

Homework 3

Rohan Thakur, Charles Kekeh and Megan Jasek

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```
# Load the dataframe
load("twoyear.RData")
desc
```

```
##      variable                                label
## 1   female                                =1 if female
## 2   phsrank  % high school rank; 100 = best
## 3     BA                                =1 if Bachelor's degree
## 4     AA                                =1 if Associate's degree
## 5   black                                =1 if African-American
## 6 hispanic                                =1 if Hispanic
## 7     id                                ID Number
## 8   exper  total (actual) work experience
## 9     jc                                total 2-year credits
## 10    univ                                total 4-year credits
## 11   lwage                                log hourly wage
## 12 stotal  total standardized test score
## 13 smcity                                =1 if small city, 1972
## 14 medcity                                =1 if med. city, 1972
## 15 submed  =1 if suburb med. city, 1972
## 16 lgcity                                =1 if large city, 1972
## 17 sublg   =1 if suburb large city, 1972
## 18 vlcity  =1 if very large city, 1972
## 19 subvlg  =1 if sub. very lge. city, 1972
## 20    ne                                =1 if northeast
## 21    nc                                =1 if north central
## 22   south                                =1 if south
## 23 totcoll                                jc + univ
```

Question 1

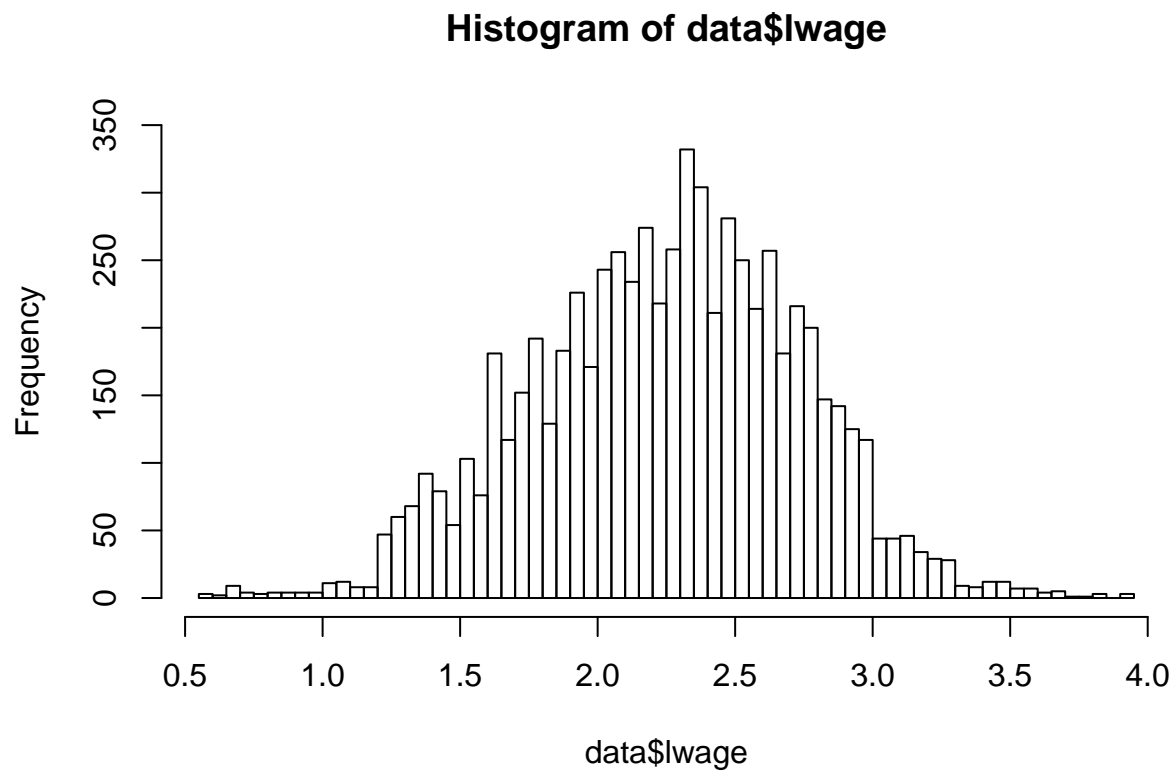
```
summary(data$lwage)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.5555  1.9250  2.2760  2.2480  2.5970  3.9120
```

```
print(quantile(data$lwage, probs = c(0.01, 0.05, 0.1,
  0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##      1%      5%      10%      25%      50%      75%      90%      95%
## 1.148702 1.398129 1.609438 1.925291 2.276300 2.596916 2.851921 2.995732
##      99%     100%
## 3.325316 3.911953
```

```
hist(data$lwage, 50, ylim = c(0, 350))
```



```
summary(data$jc)
```

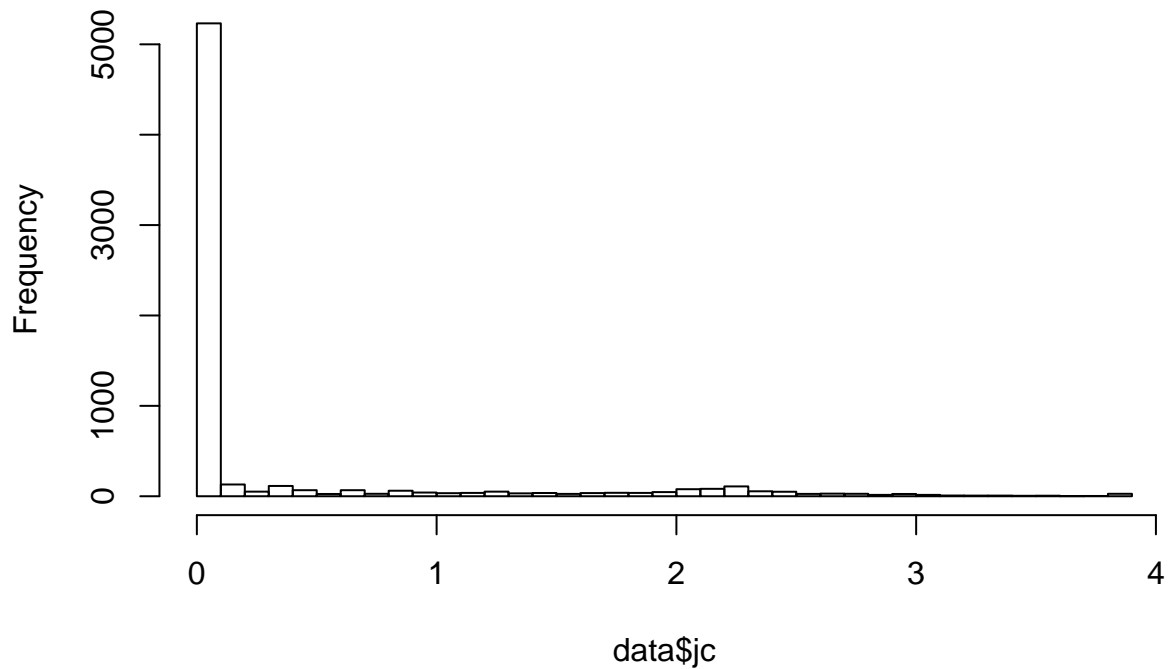
```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.00000 0.00000 0.00000 0.3389 0.00000 3.8330
```

```
print(quantile(data$jc, probs = c(0.01, 0.05, 0.1,
  0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##      1%      5%     10%     25%     50%     75%     90%     95%
## 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.766667 2.266667
##      99%     100%
## 3.089665 3.833333
```

```
hist(data$jc, 50)
```

Histogram of data\$jc



```
summary(data$univ)
```

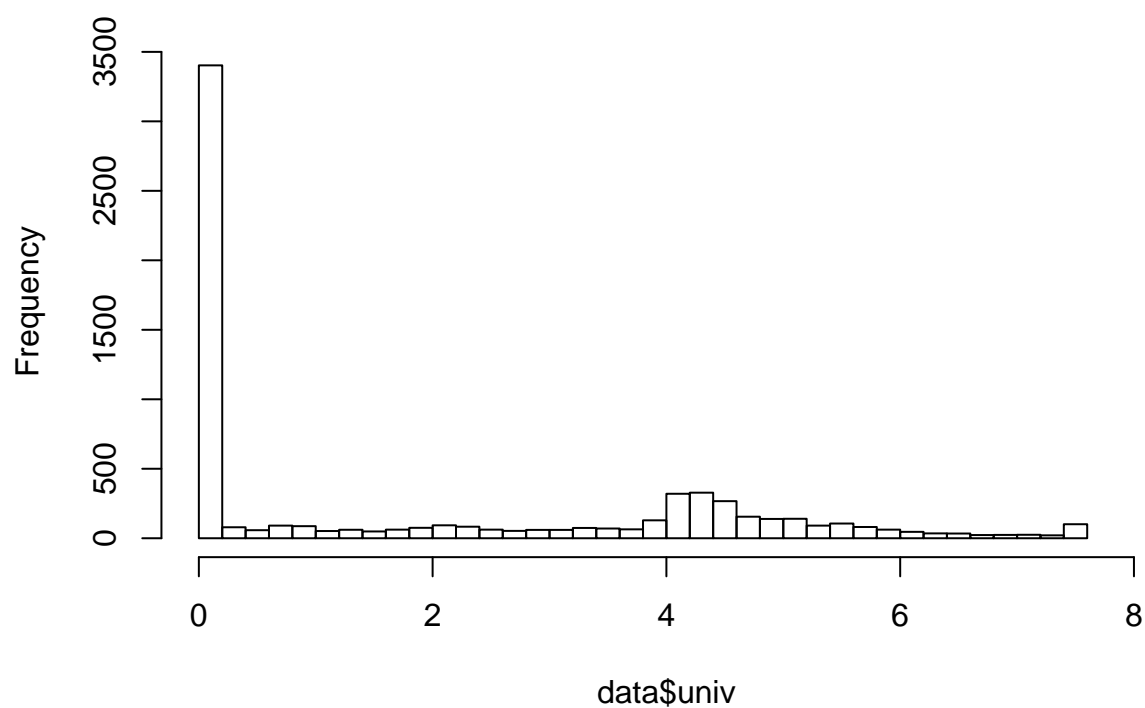
```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.000   0.000   0.200   1.926   4.200   7.500
```

```
print(quantile(data$univ, probs = c(0.01, 0.05, 0.1,
    0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##           1%           5%           10%           25%           50%           75%           90%
## 0.0000000 0.0000000 0.0000000 0.0000000 0.1999997 4.1999998 5.1777687
##           95%           99%          100%
## 5.9099934 7.5000000 7.5000000
```

```
hist(data$univ, 50, xlim = c(0, 8))
```

Histogram of data\$univ



```
summary(data$exper)
```

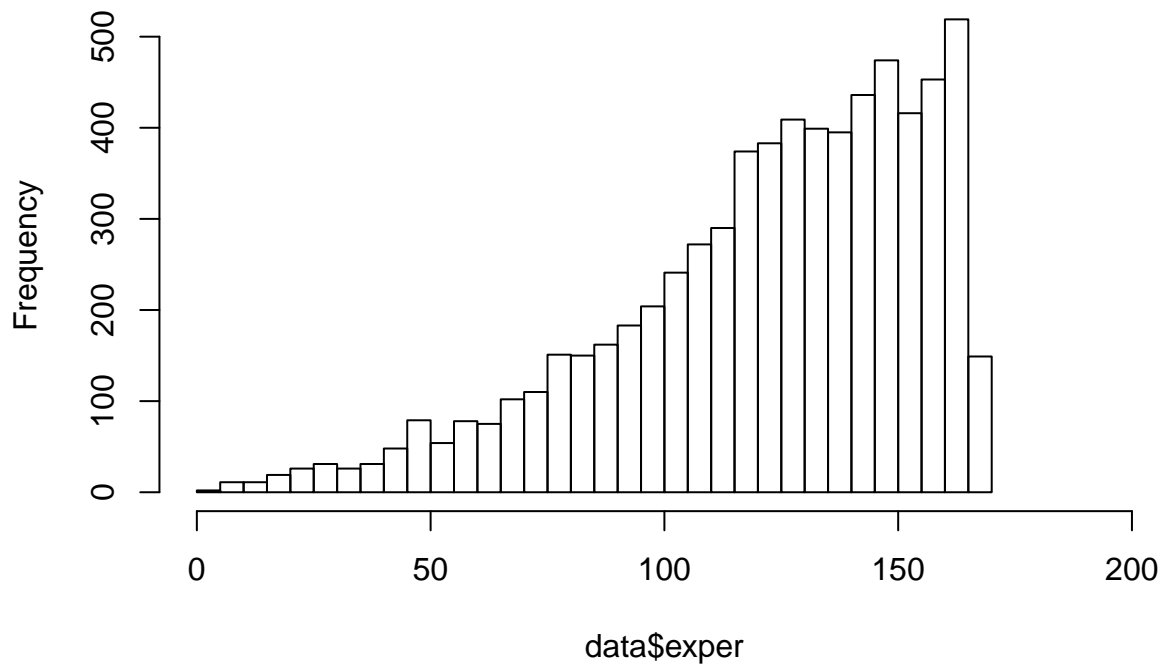
```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##       3.0   104.0   129.0   122.4   149.0   166.0
```

```
print(quantile(data$exper, probs = c(0.01, 0.05, 0.1,
    0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

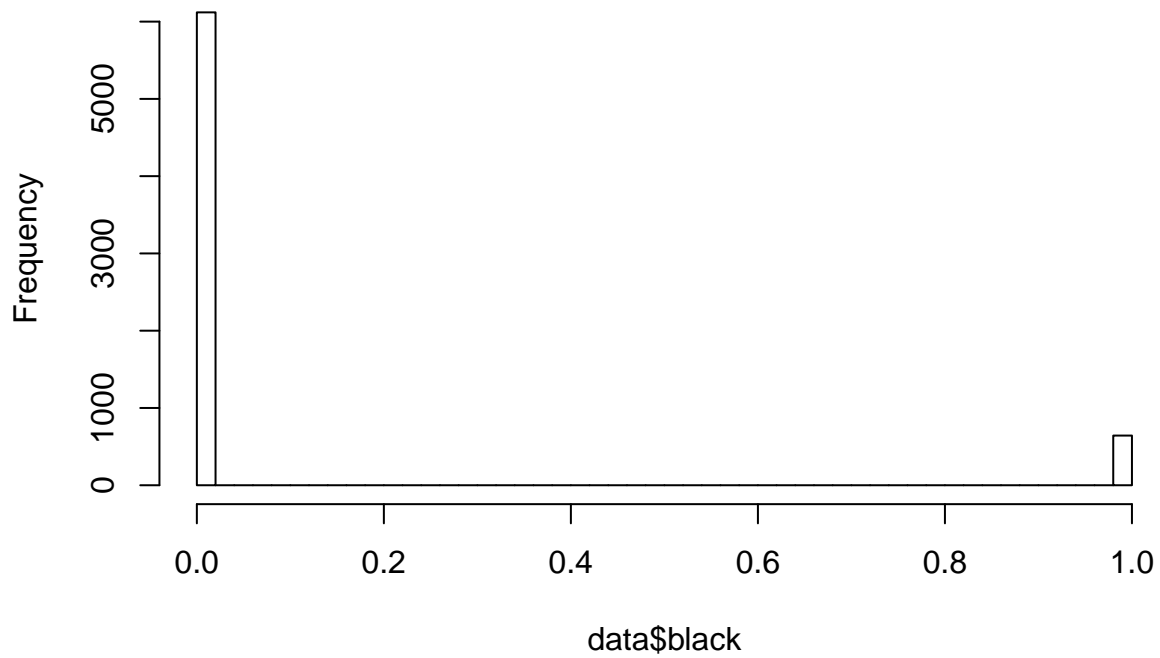
```
##      1%    5%   10%   25%   50%   75%   90%   95%   99%  100%
##      25    56    74   104   129   149   160   163   166   166
```

```
hist(data$exper, 50, xlim = c(0, 200))
```

Histogram of data\$exper



Histogram of data\$black



```
summary(data$hispanic)
```

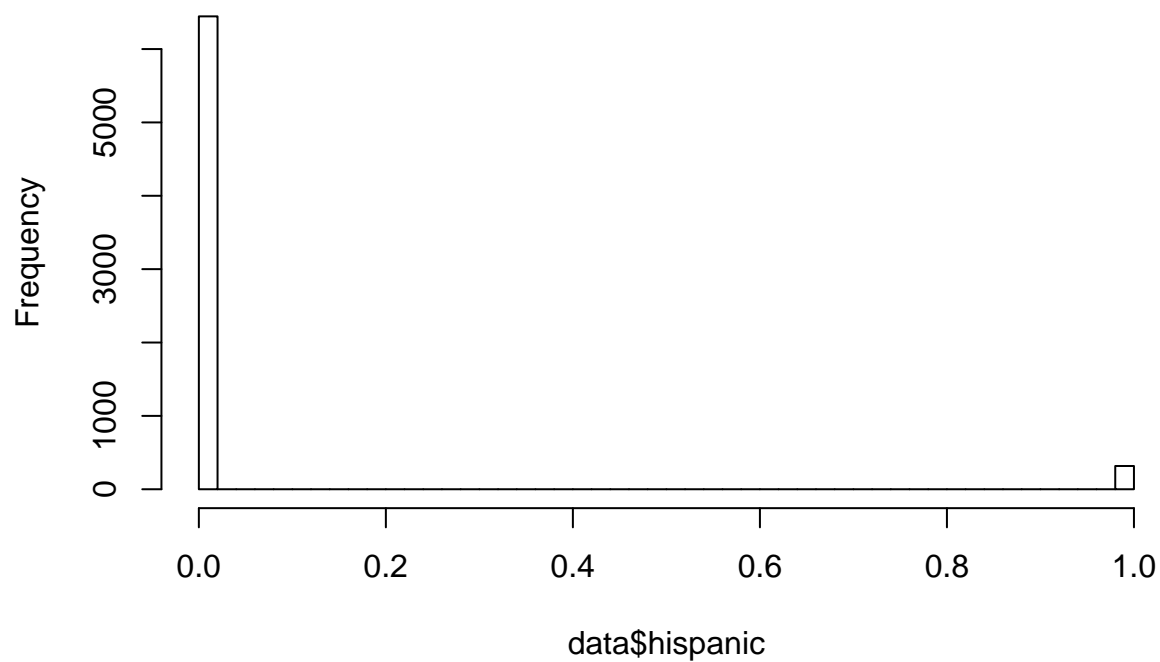
```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.00000 0.00000 0.00000 0.04687 0.00000 1.00000
```

```
print(quantile(data$hispanic, probs = c(0.01, 0.05,
    0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##      1%      5%     10%    25%    50%    75%    90%    95%    99%   100%
##       0       0       0       0       0       0       0       0       1       1
```

```
hist(data$hispanic, 50)
```

Histogram of data\$hispanic



```
summary(data$AA)
```

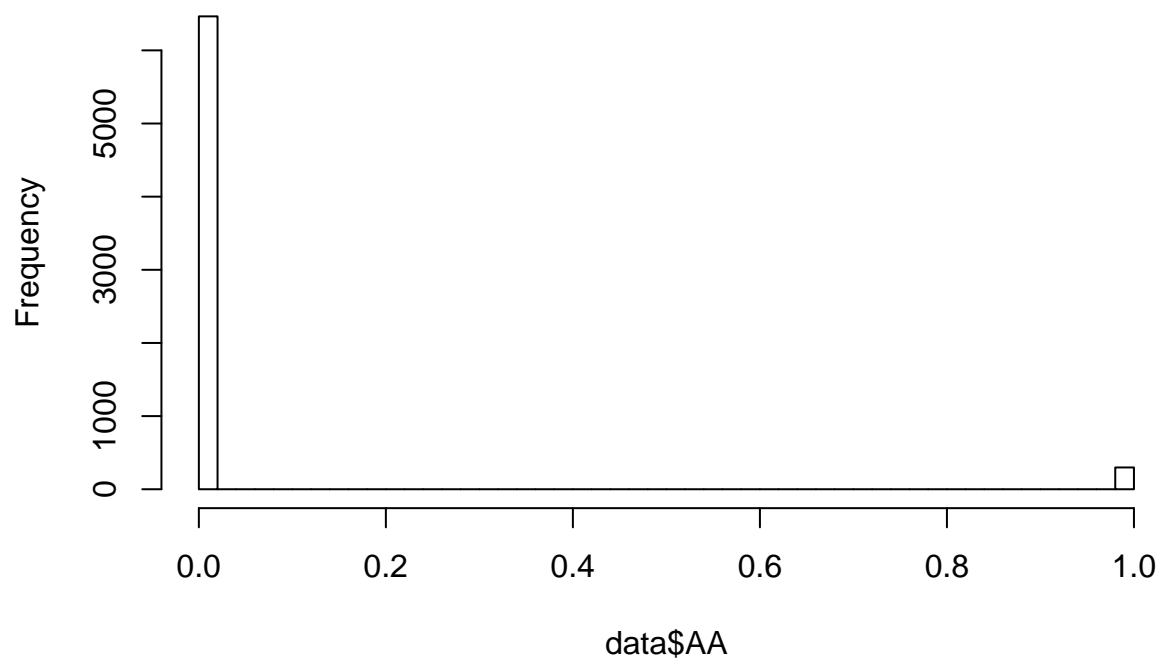
```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.00000 0.00000 0.00000 0.04406 0.00000 1.00000
```

```
print(quantile(data$AA, probs = c(0.01, 0.05, 0.1,
                                   0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##      1%    5%   10%   25%   50%   75%   90%   95%   99%  100%
##       0     0     0     0     0     0     0     0     1     1
```

```
hist(data$AA, 50)
```

Histogram of data\$AA



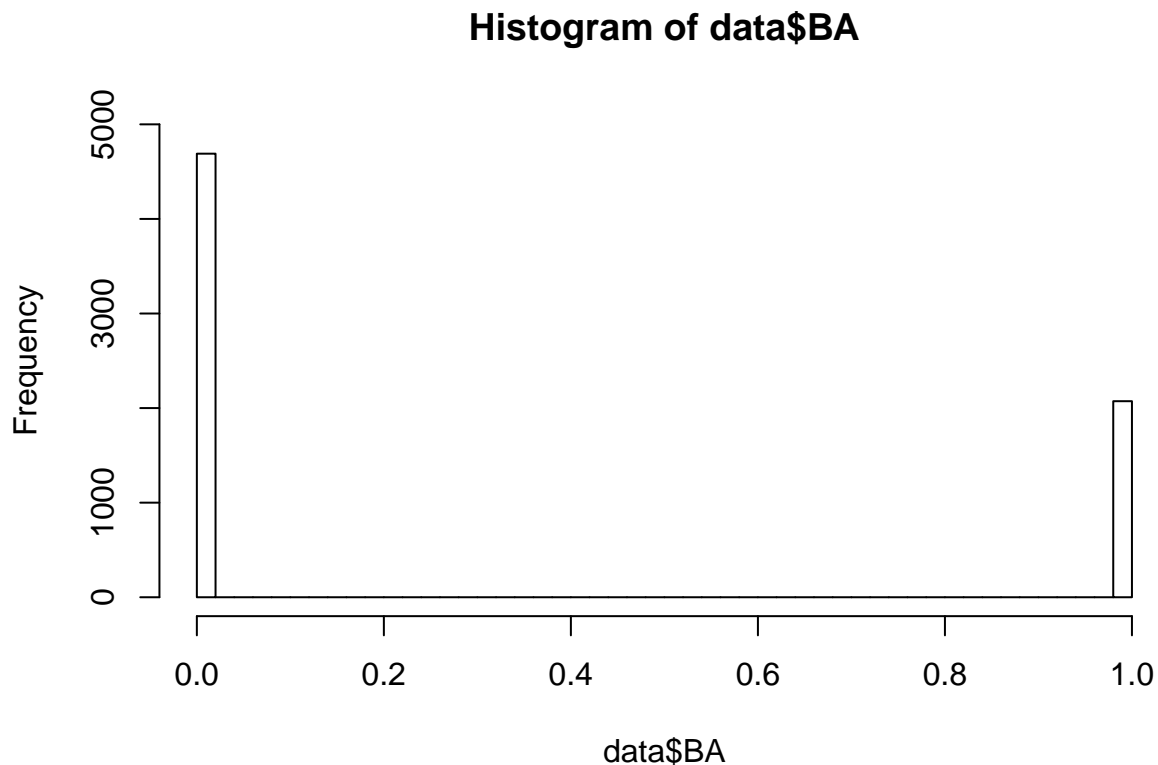
```
summary(data$BA)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.0000 0.0000 0.0000 0.3065 1.0000 1.0000
```

```
print(quantile(data$BA, probs = c(0.01, 0.05, 0.1,
  0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##      1%   5%  10%  25%  50%  75%  90%  95%  99% 100%
##      0    0   0   0   0   1   1   1   1   1
```

```
hist(data$BA, 50, ylim = c(0, 5000))
```

Basic structure of the data

There are no missing values in the data.

lwage variable has a normal-like distribution.

jc variable has values from 0 to about 4 and is heavily positively skewed with a majority of values at or near 0.

univ variable has values from 0 to 7.5 and is heavily positively skewed with a majority of values at or near 0.
exper variable has values from 0 to 166 and is negatively skewed with a hill-climb distribution from 0 to about 500.

black, **hispanic**, **AA**, **BA** variables are binary with values of 0 or 1.

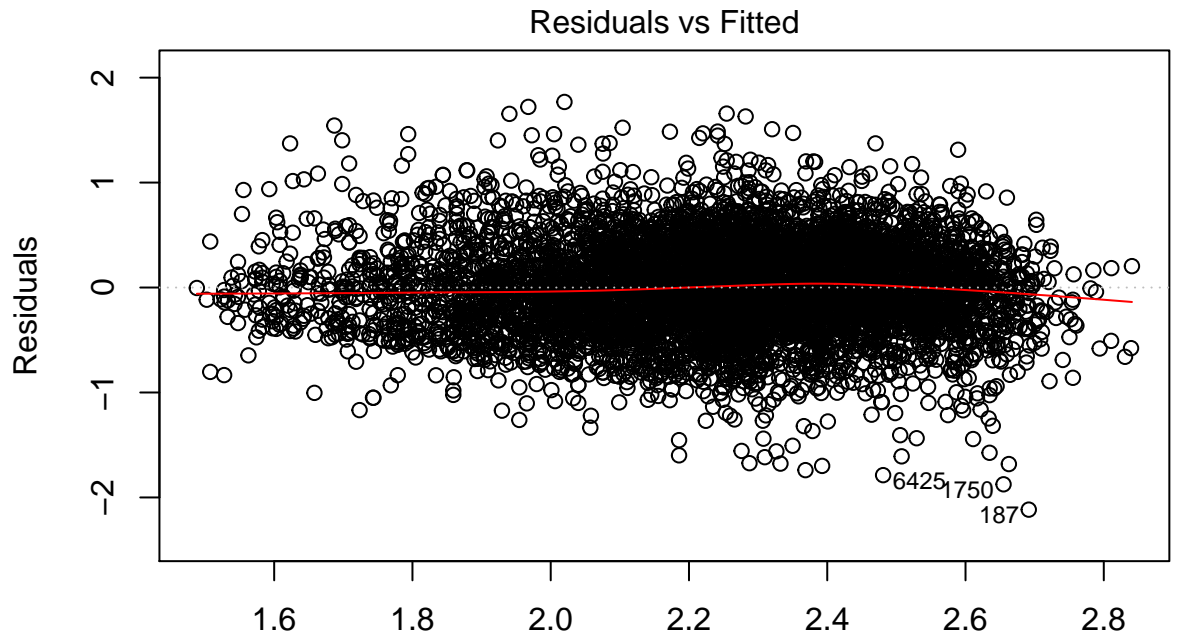
Question 2

```
# Create the experXblack variable by multiplying
# the exper and black variables.
data$experXblack = data$exper * data$black

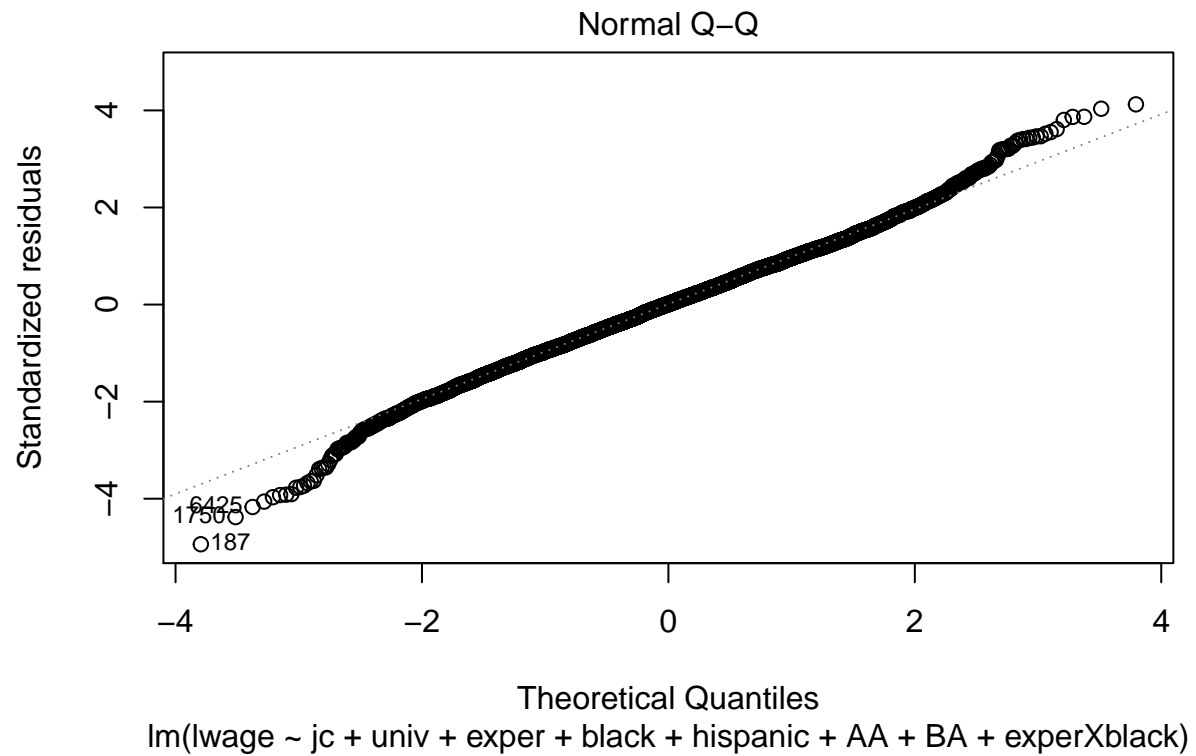
# Run the requested OLS regression.
ols.lwage.8ind = lm(lwage ~ jc + univ + exper + black +
  hispanic + AA + BA + experXblack, data = data)
summary(ols.lwage.8ind)
```

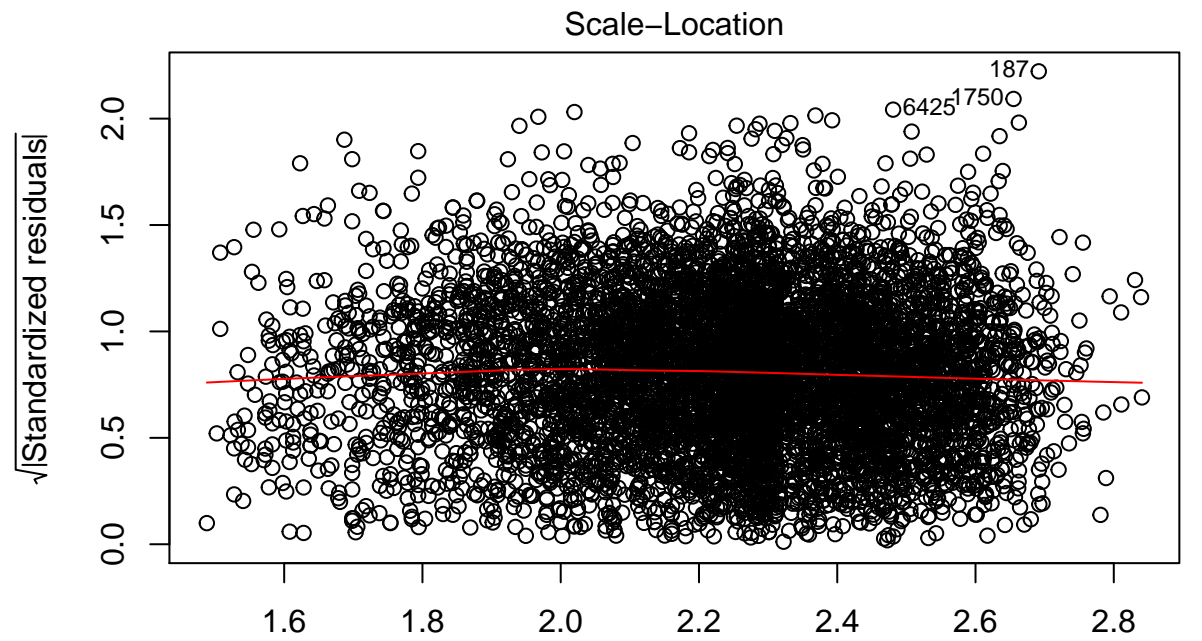
```
##
## Call:
## lm(formula = lwage ~ jc + univ + exper + black + hispanic + AA +
##      BA + experXblack, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.11612 -0.27836  0.00432  0.28676  1.76811
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.4773315  0.0223780  66.017 < 2e-16 ***
## jc           0.0637926  0.0079034   8.072 8.15e-16 ***
## univ         0.0732806  0.0031486  23.274 < 2e-16 ***
## exper        0.0050234  0.0001667  30.141 < 2e-16 ***
## black        0.0331709  0.0613984   0.540  0.5890
## hispanic     -0.0193629  0.0248914  -0.778  0.4367
## AA           -0.0077759  0.0295497  -0.263  0.7924
## BA           0.0176735  0.0156553   1.129  0.2590
## experXblack -0.0012679  0.0004991  -2.541  0.0111 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4287 on 6754 degrees of freedom
## Multiple R-squared:  0.2282, Adjusted R-squared:  0.2272
## F-statistic: 249.6 on 8 and 6754 DF,  p-value: < 2.2e-16

# Print the diagnostic plots
plot(ols.lwage.8ind)
```

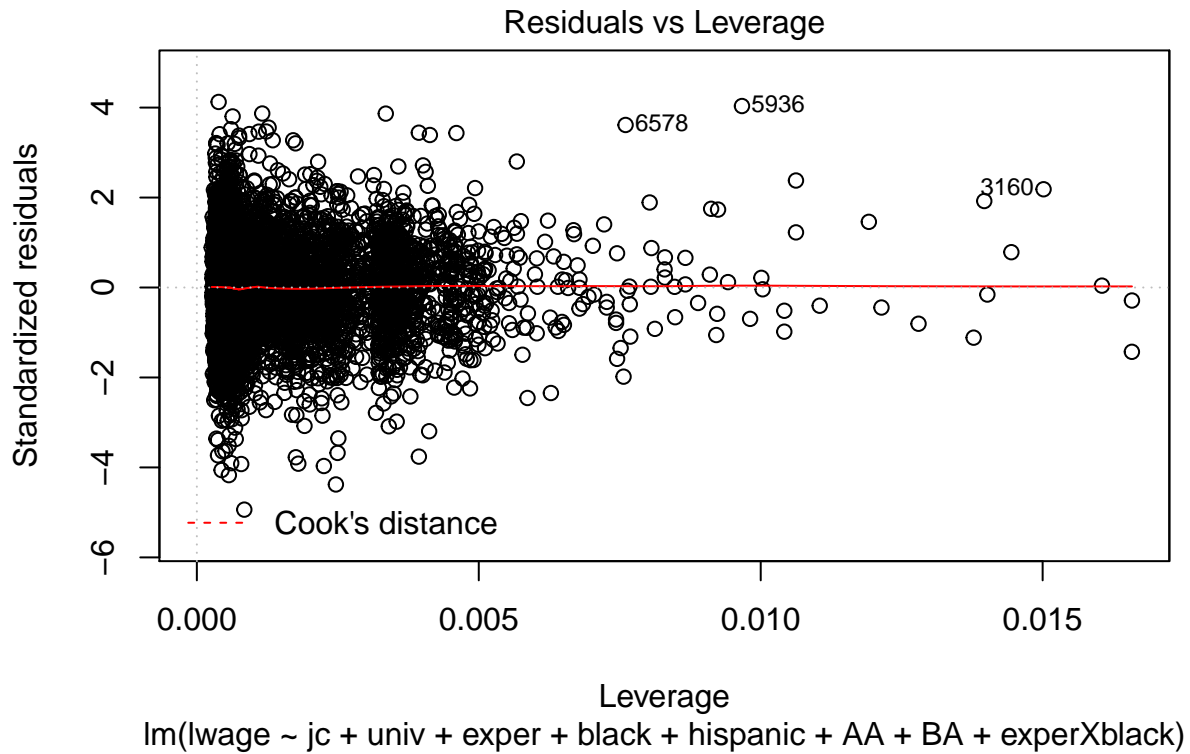


Fitted values
 $\text{lm}(\text{lwage} \sim \text{jc} + \text{univ} + \text{exper} + \text{black} + \text{hispanic} + \text{AA} + \text{BA} + \text{experXblack})$





Fitted values
 $\text{lm}(\text{lwage} \sim \text{jc} + \text{univ} + \text{exper} + \text{black} + \text{hispanic} + \text{AA} + \text{BA} + \text{experXblack})$



```
# Print the B_hat4 and B_hat8 coefficients
print(ols.lwage.8ind$coefficients[5])
```

```
##      black
## 0.03317088
```

```
print(ols.lwage.8ind$coefficients[9])
```

```
##  experXblack
## -0.001267898
```

Interpret the coefficients $\hat{\beta}_4$ and $\hat{\beta}_8$

$\hat{\beta}_4$ is the estimate for the black variable coefficient.

$\hat{\beta}_8$ is the estimate for the experXblack variable.

Do we talk about:

zero-conditional mean seems to be met

homoskedasticity seems to be met

assuming random sample

assuming linear relationship

Question 3

```
# Show the summary of the model again
```

```
summary(ols.lwage.8ind)
```

```
##
## Call:
## lm(formula = lwage ~ jc + univ + exper + black + hispanic + AA +
##     BA + experXblack, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.11612 -0.27836  0.00432  0.28676  1.76811
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.4773315   0.0223780   66.017 < 2e-16 ***
## jc           0.0637926   0.0079034    8.072 8.15e-16 ***
## univ         0.0732806   0.0031486   23.274 < 2e-16 ***
## exper        0.0050234   0.0001667   30.141 < 2e-16 ***
## black        0.0331709   0.0613984    0.540  0.5890
## hispanic     -0.0193629   0.0248914   -0.778  0.4367
## AA           -0.0077759   0.0295497   -0.263  0.7924
## BA           0.0176735   0.0156553    1.129  0.2590
## experXblack -0.0012679   0.0004991   -2.541  0.0111 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4287 on 6754 degrees of freedom
## Multiple R-squared:  0.2282, Adjusted R-squared:  0.2272
## F-statistic: 249.6 on 8 and 6754 DF,  p-value: < 2.2e-16
```

```
# Print the univ coefficient
```

```
print(ols.lwage.8ind$coefficients[3])
```

```
##      univ
## 0.07328063
```

```
(0.0733 - 0.07)/(0.0031)
```

```
## [1] 1.064516
```

```
2 * (1 - 0.8554)
```

```
## [1] 0.2892
```

Test that the return to university education is 7%.

Null Hypothesis: $H_0: \beta_2 = 0.07$.

Alternate Hypothesis: $H_1: \beta_2 \neq 0.07$.

Formula for t-statistic = $(\beta_2 - H_0)/(se) = (.0733 - .07)/(.0031) = 1.064516$

p-value = $2 * (1 - .8554) = 0.2892$

Based on the p-value, the test is not significant at the 0.05% significance level. Therefore, we can't reject the null hypothesis that the return to university education is 7%.

Question 4

Test that the return to junior college education is equal for black and non-black

Question 5

Test whether the return to university education is equal to the return to 1 year of working experience.

Original model:

$$lwage = \beta_0 + \beta_1 jc + \beta_2 univ + \beta_3 exper + \beta_4 black + \beta_5 hispanic + \beta_6 AA + \beta_7 BA + \beta_8 experXblack + \epsilon$$

Convert the experience variable from months to years by creating a new variable experYr that divides the original variable exper by 12. Replace the exper variable in the original model with this variable.

$$lwage = \beta_0 + \beta_1 jc + \beta_2 univ + \beta_3 experYr + \beta_4 black + \beta_5 hispanic + \beta_6 AA + \beta_7 BA + \beta_8 experXblack + \epsilon$$

We would like to know if the β_2 and β_3 coefficients are the same or, equivalently, if their difference is 0. We can define a variable θ such that $\theta = \beta_2 - \beta_3$ and rewrite our model like this:

$$lwage = \beta_0 + \beta_1 jc + (\theta + \beta_3)univ + \beta_3 experYr + \beta_4 black + \beta_5 hispanic + \beta_6 AA + \beta_7 BA + \beta_8 experXblack + \epsilon$$

Rewrite the model to get θ by itself as a coefficient:

$$lwage = \beta_0 + \beta_1 jc + \theta univ + \beta_3(univ + experYr) + \beta_4 black + \beta_5 hispanic + \beta_6 AA + \beta_7 BA + \beta_8 experXblack + \epsilon$$

Now our null hypothesis is $H_0 : \theta = 0$.

Alternate Hypothesis: $H_1: \theta \neq 0$.

```
# Convert the exper variable from months to years
# by dividing it by 12.
data$experYr = data$exper/12
# Create a variable that is the sum of the univ and
# experYr variables
data$univ_plus_experYr = data$univ + data$experYr
# Rerun the regression with the new variables.
ols.lwage.univ.experYr = lm(lwage ~ jc + univ + univ_plus_experYr +
  black + hispanic + AA + BA + experXblack, data = data)
# Display a summary of the new model
summary(ols.lwage.univ.experYr)
```

##

Call:


```
## lm(formula = lwage ~ jc + univ + univ_plus_experYr + black +
##     hispanic + AA + BA + experXblack, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.11612 -0.27836  0.00432  0.28676  1.76811
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   1.4773315   0.0223780   66.017 < 2e-16 ***
## jc            0.0637926   0.0079034    8.072 8.15e-16 ***
## univ          0.0129997   0.0035721    3.639 0.000276 ***
## univ_plus_experYr 0.0602810  0.0020000   30.141 < 2e-16 ***
## black         0.0331709   0.0613984    0.540 0.589038
## hispanic      -0.0193629   0.0248914   -0.778 0.436659
## AA            -0.0077759   0.0295497   -0.263 0.792446
## BA            0.0176735   0.0156553    1.129 0.258972
## experXblack    -0.0012679  0.0004991   -2.541 0.011088 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4287 on 6754 degrees of freedom
## Multiple R-squared:  0.2282, Adjusted R-squared:  0.2272
## F-statistic: 249.6 on 8 and 6754 DF,  p-value: < 2.2e-16
```

Based on the very low p-value (0.000276) for θ , the test is significant at the 0.05% significance level. And even though the value of θ is close to 0 at 0.0129997, we can reject the null hypothesis that $\theta = 0$.

Question 6

```
# Show the summary of the model again
summary(ols.lwage.8ind)
```

```
##
## Call:
## lm(formula = lwage ~ jc + univ + exper + black + hispanic + AA +
##     BA + experXblack, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.11612 -0.27836  0.00432  0.28676  1.76811
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   1.4773315   0.0223780   66.017 < 2e-16 ***
## jc            0.0637926   0.0079034    8.072 8.15e-16 ***
## univ          0.0732806   0.0031486   23.274 < 2e-16 ***
## exper         0.0050234   0.0001667   30.141 < 2e-16 ***
## black         0.0331709   0.0613984    0.540 0.5890
## hispanic      -0.0193629   0.0248914   -0.778 0.4367
## AA            -0.0077759   0.0295497   -0.263 0.7924
```

```
## BA          0.0176735  0.0156553  1.129  0.2590
## experXblack -0.0012679  0.0004991 -2.541  0.0111 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4287 on 6754 degrees of freedom
## Multiple R-squared:  0.2282, Adjusted R-squared:  0.2272
## F-statistic: 249.6 on 8 and 6754 DF,  p-value: < 2.2e-16
```

Test the overall significance of this regression.

We are testing the overall significance of the original model as stated below:

$$lwage = \beta_0 + \beta_1 jc + \beta_2 univ + \beta_3 exper + \beta_4 black + \beta_5 hispanic + \beta_6 AA + \beta_7 BA + \beta_8 experXblack + \epsilon$$

Here is the output from the summary of the original model.

Residual standard error: 0.4287 on 6754 degrees of freedom

Multiple R-squared: 0.2282, Adjusted R-squared: 0.2272

F-statistic: 249.6 on 8 and 6754 DF, p-value: < 2.2e-16

1. Our model null hypothesis is that there is no relationship among any of the independent variables and lwage variable. We are able to reject the null hypothesis since our p-value of the f-statistic of the model is significant at < 2.2e-16.
2. Practical significance: we have an R-squared value of 0.2282, indicating that 22.82% of the variation in lwage is explained by our model.

Question 7

```
# Define a square term for the exper variable
data$experXexper = data$exper * data$exper
# Add the new variable to the regression
ols.lwage.9ind = lm(lwage ~ jc + univ + exper + black +
  hispanic + AA + BA + experXblack + experXexper,
  data = data)
# Show the summary of the model
summary(ols.lwage.9ind)
```

```
##
## Call:
## lm(formula = lwage ~ jc + univ + exper + black + hispanic + AA +
##     BA + experXblack + experXexper, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.11982 -0.27743  0.00475  0.28741  1.77397
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept) 1.510e+00 4.427e-02 34.108 < 2e-16 ***
## jc          6.417e-02 7.916e-03  8.106 6.14e-16 ***
## univ        7.382e-02 3.211e-03 22.992 < 2e-16 ***
## exper       4.301e-03 8.588e-04  5.008 5.64e-07 ***
## black       2.994e-02 6.152e-02  0.487 0.6265
## hispanic    -1.932e-02 2.489e-02 -0.776 0.4378
## AA          -7.539e-03 2.955e-02 -0.255 0.7986
## BA          1.797e-02 1.566e-02  1.147 0.2513
## experXblack -1.239e-03 5.002e-04 -2.477 0.0133 *
## experXexper 3.379e-06 3.939e-06  0.858 0.3911
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4287 on 6753 degrees of freedom
## Multiple R-squared:  0.2282, Adjusted R-squared:  0.2272
## F-statistic: 221.9 on 9 and 6753 DF,  p-value: < 2.2e-16
```

Estimated return to work experience in this model

$$lwage = \beta_0 + \beta_1 jc + \beta_2 univ + \beta_3 exper + \beta_4 black + \beta_5 hispanic + \beta_6 AA + \beta_7 BA + \beta_8 experXblack + \beta_9 experXexper$$

$$\frac{\delta lwage}{exper} = \beta_3 + \beta_8 black + 2 * \beta_9 exper$$

$$\beta_3 = 4.301e - 03 = .004301, \beta_8 = .001239, \beta_9 = 0.00003379$$

Substituting these values in the the equation above we get:

$$\frac{\delta lwage}{exper} = (.004301 - .001239 * black + .000006758 * exper)$$

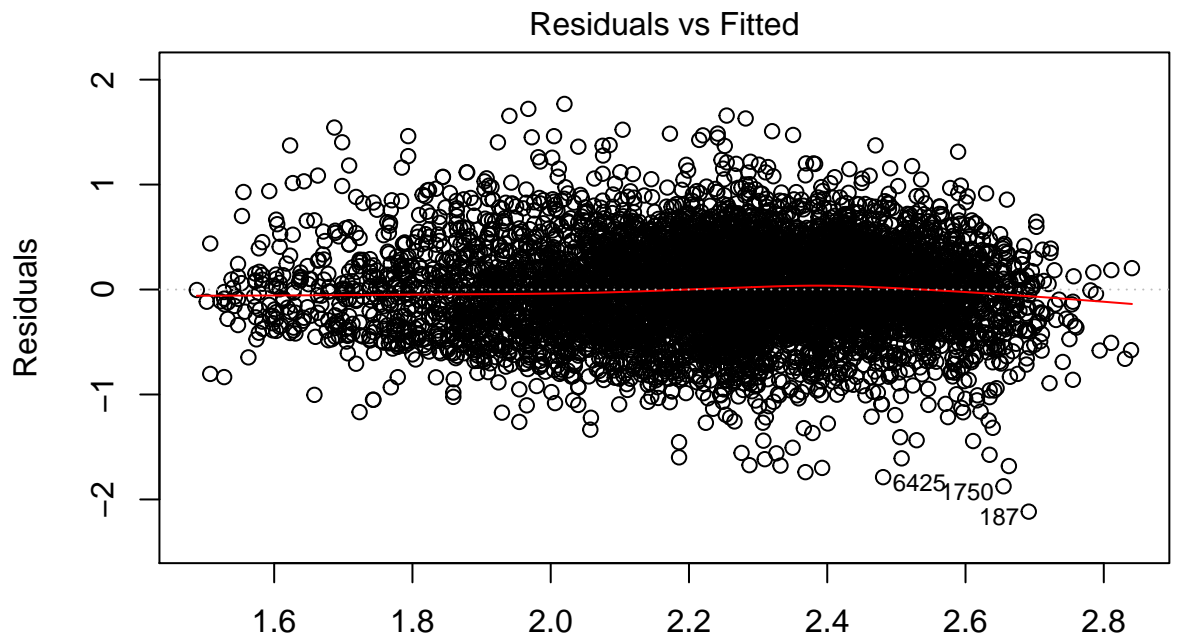
Now convert the log wage back to wage by exponentiating. This gives us a return to work experience:

$$= e^{(.004301 - .001239 * black + .000006758 * exper)}$$

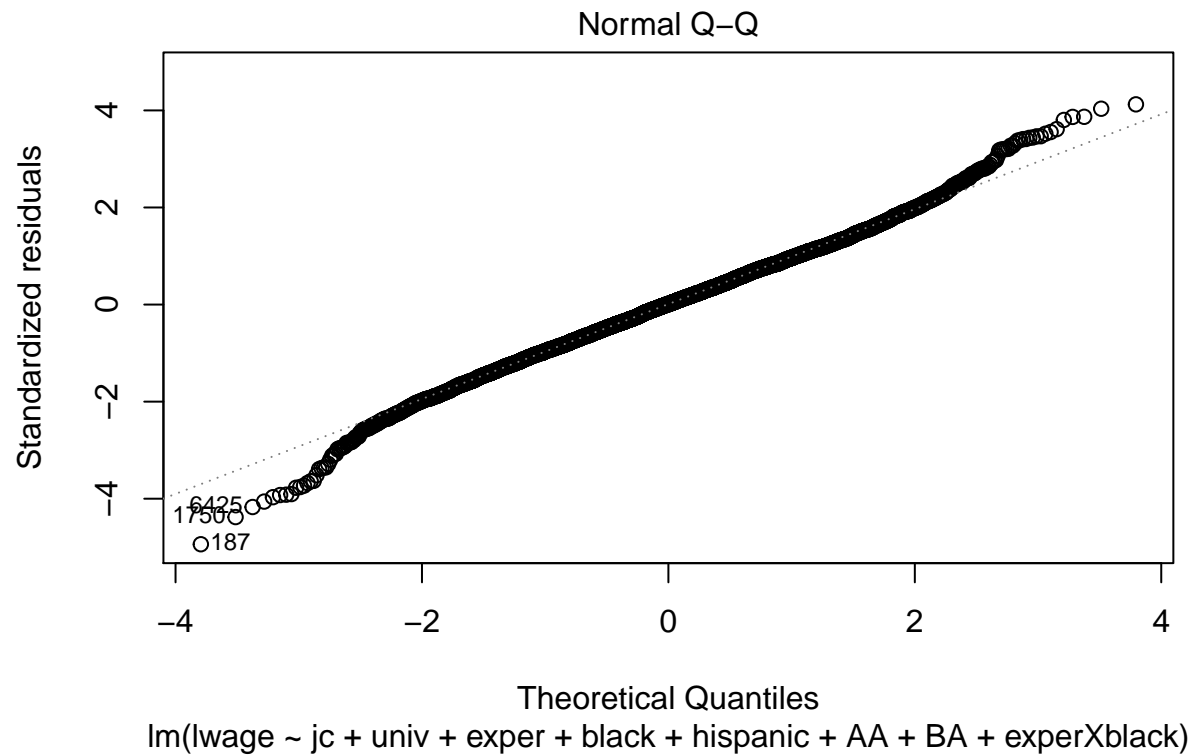
??is the correct or do we need to multiply it by something else

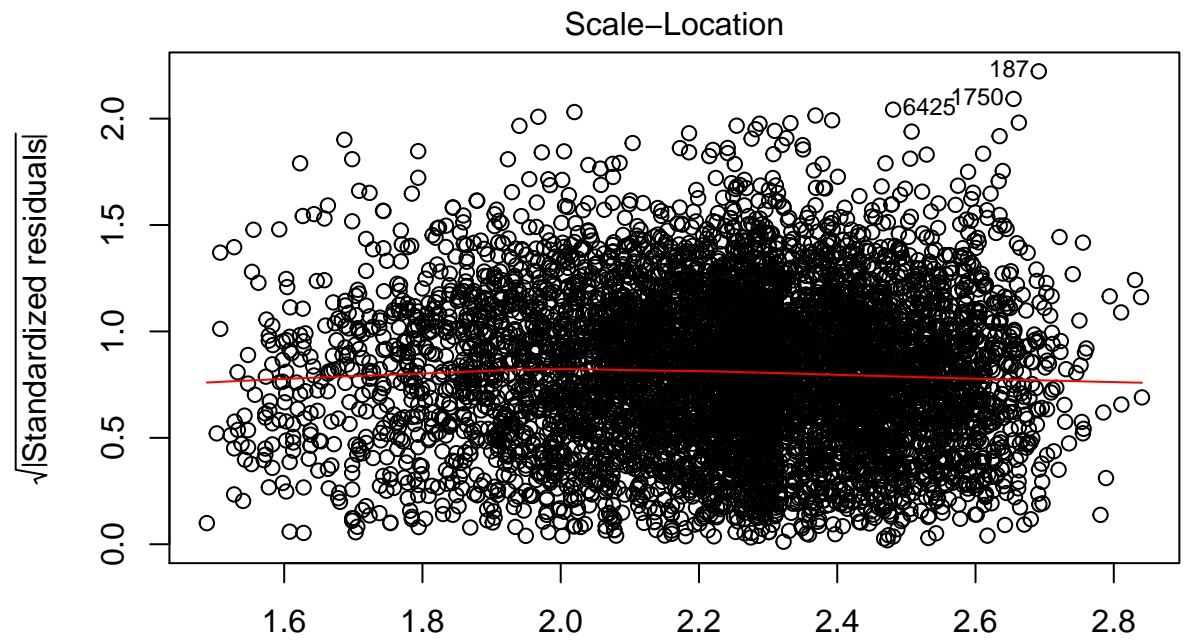
Question 8

```
# Plot the graphs from the model
plot(ols.lwage.8ind)
```

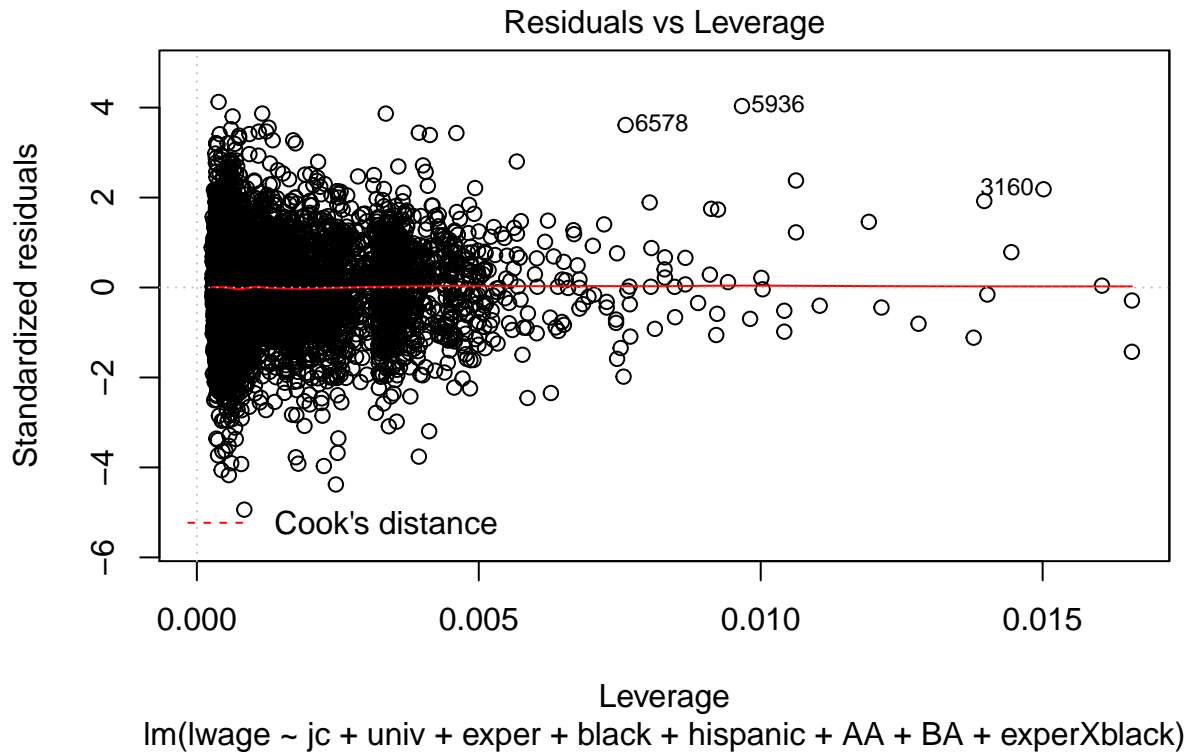


Fitted values
 $\text{lm}(\text{lwage} \sim \text{jc} + \text{univ} + \text{exper} + \text{black} + \text{hispanic} + \text{AA} + \text{BA} + \text{experXblack})$





Fitted values
 $\text{lm}(\text{lwage} \sim \text{jc} + \text{univ} + \text{exper} + \text{black} + \text{hispanic} + \text{AA} + \text{BA} + \text{experXblack})$



```
# Just as a precaution, use the robust standard
# errors
coeftest(ols.lwage.8ind, vcov = vcovHC)
```

```
##
## t test of coefficients:
##
##      Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.47733155  0.02293512  64.4135 < 2e-16 ***
## jc           0.06379261  0.00761208   8.3804 < 2e-16 ***
## univ         0.07328063  0.00336598  21.7709 < 2e-16 ***
## exper        0.00502341  0.00016840  29.8294 < 2e-16 ***
## black        0.03317088  0.06872723   0.4826  0.62936
## hispanic     -0.01936289  0.02498704  -0.7749  0.43842
## AA           -0.00777589  0.02746594  -0.2831  0.77710
## BA           0.01767355  0.01656455   1.0670  0.28603
## experXblack  -0.00126790  0.00053779  -2.3576  0.01842 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# Show the summary of the model
summary(ols.lwage.8ind)
```

```
##
## Call:
```

```
## lm(formula = lwage ~ jc + univ + exper + black + hispanic + AA +
##      BA + experXblack, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.11612 -0.27836  0.00432  0.28676  1.76811
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.4773315   0.0223780   66.017 < 2e-16 ***
## jc           0.0637926   0.0079034    8.072 8.15e-16 ***
## univ         0.0732806   0.0031486   23.274 < 2e-16 ***
## exper        0.0050234   0.0001667   30.141 < 2e-16 ***
## black        0.0331709   0.0613984    0.540  0.5890
## hispanic     -0.0193629   0.0248914   -0.778  0.4367
## AA           -0.0077759   0.0295497   -0.263  0.7924
## BA           0.0176735   0.0156553    1.129  0.2590
## experXblack -0.0012679   0.0004991   -2.541  0.0111 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4287 on 6754 degrees of freedom
## Multiple R-squared:  0.2282, Adjusted R-squared:  0.2272
## F-statistic: 249.6 on 8 and 6754 DF,  p-value: < 2.2e-16
```

Homoskedasticity analysis:

We are testing homoskedasticity of the original model as stated below:

$$lwage = \beta_0 + \beta_1 jc + \beta_2 univ + \beta_3 exper + \beta_4 black + \beta_5 hispanic + \beta_6 AA + \beta_7 BA + \beta_8 experXblack + \epsilon$$

The assumption of homoskedasticity holds:

- 1 - We can see from the residuals vs fitted plot that the variance band is about the same as we move to higher fitted values.
- 2 - The same story is told by the scale-location plot where we see that the smoothing line is almost completely horizontal, which is what we get if homoskedasticity is met.
- 3 - We do not look at the Breusch Pagan test since we have a large number of observations, therefore we know almost certainly that we will obtain significance.

The implication of homoskedasticity in the data is that the standard error of the univ coefficient (β_2) is unbiased. Unbiased standard errors will not impact the outcomes of statistical tests. Therefore, it does not affect the testing of no effect of university education on salary change.

Just as a precaution the model was rerun using robust standard errors. The β_2 coefficient was essentially unchanged. In the original model $\beta_2 = 0.0732806$ and in the model with robust standard errors $\beta_2 = 0.07328063$.