W271 Lab 3 Spring 2016

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Below, we define some functions we will be using in the problem set:

```
# Functions for Parts 2, 3, 4
get.best.arima <- function(x.ts, maxord = c(1, 1, 1)) {
    best.aic <- 1e+08
    all.aics <- vector()
    all.models <- vector()
    n <- length(x.ts)
    for (p in 0:maxord[1]) for (d in 0:maxord[2]) for (q in 0:maxord[3]) {
        fit <- arima(x.ts, order = c(p, d, q), method = "ML")
        fit.aic <- -2 * fit$loglik + (log(n) + 1) * length(fit$coef)
        if (fit.aic < best.aic) {
            best.aic <- fit.aic
            best.fit <- fit
            best.model <- c(p, d, q)
        }
        all.aics <- c(all.aics, fit.aic)
        all.models <- c(all.models, sprintf("(%d, %d, %d)", p,
            d, q))
    }
    list(best = list(best.aic, best.fit, best.model), others = data.frame(aics = all.aics,
        models = all.models))
}

get.best.sarima <- function(x.ts, maxord = c(1, 1, 1, 1, 1, 1),
    freq) {
    best.aic <- 1e+08
    all.aics <- vector()
    all.models <- vector()
    n <- length(x.ts)
    for (p in 0:maxord[1]) for (d in 0:maxord[2]) for (q in 0:maxord[3]) for (P in 0:maxord[3]) for (D i
        fit <- arima(x.ts, order = c(p, d, q), seasonal = list(order = c(P,
            D, Q), freq), method = "CSS", optim.control = list(maxit = 10000))
        fit.aic <- -2 * fit$loglik + (log(n) + 1) * length(fit$coef)
        if (fit.aic < best.aic) {
            best.aic <- fit.aic
            best.fit <- fit
            best.model <- c(p, d, q, P, D, Q)
        }
        all.aics <- c(all.aics, fit.aic)
        all.models <- c(all.models, sprintf("(%d, %d, %d, %d, %d, %d)",
            p, d, q, P, D, Q))
    }
    list(best = list(best.aic, best.fit, best.model), others = data.frame(aics = all.aics,
        models = all.models))
}
```

```r
plot.time.series <- function(x.ts, bins = 30, name) {
    str(x.ts)
    par(mfrow = c(2, 2))
    hist(x.ts, bins, main = paste("Histogram of", name, sep = " "),
        xlab = "Values")
    plot(x.ts, main = paste("Plot of", name, sep = " "), ylab = "Values",
        xlab = "Time Period")
    acf(x.ts, main = paste("ACF of", name, sep = " "))
    pacf(x.ts, main = paste("PACF of", name, sep = " "))
}

plot.residuals.ts <- function(x.mod, model_name) {
    par(mfrow = c(1, 1))
    hist(x.mod$residuals, 30, main = paste("Histogram of", model_name,
        "Residuals", sep = " "), xlab = "Values")
    par(mfrow = c(2, 2))
    plot(x.mod$residuals, fitted(x.mod), main = paste(model_name,
        "Fitted vs. Residuals", sep = " "), ylab = "Fitted Values",
        xlab = "Residuals")
    plot(x.mod$residuals, main = paste(model_name, "Residuals",
        sep = " "), ylab = paste("Residuals", sep = " "))
    acf(x.mod$residuals, main = paste("ACF of", model_name, sep = " "))
    pacf(x.mod$residuals, main = paste("PACF of", model_name,
        sep = " "))
    Box.test(x.mod$residuals, type = "Ljung-Box")
}

estimate.ar <- function(x.ts) {
    x.ar = ar(x.ts)
    print("Difference in AICs")
    print(x.ar$aic)
    print("AR parameters")
    print(x.ar$ar)
    print("AR order")
    print(x.ar$order)
    return(x.ar)
}

plot.orig.model.resid <- function(x.ts, x.mod, model_name, xlim,
    ylim) {
    df <- data.frame(cbind(x.ts, fitted(x.mod), x.mod$residuals))
    class(df)
    stargazer(df, type = "text", title = "Descriptive Stat",
        digits = 1)

    summary(x.ts)
    summary(x.mod$residuals)
    par(mfrow = c(1, 1))
    plot.ts(x.ts, col = "red", main = paste("Orivinal vs Estimated",
        model_name, "Series with Resdiauls", sep = " "), ylab = "Original and Estimated Values",
        xlim = xlim, ylim = ylim, pch = 1, lty = 2)
    par(new = T)
    plot.ts(fitted(x.mod), col = "blue", axes = T, xlab = "",
```

```
        ylab = "", xlim = xlim, ylim = ylim, lty = 1)
    leg.txt <- c("Original Series", "Estimated Series", "Residuals")
    legend("topleft", legend = leg.txt, lty = c(2, 1, 2), col = c("red",
        "blue", "green"), bty = "n", cex = 1)
    par(new = T)
    plot.ts(x.mod$residuals, axes = F, xlab = "", ylab = "",
        col = "green", xlim = xlim, ylim = ylim, lty = 2, pch = 1,
        col.axis = "green")
    axis(side = 4, col = "green")
    mtext("Residuals", side = 4, line = 2, col = "green")
}

plot.model.forecast <- function(x.mod, mod.fcast, num_steps,
    x, y) {
    par(mfrow = c(1, 1))
    plot(mod.fcast, main = paste(num_steps, "-Step Ahead Forecast and Original & Estimated Series",
        sep = ""), xlab = "Simulated Time Period", ylab = "Original, Estimated, and Forecasted Values",
        xlim = x, ylim = y, lty = 2, lwd = 1.5)
    par(new = T)
    plot.ts(fitted(x.mod), col = "blue", lty = 2, lwd = 2, xlab = "",
        ylab = "", xlim = x, ylim = y)
    leg.txt <- c("Original Series", "Estimated Series", "Forecast")
    legend("topleft", legend = leg.txt, lty = c(2, 2, 1), lwd = c(1,
        2, 2), col = c("black", "blue", "blue"), bty = "n", cex = 1)
}
```

Part 1 (25 points): Modeling House Values

Step 1 - Univariate Analysis

1. **Crime Rate** - This variable is positively skewed, with 90% of datapoints having a crime rate below 11.2%, but outliers above that going upto 89%. We take the log to create a new variable before proceeding.
2. **nonRetailBusiness** - Has a suspiciously high mode at 0.18, which may indicate that a lot of the data points come from the same neighbourhood, which would explain the high number of occurences of a single value.
3. **withWater** - This is a categorical variable. 6.75% of homes in the given sample are in neighbourhoods within 5 miles of a water body.
4. **ageHouse** - This value is in percentage terms and not in strict proportion like other variables in the dataset. Over 50% of the houses in the dataset are in neighbourhoods with a proportion of houses older than 1950 that is greater than 78%
5. **distanceToCity** - 75% of the houses are less than 15 miles away from a city, and 90% are less than 25 miles away. However, the variable has a lage outlier, which is almost 55 miles away from a city. We take a log of the variable before proceeding, in order to make it more evenly distributed.
6. **distanceToHighway** - Definition of the variable is not provided in the dictionary. We see that there are 104 datapoints, exactly 24 miles away from the highway, so we assume this variable measures distance of a neighbourhood from the highway. This is exactly the same number of points for which nonRetailBusiness has a value of 0.18, so it further strengthens the argument that a lot of the datapoints seem to be for houses in the same or very close neighbourhood.
7. **pupilTeacherRatio** - We find another variable with a high modal value of exactly 23.2 pupils per teacher. Furthers the above argument that a large part of the sample is taken from a single neighbourhood.
8. **pctLowIncome** - 90% of the homes come from neighbourhoods with less than 30% households being low-income, however we do have values going up to 49% in the dataset.
9. **homeValue** - The distribution has 95% of houses valued at well below $1 million, however, there are outliers above that value upto $1.125 million. We take the log of the variable to make it closer to a normal distribution.
10. **pollutionIndex** - Has a scattered distribution with a median of 38.8, and a large outlier at 72.1.
11. **nBedRooms** - Close to normal distribution, with the mean and median around 4.25 bedrooms on average for a single family home, however there are small, as well as large outliers in the distribution.

```
q1.dataset = read.csv("houseValueData.csv")

str(q1.dataset)
```

```
## 'data.frame':    400 obs. of  11 variables:
##  $ crimeRate_pc     : num  37.6619 0.5783 0.0429 22.5971 0.0664 ...
##  $ nonRetailBusiness: num  0.181 0.0397 0.1504 0.181 0.0405 ...
##  $ withWater        : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ ageHouse         : num  78.7 67 77.3 89.5 74.4 71.3 68.2 97.3 92.2 96.2 ...
##  $ distanceToCity   : num  2.71 4.12 7.82 1.95 5.54 ...
##  $ distanceToHighway: int  24 5 4 24 5 5 5 5 3 5 ...
##  $ pupilTeacherRatio: num  23.2 16 21.2 23.2 19.6 23.9 22.2 17.7 20.8 17.7 ...
##  $ pctLowIncome     : int  18 9 13 41 8 9 12 18 5 4 ...
##  $ homeValue        : int  245250 1125000 463500 166500 672750 596250 425250 483750 852750 1125000 .
##  $ pollutionIndex   : num  52.9 42.5 31.4 55 36 37 34.9 72.1 33.8 45.5 ...
##  $ nBedRooms        : num  4.2 6.3 4.25 3 4.86 ...
```

```r
summary(q1.dataset)
```

```
##   crimeRate_pc      nonRetailBusiness   withWater         ageHouse
##  Min.   : 0.00632   Min.   :0.0074    Min.   :0.0000   Min.   :  2.90
##  1st Qu.: 0.08260   1st Qu.:0.0513    1st Qu.:0.0000   1st Qu.: 45.67
##  Median : 0.26600   Median :0.0969    Median :0.0000   Median : 77.95
##  Mean   : 3.76256   Mean   :0.1115    Mean   :0.0675   Mean   : 68.93
##  3rd Qu.: 3.67481   3rd Qu.:0.1810    3rd Qu.:0.0000   3rd Qu.: 94.15
##  Max.   :88.97620   Max.   :0.2774    Max.   :1.0000   Max.   :100.00
##  distanceToCity   distanceToHighway pupilTeacherRatio  pctLowIncome
##  Min.   : 1.228   Min.   : 1.000    Min.   :15.60     Min.   : 2.00
##  1st Qu.: 3.240   1st Qu.: 4.000    1st Qu.:19.90     1st Qu.: 8.00
##  Median : 6.115   Median : 5.000    Median :21.90     Median :14.00
##  Mean   : 9.638   Mean   : 9.582    Mean   :21.39     Mean   :15.79
##  3rd Qu.:13.628   3rd Qu.:24.000    3rd Qu.:23.20     3rd Qu.:21.00
##  Max.   :54.197   Max.   :24.000    Max.   :25.00     Max.   :49.00
##    homeValue        pollutionIndex    nBedRooms
##  Min.   : 112500   Min.   :23.50   Min.   :1.561
##  1st Qu.: 384188   1st Qu.:29.88   1st Qu.:3.883
##  Median : 477000   Median :38.80   Median :4.193
##  Mean   : 499584   Mean   :40.61   Mean   :4.266
##  3rd Qu.: 558000   3rd Qu.:47.58   3rd Qu.:4.582
##  Max.   :1125000   Max.   :72.10   Max.   :6.780
```

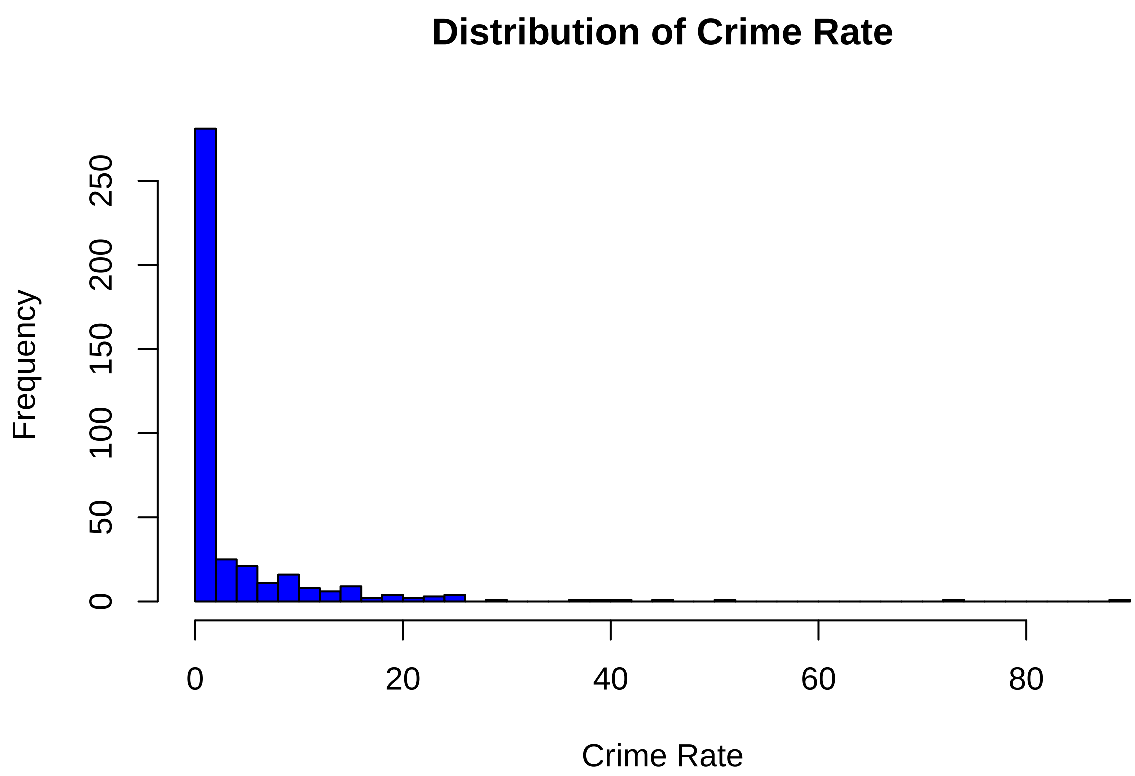
```r
# Performing univariate analysis Crime Rate
summary(q1.dataset$crimeRate_pc)
```

```
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
##  0.00632  0.08260  0.26600  3.76300  3.67500 88.98000
```

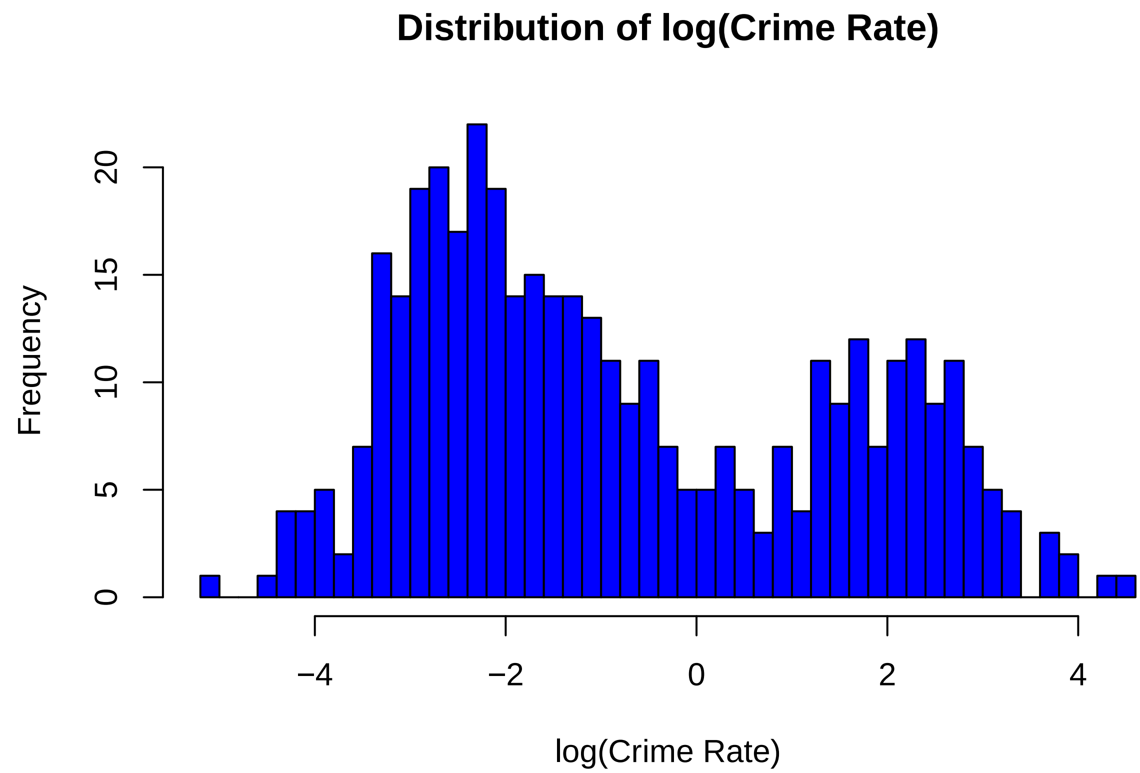
```r
quantile(q1.dataset$crimeRate_pc, probs = c(0.01, 0.05, 0.1, 0.25, 0.5,
    0.75, 0.9, 0.95, 0.99, 1))
```

```
##         1%         5%        10%        25%        50%        75%
##  0.0143128  0.0310980  0.0410280  0.0825975  0.2660050  3.6748075
##        90%        95%        99%       100%
## 11.2021500 18.1052800 41.5713690 88.9762000
```

```r
hist(q1.dataset$crimeRate_pc, breaks = 60, col = "blue", main = "Distribution of Crime Rate",
    xlab = "Crime Rate")
```




```r
# This variable is extremely +vely skewed, so we take the log
q1.dataset$logCrimeRate_pc = log(q1.dataset$crimeRate_pc)
hist(q1.dataset$logCrimeRate_pc, breaks = 60, col = "blue", main = "Distribution of log(Crime Rate)",
    xlab = "log(Crime Rate)")
```



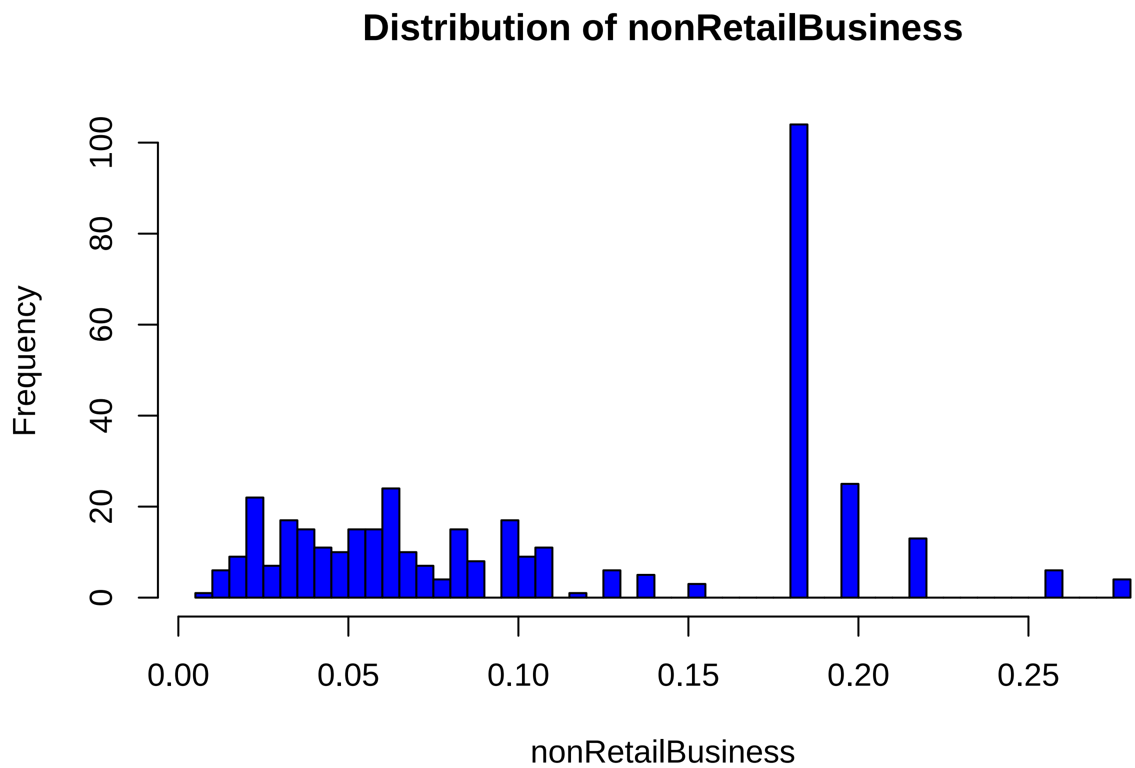

```r
# nonRetailBusiness
summary(q1.dataset$nonRetailBusiness)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.0074  0.0513  0.0969  0.1115  0.1810  0.2774
```

```r
quantile(q1.dataset$nonRetailBusiness, probs = c(0.01, 0.05, 0.1, 0.25,
    0.5, 0.75, 0.9, 0.95, 0.99, 1))
```

```
##       1%       5%      10%      25%      50%      75%      90%      95%
## 0.013794 0.021725 0.028900 0.051300 0.096900 0.181000 0.195800 0.218900
##      99%     100%
## 0.256709 0.277400
```

```r
hist(q1.dataset$nonRetailBusiness, breaks = 60, col = "blue", main = "Distribution of nonRetailBusiness
    xlab = "nonRetailBusiness")
```




```r
head(q1.dataset[order(q1.dataset$nonRetailBusiness, decreasing = TRUE),
    c("nonRetailBusiness")], n = 50)
```

```
##  [1] 0.2774 0.2774 0.2774 0.2774 0.2565 0.2565 0.2565 0.2565 0.2565 0.2565
## [11] 0.2189 0.2189 0.2189 0.2189 0.2189 0.2189 0.2189 0.2189 0.2189 0.2189
## [21] 0.2189 0.2189 0.2189 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958
## [31] 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958
## [41] 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1958 0.1810 0.1810
```

```r
tail(sort(table(q1.dataset$nonRetailBusiness)), 5)
```

```
##
## 0.2189  0.062 0.0814 0.1958  0.181
##     13     14     15     25    104
```

```r
# suspicious that this has such a large modal value. May be some coded
# val

# withWater
summary(q1.dataset$withWater)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.0000  0.0000  0.0000  0.0675  0.0000  1.0000
```

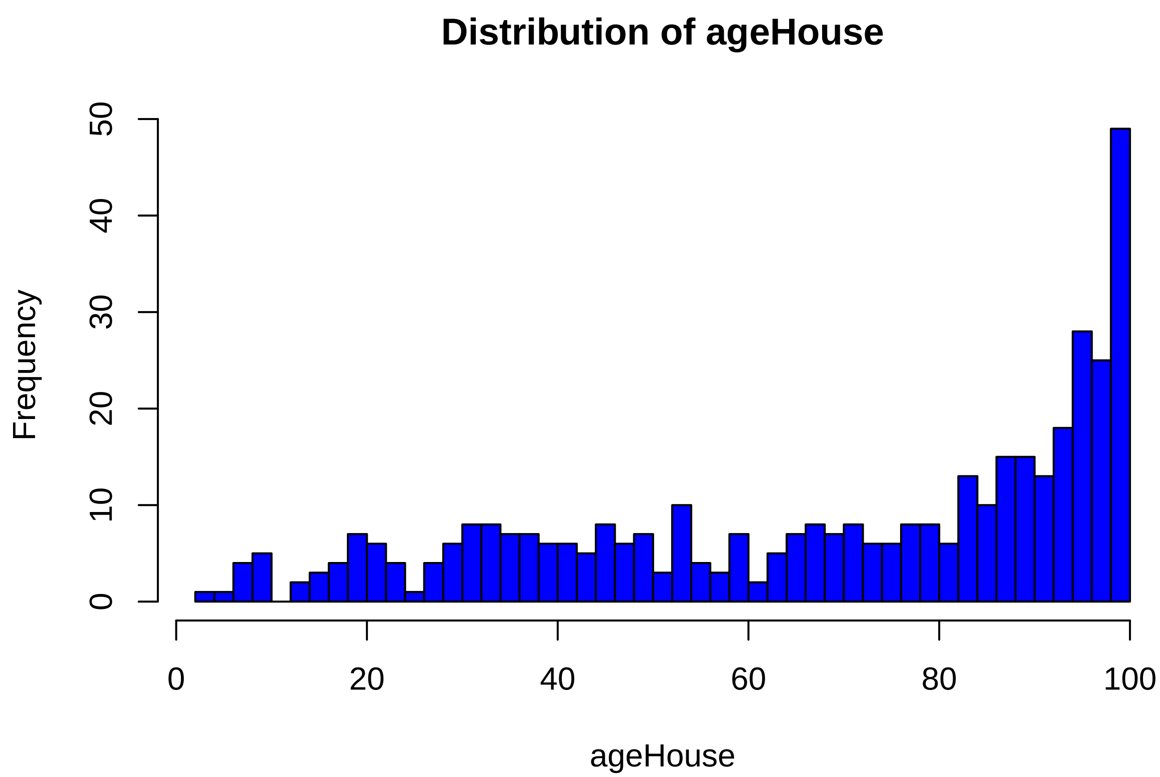
```r
# 7% have water

# ageHouse
summary(q1.dataset$ageHouse)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    2.90   45.68   77.95   68.93   94.15  100.00
```

```r
quantile(q1.dataset$ageHouse, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75,
    0.9, 0.95, 0.99, 1))
```

```
##      1%      5%     10%     25%     50%     75%     90%     95%     99%
##   7.788  18.370  27.690  45.675  77.950  94.150  98.410 100.000 100.000
##    100%
## 100.000
```

```r
hist(q1.dataset$ageHouse, breaks = 60, col = "blue", main = "Distribution of ageHouse",
    xlab = "ageHouse")
```



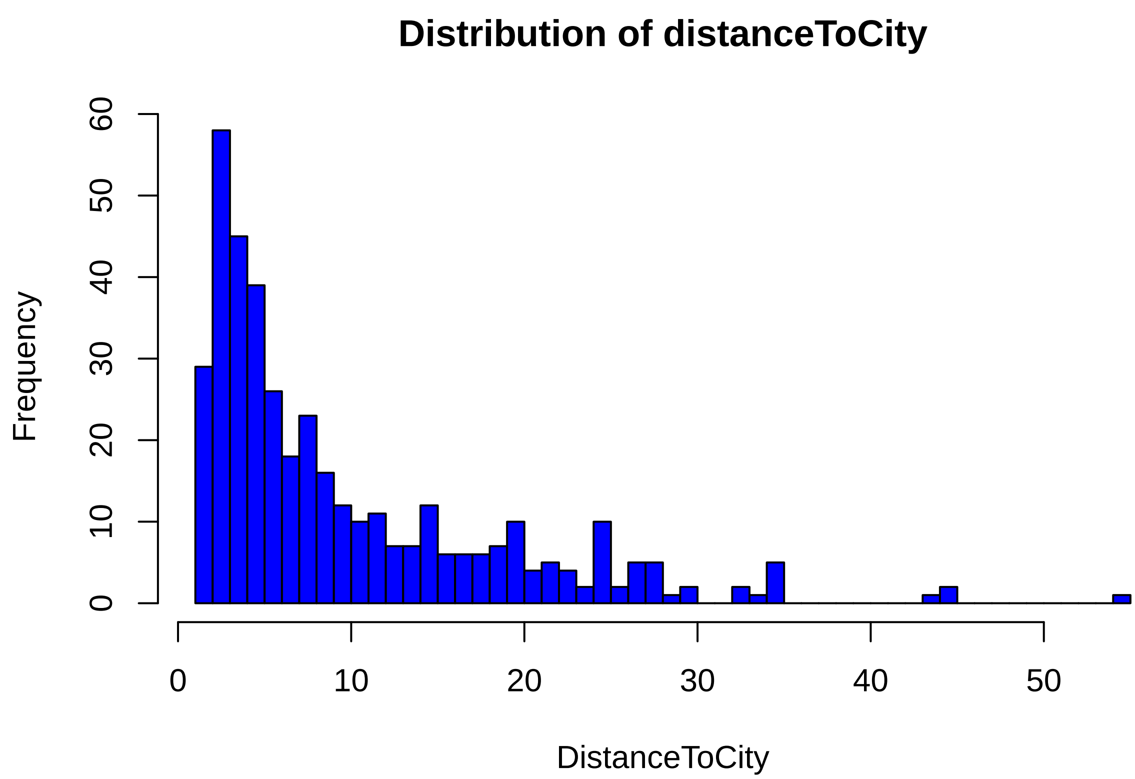

```r
# Looks like a % value.  May require a power transformation

# disttocity
summary(q1.dataset$distanceToCity)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   1.228   3.240   6.115   9.638  13.630  54.200
```

```r
quantile(q1.dataset$distanceToCity, probs = c(0.01, 0.05, 0.1, 0.25, 0.5,
    0.75, 0.9, 0.95, 0.99, 1))
```

```
##        1%        5%       10%       25%       50%       75%       90%
##  1.342576  1.889692  2.158538  3.239878  6.114617 13.627873 22.682747
##       95%       99%      100%
## 26.939533 35.063729 54.197188
```

```r
hist(q1.dataset$distanceToCity, breaks = 60, col = "blue", main = "Distribution of distanceToCity",
    xlab = "DistanceToCity")
```



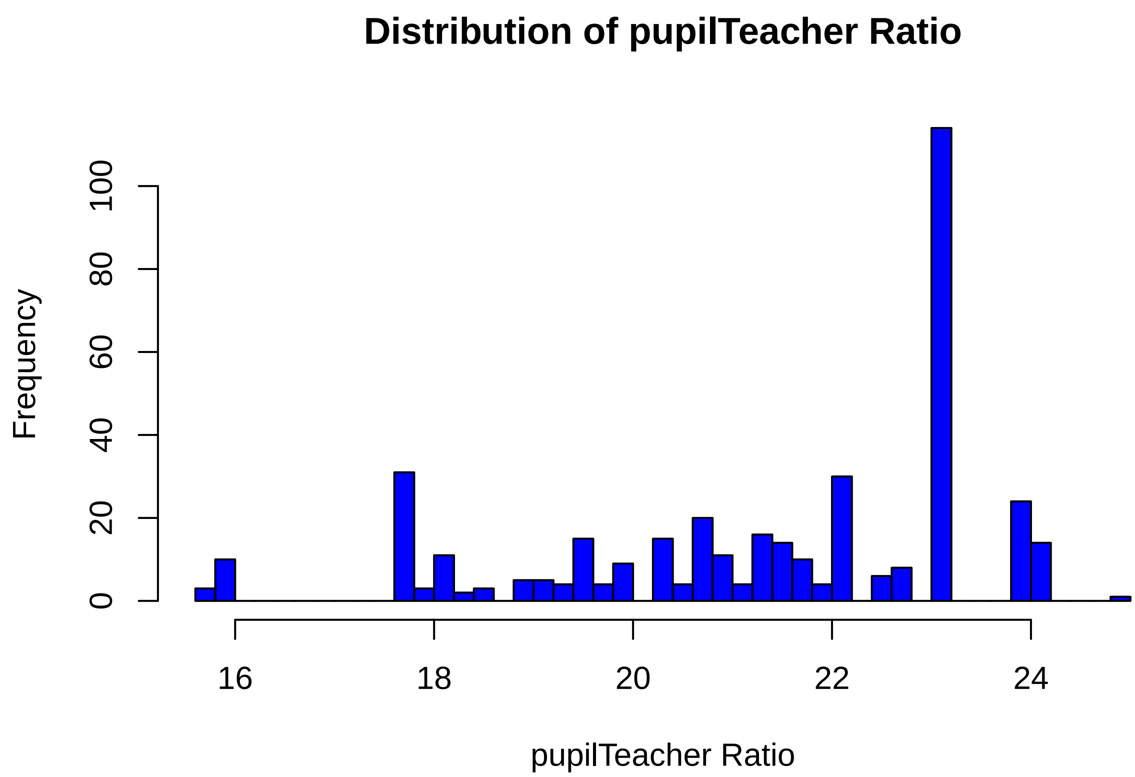

```r
q1.dataset$logDistanceToCity = log(q1.dataset$distanceToCity)
# skewed with a large outlier at the end. Keep in mind while running
# model

# pupilTeacher
summary(q1.dataset$pupilTeacherRatio)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   15.60   19.90   21.90   21.39   23.20   25.00
```

```r
quantile(q1.dataset$pupilTeacherRatio, probs = c(0.01, 0.05, 0.1, 0.25,
    0.5, 0.75, 0.9, 0.95, 0.99, 1))
```

```
##   1%   5%  10%  25%  50%  75%  90%  95%  99% 100%
## 16.0 17.7 17.7 19.9 21.9 23.2 23.2 24.0 24.2 25.0
```

```r
hist(q1.dataset$pupilTeacherRatio, breaks = 60, col = "blue", main = "Distribution of pupilTeacher Ratio
    xlab = "pupilTeacher Ratio")
```




```r
tail(sort(table(q1.dataset$pupilTeacherRatio)), 5)
```

```
##
##   24 22.2 20.8 17.7 23.2
##   16   17   20   28  110
```

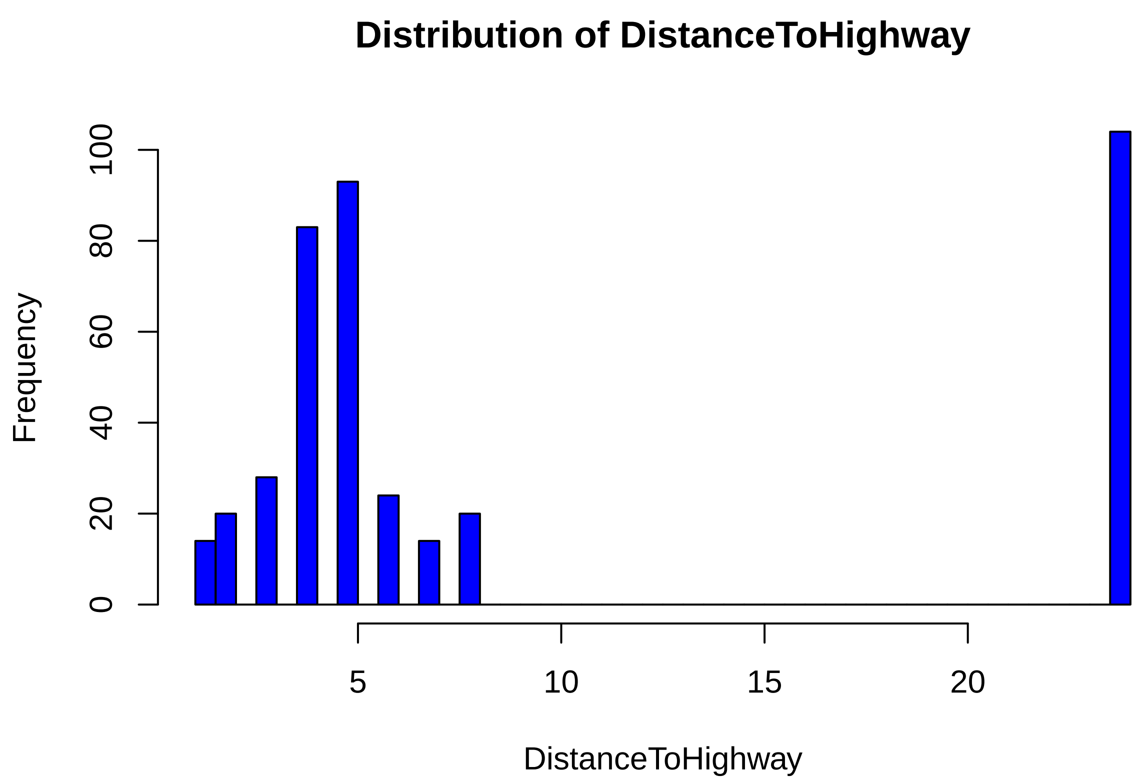
```r
# High mode at 23.2, suspicious

# dist to highway
summary(q1.dataset$distanceToHighway)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   1.000   4.000   5.000   9.582  24.000  24.000
```

```r
quantile(q1.dataset$distanceToHighway, probs = c(0.01, 0.05, 0.1, 0.25,
    0.5, 0.75, 0.9, 0.95, 0.99, 1))
```

```
##   1%   5%  10%  25%  50%  75%  90%  95%  99% 100%
##    1    2    3    4    5   24   24   24   24   24
```

```r
hist(q1.dataset$distanceToHighway, breaks = 60, col = "blue", main = "Distribution of DistanceToHighway
    xlab = "DistanceToHighway")
```




```r
tail(sort(table(q1.dataset$distanceToHighway)), 5)
```

```
##
##   6   3   4   5  24
##  24  28  83  93 104
```

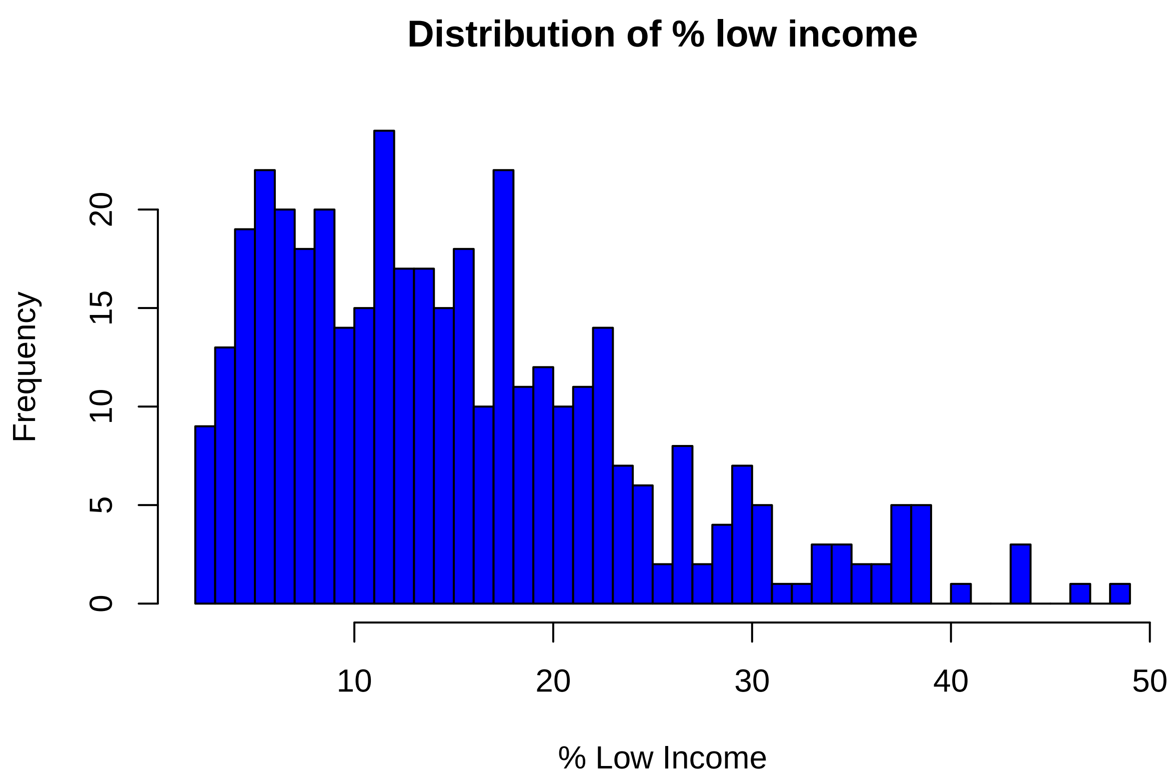
```r
# Very strange that so many values are exactly 24. May not be best
# thing for regression.

# pctlowincome
summary(q1.dataset$pctLowIncome)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##     2.0     8.0    14.0    15.8    21.0    49.0
```

```r
quantile(q1.dataset$pctLowIncome, probs = c(0.01, 0.05, 0.1, 0.25, 0.5,
    0.75, 0.9, 0.95, 0.99, 1))
```

```
##    1%    5%   10%   25%   50%   75%   90%   95%   99%  100%
##  3.00  4.00  5.00  8.00 14.00 21.00 29.10 35.05 44.00 49.00
```

```r
hist(q1.dataset$pctLowIncome, breaks = 60, col = "blue", main = "Distribution of % low income",
    xlab = "% Low Income")
```




```r
tail(sort(table(q1.dataset$pctLowIncome)), 5)
```

```
##
##  7  9  6 18 12
## 20 20 22 22 24
```

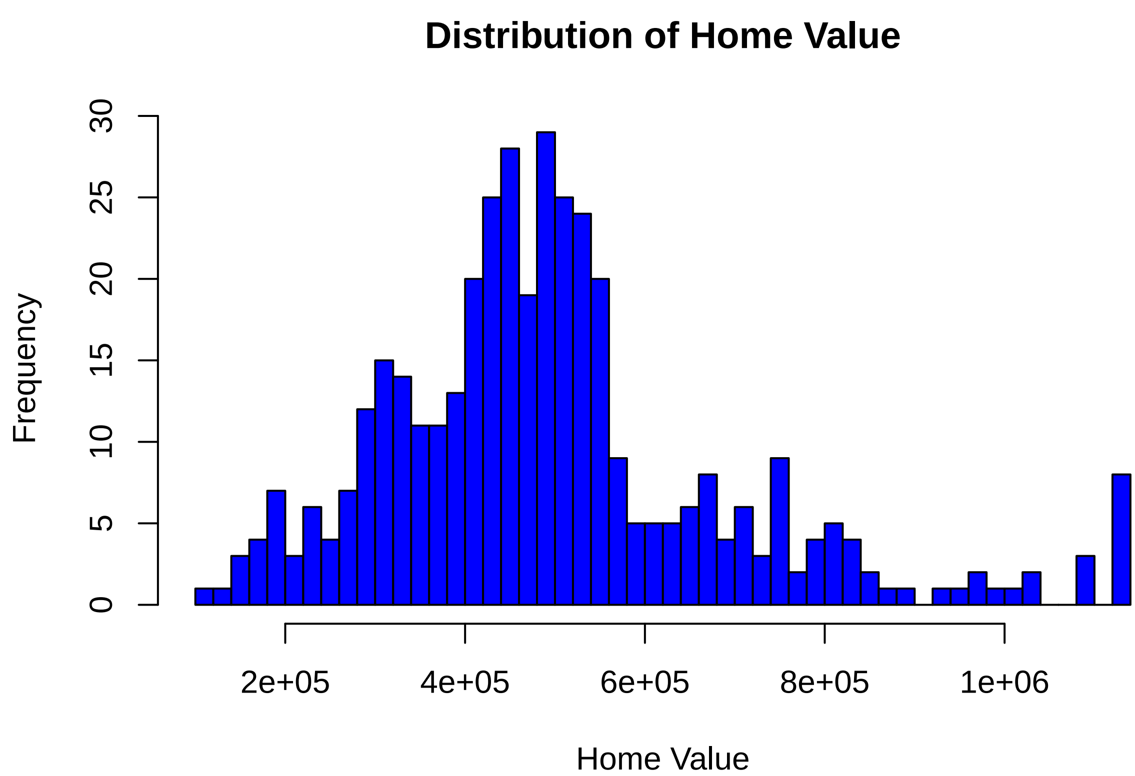
```r
# slight neg skew

# Home value
summary(q1.dataset$homeValue)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  112500  384200  477000  499600  558000 1125000
```

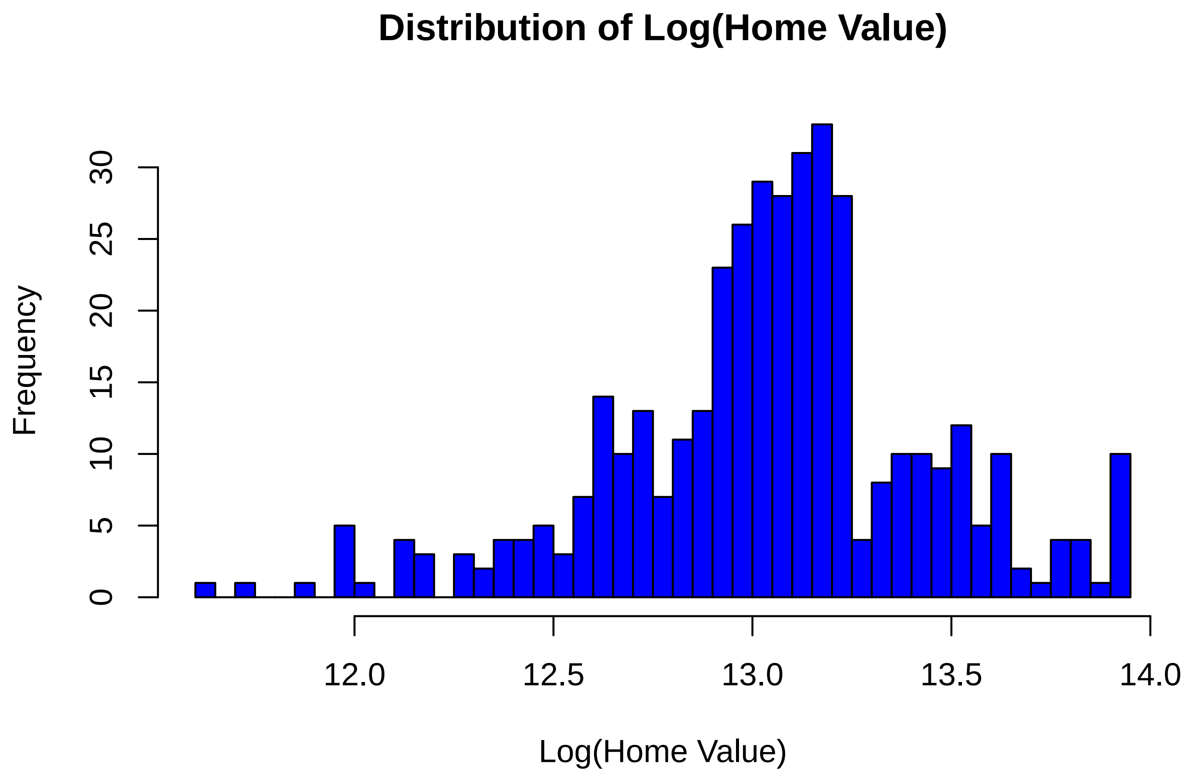
```r
quantile(q1.dataset$homeValue, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75,
    0.9, 0.95, 0.99, 1))
```

```
##        1%        5%       10%       25%       50%       75%       90%
##  157500.0  229500.0  291825.0  384187.5  477000.0  558000.0  749475.0
##       95%       99%      100%
##  871987.5 1125000.0 1125000.0
```

```r
hist(q1.dataset$homeValue, breaks = 60, col = "blue", main = "Distribution of Home Value",
    xlab = "Home Value")
```




```r
q1.dataset$logHomeValue = log(q1.dataset$homeValue)
hist(q1.dataset$logHomeValue, breaks = 60, col = "blue", main = "Distribution of Log(Home Value)",
    xlab = "Log(Home Value)")
```



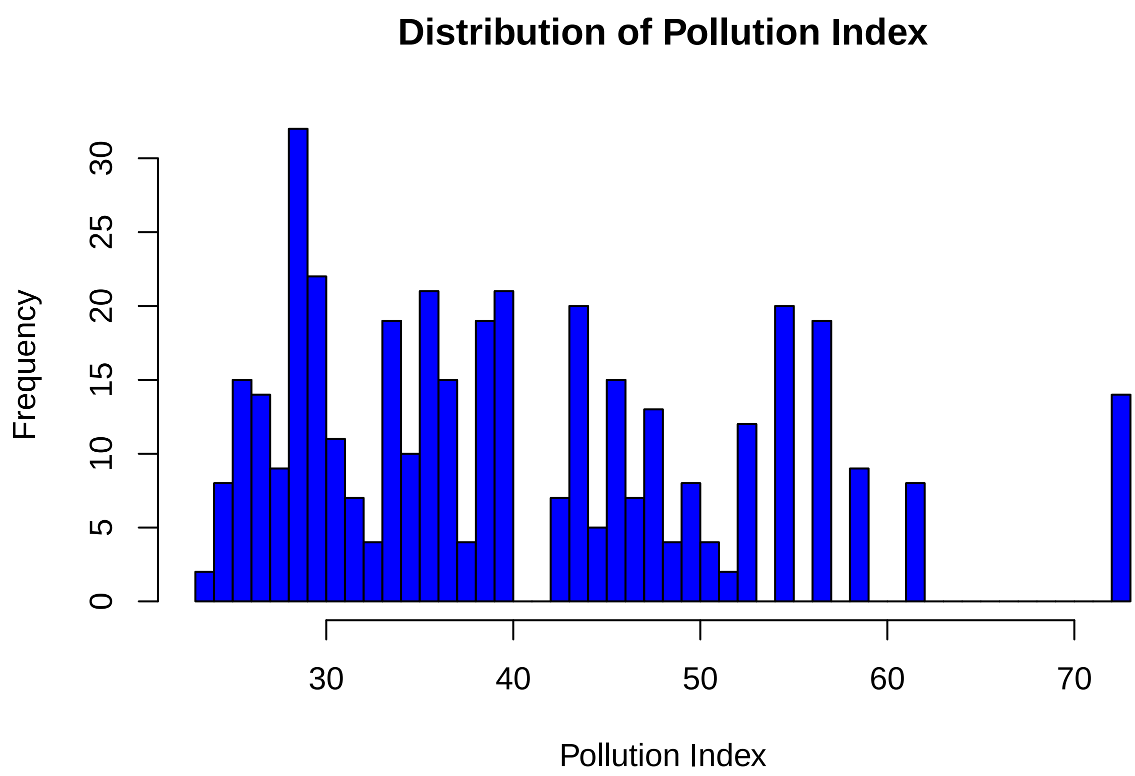

```r
# Pretty normal

# poll Index
summary(q1.dataset$pollutionIndex)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   23.50   29.87   38.80   40.61   47.58   72.10
```

```r
quantile(q1.dataset$pollutionIndex, probs = c(0.01, 0.05, 0.1, 0.25, 0.5,
    0.75, 0.9, 0.95, 0.99, 1))
```

```
##      1%      5%     10%     25%     50%     75%     90%     95%     99%    100%
## 24.398 25.880 27.600 29.875 38.800 47.575 56.300 62.000 72.100 72.100
```

```r
hist(q1.dataset$pollutionIndex, breaks = 60, col = "blue", main = "Distribution of Pollution Index",
    xlab = "Pollution Index")
```



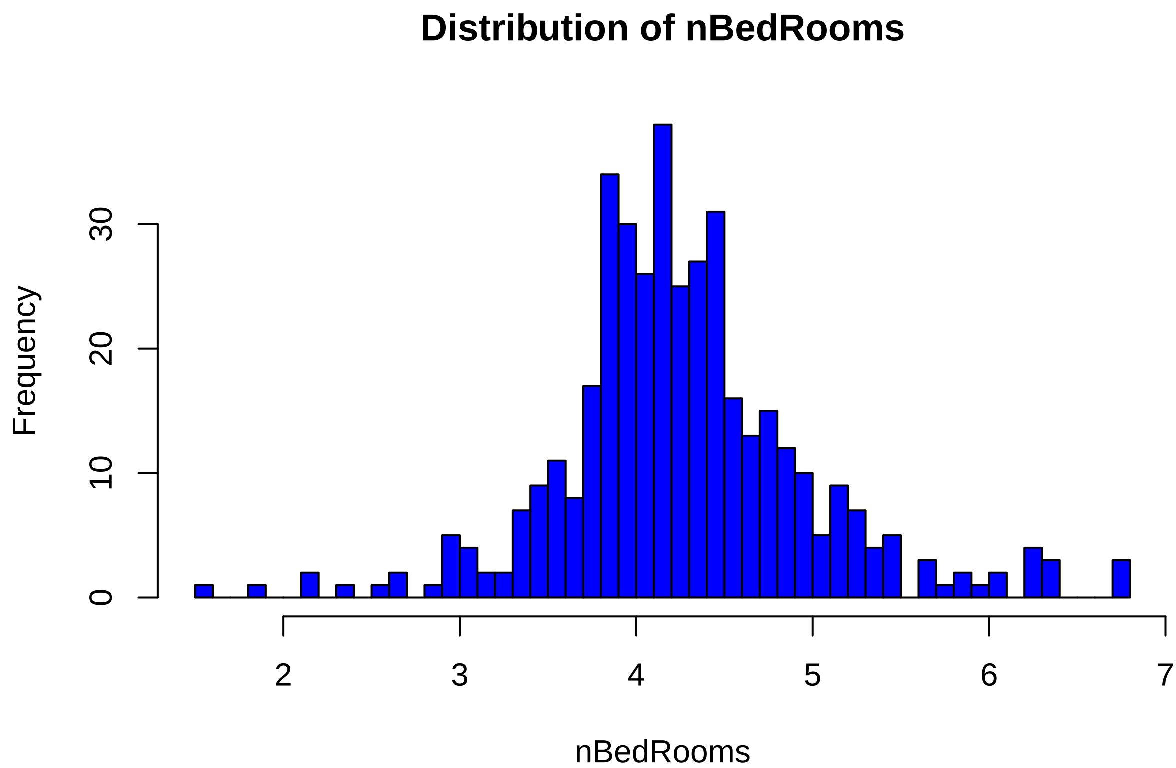

```r
# scattered dist, one high outlier at 72

# nbedrooms
summary(q1.dataset$nBedRooms)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   1.561   3.883   4.193   4.266   4.582   6.780
```

```r
quantile(q1.dataset$nBedRooms, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75,
    0.9, 0.95, 0.99, 1))
```

```
##      1%      5%     10%     25%     50%     75%     90%     95%     99%
## 2.36570 3.26770 3.53550 3.88300 4.19300 4.58175 5.14710 5.45480 6.37523
##    100%
## 6.78000
```

```r
hist(q1.dataset$nBedRooms, breaks = 60, col = "blue", main = "Distribution of nBedRooms",
    xlab = "nBedRooms")
```




```
# Pretty normal
```

Step 2 - Bivariate Analysis

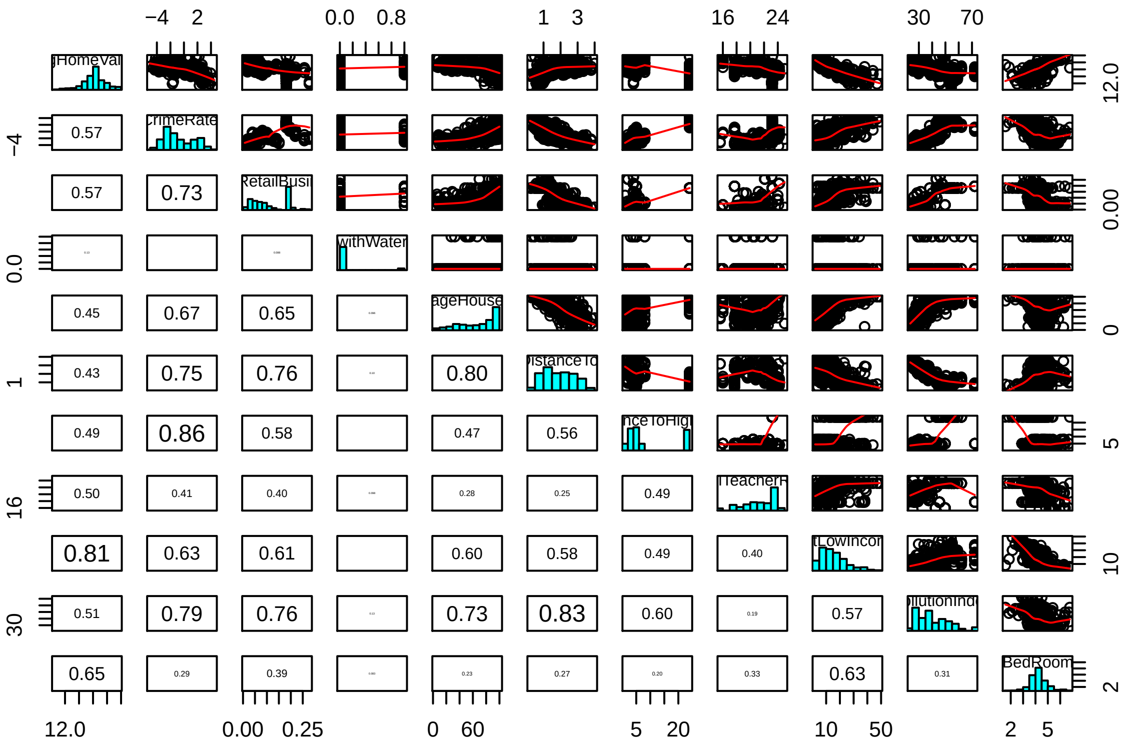
We examine bivariate correlations and scatterplots for all (transformed) variables in the dataset.

Conclusions

1. A lot of variables show strong correlations with each other in the dataset (absolute val of correlation > 0.7). We must be wary of multicollinearity when including these variables together in our regression models which would make our model coefficients lose precision. At the same time, it is important to include necessary variables in order to prevent any omitted variable bias.

- logCrimeRate shows a strong correlation with logdistanceToCity, distanceToHighway and pollutionIndex
- nonRetailBusiness shows a strong correlation with logdistanceToCity and pollutionIndex
- ageHouse also shows a strong correlation with logdistanceToCity and pollutionIndex
- logDistancetoCity shows a strong correlation with pollutionIndex

```r
panel.hist <- function(x, ...) {
    usr <- par("usr")
    on.exit(par(usr))
    par(usr = c(usr[1:2], 0, 1.5))
    h <- hist(x, plot = FALSE)
    breaks <- h$breaks
    nB <- length(breaks)
    y <- h$counts
    y <- y/max(y)
    rect(breaks[-nB], 0, breaks[-1], y, col = "cyan", ...)
}

panel.cor <- function(x, y, digits = 2, prefix = "", cex.cor, ...) {
    usr <- par("usr")
    on.exit(par(usr))
    par(usr = c(0, 1, 0, 1))
    r <- abs(cor(x, y))
    txt <- format(c(r, 0.123456789), digits = digits)[1]
    txt <- paste0(prefix, txt)
    if (missing(cex.cor))
        cex.cor <- 0.8/strwidth(txt)
    text(0.5, 0.5, txt, cex = cex.cor * r)
}

pairs(logHomeValue ~ logCrimeRate_pc + nonRetailBusiness + withWater +
    ageHouse + logDistanceToCity + distanceToHighway + pupilTeacherRatio +
    pctLowIncome + pollutionIndex + nBedRooms, data = q1.dataset, upper.panel = panel.smooth,
    lower.panel = panel.cor, diag.panel = panel.hist)
```



−4 2
0.0 0.8
1 3
16 24
30 70
12.0
0.00
0
5
10
2
−4
0.0
1
16
30
CrimeRate
RetailBusi
withWater
DistanceTo
TeacherR
LowIncon
BedRoom
0.57
0.57 0.73
0.45 0.67 0.65
0.43 0.75 0.76 0.80
0.49 0.86 0.58 0.47 0.56
0.50 0.41 0.40 0.28 0.25 0.49
0.81 0.63 0.61 0.60 0.58 0.49 0.40
0.51 0.79 0.76 0.73 0.83 0.60 0.57
0.65 0.29 0.39 0.27 0.33 0.63 0.31
12.0
0.00 0.25
0 60
5 20
10 50
2 5

Step 3 - Model Estimation

We start off with a naive approach, including all variables in the regression to observe results.

```
model.1 = lm(logHomeValue ~ logCrimeRate_pc + nonRetailBusiness + withWater +
    ageHouse + logDistanceToCity + distanceToHighway + pupilTeacherRatio +
    pctLowIncome + pollutionIndex + nBedRooms, data = q1.dataset)
summary(model.1)
```

```
##
## Call:
## lm(formula = logHomeValue ~ logCrimeRate_pc + nonRetailBusiness +
##     withWater + ageHouse + logDistanceToCity + distanceToHighway +
##     pupilTeacherRatio + pctLowIncome + pollutionIndex + nBedRooms,
##     data = q1.dataset)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.73040 -0.09641 -0.00502  0.09332  0.78653
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)       14.2334735  0.2107218  67.546  < 2e-16 ***
## logCrimeRate_pc   -0.0107344  0.0129755  -0.827 0.408585
## nonRetailBusiness -0.4128304  0.2645968  -1.560 0.119520
## withWater          0.1411053  0.0409450   3.446 0.000631 ***
## ageHouse           0.0001650  0.0006556   0.252 0.801461
## logDistanceToCity -0.1288004  0.0254045  -5.070 6.17e-07 ***
## distanceToHighway -0.0010709  0.0025088  -0.427 0.669728
## pupilTeacherRatio -0.0303139  0.0060208  -5.035 7.33e-07 ***
## pctLowIncome      -0.0238385  0.0018412 -12.947  < 2e-16 ***
## pollutionIndex    -0.0081282  0.0018766  -4.331 1.89e-05 ***
## nBedRooms          0.1028429  0.0192037   5.355 1.46e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1999 on 389 degrees of freedom
## Multiple R-squared:  0.7529, Adjusted R-squared:  0.7465
## F-statistic: 118.5 on 10 and 389 DF,  p-value: < 2.2e-16
```

```
AIC(model.1)
```

```
## [1] -140.1337
```

```
BIC(model.1)
```

```
## [1] -92.23611
```

In this model, we see that several variables do not have statistical significance. We see that the coefficients lack precision, having extremely high standard errors.

Before we move on to more parsimonious models, we will examine interaction variables to see if they add any explanatory power to our model. We have two categorical variables in our dataset: withWater and distanceToHighway (though a numerical variable, it has only nine distinct values, effectively functioning as a categorical). We add all possible interactions with these variables to see if we obtain any noteworthy result.

```
model.2 = lm(logHomeValue ~ logCrimeRate_pc + nonRetailBusiness + withWater +
    ageHouse + logDistanceToCity + distanceToHighway + pupilTeacherRatio +
    pctLowIncome + pollutionIndex + nBedRooms + distanceToHighway:logCrimeRate_pc +
    distanceToHighway:nonRetailBusiness + distanceToHighway:ageHouse +
    distanceToHighway:pupilTeacherRatio + distanceToHighway:pctLowIncome +
    distanceToHighway:pollutionIndex + distanceToHighway:nBedRooms + withWater:pollutionIndex +
    withWater:logCrimeRate_pc + withWater:nonRetailBusiness + withWater:ageHouse +
    withWater:logDistanceToCity + withWater:pupilTeacherRatio + withWater:pctLowIncome +
    withWater:nBedRooms, data = q1.dataset)
summary(model.2)
```

```
##
## Call:
## lm(formula = logHomeValue ~ logCrimeRate_pc + nonRetailBusiness +
##     withWater + ageHouse + logDistanceToCity + distanceToHighway +
##     pupilTeacherRatio + pctLowIncome + pollutionIndex + nBedRooms +
##     distanceToHighway:logCrimeRate_pc + distanceToHighway:nonRetailBusiness +
##     distanceToHighway:ageHouse + distanceToHighway:pupilTeacherRatio +
##     distanceToHighway:pctLowIncome + distanceToHighway:pollutionIndex +
##     distanceToHighway:nBedRooms + withWater:pollutionIndex +
##     withWater:logCrimeRate_pc + withWater:nonRetailBusiness +
##     withWater:ageHouse + withWater:logDistanceToCity + withWater:pupilTeacherRatio +
##     withWater:pctLowIncome + withWater:nBedRooms, data = q1.dataset)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.66948 -0.07603 -0.00873  0.06591  0.68353
##
## Coefficients:
##                                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)                          1.298e+01  4.468e-01  29.055  < 2e-16
## logCrimeRate_pc                      4.846e-02  1.559e-02   3.108 0.002027
## nonRetailBusiness                   -1.489e+00  4.294e-01  -3.468 0.000585
## withWater                            1.801e-01  8.372e-01   0.215 0.829771
## ageHouse                            -3.679e-03  7.789e-04  -4.723 3.30e-06
## logDistanceToCity                   -1.238e-01  2.274e-02  -5.445 9.40e-08
## distanceToHighway                    1.881e-02  7.910e-02   0.238 0.812165
## pupilTeacherRatio                   -3.658e-02  1.793e-02  -2.040 0.042057
## pctLowIncome                        -1.443e-03  2.692e-03  -0.536 0.592234
## pollutionIndex                      -3.252e-04  2.705e-03  -0.120 0.904352
## nBedRooms                            3.753e-01  2.812e-02  13.346  < 2e-16
## logCrimeRate_pc:distanceToHighway   -1.020e-02  1.447e-03  -7.052 8.58e-12
## nonRetailBusiness:distanceToHighway  2.281e-01  8.831e-02   2.583 0.010177
## ageHouse:distanceToHighway           2.572e-04  8.876e-05   2.898 0.003976
## distanceToHighway:pupilTeacherRatio  2.983e-03  3.840e-03   0.777 0.437728
## distanceToHighway:pctLowIncome      -1.189e-03  1.600e-04  -7.429 7.50e-13
## distanceToHighway:pollutionIndex    -7.424e-04  1.844e-04  -4.026 6.88e-05
## distanceToHighway:nBedRooms         -1.827e-02  1.668e-03 -10.949  < 2e-16
## withWater:pollutionIndex            -1.191e-02  5.861e-03  -2.032 0.042887
## logCrimeRate_pc:withWater            6.485e-02  6.100e-02   1.063 0.288479
## nonRetailBusiness:withWater          1.332e+00  1.408e+00   0.946 0.344643
```

```
## withWater:ageHouse                    1.852e-03  2.952e-03   0.627 0.530828
## withWater:logDistanceToCity           8.978e-02  1.229e-01   0.730 0.465649
## withWater:pupilTeacherRatio           2.894e-02  2.386e-02   1.213 0.225988
## withWater:pctLowIncome               -6.662e-03  6.682e-03  -0.997 0.319416
## withWater:nBedRooms                  -1.047e-01  5.846e-02  -1.790 0.074245
##
## (Intercept)                          ***
## logCrimeRate_pc                      **
## nonRetailBusiness                    ***
## withWater
## ageHouse                             ***
## logDistanceToCity                    ***
## distanceToHighway
## pupilTeacherRatio                    *
## pctLowIncome
## pollutionIndex
## nBedRooms                            ***
## logCrimeRate_pc:distanceToHighway    ***
## nonRetailBusiness:distanceToHighway *
## ageHouse:distanceToHighway           **
## distanceToHighway:pupilTeacherRatio
## distanceToHighway:pctLowIncome       ***
## distanceToHighway:pollutionIndex     ***
## distanceToHighway:nBedRooms          ***
## withWater:pollutionIndex             *
## logCrimeRate_pc:withWater
## nonRetailBusiness:withWater
## withWater:ageHouse
## withWater:logDistanceToCity
## withWater:pupilTeacherRatio
## withWater:pctLowIncome
## withWater:nBedRooms                  .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1584 on 374 degrees of freedom
## Multiple R-squared:  0.8508, Adjusted R-squared:  0.8408
## F-statistic: 85.29 on 25 and 374 DF,  p-value: < 2.2e-16
```

```r
AIC(model.2)
```

```
## [1] -311.9024
```

```r
BIC(model.2)
```

```
## [1] -204.1328
```

```r
waldtest(model.1, model.2)
```

```
## Wald test
##
## Model 1: logHomeValue ~ logCrimeRate_pc + nonRetailBusiness + withWater +
```

```
##     ageHouse + logDistanceToCity + distanceToHighway + pupilTeacherRatio +
##     pctLowIncome + pollutionIndex + nBedRooms
## Model 2: logHomeValue ~ logCrimeRate_pc + nonRetailBusiness + withWater +
##     ageHouse + logDistanceToCity + distanceToHighway + pupilTeacherRatio +
##     pctLowIncome + pollutionIndex + nBedRooms + distanceToHighway:logCrimeRate_pc +
##     distanceToHighway:nonRetailBusiness + distanceToHighway:ageHouse +
##     distanceToHighway:pupilTeacherRatio + distanceToHighway:pctLowIncome +
##     distanceToHighway:pollutionIndex + distanceToHighway:nBedRooms +
##     withWater:pollutionIndex + withWater:logCrimeRate_pc + withWater:nonRetailBusiness +
##     withWater:ageHouse + withWater:logDistanceToCity + withWater:pupilTeacherRatio +
##     withWater:pctLowIncome + withWater:nBedRooms
##   Res.Df Df      F    Pr(>F)
## 1    389
## 2    374 15 16.357 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We do see some additional explanatory power through the addition of the interaction terms as we obtain a model with higher R square, lower AIC and lower BIC, as well as a significant Wald Test p value. However, apart from having a model which is extremely difficult to interpret, we also notice that most of the coefficient estimates are extremely small, having very little practical significance.

Now, we want to narrow down our model to include only variables that really add to the explanatory power of the model, that reduce multicollinearity, that provide some valuable practical significance, while meeting the think-tank's specific ask of desirable neighbourhood features and environmental features' relation to home values.

We remove the following variables:

1. nonRetailBusiness: It has a high correlation with several of the variables in the dataset, and is not of direct interest to answering the question asked.
2. ageHouse: It is not of direct consequence to the question asked.
3. DistanceToCity has a high correlation with pollutionIndex, a variable we are definitely interested in, so we remove it to reduce the loss of precision that comes with multicollinearity
4. nBedRooms: It is not of direct consequence to the question asked.
5. distanceToHighway interactions except the interaction with log crime: This is the only interaction with a variable still in the model which has statistical and practical significance.
6. withWater interactions except the interaction with pollutionIndex: This interaction seems to have some practical as well as statistical significance.

```r
model.3 = lm(logHomeValue ~ logCrimeRate_pc + withWater + distanceToHighway +
    pctLowIncome + pollutionIndex + distanceToHighway:logCrimeRate_pc +
    withWater:pollutionIndex, data = q1.dataset)
summary(model.3)
```

```
##
## Call:
## lm(formula = logHomeValue ~ logCrimeRate_pc + withWater + distanceToHighway +
##     pctLowIncome + pollutionIndex + distanceToHighway:logCrimeRate_pc +
##     withWater:pollutionIndex, data = q1.dataset)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
```

```
## -0.66018 -0.14040 -0.02863  0.10239  0.86916
##
## Coefficients:
##                                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)                     13.643386   0.086436 157.843  < 2e-16
## logCrimeRate_pc                  0.044515   0.015860   2.807  0.00525
## withWater                        0.476923   0.144643   3.297  0.00107
## distanceToHighway                0.004164   0.003324   1.253  0.21110
## pctLowIncome                    -0.029823   0.001600 -18.640  < 2e-16
## pollutionIndex                  -0.002229   0.001718  -1.297  0.19540
## logCrimeRate_pc:distanceToHighway -0.005868   0.001226  -4.786 2.41e-06
## withWater:pollutionIndex        -0.007204   0.003038  -2.371  0.01820
##
## (Intercept)                     ***
## logCrimeRate_pc                 **
## withWater                       **
## distanceToHighway
## pctLowIncome                    ***
## pollutionIndex
## logCrimeRate_pc:distanceToHighway ***
## withWater:pollutionIndex        *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2189 on 392 degrees of freedom
## Multiple R-squared:  0.7013, Adjusted R-squared:  0.696
## F-statistic: 131.5 on 7 and 392 DF,  p-value: < 2.2e-16
```

```r
AIC(model.3)
```

```
## [1] -70.30455
```

```r
BIC(model.3)
```

```
## [1] -34.38137
```

We see that the distanceToHighway and pollutionIndex variables don't seem to have statistical significance. We will remove the distanceToHighway variable since it does not have direct consequence to the question asked. Further, we remove the logCrimeRate variable, since it has a high correlation with pollutionIndex, a variable of importance to us. We do this to observe if its removal increases the precision of the pollutonIndex variable. Note that this also means that we remove the interaction of distanceToHighway and logCrimeRate

```r
model.4 = lm(logHomeValue ~ withWater + pctLowIncome + pollutionIndex +
    withWater:pollutionIndex, data = q1.dataset)
summary(model.4)
```

```
##
## Call:
## lm(formula = logHomeValue ~ withWater + pctLowIncome + pollutionIndex +
##     withWater:pollutionIndex, data = q1.dataset)
##
## Residuals:
```

```
##      Min       1Q   Median       3Q      Max
## -0.67726 -0.14224 -0.02364  0.10665  0.85551
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)              13.639280   0.043563 313.090  < 2e-16 ***
## withWater                 0.474642   0.148998   3.186  0.00156 **
## pctLowIncome             -0.032366   0.001525 -21.218  < 2e-16 ***
## pollutionIndex           -0.002305   0.001286  -1.792  0.07382 .
## withWater:pollutionIndex -0.006498   0.003124  -2.080  0.03820 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2273 on 395 degrees of freedom
## Multiple R-squared:  0.6754, Adjusted R-squared:  0.6721
## F-statistic: 205.4 on 4 and 395 DF,  p-value: < 2.2e-16
```

```
AIC(model.4)
```

```
## [1] -43.00319
```

```
BIC(model.4)
```

```
## [1] -19.0544
```

We still obtain no significance for our pollutionIndex variable, and are unable to make any confident claims about this variable's impact on homeValue. Perhaps this variable, which was significant in earlier models with more variables, is losing precision due to an omitted variable bias.

We try adding back the pupilTeacherRatio variable to see if that makes any difference to the model.

```
model.5 = lm(logHomeValue ~ withWater + pctLowIncome + pupilTeacherRatio +
    pollutionIndex + withWater:pollutionIndex, data = q1.dataset)
summary(model.5)
```

```
##
## Call:
## lm(formula = logHomeValue ~ withWater + pctLowIncome + pupilTeacherRatio +
##     pollutionIndex + withWater:pollutionIndex, data = q1.dataset)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.66742 -0.12448 -0.01229  0.10960  0.87460
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)              14.391272   0.117353 122.632  < 2e-16 ***
## withWater                 0.493635   0.141065   3.499  0.00052 ***
## pctLowIncome             -0.028714   0.001539 -18.654  < 2e-16 ***
## pupilTeacherRatio        -0.037394   0.005463  -6.844 2.95e-11 ***
## pollutionIndex           -0.002505   0.001217  -2.057  0.04033 *
## withWater:pollutionIndex -0.007437   0.002961  -2.512  0.01240 *
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2152 on 394 degrees of freedom
## Multiple R-squared:  0.7099, Adjusted R-squared:  0.7062
## F-statistic: 192.8 on 5 and 394 DF,  p-value: < 2.2e-16
```

```r
AIC(model.5)
```

```
## [1] -85.94111
```

```r
BIC(model.5)
```

```
## [1] -58.00086
```

We obtain statistical significance for all our coefficients, while maintaining a high R square value. While AIC and BIC values are not the lowest, we prefer the parsimony of this model, and choose this as our final model to present to the think-tank.

We do not consider introducing instrument variables to the model for the following reasons:

1. None of our predictors seems to have any correlation with the error term, making them all exogenous (shown above).
2. While some variables do meet the criteria for instrument relevance, they are correlated with multiple variables in the model, so they do not make good overall candidates for instruments as they would introduct multicollinearity to the prediction of the variable for which they would serve as instruments.

```r
cor(q1.dataset$pctLowIncome, model.5$residuals)
```

```
## [1] -9.991986e-18
```

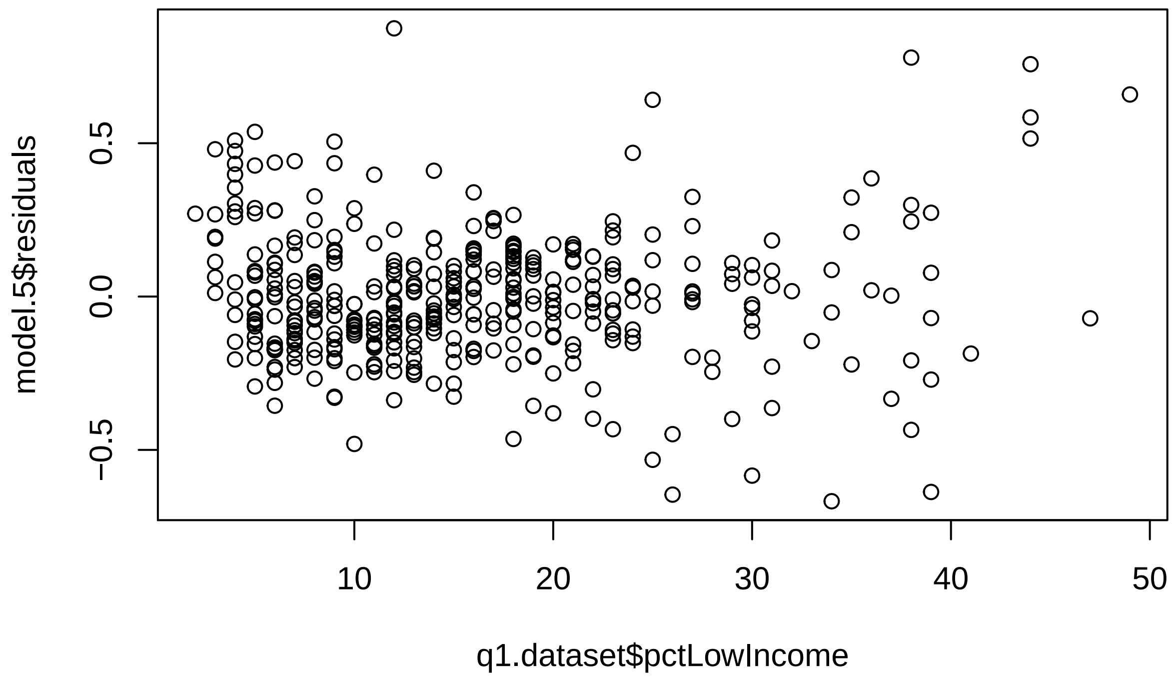
```r
cor(q1.dataset$pollutionIndex, model.5$residuals)
```

```
## [1] 5.264698e-16
```

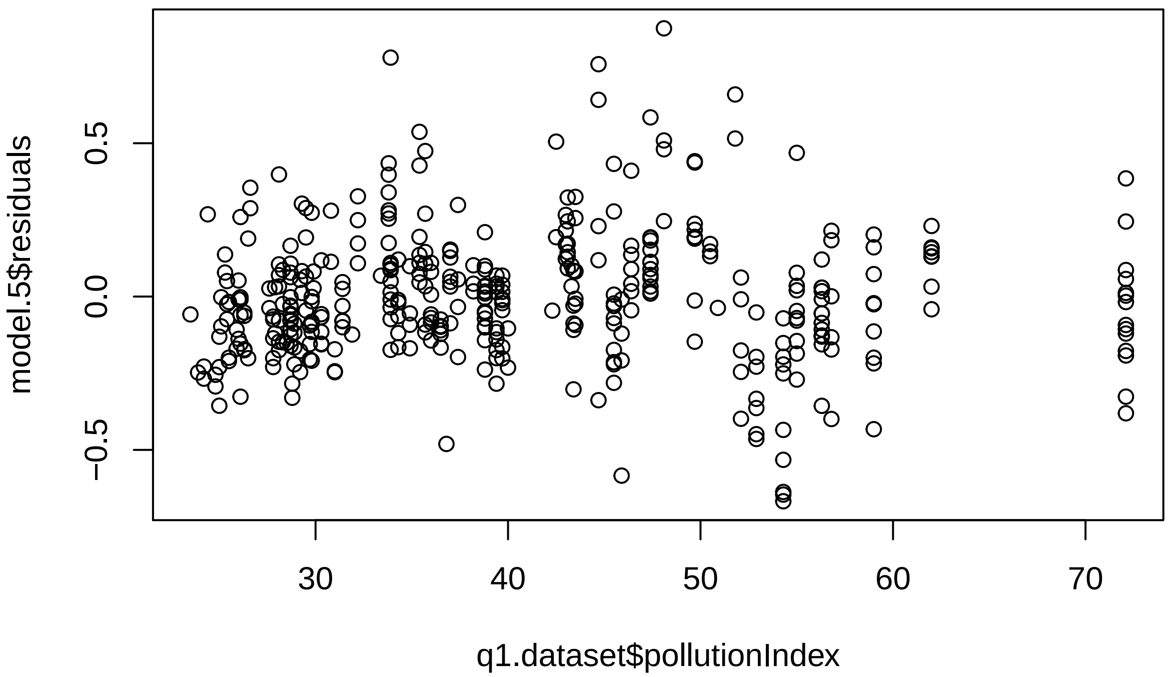
```r
cor(q1.dataset$pupilTeacherRatio, model.5$residuals)
```

```
## [1] 2.849975e-15
```

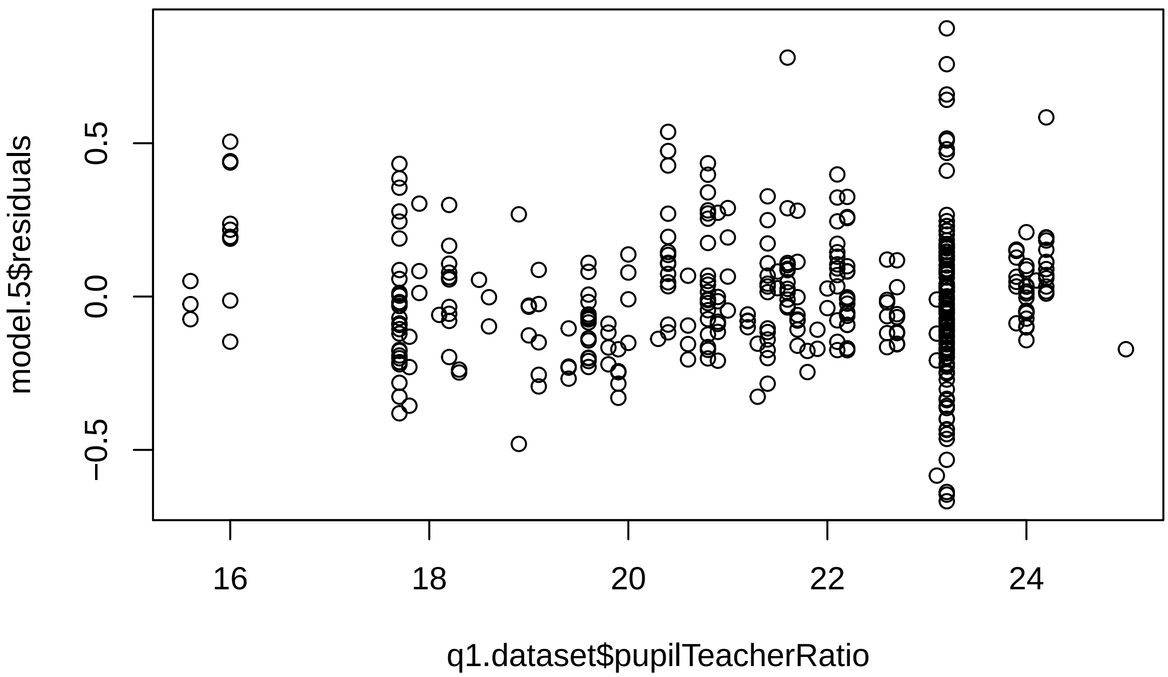
```r
plot(q1.dataset$pctLowIncome, model.5$residuals)
```




```
plot(q1.dataset$pollutionIndex, model.5$residuals)
```




```r
plot(q1.dataset$pupilTeacherRatio, model.5$residuals)
```



Now, we look at diagnostics for our final model.

1. The residuals vs fitted plot shows a very slight upward trend. The slope is negligible, so we assume zero conditional mean to hold.
2. Errors follow a close to normal distribution. In either case, we have 400 observations, enabling us to rely on OLS asymptotics
3. The scale-location plot shows some trend which is not a significant cause for concern. It shows some heteroskedasticity, which we account for by taking robust standard errors below. Variables in our model remain significant. The Wald-Test shows that the model also remains significant.
4. The Residuals vs Leverrage plot shows some outliers but no major cause for concern.

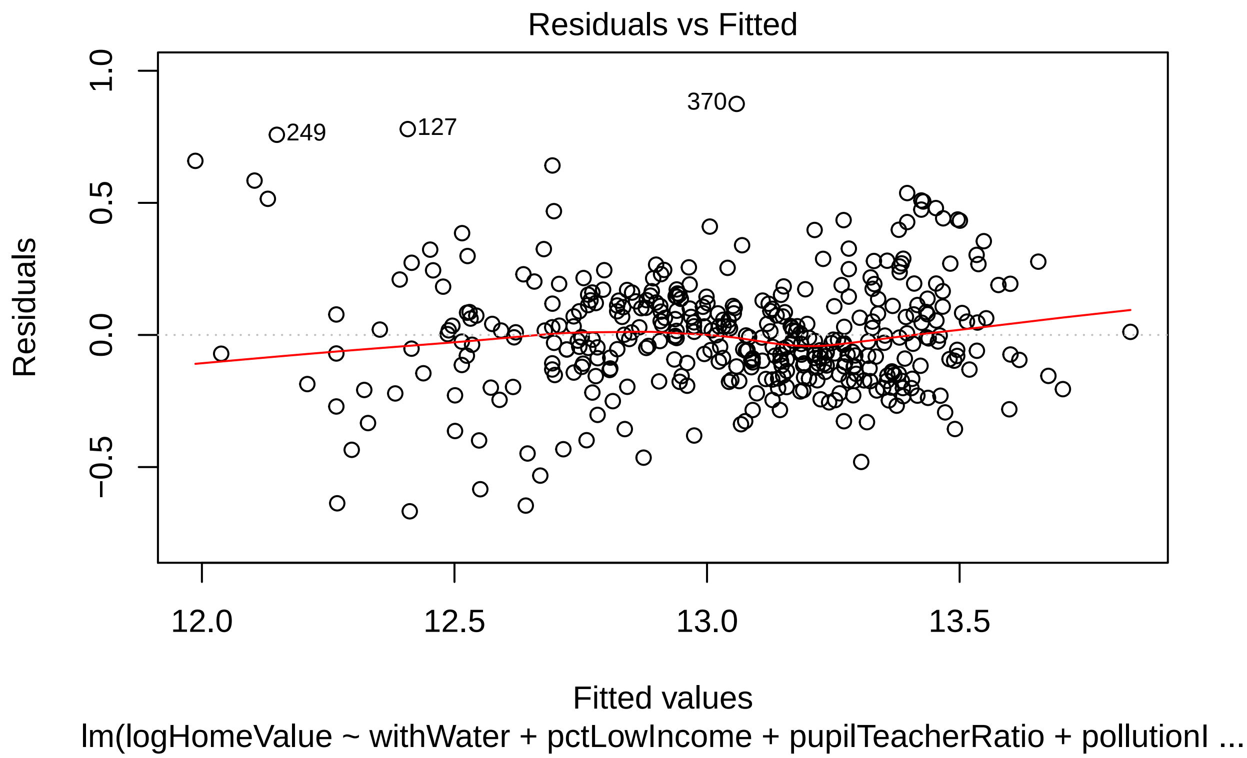
```
coeftest(model.5)
```

```
##
## t test of coefficients:
##
##                            Estimate Std. Error  t value  Pr(>|t|)
## (Intercept)              14.3912717  0.1173529 122.6325 < 2.2e-16 ***
## withWater                 0.4936346  0.1410650   3.4993 0.0005197 ***
## pctLowIncome             -0.0287143  0.0015393 -18.6535 < 2.2e-16 ***
## pupilTeacherRatio        -0.0373937  0.0054634  -6.8444 2.952e-11 ***
## pollutionIndex           -0.0025045  0.0012175  -2.0572 0.0403278 *
## withWater:pollutionIndex -0.0074373  0.0029605  -2.5122 0.0123980 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

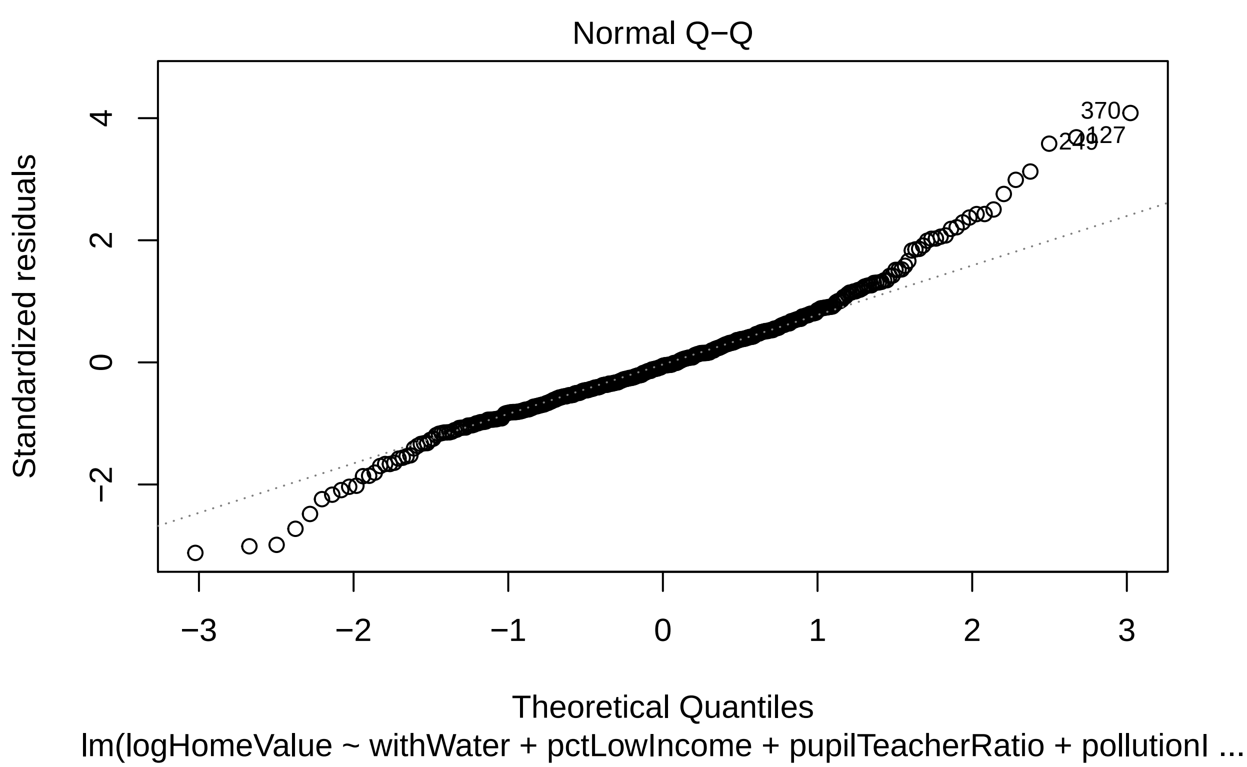
```r
waldtest(model.5, vcov = vcovHC)
```

```
## Wald test
##
## Model 1: logHomeValue ~ withWater + pctLowIncome + pupilTeacherRatio +
##     pollutionIndex + withWater:pollutionIndex
## Model 2: logHomeValue ~ 1
##   Res.Df Df      F    Pr(>F)
## 1    394
## 2    399 -5 123.26 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

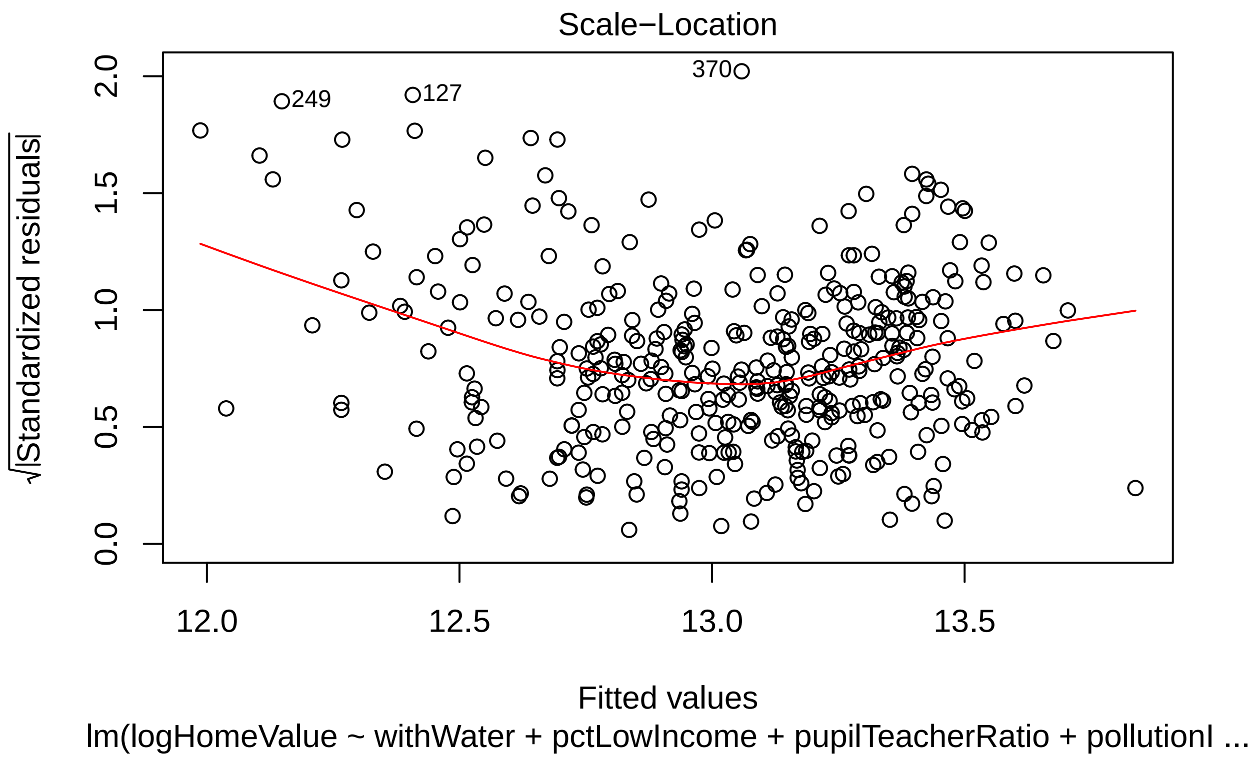
```r
plot(model.5)
```

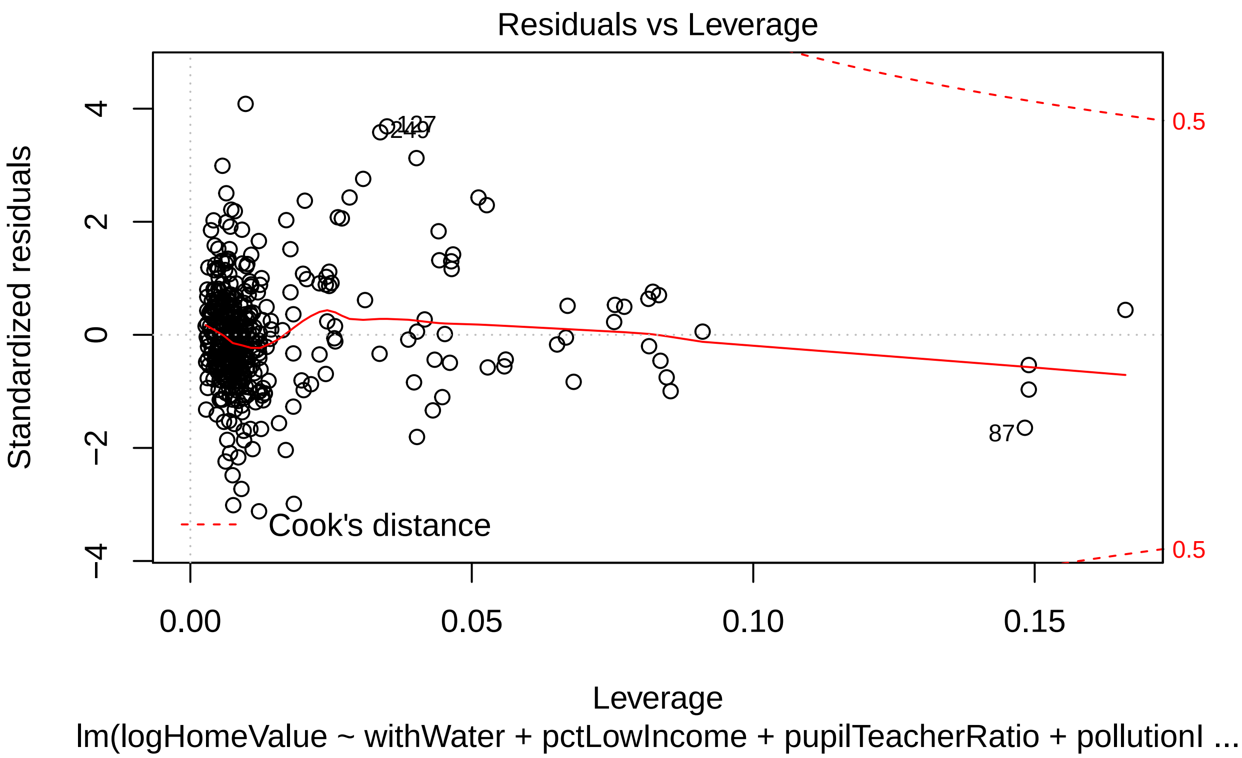


Residuals vs Fitted
Residuals
Fitted values
lm(logHomeValue ~ withWater + pctLowIncome + pupilTeacherRatio + pollutionI ...
370
249
127
1.0
0.5
0.0
−0.5
12.0
12.5
13.0
13.5



Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(logHomeValue ~ withWater + pctLowIncome + pupilTeacherRatio + pollutionI ...
370
249
127
4
2
0
−2
−3
−2
−1
0
1
2
3


Scale−Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
12.0
12.5
13.0
13.5
Fitted values
lm(logHomeValue ~ withWater + pctLowIncome + pupilTeacherRatio + pollutionI ...
249
127
370



Residuals vs Leverage
Standardized residuals
Leverage
lm(logHomeValue ~ withWater + pctLowIncome + pupilTeacherRatio + pollutionI ...
Cook's distance
0.5
0.5
127
249
87
4
2
0
−2
−4
0.00
0.05
0.10
0.15

Step 4 - Final Model Conclusions

Conclusion

Below we present the a comparison of initial and the final models run. We see that despite removing 6 variables, we see a reduction of only around 4% in R square, implying that we have preserved most of the explanatory power of the model, while sticking with parsimony.

```
stargazer(model.1, model.5, type = "text")
```

```
##
## ===========================================================================
##                                           Dependent variable:
##                              ----------------------------------------------
##                                               logHomeValue
##                                         (1)                    (2)
## ---------------------------------------------------------------------------
## logCrimeRate_pc                       -0.011
##                                       (0.013)
##
## nonRetailBusiness                     -0.413
##                                       (0.265)
##
## withWater                             0.141***               0.494***
##                                       (0.041)                (0.141)
##
## ageHouse                              0.0002
##                                       (0.001)
##
## logDistanceToCity                    -0.129***
##                                       (0.025)
##
## distanceToHighway                     -0.001
##                                       (0.003)
##
## pupilTeacherRatio                    -0.030***              -0.037***
##                                       (0.006)                (0.005)
##
## pctLowIncome                         -0.024***              -0.029***
##                                       (0.002)                (0.002)
##
## pollutionIndex                       -0.008***               -0.003**
##                                       (0.002)                (0.001)
##
## nBedRooms                             0.103***
##                                       (0.019)
##
## withWater:pollutionIndex                                     -0.007**
##                                                              (0.003)
##
## Constant                             14.233***              14.391***
##                                       (0.211)                (0.117)
##
```

```
## -----------------------------------------------------------------------------
## Observations                           400                      400
## R2                                    0.753                    0.710
## Adjusted R2                           0.747                    0.706
## Residual Std. Error             0.200 (df = 389)          0.215 (df = 394)
## F Statistic                 118.510*** (df = 10; 389) 192.789*** (df = 5; 394)
## =============================================================================
## Note:                                             *p<0.1; **p<0.05; ***p<0.01
```

Explanation of the Model

The final results from the model we present to the think-tank are as follows:

1. Home values are 50% greater for homes located within 5 miles of water.
2. For every unit percentage increase in low-income households, home values are close to 3% lower.
3. For every one unit increase in the pupilTeacherRatio in a neighbourhood, home values are close to 4% lower.
4. For an additional unit on the pollutionIndex, we expect to see a decrease in home value of 0.2% if it is in a neighbourhood not within 5 miles of a water body. However, if it is within 5 miles of a water body, we expect an almost 1% decrease in home value.

Part 2 (25 points): Modeling and Forecasting a Real-World Macroeconomic / Financial time series

The series appears to be a time series of financial data, presumably one of a daily closing price of some financial instrumemt or index.

We observe that the time series is non-stationary in the mean. That observation, is coupled with the fact that the PACF of the time series indicates a correlation at lag 1 that ressembles and AR(1) series. Therefore we can attempt to diff the time series to see if the resulting series is stationary in the mean.

The plot of the original financial series does not indicate any amount of seasonality. The observation is confirmed by the ACF which doesn't display any significant amount of correlation at any lag. We will therefore not try to fit a model that includes a seasonal component to the data. We now proceed to analyze the first difference of the time series.

We get confirmation from the 1st difference series obtained that the original financial series likely has an AR(1) component as the first difference series is stationary in the mean. We also observe that after the first difference is taken, the ACF of the series, just like the PACF suggest white noise dynamics. We have no reason to suspect an additional MA component in the original series. But the first differential series does show clustered volatility in the variance.

```r
# Load the data and describe it
data <- read.csv(file.path("lab3_series02.csv"))

# Describing Series
str(data)
```

```
## 'data.frame':    2332 obs. of  2 variables:
##  $ X         : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ DXCM.Close: num  9.88 9.79 9.68 9.64 9.42 9.47 9.16 8.99 8.6 8.81 ...
```

```r
summary(data)
```

```
##        X            DXCM.Close
##  Min.   :   1.0   Min.   :  1.390
##  1st Qu.: 583.8   1st Qu.:  8.188
##  Median :1166.5   Median : 12.355
##  Mean   :1166.5   Mean   : 23.210
##  3rd Qu.:1749.2   3rd Qu.: 32.565
##  Max.   :2332.0   Max.   :101.910
```

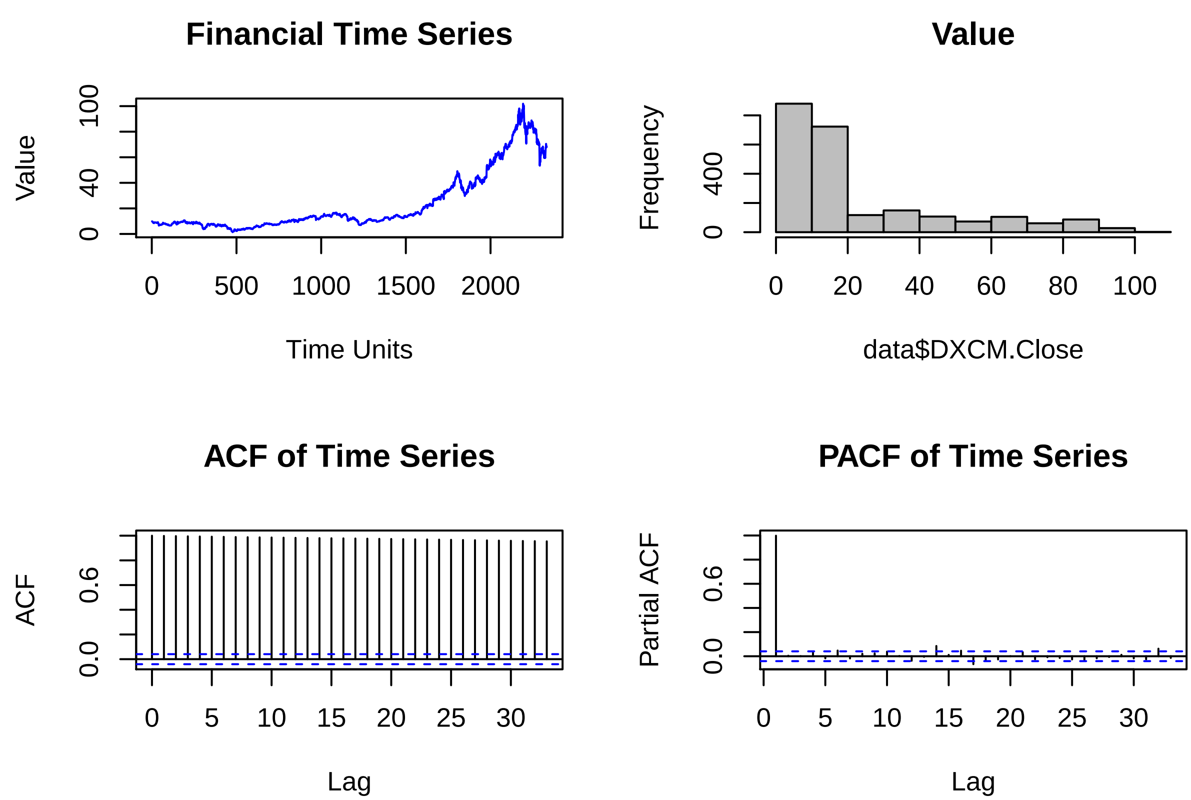
```r
cbind(head(data), tail(data))
```

```
##   X DXCM.Close    X DXCM.Close
## 1 1       9.88 2327      67.63
## 2 2       9.79 2328      70.49
## 3 3       9.68 2329      67.79
## 4 4       9.64 2330      68.72
## 5 5       9.42 2331      68.43
## 6 6       9.47 2332      68.08
```

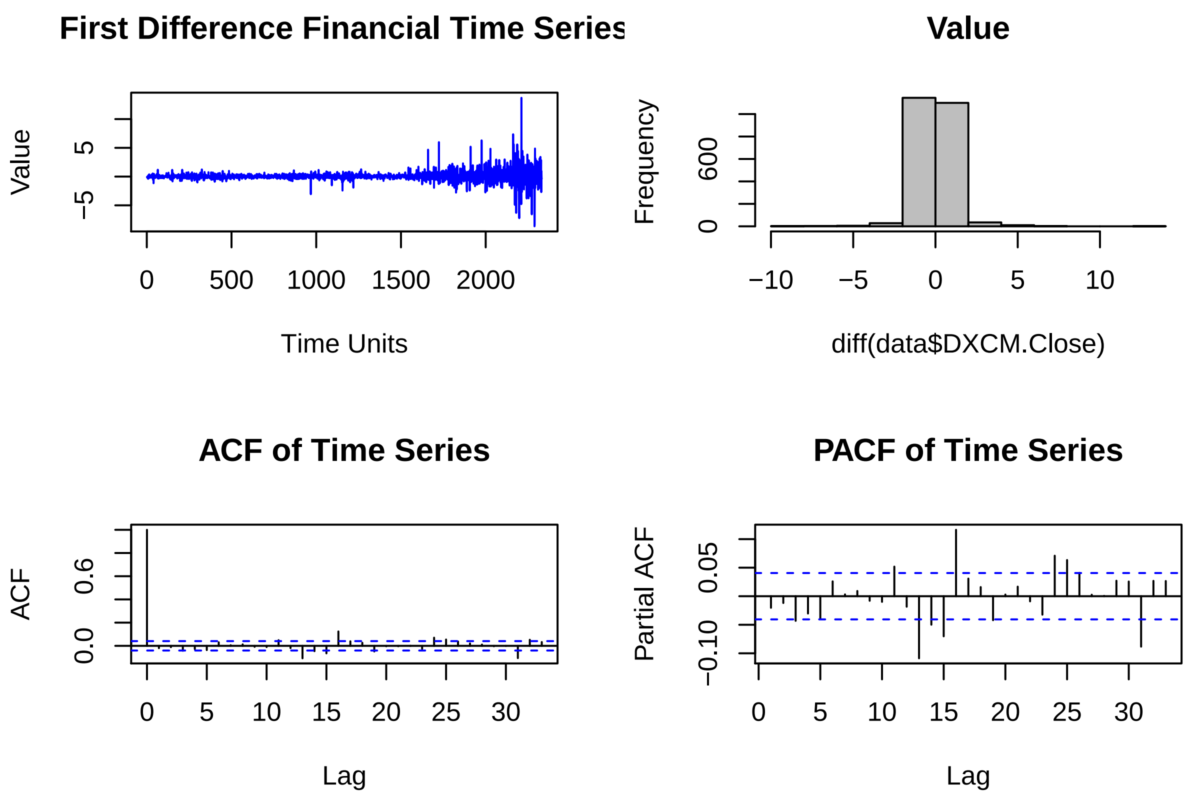
```r
quantile(as.numeric(data$DXCM.Close), c(0, 0.01, 0.05, 0.1, 0.25,
    0.5, 0.75, 0.9, 0.95, 0.99, 1))
```

```
##       0%       1%       5%      10%      25%      50%      75%      90% 
##   1.3900   2.7531   4.1700   6.1630   8.1875  12.3550  32.5650  63.5210 
##      95%      99%     100% 
##  80.0430  91.6887 101.9100
```

```r
# EDA on series
par(mfrow = c(2, 2))
plot.ts(data$DXCM.Close, main = "Financial Time Series", ylab = "Value",
    xlab = "Time Units", col = "blue")
hist(data$DXCM.Close, col = "gray", main = "Value")
acf(data$DXCM.Close, main = "ACF of Time Series")
pacf(data$DXCM.Close, main = "PACF of Time Series")
```




```r
# EDA on the first difference series
par(mfrow = c(2, 2))
plot.ts(diff(data$DXCM.Close), main = "First Difference Financial Time Series",
    ylab = "Value", xlab = "Time Units", col = "blue")
hist(diff(data$DXCM.Close), col = "gray", main = "Value")
acf(diff(data$DXCM.Close), main = "ACF of Time Series")
pacf(diff(data$DXCM.Close), main = "PACF of Time Series")
```



We now try to determine a best SARIMA model based on the best AIC for that series. Interestingly, the best model based on AIC is a seasonal model with (p,d,q,P,D,Q) of values (0, 0, 0, 0, 1, 0). If we assume a frequency of one, that model is the same as the second best model, which has orders (p,d,q,P,D,Q) of (0, 1, 0, 0, 0, 0). The results confirm the observation we made of a lag 1 correlation looking at the PACF of the financial time series We therefore decide to use (p,d,q,P,D,Q)=(0, 1, 0, 0, 0, 0) as our fitted model going forward.

```r
# data.best <- get.best.sarima(data$DXCM, maxord=rep(3,6),1)
# data.best$best
# data.best$others[order(data.best$others$aics)[1:20],]
```

The fitted model is simply a first differential series, and therefore has no parameters which need 95 confidence intervals that we need to validate.

```r
# Fitting a first difference series to our model
data.fit <- Arima(data$DXCM, order = c(1, 0, 0), seasonal = list(order = c(0,
    0, 0)), method = "CSS-ML")
data.res <- data.fit$resid
quantile(as.numeric(data.res), c(0, 0.01, 0.05, 0.1, 0.25, 0.5,
    0.75, 0.9, 0.95, 0.99, 1))
```

```
##           0%           1%           5%          10%          25% 
## -8.624611999 -2.432495919 -1.092944118 -0.561843680 -0.235516419 
##          50%          75%          90%          95%          99% 
## -0.008677486  0.247368331  0.661566155  1.151658704  2.879518631 
##         100%
```

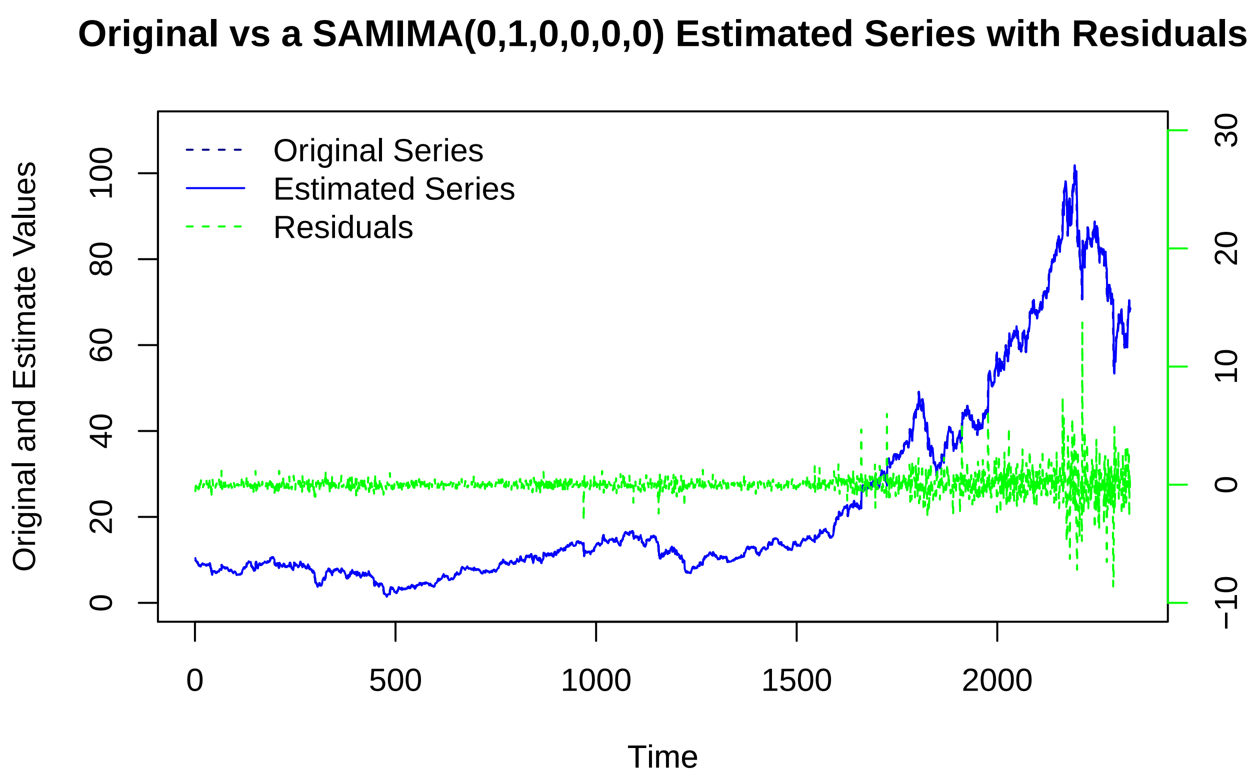
```
## 13.725399392
```

```r
t(confint(data.fit))
```

```
##                ar1  intercept
## 2.5 %  0.9982756 -17.00436
## 97.5 % 1.0005853  69.11511
```

We now perform in-sample fit using the fitted series to assess our fitted model. The fitted series models the original series very well and the model selection seems appropriat based on in-sample fit.

```r
# Performing in-sample fit using our fitted series
par(mfrow = c(1, 1))
plot.ts(data$DXCM, col = "navy", lty = 2, main = "Original vs a SAMIMA(0,1,0,0,0,0) Estimated Series wi
    ylab = "Original and Estimate Values", ylim = c(0, 110))
par(new = T)
plot(fitted(data.fit), col = "blue", axes = F, ylab = "", ylim = c(0,
    110))
leg.txt <- c("Original Series", "Estimated Series", "Residuals")
legend("topleft", legend = leg.txt, lty = c(2, 1, 2), col = c("navy",
    "blue", "green"), bty = "n", cex = 1)
par(new = T)
plot.ts(data.res, axes = F, xlab = "", ylab = "", col = "green",
    ylim = c(-10, 30), pch = 1, lty = 2)
axis(side = 4, col = "green")
mtext("Residuals", side = 4, line = 2, col = "green")
```



To further assess the fitted series, We now perform 36 steps backtesting. What the out of sample forecast shows is that the original series falls for the most part within the 80% confidence interval and almost totally within the 95% confidence interval of the prediction. However, we can see from the plot of the financial time series that the volatility of the series seems to increase over time. Clustered volatility in the variance likely explain this dynamic. We next turn our eyes to the residuals of the fitted series and to analysis of the dynamics of its variance.

```
# Performing 36 steps backtesting using our fitted series
data.fit.back <- Arima(data$DXCM[1:(length(data$DXCM) - 36)],
    order = c(0, 1, 0), seasonal = list(order = c(0, 0, 0)),
    method = "CSS-ML")
summary(data.fit.back)
```

```
## Series: data$DXCM[1:(length(data$DXCM) - 36)]
## ARIMA(0,1,0)
##
## sigma^2 estimated as 0.8223:  log likelihood=-3031.91
## AIC=6065.83   AICc=6065.83   BIC=6071.57
##
## Training set error measures:
##                      ME      RMSE       MAE         MPE     MAPE      MASE
## Training set 0.02262625 0.9065937 0.4574782 0.007157606 2.477381 0.9995739
##                     ACF1
## Training set -0.01924297
```

```
length(fitted(data.fit.back))
```

```
## [1] 2296
```

```
length(data.fit.back$resid)
```

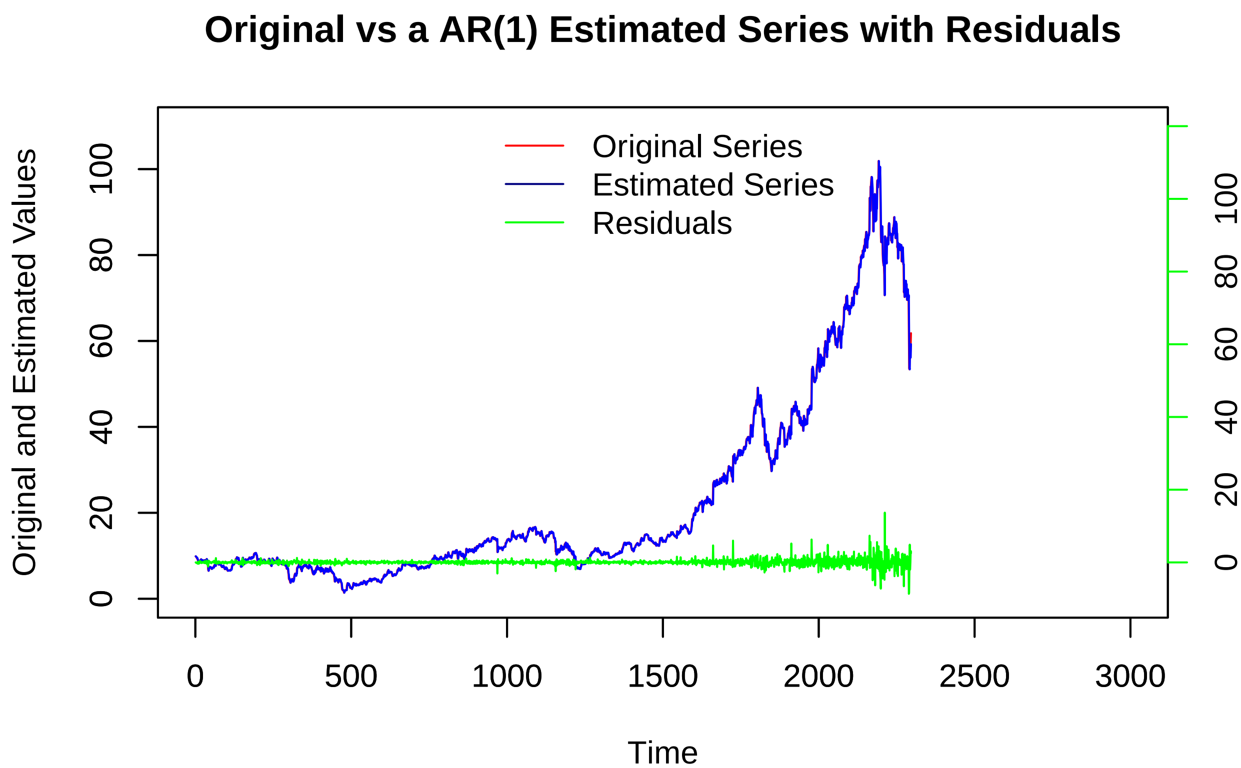
```
## [1] 2296
```

```
df = cbind(data$DXCM[1:(length(data$DXCM) - 36)], fitted(data.fit.back),
    data.fit.back$resid)
colnames(df) = c("orig_series", "fitted_vals", "resid")
head(df)
```

```
##      orig_series fitted_vals        resid
## [1,]        9.88     9.87012  0.009879995
## [2,]        9.79     9.88000 -0.090000000
## [3,]        9.68     9.79000 -0.110000000
## [4,]        9.64     9.68000 -0.040000000
## [5,]        9.42     9.64000 -0.220000000
## [6,]        9.47     9.42000  0.050000000
```

```
# Step 1: Plot the original and estimate series
par(mfrow = c(1, 1))
plot.ts(df[, "orig_series"], col = "red", main = "Original vs a AR(1) Estimated Series with Residuals",
    ylab = "Original and Estimated Values", xlim = c(0, 3000),
    ylim = c(0, 110))
```

```r
par(new = T)
plot.ts(df[, "fitted_vals"], col = "blue", axes = T, xlab = "",
    ylab = "", xlim = c(0, 3000), ylim = c(0, 110))
leg.txt <- c("Original Series", "Estimated Series", "Residuals")
legend("top", legend = leg.txt, lty = 1, col = c("red", "navy",
    "green"), bty = "n", cex = 1)
par(new = T)
plot.ts(df[, "resid"], axes = F, xlab = "", ylab = "", col = "green",
    xlim = c(0, 3000), ylim = c(-10, 120), pch = 1)
axis(side = 4, col = "green")
mtext("Residuals", side = 4, line = 2, col = "green")
```

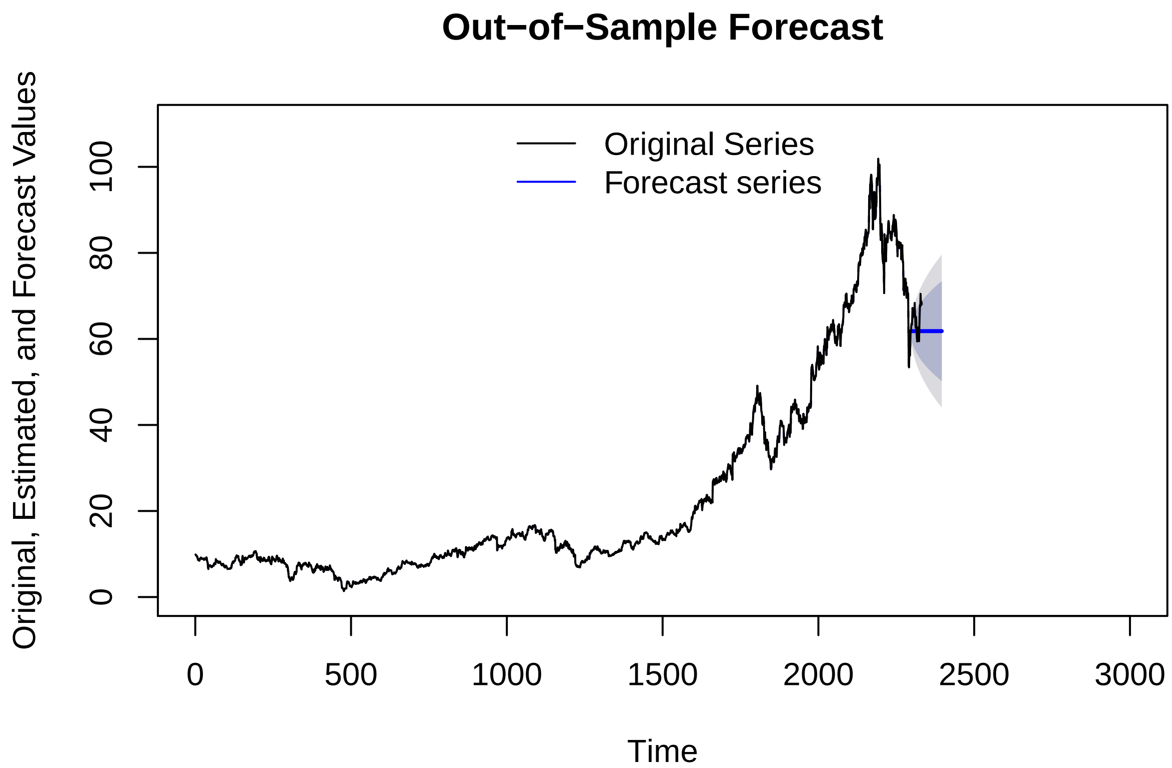



```r
# Step 2: Out of sample forecast
data.fit.back.fcast <- forecast.Arima(data.fit.back, h = 100)
length(data.fit.back.fcast$mean)
```

```
## [1] 100
```

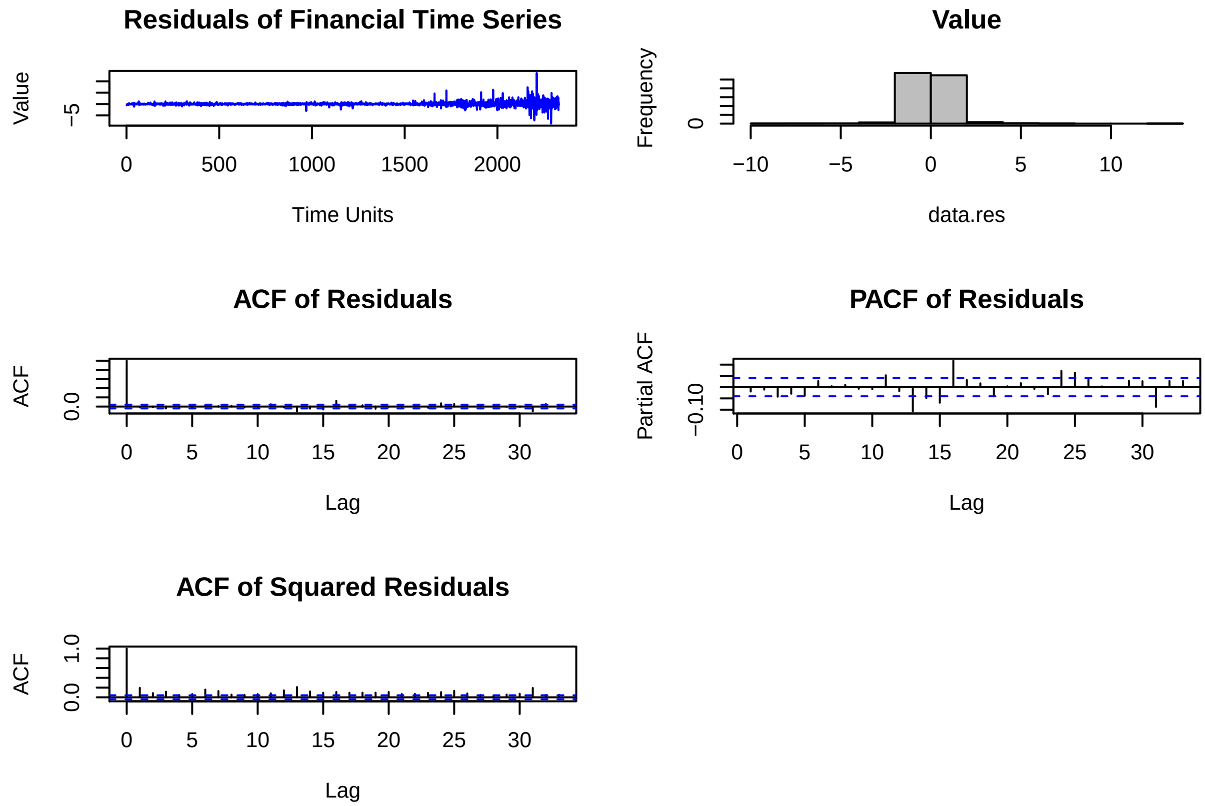
```r
par(mfrow = c(1, 1))
plot(data.fit.back.fcast, lty = 2, col = "navy", main = "Out-of-Sample Forecast",
    ylab = "Original, Estimated, and Forecast Values", xlim = c(0,
        3000), ylim = c(0, 110))
par(new = T)
plot.ts(data$DXCM, axes = F, lty = 1, xlim = c(0, 3000), ylim = c(0,
```

```
    110), ylab = "")
leg.txt <- c("Original Series", "Forecast series")
legend("top", legend = leg.txt, lty = 1, col = c("black", "blue"),
    bty = "n", cex = 1)
```



We observe from the residual time series that the variance of the series is non-stationary. The series exhibits volatility with a variance changing in a regular way. It exhibits conditional heteroskedasticity. An observation of the PACF of the squared residuals series provides confirmation of the variance dynamics. Therefore, we decide to model its residuals using GARCH

```
# Plot the residuals time series
par(mfrow = c(3, 2))
plot.ts(data.res, main = "Residuals of Financial Time Series",
    ylab = "Value", xlab = "Time Units", col = "blue")
hist(data.res, col = "gray", main = "Value")
acf(data.res, main = "ACF of Residuals")
pacf(data.res, main = "PACF of Residuals")
acf(data.res^2, main = "ACF of Squared Residuals")
```



We chose the default (p, q) = (1, 1) parameters of the function for our GARCH model. The parameters of the model are all significant based on a 95% confidence interval.

We observe from the ACF of the residuals of the GARCH fitted series that they have the charactersitics of white noise with mostly non-significant correlations at all lags of the ACF. What the GARCH model of the residuals tells is that we can expect more or less volatility through the forecast of the point series that invalidate the confidence interval of our predictions since those were made with the assumption of a stationary variance.

```
# Model the residuala of the financila time series using
# GARCH
data.garch <- garch(data.res, trace = F)
```

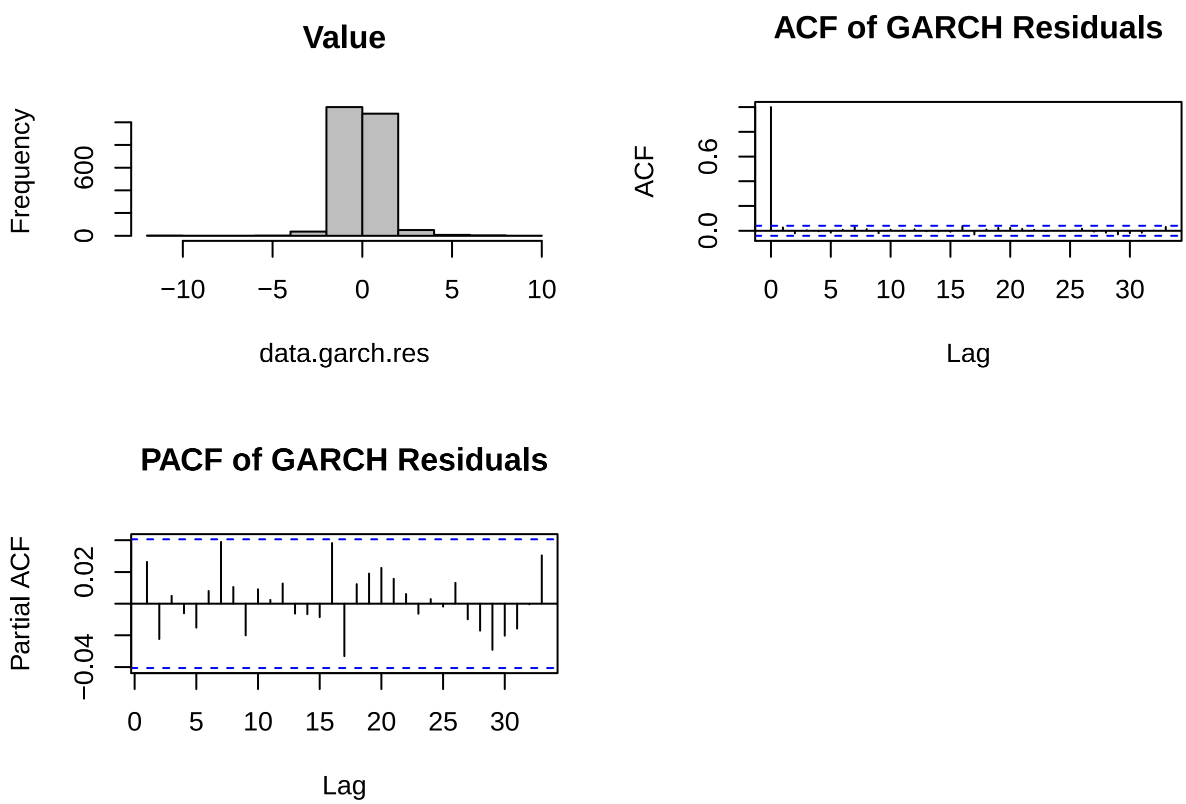
```
## Warning in sqrt(pred$e): NaNs produced
```

```
t(confint(data.garch))
```

```
##                  a0         a1        b1
## 2.5 %  5.785885e-05 0.03064265 0.9652906
## 97.5 % 4.276349e-04 0.03947031 0.9732850
```

```
data.garch.res <- resid(data.garch)[-1]

# Plot a histogram, ACF and PACF of the residuals after
# fitting a GARCH model
```

```r
par(mfrow = c(2, 2))
hist(data.garch.res, col = "gray", main = "Value")
acf(data.garch.res, na.action = na.pass, main = "ACF of GARCH Residuals")
pacf(data.garch.res, na.action = na.pass, main = "PACF of GARCH Residuals")
```



With the previous observations in mind, we still use the fitted series to predict 36 steps ahead. We will later adjust the confidence intervals of the predictions using our fitted GARCH model.

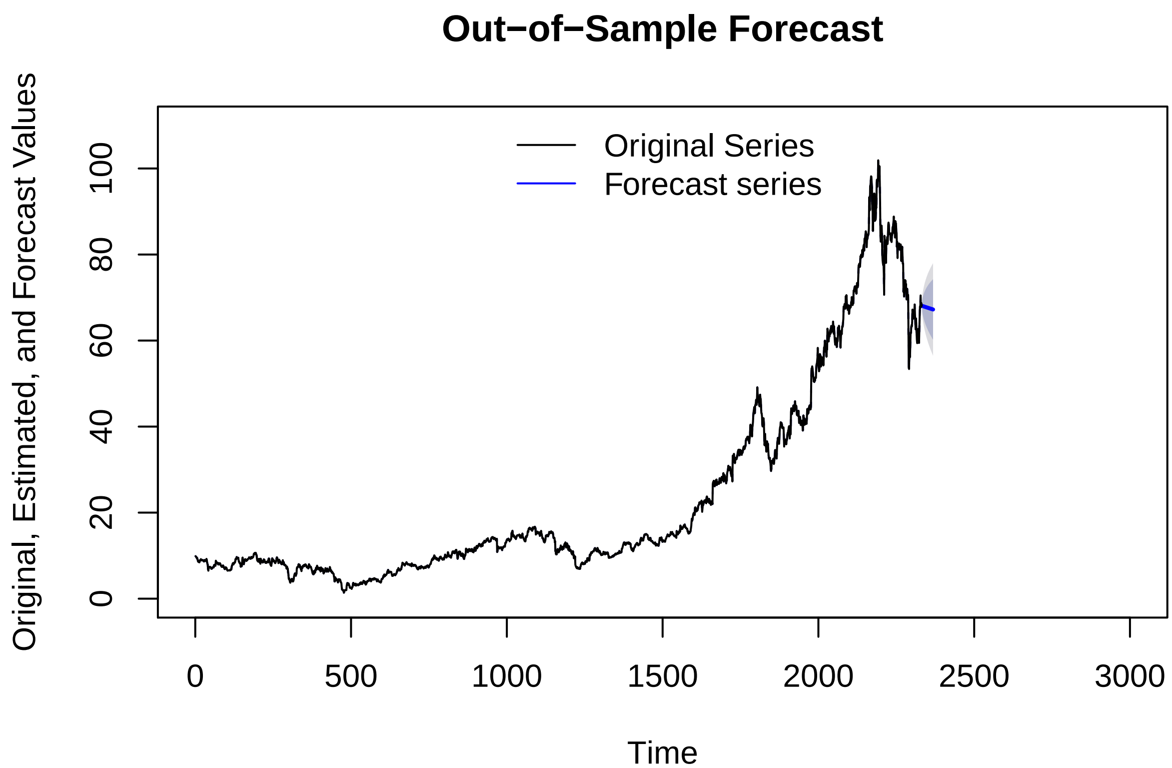
To perform a 36 steps ahead forecast of our series, we use the original point estimate of the series, that being the (0,1,0,0,0,0) SARIMA model initially estimated. The GARCH model of the residuals will additionally be used to forcast the variance of the series, and help us adjust the confidence interval of the prediction.

```r
# 36 steps ahead sample forecast of the financial time series
data.fit.ahead.fcast <- forecast.Arima(data.fit, h = 36)
length(data.fit.ahead.fcast$mean)
```

```
## [1] 36
```

```r
par(mfrow = c(1, 1))
plot(data.fit.ahead.fcast, lty = 2, col = "navy", main = "Out-of-Sample Forecast",
    ylab = "Original, Estimated, and Forecast Values", xlim = c(0,
        3000), ylim = c(0, 110))
par(new = T)
plot.ts(data$DXCM, axes = F, lty = 1, xlim = c(0, 3000), ylim = c(0,
    110), ylab = "")
```

```r
leg.txt <- c("Original Series", "Forecast series")
legend("top", legend = leg.txt, lty = 1, col = c("black", "blue"),
    bty = "n", cex = 1)
```



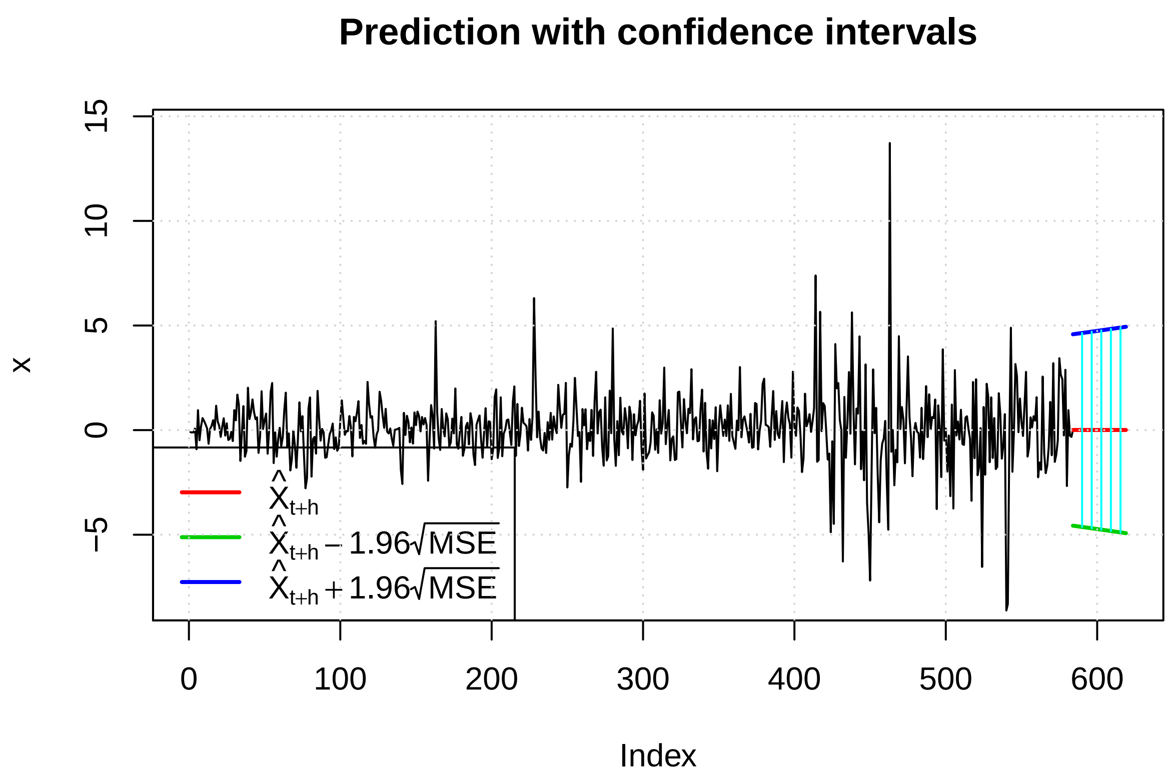
Having acknowledged the confidence interval problem on the prediction caused by the non-stationary variance of the financial search time series, we want to use our fitted GARCH model to predict the mean and variance of the residuals of the series.

The results of this prediction are a better estimate of the 95%confidence interval of the residuals of the global warming time series after modeling with our SAMIMA (1,1,1,0,0,0) model. We note that the volatility is predicted by the GARCH model to be in the range of -5 to 5, far wider than the predicted by the SARIMA model but consistent with the volatility observed towards the end of the original time series.

```r
data.garch.fit = garchFit(~garch(1, 1), data = data.res, trace = FALSE)
```

```
## Warning in sqrt(diag(fit$cvar)): NaNs produced
```

```
gw.garch.pred <- predict(data.garch.fit, n.ahead = 36, plot = TRUE)
```



Using our GARCH model fitted on the residuals, we now plot the predicted confidence intervals obtained with GARCH modeling over the original fitted SARIMA(0,1,0,0,0,0) series. Replace the 95% confidence interval of the fitted SARIMA with the GARCH confidence interval The mean series of the 36 steps ahead predctions obtained from the fitted first difference model are:

```
data.fit.ahead.fcast$mean
```

```
## Time Series:
## Start = 2333
## End = 2368
## Frequency = 1
##  [1] 68.05606 68.03214 68.00823 67.98434 67.96046 67.93659 67.91274
##  [8] 67.88890 67.86507 67.84126 67.81746 67.79367 67.76990 67.74614
## [15] 67.72239 67.69866 67.67494 67.65124 67.62755 67.60387 67.58020
## [22] 67.55655 67.53292 67.50929 67.48568 67.46208 67.43850 67.41493
## [29] 67.39137 67.36783 67.34430 67.32078 67.29728 67.27379 67.25031
## [36] 67.22685
```

The corresponding lower and upper confidence intervals after substituting for the conditional variance obtained from GARCH model are:

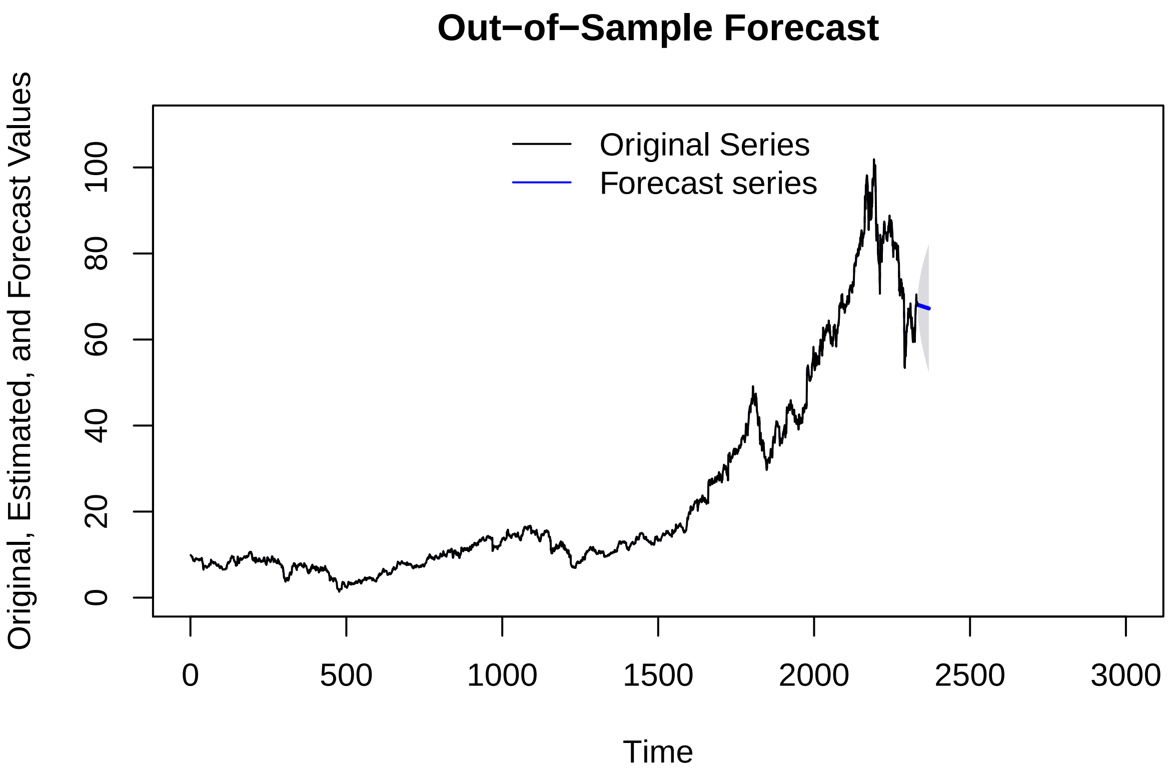
```r
# Reset the 95% confidence interval using the conditional
# variance from GARCH
data.fit.ahead.fcast$lower[, 2] <- -(((data.fit.ahead.fcast$mean -
    data.fit.ahead.fcast$lower[, 2])/(1.96 * sqrt(data.fit.ahead.fcast$model$sigma2))) *
    gw.garch.pred$standardDeviation) + data.fit.ahead.fcast$mean
data.fit.ahead.fcast$lower[, 2]
```

```
##  [1] 65.72165 64.72459 63.94974 63.28922 62.70132 62.16470 61.66671
##  [8] 61.19910 60.75618 60.33380 59.92882 59.53883 59.16189 58.79646
## [15] 58.44125 58.09519 57.75737 57.42703 57.10351 56.78621 56.47465
## [22] 56.16837 55.86699 55.57014 55.27752 54.98885 54.70386 54.42233
## [29] 54.14404 53.86881 53.59645 53.32680 53.05972 52.79507 52.53272
## [36] 52.27254
```

```r
data.fit.ahead.fcast$upper[, 2] <- (((data.fit.ahead.fcast$upper[,
    2] - data.fit.ahead.fcast$mean)/(1.96 * sqrt(data.fit.ahead.fcast$model$sigma2))) *
    gw.garch.pred$standardDeviation) + data.fit.ahead.fcast$mean
data.fit.ahead.fcast$upper[, 2]
```

```
##  [1] 70.39048 71.33969 72.06673 72.67946 73.21960 73.70848 74.15877
##  [8] 74.57869 74.97396 75.34872 75.70609 76.04851 76.37790 76.69581
## [15] 77.00354 77.30213 77.59251 77.87544 78.15159 78.42153 78.68576
## [22] 78.94473 79.19885 79.44844 79.69384 79.93532 80.17314 80.40753
## [29] 80.63871 80.86685 81.09215 81.31476 81.53484 81.75251 81.96791
## [36] 82.18116
```

```r
# Clear the 80% confidence interval
data.fit.ahead.fcast$lower[, 1] <- data.fit.ahead.fcast$mean
data.fit.ahead.fcast$upper[, 1] <- data.fit.ahead.fcast$mean

# Plot the forecast with the updated
par(mfrow = c(1, 1))
plot(data.fit.ahead.fcast, lty = 2, col = "navy", main = "Out-of-Sample Forecast",
    ylab = "Original, Estimated, and Forecast Values", xlim = c(0,
        3000), ylim = c(0, 110))
par(new = T)
plot.ts(data$DXCM, axes = F, lty = 1, xlim = c(0, 3000), ylim = c(0,
    110), ylab = "")
leg.txt <- c("Original Series", "Forecast series")
legend("top", legend = leg.txt, lty = 1, col = c("black", "blue"),
    bty = "n", cex = 1)
```


Out−of−Sample Forecast
Original Series
Forecast series
Original, Estimated, and Forecast Values
0
20
40
60
80
100
0
500
1000
1500
2000
2500
3000
Time

```
par(new = T)
```

Part 3 (25 points): Forecast the Web Search Activity for global Warming

Data Analysis

1. The time series has weekly 630 values starting at 1/4/04 and ending at 1/24/16. The minimum value is -0.551 and the maximum value is 4.104.
2. Time series plot shows that the series is very persistent, The series is basically flat from 2004 to 2012. After 2012, there is a sharp trend upward. There is more volatility after 2012. There are spikes and dips which could be seasonal with a yearly frequency. The series is not stationary.
3. Histogram shows is heavily positively skewed with most values between -0.551 and -0.3.
4. ACF of the series has correlations at around 0.75 for almost 25 lags.
5. PACF drops off immediately after first lag. There are 4 points that fall outside the 95% confidence interval (blue lines) at lags 3, 5, 11 and 14.

Model Selection Process

1. **Try AR models.** Use the ar() command in R to find AR(p) models or order p that potentially fit the time series. This command output a model or order 15, but looking at the difference in AICs, the AIC for the AR(1) model is not that different from the AIC of the AR(15), so for parsimony we will try using that one. Check if the residuals look like white noise.

- Histogram: Yes. This looks like a normal distribution.
- Fitted vs. Residuals: No. The plot does not look like an evenly distributed cloud.
- Plot: No. The plot does not look random, there is a lot of volatility on the right hand side of the graph.
- ACF: No. The ACF drops off after lag 0, but has only a few lags where the correlation comes out of the 95% CI.
- PACF: No. The PACF shows correlation with several values outside of the 9%% CI. In summary, the residuals for this model do not look like white noise, so there is more variation that could be explained by our model.
  The In-Sample fit of this estimated model matches the original model very well as evidenced in the plot.

2. **Try ARIMA models.** Use the get.best.arima() function which will try models with c(p,d,q) where p=0-4, d=0-2 and q=0-2. And then we can print out a list in ascending order by AIC of the 20 models with the lowest AIC. Inspecty these models for parsimony and select one with a good AIC and a small number of parameters. The best model output from the function had an AIC of -1058.794 with parameters = c(1, 2, 2). For parsimony a model of c(1,1,1) was chosen with an AIC of -1032.364 which is not that different from the best AIC. Check if the residuals look like white noise. No, the residuals do not look like white noise. They exhibit the same characteristics as the AR(1) model from step 1.
   The In-Sample fit of this estimated model matches the original model very well as evidenced in the plot.
3. **Try SARIMA models.** From the plot of the original series, it looks like this series has a seasonal component with a 52-week periodicity. Use the get.best.sarima() function with parameters c(2,2,2,2,2,2). The best AIC output is -1276.817 with a model of c(1, 2, 2, 1, 0, 2). For parsimony try running get.best.sarima() with c(1,1,1,1,1,1). A parsimonious model from this output is c(0, 1, 1, 1, 0, 1) with AIC -1246.412 which is very close to the AIC output from c(2,2,2,2,2,2). For parsimony we will choose c(0, 1, 1, 1, 0, 1) and check the residuals. No, the residuals do not look like white noise. They exhibit the same characteristics as the AR(1) model from step 1. The residuals exhibit evidence of time
   The In-Sample fit of this estimated model matches the original model very well as evidenced in the plot.
4. **Backtesting.**

5. **Forecast the model.** Using the SARIMA model, we will make the requested 12-step ahead forecast of the model. The forecast looks like it captures the seasonality of the model as it matches the upward trend and the seasonal volatility.

6. **GARCH.** Since the residuals look like they have time-varying volatility, we will use GARCH to model the residuals. The ACF of the squared residuals shows no obvious patterns or significant values. The model has reached and acceptable fit to the original time series.

7. **Update the forecast with GARCH.** To add the GARCH information to the forecast, we create a GARCH forecast. We then take the forecast from the SARIMA model and update it with the GARCH confidence intervals.

Conclusion

This time series is satisfactorily modeled with a SARMIMA(x,x,x,x,x,x) model to hande trends and seasonality and a GARCH model to handle the time-varying volatility. We observe that the forecasts of the model are consisent with the seasonality of the original series and the forecasts are within the 95% confidence interval produced by GARCH. This gives us confidence that this model will give us decent forecasts for the original time series.

```r
# Read in the time series data
glob.warm = read.csv("globalWarming.csv", header = TRUE)
glob.warm.ts = ts(glob.warm$data.science, start = 2004, frequency = 52)
# glob.warm.ts = ts(glob.warm$data.science) Print descriptive
# statistics
str(glob.warm.ts)
```

```
##  Time-Series [1:630] from 2004 to 2016: -0.44 -0.474 -0.423 -0.551 -0.486 -0.551 -0.453 -0.462 -0.55
```

```r
summary(glob.warm.ts)
```

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## -0.551000 -0.506000 -0.485000  0.000038 -0.200000  4.104000
```

```r
cbind(head(glob.warm.ts), tail(glob.warm.ts))
```

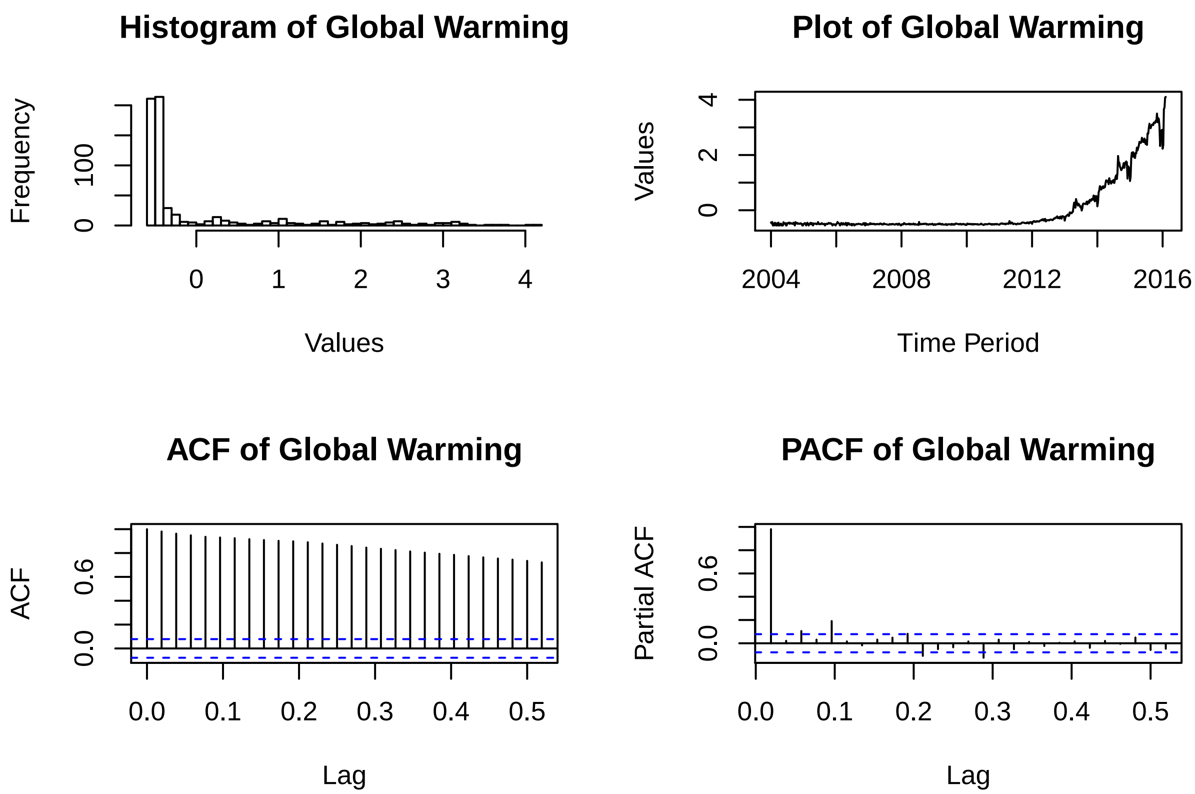
```
##        [,1]  [,2]
## [1,] -0.440 2.227
## [2,] -0.474 2.360
## [3,] -0.423 3.662
## [4,] -0.551 3.721
## [5,] -0.486 4.087
## [6,] -0.551 4.104
```

```r
quantile(as.numeric(glob.warm.ts), c(0.01, 0.05, 0.1, 0.25, 0.5,
    0.75, 0.9, 0.95, 0.99))
```

```
##       1%       5%      10%      25%      50%      75%      90%      95%
## -0.55100 -0.53220 -0.51900 -0.50600 -0.48500 -0.20000  1.68410  2.48055
##      99%
##  3.28021
```

```r
# Plot the time series
plot.time.series(glob.warm.ts, 50, "Global Warming")
```

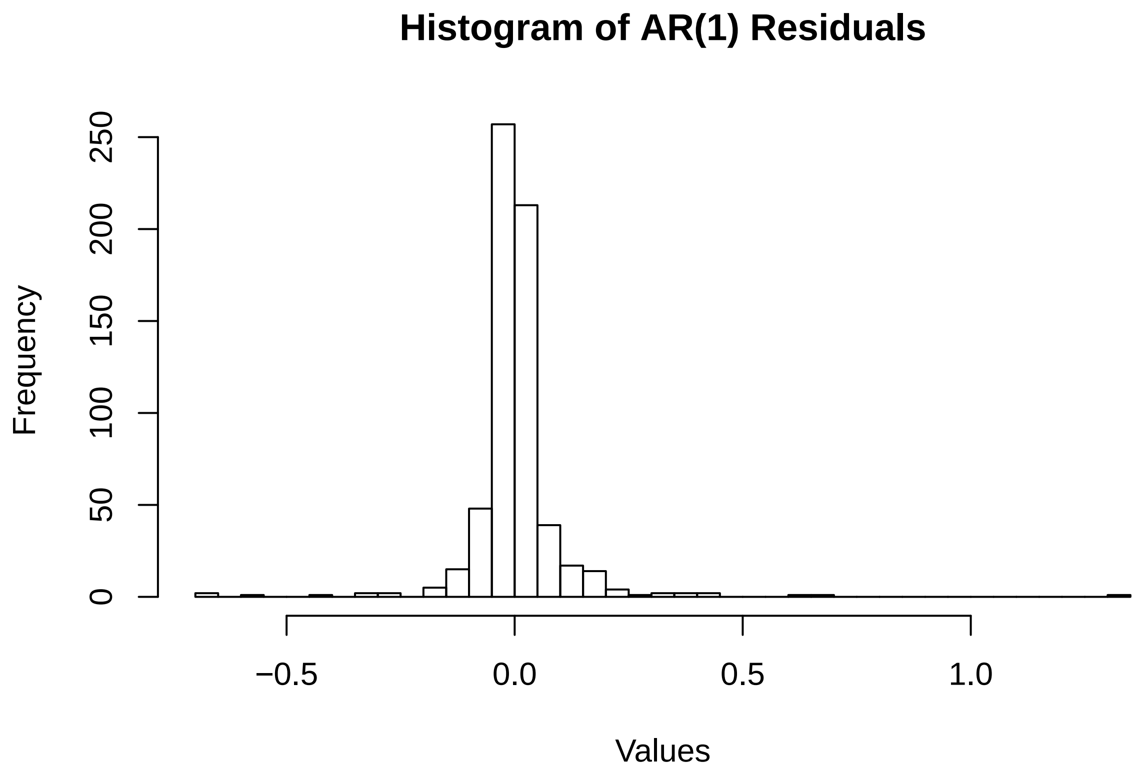
```
##  Time-Series [1:630] from 2004 to 2016: -0.44 -0.474 -0.423 -0.551 -0.486 -0.551 -0.453 -0.462 -0.55
```

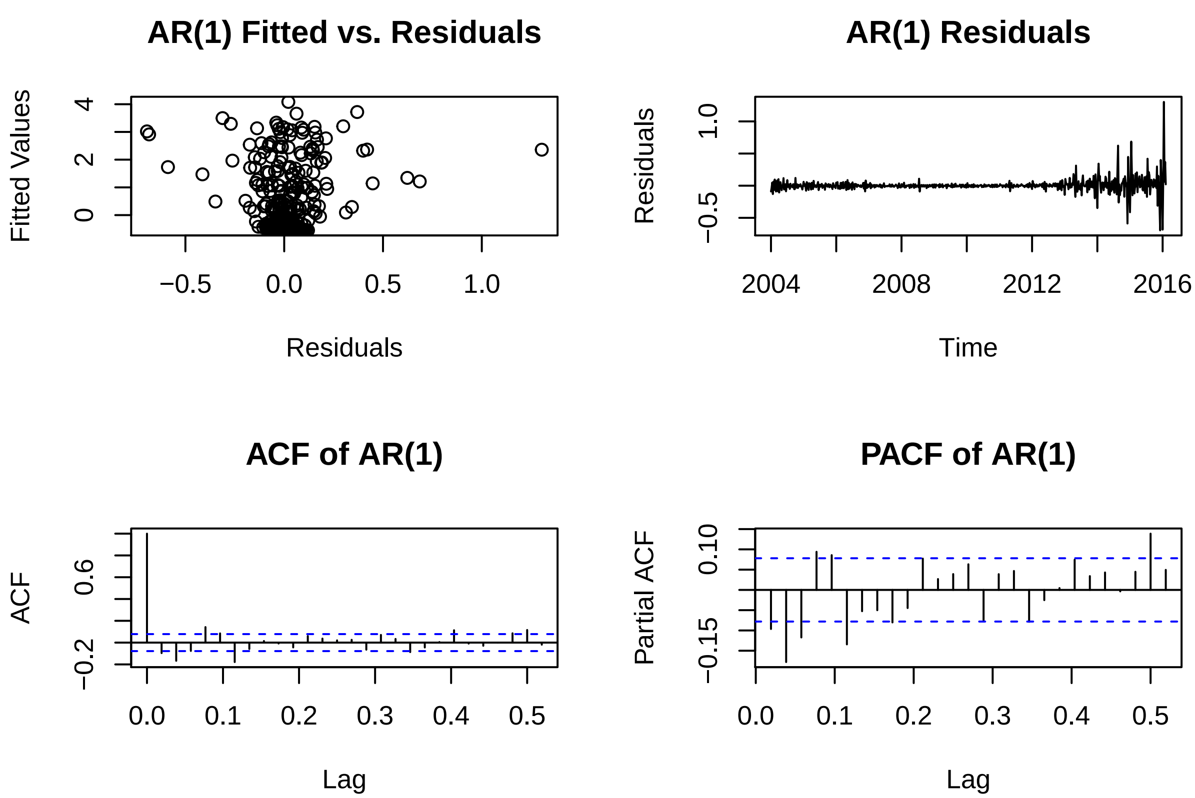



```r
### 1. Try AR models
glob.warm.ar = estimate.ar(glob.warm.ts)
```

```
## [1] "Difference in AICs"
##           0           1           2           3           4           5
## 2084.447812   29.847743   31.560248   26.579621   27.960796    6.553854
##           6           7           8           9          10          11
##    8.386263   10.176681   11.540473   12.035569    9.848063    4.382889
##          12          13          14          15          16          17
##    4.754476    5.996066    7.842039    0.000000    1.380591    1.728222
##          18          19          20          21          22          23
##    3.638626    5.291781    7.280008    9.104280   10.136039   11.875658
##          24          25          26          27
##   13.856501   14.302766   14.191716   14.781132
## [1] "AR parameters"
##  [1]  0.944522755 -0.084770519  0.084153344 -0.171500315  0.188422207
##  [6]  0.058499722 -0.055671998 -0.008980095 -0.033122819  0.204945468
## [11] -0.084654024 -0.006099723 -0.059202741  0.132988289 -0.124502569
## [1] "AR order"
## [1] 15
```

```r
glob.warm.ar1 = arima(glob.warm.ts, order = c(1, 0, 0))
# Plot the residuals
plot.residuals.ts(glob.warm.ar1, "AR(1)")
```

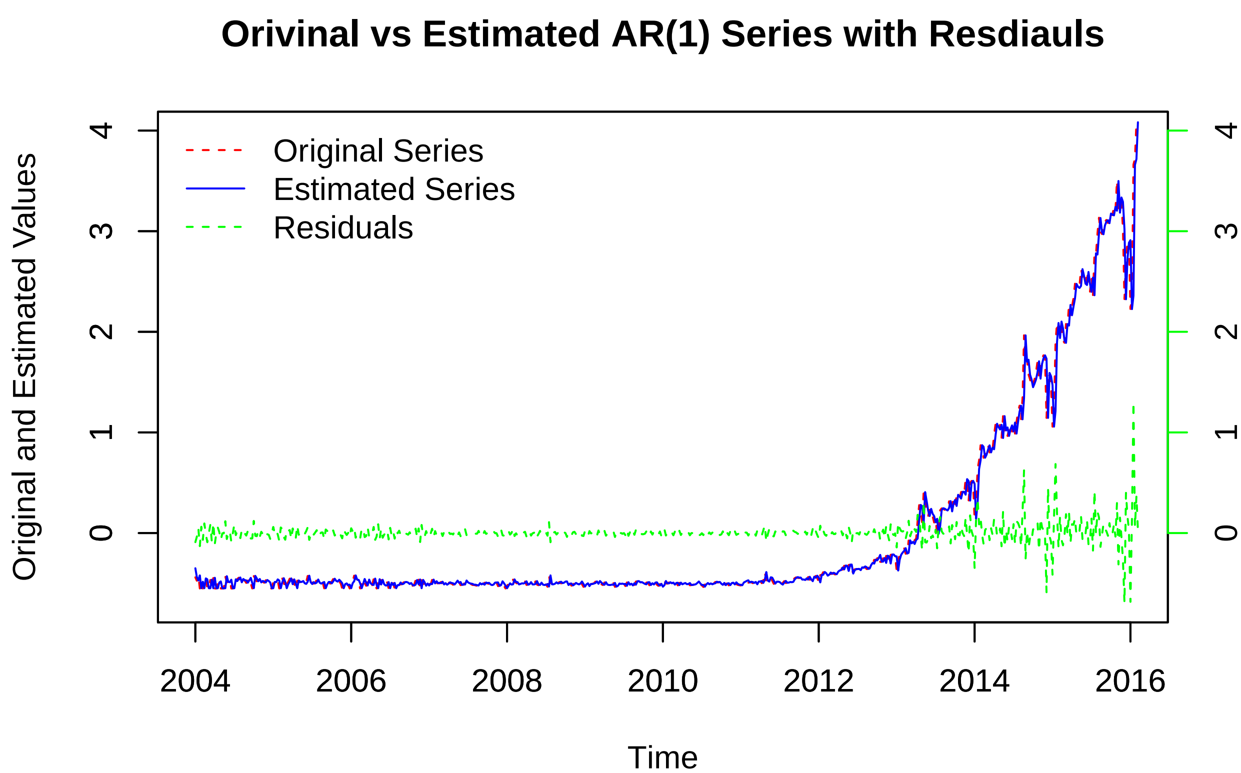

Histogram of AR(1) Residuals
Frequency
0
50
100
150
200
250
−0.5
0.0
0.5
1.0
Values


AR(1) Fitted vs. Residuals
Fitted Values
0
2
4
−0.5
0.0
0.5
1.0
Residuals
AR(1) Residuals
Residuals
−0.5
1.0
2004
2008
2012
2016
Time
ACF of AR(1)
ACF
−0.2
0.6
0.0
0.1
0.2
0.3
0.4
0.5
Lag
PACF of AR(1)
Partial ACF
−0.15
0.10
0.0
0.1
0.2
0.3
0.4
0.5
Lag

```
##
##  Box-Ljung test
##
## data:  x.mod$residuals
## X-squared = 5.8789, df = 1, p-value = 0.01532
```

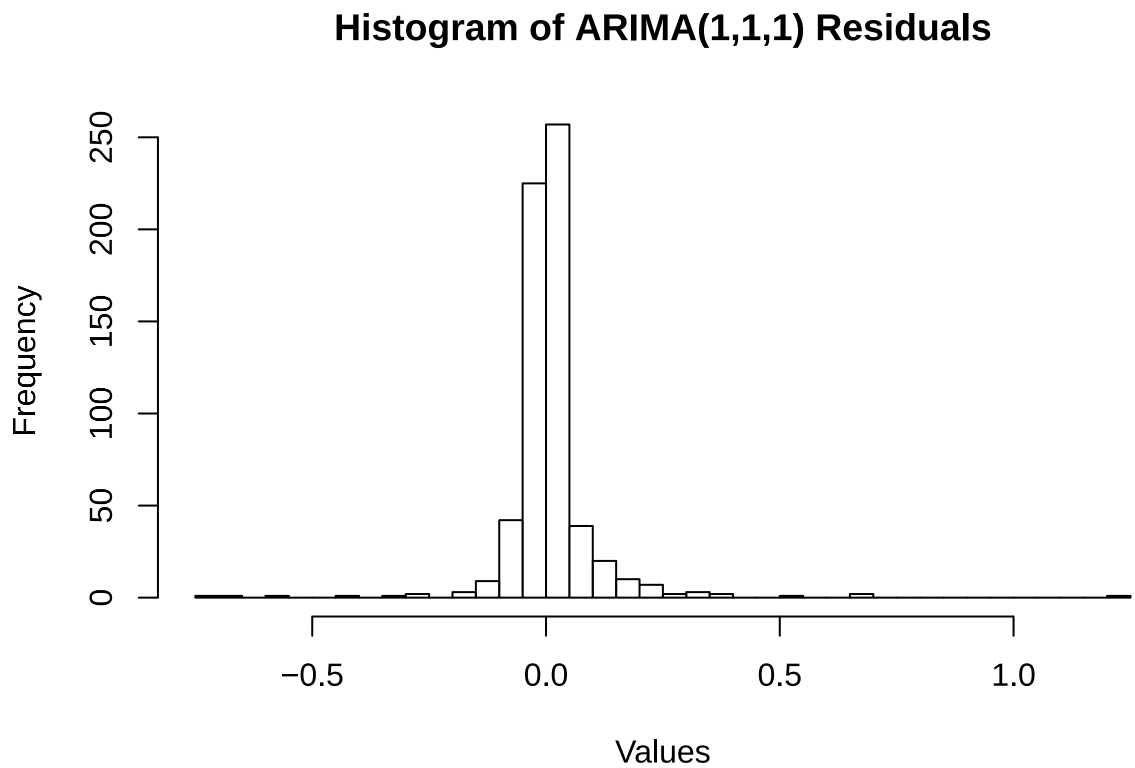
```
# Plot the In-sample fit
plot.orig.model.resid(glob.warm.ts, glob.warm.ar1, "AR(1)", c(2004,
    2016), c(-0.7, 4))
```

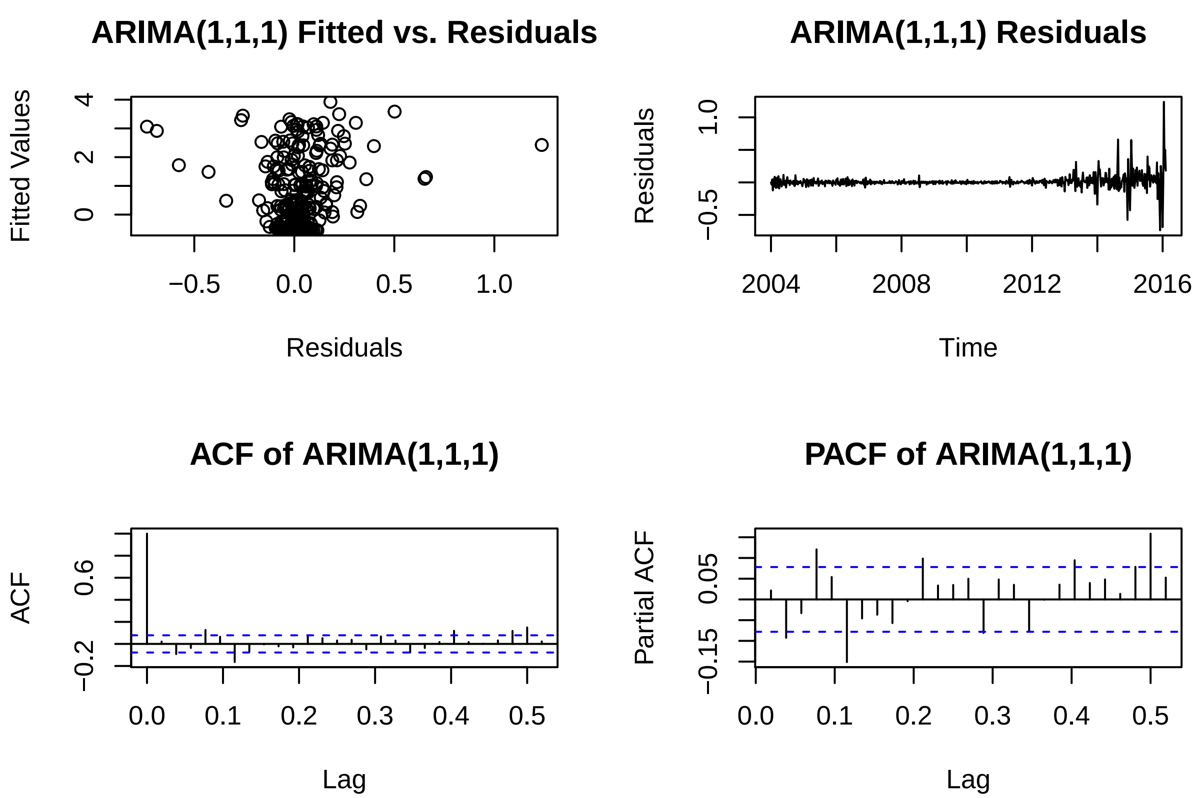
```
##
## Descriptive Stat
## ===========================================
## Statistic        N   Mean   St. Dev. Min  Max
## -------------------------------------------
## x.ts             630 0.000    1.0    -0.6 4.1
## fitted.x.mod.    630 -0.01    1.0    -0.5 4.1
## x.mod.residuals 630 0.01     0.1    -0.7 1.3
## -------------------------------------------
```




```
### 2. Try ARIMA models gw.arima.best <-
### get.best.arima(glob.warm.ts, maxord=c(4,2,2)) Print the top
### 20 best models based on AIC
### gw.arima.best$others[order(gw.arima.best$others$aics)[1:20],]
glob.warm.arima = arima(glob.warm.ts, order = c(1, 1, 1))
```

```r
# Plot the residuals
plot.residuals.ts(glob.warm.arima, "ARIMA(1,1,1)")
```

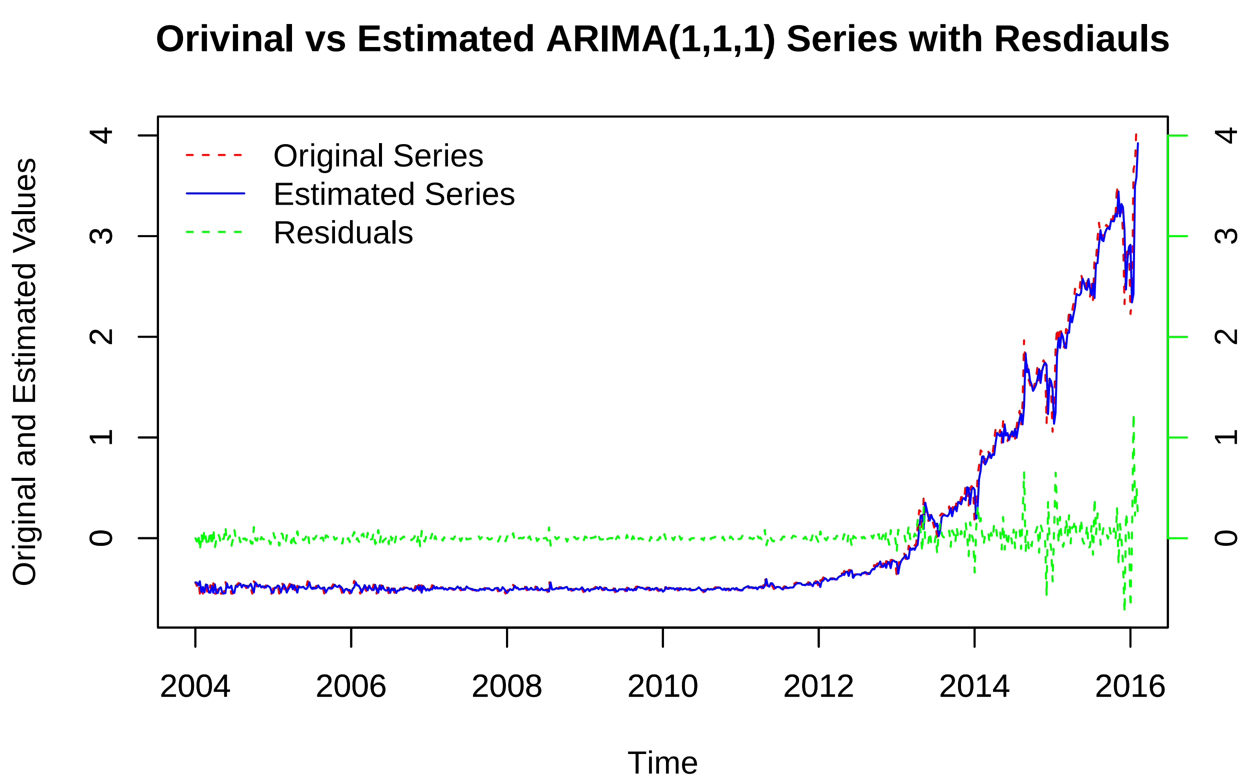

Histogram of ARIMA(1,1,1) Residuals
Frequency
0
50
100
150
200
250
−0.5
0.0
0.5
1.0
Values


ARIMA(1,1,1) Fitted vs. Residuals
Fitted Values
0
2
4
−0.5
0.0
0.5
1.0
Residuals
ARIMA(1,1,1) Residuals
Residuals
−0.5
1.0
2004
2008
2012
2016
Time
ACF of ARIMA(1,1,1)
ACF
−0.2
0.6
0.0
0.1
0.2
0.3
0.4
0.5
Lag
PACF of ARIMA(1,1,1)
Partial ACF
−0.15
0.05
0.0
0.1
0.2
0.3
0.4
0.5
Lag

```
##
##  Box-Ljung test
##
## data:  x.mod$residuals
## X-squared = 0.29725, df = 1, p-value = 0.5856
```

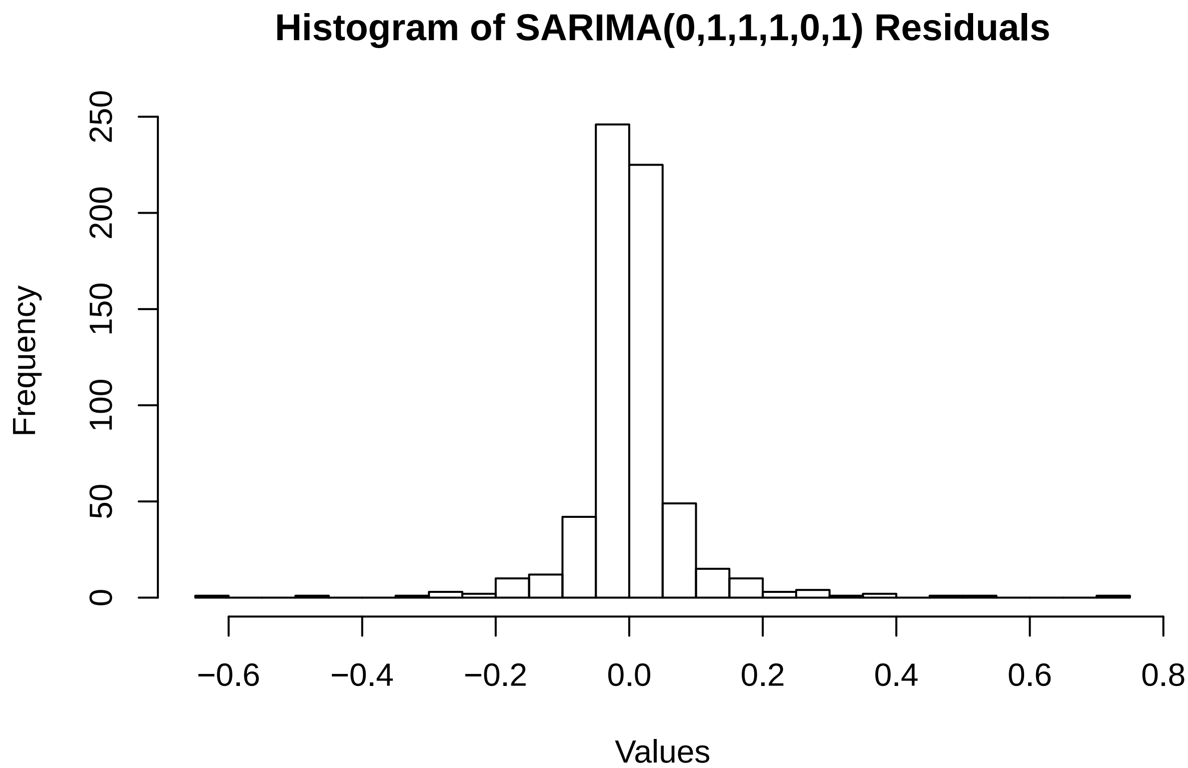
```
# Plot the In-sample fit
plot.orig.model.resid(glob.warm.ts, glob.warm.arima, "ARIMA(1,1,1)",
    c(2004, 2016), c(-0.7, 4))
```

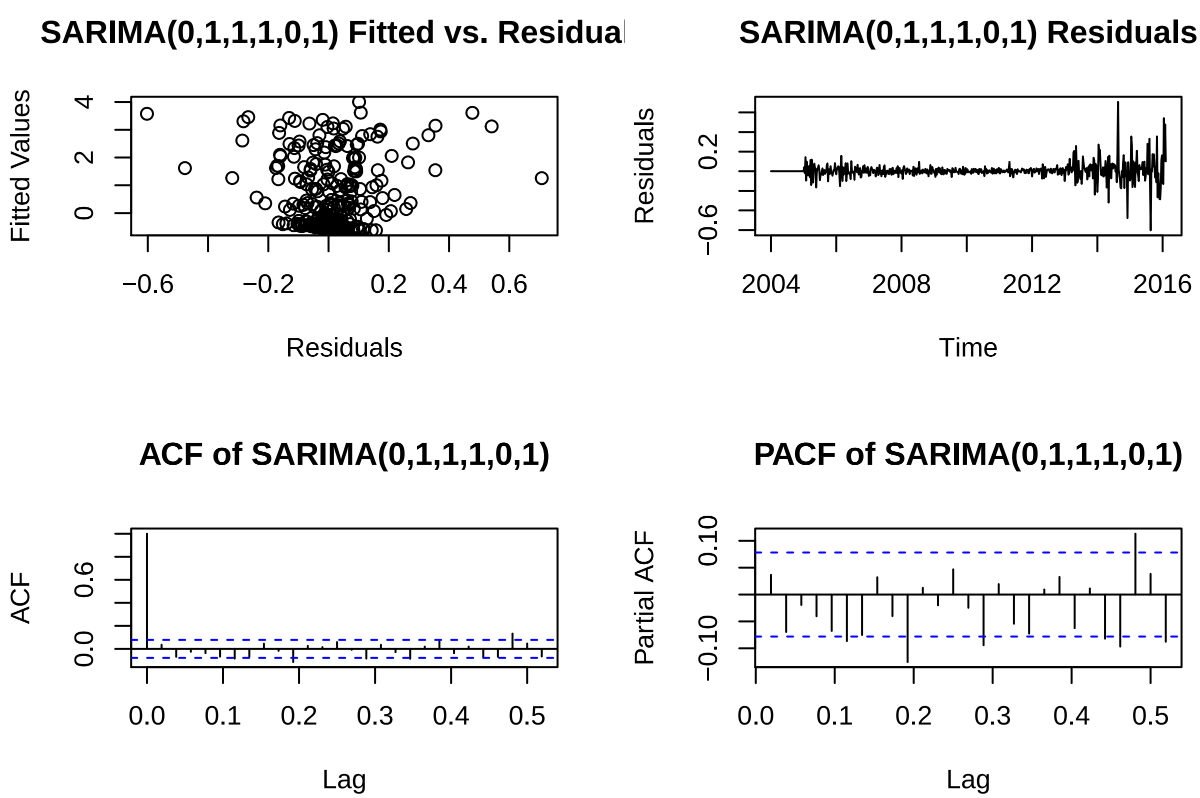
```
##
## Descriptive Stat
## ===========================================
## Statistic         N   Mean  St. Dev. Min  Max
## -------------------------------------------
## x.ts              630 0.000   1.0     -0.6 4.1
## fitted.x.mod.     630 -0.01   1.0     -0.5 3.9
## x.mod.residuals 630 0.01    0.1     -0.7 1.2
## -------------------------------------------
```




```
### 3. Try SARIMA models gw.seas.best <-
### get.best.sarima(glob.warm.ts, maxord=c(2,2,2,2,2,2), 52)
### Print the top 20 best models based on AIC
### gw.seas.best$others[order(gw.seas.best$others$aics)[1:20],]
```

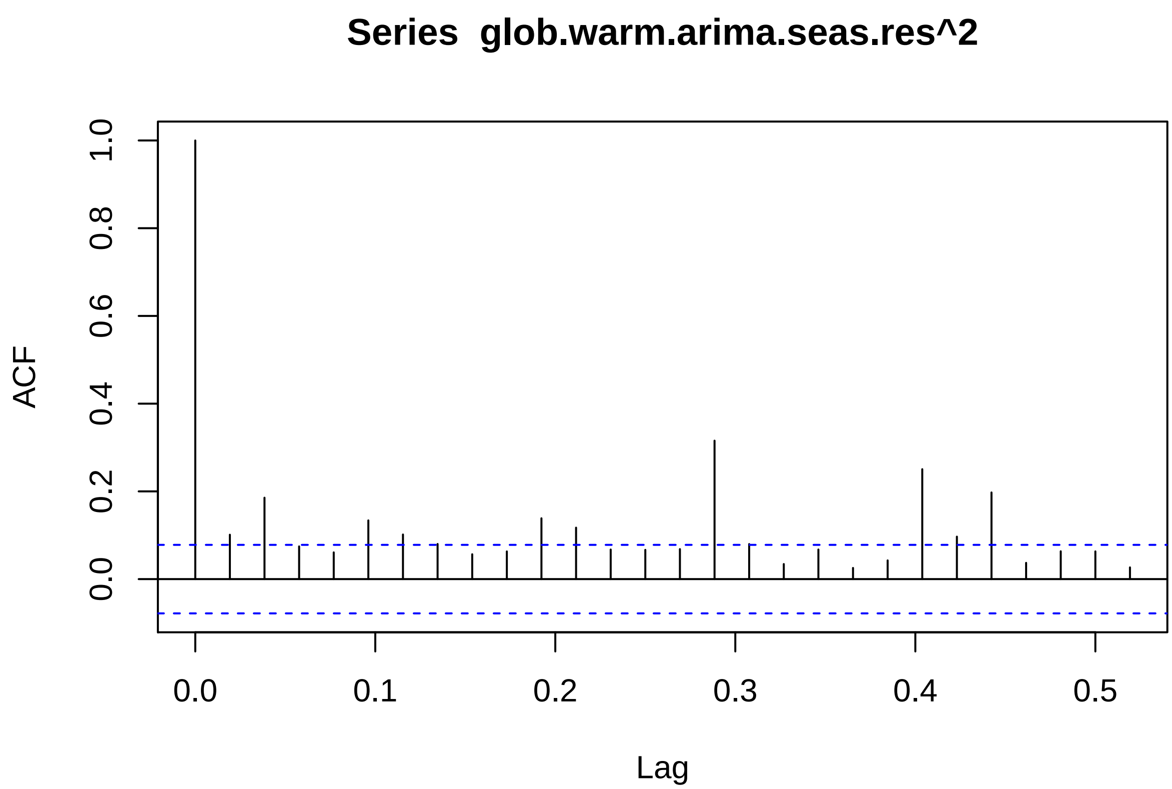
```r
# gw.seas.best1 <- get.best.sarima(glob.warm.ts,
# maxord=c(1,1,1,1,1,1), 52) Print the top 20 best models
# based on AIC
# gw.seas.best1$others[order(gw.seas.best1$others$aics)[1:20],]
glob.warm.arima.seas = arima(glob.warm.ts, order = c(0, 1, 1),
    seas = list(order = c(1, 0, 1), 52), method = "CSS")
# Plot the residuals
glob.warm.arima.seas.res = glob.warm.arima.seas$residuals
plot.residuals.ts(glob.warm.arima.seas, "SARIMA(0,1,1,1,0,1)")
```


Histogram of SARIMA(0,1,1,1,0,1) Residuals
Frequency
0
50
100
150
200
250
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
Values


SARIMA(0,1,1,1,0,1) Fitted vs. Residual
Fitted Values
0
2
4
−0.6
−0.2
0.2
0.4
0.6
Residuals
SARIMA(0,1,1,1,0,1) Residuals
Residuals
−0.6
0.2
2004
2008
2012
2016
Time
ACF of SARIMA(0,1,1,1,0,1)
ACF
0.0
0.6
0.0
0.1
0.2
0.3
0.4
0.5
Lag
PACF of SARIMA(0,1,1,1,0,1)
Partial ACF
−0.10
0.10
0.0
0.1
0.2
0.3
0.4
0.5
Lag

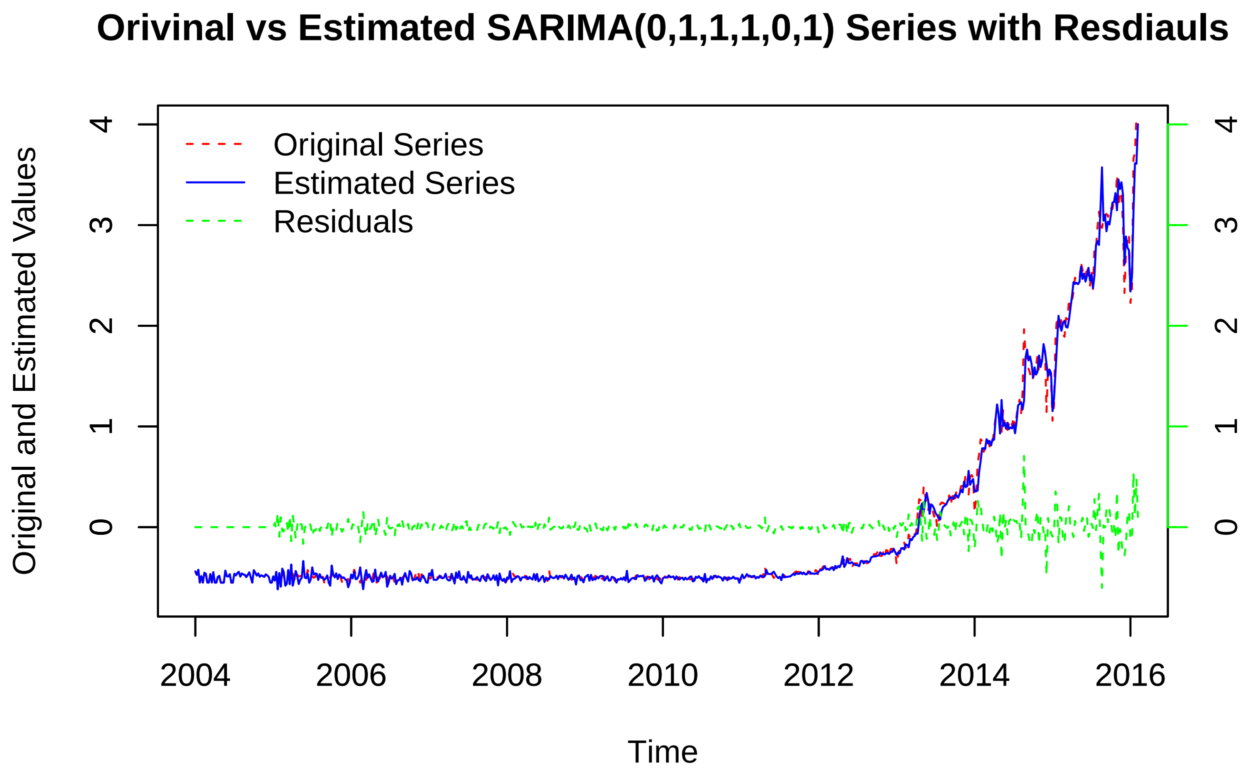
```
##
##  Box-Ljung test
##
## data:  x.mod$residuals
## X-squared = 0.8408, df = 1, p-value = 0.3592
```

```
par(mfrow = c(1, 1))
acf(glob.warm.arima.seas.res^2)
```




```
# Plot the In-sample fit
plot.orig.model.resid(glob.warm.ts, glob.warm.arima.seas, "SARIMA(0,1,1,1,0,1)",
    c(2004, 2016), c(-0.7, 4))
```

```
##
## Descriptive Stat
## ===========================================
## Statistic         N   Mean  St. Dev. Min  Max
## -------------------------------------------
## x.ts             630 0.000   1.0    -0.6 4.1
## fitted.x.mod.    630 -0.01   1.0    -0.6 4.0
## x.mod.residuals 630 0.01    0.1    -0.6 0.7
## -------------------------------------------
```

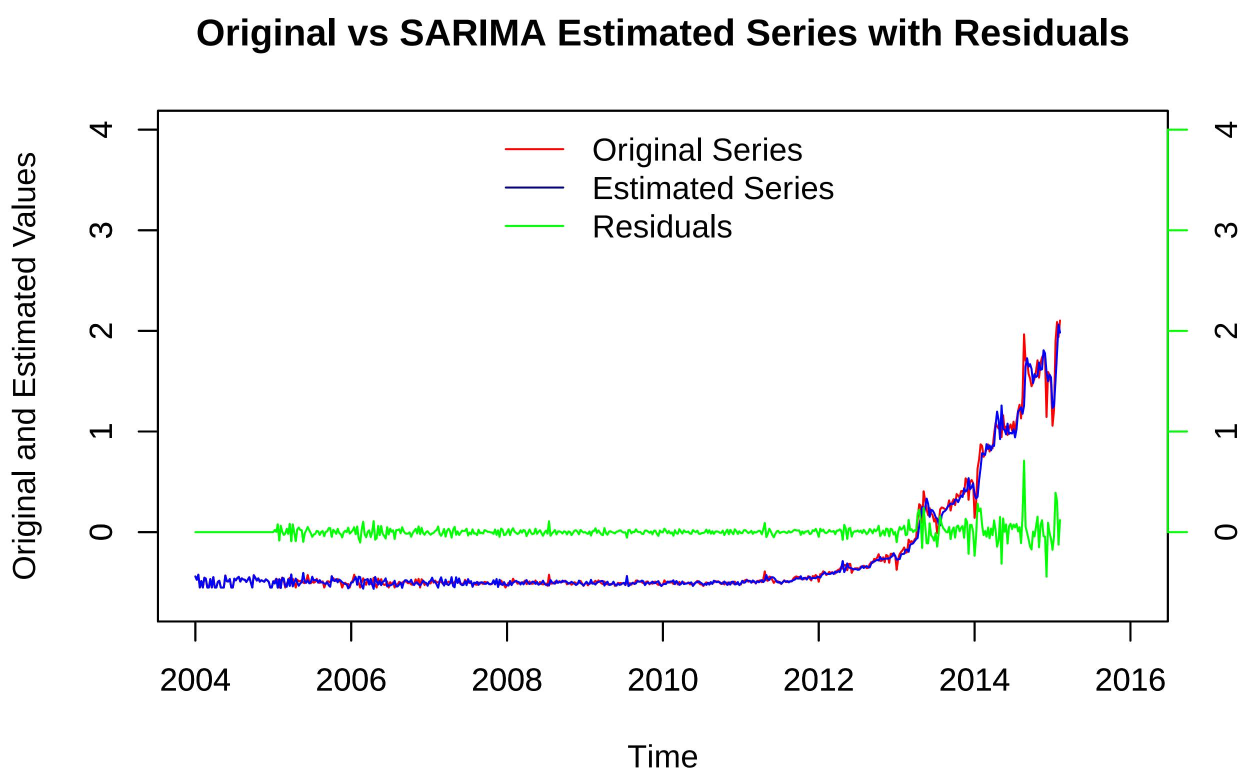



```
### 4. Backtesting
glob.warm.bt.ts = ts(glob.warm.ts[1:(length(glob.warm.ts) - 52)],
    start = 2004, frequency = 52)
glob.warm.arima.seas.bt = arima(glob.warm.bt.ts, order = c(0,
    1, 1), seas = list(order = c(1, 0, 1), 52), method = "CSS")
df = cbind(glob.warm.bt.ts, fitted(glob.warm.arima.seas.bt),
    glob.warm.arima.seas.bt$resid)
colnames(df) = c("orig_series", "fitted_vals", "resid")
head(df)
```

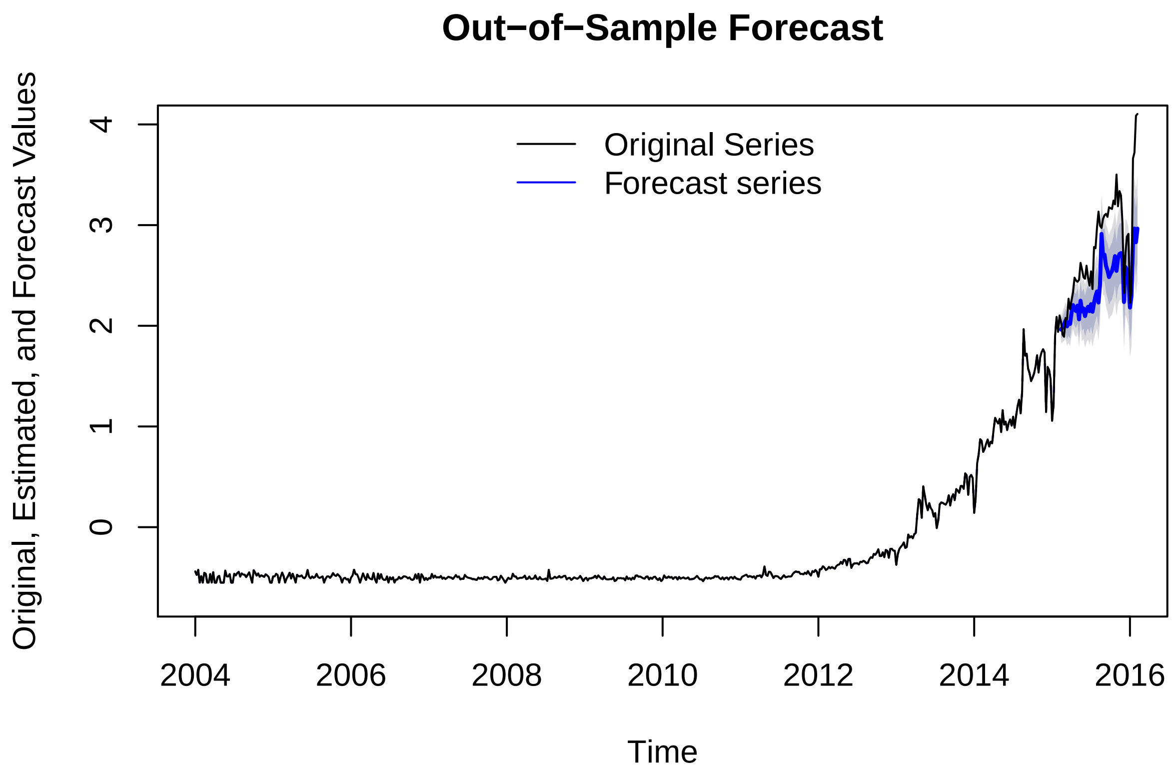
```
##      orig_series fitted_vals resid
## [1,]      -0.440      -0.440     0
## [2,]      -0.474      -0.474     0
## [3,]      -0.423      -0.423     0
## [4,]      -0.551      -0.551     0
## [5,]      -0.486      -0.486     0
## [6,]      -0.551      -0.551     0
```

```
# Plot the original and estimate series
par(mfrow = c(1, 1))
plot.ts(df[, "orig_series"], col = "red", main = "Original vs SARIMA Estimated Series with Residuals",
    ylab = "Original and Estimated Values", xlim = c(2004, 2016),
    ylim = c(-0.7, 4))
par(new = T)
plot.ts(df[, "fitted_vals"], col = "blue", axes = T, xlab = "",
```

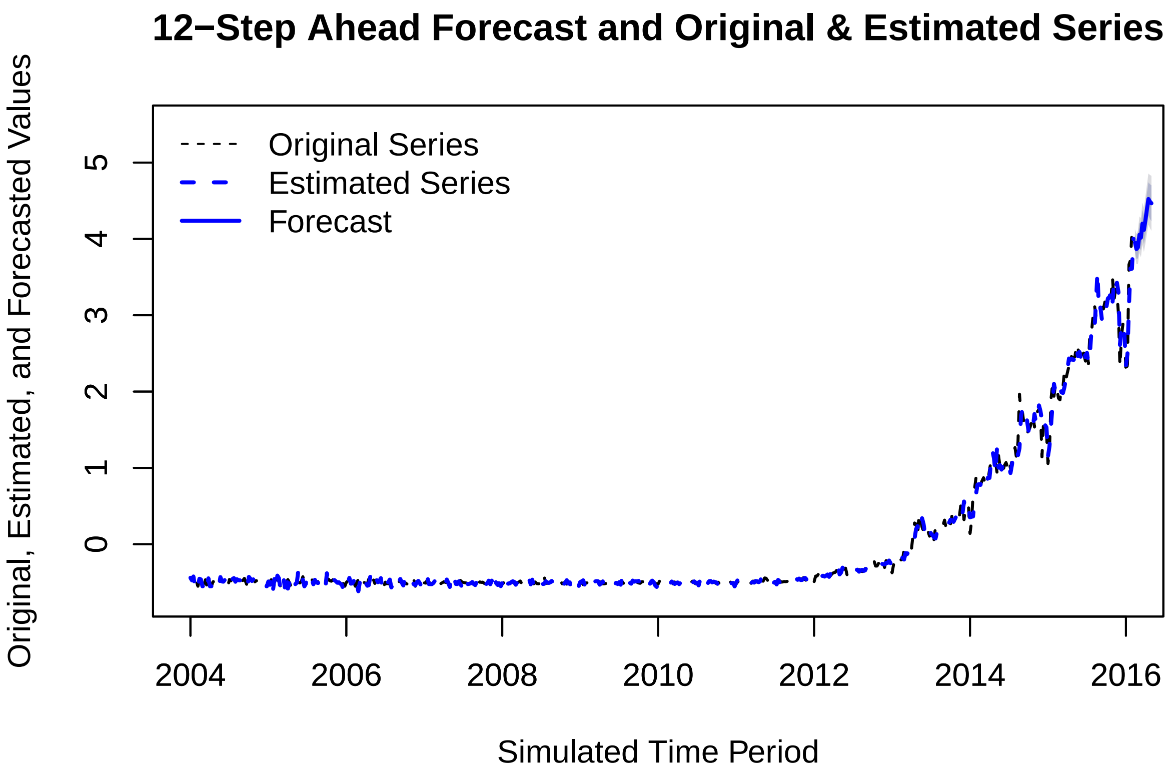
```
    ylab = "", xlim = c(2004, 2016), ylim = c(-0.7, 4))
leg.txt <- c("Original Series", "Estimated Series", "Residuals")
legend("top", legend = leg.txt, lty = 1, col = c("red", "navy",
    "green"), bty = "n", cex = 1)
par(new = T)
plot.ts(df[, "resid"], axes = F, xlab = "", ylab = "", col = "green",
    xlim = c(2004, 2016), ylim = c(-0.7, 4), pch = 1)
axis(side = 4, col = "green")
mtext("Residuals", side = 4, line = 2, col = "green")
```



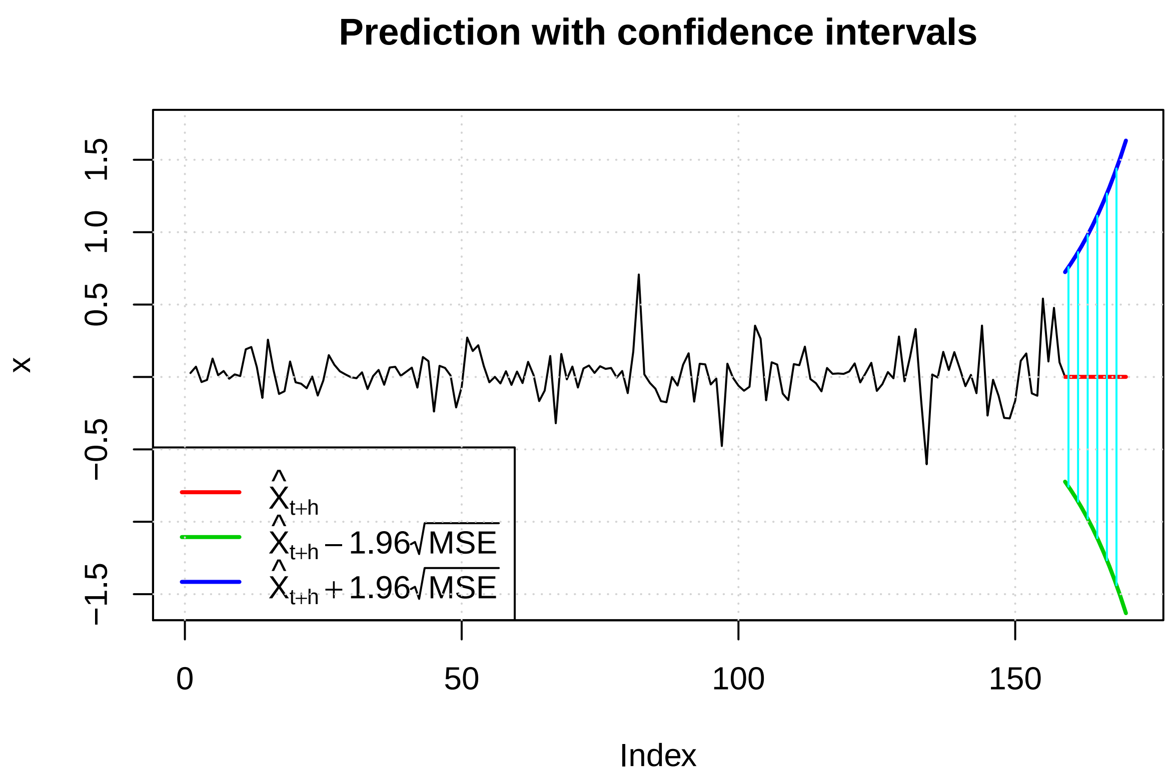

```
glob.warm.arima.bt.fcast = forecast.Arima(glob.warm.arima.seas.bt,
    h = 52)
par(mfrow = c(1, 1))
plot(glob.warm.arima.bt.fcast, lty = 2, col = "navy", main = "Out-of-Sample Forecast",
    ylab = "Original, Estimated, and Forecast Values", xlim = c(2004,
        2016), ylim = c(-0.7, 4))
par(new = T)
plot.ts(glob.warm.ts, axes = F, lty = 1, col = "black", xlim = c(2004,
    2016), ylim = c(-0.7, 4), ylab = "")
leg.txt <- c("Original Series", "Forecast series")
legend("top", legend = leg.txt, lty = 1, col = c("black", "blue"),
    bty = "n", cex = 1)
```



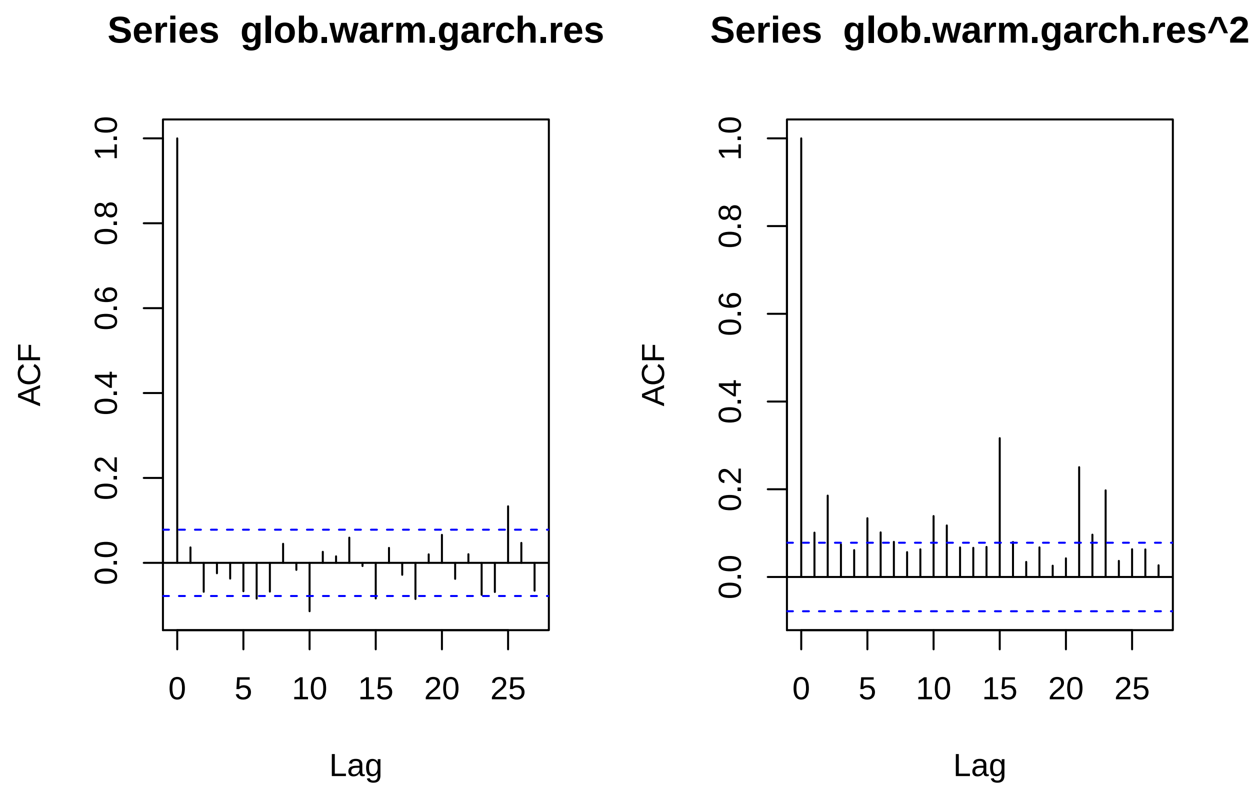

```
### 5. Forecast the model
glob.warm.arima.fcast = forecast.Arima(glob.warm.arima.seas,
    h = 12)
plot.model.forecast(glob.warm.arima.seas, glob.warm.arima.fcast,
    "12", c(2004, 2016), c(-0.7, 5.5))
```



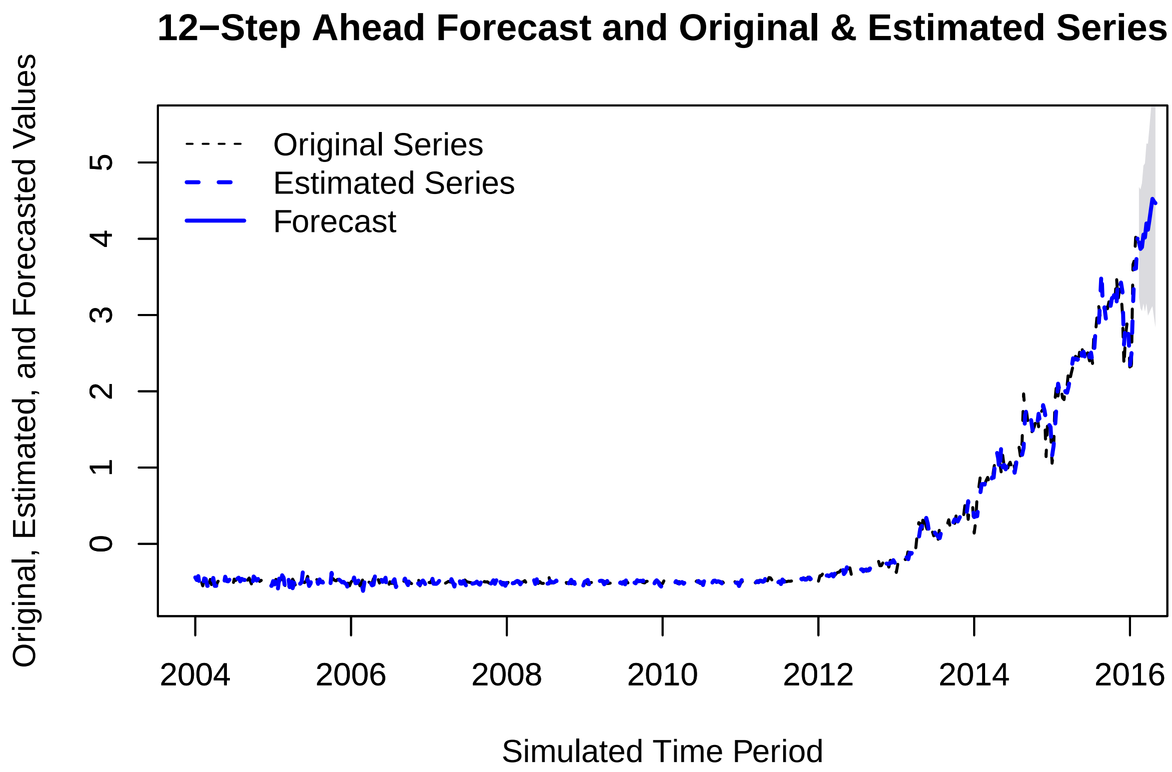

```
### 6. GARCH
glob.warm.garch.fit = garchFit(~garch(1, 1), data = glob.warm.arima.seas.res,
    trace = FALSE)
gw.garch.pred <- predict(glob.warm.garch.fit, n.ahead = 12, plot = TRUE)
```




```
glob.warm.garch.res <- glob.warm.garch.fit@residuals
par(mfrow = c(1, 2))
acf(glob.warm.garch.res)
acf(glob.warm.garch.res^2)
```




```
### 7. Update the forecast with GARCH Clear the 80% confident
### interval of the fitted ARIMA by setting all values to the
### predicted mean of it And set the values of the 95%
### confidence interval to those that came out of the GARCH
### model
glob.warm.arima.fcast$lower[, 1] <- glob.warm.arima.fcast$mean
glob.warm.arima.fcast$upper[, 1] <- glob.warm.arima.fcast$mean
glob.warm.arima.fcast$lower[, 2] <- gw.garch.pred$lowerInterval +
    glob.warm.arima.fcast$mean
glob.warm.arima.fcast$upper[, 2] <- gw.garch.pred$upperInterval +
    glob.warm.arima.fcast$mean

plot.model.forecast(glob.warm.arima.seas, glob.warm.arima.fcast,
    "12", c(2004, 2016), c(-0.7, 5.5))
```


12−Step Ahead Forecast and Original & Estimated Series
Original Series
Estimated Series
Forecast
Original, Estimated, and Forecasted Values
0
1
2
3
4
5
2004
2006
2008
2010
2012
2014
2016
Simulated Time Period