## Homework 3

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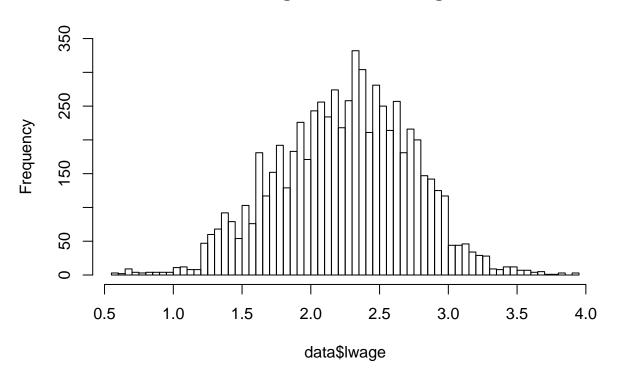
```
# Load the dataframe
load("twoyear.RData")
desc
```

```
##
      variable
                                           label
                                   =1 if female
## 1
        female
## 2
       phsrank
                % high school rank; 100 = best
## 3
                        =1 if Bachelor's degree
            BA
## 4
            AA
                      =1 if Associate's degree
## 5
                         =1 if African-American
         black
## 6
      hispanic
                                 =1 if Hispanic
                                      ID Number
## 7
## 8
                total (actual) work experience
         exper
## 9
            jс
                           total 2-year credits
## 10
                           total 4-year credits
          univ
## 11
         lwage
                                log hourly wage
## 12
        stotal
                 total standardized test score
## 13
        smcity
                         =1 if small city, 1972
## 14
       medcity
                          =1 if med. city, 1972
## 15
        submed
                  =1 if suburb med. city, 1972
                         =1 if large city, 1972
## 16
        lgcity
## 17
         sublg
                 =1 if suburb large city, 1972
## 18
       vlgcity
                   =1 if very large city, 1972
## 19
        subvlg =1 if sub. very lge. city, 1972
## 20
                                =1 if northeast
            ne
## 21
                            =1 if north central
            nc
## 22
                                    =1 if south
         south
## 23
       totcoll
                                      jc + univ
```

## Question 1

```
summary(data$lwage)
      Min. 1st Qu.
                    Median
                              Mean 3rd Qu.
                                               Max.
   0.5555 1.9250
                   2.2760
                            2.2480 2.5970
                                            3.9120
print(quantile(data$lwage, probs = c(0.01, 0.05, 0.1,
   0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
         1%
                  5%
                          10%
                                    25%
                                             50%
                                                      75%
                                                                90%
                                                                         95%
## 1.148702 1.398129 1.609438 1.925291 2.276300 2.596916 2.851921 2.995732
        99%
## 3.325316 3.911953
```

# Histogram of data\$Iwage



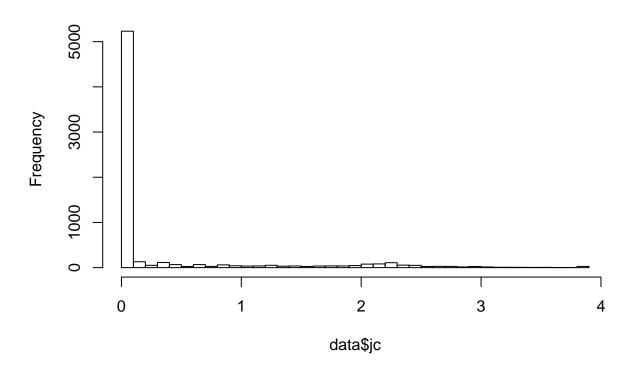
#### summary(data\$jc)

```
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 0.0000 0.0000 0.0000 0.3389 0.0000 3.8330
```

```
## 1% 5% 10% 25% 50% 75% 90% 95% ## 0.000000 0.000000 0.000000 0.000000 1.766667 2.266667 ## 99% 100% ## 3.089665 3.833333
```

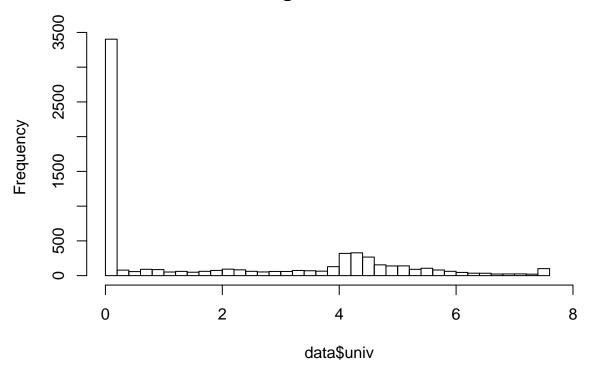
hist(data\$jc, 50)

# Histogram of data\$jc



```
summary(data$univ)
##
     Min. 1st Qu.
                Median
                        Mean 3rd Qu.
                                      Max.
    0.000
          0.000
                 0.200
                        1.926
                              4.200
                                     7.500
print(quantile(data$univ, probs = c(0.01, 0.05, 0.1,
   0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
                                        50%
##
                        10%
                                25%
                                                 75%
                                                         90%
        1%
                5%
95%
               99%
                       100%
## 5.9099934 7.5000000 7.5000000
hist(data\$univ, 50, xlim = c(0, 8))
```

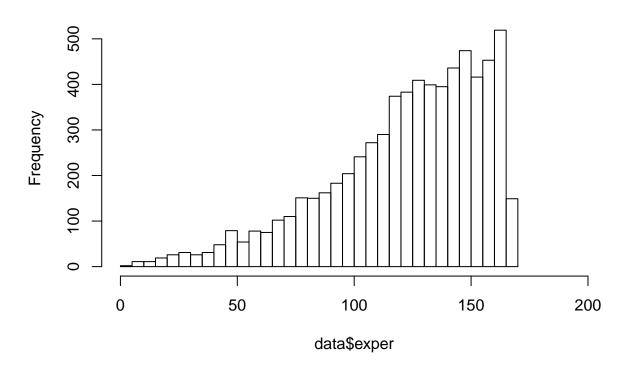
# Histogram of data\$univ



```
summary(data$exper)
##
      Min. 1st Qu.
                    Median
                              Mean 3rd Qu.
                                              Max.
##
             104.0
                     129.0
                             122.4
                                     149.0
                                             166.0
print(quantile(data$exper, probs = c(0.01, 0.05, 0.1,
    0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
     1%
##
              10%
                        50%
                             75%
                                  90%
                                       95%
                                            99% 100%
##
     25
          56
               74
                  104 129
                             149
                                  160
                                       163
                                            166 166
```

hist(data\$exper, 50, xlim = c(0, 200))

# Histogram of data\$exper



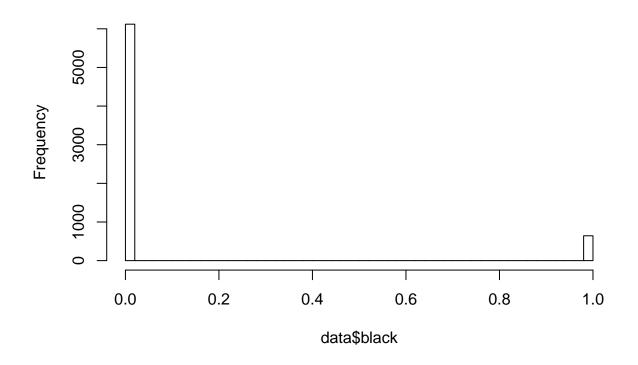
### summary(data\$black)

```
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 0.00000 0.00000 0.00000 0.09508 0.00000 1.00000
```

```
## 1% 5% 10% 25% 50% 75% 90% 95% 99% 100% ## 0 0 0 0 0 0 1 1 1
```

hist(data\$black, 50)

# Histogram of data\$black



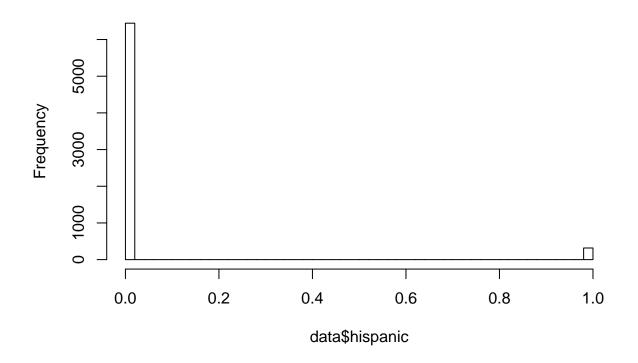
### summary(data\$hispanic)

```
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 0.00000 0.00000 0.00000 0.04687 0.00000 1.00000
```

```
## 1% 5% 10% 25% 50% 75% 90% 95% 99% 100% ## 0 0 0 0 0 0 0 0 1 1
```

hist(data\$hispanic, 50)

# Histogram of data\$hispanic



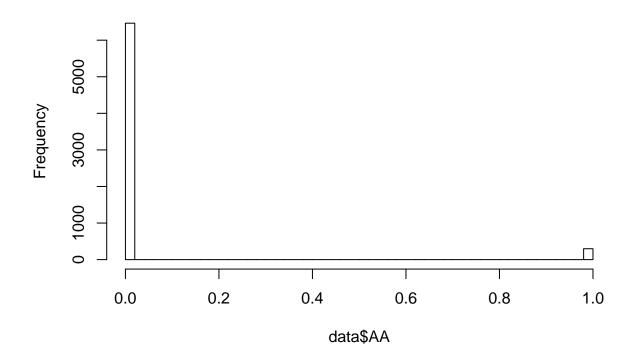
### summary(data\$AA)

```
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 0.00000 0.00000 0.00000 0.04406 0.00000 1.00000
```

```
## 1% 5% 10% 25% 50% 75% 90% 95% 99% 100% ## 0 0 0 0 0 0 0 0 1 1
```

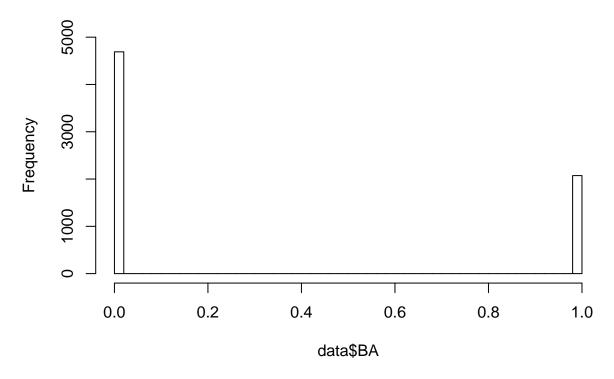
hist(data\$AA, 50)

# Histogram of data\$AA



```
summary(data$BA)
##
     Min. 1st Qu. Median
                             Mean 3rd Qu.
                                            Max.
  0.0000 0.0000 0.0000 0.3065 1.0000 1.0000
print(quantile(data$BA, probs = c(0.01, 0.05, 0.1,
   0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
##
     1%
            10% 25% 50% 75% 90%
                                     95%
                                          99% 100%
                         0
                              1
                                   1
##
                    0
hist(data\$BA, 50, ylim = c(0, 5000))
```

## Histogram of data\$BA



#### Basic structure of the data

There are no missing values in the data.

lwage variable has a normal-like distribution.

 $\mathbf{jc}$  variable has values from 0 to about 4 and is heavily positively skewed with a majority of values at or near 0

**univ** variable has values from 0 to 7.5 and is heavily positively skewed with a majority of values at or near 0. **exper** variable has values from 0 to 166 and is negatively skewed with a hill-climb distribution from 0 to about 500.

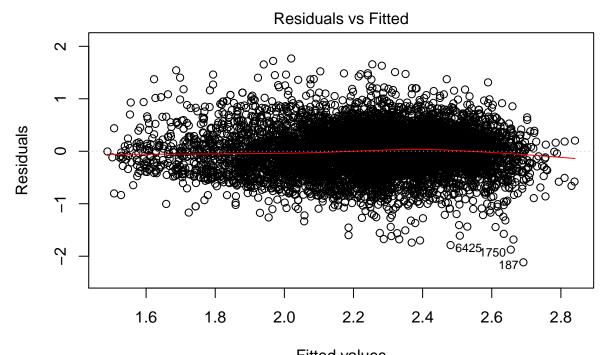
black, hispanic, AA, BA variables are binary with values of 0 or 1.

## Question 2

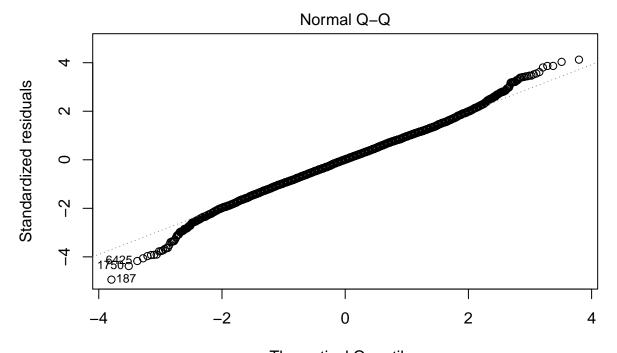
```
# Create the experXblack variable by multiplying
# the exper and black variables.
data$experXblack = data$exper * data$black

# Run the requested OLS regression.
ols.lwage.8ind = lm(lwage ~ jc + univ + exper + black +
    hispanic + AA + BA + experXblack, data = data)
summary(ols.lwage.8ind)
```

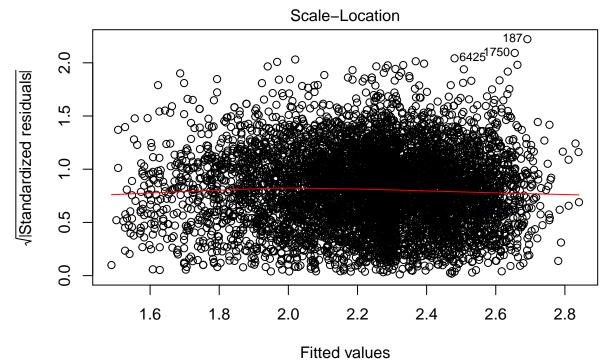
```
##
## Call:
## lm(formula = lwage ~ jc + univ + exper + black + hispanic + AA +
      BA + experXblack, data = data)
## Residuals:
                1Q
                   Median
                                30
## -2.11612 -0.27836 0.00432 0.28676 1.76811
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.4773315 0.0223780 66.017 < 2e-16 ***
             0.0637926 0.0079034 8.072 8.15e-16 ***
## jc
## univ
             ## exper
             0.0050234 0.0001667 30.141 < 2e-16 ***
## black
             0.0331709 0.0613984
                                  0.540
                                          0.5890
## hispanic
             -0.0193629 0.0248914 -0.778
                                         0.4367
## AA
             -0.0077759 0.0295497 -0.263
                                         0.7924
## BA
             0.0176735 0.0156553
                                  1.129
                                          0.2590
## experXblack -0.0012679 0.0004991 -2.541
                                         0.0111 *
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4287 on 6754 degrees of freedom
## Multiple R-squared: 0.2282, Adjusted R-squared: 0.2272
## F-statistic: 249.6 on 8 and 6754 DF, p-value: < 2.2e-16
# Print the diagnostic plots
plot(ols.lwage.8ind)
```



Fitted values
Im(Iwage ~ jc + univ + exper + black + hispanic + AA + BA + experXblack)

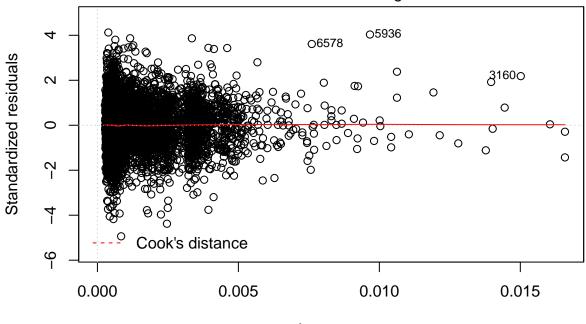


Theoretical Quantiles
Im(Iwage ~ jc + univ + exper + black + hispanic + AA + BA + experXblack)



Im(Iwage ~ jc + univ + exper + black + hispanic + AA + BA + experXblack)

### Residuals vs Leverage



Leverage Im(Iwage ~ jc + univ + exper + black + hispanic + AA + BA + experXblack)

```
# Print the B_hat4 and B_hat8 coefficients
print(ols.lwage.8ind$coefficients["black"])
```

```
## black
## 0.03317088
```

```
print(ols.lwage.8ind$coefficients["experXblack"])
```

```
## experXblack
## -0.001267898
```

## Interpret the coefficients $\hat{\beta}4$ and $\hat{\beta}8$

 $\hat{\beta}4$  is the estimate for the black variable coefficient.  $\hat{\beta}8$  is the estimate for the experXblack variable.

The  $\beta_4$  coefficient captures the effect of being black on the log of wage, holding all other variables in the model fix and assuming zero-employment experience. It provides an indication of how much percentage point change to expect in the wage of an individual when they move from the baseline (white) to being black. Hence the coefficient captures the difference at the intercept of the log(wage) vs experience plot between black and white respondants.

The  $\beta_8$  coefficient captures the effect of being black on the impact of experience over the years on wage. It is better explained in terms of derivatives with:

$$\frac{\delta log(wage)}{\delta exper} = \beta_3 + \beta_8 * black$$

In the previous formulation, we can see that the  $\beta_8$  coefficient captures the impact of being back on the slope of the log(wage) vs experience curve. In other words, the coefficient describes how much more or less experience impacts the log of wage for back people over the years, vs the baseline (while people). And because the outcome variable is the log of wage, the impact above can actually be formulated in terms of impact of ethnicity on percentage changes on the actual wage of individuals.

### Question 3

univ

## 0.07328063

```
# Show the summary of the model again
summary(ols.lwage.8ind)
##
## Call:
## lm(formula = lwage ~ jc + univ + exper + black + hispanic + AA +
       BA + experXblack, data = data)
##
## Residuals:
##
        Min
                       Median
                  1Q
                                    3Q
                                            Max
  -2.11612 -0.27836 0.00432 0.28676
                                        1.76811
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 1.4773315
                           0.0223780
                                      66.017 < 2e-16 ***
                                       8.072 8.15e-16 ***
## jc
                0.0637926
                           0.0079034
## univ
                0.0732806
                           0.0031486
                                      23.274
                                              < 2e-16 ***
## exper
                0.0050234
                           0.0001667
                                      30.141
                                              < 2e-16 ***
## black
                0.0331709
                           0.0613984
                                       0.540
                                               0.5890
## hispanic
               -0.0193629
                           0.0248914
                                      -0.778
                                               0.4367
               -0.0077759
                           0.0295497
                                      -0.263
                                               0.7924
## AA
## BA
                0.0176735
                           0.0156553
                                       1.129
                                               0.2590
## experXblack -0.0012679
                           0.0004991
                                      -2.541
                                               0.0111 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.4287 on 6754 degrees of freedom
## Multiple R-squared: 0.2282, Adjusted R-squared: 0.2272
## F-statistic: 249.6 on 8 and 6754 DF, p-value: < 2.2e-16
# Print the univ coefficient
print(ols.lwage.8ind$coefficients["univ"])
```

Test that the return to university education is 7%.

```
Null Hypothesis: H0: \beta 2 = 0.07.
Alternate Hypothesis: H1: \beta 2 \neq 0.07.
```

Using the linear model and summary statistics, we compute the pvalue for the test above as:

```
## univ
## 0.3246284
```

Based on the p-value, the test is not significant at the 0.05% significance level. Therefore, we can't reject the null hypothesis that the return to university education is 7%.

### Question 4

Test that the return to junior college education is equal for black and non-black

```
From the linear model, we derive: \frac{\delta log(wage)}{\delta jc} = \beta_1
```

The model is specified in a way that the return to junior college education, is  $\beta_1$ , and is independent of ethnicity. Therefore without additional computation, we can immediately answer that the return on junior college education is the same for all ethnicities.

### Question 5

Test whether the return to university education is equal to the return to 1 year of working experience.

Original model:

```
lwage = \beta 0 + \beta 1jc + \beta 2univ + \beta 3exper + \beta 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \beta 8experXblack + \epsilon 6AA + \beta 7BA + \delta 8experXblack + \epsilon 6AA + \delta 7BA + \delta 8experXblack + \epsilon 6AA + \delta 7BA + \delta 8experXblack + \epsilon 6AA + \delta 7BA + \delta 8experXblack + \epsilon 6AA + \delta 7BA + \delta 8experXblack + \epsilon 6AA + \delta 7BA + \delta 8experXblack + \delta 8AA + \delta 7BA + \delta 8AA + \delta 7AA +
```

Convert the experience variable from months to years by creating a new variable experYr that divides the original variable exper by 12. Replace the exper variable in the original model with this variable.

```
lwage = \beta 0 + \beta 1jc + \beta 2univ + \beta 3experYr + \beta 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 7BA + \beta 7BA
```

We would like to know if the  $\beta 2$  and  $\beta 3$  coefficients are the same or, equivalently, if their difference is 0. We can define a variable  $\theta$  such that  $\theta = \beta 2 - \beta 3$  and rewrite our model like this:

```
lwage = \beta 0 + \beta 1jc + (\theta + \beta 3)univ + \beta 3experYr + \beta 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \delta 6AA + \beta 7BA + \beta 8experXblack + \delta 6AA + \beta 7BA + \beta 8experXblack + \delta 6AA + \beta 7BA + \delta 6AA + \delta 6AA
```

Rewrite the model to get  $\theta$  by itself as a coefficient:

```
Now our null hypothesis is H0: \theta = 0.
```

Alternate Hypothesis: H1:  $\theta \neq 0$ .

```
##
## Call:
## lm(formula = lwage ~ jc + univ + univ_plus_experYr + black +
##
      hispanic + AA + BA + experXblack, data = data)
##
## Residuals:
##
       Min
                1Q
                    Median
                                 3Q
                                         Max
## -2.11612 -0.27836  0.00432  0.28676  1.76811
## Coefficients:
##
                     Estimate Std. Error t value Pr(>|t|)
                    1.4773315 0.0223780 66.017 < 2e-16 ***
## (Intercept)
## jc
                    0.0637926  0.0079034  8.072  8.15e-16 ***
                    ## univ
## univ_plus_experYr 0.0602810 0.0020000 30.141 < 2e-16 ***
## black
                   0.0331709 0.0613984 0.540 0.589038
## hispanic
                  -0.0193629 0.0248914 -0.778 0.436659
## AA
                   -0.0077759 0.0295497 -0.263 0.792446
                   0.0176735 0.0156553 1.129 0.258972
## BA
## experXblack
                   -0.0012679 0.0004991 -2.541 0.011088 *
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.4287 on 6754 degrees of freedom
## Multiple R-squared: 0.2282, Adjusted R-squared: 0.2272
## F-statistic: 249.6 on 8 and 6754 DF, p-value: < 2.2e-16
```

Based on the very low p-value (0.000276) for  $\theta$ , the test is significant at the 0.05% significance level. And even though the value of  $\theta$  is close to 0 at 0.0129997, we can reject the null hypothesis that  $\theta = 0$ .

### Question 6

```
# Show the summary of the model again
summary(ols.lwage.8ind)
##
## Call:
```

## lm(formula = lwage ~ jc + univ + exper + black + hispanic + AA +

```
##
       BA + experXblack, data = data)
##
## Residuals:
##
                                    3Q
       Min
                  1Q
                       Median
                                             Max
##
   -2.11612 -0.27836
                      0.00432
                               0.28676
                                        1.76811
##
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                1.4773315
                           0.0223780
                                      66.017
                                               < 2e-16 ***
## jc
                0.0637926
                           0.0079034
                                       8.072 8.15e-16 ***
## univ
                0.0732806
                           0.0031486
                                       23.274
                                               < 2e-16 ***
                                               < 2e-16 ***
## exper
                0.0050234
                           0.0001667
                                       30.141
                0.0331709
                           0.0613984
                                       0.540
                                                0.5890
## black
                           0.0248914
                                      -0.778
                                                0.4367
## hispanic
               -0.0193629
                                                0.7924
## AA
               -0.0077759
                           0.0295497
                                       -0.263
## BA
                0.0176735
                           0.0156553
                                       1.129
                                                0.2590
## experXblack -0.0012679
                           0.0004991
                                      -2.541
                                                0.0111 *
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4287 on 6754 degrees of freedom
## Multiple R-squared: 0.2282, Adjusted R-squared: 0.2272
## F-statistic: 249.6 on 8 and 6754 DF, p-value: < 2.2e-16
```

#### Test the overall significance of this regression.

We are testing the overall significance of the original model as stated below:

```
lwage = \beta 0 + \beta 1jc + \beta 2univ + \beta 3exper + \beta 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 6AA + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 7BA + \beta 8experXblack + \epsilon 4black + \beta 7BA + \beta 7B
```

Here is the output from the summary of the original model. Residual standard error: 0.4287 on 6754 degrees of freedom Multiple R-squared: 0.2282, Adjusted R-squared: 0.2272 F-statistic: 249.6 on 8 and 6754 DF, p-value: < 2.2e-16

- 1. Our model null hypothesis is that there is no relationship among any of the independent variables and lwage variable. We are able to reject the null hypothesis since our p-value of the f-statistic of the model is significant at < 2.2e-16.
- 2. Practical significance: we have an R-squared value of 0.2282, indicating that 22.82% of the variation in lwage is explained by our model.

## Question 7

```
# Define a square term for the exper variable
data$experXexper = data$exper * data$exper
# Add the new variable to the regression
ols.lwage.9ind = lm(lwage ~ jc + univ + exper + black +
    hispanic + AA + BA + experXblack + experXexper,
```

```
data = data)
# Show the summary of the model
summary(ols.lwage.9ind)
##
## Call:
## lm(formula = lwage ~ jc + univ + exper + black + hispanic + AA +
##
      BA + experXblack + experXexper, data = data)
##
## Residuals:
##
       Min
                 1Q
                      Median
                                   3Q
## -2.11982 -0.27743 0.00475 0.28741 1.77397
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.510e+00 4.427e-02 34.108 < 2e-16 ***
## jc
               6.417e-02 7.916e-03
                                     8.106 6.14e-16 ***
               7.382e-02 3.211e-03 22.992 < 2e-16 ***
## univ
## exper
               4.301e-03 8.588e-04
                                     5.008 5.64e-07 ***
## black
               2.994e-02 6.152e-02 0.487
                                             0.6265
              -1.932e-02 2.489e-02 -0.776
                                             0.4378
## hispanic
              -7.539e-03 2.955e-02 -0.255
                                             0.7986
## AA
               1.797e-02 1.566e-02
                                             0.2513
## BA
                                     1.147
## experXblack -1.239e-03 5.002e-04 -2.477
                                             0.0133 *
## experXexper 3.379e-06 3.939e-06
                                     0.858
                                             0.3911
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.4287 on 6753 degrees of freedom
## Multiple R-squared: 0.2282, Adjusted R-squared: 0.2272
## F-statistic: 221.9 on 9 and 6753 DF, p-value: < 2.2e-16
```

#### Estimated return to work experience in this model

 $lwage = \beta 0 + \beta 1jc + \beta 2univ + \beta 3exper + \beta 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 9experXexper$ 

We obtain the return on a year of experience by evaluating:

$$\frac{\delta lwage}{exper} = \beta 3 + \beta 8black + 2 * \beta 9exper$$

 $\beta 3 = 4.301e - 03 = .004301$ ,  $\beta 8 = .001239$ ,  $\beta 9 = 000003379$  Substituting these values in the equation above we get:

$$\frac{\delta lwage}{exper} = (.004301 - .001239*black + .000006758*exper)$$

Thus for blacks, the return to one year of working experience is:

```
.004301 - .001239 + .000006758
## [1] 0.003068758
```

We interpret is as  $3/10\mathrm{th}$  of a percent of increase in wage per year of experience.

And for whites, that return is:

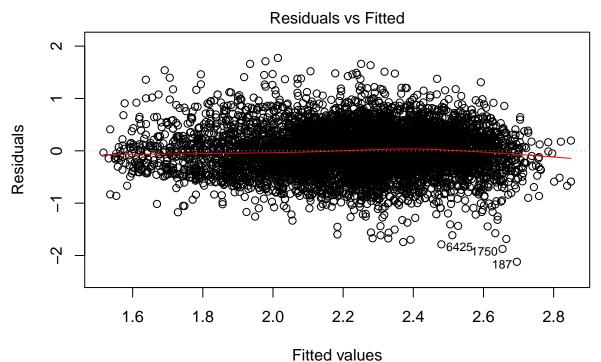
```
.004301 + .000006758
```

## [1] 0.004307758

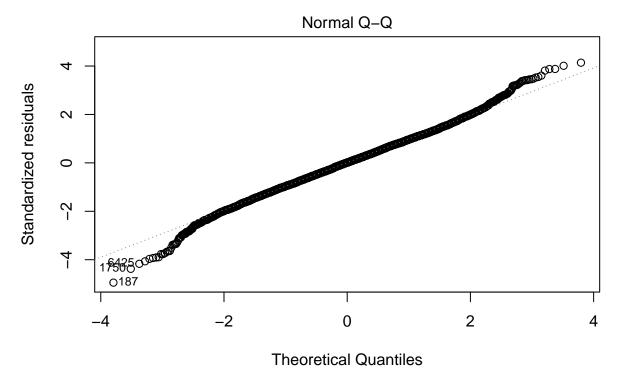
We interpret is as 4.3/10th of a percent of increase in wage per year of experience.

# Question 8

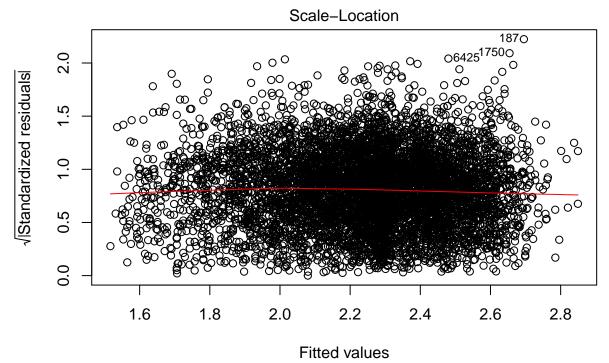
# Plot the graphs from the model
plot(ols.lwage.9ind)



Im(lwage ~ jc + univ + exper + black + hispanic + AA + BA + experXblack + e ...

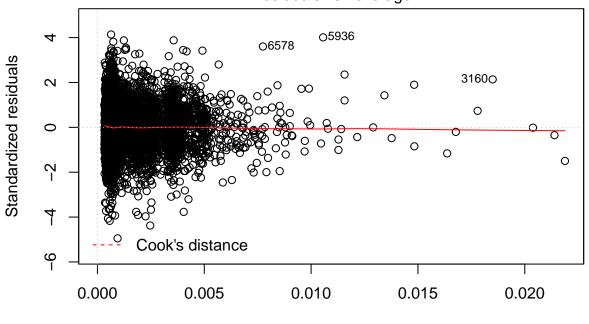


Im(Iwage ~ jc + univ + exper + black + hispanic + AA + BA + experXblack + e ...



Im(Iwage ~ jc + univ + exper + black + hispanic + AA + BA + experXblack + e ...

#### Residuals vs Leverage



Leverage Im(Iwage ~ jc + univ + exper + black + hispanic + AA + BA + experXblack + e ...

```
# Show the summary of the model
summary(ols.lwage.9ind)
```

```
##
## Call:
  lm(formula = lwage ~ jc + univ + exper + black + hispanic + AA +
       BA + experXblack + experXexper, data = data)
##
##
## Residuals:
       Min
                  1Q
                       Median
                                    3Q
                                             Max
   -2.11982 -0.27743
                      0.00475
                               0.28741
                                        1.77397
##
##
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
                           4.427e-02
                                      34.108 < 2e-16 ***
## (Intercept) 1.510e+00
## jc
                6.417e-02
                           7.916e-03
                                       8.106 6.14e-16 ***
## univ
                7.382e-02
                           3.211e-03
                                      22.992
                                              < 2e-16 ***
                4.301e-03
                           8.588e-04
                                       5.008 5.64e-07 ***
## exper
## black
                2.994e-02
                           6.152e-02
                                       0.487
                                                0.6265
               -1.932e-02
                           2.489e-02
                                      -0.776
                                               0.4378
## hispanic
## AA
               -7.539e-03
                           2.955e-02
                                      -0.255
                                               0.7986
                                               0.2513
## BA
                1.797e-02
                           1.566e-02
                                       1.147
## experXblack -1.239e-03
                           5.002e-04
                                      -2.477
                                                0.0133 *
## experXexper 3.379e-06
                          3.939e-06
                                       0.858
                                               0.3911
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 0.4287 on 6753 degrees of freedom
## Multiple R-squared: 0.2282, Adjusted R-squared: 0.2272
## F-statistic: 221.9 on 9 and 6753 DF, p-value: < 2.2e-16
# Use the robust standard errors
coeftest(ols.lwage.9ind, vcov = vcovHC)
##
## t test of coefficients:
##
                  Estimate Std. Error t value Pr(>|t|)
##
               1.5101e+00 4.3591e-02 34.6427 < 2.2e-16 ***
## (Intercept)
                6.4168e-02 7.6224e-03 8.4183 < 2.2e-16 ***
## jc
## univ
                7.3819e-02 3.4501e-03 21.3963 < 2.2e-16 ***
## exper
                           8.4541e-04 5.0873 3.731e-07 ***
                4.3008e-03
## black
                2.9937e-02
                           6.8436e-02 0.4374
                                                 0.66180
## hispanic
               -1.9317e-02 2.4985e-02 -0.7731
                                                 0.43947
               -7.5392e-03
                           2.7481e-02 -0.2743
                                                 0.78383
## AA
## BA
                1.7967e-02
                           1.6579e-02
                                       1.0837
                                                 0.27853
## experXblack -1.2388e-03 5.3539e-04 -2.3139
                                                 0.02071 *
## experXexper 3.3790e-06 3.8745e-06 0.8721
                                                 0.38318
## ---
```

#### Homoskedasticity analysis:

We are testing homoskedasticity of the model with the quadratic experience term, expressed as:

## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.05 '.' 0.1 ' ' 1

 $lwaqe = \beta 0 + \beta 1jc + \beta 2univ + \beta 3exper + \beta 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 9experXexper + \epsilon 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 9experXexper + \epsilon 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 9experXexper + \epsilon 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 9experXexper + \epsilon 4black + \beta 5hispanic + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7BA + \beta 8experXblack + \beta 6AA + \beta 7AA + \beta 8experXblack + \beta 6AA + \beta 7AA + \beta 8experXblack + \beta 7AA + \beta$ 

We observe a small amount of heteroskedasticity from the plot:

- 1 We can see from the residuals vs fitted plot that the variance band changes very slightly as we move to higher fitted values.
- 2 The same story is told by the scale-location plot where we see that the smoothing line is not quite completely horizontal. Therefore we conclude a very small amount of heteroskedasticity and decide to use heteroskedasticity-robust methods for coefficient estimation.
- 3 We do not look at the Breusch Pagan test since we have a large number of observations, therefore we know almost certainly that we will obtain significance.

The implication of heteroskedastcity (even small) in the data is that the standard error of the univ coefficient ( $\beta$ 2) may be biased. Biased standard errors can impact the outcomes of statistical tests. Therefore, it can affect the testing of no effect of university education on salary, which is the t-test on the coefficient  $\beta$ 2

The  $\beta 2$  coefficient from the robust method was essentially unchanged, going from a value of  $\beta 2 = 7.382e - 02$  using the non-robust estimation to  $\beta 2 = 7.3819e - 02$  with robust estimation. The standard error of the  $\beta 2$  coefficient was changed, going from 3.211e-03 using the non-robust estimation to 3.4501e-03 using the robust estimation. However, even using the robust estimation, the p-value for our  $\beta 2$  coefficient remains significant at the 0.05, and we confirm that we must reject the null hypothesis that the coefficient value is null, with the meaning of that null hypothesis being that there is no relationship between time at university and wages.