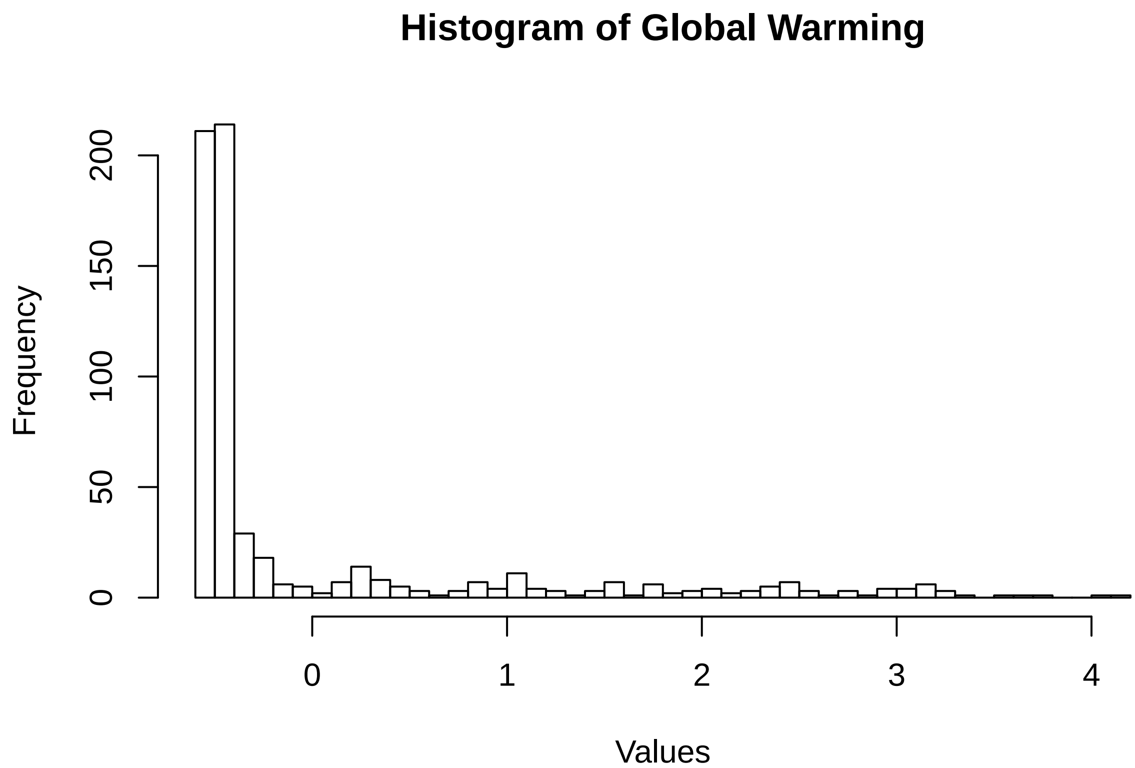
```
##        [,1]  [,2]
## [1,] -0.440 2.227
## [2,] -0.474 2.360
## [3,] -0.423 3.662
## [4,] -0.551 3.721
## [5,] -0.486 4.087
## [6,] -0.551 4.104
```

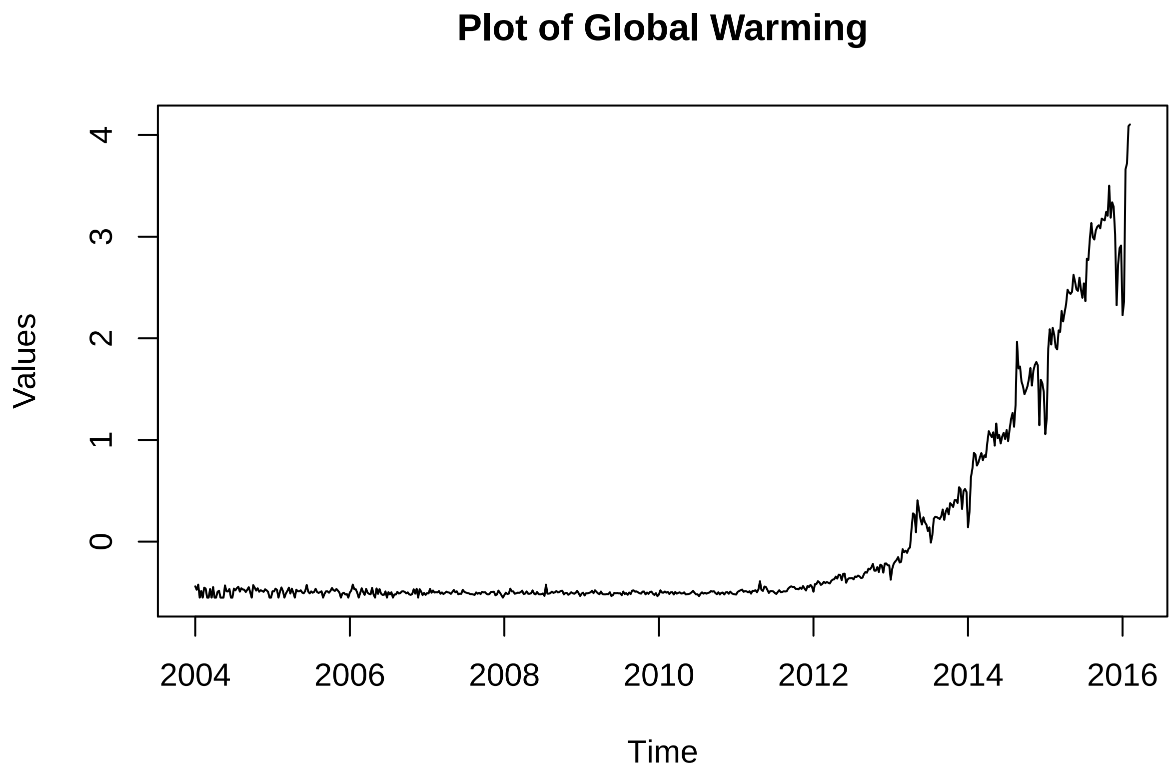
```r
quantile(as.numeric(glob.warm.ts), c(0.01, 0.05, 0.1, 0.25, 0.5,
    0.75, 0.9, 0.95, 0.99))
```

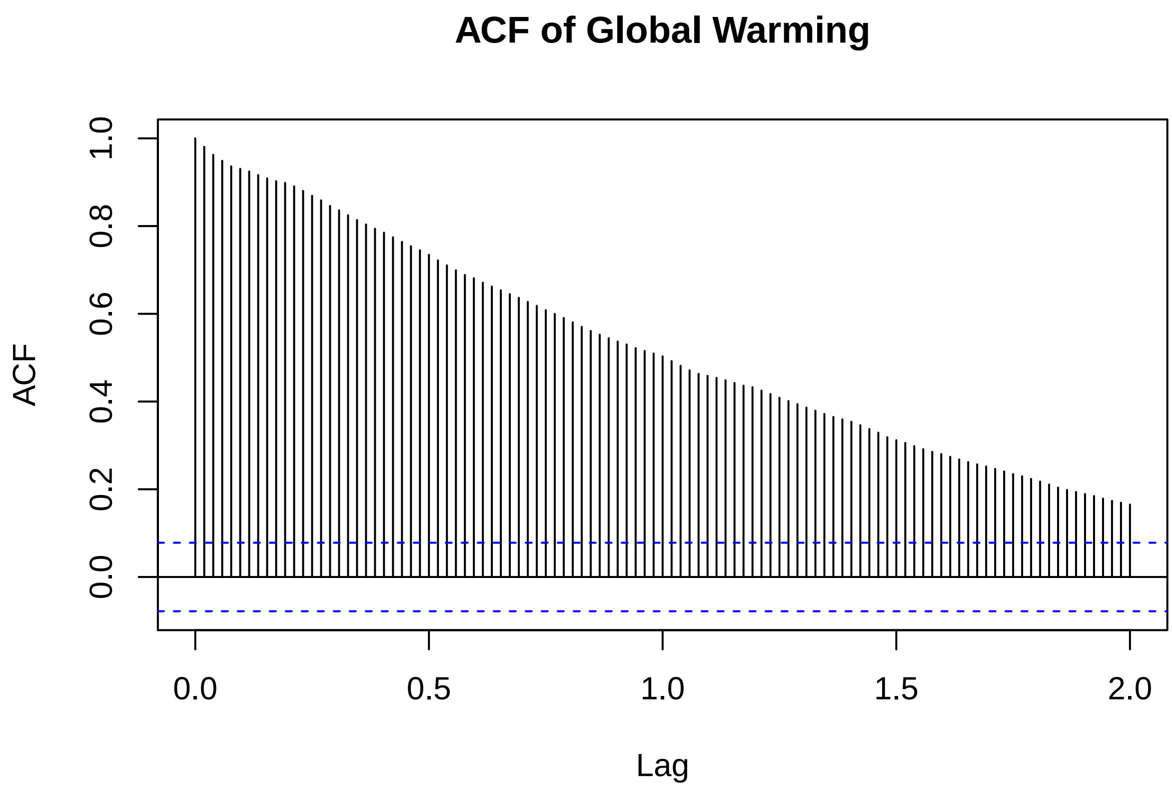
```
##       1%       5%      10%      25%      50%      75%      90%      95%
## -0.55100 -0.53220 -0.51900 -0.50600 -0.48500 -0.20000  1.68410  2.48055
##      99%
##  3.28021
```

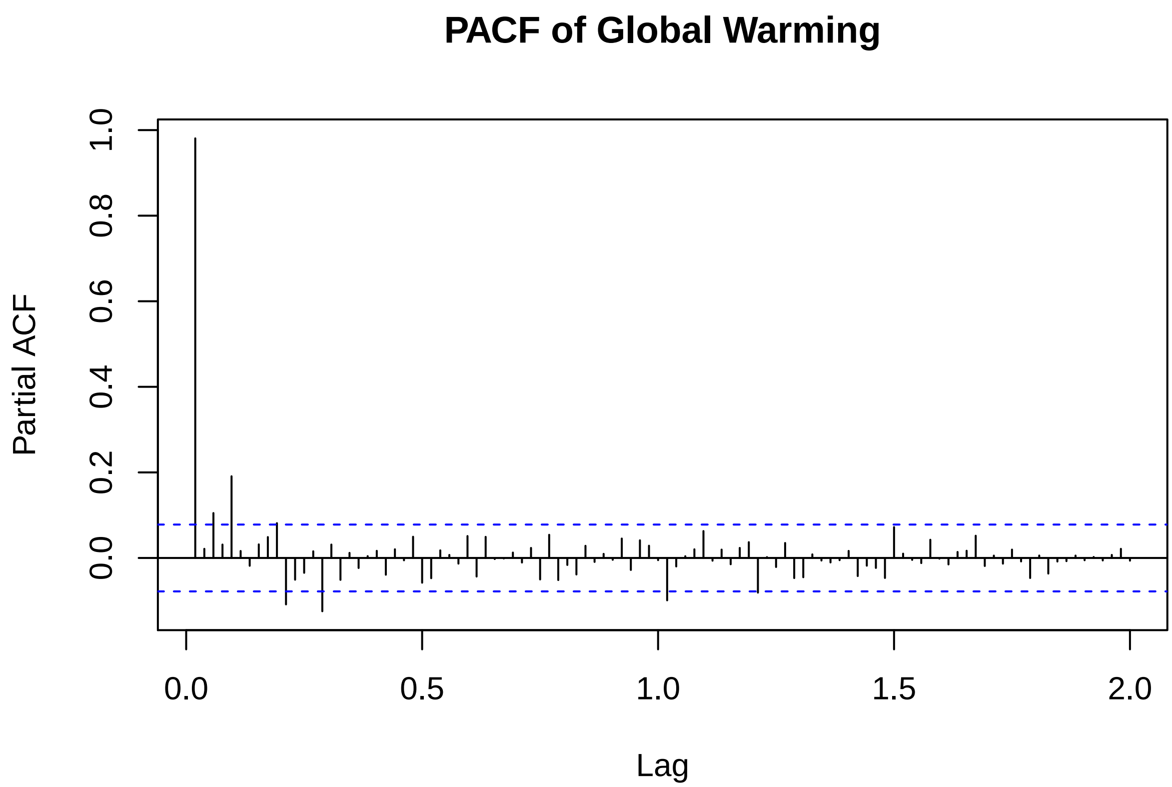
```r
# Plot the time series
plot.time.series(glob.warm.ts, 50, "Global Warming", 104, "single")
```

```
##  Time-Series [1:630] from 2004 to 2016: -0.44 -0.474 -0.423 -0.551 -0.486 -0.551 -0.453 -0.462 -0.55
```


Histogram of Global Warming
Frequency
0
50
100
150
200
0
1
2
3
4
Values

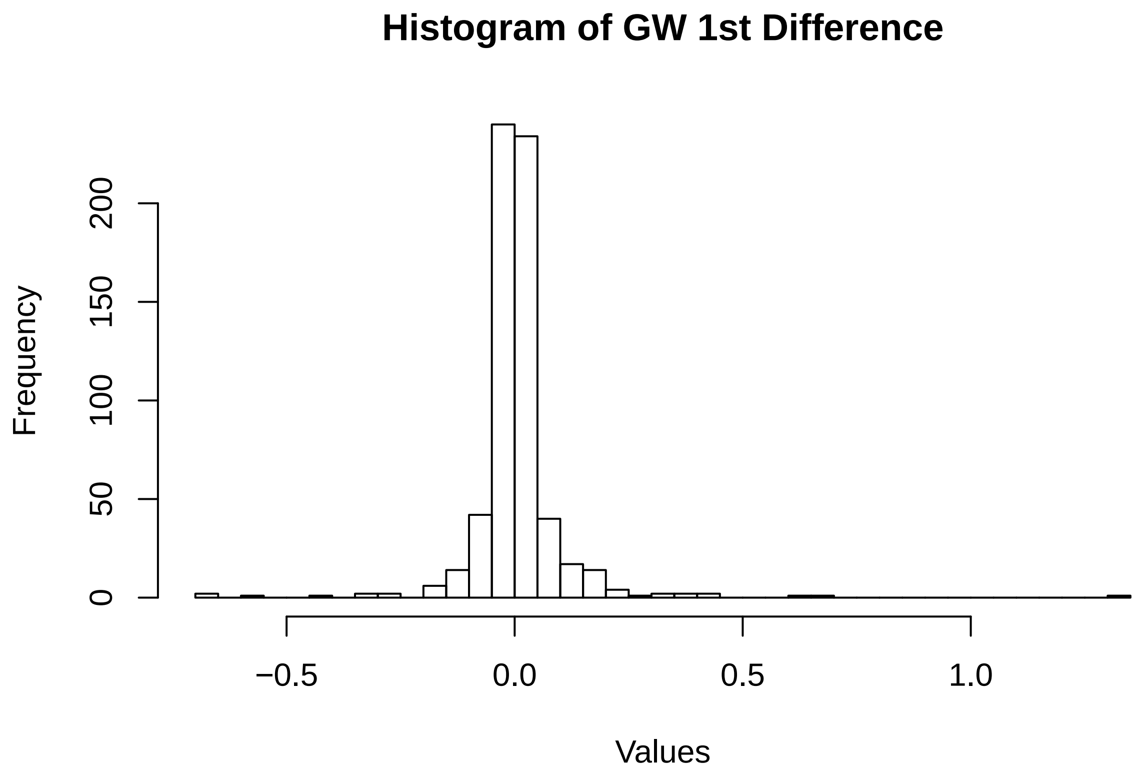

Plot of Global Warming
Values
0
1
2
3
4
2004
2006
2008
2010
2012
2014
2016
Time

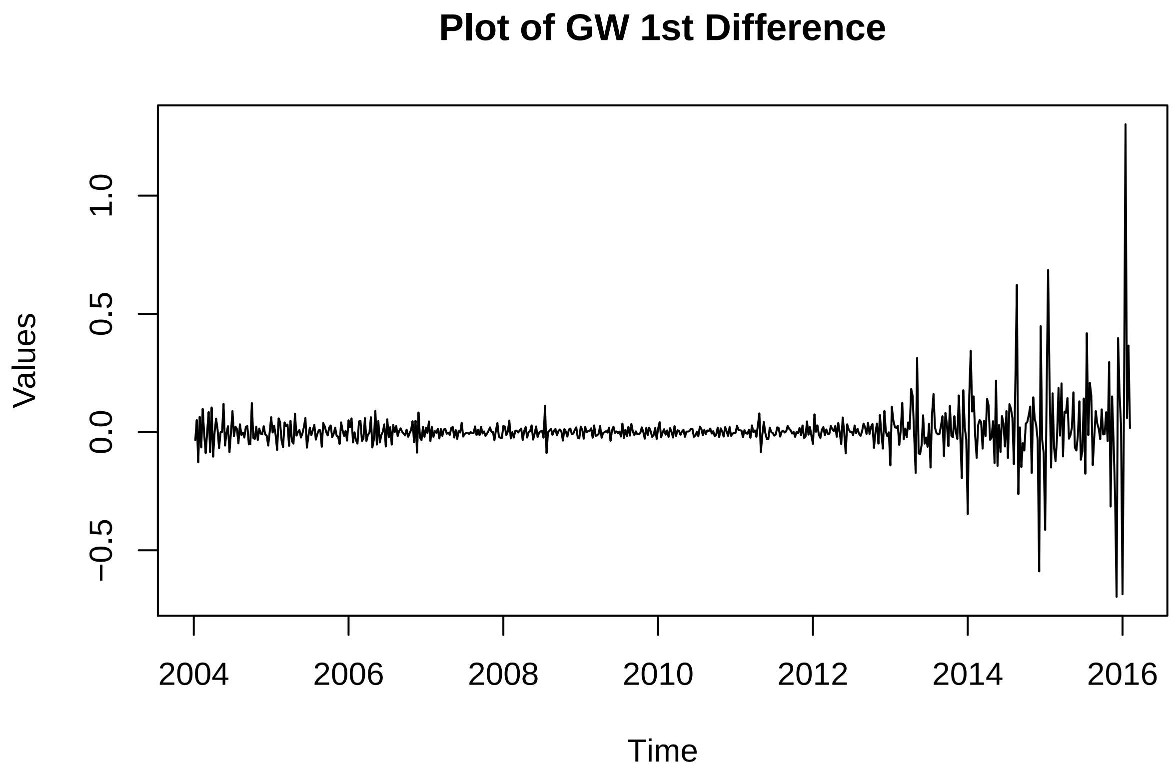

ACF of Global Warming
ACF
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
Lag

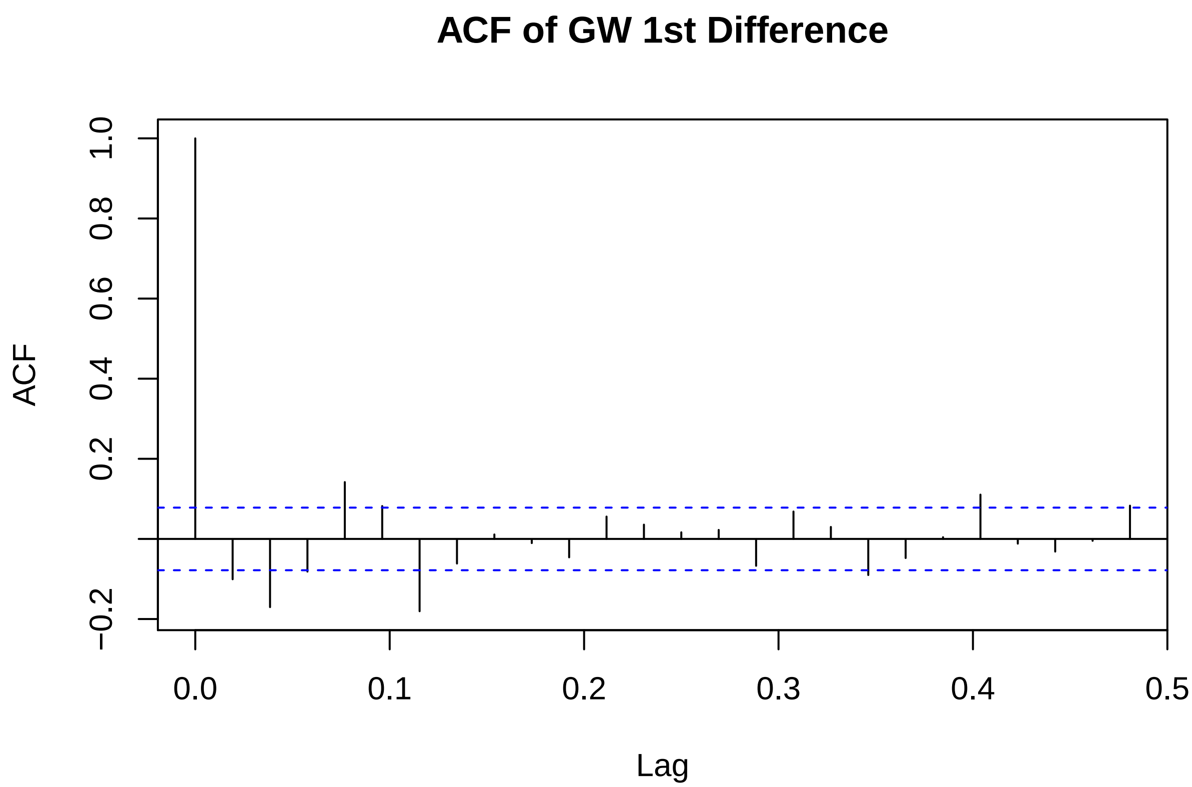


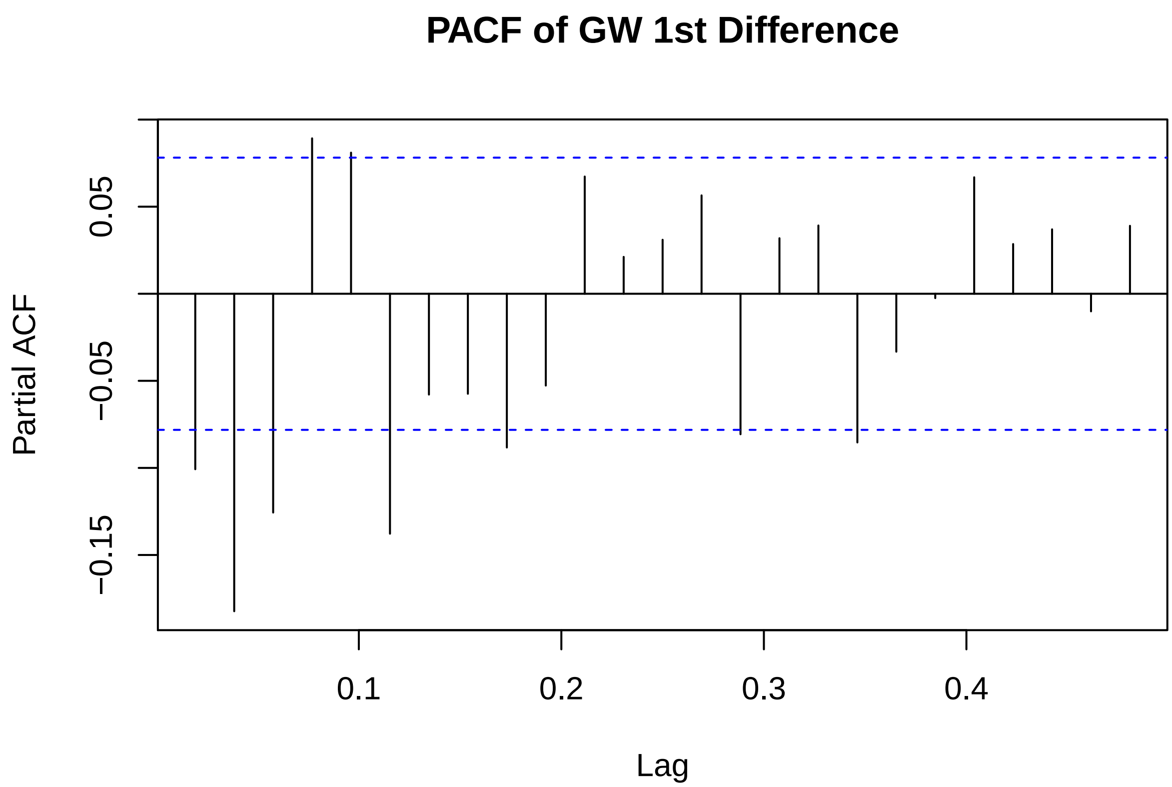

```
### Data Analysis - 1st Difference of Original Series Plot the
### 1st difference of time series
plot.time.series(diff(glob.warm.ts), 50, "GW 1st Difference",
    25, "single")
```

```
##  Time-Series [1:629] from 2004 to 2016: -0.034 0.051 -0.128 0.065 -0.065 ...
```


Histogram of GW 1st Difference
Frequency
0
50
100
150
200
−0.5
0.0
0.5
1.0
Values


Plot of GW 1st Difference
Values
1.0
0.5
0.0
−0.5
2004
2006
2008
2010
2012
2014
2016
Time


ACF of GW 1st Difference
ACF
1.0
0.8
0.6
0.4
0.2
−0.2
0.0
0.1
0.2
0.3
0.4
0.5
Lag




```r
### 3. Try SARIMA models
gw.seas.best <- get.best.sarima(glob.warm.ts, maxord = c(2, 2,
    2, 2, 2, 2), 52)
# Print the top 20 best models based on AIC
gw.seas.best$others[order(gw.seas.best$others$aics)[1:20], ]
```

```
##          aics            models
## 471 -1276.817 (1, 2, 2, 1, 0, 2)
## 525 -1256.987 (2, 0, 1, 1, 0, 2)
## 363 -1256.930 (1, 1, 1, 1, 0, 2)
## 120 -1254.855 (0, 1, 1, 1, 0, 2)
## 390 -1254.580 (1, 1, 2, 1, 0, 2)
## 714 -1253.765 (2, 2, 2, 1, 0, 2)
## 147 -1253.291 (0, 1, 2, 1, 0, 2)
## 516 -1253.199 (2, 0, 1, 0, 0, 2)
## 524 -1252.512 (2, 0, 1, 1, 0, 1)
## 606 -1251.219 (2, 1, 1, 1, 0, 2)
## 362 -1249.703 (1, 1, 1, 1, 0, 1)
## 119 -1246.412 (0, 1, 1, 1, 0, 1)
## 389 -1246.392 (1, 1, 2, 1, 0, 1)
## 146 -1246.162 (0, 1, 2, 1, 0, 1)
## 309 -1245.588 (1, 0, 2, 1, 0, 2)
## 282 -1244.671 (1, 0, 1, 1, 0, 2)
## 633 -1244.117 (2, 1, 2, 1, 0, 2)
## 605 -1242.534 (2, 1, 1, 1, 0, 1)
## 579 -1241.850 (2, 1, 0, 1, 0, 2)
```

```
## 308 -1239.717 (1, 0, 2, 1, 0, 1)
```

```r
gw.seas.best1 <- get.best.sarima(glob.warm.ts, maxord = c(1,
    1, 1, 1, 1, 1), 52)
# Print the top 20 best models based on AIC
gw.seas.best1$others[order(gw.seas.best1$others$aics)[1:20],
    ]
```

```
##         aics             models
## 62 -1249.703 (1, 1, 1, 1, 0, 1)
## 30 -1246.412 (0, 1, 1, 1, 0, 1)
## 46 -1237.091 (1, 0, 1, 1, 0, 1)
## 61 -1221.391 (1, 1, 1, 1, 0, 0)
## 54 -1219.342 (1, 1, 0, 1, 0, 1)
## 29 -1217.166 (0, 1, 1, 1, 0, 0)
## 45 -1208.780 (1, 0, 1, 1, 0, 0)
## 53 -1194.411 (1, 1, 0, 1, 0, 0)
## 22 -1171.591 (0, 1, 0, 1, 0, 1)
## 21 -1159.063 (0, 1, 0, 1, 0, 0)
## 38 -1158.699 (1, 0, 0, 1, 0, 1)
## 58 -1151.999 (1, 1, 1, 0, 0, 1)
## 26 -1146.658 (0, 1, 1, 0, 0, 1)
## 37 -1146.231 (1, 0, 0, 1, 0, 0)
## 42 -1144.566 (1, 0, 1, 0, 0, 1)
## 60 -1136.810 (1, 1, 1, 0, 1, 1)
## 50 -1136.063 (1, 1, 0, 0, 0, 1)
## 28 -1134.650 (0, 1, 1, 0, 1, 1)
## 44 -1133.989 (1, 0, 1, 0, 1, 1)
## 18 -1125.428 (0, 1, 0, 0, 0, 1)
```

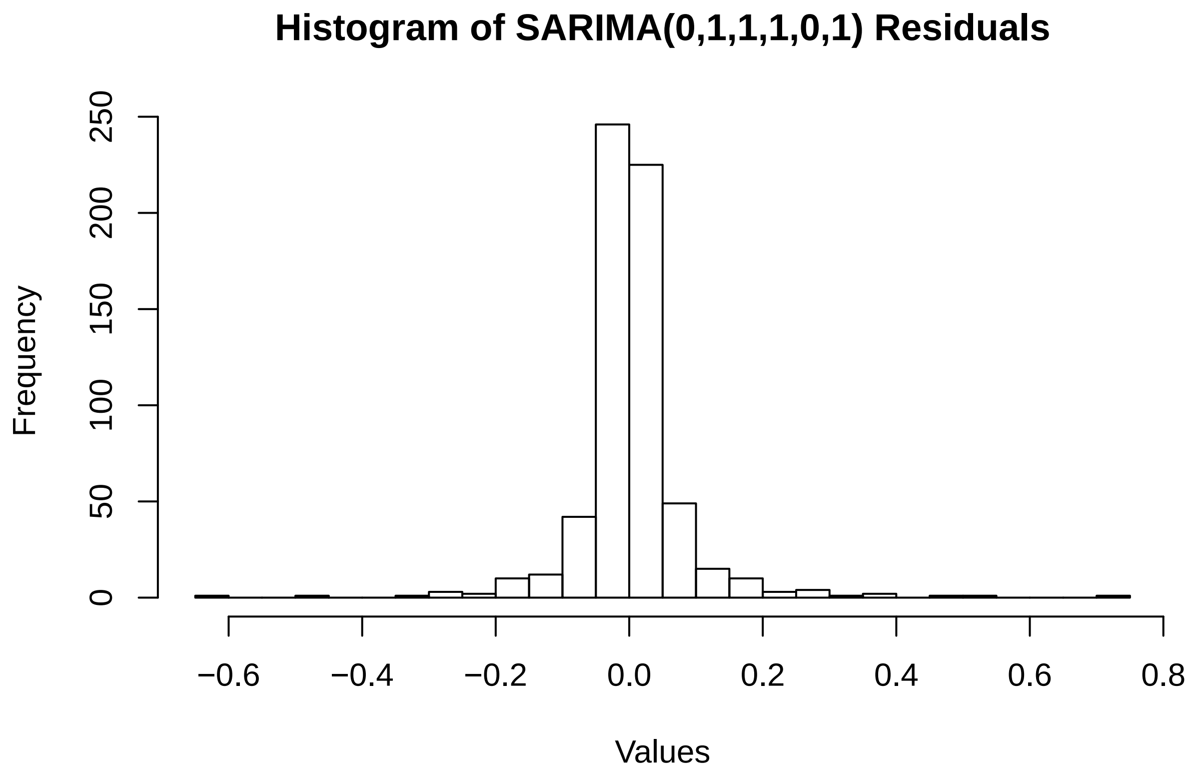
```r
glob.warm.arima.seas = arima(glob.warm.ts, order = c(0, 1, 1),
    seas = list(order = c(1, 0, 1), 52), method = "CSS")
# Examine the coefficients.  The coefficients fall in the 95%
# CI.
glob.warm.arima.seas$coef
```

```
##        ma1       sar1       sma1
## -0.4493386  1.2826553 -0.4615329
```

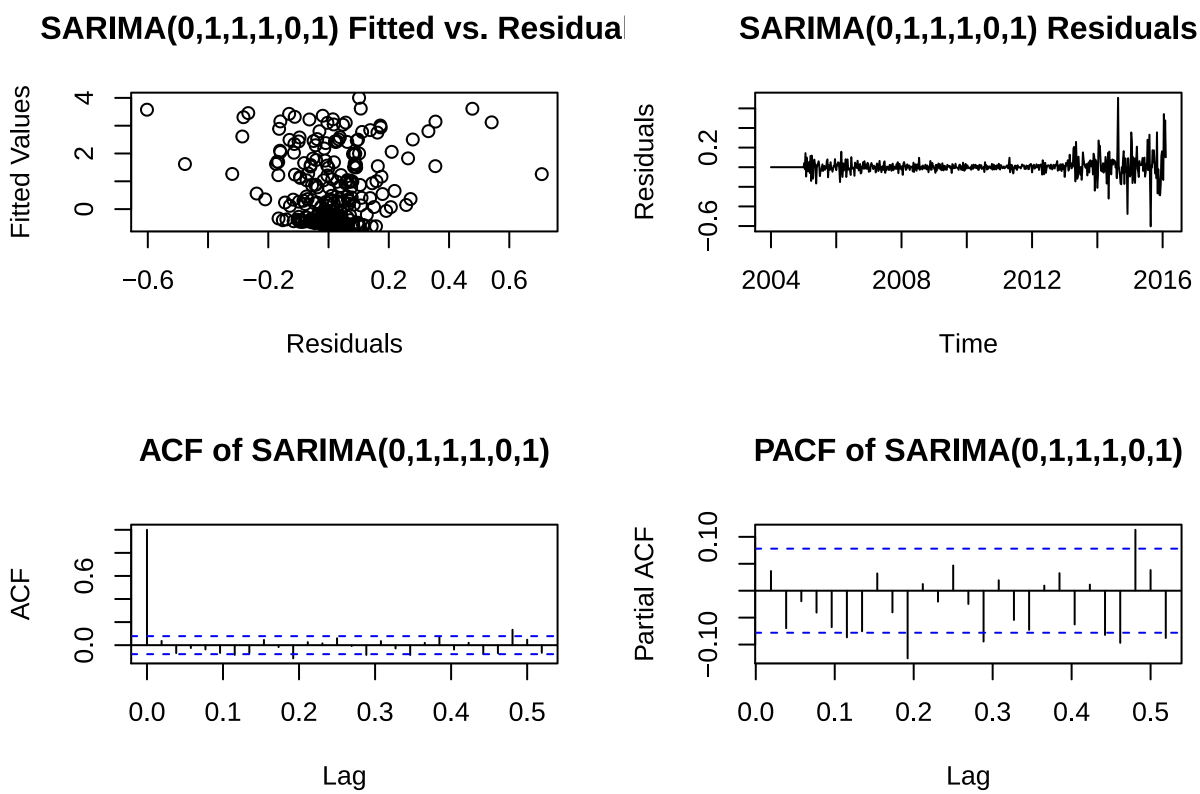
```r
t(confint(glob.warm.arima.seas))
```

```
##               ma1     sar1       sma1
## 2.5 %  -0.5391474 1.162581 -0.5812471
## 97.5 % -0.3595299 1.402729 -0.3418188
```

```r
# Plot the residuals
glob.warm.arima.seas.res = glob.warm.arima.seas$residuals
plot.residuals.ts(glob.warm.arima.seas, "SARIMA(0,1,1,1,0,1)")
```

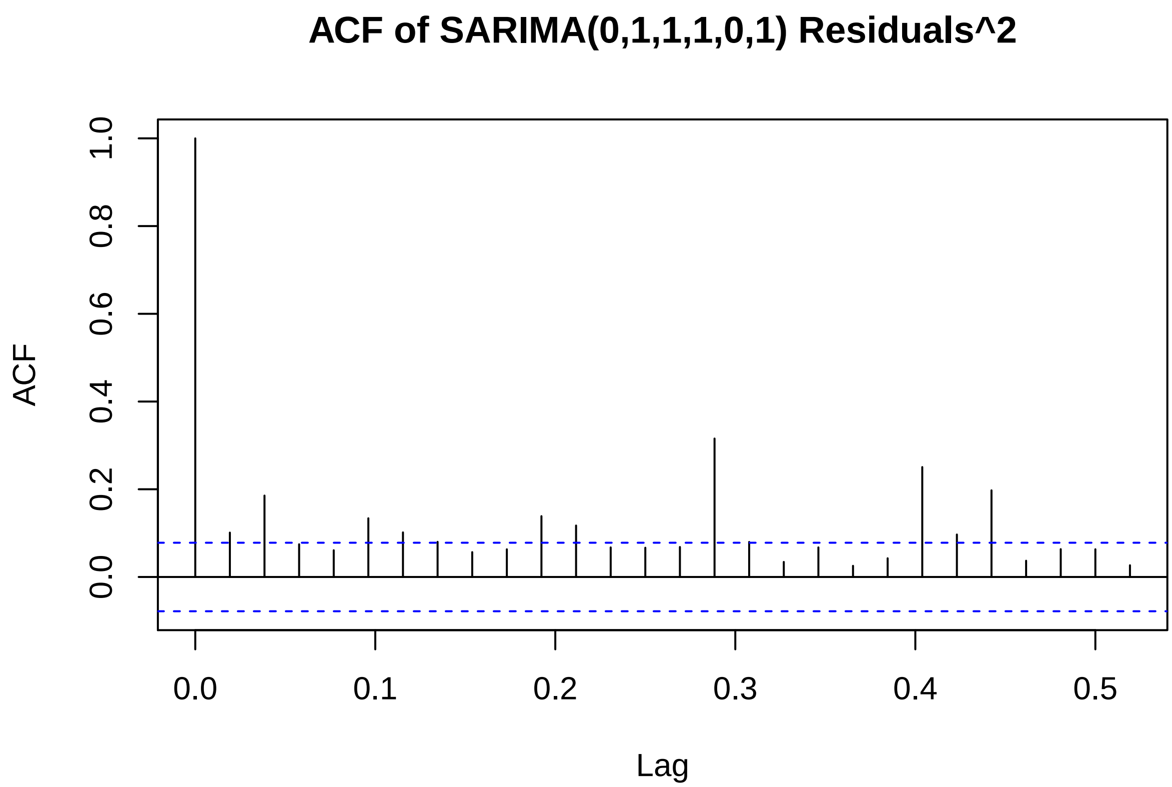


Histogram of SARIMA(0,1,1,1,0,1) Residuals
Frequency
0
50
100
150
200
250
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
Values



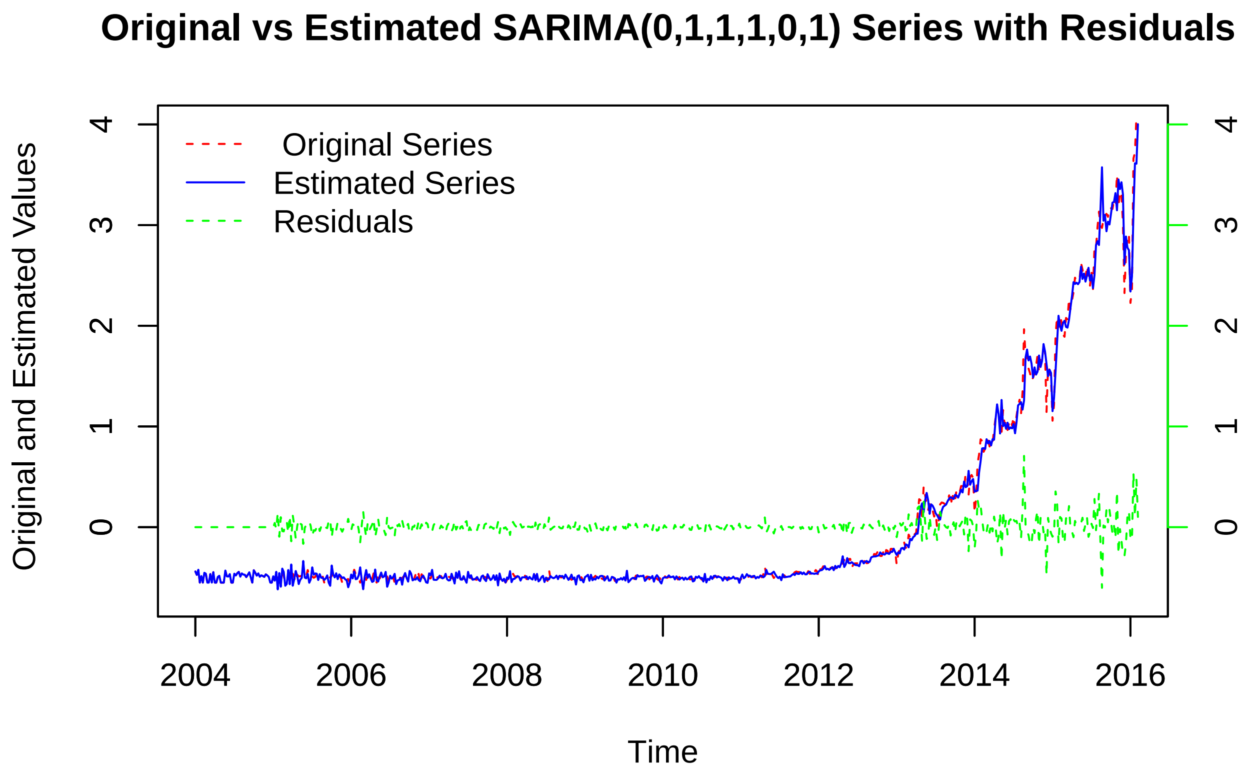

```
##
##  Box-Ljung test
##
## data:  x.mod$residuals
## X-squared = 0.8408, df = 1, p-value = 0.3592
```

```r
par(mfrow = c(1, 1))
acf(glob.warm.arima.seas.res^2, main = "ACF of SARIMA(0,1,1,1,0,1) Residuals^2")
```




```r
# Plot the In-sample fit
plot.orig.model.resid(glob.warm.ts, glob.warm.arima.seas, "",
    "SARIMA(0,1,1,1,0,1)", c(2004, 2016), c(-0.7, 4))
```

```
##
## Descriptive Stat
## ===========================================
## Statistic        N   Mean  St. Dev. Min  Max
## -------------------------------------------
## x.ts            630 0.000   1.0    -0.6 4.1
## fitted.x.mod.   630 -0.01   1.0    -0.6 4.0
## x.mod.residuals 630 0.01    0.1    -0.6 0.7
## -------------------------------------------
```



Part B - Abbreviated Dataset

Summary: We decided to use 2013 onwards as our abberviated dataset and were able to produce a much more suitable forecast. The Abbreviated 2013-2016 Global Warming time series is satisfactorily modeled with a SARMIMA(0,1,0,0,0,1) model to handle trends and seasonality. The residuals are close enough to white noise and the seasonality is modeled.

4. **Using a portion of the data.** The data has a clear split around 2012 or 2013 where it goes from being stationary in the mean to being non-stationary in the mean. The behavior of the series before this split is very different than after the split. The series before the split has no trend and the series after the split has a clear upward trend and what looks like yearly seasonality. Since we are trying to make a forecast for data after 2016, we are going to use the part of the series after the split. Since that is the data closest in time to 2016. 2012 could have been chosen, but it still had some of the non-trending data contained in it, so 2013 was chosen as a start year. When graphs are created with this Abbreviated 2013-2016 series, they will be denoted by the phrase "Abrv. Original" instead of "Original".

5. **Data Analysis of Abbreviated Series.**

- The time series has weekly values (160 of them) starting at 1/6/13 and ending at 1/24/16. The minimum value is -0.2180 and the maximum value is 4.104.
- Time series plot shows that the series is very persistent. There is a sharp trend upward. There is more volatility after 2012. There are spikes and dips which could be seasonal with a yearly frequency. The series is not stationary in the mean.
- Histogram is multimodal showing about 5 hills over the range of values.
- ACF of the series has a gradual decay toward 0. At lag 25, there is still a correlation of about 0.6. This gradual decay is an indication that the series likely has an AR(p) component. There is also a slight dip in the correlation just after lag 0 which supports a theory that there is seasonality in the time series.

- PACF drops off immediately after first lag. This gives an indication that the series likely has an AR(1) component. After lag 1, there are no points that fall outside of the 95% confidence interval. The PACF shows some signs of seasonality.

6. **Data Analysis of Abbreviated 1st Differenced Series.**

- Plot. The plot of the 1st differenced time series shows that it is stationary in the mean. It looks a lot like white noise. It has a lot more volatility on the right hand side of the equation.
- Histogram. This has a normal distribution like white noise.
- ACF. Mostly drops off after 1 lag with several lags out of the 95% confidence interval.
- PACF. Mostly drops off after 0 lags and slowly oscillates to zero.

7. **Try AR models.** Since AR models are used to model time series that are stationary in the mean and the non-differenced abbreviated version of this model is not stationary in the mean, then we will not estimate an AR model for this series.

8. **Try ARIMA models.** Since we have identified that there is a seasonal component in this series, we will skip estimating models that do not incorporate a seasonal component and move to SARIMA models.

9. **Try SARIMA models.** Use the get.best.sarima() function. The best AIC output is -90.17105 with parameters c(0,1,1,1,0,1), but for parsimony we will choose a SARIMA(0,1,0,0,0,1) with an AIC of -76.45286 which is very close to the other model. Check the residuals. Yes, the residuals look basically like white noise. There is one place in the squared residuals where the value exceeds the 95% confidence interval. The In-Sample fit of this estimated model. The model now has a satisfactory fit and we will move on to backtesting and forecasting. The coefficient sma1=5.793727 is significant and falls in to the 95% confidence interval as shown below in the R code.

10. **Backtesting.** For backtesting, 10% of the values from the end of the 2013-2016 time series were withheld, in this case 10 values. The backtesting model shows mean predicted values that follow the up and down changes of the original time series, but the mean predicted values are are not as extreme as the orignal values. The seasonality of the original series is being modeled to some extent. We also note that the original series, for the most part, falls within the 95% confidence interval of the forecast, giving us confidence that this model could be used as a decent predictive model for the original time series.

11. **Forecast the model.** Using the SARIMA(0,1,0,0,0,1) model with the 2013-2016 version of the time series, we made the requested 12-step ahead forecast of the model. The forecast looks like it captures the seasonality of the model as it matches the upward trend and the seasonal volatility. We also note that all of the forecasted values are within the 80% confidence interval of the prediction from the graph and table show below.

```r
### 4. Using a portion of the data Create 2013 to 2016 series
glob.warm.2013.ts = ts(glob.warm.ts[471:length(glob.warm.ts)],
    start = 2013, frequency = 52)
# Create a string label to prepend to the word 'Original' for
# proper labeling in graphs
part_name = "Abrv."
# Rename series
glob.warm.part.ts = glob.warm.2013.ts

### 5. Data Analysis of Abbreviated Series.  Descriptive
### statistics
str(glob.warm.part.ts)
```

```
##  Time-Series [1:160] from 2013 to 2016: -0.218 -0.196 -0.179 -0.152 -0.206 -0.198 -0.074 -0.104 -0.08
```

```r
summary(glob.warm.part.ts)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -0.2180  0.3378  1.1370  1.4080  2.3610  4.1040
```

```r
cbind(head(glob.warm.part.ts), tail(glob.warm.part.ts))
```

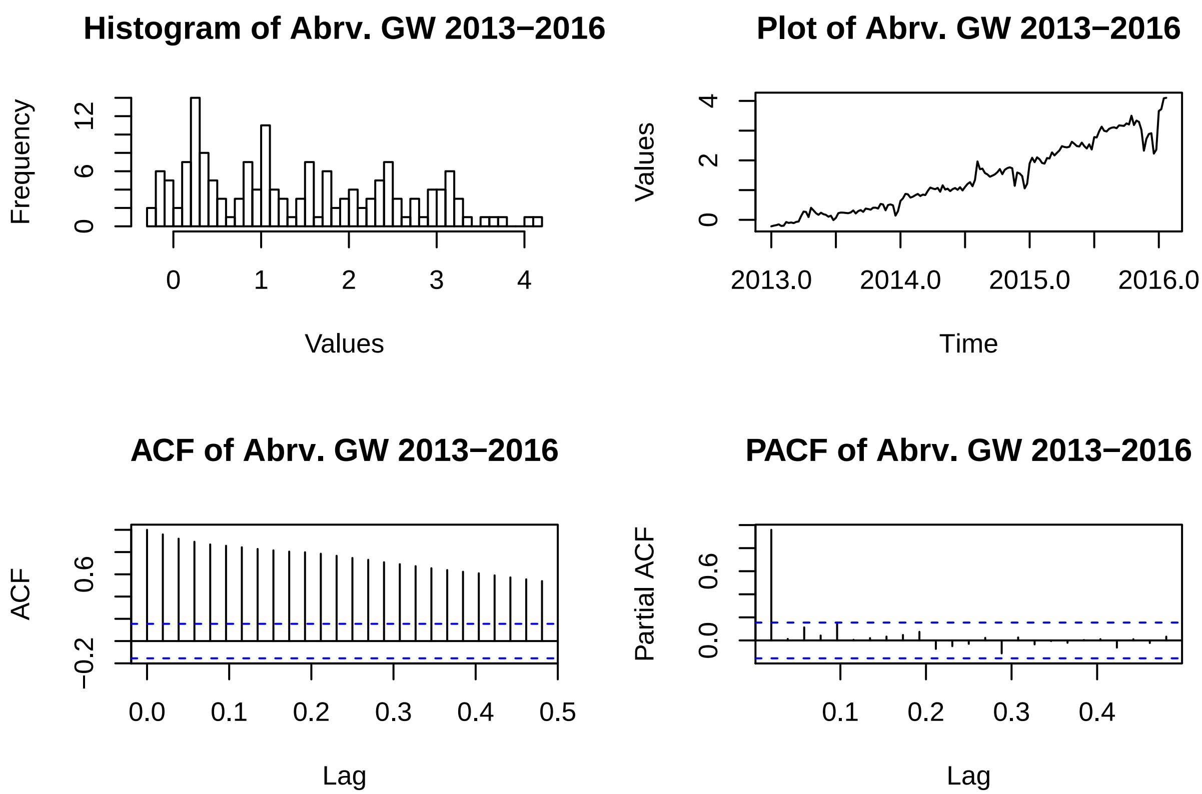
```
##        [,1]  [,2]
## [1,] -0.218 2.227
## [2,] -0.196 2.360
## [3,] -0.179 3.662
## [4,] -0.152 3.721
## [5,] -0.206 4.087
## [6,] -0.198 4.104
```

```r
quantile(as.numeric(glob.warm.part.ts), c(0.01, 0.05, 0.1, 0.25,
    0.5, 0.75, 0.9, 0.95, 0.99))
```

```
##       1%       5%      10%      25%      50%      75%      90%      95%
## -0.20128 -0.08975  0.12390  0.33775  1.13700  2.36125  3.08360  3.20790
##      99%
##  3.87106
```

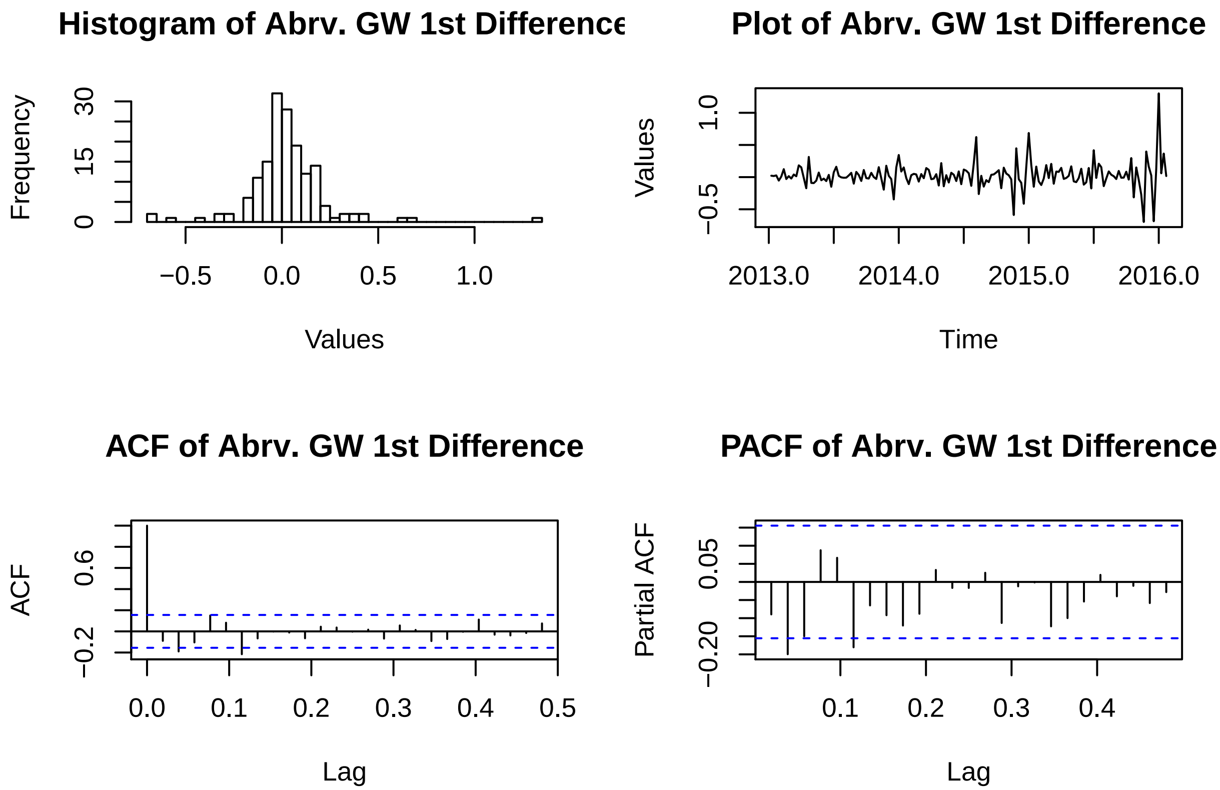
```r
# Plot the time series
plot.time.series(glob.warm.part.ts, 50, "Abrv. GW 2013-2016")
```

```
##  Time-Series [1:160] from 2013 to 2016: -0.218 -0.196 -0.179 -0.152 -0.206 -0.198 -0.074 -0.104 -0.08
```




```
### 6. Data Analysis of Abbreviated 1st Differenced Series.
### Plot the 1st difference of time series
plot.time.series(diff(glob.warm.part.ts), 50, "Abrv. GW 1st Difference")
```

```
##  Time-Series [1:159] from 2013 to 2016: 0.022 0.017 0.027 -0.054 0.008 ...
```




```
### 9.  Try SARIMA models
gw.part.seas.best <- get.best.sarima(glob.warm.part.ts, maxord = c(1,
    1, 1, 1, 1, 1), 52)
```

```
## Warning in arima(x.ts, order = c(p, d, q), seasonal = list(order = c(P, :
## possible convergence problem: optim gave code = 1
```

```
# Print the top 20 best models based on AIC
gw.part.seas.best$others[order(gw.part.seas.best$others$aics)[1:5],
    ]
```

```
##          aics             models
## 30 -90.17105 (0, 1, 1, 1, 0, 1)
## 54 -87.66116 (1, 1, 0, 1, 0, 1)
## 62 -84.61532 (1, 1, 1, 1, 0, 1)
## 46 -78.92479 (1, 0, 1, 1, 0, 1)
## 18 -76.45286 (0, 1, 0, 0, 0, 1)
```

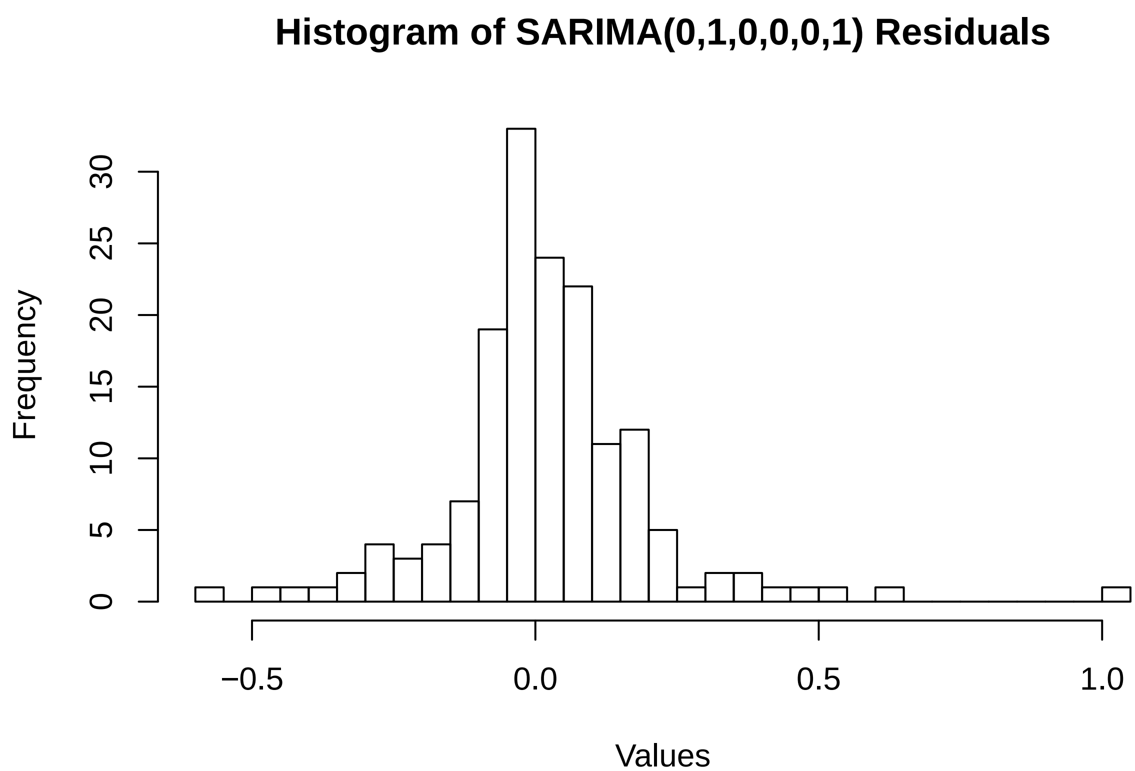
```
# Create SARIMA(0,1,0,0,0,1) model
glob.warm.part.sarima = arima(glob.warm.part.ts, order = c(0,
    1, 0), seas = list(order = c(0, 0, 1), 52), method = "CSS")
# Examine the coefficients.  The coefficients fall in the 95%
# CI.
glob.warm.part.sarima$coef
```

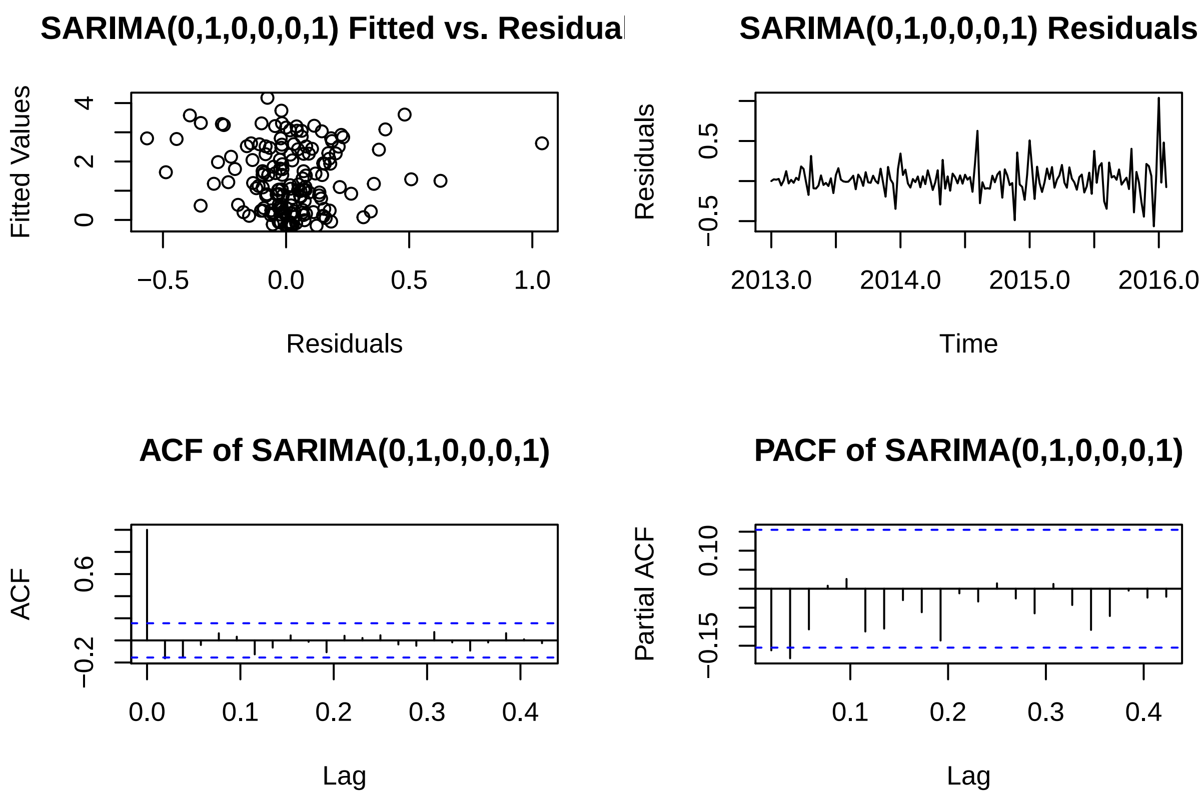
```
##      sma1
## 0.5168062
```

```r
t(confint(glob.warm.part.sarima))
```

```
##              sma1
## 2.5 %  0.3419755
## 97.5 % 0.6916369
```

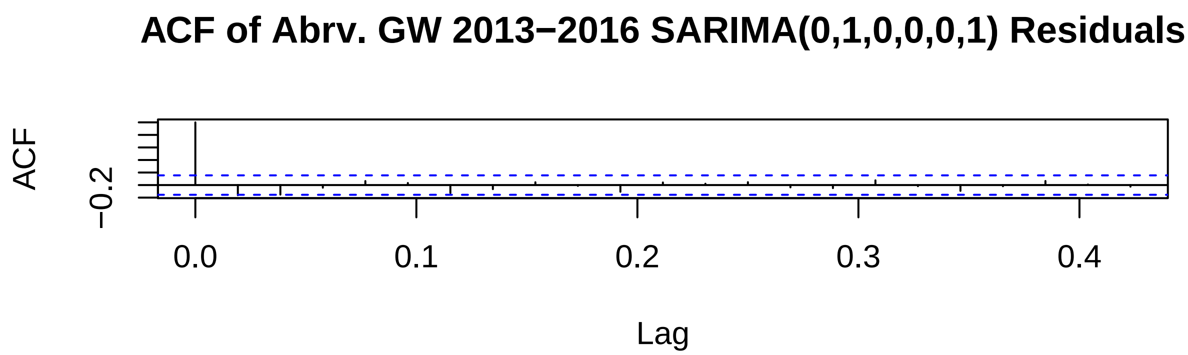
```r
# Plot the residuals
plot.residuals.ts(glob.warm.part.sarima, "SARIMA(0,1,0,0,0,1)")
```

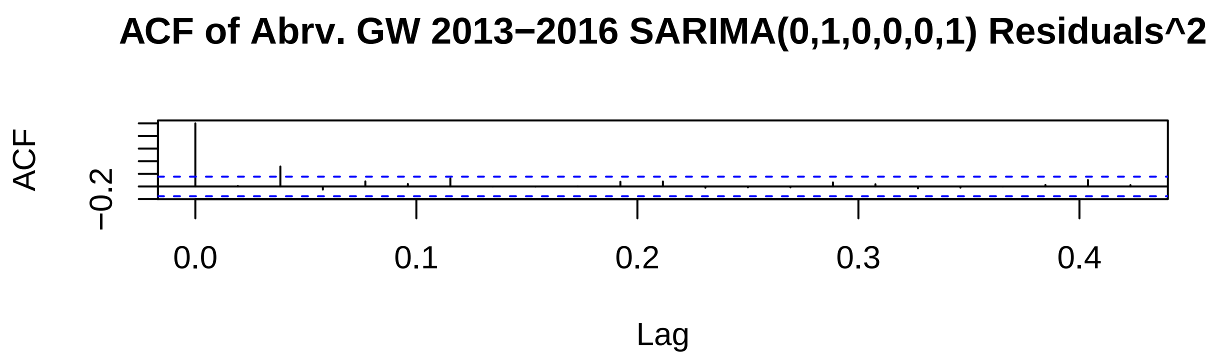





```
##
##  Box-Ljung test
##
## data:  x.mod$residuals
## X-squared = 4.2839, df = 1, p-value = 0.03847
```

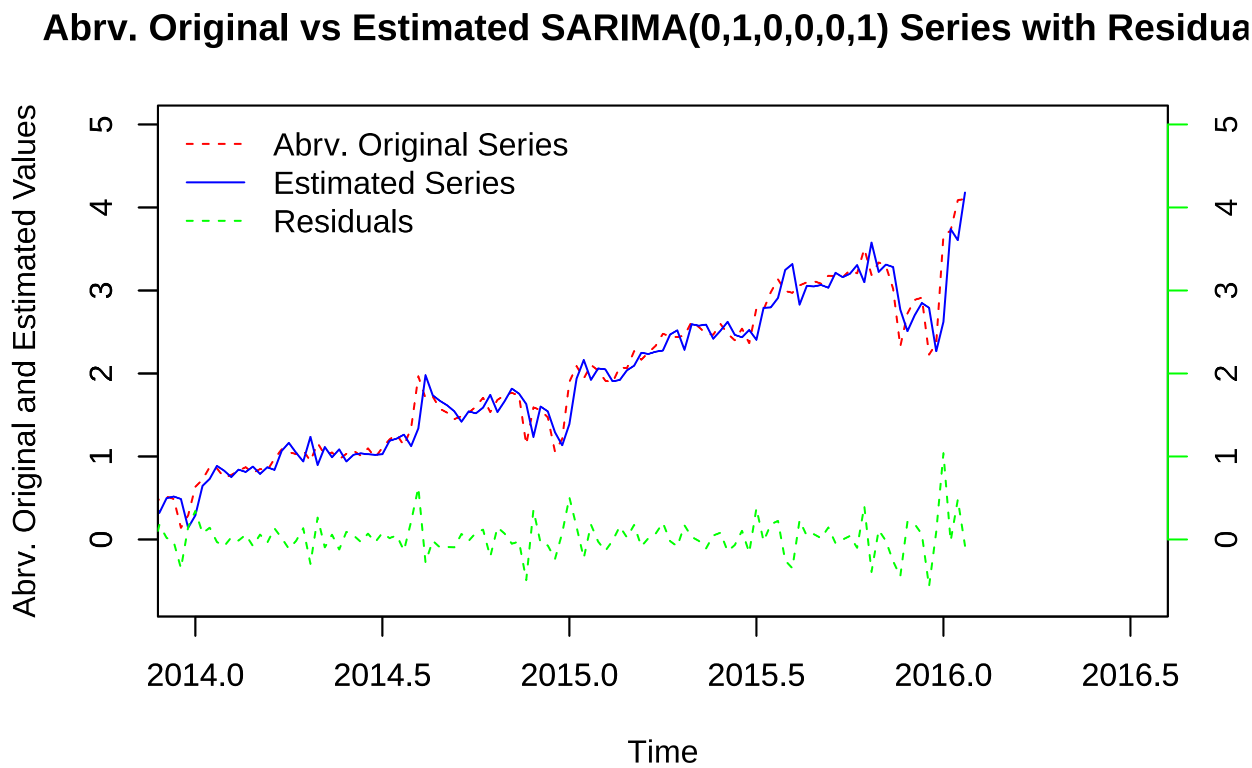
```
par(mfrow = c(2, 1))
acf(glob.warm.part.sarima$residuals, main = "ACF of Abrv. GW 2013-2016 SARIMA(0,1,0,0,0,1) Residuals")
acf(glob.warm.part.sarima$residuals^2, main = "ACF of Abrv. GW 2013-2016 SARIMA(0,1,0,0,0,1) Residuals^2
```






```
# Plot the In-sample fit
plot.orig.model.resid(glob.warm.part.ts, glob.warm.part.sarima,
    part_name, "SARIMA(0,1,0,0,0,1)", c(2014, 2016.5), c(-0.7,
        5))
```

```
##
## Descriptive Stat
## ==========================================
## Statistic        N   Mean St. Dev. Min  Max
## ------------------------------------------
## x.ts             160 1.4     1.1    -0.2 4.1
## fitted.x.mod.    160 1.4     1.1    -0.2 4.2
## x.mod.residuals 160 0.02    0.2    -0.6 1.0
## ------------------------------------------
```

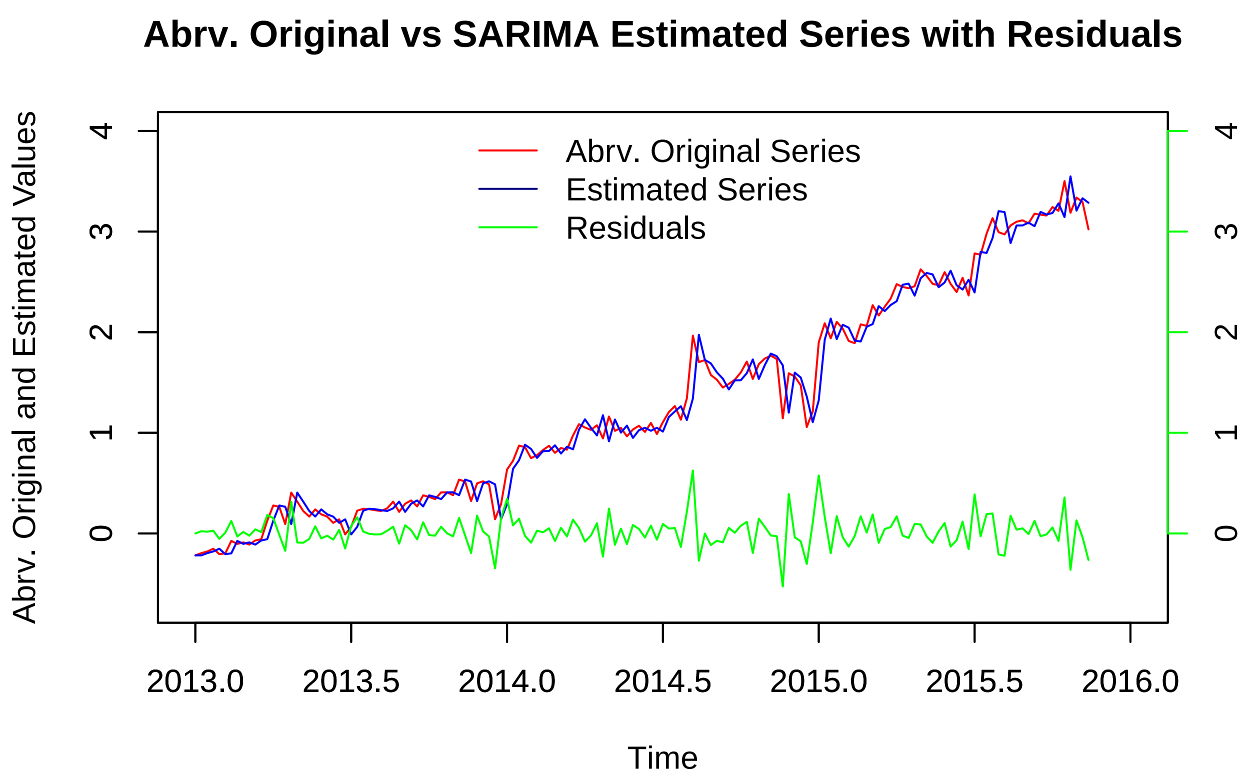



```r
#### 10.  Backtesting Create a time series cutting off the last
#### 10% of the values
glob.warm.part.bt.ts = ts(glob.warm.part.ts[1:(length(glob.warm.part.ts) -
    10)], start = 2013, frequency = 52)
# Recreate SARIMA(0,1,0,0,0,1) model
glob.warm.part.sarima.bt = arima(glob.warm.part.bt.ts, order = c(0,
    1, 0), seas = list(order = c(0, 0, 1), 52), method = "CSS")
# Combine the cut-off time series with the fitted values from
# the model and the residuals from the model
df.part = cbind(glob.warm.part.bt.ts, fitted(glob.warm.part.sarima.bt),
    glob.warm.part.sarima.bt$resid)
colnames(df.part) = c("orig_series", "fitted_vals", "resid")
head(df.part)
```

```
##      orig_series fitted_vals  resid
## [1,]      -0.218      -0.218  0.000
## [2,]      -0.196      -0.218  0.022
## [3,]      -0.179      -0.196  0.017
## [4,]      -0.152      -0.179  0.027
## [5,]      -0.206      -0.152 -0.054
## [6,]      -0.198      -0.206  0.008
```

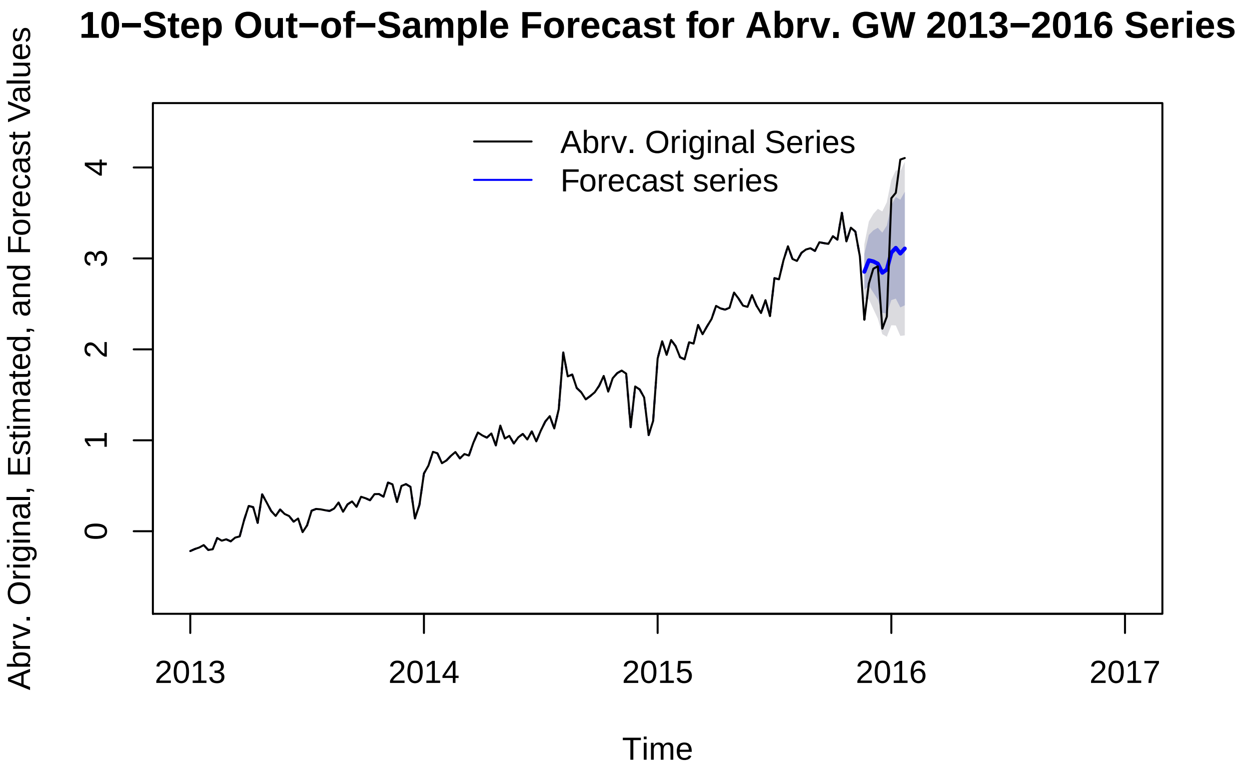
```r
# Plot the Abrv. Original and estimate series with residuals
par(mfrow = c(1, 1))
plot.ts(df.part[, "orig_series"], col = "red", main = "Abrv. Original vs SARIMA Estimated Series with Re
```

```
    ylab = "Abrv. Original and Estimated Values", xlim = c(2013,
        2016), ylim = c(-0.7, 4))
par(new = T)
plot.ts(df.part[, "fitted_vals"], col = "blue", axes = T, xlab = "",
    ylab = "", xlim = c(2013, 2016), ylim = c(-0.7, 4))
leg.txt <- c("Abrv. Original Series", "Estimated Series", "Residuals")
legend("top", legend = leg.txt, lty = 1, col = c("red", "navy",
    "green"), bty = "n", cex = 1)
par(new = T)
plot.ts(df.part[, "resid"], axes = F, xlab = "", ylab = "", col = "green",
    xlim = c(2013, 2016), ylim = c(-0.7, 4), pch = 1)
axis(side = 4, col = "green")
mtext("Residuals", side = 4, line = 2, col = "green")
```




```
# Create a forecast for backtesting and plot it
glob.warm.part.sarima.bt.fcast = forecast.Arima(glob.warm.part.sarima.bt,
    h = 10)
par(mfrow = c(1, 1))
plot(glob.warm.part.sarima.bt.fcast, lty = 2, col = "navy", main = "10-Step Out-of-Sample Forecast for A
    ylab = "Abrv. Original, Estimated, and Forecast Values",
    xlim = c(2013, 2017), ylim = c(-0.7, 4.5))
par(new = T)
plot.ts(glob.warm.part.ts, axes = F, lty = 1, col = "black",
    xlim = c(2013, 2017), ylim = c(-0.7, 4.5), ylab = "")
leg.txt <- c("Abrv. Original Series", "Forecast series")
```

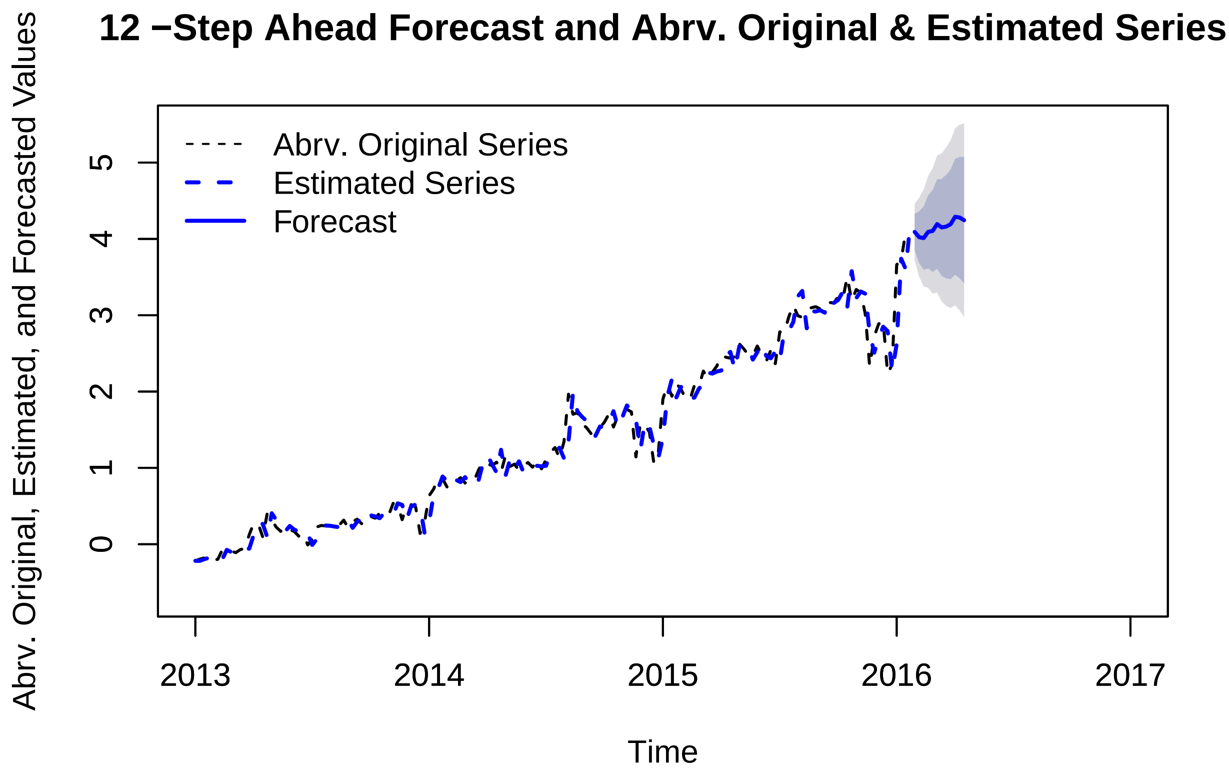
```r
legend("top", legend = leg.txt, lty = 1, col = c("black", "blue"),
    bty = "n", cex = 1)
```




```r
#### 11.  Forecasting - Forecast the request 12-step ahead
#### forecast
glob.warm.part.sarima.fcast = forecast.Arima(glob.warm.part.sarima,
    h = 12)
print(summary(glob.warm.part.sarima.fcast$mean))
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   4.011   4.092   4.156   4.153   4.207   4.290
```

```r
plot.model.forecast(glob.warm.part.sarima, glob.warm.part.sarima.fcast,
    part_name, "12", c(2013, 2017), c(-0.7, 5.5))
```




```r
# Print the forecasted values and 80% CI.
df = data.frame(cbind(glob.warm.part.sarima.fcast$mean, glob.warm.part.sarima.fcast$lower[,
    1], glob.warm.part.sarima.fcast$upper[, 1]))
names(df) = c("Mean", "Lower 80% conf", "Upper 80% conf")
stargazer(df, type = "text", summary = FALSE)
```

```
##
## ======================================
##     Mean  Lower 80% conf Upper 80% conf
## --------------------------------------
## 1  4.092     3.852          4.331
## 2  4.022     3.683          4.361
## 3  4.011     3.596          4.426
## 4  4.093     3.613          4.572
## 5  4.104     3.568          4.640
## 6  4.195     3.608          4.782
## 7  4.151     3.517          4.785
## 8  4.161     3.483          4.839
## 9  4.193     3.474          4.912
## 10 4.290     3.532          5.048
## 11 4.280     3.485          5.075
## 12 4.245     3.414          5.075
## --------------------------------------
```

Part 4 (25 points): Forecast Inflation-Adjusted Gas Price

The dataframe contains three variables. Date, Production and Price. It consists of 410 observations of those variables. The Date variable indicates that the data ranges from January 01 1978 to February 01 2012. We next perform some exploratory data analysis of those variables.

1 **Variable Production** If one were to consider the data in the time series as independent observations, as it seems the AP analysis did, they would observe that the histogram shows a data distribution of the variable that appears to be multimodal. It does not appear that the underlying distribution of the data is a normal distribution. However, there are no outliers or singularities in the data.

2 **Variable Price** Similarly, if one were to consider the data in this time series as independent observations, they would observe that the histogram shows a data distribution that is positively skewed. There are no indications from the histogram that the data follow a Normal distribution. However, there are no outliers or singularities in the data.

The data being from time series, we know that it should not be considered as observations of random samples from the same distributions, but rather as time series data. Our side by side plot of both series shows trends up and down and volatility. Both series appear to be non-stationary in the mean.

```r
# Load the data
gas.data <- load(file.path("gasOil.Rdata"))

# Summary information about the data
str(gasOil)
```

```
## 'data.frame':    410 obs. of  3 variables:
##  $ Date      : chr  "1978-01-01" "1978-02-01" "1978-03-01" "1978-04-01" ...
##  $ Production: num  259 235 270 265 274 ...
##  $ Price     : num  2.46 2.44 2.43 2.41 2.41 ...
```

```r
summary(gasOil)
```

```
##      Date             Production        Price      
##  Length:410         Min.   :119.4   Min.   :1.329  
##  Class :character   1st Qu.:173.0   1st Qu.:1.823  
##  Mode  :character   Median :201.4   Median :2.096  
##                     Mean   :210.0   Mean   :2.391  
##                     3rd Qu.:255.8   3rd Qu.:2.909  
##                     Max.   :283.2   Max.   :4.432  
```

```r
cbind(head(gasOil$Date), head(gasOil$Price), head(gasOil$Production),
    tail(gasOil$Date), tail(gasOil$Price), tail(gasOil$Production))
```

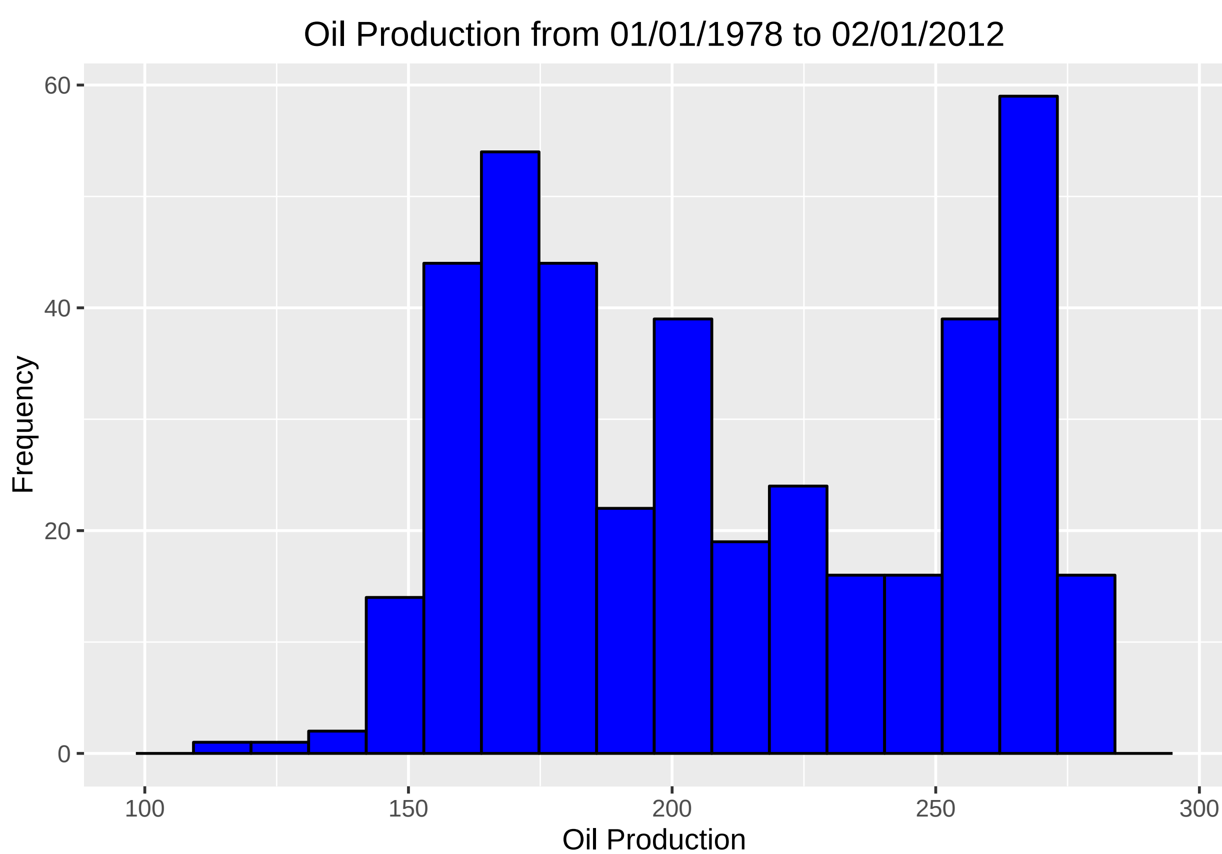
```
##      [,1]         [,2]               [,3]      [,4]        
## [1,] "1978-01-01" "2.45669201913876" "259.15"  "2011-09-01"
## [2,] "1978-02-01" "2.44122034761905" "234.544" "2011-10-01"
## [3,] "1978-03-01" "2.42581832649842" "270.324" "2011-11-01"
## [4,] "1978-04-01" "2.41427695305164" "264.526" "2011-12-01"
## [5,] "1978-05-01" "2.41393090697674" "273.583" "2012-01-01"
## [6,] "1978-06-01" "2.42461854"       "264.974" "2012-02-01"
##      [,5]               [,6]     
## [1,] "3.78699964541901" "166.849"
```

```
## [2,] "3.61365255477027" "181.493"
## [3,] "3.54091412492241" "179.099"
## [4,] "3.41761395265139" "185.712"
## [5,] "3.52764117334551" "190.358"
## [6,] "3.72698725376175" "180.969"
```

```
# EDA for variable Production
print(quantile(gasOil$Production, probs = c(0.01, 0.05, 0.1,
    0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##       1%       5%      10%      25%      50%      75%      90%      95%
## 143.5285 154.5767 159.3485 173.0135 201.4405 255.7722 267.6471 271.1311
##      99%     100%
## 279.2919 283.2480
```

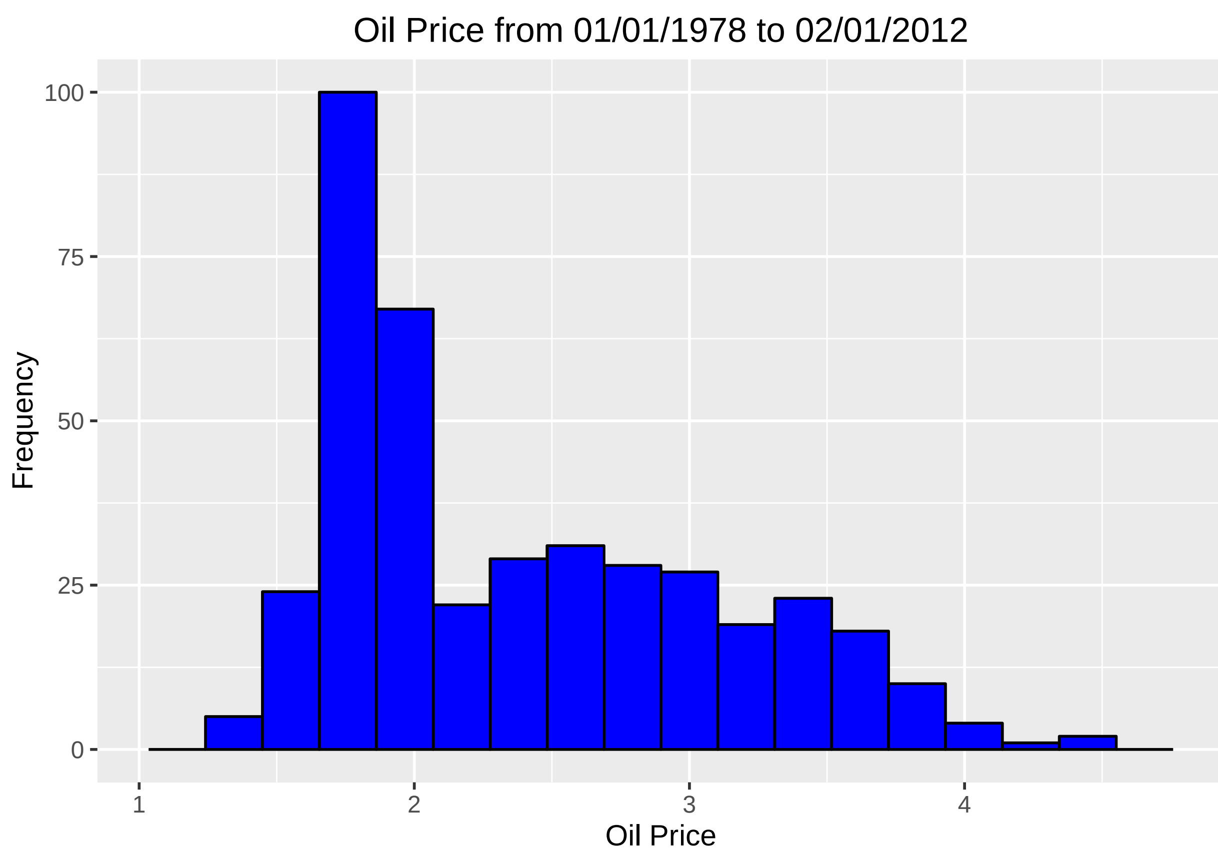
```
# Plot the histogram of Production at 15 bins
gasOil.prod.hist <- ggplot(gasOil, aes(Production)) + theme(legend.position = "none") +
    geom_histogram(fill = "Blue", colour = "Black", binwidth = (range(gasOil$Production)[2] -
        range(gasOil$Production)[1])/15) + labs(title = "Oil Production from 01/01/1978 to 02/01/2012",
    x = "Oil Production", y = "Frequency")

plot(gasOil.prod.hist)
```



```r
# EDA for variable Price
print(quantile(gasOil$Price, probs = c(0.01, 0.05, 0.1, 0.25,
    0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

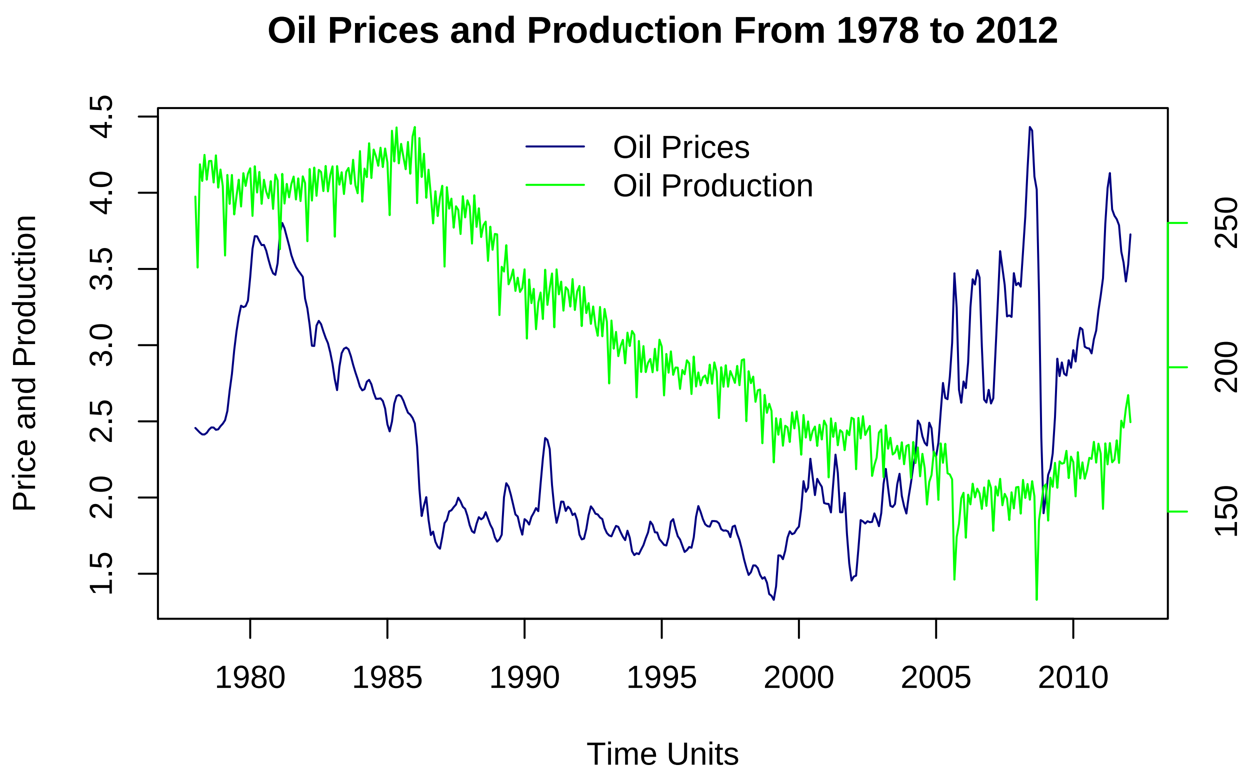
```
##       1%       5%      10%      25%      50%      75%      90%      95%
## 1.443087 1.619687 1.709716 1.823093 2.096003 2.908782 3.471567 3.671866
##      99%     100%
## 4.100232 4.431625
```

```r
# Plot the histogram of Price at 15 bins
gasOil.price.hist <- ggplot(gasOil, aes(Price)) + theme(legend.position = "none") +
    geom_histogram(fill = "Blue", colour = "Black", binwidth = (range(gasOil$Price)[2] -
        range(gasOil$Price)[1])/15) + labs(title = "Oil Price from 01/01/1978 to 02/01/2012",
    x = "Oil Price", y = "Frequency")

plot(gasOil.price.hist)
```




```r
# Side by side plots of Price and Production time series
# First convert the price data to a time series:
price.ts <- ts(gasOil$Price, start = c(1978, 1), end = c(2012,
    2), frequency = 12)
# Convert the production data to a time series
production.ts <- ts(gasOil$Production, start = c(1978, 1), end = c(2012,
    2), frequency = 12)
```

```
# Plot the two time series
par(mfrow = c(1, 1))
plot.ts(price.ts, main = "Oil Prices and Production From 1978 to 2012",
    ylab = "Price and Production", xlab = "Time Units", col = "navy")
par(new = T)
plot.ts(production.ts, ylab = "", xlab = "", col = "green", axes = F)
axis(side = 4, col = "green")
leg.txt <- c("Oil Prices", "Oil Production")
legend("top", legend = leg.txt, lty = 1, col = c("navy", "green"),
    bty = "n", cex = 1)
```



Task1

We can assume that the AP tested the correlation of the time series of Oil Price and Oil Production. We can replicate the calculation of the reported p-value with a test of the correlation of the variables Price and Production. The test reports a p-value of 0.5752, which is non-significant. The reported 95% confidence interval for the correlation is [-0.06927648 0.12427029]. Since the p-value for the test is non-significant, the confidence interval non-surprisingly spans the zero value.

When computing the correlation of two sets of observations of data, we assume that the data is from random samples each drawn from their respective population with distributions that have constant means and variances. We know that our data is from a time time series. We have seen from the side by side plots that these series are non-stationary in the mean. Therefore the assumption of constant mean in the calculation of the correlation does not hold and the calculated p-value is flawed. Another assumtion made with correlations is the assumption of independence of variables in each samples. As stated before, we usually assume that the samples are random draws from a population. For a time series the assumption of independence between the data observations must be rejected. For times series, the observation of x_t is dependent on previous observations of $x_{t??????1}$, $x_{t??????2}$,.... That dependency is captured in a joint probability distribution which is unavailable to us, as the time series represents the single instance of the realisation of a stochastic process that we are able to observe.

We next turn to studying the time series of gas prices.

```
cor.test(gasOil$Price, gasOil$Production)
```

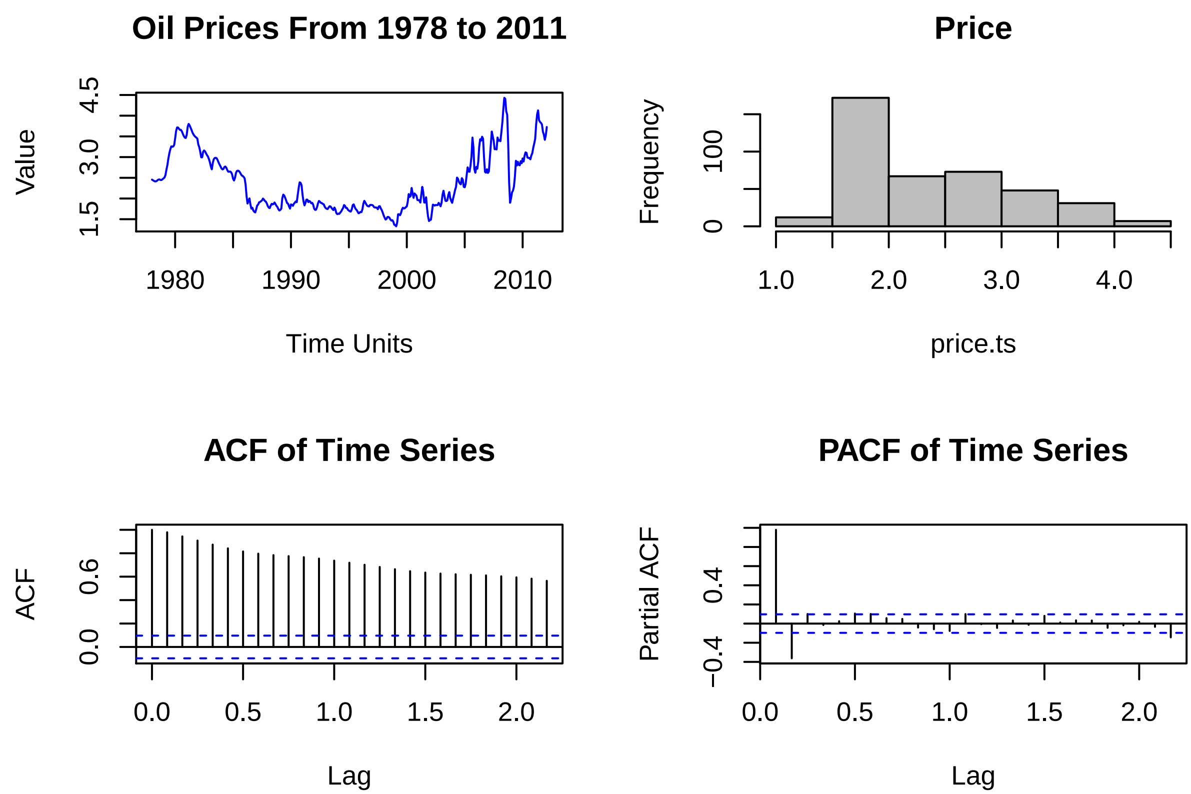
```
##
##  Pearson's product-moment correlation
##
## data:  gasOil$Price and gasOil$Production
## t = 0.56088, df = 408, p-value = 0.5752
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  -0.06927648  0.12427029
## sample estimates:
##        cor
## 0.02775705
```

Task 2

Approach summary
We fit the series with a SARIMA(0, 1, 1, 1, 0, 3) model, followed by fitting the residuals of our fitted model with GARCH and using the results of both fitted series to make our forecast with accurate confidence intervals depicting the series. The next sections provide the details of our approach.

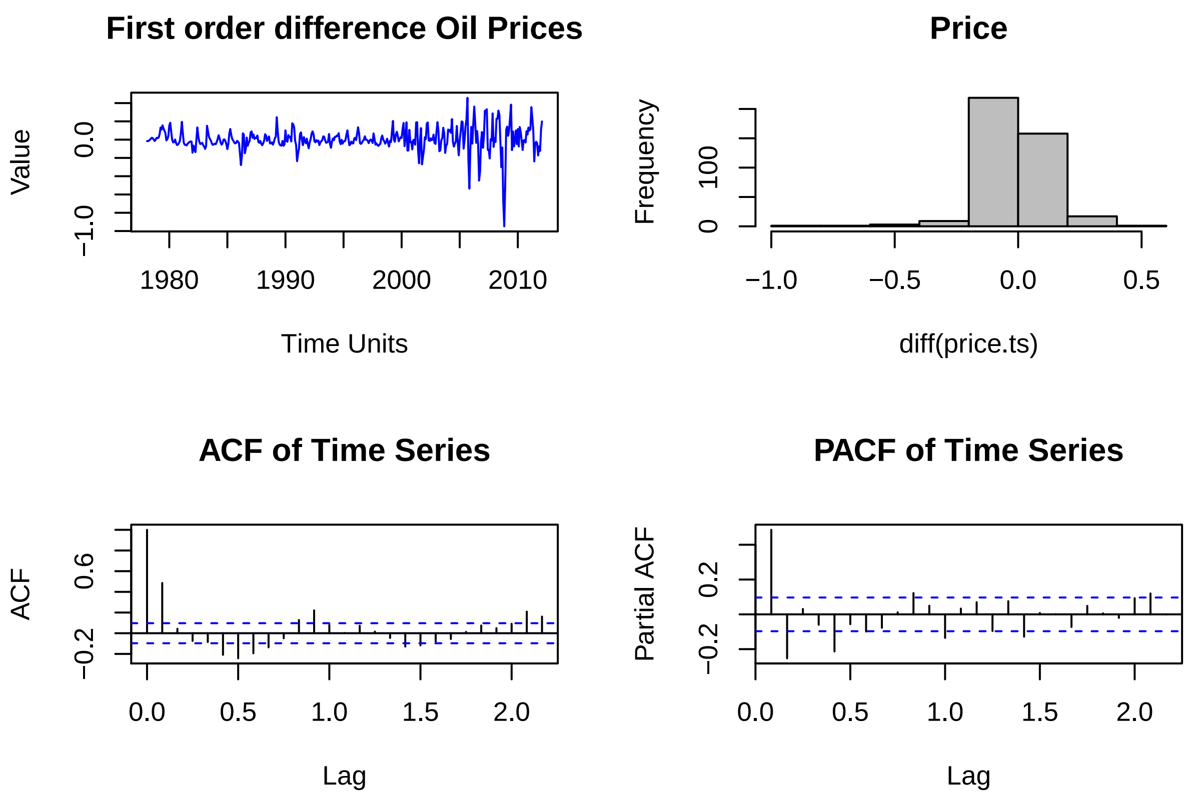
EDA on the series
The series exhibits trends up and down and a lot of volatility. It appears to be non-stationary in the mean. We can also see that ACF is gradually descending, indicating a possible ARIMA or SARIMA dynamics. Knowing the series to be that of oil prices, we can speculate that it incorporates seasonality as we'd expect prices to follow the seasons of the year and the corresponding changes in oil compsumption. We would expect yearly seasonality. The PACF shows significant correlations at lags 1 and less so at lag 2, suggesting that the series might have characteristics of an AR(1) or AR(2) component. To verify our observations, we next study 1 and 2 order differences of the series.

```
# EDA on series
par(mfrow = c(2, 2))
plot.ts(price.ts, main = "Oil Prices From 1978 to 2011", ylab = "Value",
    xlab = "Time Units", col = "blue")
hist(price.ts, col = "gray", main = "Price")
acf(price.ts, main = "ACF of Time Series")
pacf(price.ts, main = "PACF of Time Series")
```

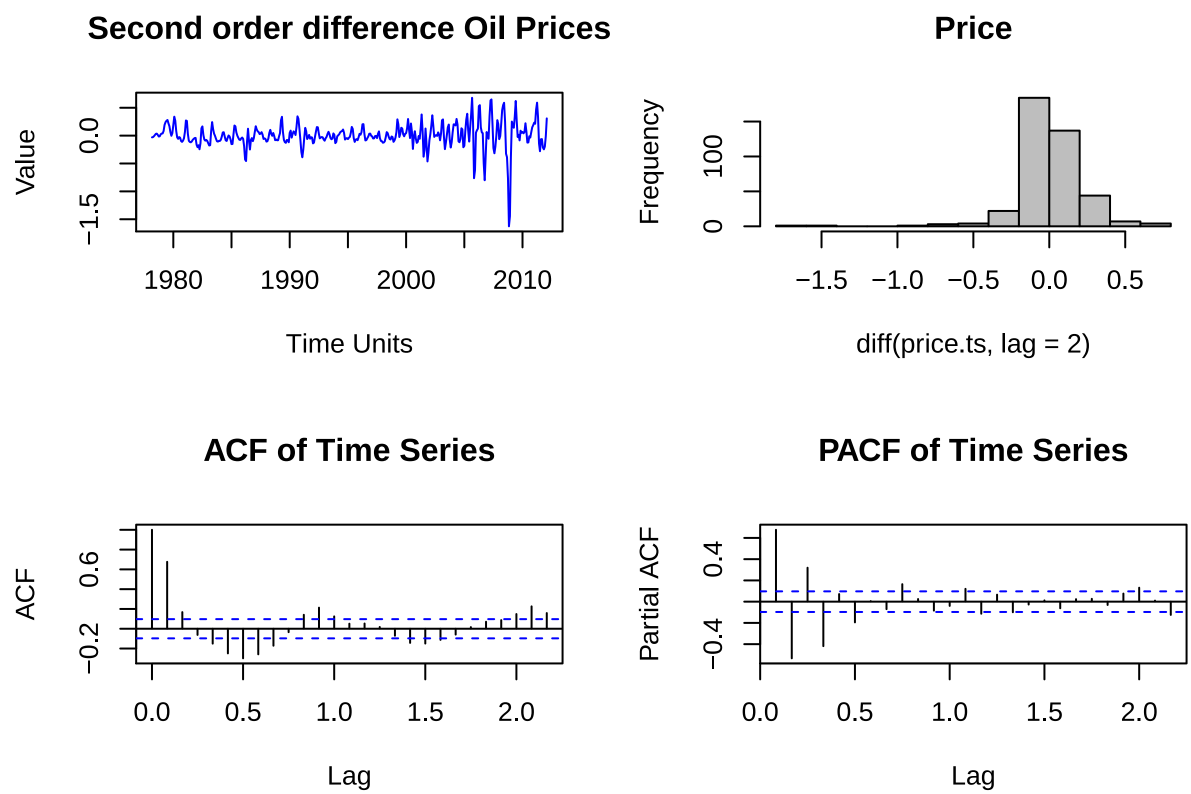


As expected, the diff(1) and diff(2) series are stationary in the mean. They appear to show conditional volatility that will need to be analyzed with a GARCH model. We can also observe patterns that appear to be seasonal patterns of repetition on yearly basis in the series. That observation would support the ituition that weather cycles and corresponding consuption changes may affect gas prices. Because the first difference series is stationary in the mean, we will not need the second order differencing of the series when we study it furter. We can also see that the ACF of the first and second order series drop sharply after lag 1 indicating the presence of an MA(1) component in the series. Similarly, the PACF of the first and second difference series have a single significant correlation at lag 1, indicating the possible presence of an AR(1) component in the series. We next perform a systematic search of the best model fit for the series based on the AIC.

```r
par(mfrow = c(2, 2))
plot.ts(diff(price.ts), main = "First order difference Oil Prices",
    ylab = "Value", xlab = "Time Units", col = "blue")
hist(diff(price.ts), col = "gray", main = "Price")
acf(diff(price.ts), main = "ACF of Time Series")
pacf(diff(price.ts), main = "PACF of Time Series")
```




```r
par(mfrow = c(2, 2))
plot.ts(diff(price.ts, lag = 2), main = "Second order difference Oil Prices",
    ylab = "Value", xlab = "Time Units", col = "blue")
hist(diff(price.ts, lag = 2), col = "gray", main = "Price")
acf(diff(price.ts, lag = 2), main = "ACF of Time Series")
pacf(diff(price.ts, lag = 2), main = "PACF of Time Series")
```



Fitting a model to the time series data

The best SARIMA model fitted is a (p,d,q,P,D,Q) of orders (0, 1, 2, 1, 0, 3) with an AIC of -672.50. We note that immediately following that model, is a SARIMA of parameters (0, 1, 1, 1, 0, 3) with an AIC of -672.19. With parsimony in mind, we select that model to fit our data and proceed to assess the in-sample fit of that model. It matches our earlier observations of a stationary first difference series, and of the possible presence of AR(1) and MA(1) components in the time series, along with that of a seasonal component. An inspection of the confidence intervals of the paramaters of this model indicate that the parameter for the second MA component of the seasonal model is not significant. But the third parameter of the same seasonal component is significant. We next perform in-sample fit using the fitted series to assess our fitted model.

```r
price.best <- get.best.sarima(price.ts, maxord = rep(3, 6))
price.best$best
```

```
## [[1]]
## [1] -672.5026
##
## [[2]]
##
## Call:
## arima(x = x.ts, order = c(p, d, q), seasonal = list(order = c(P, D, Q), freq),
##     method = "CSS", optim.control = list(maxit = 10000))
##
## Coefficients:
##          ma1     ma2    sar1     sma1    sma2    sma3
##       0.6226  0.1467  0.9866  -1.0817  0.0138  0.2462
## s.e.  0.0530  0.0536  0.0362   0.0588  0.0739  0.0563
```

```
##
## sigma^2 estimated as 0.0102:  part log likelihood = 357.3
##
## [[3]]
## [1] 0 1 2 1 0 3
```

```
price.best$others[order(price.best$others$aics)[1:20], ]
```

```
##            aics             models
## 404  -672.5026 (0, 1, 2, 1, 0, 3)
## 340  -672.1951 (0, 1, 1, 1, 0, 3)
## 1364 -671.5471 (1, 1, 1, 1, 0, 3)
## 1428 -669.4694 (1, 1, 2, 1, 0, 3)
## 2324 -669.3623 (2, 1, 0, 1, 0, 3)
## 468  -668.7218 (0, 1, 3, 1, 0, 3)
## 1492 -664.0486 (1, 1, 3, 1, 0, 3)
## 1172 -664.0404 (1, 0, 2, 1, 0, 3)
## 2132 -663.4126 (2, 0, 1, 1, 0, 3)
## 339  -663.4071 (0, 1, 1, 1, 0, 2)
## 2388 -663.0372 (2, 1, 1, 1, 0, 3)
## 1108 -662.7184 (1, 0, 1, 1, 0, 3)
## 467  -661.1375 (0, 1, 3, 1, 0, 2)
## 3092 -661.0822 (3, 0, 0, 1, 0, 3)
## 2196 -660.6719 (2, 0, 2, 1, 0, 3)
## 1363 -660.5172 (1, 1, 1, 1, 0, 2)
## 1427 -660.2191 (1, 1, 2, 1, 0, 2)
## 385  -659.7950 (0, 1, 2, 0, 0, 0)
## 1236 -659.4135 (1, 0, 3, 1, 0, 3)
## 3348 -658.9226 (3, 1, 0, 1, 0, 3)
```

The fitted series models the original series very well and the model selection seems appropriate based on in-sample fit. To further assess the fitted series, We next perform 48 steps backtesting.

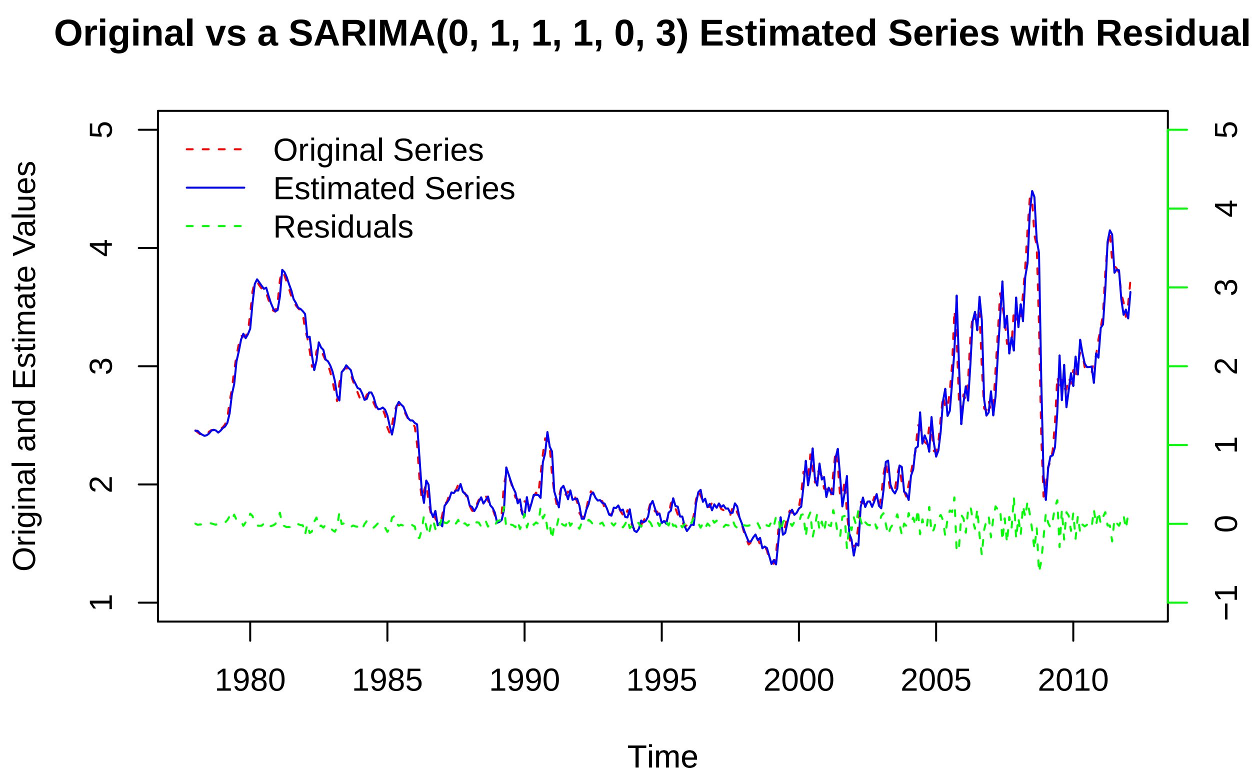
```
# Fit the selected ARIMA model to the time series data.
price.fit <- Arima(price.ts, order = c(0, 1, 1), seasonal = list(order = c(1,
    0, 3), period = 12), method = "CSS-ML")
price.res <- price.fit$resid
t(confint(price.fit))
```

```
##               ma1      sar1       sma1       sma2      sma3
## 2.5 %  0.4570524 0.855634 -1.1133820 -0.1289343 0.1123691
## 97.5 % 0.5999814 0.992224 -0.8704157  0.1411505 0.3294568
```

```
quantile(as.numeric(price.res), c(0, 0.01, 0.05, 0.1, 0.25, 0.5,
    0.75, 0.9, 0.95, 0.99, 1))
```

```
##           0%           1%           5%          10%          25%
## -0.616577523 -0.323314392 -0.164782375 -0.083391460 -0.030337106
##          50%          75%          90%          95%          99%
## -0.003213021  0.044032156  0.114419059  0.163404708  0.231930102
##         100%
##  0.340313336
```

```r
# We now perform in-sample fit using the fitted series to
# assess our fitted model.
par(mfrow = c(1, 1))
plot.ts(price.ts, col = "red", lty = 2, main = "Original vs a SARIMA(0, 1, 1, 1, 0, 3) Estimated Series
    ylab = "Original and Estimate Values", ylim = c(1, 5))
par(new = T)
plot(fitted(price.fit), col = "blue", axes = F, ylab = "", ylim = c(1,
    5))
leg.txt <- c("Original Series", "Estimated Series", "Residuals")
legend("topleft", legend = leg.txt, lty = c(2, 1, 2), col = c("red",
    "blue", "green"), bty = "n", cex = 1)
par(new = T)
plot.ts(price.res, axes = F, xlab = "", ylab = "", col = "green",
    ylim = c(-1, 5), pch = 1, lty = 2)
axis(side = 4, col = "green")
mtext("Residuals", side = 4, line = 2, col = "green")
```



Our out of sample forecast appears to be reasonable. The 80 and 95 % confidence intervals of the forecast partially include the actual values of the series. We had previously observed that the variance of the residuals seemed to show some volatility that we know is not modeled by the fitted point series. Therefore, we next turn our eyes to the residuals of the fitted series and to the analysis of the dynamics of its variance.

```r
price.fit.back <- Arima(price.ts[1:(length(price.ts) - 48)],
    order = c(0, 1, 1), seasonal = list(order = c(1, 0, 3), period = 12),
    method = "CSS-ML")
summary(price.fit.back)
```

```
## Series: price.ts[1:(length(price.ts) - 48)]
## ARIMA(0,1,1)(1,0,3)[12]
##
## Coefficients:
##          ma1    sar1     sma1    sma2    sma3
##       0.5370  0.8933  -0.9756  0.0840  0.1450
## s.e.  0.0408  0.0564   0.0799  0.0794  0.0673
##
## sigma^2 estimated as 0.007341:  log likelihood=374.29
## AIC=-736.58   AICc=-736.34   BIC=-713.25
##
## Training set error measures:
##                       ME       RMSE        MAE        MPE     MAPE
## Training set 0.001656362 0.08496843 0.05633002 0.06153449 2.437364
##                   MASE       ACF1
## Training set 0.8325864 0.01216812
```

```
length(fitted(price.fit.back))
```

```
## [1] 362
```

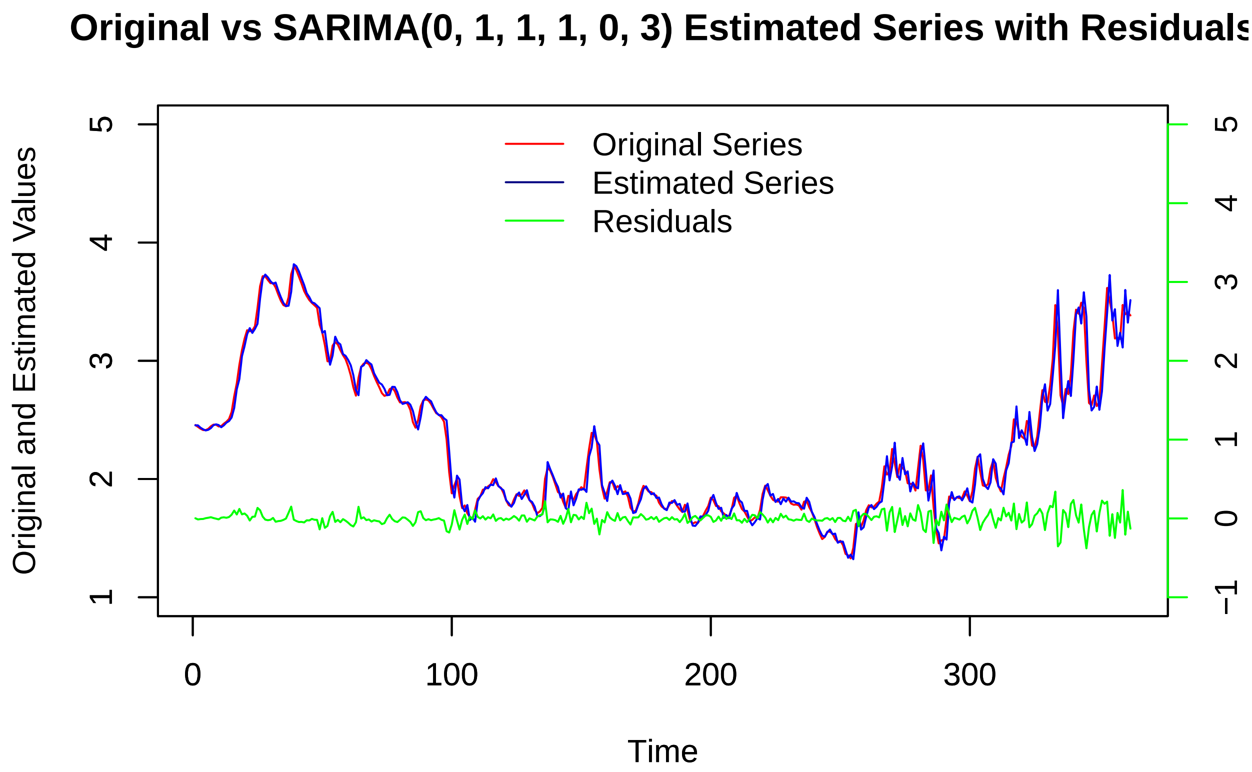
```
length(price.fit.back$resid)
```

```
## [1] 362
```

```
df = cbind(price.ts[1:(length(price.ts) - 48)], fitted(price.fit.back),
    price.fit.back$resid)
colnames(df) = c("orig_series", "fitted_vals", "resid")
head(df)
```

```
##      orig_series fitted_vals        resid
## [1,]    2.456692    2.454235  0.002456690
## [2,]    2.441220    2.454073 -0.012852734
## [3,]    2.425818    2.433999 -0.008180921
## [4,]    2.414277    2.420843 -0.006566487
## [5,]    2.413931    2.410769  0.003161880
## [6,]    2.424619    2.416242  0.008377038
```

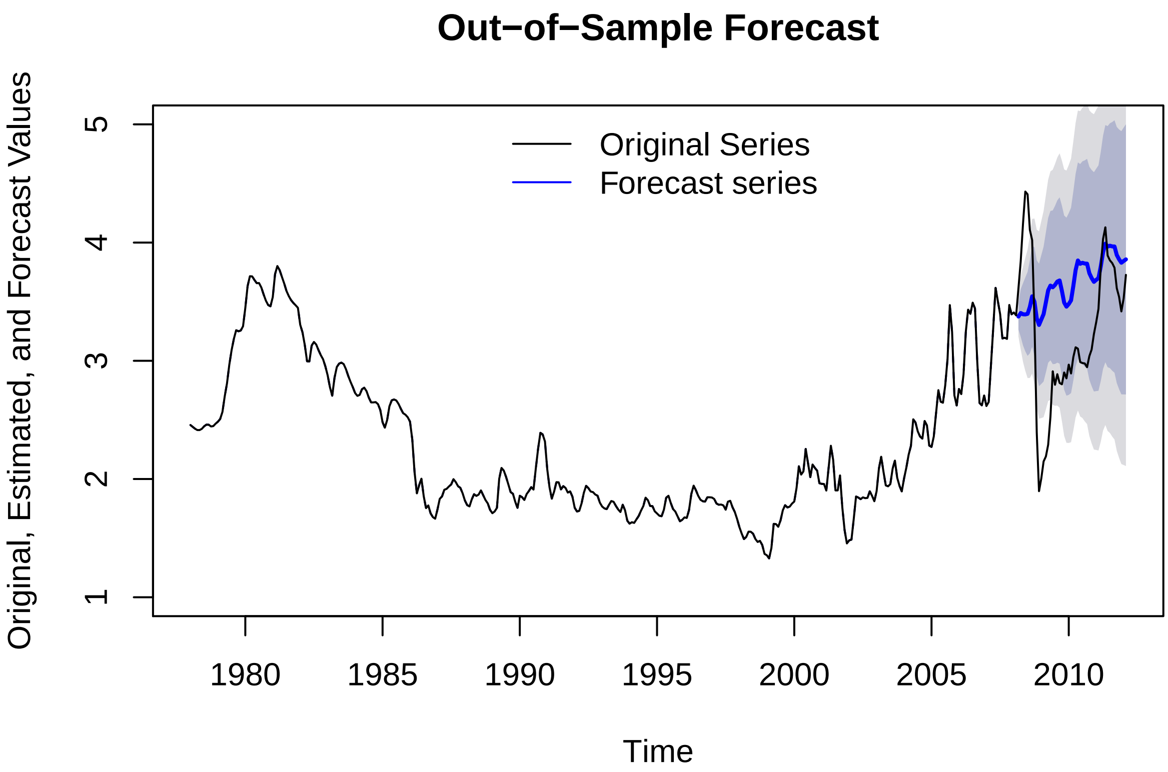
```
# Step 1: Plot the original and estimate series
par(mfrow = c(1, 1))
plot.ts(df[, "orig_series"], col = "red", main = "Original vs SARIMA(0, 1, 1, 1, 0, 3) Estimated Series
    ylab = "Original and Estimated Values", ylim = c(1, 5))
par(new = T)
plot.ts(df[, "fitted_vals"], col = "blue", axes = T, ylab = "",
    ylim = c(1, 5))
leg.txt <- c("Original Series", "Estimated Series", "Residuals")
legend("top", legend = leg.txt, lty = 1, col = c("red", "navy",
    "green"), bty = "n", cex = 1)
par(new = T)
plot.ts(df[, "resid"], axes = F, xlab = "", ylab = "", col = "green",
    ylim = c(-1, 5), pch = 1)
axis(side = 4, col = "green")
mtext("Residuals", side = 4, line = 2, col = "green")
```




```
# Step 2: Out of sample forecast
price.fit.back.fcast <- forecast.Arima(price.fit.back, h = 48)
length(price.fit.back.fcast$mean)
```

```
## [1] 48
```

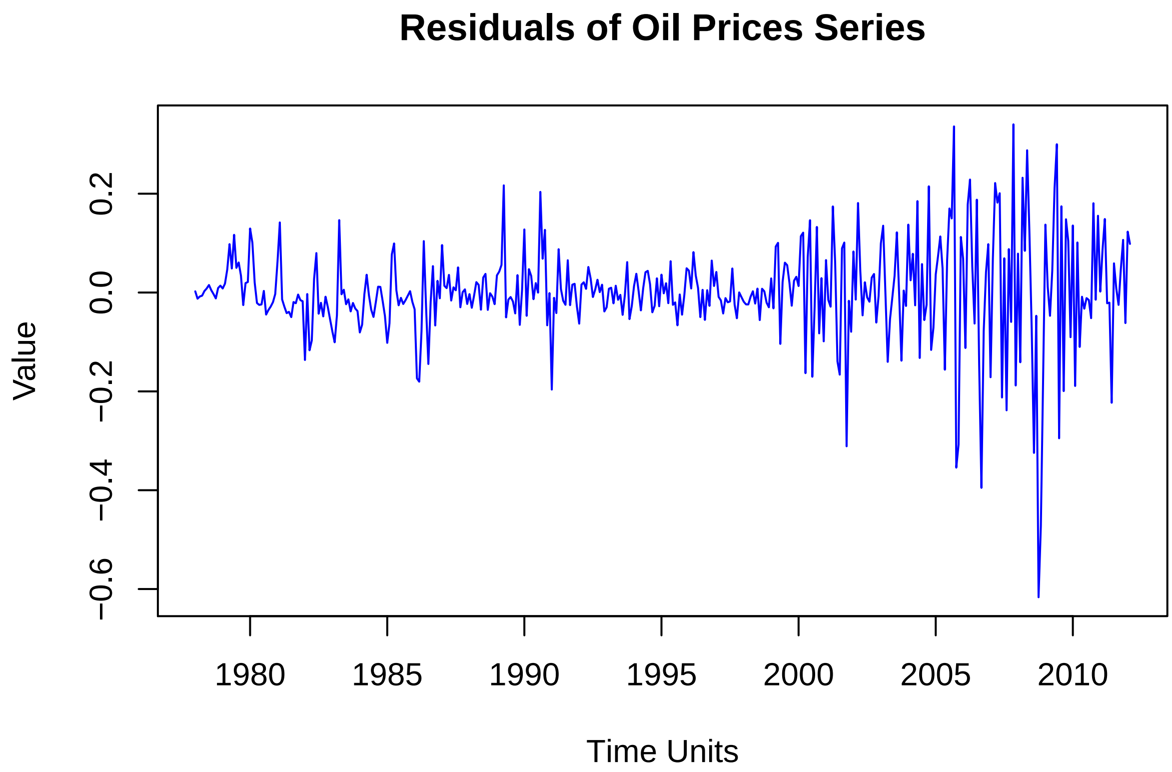
```
par(mfrow = c(1, 1))
plot(price.fit.back.fcast, lty = 2, col = "navy", main = "Out-of-Sample Forecast",
    ylab = "Original, Estimated, and Forecast Values", ylim = c(1,
        5), , axes = F)
par(new = T)
plot.ts(price.ts, axes = T, lty = 1, ylim = c(1, 5), ylab = "")
leg.txt <- c("Original Series", "Forecast series")
legend("top", legend = leg.txt, lty = 1, col = c("black", "blue"),
    bty = "n", cex = 1)
```



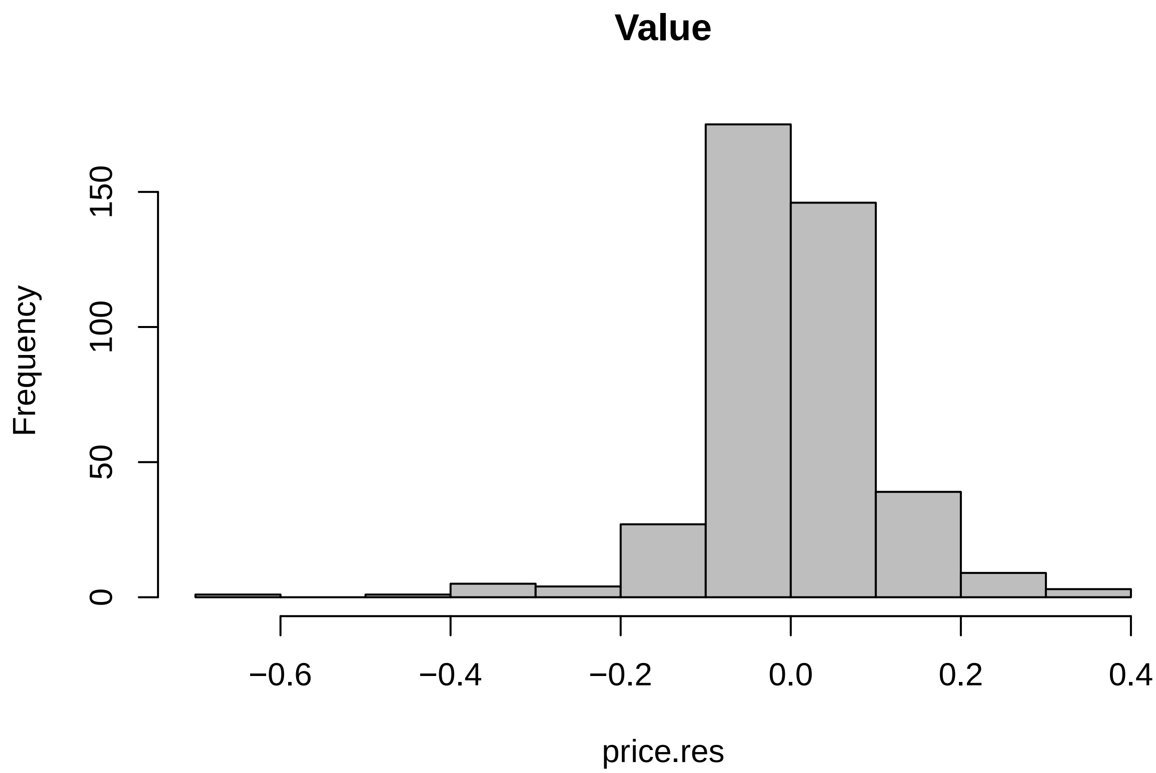
The ACF and PACF of the residual series resemble those of a white noise series. But we observe from the squared residuals time series that the variance of the series is non-stationary. The series exhibits volatility with a variance changing in a regular way. It exhibits conditional heteroskedasticity. Therefore, we will model its residuals using GARCH.

48 steps ahead forecasting
With this in mind, we proceed to an initial 48 steps ahead forecast of the oil price series.

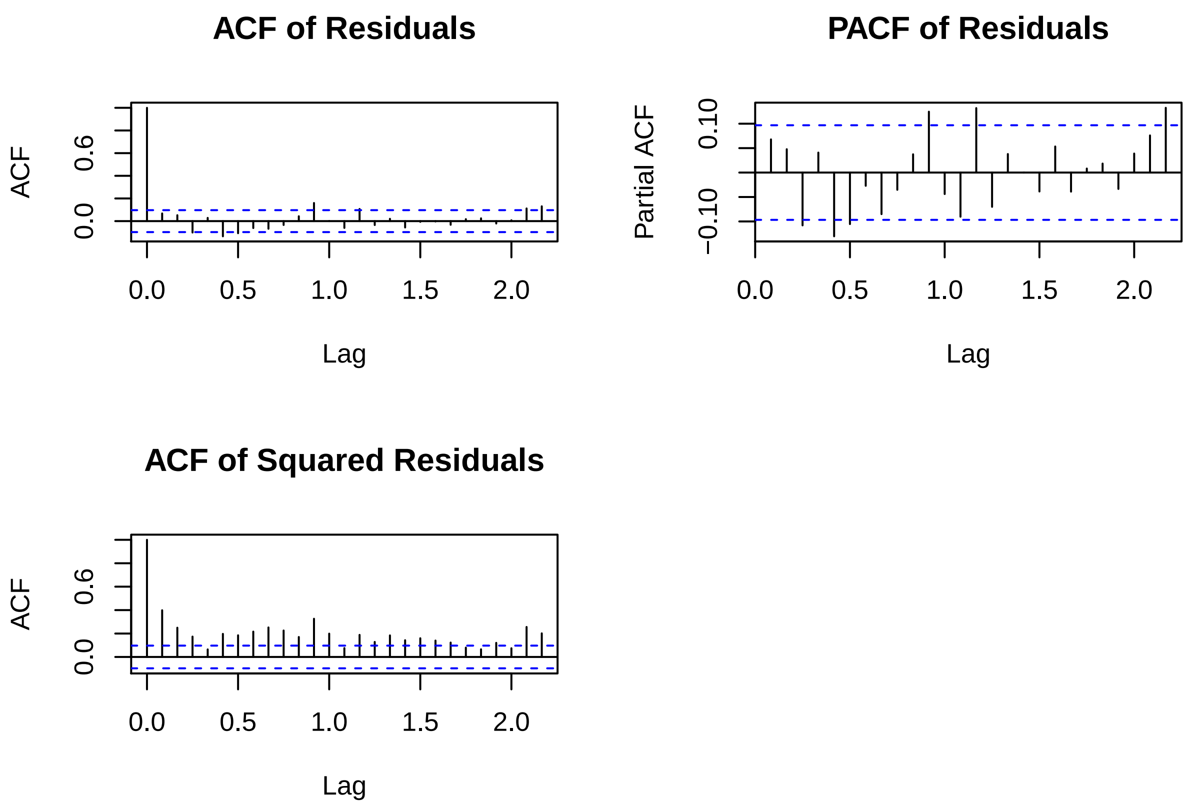
```r
# Plot the residuals time series
par(mfrow = c(1, 1))
plot.ts(price.res, main = "Residuals of Oil Prices Series", ylab = "Value",
    xlab = "Time Units", col = "blue")
```




```r
par(mfrow = c(1, 1))
hist(price.res, col = "gray", main = "Value")
```




```r
par(mfrow = c(2, 2))
acf(price.res, main = "ACF of Residuals")
pacf(price.res, main = "PACF of Residuals")
acf(price.res^2, main = "ACF of Squared Residuals")
```

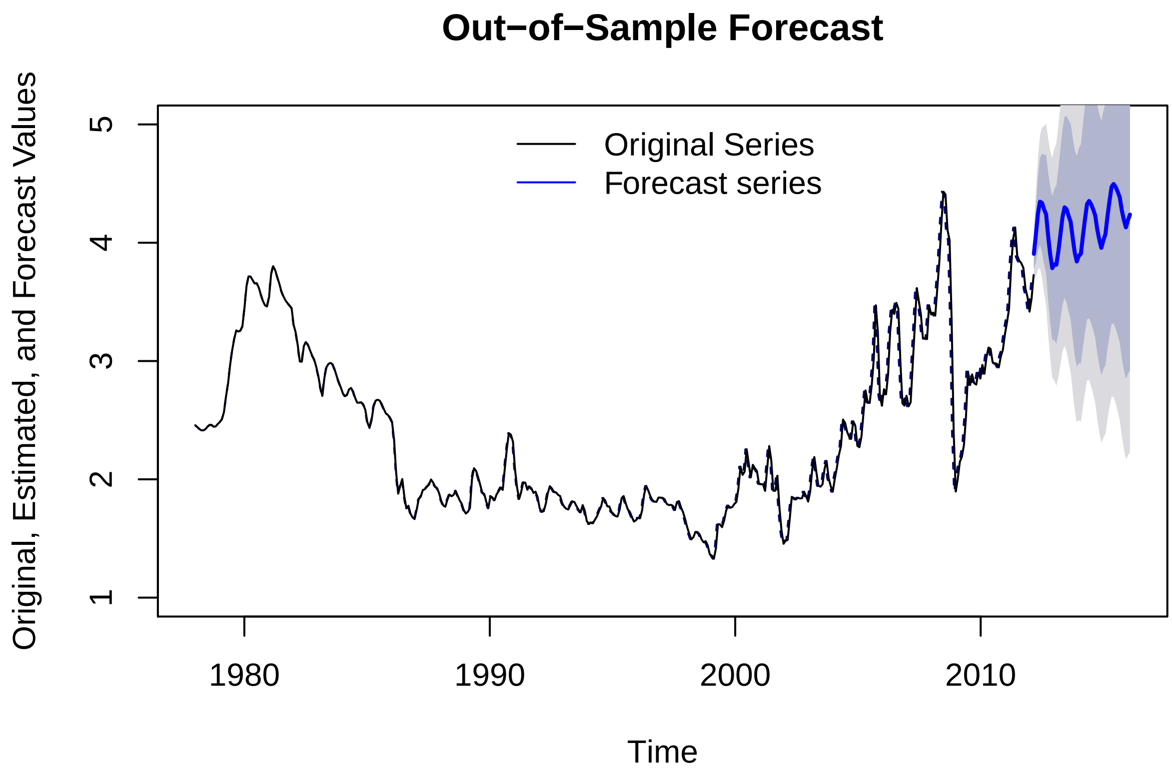


The point estimates of our forecasts look reasonable. We are aware that the model used for this point forecast assumes a stationary mean and variance. We have observed a stationary mean, but not a variance. The consequence of the non-stationary variance is that the confidence intervals around our estimates are inaccurate. Having acknowledged the confidence interval problem on the prediction caused by the non-stationary variance of the financial search time series, we want to use our fitted GARCH model to predict the mean and variance of the residuals of the point series.

```
# 2012-2016 steps ahead sample forecast
price.fit.ahead.fcast <- forecast.Arima(price.fit, h = 48)
length(price.fit.ahead.fcast$mean)
```

```
## [1] 48
```

```
par(mfrow = c(1, 1))
plot(price.fit.ahead.fcast, lty = 2, col = "navy", main = "Out-of-Sample Forecast",
    ylab = "Original, Estimated, and Forecast Values", ylim = c(1,
        5))
par(new = T)
plot.ts(price.ts, axes = F, lty = 1, ylim = c(1, 5), xlim = c(1978,
    2016), ylab = "")
leg.txt <- c("Original Series", "Forecast series")
legend("top", legend = leg.txt, lty = 1, col = c("black", "blue"),
    bty = "n", cex = 1)
```



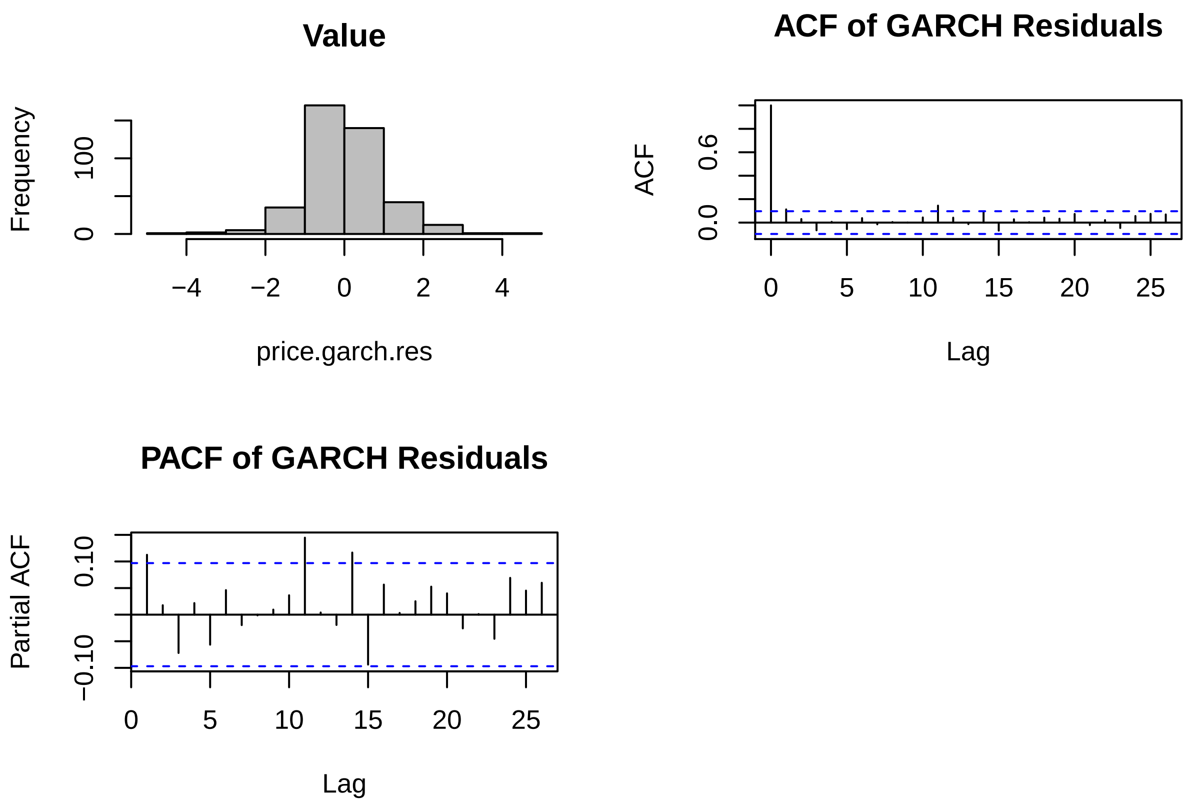
We observe from the ACF of the residuals of the GARCH fitted series that they have characteristics of white noise with mostly non-significant correlations at all lags of the ACF. What the GARCH model of the residuals tells is that we can expect more or less volatility through the forecast of the point series. That volatility affects the confidence intervals of the estimates of our SARIMA model as previously observed with backstesting. Using our fitted GARCH model, we can now better predict the variance of the point estimates from 2012 to 2016.

```
# Fit a GARCH model to the residuals of the fitted time
# series
price.garch <- garch(price.res, order = c(1, 1), trace = F)
t(confint(price.garch))
```

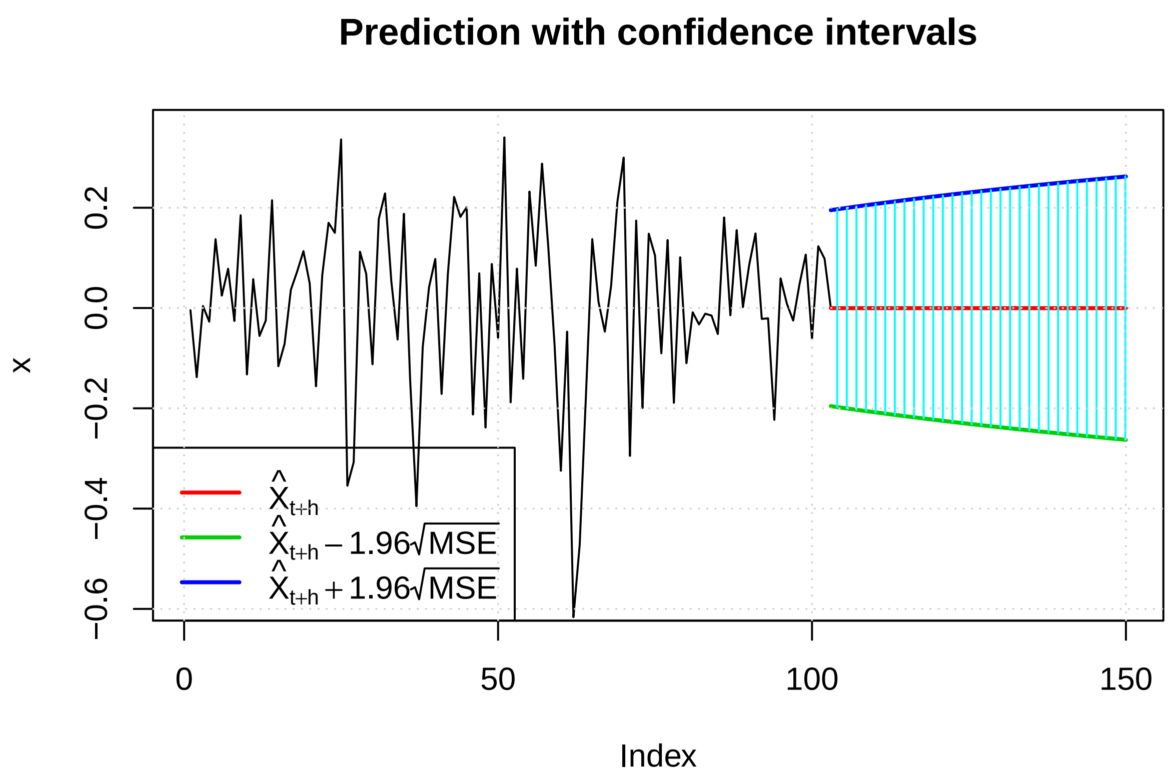
```
##                   a0        a1        b1
## 2.5 %  9.844544e-05 0.1557701 0.7131228
## 97.5 % 3.329228e-04 0.3048951 0.8252643
```

```
price.garch.res <- resid(price.garch)[-1]

# Perform EDA on residuals of fit
par(mfrow = c(2, 2))
hist(price.garch.res, col = "gray", main = "Value")
acf(price.garch.res, na.action = na.pass, main = "ACF of GARCH Residuals")
pacf(price.garch.res, na.action = na.pass, main = "PACF of GARCH Residuals")

# Predict the residuals variance 4 years ahead
```

```r
price.garch.fit <- garchFit(~garch(1, 1), data = price.res, trace = FALSE)
par(mfrow = c(1, 1))
```




```r
price.garch.pred <- predict(price.garch.fit, n.ahead = 48, plot = TRUE)
```



Having better estimates of the variance with GARCH, we next update our estimated SARIMA model's confidence interval with those.

```
# Recompute the lower and upper 95% confidence intervals
# based on GARCH variance
price.fit.ahead.fcast$lower[, 1] <- price.fit.ahead.fcast$mean
price.fit.ahead.fcast$upper[, 1] <- price.fit.ahead.fcast$mean

price.fit.ahead.fcast$lower[, 2] <- -(((price.fit.ahead.fcast$mean -
    price.fit.ahead.fcast$lower[, 2])/(sqrt(price.fit.ahead.fcast$model$sigma2))) *
    price.garch.pred$standardDeviation) + price.fit.ahead.fcast$mean
price.fit.ahead.fcast$upper[, 2] <- (((price.fit.ahead.fcast$upper[,
    2] - price.fit.ahead.fcast$mean)/(sqrt(price.fit.ahead.fcast$model$sigma2))) *
    price.garch.pred$standardDeviation) + price.fit.ahead.fcast$mean

df <- data.frame(cbind(price.fit.ahead.fcast$mean, price.fit.ahead.fcast$lower[,
    2], price.fit.ahead.fcast$upper[, 2]))
names(df) <- c("Mean", "Lower 95% conf", "Upper 95% conf")
stargazer(df, type = "text", summary = FALSE)
```

```
##
## ======================================
##    Mean  Lower 95% conf Upper 95% conf
## --------------------------------------
## 1  3.906     3.710          4.101
## 2  4.071     3.711          4.431
```

```
## 3  4.245     3.771          4.719
## 4  4.347     3.779          4.915
## 5  4.337     3.685          4.988
## 6  4.281     3.553          5.008
## 7  4.240     3.442          5.038
## 8  4.057     3.192          4.922
## 9  3.900     2.972          4.829
## 10 3.785     2.795          4.775
## 11 3.817     2.768          4.866
## 12 3.813     2.707          4.920
## 13 3.929     2.770          5.087
## 14 4.069     2.862          5.276
## 15 4.215     2.960          5.470
## 16 4.300     2.997          5.602
## 17 4.283     2.935          5.632
## 18 4.227     2.832          5.622
## 19 4.177     2.736          5.617
## 20 4.041     2.556          5.526
## 21 3.915     2.386          5.445
## 22 3.841     2.268          5.415
## 23 3.890     2.273          5.507
## 24 3.907     2.247          5.567
## 25 4.056     2.356          5.757
## 26 4.199     2.460          5.938
## 27 4.326     2.549          6.104
## 28 4.353     2.537          6.169
## 29 4.325     2.471          6.179
## 30 4.282     2.390          6.174
## 31 4.231     2.301          6.160
## 32 4.113     2.146          6.080
## 33 4.026     2.021          6.030
## 34 3.956     1.914          5.998
## 35 4.021     1.942          6.100
## 36 4.073     1.957          6.189
## 37 4.223     2.063          6.382
## 38 4.354     2.147          6.561
## 39 4.472     2.218          6.726
## 40 4.497     2.196          6.797
## 41 4.471     2.124          6.818
## 42 4.431     2.038          6.824
## 43 4.384     1.945          6.822
## 44 4.275     1.791          6.760
## 45 4.194     1.664          6.724
## 46 4.130     1.555          6.705
## 47 4.190     1.570          6.810
## 48 4.238     1.573          6.902
## --------------------------------------
```

We can now plot our 2012 to 2016 point estimates with the proper 95% confidence interval.

```
# 2012-2016 steps ahead sample forecast
par(mfrow = c(1, 1))
plot(price.fit.ahead.fcast, lty = 2, col = "navy", main = "Out-of-Sample Forecast",
    ylab = "Original, Estimated, and Forecast Values", ylim = c(1,
```

```
        7))
par(new = T)
plot.ts(price.ts, axes = F, lty = 1, ylim = c(1, 7), xlim = c(1978,
    2016), ylab = "")
leg.txt <- c("Original Series", "Forecast series")
legend("top", legend = leg.txt, lty = 1, col = c("black", "blue"),
    bty = "n", cex = 1)
```