Lab2

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Question 1

Part 1

$$E(Y|X) = \int_0^x y * \frac{1}{x}\ dy = \frac{y^2}{2x}|_0^x = \frac{x}{2} - 0$$
$$\mathbf{E(Y|X) = \frac{x}{2}}$$

Part 2

$$E(Y) = E(E(Y|X)) = E(\frac{x}{2}) = \int_0^1 \frac{x}{2}\ dx = \frac{x^2}{4}|_0^1 = \frac{1}{4} - 0$$
$$\mathbf{E(Y) = \frac{1}{4}}$$

Part 3

$$f_{X,Y}(x,y) = f_{Y|X}(y|x) * f_X(x)$$

We know that

$$f_{Y|X}(y|x) = \frac{1}{x}\ and\ f_X(x) = 1$$

Substituting these values in to the equation, we get

$$\mathbf{f_{X,Y}(x,y) = \frac{1}{x}}$$

Part 4

$$f_Y(y) = \int_y^1 f_{Y|X}(y|x) * f_X(x)dx = \int_y^1 \frac{1}{x} * 1\ dx$$
$$= \log(x)|_y^1 = \log(1) - \log(y) = 0 - \log(y) = \log(\frac{1}{y})$$
$$f_Y(y) = \log(\frac{1}{y})$$

We know that

$$f_{X,Y}(x,y) = f_{X|Y}(x|y) * f_Y(y)$$

Solving for $f_{X|Y}(x|y)$, we get

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

Substituting, we get

$$f_{X|Y}(x|y) = \frac{\frac{1}{x}}{\log(\frac{1}{y})}$$
$$\mathbf{f_{X|Y}(x|y) = \frac{1}{x\log(\frac{1}{y})}}$$

Part 5

$$E(X|Y=\frac{1}{2}) = \int_{\frac{1}{2}}^{1} \frac{1}{x\log(2)}dx = \frac{1}{\log(2)}\int_{\frac{1}{2}}^{1}\frac{1}{x}dx$$

$$= \frac{1}{\log(2)} * (\log(x)|_{\frac{1}{2}}^{1}) = \frac{1}{\log(2)} * (\log(1) - \log(\frac{1}{2}))$$

$$= \frac{1}{\log(2)} * (0 + \log(2)) = \frac{1}{\log(2)} * \log(2)$$

$$\mathbf{E(X|Y = \frac{1}{2}) = 1}$$

Question 2

Question 3

$y_i, i = 1, \cdots, n$ random uniform variables.

Part 1 - Likelihood Function

$L(\theta)$ being the likelihood function, we know we have:

$$L(\theta) = f(y_1, \cdots, y_n|\theta) = f(y_1|\theta)f(y_2|\theta)\cdots f(y_n|\theta)$$

Where f is the uniform probablity density function with parameter θ.

$$f(y_i,\theta) = \begin{cases} \frac{1}{\theta} & \text{for } 0 \le y_i \le \theta \\ 0 & \text{otherwise} \end{cases}$$

Making

$$L(\theta) = \begin{cases} \frac{1}{\theta^n} & \text{for } 0 \le y_i \le \theta, i \in 1, \cdots, n \\ 0 & \text{otherwise} \end{cases}$$

Part 2 - MLE

Based on $L(\theta)$ The MLE of θ is a value of θ for which $\theta \ge y_i fori \in 1, \cdots, n$ and which maximizes $1/\theta^n$. $MLE(\theta)$ is the smallest of such values of θ such that $\theta \ge y_i fori \in 1, \cdots, n$. Therefore:

$$\mathbf{MLE}(\theta) = \hat{\theta} = \mathbf{max}(\mathbf{y_1}, \cdots, \mathbf{y_n})$$

Part 3 - Expectation n=1

Taking $\hat{\theta} = max(y_1, \cdots, n)$ and n=1
We have

$$\hat{\theta} = y_1$$

And

$$\mathbf{E}[\hat{\theta}] = \mathbf{E}[\mathbf{y_1}] = \frac{\theta}{\mathbf{2}}$$

Knowing that y_i is from a random uniform distribution over $[0, \theta]$

Part 4 - Bias

Yes, from the above, $\hat{\theta}$ is biased. For any $y_1, \cdots, y_n$, we expect $maxy_1, \cdots, n < \theta$ with probability 1. Hence $\hat{\theta}$ underestimates θ and we have just proven that for n=1, $E(\hat{\theta}) \neq \theta$.

Part 5 - Expectation general case

Taking $\hat{\theta} = max(y_1, \cdots, y_n)$ and assuming $n \geq 1$.

$$E(\hat{\theta}) = E[max(y_1, \cdots, y_n)]$$

Let's define $x = max(y_i), i \in 1, \cdots, n$.

$$CDF(x) = P(max(y_i, \cdots, y_n) < x), i \in 1, \cdots, n$$

$$CDF(x) = P(y_1 < x, y_2 < x, \cdots, y_n < x)$$

$$CDF(x) = \prod P(y_i < x), i \in 1, \cdots, n$$

$$CDF(x) = (\frac{x}{\theta})^n$$

From CDF(x), which is the cumulative distribution of x, we deterne the desnsity probability as

$$PDF(x) = \frac{\delta}{\delta x}(\frac{\theta}{x})^n$$

$$PDF(x) = \frac{nx^{n-1}}{\theta^n}$$

From PDF(x), we can now compute E(x) as:

$$E(x) = \int_{x=0}^{\theta} \frac{n(x^{n-1})}{\theta} x dx$$

and

$$\mathbf{E(x)} = \hat{\theta} = \frac{\mathbf{n}}{\mathbf{n+1}}\theta$$

Part 6 - Expectation general case

From the previous compututation of the general case of $n \geq 1$, we can state that

$$\lim_{\mathbf{n} \to \infty} \hat{\theta} = \theta$$

and $\hat{\theta}$ is a consistent estimator of θ.

Question 4

4.1 Univariate Analysis

- **wage** - The wage variable has a range from \$127 to \$2,404 with a mean of \$579 and median of \$543 with most values occuring between \$250 and \$750. The histogram shows a data distribution that's positevely skewed.

- **logWage** - The logWage variable has a range from \$4.844 to \$7.785 with a mean of \$6.263 and median of \$6.297. The histogram shows a data distribution that's approximately normal.
- **education** - The education variable is an integer and has a range from 2 to 18 with a mean of 12 and median of 12. The histogram shows a data distribution that is slightly negatively skewed. There is a spike at 12 and a smaller spike at 16.
- **experience** - The experience variable is an integer and has a range from 0 to 23 with a mean of 8.788 and median of 8. The histogram shows a data distribution that is slightly positively skewed.
- **experienceSquare** - The experience variable is an integer and has a range from 0 to 529 with a mean of 95.03 and median of 64. The histogram shows a data distribution that is positively skewed. There is a spike at about 50.
- **IQscore** - The IQscore variable is an integer and has a range from 50 to 144 with a mean of 102.3 and median of 103. The histogram shows a data distribution that is approximately normal. There are 316 missing values.
- **dad_education** - The dad_education variable is an integer and has a range from 0 to 18 with a mean of 10.18 and median of 11. The histogram shows a data distribution that has many frequencies at about count 30 and spikes at 8 and 12. These spikes make intuitive sense because these are natural educaiton breakpoints for people. Eight years signifying the end of middle school and 12 years indicating the end of high school. There are 239 missing values.
- **mom_education** - The mom_education variable is an integer and has a range from 0 to 18 with a mean of 10.45 and median of 12. The histogram shows a data distribution that has many frequencies at about count 50 and spikes at 12. This spike makes intuitive sense because 12 years indicates the end of high school which is a natural education break point for people. There are 128 missing values.
- **age** - The age variable is an integer and has a range from 24 to 34 with a mean of 28.01 and median of 27. For the ages between 24 and 28, the fequency is around 105. For the ages between 29 and 34, the frequency is around 65.
- **raceColor** - The raceColor variable is a binary variable with values 0 or 1 and mean 0.238. This means that there are about 24% 1's and 76% 0's.
- **rural** - The rural variable is a binary variable with values 0 or 1 and mean 0.391. This means that there are about 39% 1's and 61% 0's. 39% of the participants live in a rural area and 61% do not.
- **city** - The rural variable is a binary variable with values 0 or 1 and mean 0.712. This means that there are about 71% 1's and 29% 0's. 71% of the participants live in a city and 29% do not.
- **z1** - The z1 variable is a binary variable with values 0 or 1 and mean 0.44. This means that there are about 44% 1's and 56% 0's.
- **z2** - The z2 variable is a binary variable with values 0 or 1 and mean 0.686. This means that there are about 69% 1's and 31% 0's.

```r
# Load the data in to the df dataframe
data = read.csv("WageData2.csv", header = TRUE)
# There was already a logWage variable in the dataset, so set that one
# to logWageOLD
data$logWageOLD = data$logWage
# Create a logWage variable to use for the rest of the problem
data$logWage = log(data$wage)
# Create the experienceSquare variable
data$experienceSquare = data$experience * data$experience
```

```r
# wage variable
summary(data$wage)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   127.0   400.0   543.0   578.8   702.5  2404.0
```

```r
print(quantile(data$wage, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9,
    0.95, 0.99, 1)))
```

```
##      1%      5%     10%     25%     50%     75%     90%     95%     99%
##  187.92  244.90  289.00  400.00  543.00  702.50  914.00 1068.70 1402.23
##    100%
## 2404.00
```

```r
# Plot the histogram of apps at 30 bins
wage.hist <- ggplot(data, aes(wage)) + theme(legend.position = "none") +
    geom_histogram(fill = "Blue", colour = "Black", binwidth = (range(data$wage)[2] -
        range(data$wage)[1])/30) + labs(title = "Distribution of wage",
    x = "wage ($)", y = "Frequency")

plot(wage.hist)
```


```r
# logWage variable
summary(data$logWage)
```

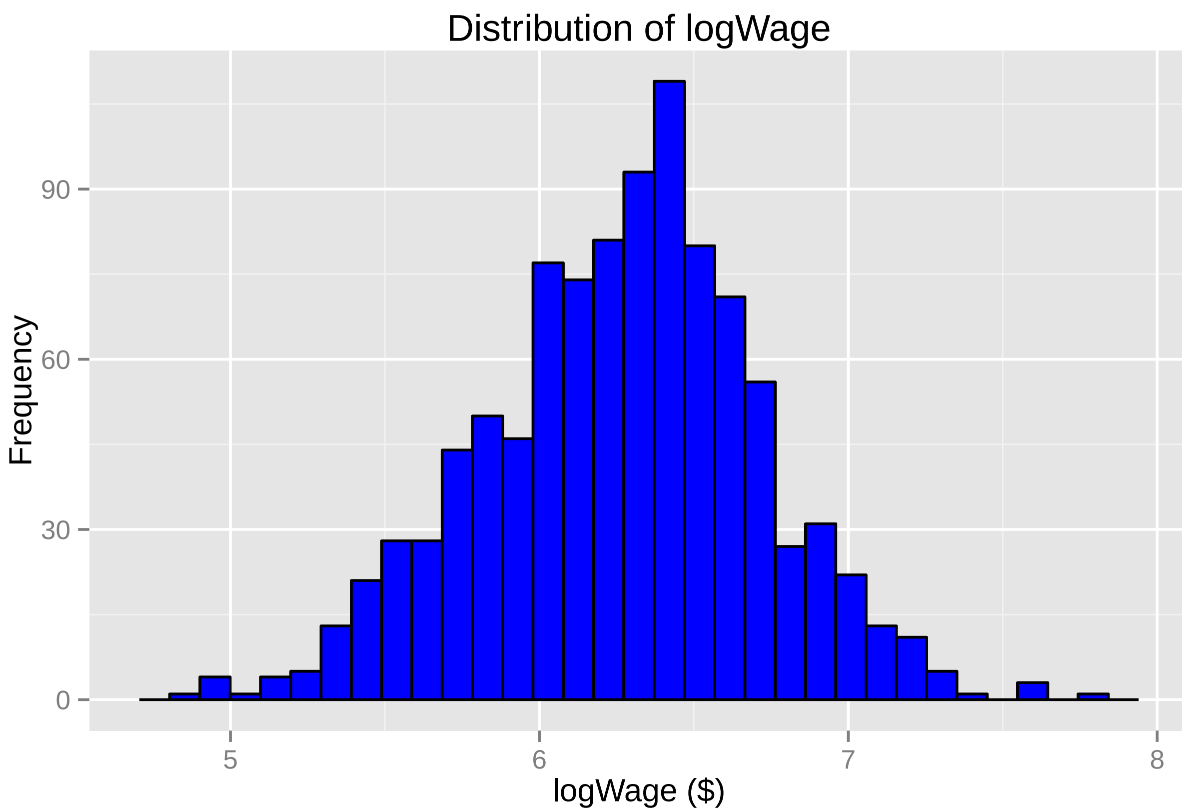
```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   4.844   5.991   6.297   6.263   6.555   7.785
```

```r
print(quantile(data$logWage, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75,
    0.9, 0.95, 0.99, 1)))
```

```
##       1%       5%      10%      25%      50%      75%      90%      95%
## 5.236007 5.500848 5.666427 5.991465 6.297109 6.554645 6.817825 6.974194
##      99%     100%
## 7.245818 7.784889
```

```r
# Plot the histogram of apps at 30 bins
logWage.hist <- ggplot(data, aes(logWage)) + theme(legend.position = "none") +
    geom_histogram(fill = "Blue", colour = "Black", binwidth = (range(data$logWage)[2] -
        range(data$logWage)[1])/30) + labs(title = "Distribution of logWage",
    x = "logWage ($)", y = "Frequency")

plot(logWage.hist)
```




```r
# education variable
summary(data$education)
```

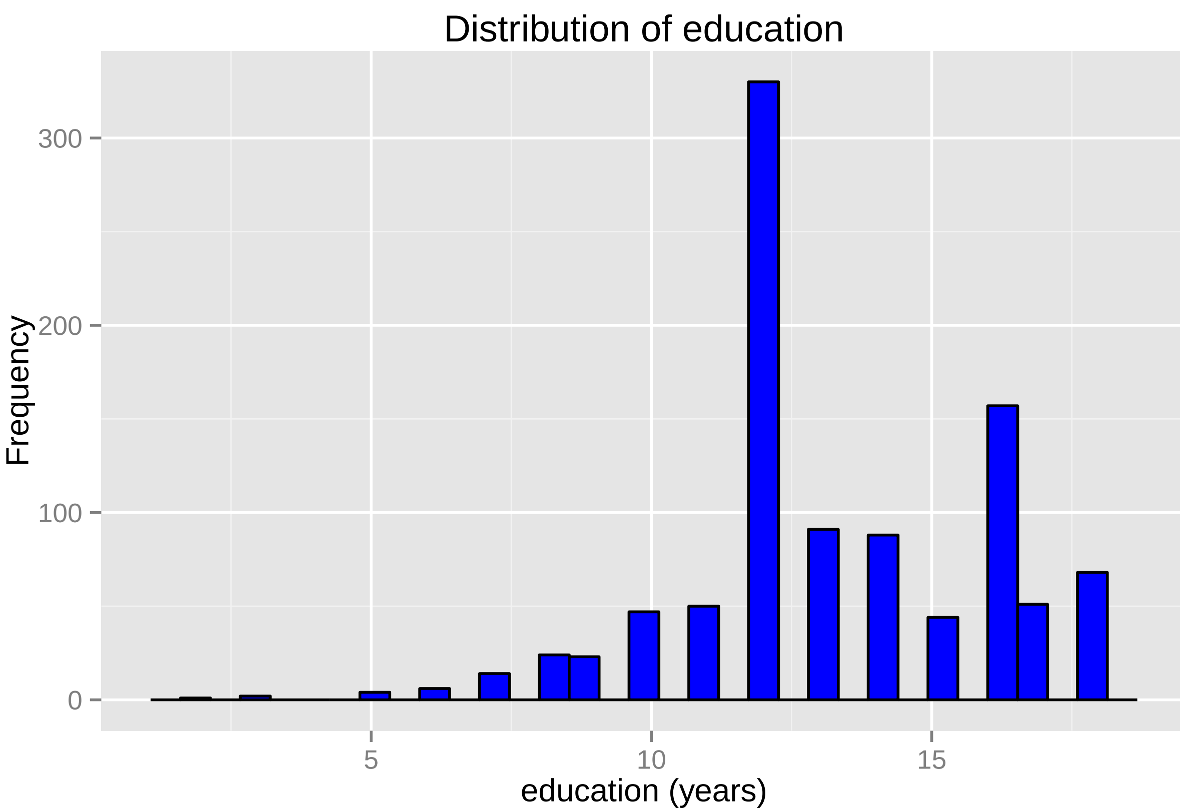
```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    2.00   12.00   12.00   13.22   16.00   18.00
```

```r
print(quantile(data$education, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75,
    0.9, 0.95, 0.99, 1)))
```

```
##   1%   5%  10%  25%  50%  75%  90%  95%  99% 100%
##    6    8   10   12   12   16   17   18   18   18
```

```r
# Plot the histogram of apps at 30 bins
education.hist <- ggplot(data, aes(education)) + theme(legend.position = "none") +
    geom_histogram(fill = "Blue", colour = "Black", binwidth = (range(data$education)[2] -
        range(data$education)[1])/30) + labs(title = "Distribution of education",
    x = "education (years)", y = "Frequency")

plot(education.hist)
```




```r
# experience variable
summary(data$experience)
```

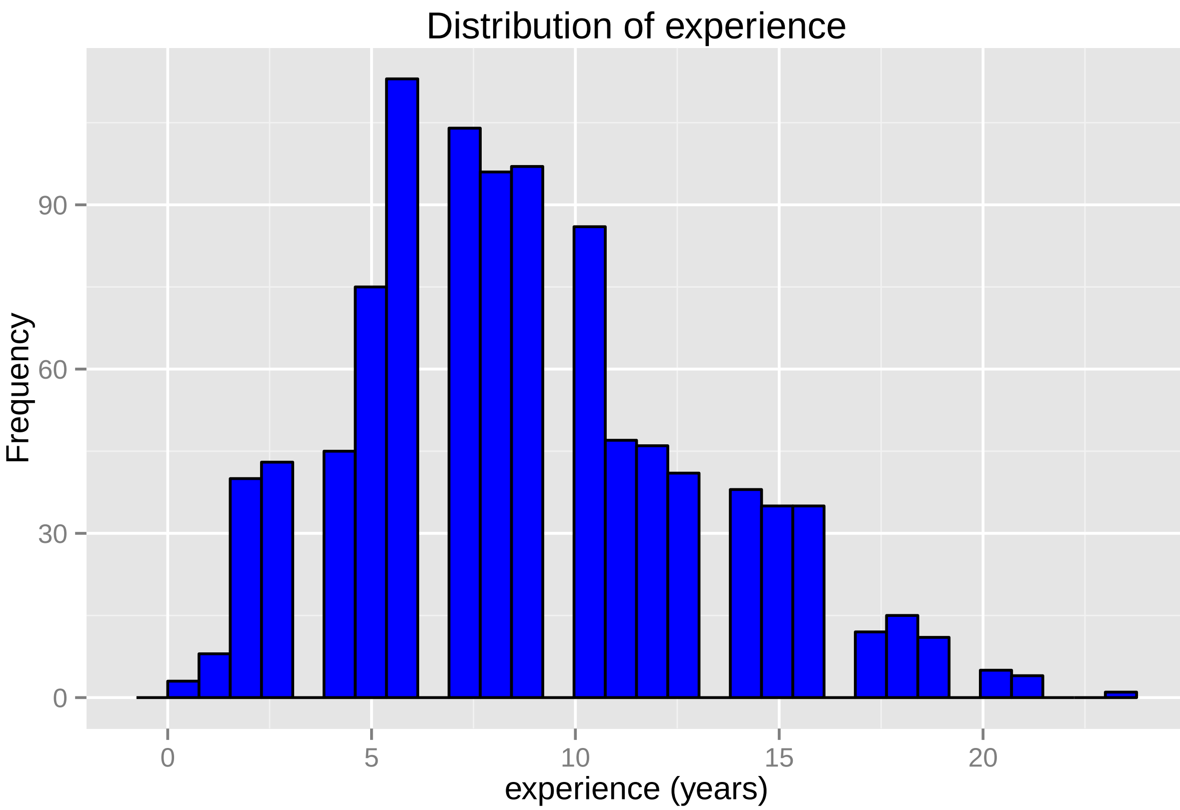
```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.000   6.000   8.000   8.788  11.000  23.000
```

```r
print(quantile(data$experience, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75,
    0.9, 0.95, 0.99, 1)))
```

```
##    1%    5%   10%   25%   50%   75%   90%   95%   99%  100%
##  1.00  2.00  4.00  6.00  8.00 11.00 15.00 16.00 19.01 23.00
```

```r
# Plot the histogram of apps at 30 bins
experience.hist <- ggplot(data, aes(experience)) + theme(legend.position = "none") +
    geom_histogram(fill = "Blue", colour = "Black", binwidth = (range(data$experience)[2] -
        range(data$experience)[1])/30) + labs(title = "Distribution of experience",
    x = "experience (years)", y = "Frequency")

plot(experience.hist)
```




```r
# experienceSquare variable
summary(data$experienceSquare)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    0.00   36.00   64.00   95.03  121.00  529.00
```

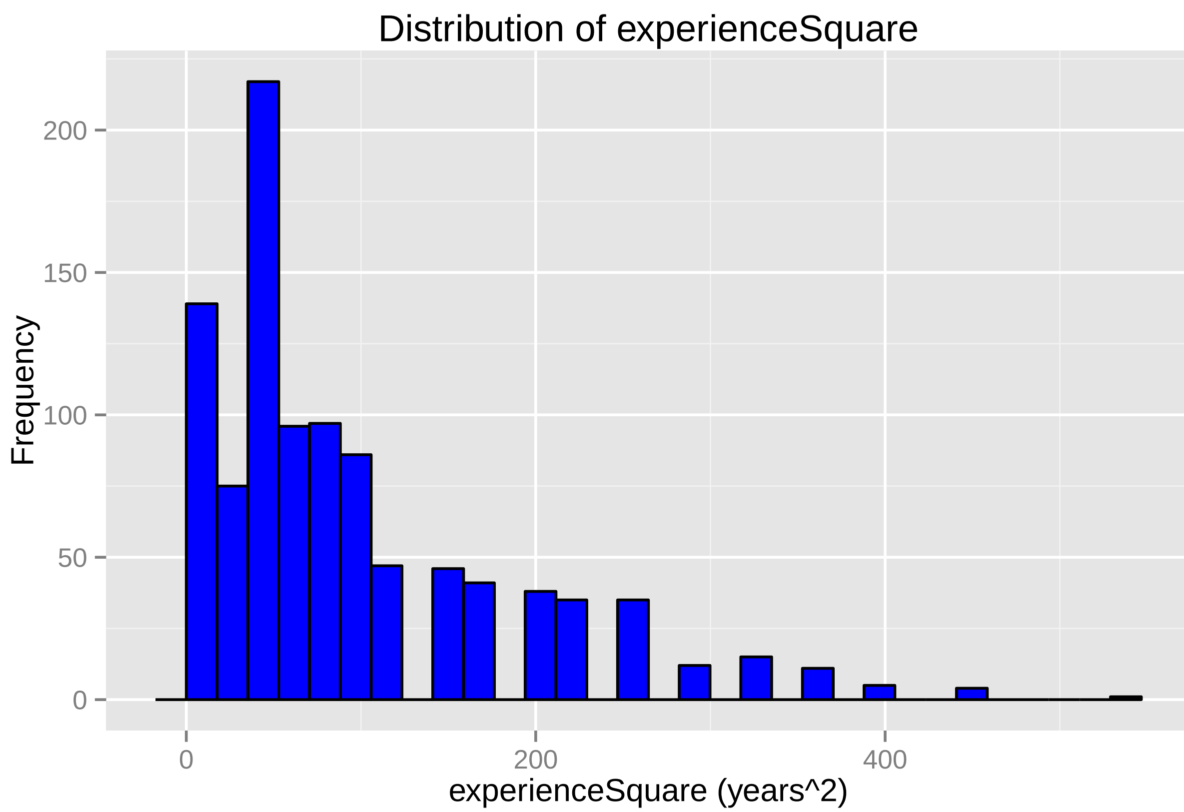
```r
print(quantile(data$experienceSquare, probs = c(0.01, 0.05, 0.1, 0.25,
    0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##     1%      5%     10%     25%     50%     75%     90%     95%     99%    100%
##   1.00    4.00   16.00   36.00   64.00  121.00  225.00  256.00  361.39  529.00
```

```r
# Plot the histogram of apps at 30 bins
experienceSquare.hist <- ggplot(data, aes(experienceSquare)) + theme(legend.position = "none") +
    geom_histogram(fill = "Blue", colour = "Black", binwidth = (range(data$experienceSquare)[2] -
```

```r
        range(data$experienceSquare)[1])/30) + labs(title = "Distribution of experienceSquare",
    x = "experienceSquare (years^2)", y = "Frequency")

plot(experienceSquare.hist)
```




```r
# IQscore variable
summary(data$IQscore)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##    50.0    93.0   103.0   102.3   113.0   144.0     316
```

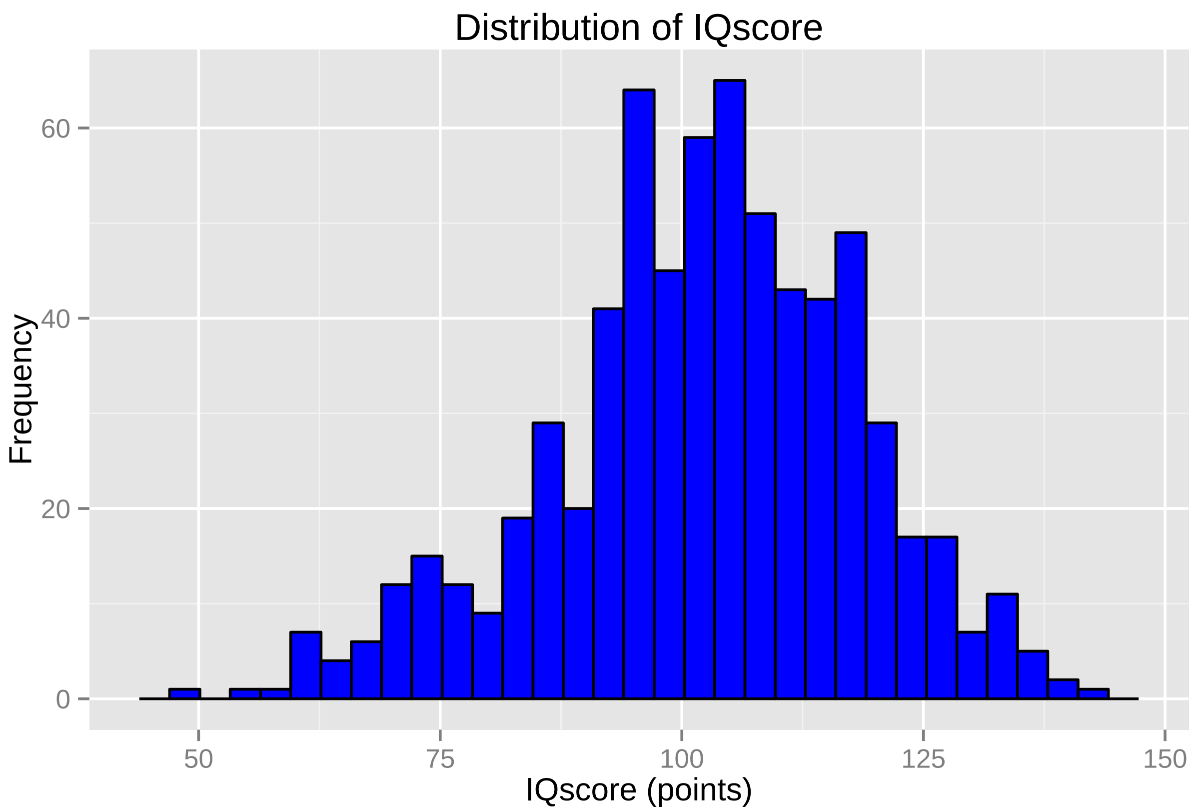
```r
print(quantile(data$IQscore, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75,
    0.9, 0.95, 0.99, 1), na.rm = TRUE))
```

```
##     1%      5%     10%     25%     50%     75%     90%     95%     99%    100%
##  61.83   73.15   82.00   93.00  103.00  113.00  122.00  126.85  135.00  144.00
```

```r
# Plot the histogram of apps at 30 bins
IQscore.hist <- ggplot(data, aes(IQscore)) + theme(legend.position = "none") +
    geom_histogram(fill = "Blue", colour = "Black") + labs(title = "Distribution of IQscore",
    x = "IQscore (points)", y = "Frequency")

plot(IQscore.hist)
```

```
## stat_bin: binwidth defaulted to range/30. Use 'binwidth = x' to adjust this.
```




```r
# dad_education variable
summary(data$dad_education)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##    0.00    8.00   11.00   10.18   12.00   18.00     239
```

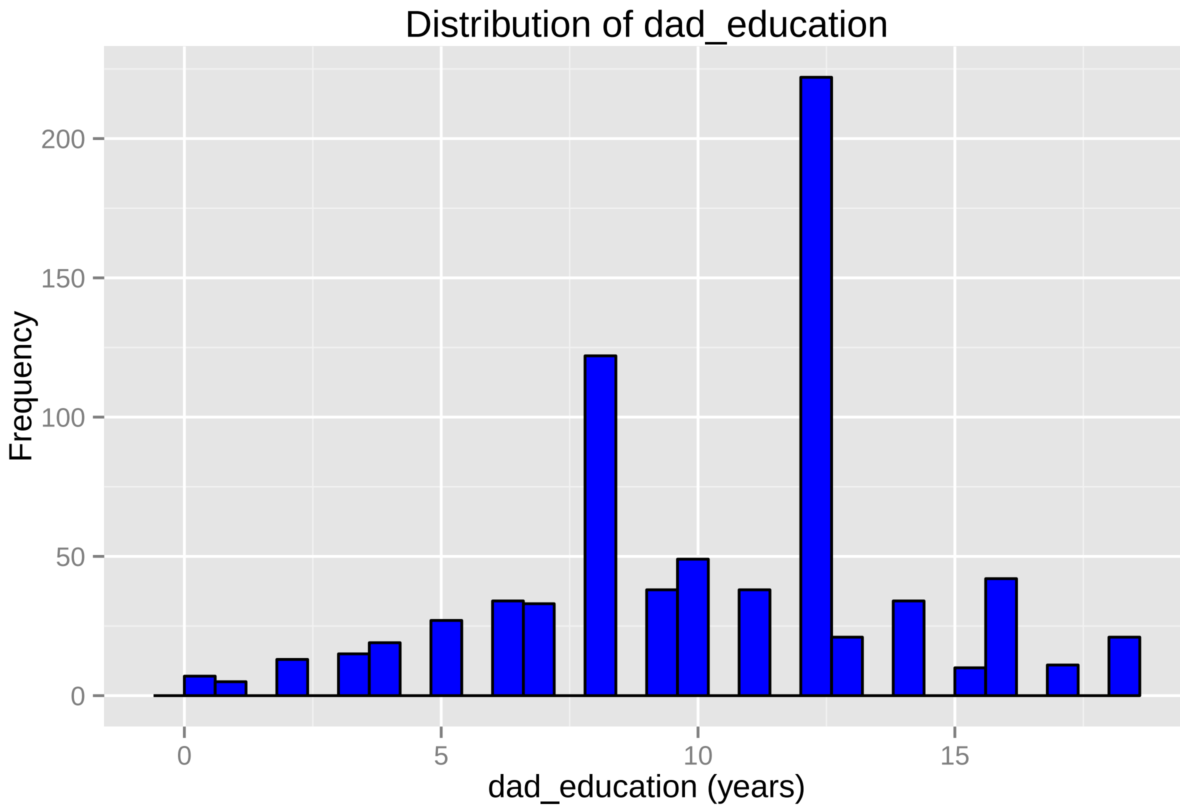
```r
print(quantile(data$dad_education, probs = c(0.01, 0.05, 0.1, 0.25, 0.5,
    0.75, 0.9, 0.95, 0.99, 1), na.rm = TRUE))
```

```
##   1%   5%  10%  25%  50%  75%  90%  95%  99% 100%
##    1    3    5    8   11   12   15   16   18   18
```

```r
# Plot the histogram of apps at 30 bins
dad_education.hist <- ggplot(data, aes(dad_education)) + theme(legend.position = "none") +
    geom_histogram(fill = "Blue", colour = "Black") + labs(title = "Distribution of dad_education",
    x = "dad_education (years)", y = "Frequency")

plot(dad_education.hist)
```

```
## stat_bin: binwidth defaulted to range/30. Use 'binwidth = x' to adjust this.
```




```
# mom_education variable
summary(data$mom_education)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##    0.00    8.00   12.00   10.45   12.00   18.00     128
```

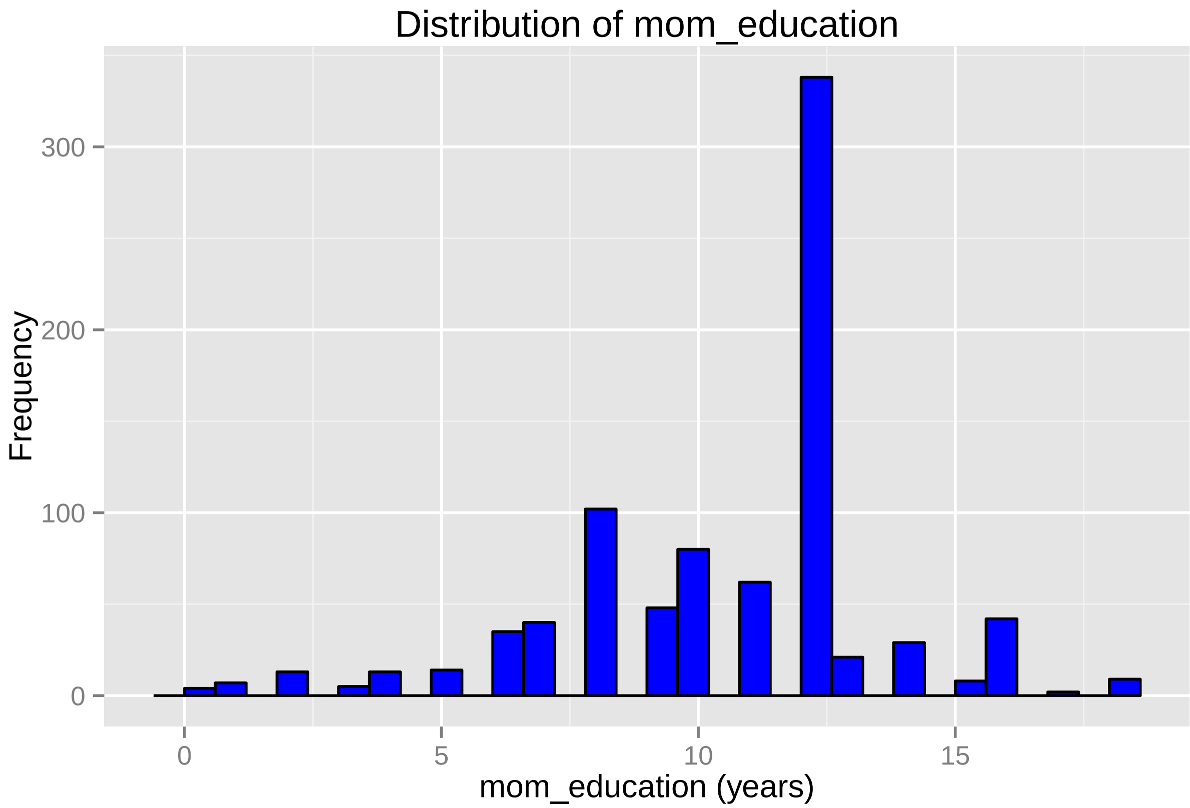
```
print(quantile(data$mom_education, probs = c(0.01, 0.05, 0.1, 0.25, 0.5,
    0.75, 0.9, 0.95, 0.99, 1), na.rm = TRUE))
```

```
##    1%    5%   10%   25%   50%   75%   90%   95%   99%  100%
##  1.00  5.00  6.00  8.00 12.00 12.00 14.00 16.00 17.29 18.00
```

```
# Plot the histogram of apps at 30 bins
mom_education.hist <- ggplot(data, aes(mom_education)) + theme(legend.position = "none") +
    geom_histogram(fill = "Blue", colour = "Black") + labs(title = "Distribution of mom_education",
    x = "mom_education (years)", y = "Frequency")

plot(mom_education.hist)
```

```
## stat_bin: binwidth defaulted to range/30. Use 'binwidth = x' to adjust this.
```




```r
# age variable
summary(data$age)
```

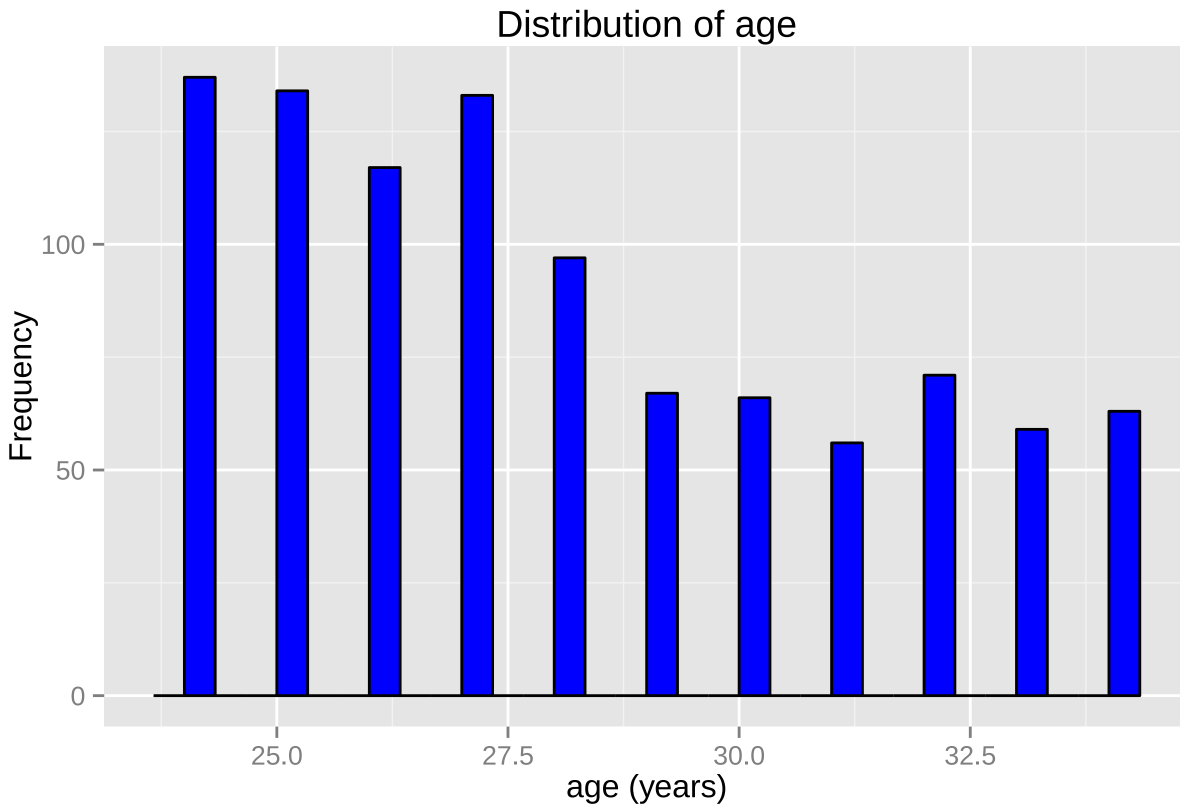
```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   24.00   25.00   27.00   28.01   30.00   34.00
```

```r
print(quantile(data$age, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9,
    0.95, 0.99, 1), na.rm = TRUE))
```

```
##   1%   5%  10%  25%  50%  75%  90%  95%  99% 100%
##   24   24   24   25   27   30   33   34   34   34
```

```r
# Plot the histogram of apps at 30 bins
age.hist <- ggplot(data, aes(age)) + theme(legend.position = "none") +
    geom_histogram(fill = "Blue", colour = "Black", binwidth = (range(data$age)[2] -
        range(data$age)[1])/30) + labs(title = "Distribution of age", x = "age (years)",
    y = "Frequency")

plot(age.hist)
```




```r
# raceColor variable
summary(data$raceColor)
```

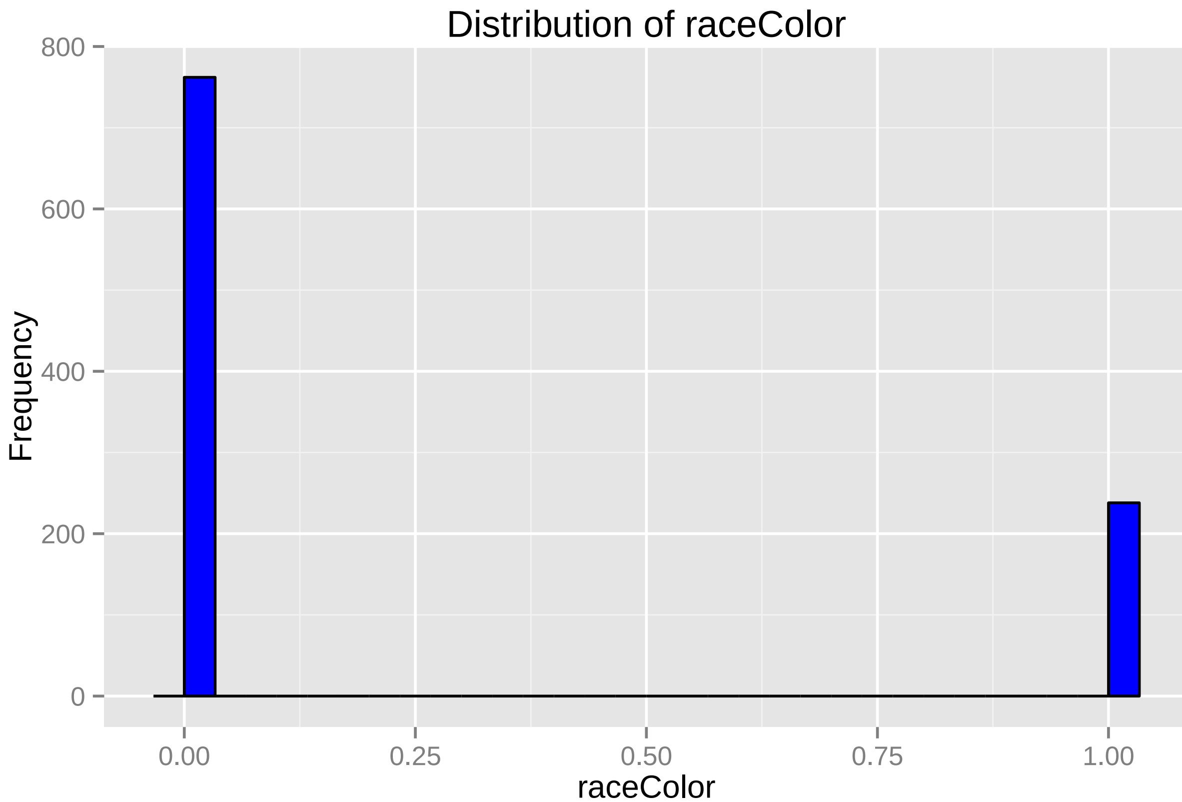
```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.000   0.000   0.000   0.238   0.000   1.000
```

```r
print(quantile(data$raceColor, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75,
    0.9, 0.95, 0.99, 1), na.rm = TRUE))
```

```
##   1%   5%  10%  25%  50%  75%  90%  95%  99% 100%
##    0    0    0    0    0    0    1    1    1    1
```

```r
# Plot the histogram of apps at 30 bins
raceColor.hist <- ggplot(data, aes(raceColor)) + theme(legend.position = "none") +
    geom_histogram(fill = "Blue", colour = "Black", binwidth = (range(data$raceColor)[2] -
        range(data$raceColor)[1])/30) + labs(title = "Distribution of raceColor",
    x = "raceColor", y = "Frequency")

plot(raceColor.hist)
```




```
# rural variable
summary(data$rural)
```

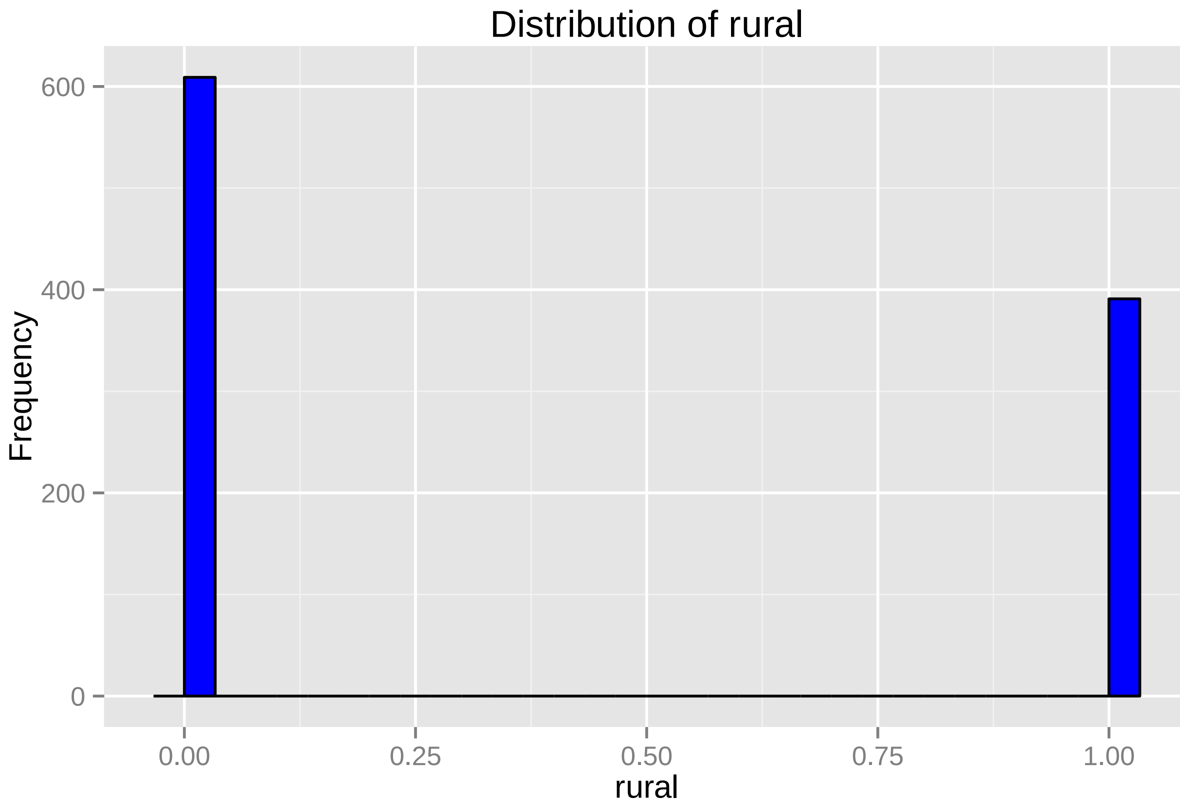
```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.000   0.000   0.000   0.391   1.000   1.000
```

```
print(quantile(data$rural, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75,
    0.9, 0.95, 0.99, 1), na.rm = TRUE))
```

```
##   1%   5%  10%  25%  50%  75%  90%  95%  99% 100%
##    0    0    0    0    0    1    1    1    1    1
```

```
# Plot the histogram of apps at 30 bins
rural.hist <- ggplot(data, aes(rural)) + theme(legend.position = "none") +
    geom_histogram(fill = "Blue", colour = "Black", binwidth = (range(data$rural)[2] -
        range(data$rural)[1])/30) + labs(title = "Distribution of rural",
    x = "rural", y = "Frequency")

plot(rural.hist)
```




```
# city variable
summary(data$city)
```

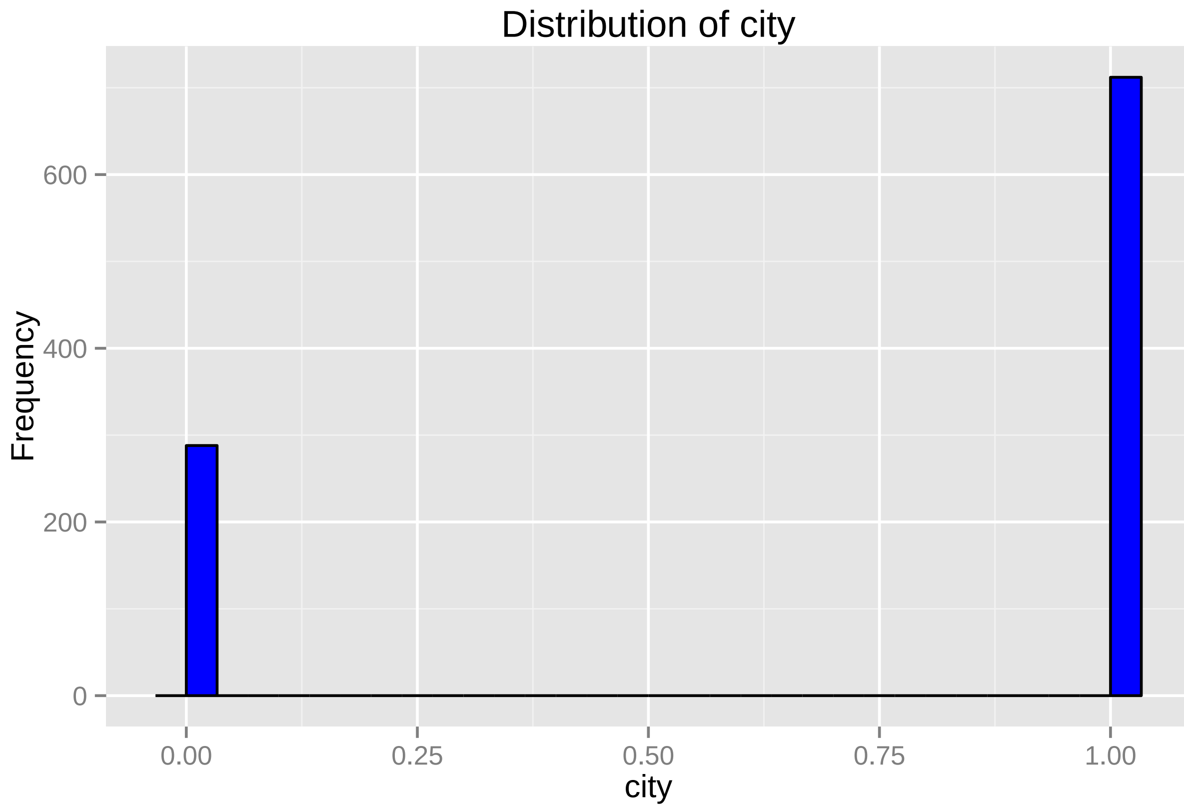
```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.000   0.000   1.000   0.712   1.000   1.000
```

```
print(quantile(data$city, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9,
    0.95, 0.99, 1), na.rm = TRUE))
```

```
##   1%   5%  10%  25%  50%  75%  90%  95%  99% 100%
##    0    0    0    0    1    1    1    1    1    1
```

```
# Plot the histogram of apps at 30 bins
city.hist <- ggplot(data, aes(city)) + theme(legend.position = "none") +
    geom_histogram(fill = "Blue", colour = "Black", binwidth = (range(data$city)[2] -
        range(data$city)[1])/30) + labs(title = "Distribution of city",
    x = "city", y = "Frequency")

plot(city.hist)
```




```
# z1 variable
summary(data$z1)
```

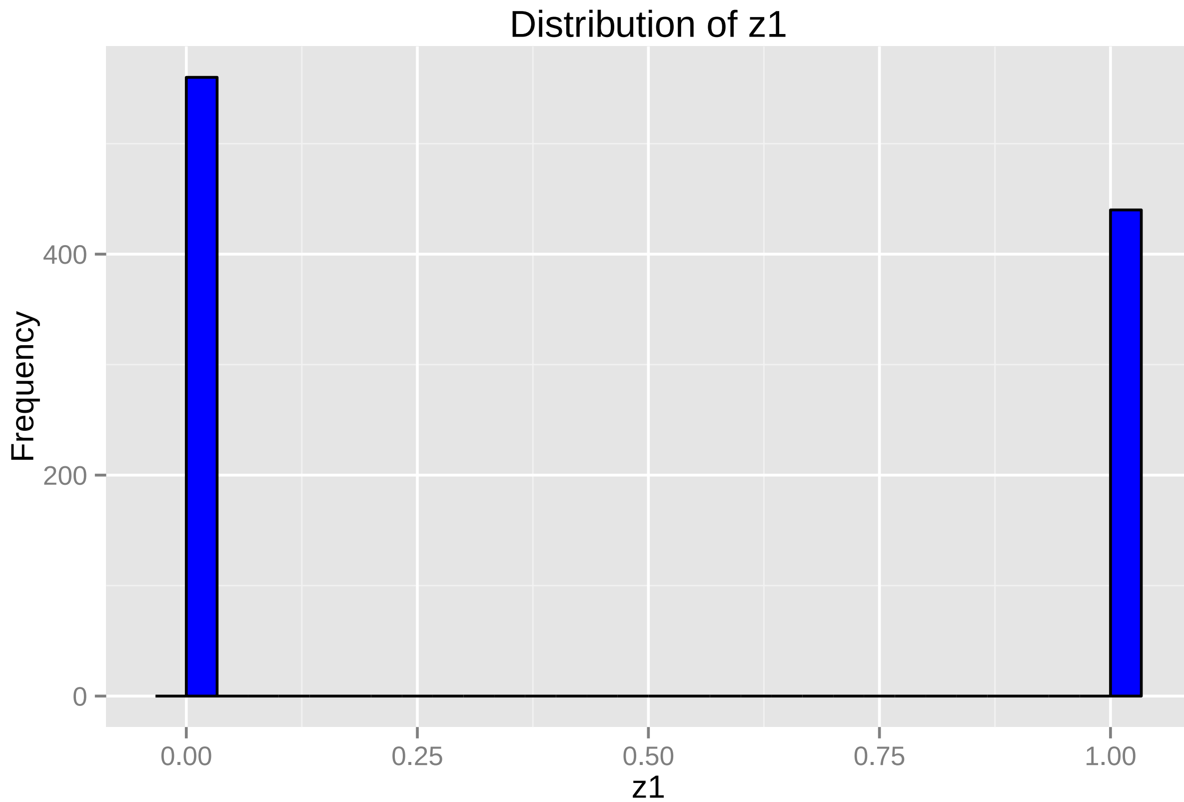
```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    0.00    0.00    0.00    0.44    1.00    1.00
```

```
print(quantile(data$z1, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9,
    0.95, 0.99, 1), na.rm = TRUE))
```

```
##   1%   5%  10%  25%  50%  75%  90%  95%  99% 100%
##    0    0    0    0    0    1    1    1    1    1
```

```
# Plot the histogram of apps at 30 bins
z1.hist <- ggplot(data, aes(z1)) + theme(legend.position = "none") + geom_histogram(fill = "Blue",
    colour = "Black", binwidth = (range(data$z1)[2] - range(data$z1)[1])/30) +
    labs(title = "Distribution of z1", x = "z1", y = "Frequency")

plot(z1.hist)
```




```r
# z2 variable
summary(data$z2)
```

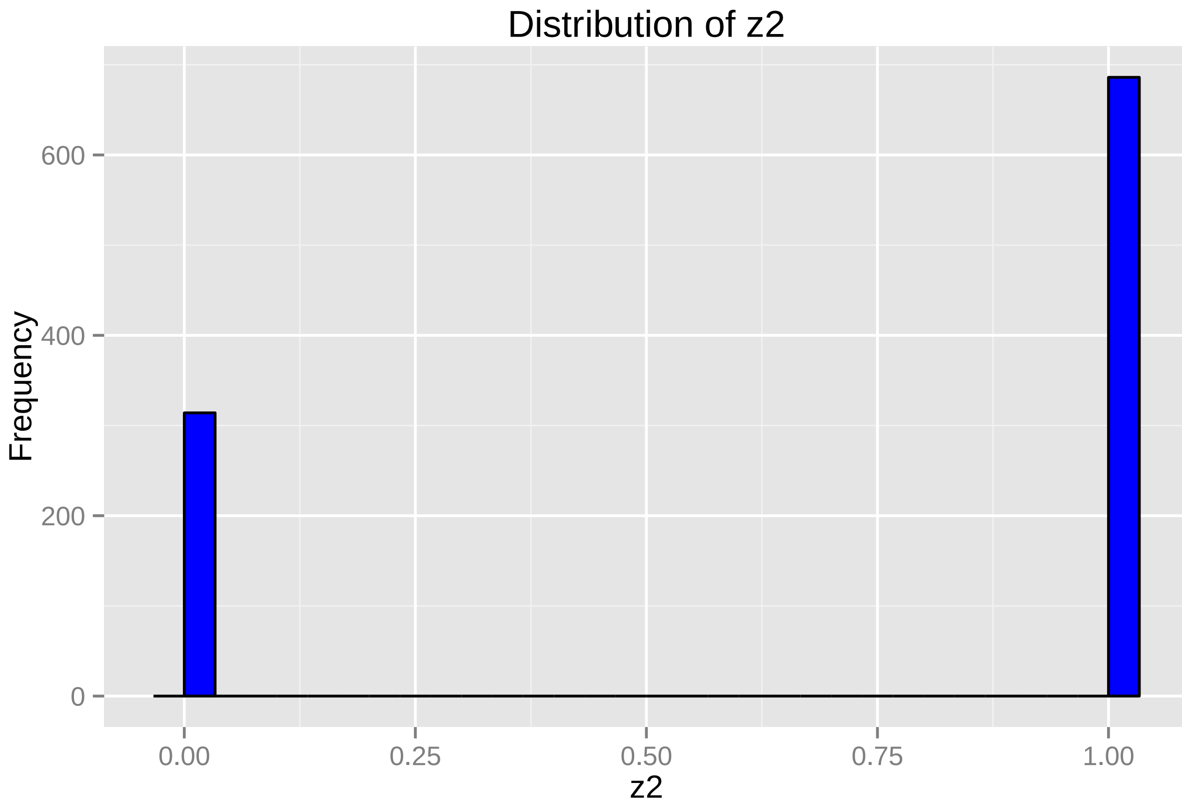
```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.000   0.000   1.000   0.686   1.000   1.000
```

```r
print(quantile(data$z2, probs = c(0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9,
    0.95, 0.99, 1), na.rm = TRUE))
```

```
##   1%   5%  10%  25%  50%  75%  90%  95%  99% 100%
##    0    0    0    0    1    1    1    1    1    1
```

```r
# Plot the histogram of apps at 30 bins
z2.hist <- ggplot(data, aes(z2)) + theme(legend.position = "none") + geom_histogram(fill = "Blue",
    colour = "Black", binwidth = (range(data$z2)[2] - range(data$z2)[1])/30) +
    labs(title = "Distribution of z2", x = "z2", y = "Frequency")

plot(z2.hist)
```

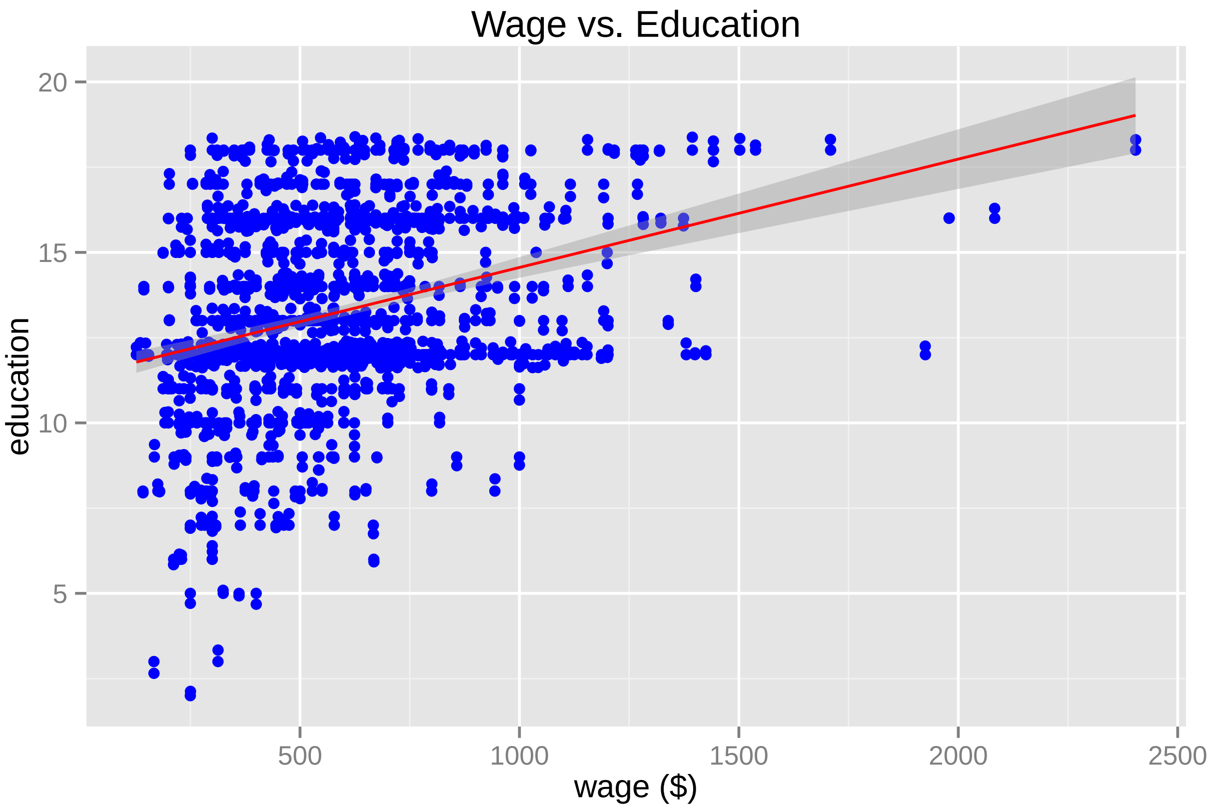


4.2 Bivariate Analysis

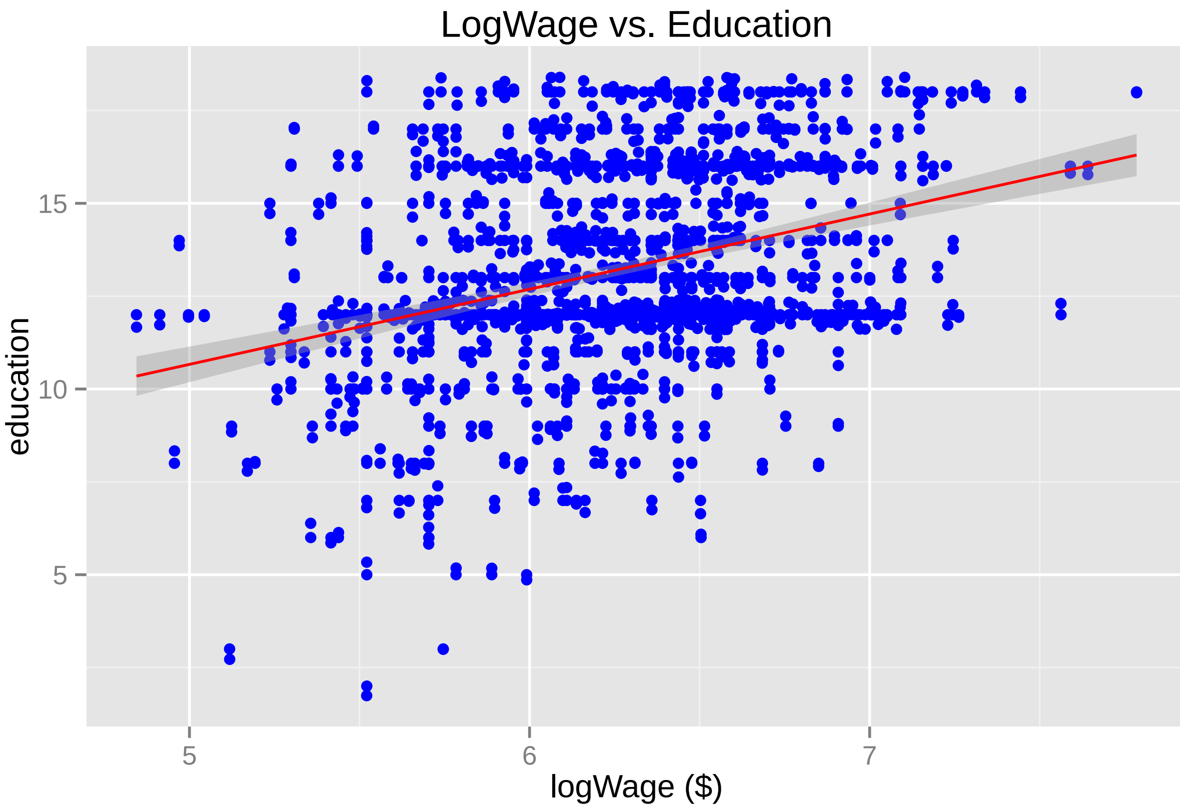
- **wage, logWage vs. education** - Both wage and logWage are weakly correlated with education with a correlation value of about 0.3. The wage vs. education scatterplot shows a possible linear trend.
- **wage, logWage vs. experience** - Both wage and logWage appear uncorrelated with experience with very low correlation values of -0.0060 and -0.0290, respectively. The wage vs. experience scatterplot shows that experience is not affected by wage for the most part. The logWage vs. experience scatterplot shows that experience is not affected by logWage as well.
- **wage, logWage vs. experienceSquare** - Both wage and logWage appear uncorrelated with experienceSquare with very low correlation values of -0.043 and -0.065, respectively. The wage vs. experienceSquare scatterplot shows that experienceSquare is not affected by wage for the most part. The logWage vs. experienceSquare scatterplot shows that experience is not affected by logWage as well.
- **wage, logWage vs. IQscore** - Both wage and logWage are weakly correlated with IQscoare with low correlation values of 0.186 and 0.201, respectively. The wage and logWage vs. IQscore scatterplots show that IQscoare affects wage and logWage slightly. As wage or logWage go up, IQscore increases by a small amount.
- **wage, logWage vs. dad_education** - Both wage and logWage are weakly correlated with dad_education with low correlation values of 0.19 and 0.19, respectively. The wage and logWage vs. dad_education scatterplots show that dad_education affects wage and logWage slightly. As wage or logWage go up, dad_education increases by a small amount.
- **wage, logWage vs. mom_education** - Both wage and logWage are weakly correlated with mom_education with low correlation values of 0.20 and 0.21, respectively. The wage and logWage vs. mom_education scatterplots show that mom_education affects wage and logWage slightly. As wage or logWage go up, mom_education increases by a small amount.

- **wage, logWage vs. age** - Both wage and logWage are weakly correlated with age with low correlation values of 0.26 and 0.25, respectively. The wage and logWage vs. age scatterplots show that age affects wage and logWage slightly. As wage or logWage go up, age increases by a small amount.
- **wage, logWage vs. raceColor** - Both wage and logWage are weakly correlated with raceColor with low correlation values of -0.30 and -0.34, respectively. The wage and logWage vs. raceColor scatterplots show that raceColor affects wage and logWage slightly. As wage or logWage go up, there are fewer people that have the raceColor variable set to 1.
- **wage, logWage vs. rural** - Both wage and logWage are weakly correlated with rural with low correlation values of -0.22 and -0.25, respectively. The wage and logWage vs. rural scatterplots show that rural affects wage and logWage slightly. As wage or logWage go up, there are fewer people that have the rural variable set to 1.
- **wage, logWage vs. city** - Both wage and logWage are weakly correlated with city with low correlation values of 0.22 and 0.24, respectively. The wage and logWage vs. rural scatterplots show that city affects wage and logWage slightly. As wage or logWage go up, there are more people that have the city variable set to 1.
- **wage, logWage vs. z1** - Both wage and logWage are weakly correlated with z1 with low correlation values of 0.101 and 0.087, respectively. The wage and logWage vs. z1 scatterplots show that z1 affects wage and logWage slightly. As wage or logWage go up, there are more people that have the z1 variable set to 1.
- **wage, logWage vs. z2** - Both wage and logWage are weakly correlated with z2 with low correlation values of 0.17 and 0.18, respectively. The wage and logWage vs. z2 scatterplots show that z2 affects wage and logWage slightly. As wage or logWage go up, there are more people that have the z2 variable set to 1. z2 shows a slightly stronger correlation with wage and logWage than z1.

```r
# Scatter plot with wage variable
wage.education.plot = ggplot(data, aes(x = wage, y = education)) + theme(legend.position = "none") +
    geom_point(colour = "Blue") + geom_jitter(colour = "Blue") + geom_smooth(colour = "red",
    method = "lm") + labs(title = "Wage vs. Education", x = "wage ($)",
    y = "education")
plot(wage.education.plot)
```




```
# Scatter plot with logWage variable
lwage.education.plot = ggplot(data, aes(x = logWage, y = education)) +
    theme(legend.position = "none") + geom_point(colour = "Blue") + geom_jitter(colour = "Blue") +
    geom_smooth(colour = "red", method = "lm") + labs(title = "LogWage vs. Education",
    x = "logWage ($)", y = "education")
plot(lwage.education.plot)
```




```r
# Run correlations with wage and logWage variables
cor(data$wage, data$education)
```

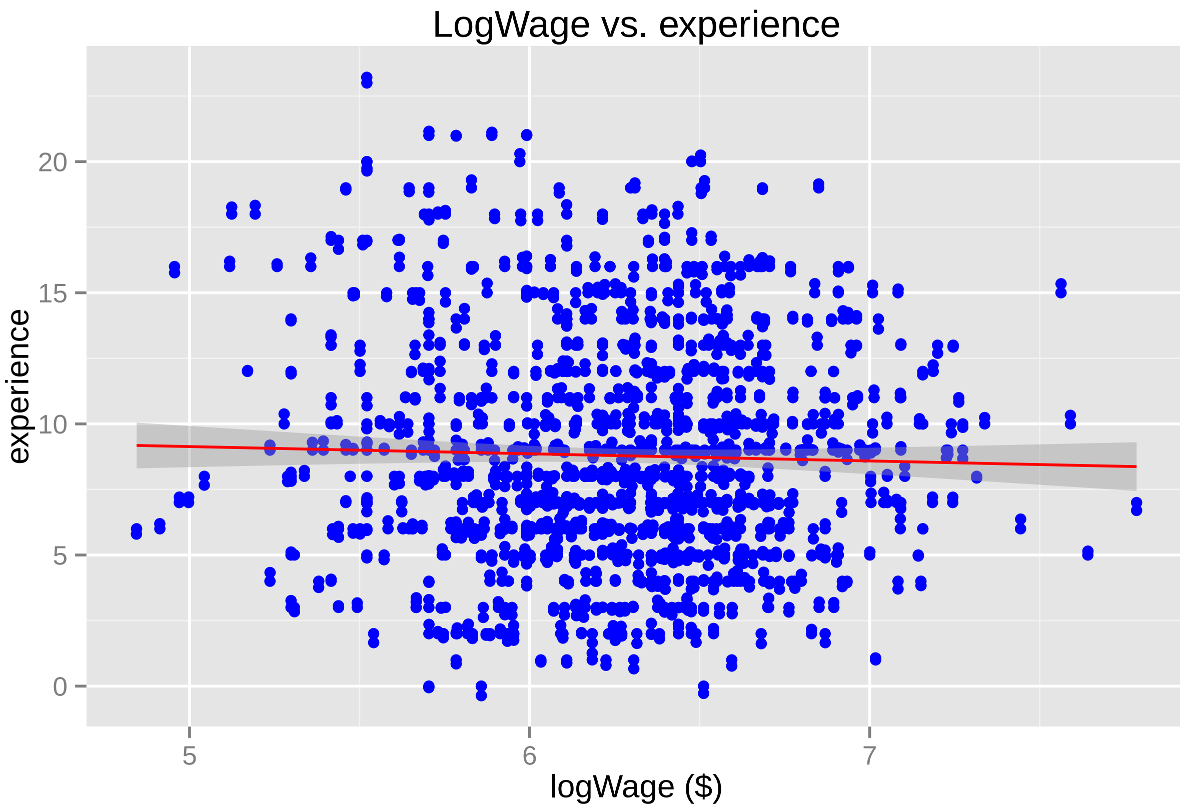
```
## [1] 0.3103986
```

```r
cor(data$logWage, data$education)
```

```
## [1] 0.3318494
```

```r
# Scatter plot with wage variable
wage.experience.plot = ggplot(data, aes(x = wage, y = experience)) + theme(legend.position = "none") +
    geom_point(colour = "Blue") + geom_jitter(colour = "Blue") + geom_smooth(colour = "red",
    method = "lm") + labs(title = "Wage vs. experience", x = "wage ($)",
    y = "experience")
plot(wage.experience.plot)
```


```
# Scatter plot with logWage variable
lwage.experience.plot = ggplot(data, aes(x = logWage, y = experience)) +
    theme(legend.position = "none") + geom_point(colour = "Blue") + geom_jitter(colour = "Blue") +
    geom_smooth(colour = "red", method = "lm") + labs(title = "LogWage vs. experience",
    x = "logWage ($)", y = "experience")
plot(lwage.experience.plot)
```




```
# Run correlations with wage and logWage variables
cor(data$wage, data$experience)
```

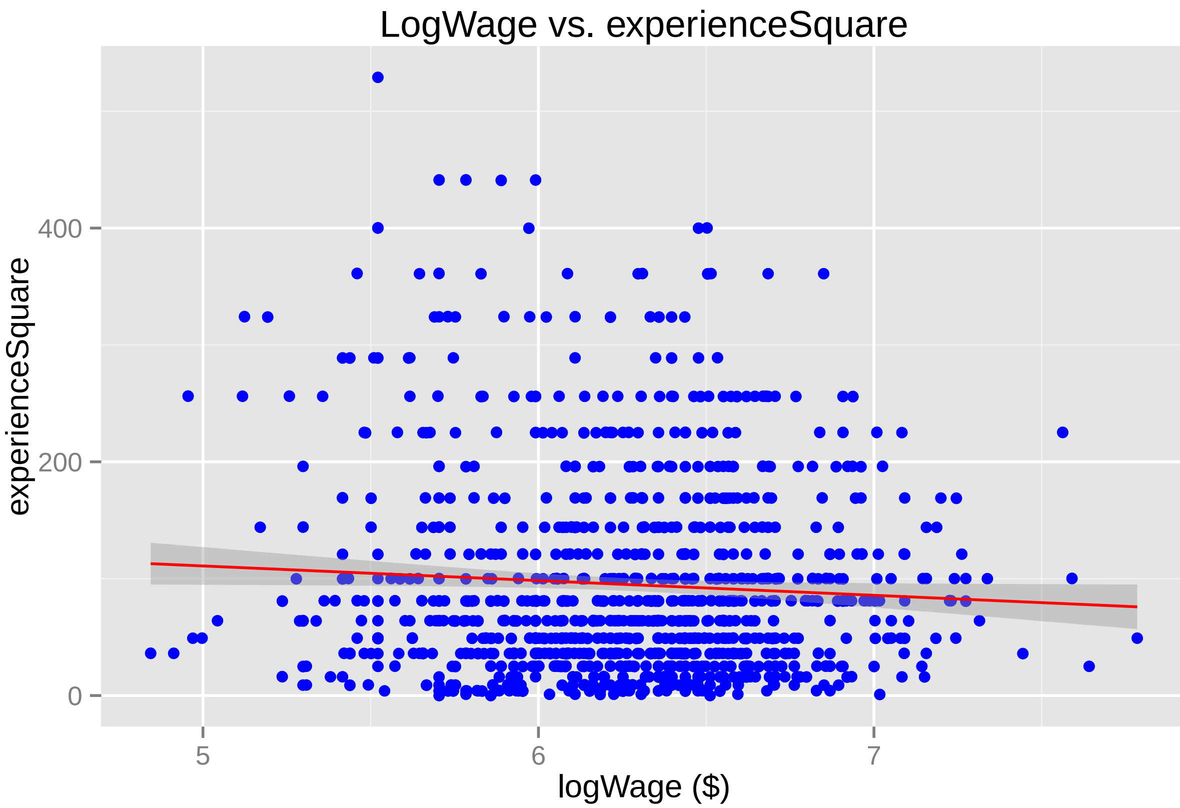
```
## [1] -0.005985988
```

```
cor(data$logWage, data$experience)
```

```
## [1] -0.02905727
```

```
# Scatter plot with wage variable
wage.experienceSquare.plot = ggplot(data, aes(x = wage, y = experienceSquare)) +
    theme(legend.position = "none") + geom_point(colour = "Blue") + geom_jitter(colour = "Blue") +
    geom_smooth(colour = "red", method = "lm") + labs(title = "Wage vs. experienceSquare",
    x = "wage ($)", y = "experienceSquare")
plot(wage.experienceSquare.plot)
```


```
# Scatter plot with logWage variable
lwage.experienceSquare.plot = ggplot(data, aes(x = logWage, y = experienceSquare)) +
    theme(legend.position = "none") + geom_point(colour = "Blue") + geom_jitter(colour = "Blue") +
    geom_smooth(colour = "red", method = "lm") + labs(title = "LogWage vs. experienceSquare",
    x = "logWage ($)", y = "experienceSquare")
plot(lwage.experienceSquare.plot)
```




```
# Run correlations with wage and logWage variables
cor(data$wage, data$experienceSquare)
```

```
## [1] -0.04270455
```

```
cor(data$logWage, data$experienceSquare)
```

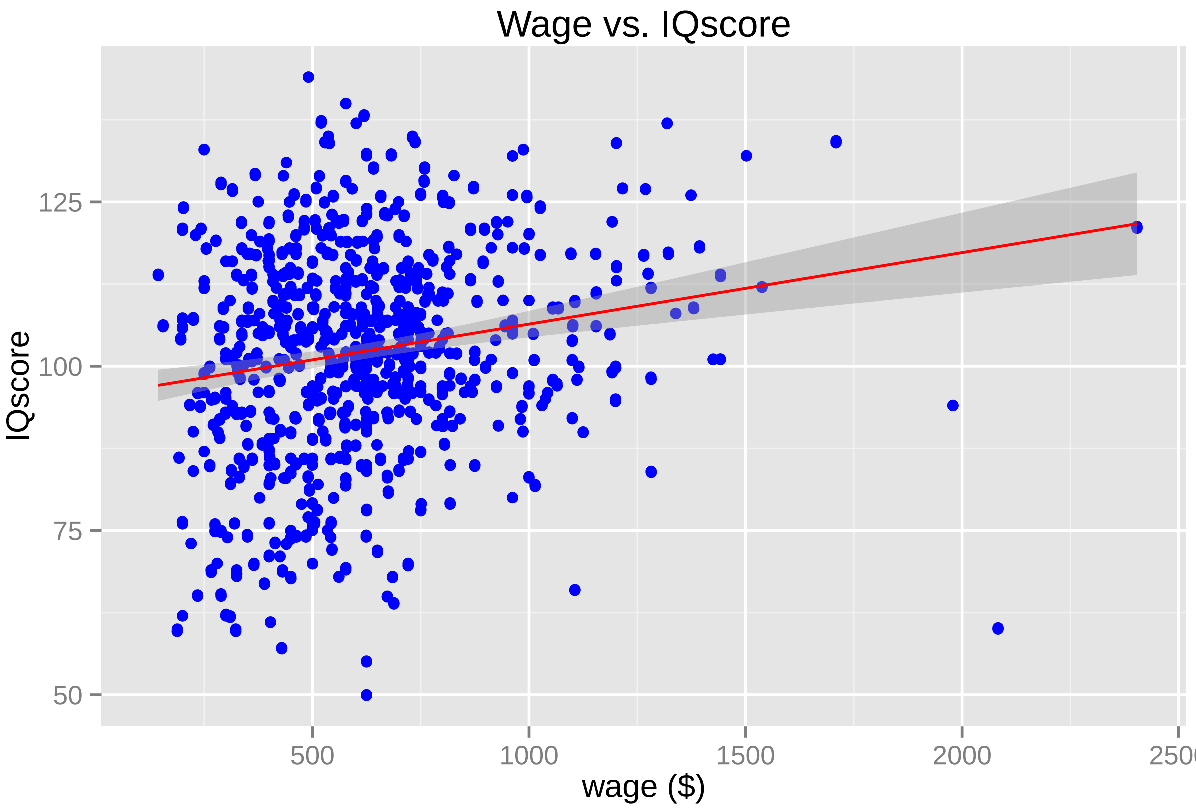
```
## [1] -0.0647476
```

```
# Scatter plot with wage variable
wage.IQscore.plot = ggplot(data, aes(x = wage, y = IQscore)) + theme(legend.position = "none") +
    geom_point(colour = "Blue") + geom_jitter(colour = "Blue") + geom_smooth(colour = "red",
    method = "lm") + labs(title = "Wage vs. IQscore", x = "wage ($)", y = "IQscore")
plot(wage.IQscore.plot)
```

```
## Warning: Removed 316 rows containing missing values (stat_smooth).
```

```
## Warning: Removed 316 rows containing missing values (geom_point).
```

```
## Warning: Removed 316 rows containing missing values (geom_point).
```

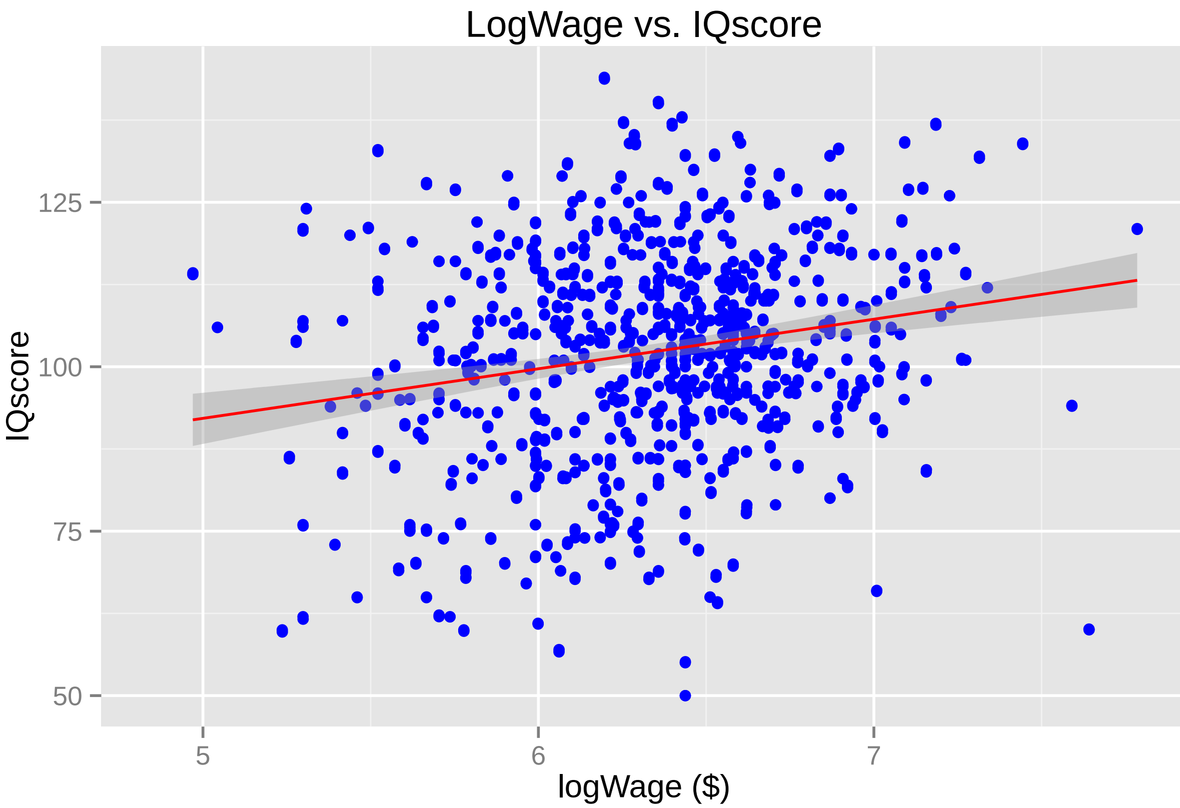



```
# Scatter plot with logWage variable
lwage.IQscore.plot = ggplot(data, aes(x = logWage, y = IQscore)) + theme(legend.position = "none") +
    geom_point(colour = "Blue") + geom_jitter(colour = "Blue") + geom_smooth(colour = "red",
    method = "lm") + labs(title = "LogWage vs. IQscore", x = "logWage ($)",
    y = "IQscore")
plot(lwage.IQscore.plot)
```

```
## Warning: Removed 316 rows containing missing values (stat_smooth).

## Warning: Removed 316 rows containing missing values (geom_point).

## Warning: Removed 316 rows containing missing values (geom_point).
```




```
# Run correlations with wage and logWage variables
cor(data$wage, data$IQscore, use = "complete.obs")
```

```
## [1] 0.1858557
```

```
cor(data$logWage, data$IQscore, use = "complete.obs")
```

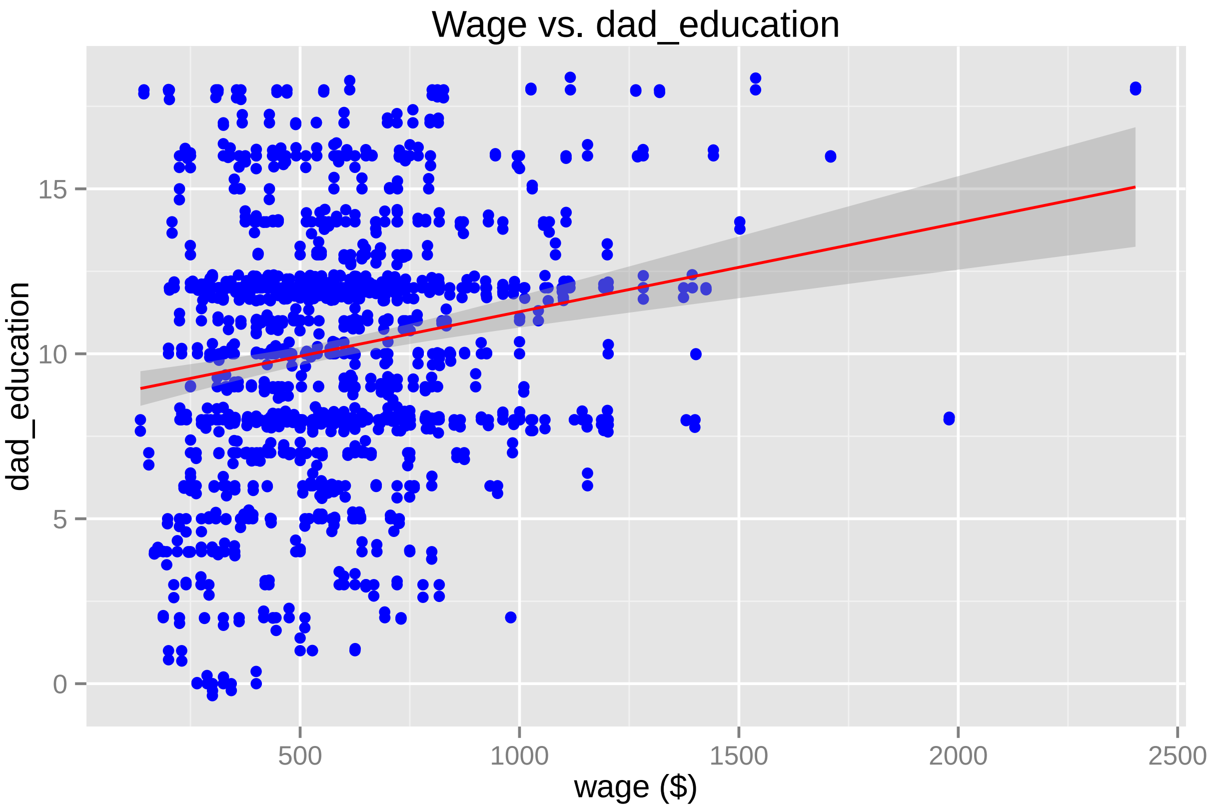
```
## [1] 0.2009578
```

```
# Scatter plot with wage variable
wage.dad_education.plot = ggplot(data, aes(x = wage, y = dad_education)) +
    theme(legend.position = "none") + geom_point(colour = "Blue") + geom_jitter(colour = "Blue") +
    geom_smooth(colour = "red", method = "lm") + labs(title = "Wage vs. dad_education",
    x = "wage ($)", y = "dad_education")
plot(wage.dad_education.plot)
```

```
## Warning: Removed 239 rows containing missing values (stat_smooth).
```

```
## Warning: Removed 239 rows containing missing values (geom_point).
```

```
## Warning: Removed 239 rows containing missing values (geom_point).
```

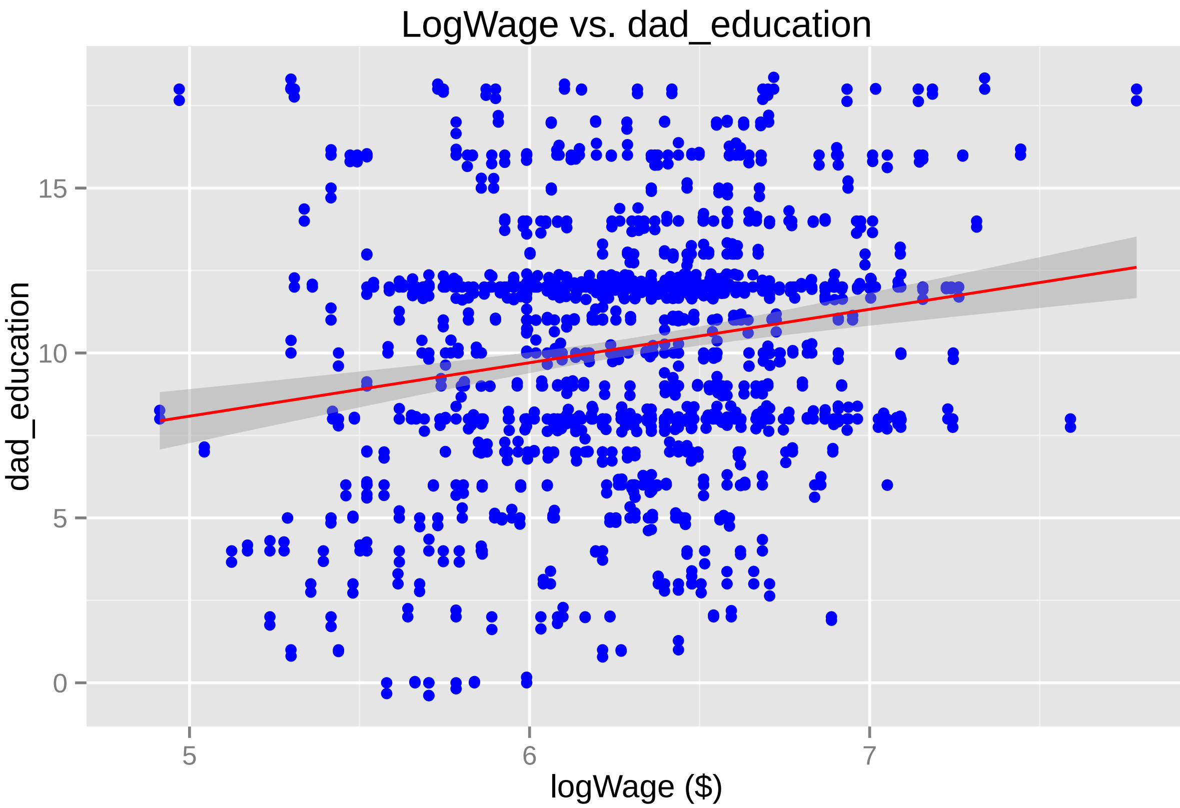



```
# Scatter plot with logWage variable
lwage.dad_education.plot = ggplot(data, aes(x = logWage, y = dad_education)) +
    theme(legend.position = "none") + geom_point(colour = "Blue") + geom_jitter(colour = "Blue") +
    geom_smooth(colour = "red", method = "lm") + labs(title = "LogWage vs. dad_education",
    x = "logWage ($)", y = "dad_education")
plot(lwage.dad_education.plot)
```

```
## Warning: Removed 239 rows containing missing values (stat_smooth).

## Warning: Removed 239 rows containing missing values (geom_point).

## Warning: Removed 239 rows containing missing values (geom_point).
```




```
# Run correlations with wage and logWage variables
cor(data$wage, data$dad_education, use = "complete.obs")
```

```
## [1] 0.1901681
```

```
cor(data$logWage, data$dad_education, use = "complete.obs")
```

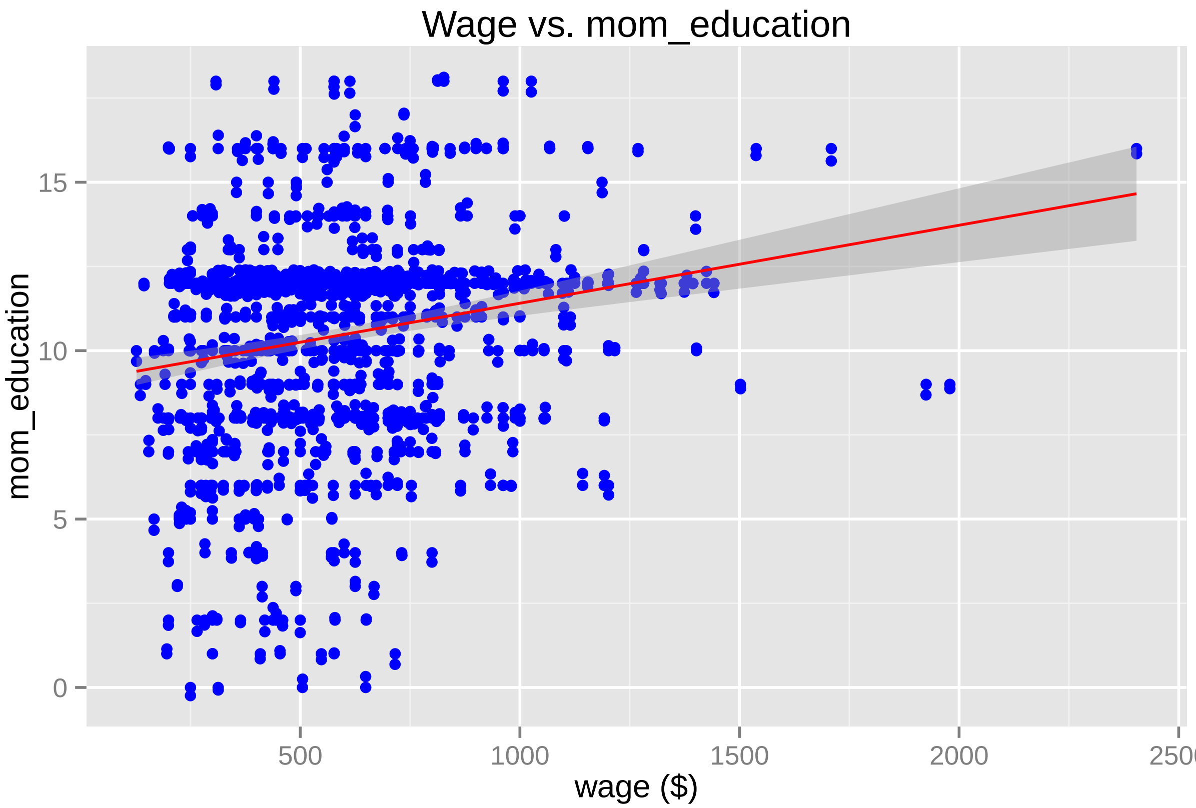
```
## [1] 0.18908
```

```
# Scatter plot with wage variable
wage.mom_education.plot = ggplot(data, aes(x = wage, y = mom_education)) +
    theme(legend.position = "none") + geom_point(colour = "Blue") + geom_jitter(colour = "Blue") +
    geom_smooth(colour = "red", method = "lm") + labs(title = "Wage vs. mom_education",
    x = "wage ($)", y = "mom_education")
plot(wage.mom_education.plot)
```

```
## Warning: Removed 128 rows containing missing values (stat_smooth).

## Warning: Removed 128 rows containing missing values (geom_point).

## Warning: Removed 128 rows containing missing values (geom_point).
```

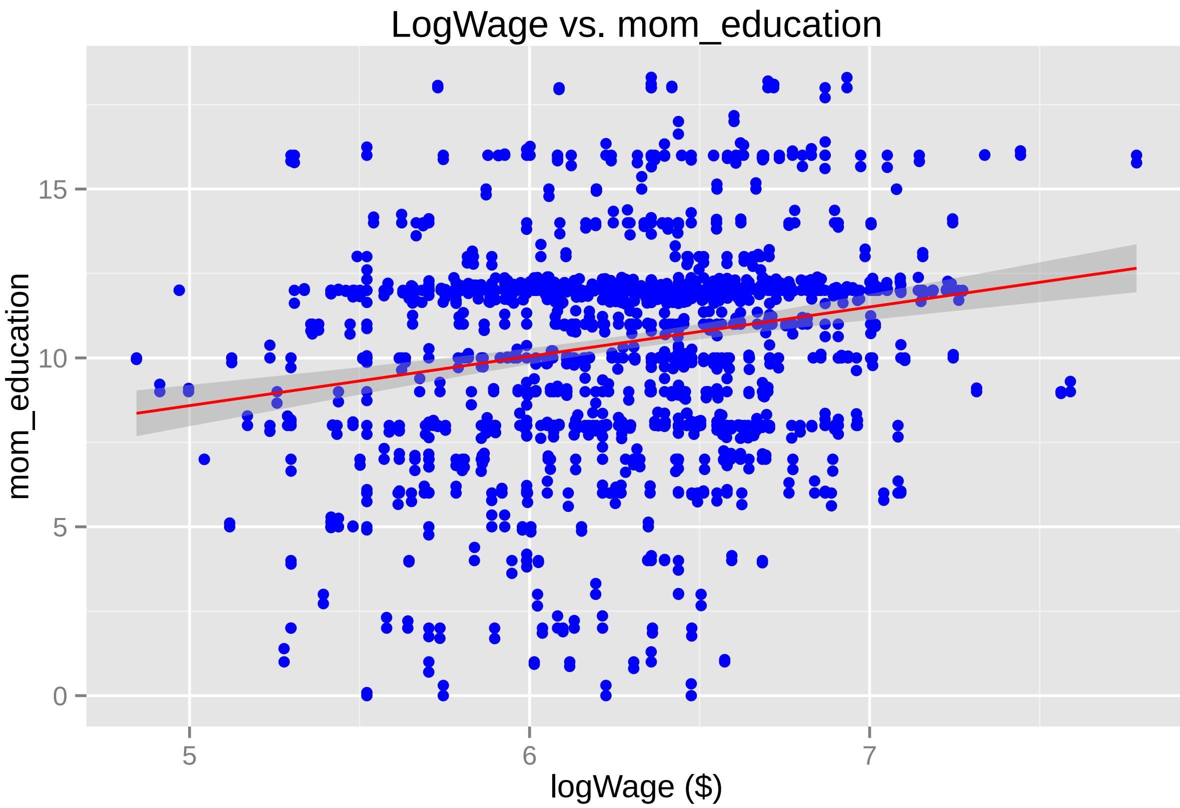



```
# Scatter plot with logWage variable
lwage.mom_education.plot = ggplot(data, aes(x = logWage, y = mom_education)) +
    theme(legend.position = "none") + geom_point(colour = "Blue") + geom_jitter(colour = "Blue") +
    geom_smooth(colour = "red", method = "lm") + labs(title = "LogWage vs. mom_education",
    x = "logWage ($)", y = "mom_education")
plot(lwage.mom_education.plot)
```

```
## Warning: Removed 128 rows containing missing values (stat_smooth).
```

```
## Warning: Removed 128 rows containing missing values (geom_point).
```

```
## Warning: Removed 128 rows containing missing values (geom_point).
```



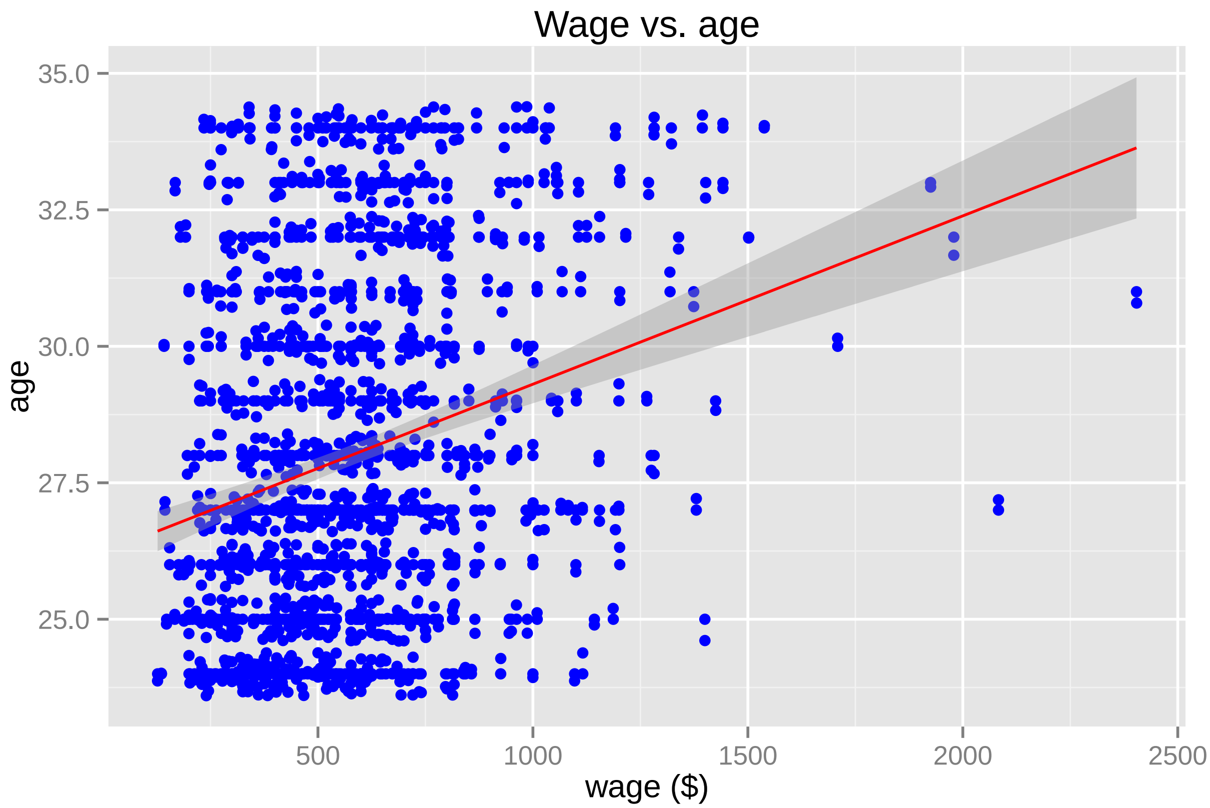

```
# Run correlations with wage and logWage variables
cor(data$wage, data$mom_education, use = "complete.obs")
```

```
## [1] 0.1983845
```

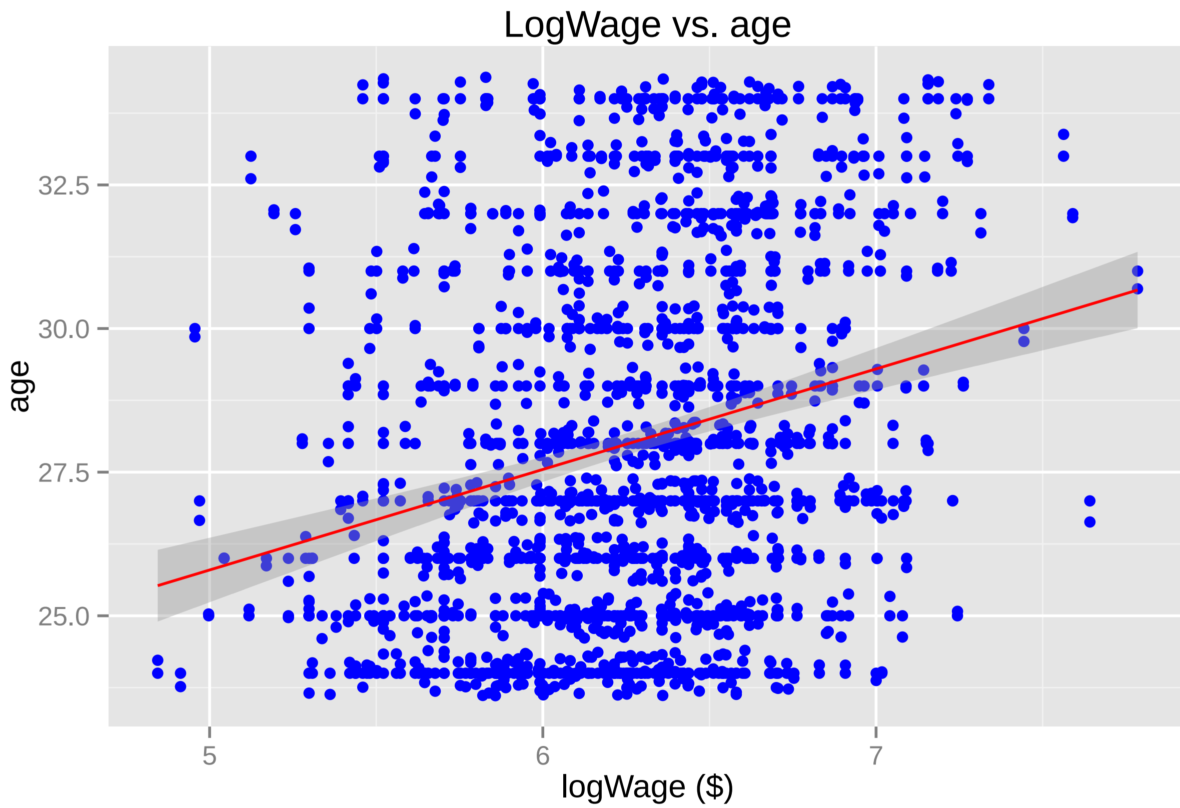
```
cor(data$logWage, data$mom_education, use = "complete.obs")
```

```
## [1] 0.2104614
```

```
# Scatter plot with wage variable
wage.age.plot = ggplot(data, aes(x = wage, y = age)) + theme(legend.position = "none") +
    geom_point(colour = "Blue") + geom_jitter(colour = "Blue") + geom_smooth(colour = "red",
    method = "lm") + labs(title = "Wage vs. age", x = "wage ($)", y = "age")
plot(wage.age.plot)
```




```
# Scatter plot with logWage variable
lwage.age.plot = ggplot(data, aes(x = logWage, y = age)) + theme(legend.position = "none") +
    geom_point(colour = "Blue") + geom_jitter(colour = "Blue") + geom_smooth(colour = "red",
    method = "lm") + labs(title = "LogWage vs. age", x = "logWage ($)",
    y = "age")
plot(lwage.age.plot)
```



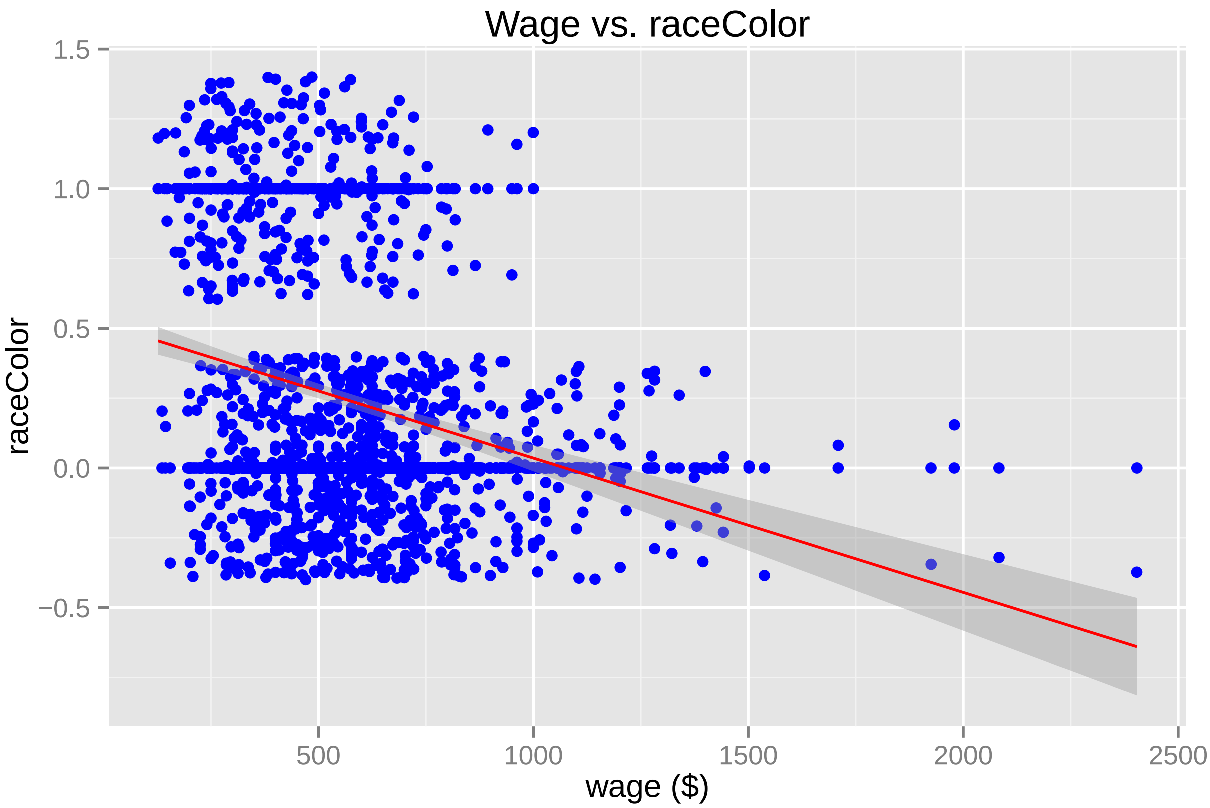

```
# Run correlations with wage and logWage variables
cor(data$wage, data$age)
```

```
## [1] 0.2635783
```

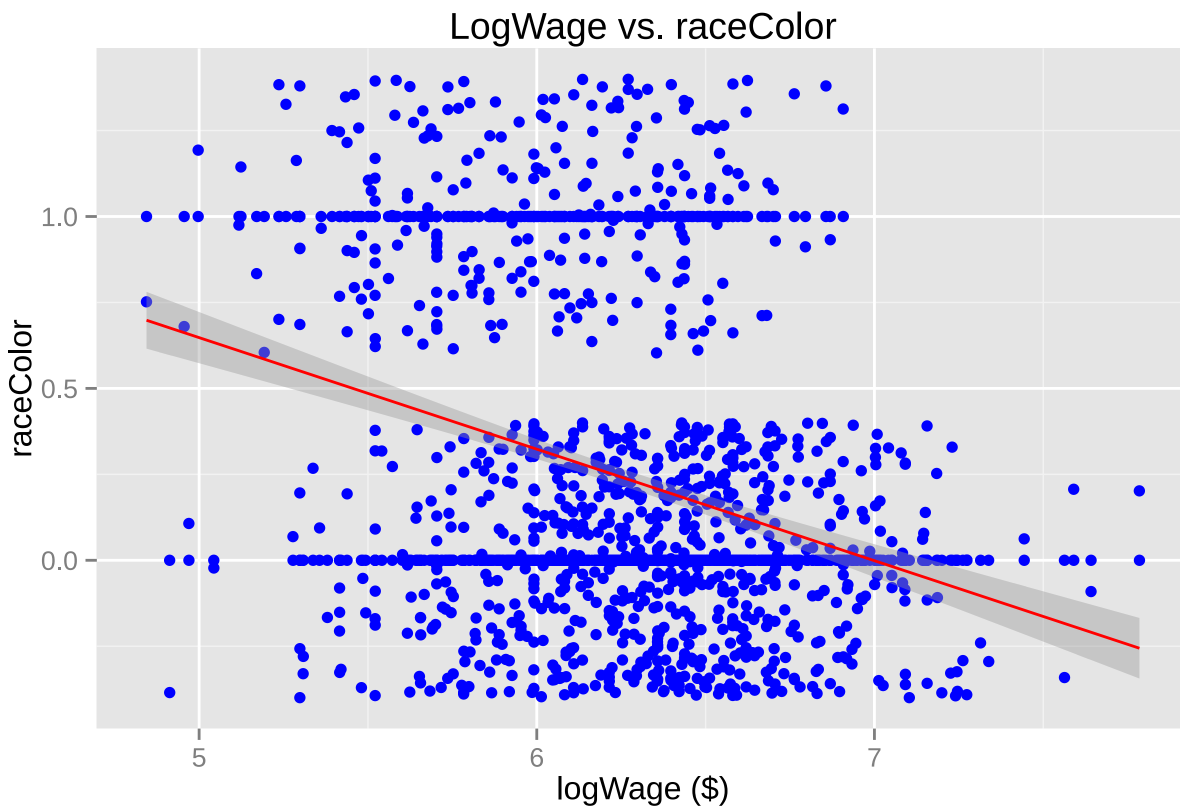
```
cor(data$logWage, data$age)
```

```
## [1] 0.2511202
```

```
# Scatter plot with wage variable
wage.raceColor.plot = ggplot(data, aes(x = wage, y = raceColor)) + theme(legend.position = "none") +
    geom_point(colour = "Blue") + geom_jitter(colour = "Blue") + geom_smooth(colour = "red",
    method = "lm") + labs(title = "Wage vs. raceColor", x = "wage ($)",
    y = "raceColor")
plot(wage.raceColor.plot)
```




```r
# Scatter plot with logWage variable
lwage.raceColor.plot = ggplot(data, aes(x = logWage, y = raceColor)) +
    theme(legend.position = "none") + geom_point(colour = "Blue") + geom_jitter(colour = "Blue") +
    geom_smooth(colour = "red", method = "lm") + labs(title = "LogWage vs. raceColor",
    x = "logWage ($)", y = "raceColor")
plot(lwage.raceColor.plot)
```



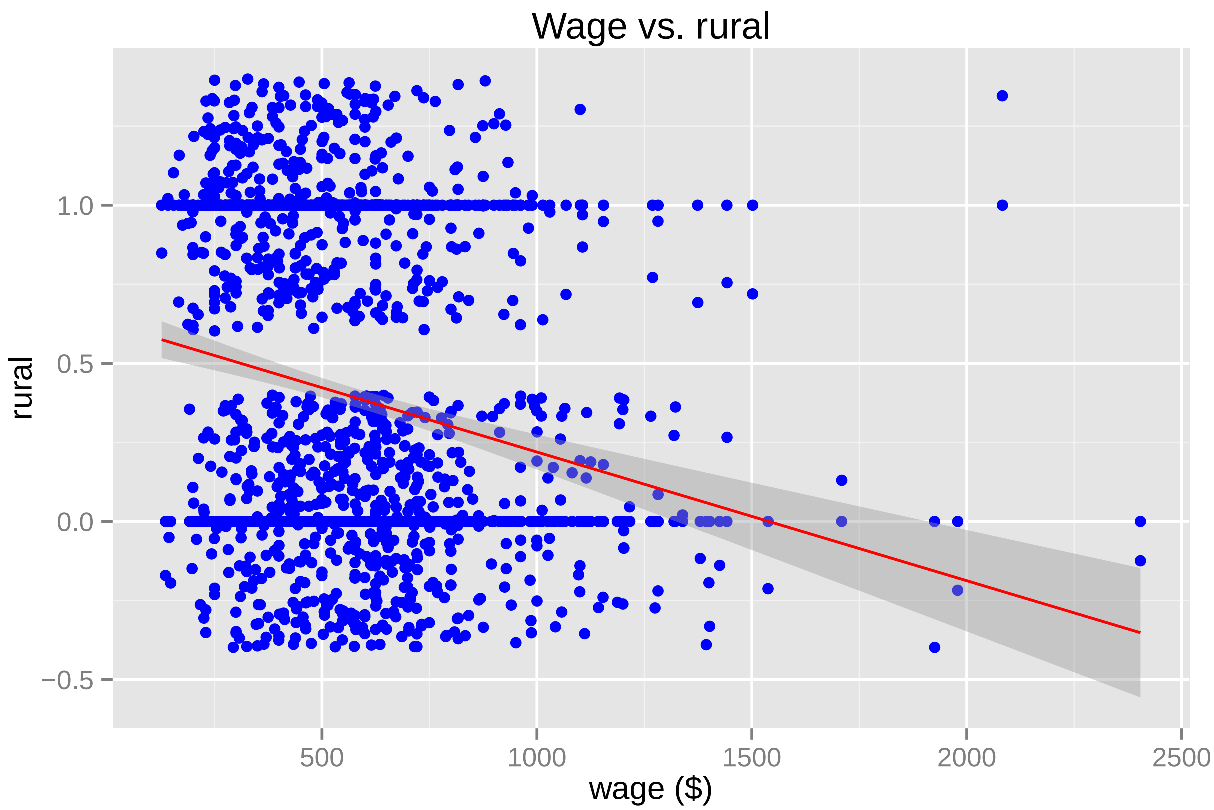

```r
# Run correlations with wage and logWage variables
cor(data$wage, data$raceColor)
```

```
## [1] -0.3008475
```

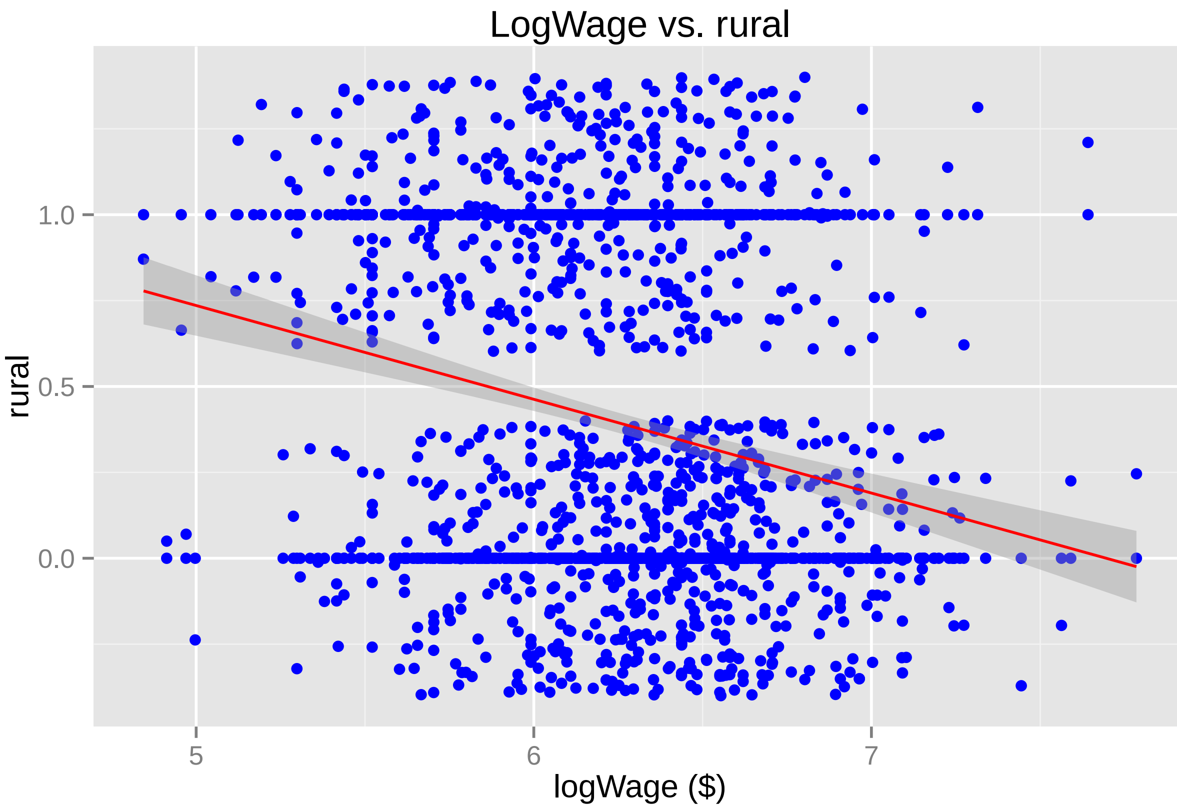
```r
cor(data$logWage, data$raceColor)
```

```
## [1] -0.3407361
```

```r
# Scatter plot with wage variable
wage.rural.plot = ggplot(data, aes(x = wage, y = rural)) + theme(legend.position = "none") +
    geom_point(colour = "Blue") + geom_jitter(colour = "Blue") + geom_smooth(colour = "red",
    method = "lm") + labs(title = "Wage vs. rural", x = "wage ($)", y = "rural")
plot(wage.rural.plot)
```




```
# Scatter plot with logWage variable
lwage.rural.plot = ggplot(data, aes(x = logWage, y = rural)) + theme(legend.position = "none") +
    geom_point(colour = "Blue") + geom_jitter(colour = "Blue") + geom_smooth(colour = "red",
    method = "lm") + labs(title = "LogWage vs. rural", x = "logWage ($)",
    y = "rural")
plot(lwage.rural.plot)
```



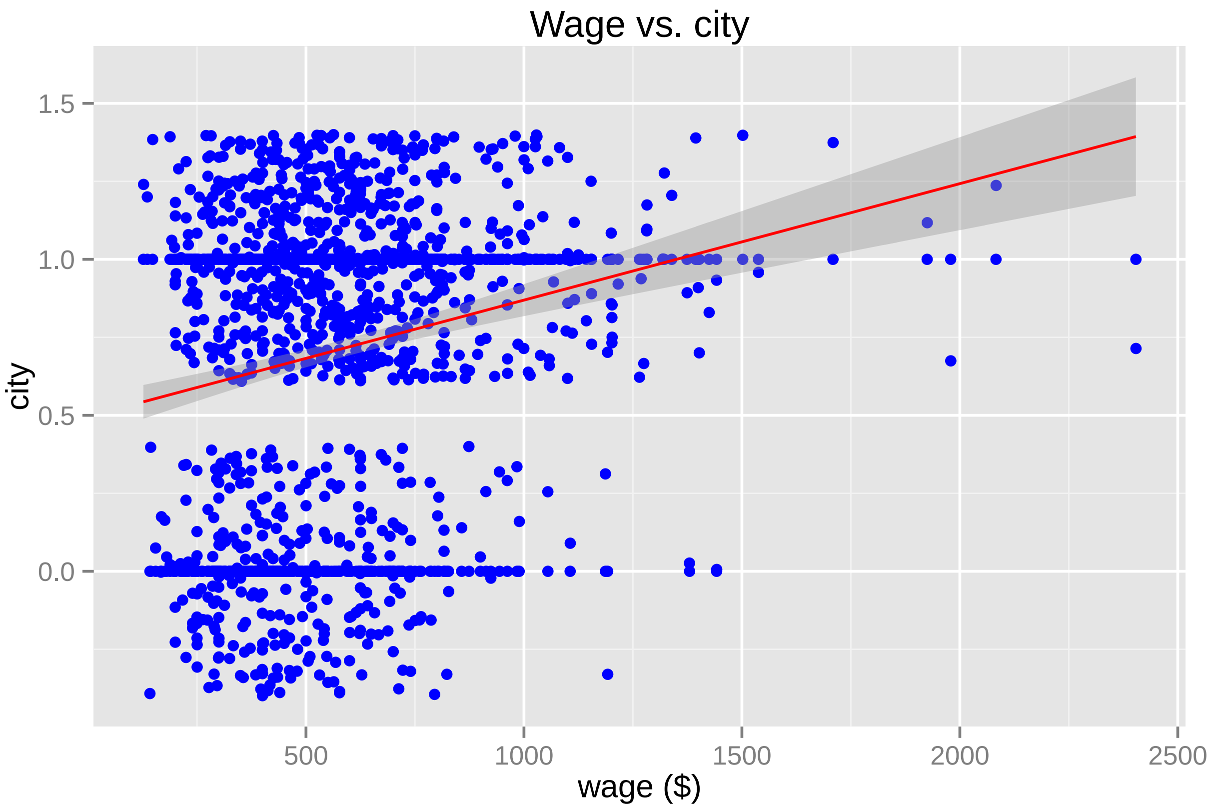

```r
# Run correlations with wage and logWage variables
cor(data$wage, data$rural)
```

```
## [1] -0.2222085
```

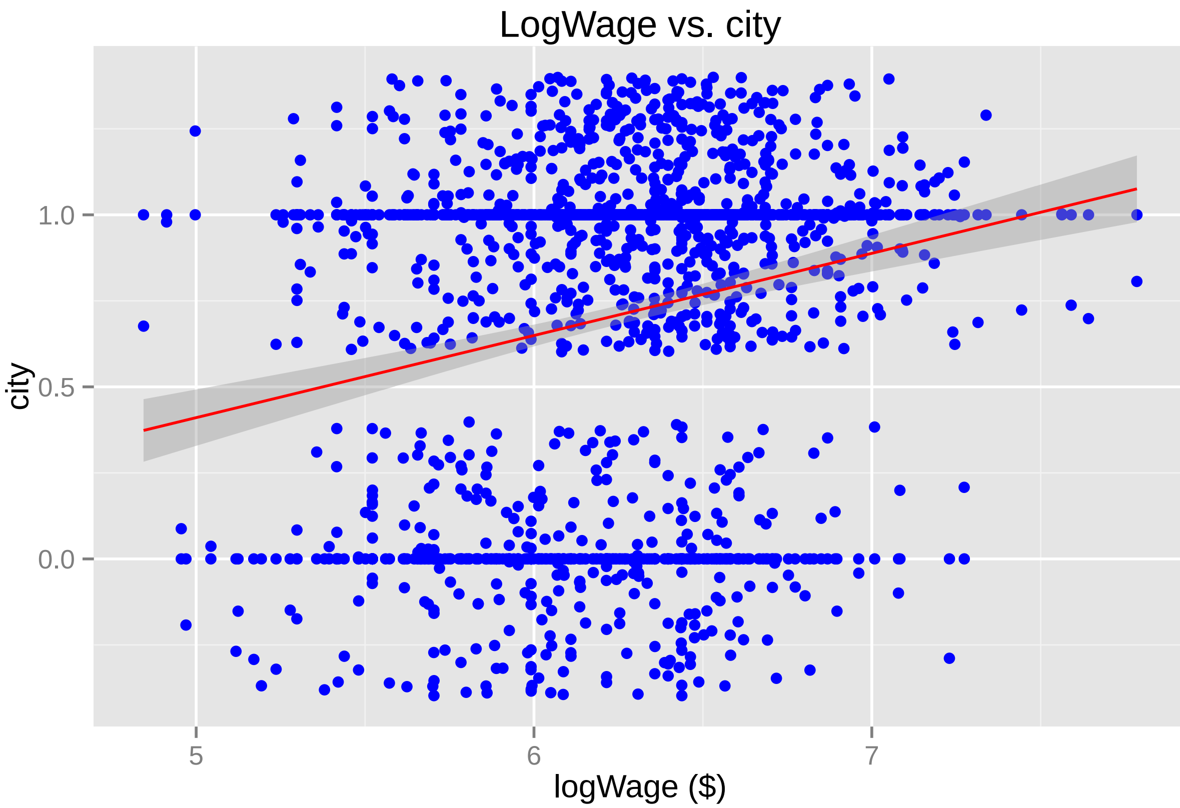
```r
cor(data$logWage, data$rural)
```

```
## [1] -0.2501131
```

```r
# Scatter plot with wage variable
wage.city.plot = ggplot(data, aes(x = wage, y = city)) + theme(legend.position = "none") +
    geom_point(colour = "Blue") + geom_jitter(colour = "Blue") + geom_smooth(colour = "red",
    method = "lm") + labs(title = "Wage vs. city", x = "wage ($)", y = "city")
plot(wage.city.plot)
```




```r
# Scatter plot with logWage variable
lwage.city.plot = ggplot(data, aes(x = logWage, y = city)) + theme(legend.position = "none") +
    geom_point(colour = "Blue") + geom_jitter(colour = "Blue") + geom_smooth(colour = "red",
    method = "lm") + labs(title = "LogWage vs. city", x = "logWage ($)",
    y = "city")
plot(lwage.city.plot)
```




```
# Run correlations with wage and logWage variables
cor(data$wage, data$city)
```

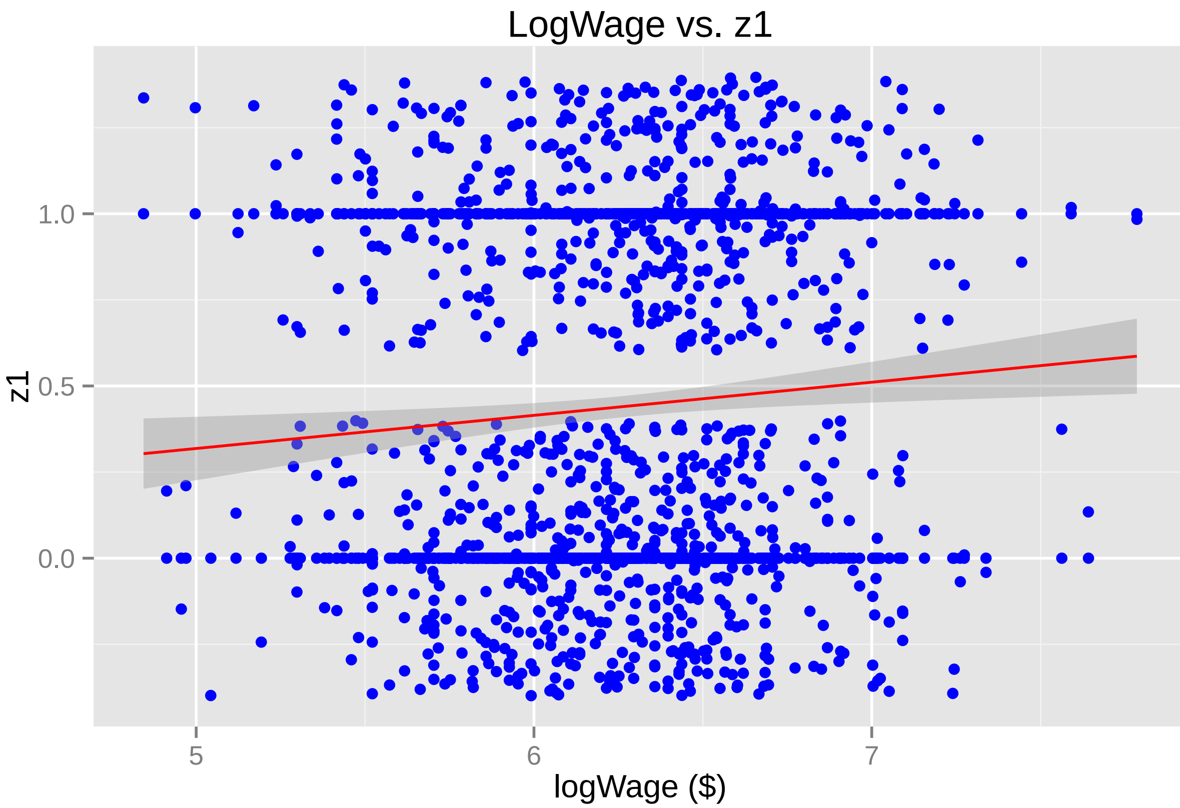
```
## [1] 0.2196804
```

```
cor(data$logWage, data$city)
```

```
## [1] 0.2358269
```

```
# Scatter plot with wage variable
wage.z1.plot = ggplot(data, aes(x = wage, y = z1)) + theme(legend.position = "none") +
    geom_point(colour = "Blue") + geom_jitter(colour = "Blue") + geom_smooth(colour = "red",
    method = "lm") + labs(title = "Wage vs. z1", x = "wage ($)", y = "z1")
plot(wage.z1.plot)
```


```
# Scatter plot with logWage variable
lwage.z1.plot = ggplot(data, aes(x = logWage, y = z1)) + theme(legend.position = "none") +
    geom_point(colour = "Blue") + geom_jitter(colour = "Blue") + geom_smooth(colour = "red",
    method = "lm") + labs(title = "LogWage vs. z1", x = "logWage ($)",
    y = "z1")
plot(lwage.z1.plot)
```




```r
# Run correlations with wage and logWage variables
cor(data$wage, data$z1)
```

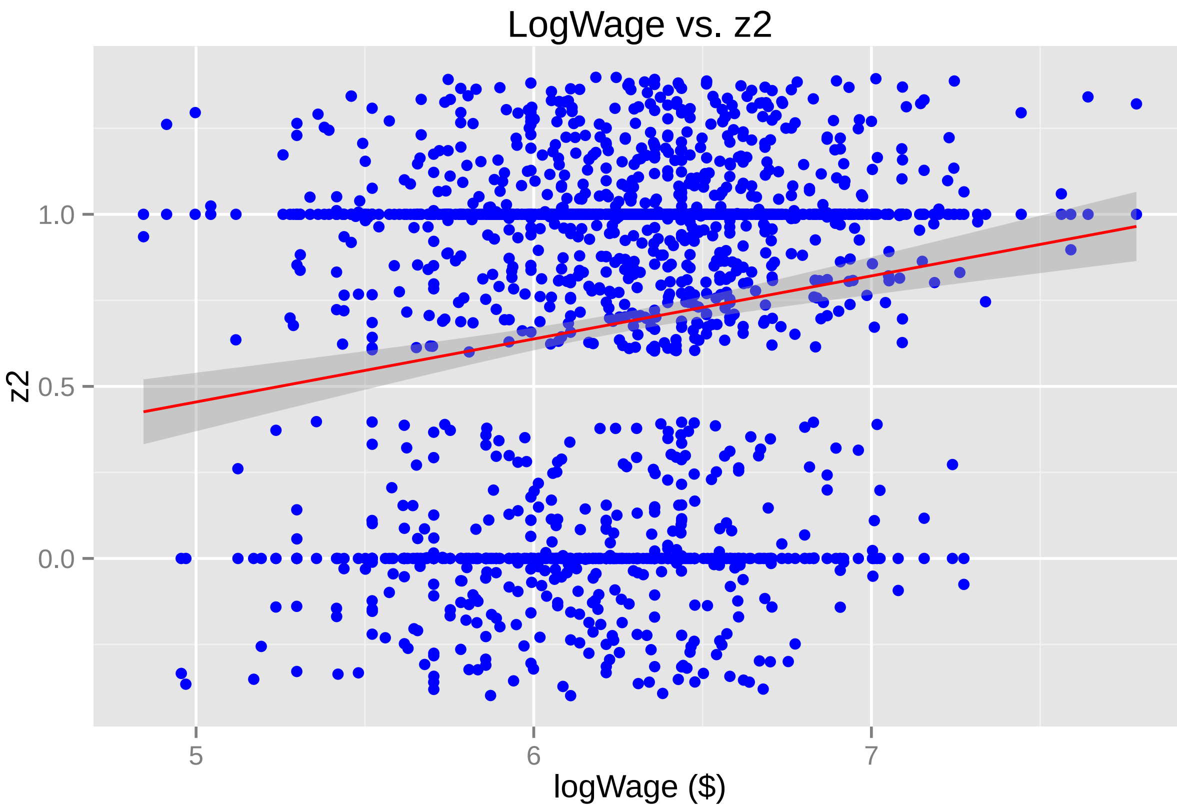
```
## [1] 0.1005669
```

```r
cor(data$logWage, data$z1)
```

```
## [1] 0.08668558
```

```r
# Scatter plot with wage variable
wage.z2.plot = ggplot(data, aes(x = wage, y = z2)) + theme(legend.position = "none") +
    geom_point(colour = "Blue") + geom_jitter(colour = "Blue") + geom_smooth(colour = "red",
    method = "lm") + labs(title = "Wage vs. z2", x = "wage ($)", y = "z2")
plot(wage.z2.plot)
```


```
# Scatter plot with logWage variable
lwage.z2.plot = ggplot(data, aes(x = logWage, y = z2)) + theme(legend.position = "none") +
    geom_point(colour = "Blue") + geom_jitter(colour = "Blue") + geom_smooth(colour = "red",
    method = "lm") + labs(title = "LogWage vs. z2", x = "logWage ($)",
    y = "z2")
plot(lwage.z2.plot)
```




```r
# Run correlations with wage and logWage variables
cor(data$wage, data$z2)
```

```
## [1] 0.1711982
```

```r
cor(data$logWage, data$z2)
```

```
## [1] 0.1765267
```

4.3 Regress log(wage) on education, experience, age, and raceColor

Part 1

Report all the estimated coefficients, their standard errors, t-statistics, F-statistic of the regression, R2, adjustedR2, and degrees of freedom
The requested information is shown in the summary information below.

```r
OLS.logWage.educ.exper.age.race = lm(logWage ~ education + experience +
    age + raceColor, data = data)
summary(OLS.logWage.educ.exper.age.race)
```

```
##
## Call:
```

```
## lm(formula = logWage ~ education + experience + age + raceColor,
##     data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.35396 -0.25550  0.01074  0.24867  1.22932
##
## Coefficients: (1 not defined because of singularities)
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.961661   0.113346  43.774   <2e-16 ***
## education    0.079608   0.006376  12.486   <2e-16 ***
## experience   0.035372   0.003988   8.869   <2e-16 ***
## age                NA         NA      NA       NA
## raceColor   -0.260813   0.030453  -8.564   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3917 on 996 degrees of freedom
## Multiple R-squared:  0.236,  Adjusted R-squared:  0.2337
## F-statistic: 102.6 on 3 and 996 DF,  p-value: < 2.2e-16
```

Part 2

Degress of freedom = 996. This value is calculated from the following formula $df = n - k - 1$ where n is the number of observations (n=1000). k is the number of independent varialbes (k=4). Plugging in these values we get, $996 = 1000 - 4 - 1$.

Part 3

The unexpected results from the regression are that the age variable has coeffiecient estimates that are NA. This is because age is a linear combination of the education and experience variables as expressed by the formula $age = education + experience + 6$. To resolve this issue one of these 3 varialbes needs to be removed from the regression. Since the intent is to estimate return to education on race and experience, then the age variable can be removed.

```r
# Create a new variable that represents the linear combination of age
# with education and experience.
data$age.formula = data$education + data$experience + 6
# Show that this new variable isdataeed the same as the age variable to
# subtracting the two variables.
data$age.difference = data$age - data$age.formula
# Now in the summary of the difference variable, all of the values are
# 0 indicating that the age.formula variable is the same as the age
# variable.
summary(data$age.difference)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##       0       0       0       0       0       0
```

Part 4 - Interpret the coefficient estimate associated with education

The estimate for the education coefficient is 0.079608. This means that for every unit change in education, there is an 8.00% change in logWage. This value is significant at the 0.1% significance level. This is a small practical effect.

Part 5 - Interpret the coefficient estimate associated with experience

The estimate for the experience coefficient is 0.035372. This means that for every unit change in experience, there is a 3.53% change in logWage. This value is significant at the 0.1% significance level. This is a small practical effect.

Question 4.4

Part 1

See graph below of the estimated effect of experience on wage.

$$\frac{\delta logWage}{\delta experience} = 0.0924 - 2 * (0.00288) * experience$$

Part 2

$$dlogWage10 = 0.0924 - 2 * (0.00288) * 10 = 0.0348$$

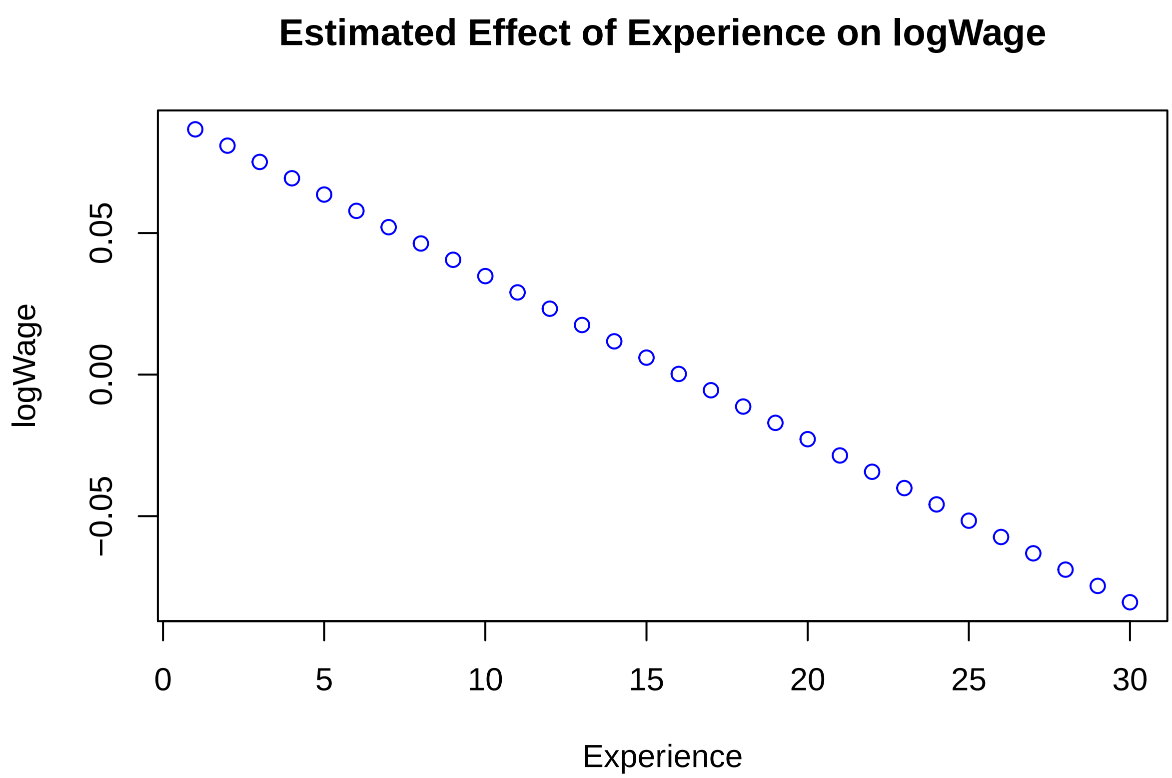
The estimated effect of experience on wage when experience is 10 years is 3.48%.

```
# Create the model
OLS.logWage.educ.exper.exper2.race = lm(logWage ~ education + experience +
    experienceSquare + raceColor, data = data)
# Print the summary of the model
summary(OLS.logWage.educ.exper.exper2.race)
```

```
##
## Call:
## lm(formula = logWage ~ education + experience + experienceSquare +
##     raceColor, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.38464 -0.25558  0.01909  0.25782  1.24410
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)       4.7355175  0.1197719  39.538  < 2e-16 ***
## education         0.0794641  0.0062917  12.630  < 2e-16 ***
## experience        0.0924930  0.0115147   8.033 2.68e-15 ***
## experienceSquare -0.0028779  0.0005452  -5.279 1.60e-07 ***
## raceColor        -0.2627226  0.0300528  -8.742  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.3865 on 995 degrees of freedom
## Multiple R-squared:  0.2569, Adjusted R-squared:  0.2539
## F-statistic: 85.98 on 4 and 995 DF,  p-value: < 2.2e-16
```

```r
# Create a variable dlogWage the represents the line created by the
# change in logWage with respect to a change in experience
dlogWage = 0
for (experience in 1:30) {
    dlogWage[experience] = 0.0924 - 2 * (0.00288) * experience
}
# Graph the line
plot(dlogWage, lty = "dashed", main = "Estimated Effect of Experience on logWage",
    col = "blue", ylab = "logWage", xlab = "Experience")
```




```r
# Calculate the value of the effect of experience on wage when
# experience is 10 years.
dlogWage10 = 0.0924 - 2 * (0.00288) * 10
dlogWage10
```

```
## [1] 0.0348
```

Question 4.5

Part 1

The number of observations used in this regression 723 (out of 1,000). The participants with missing mom_education or dad_education (mdMiss) values compare to participants that have both a mom_education and a dad_education (mdHave) value as follows.
- **wage** - The mdMiss participants have lower wages than the mdHave participants. The minimum value for wage for mdMiss is $127 vs. $136 for mdHave. The median and mean for values for mdMiss are lower at $481 vs. $570 and $531 vs. $597, respectively. The maximum values are also lower at $2,083 vs. $2,404.
- **education** - The mdMiss participants have less education than the mdHave participants. This could indicate that they stopped education sooner and went to work earlier than the mdHave participants. The mean value for mdMiss is 12.09 vs. 13.65 for mdHave, a difference of 1.56 years or a 11.4% decrease.
- **experience** - The mdMiss participants have more experience than the mdHave participants. This is further evidence that they stopped education sooner and went to work earlier than the mdHave participants. The mean value for mdMiss is 10.47 vs. 8.145 for mdHave, a difference of 2.32 years or a 28.5% increase.
- **raceColor** - The mdMiss participants have a much higher percentage of people with the raceColor variable set to 1 than mdHave. 44.77% (124 people) with 1's for mdMiss vs. 15.77% (114 people) with 1's for mdHave.

Part 2

We do not think we can just throw away the participants with the missing values. They are important to the analysis since they represent a disproportional amount of people with lower wages, less education, more experience and more raceColor variables equal to 1 than participants without missing values.

Part 3

Blindly replace all of the missing values with the average of the observed values of the corresponding variable. See the re-run of the original regression using all of the observations below.

Part 4

Regress the variable(s) with missing values on education, experience, and raceColor, and use this regression(s) to predict (i.e. "impute") the missing values. See the re-run of the original regression using all of the observations below.

Part 5

We would not use any of the previous 3 models that included the mom_education and dad_education variables. The mom_education and dad_education are not significant to even the 10% significance level in the models. We would take them out and use a 6 variable model without them. The Adjusted R-squared goes up slightly when we do this from 0.2925 to 0.2935 even though we are using fewer variables.

```r
# Part 1 Create the model
OLS.logWage.8var = lm(logWage ~ education + experience + experienceSquare +
    raceColor + dad_education + mom_education + rural + city, data = data)
# Print the model
summary(OLS.logWage.8var)
```

```
##
## Call:
## lm(formula = logWage ~ education + experience + experienceSquare +
##     raceColor + dad_education + mom_education + rural + city,
##     data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.2961 -0.2240  0.0160  0.2454  1.0404
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)       4.6422296  0.1408825  32.951  < 2e-16 ***
## education         0.0681701  0.0077409   8.806  < 2e-16 ***
## experience        0.0973419  0.0133133   7.312  7.1e-13 ***
## experienceSquare -0.0029568  0.0006678  -4.428  1.1e-05 ***
## raceColor        -0.2130226  0.0425014  -5.012  6.8e-07 ***
## dad_education    -0.0011474  0.0050988  -0.225  0.82202
## mom_education     0.0113176  0.0061886   1.829  0.06785 .
## rural            -0.0919377  0.0314151  -2.927  0.00354 **
## city              0.1782137  0.0323826   5.503  5.2e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3786 on 714 degrees of freedom
##   (277 observations deleted due to missingness)
## Multiple R-squared:  0.2746, Adjusted R-squared:  0.2665
## F-statistic: 33.79 on 8 and 714 DF,  p-value: < 2.2e-16
```

```r
# Create 2 temporary datasets.  mdMiss contains all of the rows with
# either mom_education or dad_education equal to NA.  mdHave contains
# all of the rows with both mom_education and dad_education equal to a
# non-NA value.
mdMiss = data[(is.na(data$mom_education) | is.na(data$dad_education)),
    ]
mdHave = data[(!is.na(data$mom_education) & !is.na(data$dad_education)),
    ]
# Use summary to confirm that mdMiss has the correct number of NA's for
# mom_education and dad_education
summary(mdMiss)
```

```
##        X              wage         education       experience
##  Min.   :  15   Min.   : 127   Min.   : 2.00   Min.   : 0.00
##  1st Qu.: 842   1st Qu.: 358   1st Qu.:11.00   1st Qu.: 7.00
##  Median :1688   Median : 481   Median :12.00   Median :10.00
##  Mean   :1643   Mean   : 531   Mean   :12.09   Mean   :10.47
##  3rd Qu.:2495   3rd Qu.: 640   3rd Qu.:13.00   3rd Qu.:14.00
##  Max.   :3009   Max.   :2083   Max.   :18.00   Max.   :23.00
##
##       age          raceColor      dad_education    mom_education
##  Min.   :24.00   Min.   :0.0000   Min.   : 2.000   Min.   : 0.000
##  1st Qu.:26.00   1st Qu.:0.0000   1st Qu.: 6.000   1st Qu.: 7.000
##  Median :28.00   Median :0.0000   Median : 9.500   Median : 9.000
##  Mean   :28.56   Mean   :0.4477   Mean   : 9.184   Mean   : 8.987
```

```
##  3rd Qu.:31.00   3rd Qu.:1.0000   3rd Qu.:12.000   3rd Qu.:12.000
##  Max.   :34.00   Max.   :1.0000   Max.   :16.000   Max.   :18.000
##                                   NA's   :239      NA's   :128
##      rural            city              z1               z2
##  Min.   :0.000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:0.000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000
##  Median :1.000   Median :1.0000   Median :0.0000   Median :1.0000
##  Mean   :0.509   Mean   :0.6643   Mean   :0.4188   Mean   :0.6895
##  3rd Qu.:1.000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000
##  Max.   :1.000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000
##
##     IQscore       logWage        logWageOLD    experienceSquare
##  Min.   : 50   Min.   :4.844   Min.   :4.844   Min.   :  0.0
##  1st Qu.: 85   1st Qu.:5.881   1st Qu.:5.881   1st Qu.: 49.0
##  Median : 98   Median :6.176   Median :6.176   Median :100.0
##  Mean   : 96   Mean   :6.174   Mean   :6.174   Mean   :128.1
##  3rd Qu.:107   3rd Qu.:6.461   3rd Qu.:6.461   3rd Qu.:196.0
##  Max.   :135   Max.   :7.642   Max.   :7.642   Max.   :529.0
##  NA's   :124
##   age.formula    age.difference
##  Min.   :24.00   Min.   :0
##  1st Qu.:26.00   1st Qu.:0
##  Median :28.00   Median :0
##  Mean   :28.56   Mean   :0
##  3rd Qu.:31.00   3rd Qu.:0
##  Max.   :34.00   Max.   :0
##
```

```r
summary(mdHave)
```

```
##        X              wage          education       experience
##  Min.   :   5.0   Min.   : 136.0   Min.   : 3.00   Min.   : 0.000
##  1st Qu.: 680.5   1st Qu.: 409.0   1st Qu.:12.00   1st Qu.: 5.000
##  Median :1314.0   Median : 570.0   Median :13.00   Median : 8.000
##  Mean   :1399.0   Mean   : 597.1   Mean   :13.65   Mean   : 8.145
##  3rd Qu.:2125.0   3rd Qu.: 721.0   3rd Qu.:16.00   3rd Qu.:10.000
##  Max.   :2998.0   Max.   :2404.0   Max.   :18.00   Max.   :21.000
##
##       age         raceColor      dad_education   mom_education
##  Min.   :24.0   Min.   :0.0000   Min.   : 0.00   Min.   : 0.00
##  1st Qu.:25.0   1st Qu.:0.0000   1st Qu.: 8.00   1st Qu.: 9.00
##  Median :27.0   Median :0.0000   Median :11.00   Median :12.00
##  Mean   :27.8   Mean   :0.1577   Mean   :10.23   Mean   :10.75
##  3rd Qu.:30.0   3rd Qu.:0.0000   3rd Qu.:12.00   3rd Qu.:12.00
##  Max.   :34.0   Max.   :1.0000   Max.   :18.00   Max.   :18.00
##
##      rural             city              z1               z2
##  Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000
##  Median :0.0000   Median :1.0000   Median :0.0000   Median :1.0000
##  Mean   :0.3458   Mean   :0.7303   Mean   :0.4481   Mean   :0.6846
##  3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.0000
##  Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000
##
```

```
##     IQscore         logWage        logWageOLD    experienceSquare
##  Min.   : 60.0   Min.   :4.913   Min.   :4.913   Min.   :  0.00
##  1st Qu.: 95.0   1st Qu.:6.014   1st Qu.:6.014   1st Qu.: 25.00
##  Median :105.0   Median :6.346   Median :6.346   Median : 64.00
##  Mean   :104.1   Mean   :6.297   Mean   :6.297   Mean   : 82.37
##  3rd Qu.:114.0   3rd Qu.:6.581   3rd Qu.:6.581   3rd Qu.:100.00
##  Max.   :144.0   Max.   :7.785   Max.   :7.785   Max.   :441.00
##  NA's   :192
##   age.formula   age.difference
##  Min.   :24.0   Min.   :0
##  1st Qu.:25.0   1st Qu.:0
##  Median :27.0   Median :0
##  Mean   :27.8   Mean   :0
##  3rd Qu.:30.0   3rd Qu.:0
##  Max.   :34.0   Max.   :0
##
```

```r
# Use str to confirm that the datasets have the appropriate number of
# observations
str(mdMiss)
```

```
## 'data.frame':    277 obs. of  18 variables:
##  $ X               : int  191 2059 1927 1481 1484 2548 574 2061 2700 2689 ...
##  $ wage            : int  951 288 454 565 670 624 400 673 575 340 ...
##  $ education       : int  12 8 10 12 13 9 12 12 12 9 ...
##  $ experience      : int  10 11 11 10 8 9 8 14 8 19 ...
##  $ age             : int  28 25 27 28 27 24 26 32 26 34 ...
##  $ raceColor       : int  0 1 1 1 1 1 0 0 1 1 ...
##  $ dad_education   : int  NA NA NA NA NA NA NA NA NA NA ...
##  $ mom_education   : int  12 7 1 NA NA 7 12 6 NA NA ...
##  $ rural           : int  0 1 1 1 1 1 0 1 0 1 ...
##  $ city            : int  1 0 0 1 1 0 0 0 1 0 ...
##  $ z1              : int  1 0 0 0 0 1 0 0 1 1 ...
##  $ z2              : int  1 1 1 1 1 0 1 1 0 0 ...
##  $ IQscore         : int  122 NA NA NA 99 NA 117 93 NA NA ...
##  $ logWage         : num  6.86 5.66 6.12 6.34 6.51 ...
##  $ logWageOLD      : num  6.86 5.66 6.12 6.34 6.51 ...
##  $ experienceSquare: int  100 121 121 100 64 81 64 196 64 361 ...
##  $ age.formula     : num  28 25 27 28 27 24 26 32 26 34 ...
##  $ age.difference  : num  0 0 0 0 0 0 0 0 0 0 ...
```

```r
str(mdHave)
```

```
## 'data.frame':    723 obs. of  18 variables:
##  $ X               : int  2072 945 1920 2571 437 1265 603 2936 1123 2080 ...
##  $ wage            : int  509 647 225 479 615 641 740 619 583 813 ...
##  $ education       : int  12 18 10 13 16 12 13 17 12 12 ...
##  $ experience      : int  6 5 11 15 7 16 10 6 10 6 ...
##  $ age             : int  24 29 27 34 29 34 29 29 28 24 ...
##  $ raceColor       : int  0 0 1 0 0 0 0 0 0 0 ...
##  $ dad_education   : int  12 12 5 7 12 4 16 8 14 9 ...
##  $ mom_education   : int  9 12 5 12 12 8 16 13 8 9 ...
##  $ rural           : int  1 0 1 1 0 0 0 0 0 1 ...
```

```
##  $ city            : int  1 1 0 1 1 0 0 1 1 1 ...
##  $ z1              : int  0 0 0 0 1 0 0 0 1 1 ...
##  $ z2              : int  0 1 1 1 1 1 0 0 1 1 ...
##  $ IQscore         : int  127 110 NA NA 113 92 108 138 94 NA ...
##  $ logWage         : num  6.23 6.47 5.42 6.17 6.42 ...
##  $ logWageOLD      : num  6.23 6.47 5.42 6.17 6.42 ...
##  $ experienceSquare: int  36 25 121 225 49 256 100 36 100 36 ...
##  $ age.formula     : num  24 29 27 34 29 34 29 29 28 24 ...
##  $ age.difference  : num  0 0 0 0 0 0 0 0 0 0 ...
```

```
# Compare the summaries of the mdMiss and mdHave datasets to see the
# differences in wage, eduction, experience and raceColor
summary(mdMiss$wage)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##     127     358     481     531     640    2083
```

```
summary(mdHave$wage)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   136.0   409.0   570.0   597.1   721.0  2404.0
```

```
summary(mdMiss$education)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    2.00   11.00   12.00   12.09   13.00   18.00
```

```
summary(mdHave$education)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    3.00   12.00   13.00   13.65   16.00   18.00
```

```
summary(mdMiss$experience)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    0.00    7.00   10.00   10.47   14.00   23.00
```

```
summary(mdHave$experience)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.000   5.000   8.000   8.145  10.000  21.000
```

```
summary(mdMiss$raceColor)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.0000  0.0000  0.0000  0.4477  1.0000  1.0000
```

```r
summary(mdHave$raceColor)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.0000  0.0000  0.0000  0.1577  0.0000  1.0000
```

```r
# Part 3 Copy the dataset to a new variable
data.avgForNA = data
# Set all of the values with dad_education = NA to the mean of
# dad_education
data.avgForNA$dad_education[is.na(data.avgForNA$dad_education)] = mean(data.avgForNA$dad_education,
    na.rm = TRUE)
# Set all of the values with mom_education = NA to the mean of
# mom_education
data.avgForNA$mom_education[is.na(data.avgForNA$mom_education)] = mean(data.avgForNA$mom_education,
    na.rm = TRUE)
# Rerun the regression
OLS.logWage.8var.avgNA = lm(logWage ~ education + experience + experienceSquare +
    raceColor + dad_education + mom_education + rural + city, data = data.avgForNA)

# Part 4 Copy the dataset to a new variable
data.regressForNA = data
# Regress dad_education on the education, experience and raceColor
# variables
m1 = lm(dad_education ~ education + experience + raceColor, data = data)
# Regress mom_education on the education, experience and raceColor
# variables
m2 = lm(mom_education ~ education + experience + raceColor, data = data)

# Set all of the values with dad_education = NA to the value output
# from using the regression coefficients from m1 above.
data.regressForNA$dad_education[is.na(data.regressForNA$dad_education)] = m1$coefficients[1] +
    m1$coefficients[2] * data.regressForNA$education + m1$coefficients[3] *
    data.regressForNA$experience + m1$coefficients[4] * data.regressForNA$raceColor
```

```
## Warning in data.regressForNA$dad_education[is.na(data.regressForNA
## $dad_education)] = m1$coefficients[1] + : number of items to replace is not
## a multiple of replacement length
```

```r
# Set all of the values with mom_education = NA to the value output
# from using the regression coefficients from m2 above.
data.regressForNA$mom_education[is.na(data.regressForNA$mom_education)] = m2$coefficients[1] +
    m2$coefficients[2] * data.regressForNA$education + m2$coefficients[3] *
    data.regressForNA$experience + m2$coefficients[4] * data.regressForNA$raceColor
```

```
## Warning in data.regressForNA$mom_education[is.na(data.regressForNA
## $mom_education)] = m2$coefficients[1] + : number of items to replace is not
## a multiple of replacement length
```

```r
# Rerun the regression
OLS.logWage.8var.regressNA = lm(logWage ~ education + experience + experienceSquare +
    raceColor + dad_education + mom_education + rural + city, data = data.regressForNA)
```

```r
# Part 5 Print the summaries of the 2 new models
summary(OLS.logWage.8var.avgNA)
```

```
##
## Call:
## lm(formula = logWage ~ education + experience + experienceSquare +
##     raceColor + dad_education + mom_education + rural + city,
##     data = data.avgForNA)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.30741 -0.23286  0.01943  0.24786  1.28807
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)       4.729e+00  1.226e-01  38.584  < 2e-16 ***
## education         7.097e-02  6.499e-03  10.920  < 2e-16 ***
## experience        8.958e-02  1.124e-02   7.970 4.36e-15 ***
## experienceSquare -2.678e-03  5.318e-04  -5.036 5.65e-07 ***
## raceColor        -2.313e-01  3.099e-02  -7.464 1.84e-13 ***
## dad_education    -3.513e-05  4.416e-03  -0.008 0.993656
## mom_education     3.485e-03  5.009e-03   0.696 0.486742
## rural            -9.529e-02  2.638e-02  -3.612 0.000319 ***
## city              1.671e-01  2.703e-02   6.183 9.21e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3764 on 991 degrees of freedom
## Multiple R-squared:  0.2981, Adjusted R-squared:  0.2925
## F-statistic: 52.62 on 8 and 991 DF,  p-value: < 2.2e-16
```

```r
summary(OLS.logWage.8var.regressNA)
```

```
##
## Call:
## lm(formula = logWage ~ education + experience + experienceSquare +
##     raceColor + dad_education + mom_education + rural + city,
##     data = data.regressForNA)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.30770 -0.23222  0.02095  0.24785  1.29770
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)       4.7278751  0.1228090  38.498  < 2e-16 ***
## education         0.0710341  0.0064659  10.986  < 2e-16 ***
## experience        0.0896724  0.0112433   7.976 4.16e-15 ***
## experienceSquare -0.0026820  0.0005318  -5.043 5.45e-07 ***
## raceColor        -0.2313406  0.0311112  -7.436 2.24e-13 ***
## dad_education    -0.0003385  0.0041318  -0.082 0.934718
## mom_education     0.0037753  0.0047649   0.792 0.428365
```

```
## rural            -0.0952834  0.0263780  -3.612 0.000319 ***
## city              0.1673210  0.0270228   6.192 8.70e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3764 on 991 degrees of freedom
## Multiple R-squared:  0.2982, Adjusted R-squared:  0.2925
## F-statistic: 52.64 on 8 and 991 DF,  p-value: < 2.2e-16
```

```r
# Run a 6-variable model without dad_education and mom_education
OLS.logWage.6var = lm(logWage ~ education + experience + experienceSquare +
    raceColor + rural + city, data = data)
# Print the summary of the new model
summary(OLS.logWage.6var)
```

```
##
## Call:
## lm(formula = logWage ~ education + experience + experienceSquare +
##     raceColor + rural + city, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.31258 -0.23242  0.02192  0.24694  1.28360
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)       4.7510008  0.1177400  40.352  < 2e-16 ***
## education         0.0722416  0.0061959  11.660  < 2e-16 ***
## experience        0.0892966  0.0112131   7.964 4.55e-15 ***
## experienceSquare -0.0026714  0.0005312  -5.029 5.86e-07 ***
## raceColor        -0.2345897  0.0305852  -7.670 4.09e-14 ***
## rural            -0.0963238  0.0263220  -3.659 0.000266 ***
## city              0.1677263  0.0269991   6.212 7.67e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3761 on 993 degrees of freedom
## Multiple R-squared:  0.2977, Adjusted R-squared:  0.2935
## F-statistic: 70.16 on 6 and 993 DF,  p-value: < 2.2e-16
```

Question 4.6

Part 1

The assumptions needed are Cov(z1, education) != 0 and Cov(z1, u) = 0.

Part 2

Suppose z1 is an indicator representing whether or not an individual lives in an area in which there was a recent policy change to promote the importance of education. Yes, z1 could be correlated with other unobservables captured in the error term. Some examples are 1. Income. People with higher incomes might be more educated and thus might place a higher importance on eduction and thus be more likely to live in an area that promotes education, 2. Political party. A particular political party might be more aligned with education and therefore people in that polical party might be more inclined to live in an area that promotes education, and 3. Whether you voted or not. It's possible that people who vote might be more educated and more likely to live in an area that promotes education. These are just a few examples. There could be many more.

Part 3

Using the same specification as that in question 4.5, estimate the equation by 2SLS, using both z1 and z2 as instrument variables.

The coefficient estimate on education goes from 0.0681701 in the original model to 0.0950302, however, in the new model, the education estimate is not significant at the 5% level, so the increase in the coefficient can no longer be used in our interpretation.

However, if we remove mom_education and dad_education from both the TSLS and original models, the education coefficient becomes significant again at the 5% level. The value of the education coefficient now goes from 0.0722416 in the original model to 0.1042749 in the TSLS model. This means that using z1 and z2 as instrumental variables the effect of educaiton on logWage increases from about 7.2% to 10.4% (an increase of about 3 percentage points). This is a 44% increase which is a large practical effect.

```r
# Run the IV TSLS regression with z1 and z2
TSLS.logWage.8var = ivreg(logWage ~ education + experience + experienceSquare +
    raceColor + dad_education + mom_education + rural + city | z1 * z2 +
    experience + experienceSquare + raceColor + dad_education + mom_education +
    rural + city, data = data)
# Print the summary of TSLS the model
summary(TSLS.logWage.8var)
```

```
##
## Call:
## ivreg(formula = logWage ~ education + experience + experienceSquare +
##     raceColor + dad_education + mom_education + rural + city |
##     z1 * z2 + experience + experienceSquare + raceColor + dad_education +
##         mom_education + rural + city, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.31628 -0.23169  0.03689  0.23949  1.03574
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)       4.2815365  0.8256004   5.186 2.80e-07 ***
## education         0.0950302  0.0610647   1.556  0.12010
## experience        0.1069713  0.0255275   4.190 3.13e-05 ***
## experienceSquare -0.0030032  0.0006815  -4.407 1.21e-05 ***
## raceColor        -0.2001502  0.0517616  -3.867  0.00012 ***
## dad_education    -0.0041758  0.0085477  -0.489  0.62533
## mom_education     0.0071767  0.0112304   0.639  0.52300
## rural            -0.0888567  0.0324316  -2.740  0.00630 **
## city              0.1670192  0.0412727   4.047 5.76e-05 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3818 on 714 degrees of freedom
## Multiple R-Squared: 0.2624,  Adjusted R-squared: 0.2541
## Wald test:    24 on 8 and 714 DF,  p-value: < 2.2e-16
```

```r
# Print the summary of the original model
summary(OLS.logWage.8var)
```

```
##
## Call:
## lm(formula = logWage ~ education + experience + experienceSquare +
##     raceColor + dad_education + mom_education + rural + city,
##     data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.2961 -0.2240  0.0160  0.2454  1.0404
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)       4.6422296  0.1408825  32.951  < 2e-16 ***
## education         0.0681701  0.0077409   8.806  < 2e-16 ***
## experience        0.0973419  0.0133133   7.312  7.1e-13 ***
## experienceSquare -0.0029568  0.0006678  -4.428  1.1e-05 ***
## raceColor        -0.2130226  0.0425014  -5.012  6.8e-07 ***
## dad_education    -0.0011474  0.0050988  -0.225  0.82202
## mom_education     0.0113176  0.0061886   1.829  0.06785 .
## rural            -0.0919377  0.0314151  -2.927  0.00354 **
## city              0.1782137  0.0323826   5.503  5.2e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3786 on 714 degrees of freedom
##   (277 observations deleted due to missingness)
## Multiple R-squared:  0.2746, Adjusted R-squared:  0.2665
## F-statistic: 33.79 on 8 and 714 DF,  p-value: < 2.2e-16
```

```r
# Run the IV TSLS regression with z1 and z2 with only 6 variables,
# removing mom_education and dad_education.
TSLS.logWage.6var = ivreg(logWage ~ education + experience + experienceSquare +
    raceColor + rural + city | z1 * z2 + experience + experienceSquare +
    raceColor + rural + city, data = data)
# Print the summary of the 6 variable TSLS model
summary(TSLS.logWage.6var)
```

```
##
## Call:
## ivreg(formula = logWage ~ education + experience + experienceSquare +
##     raceColor + rural + city | z1 * z2 + experience + experienceSquare +
##     raceColor + rural + city, data = data)
##
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.33618 -0.23434  0.02741  0.23425  1.25226
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)       4.2132904  0.7968306   5.288 1.52e-07 ***
## education         0.1042749  0.0473529   2.202  0.02789 *
## experience        0.1024823  0.0224135   4.572 5.43e-06 ***
## experienceSquare -0.0026811  0.0005385  -4.979 7.55e-07 ***
## raceColor        -0.1978820  0.0620751  -3.188  0.00148 **
## rural            -0.0873090  0.0297651  -2.933  0.00343 **
## city              0.1495403  0.0381914   3.916 9.64e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3811 on 993 degrees of freedom
## Multiple R-Squared: 0.2788,  Adjusted R-squared: 0.2745
## Wald test: 47.07 on 6 and 993 DF,  p-value: < 2.2e-16
```

```r
# Print the summary of the original 6 variable model
summary(OLS.logWage.6var)
```

```
##
## Call:
## lm(formula = logWage ~ education + experience + experienceSquare +
##     raceColor + rural + city, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.31258 -0.23242  0.02192  0.24694  1.28360
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)       4.7510008  0.1177400  40.352  < 2e-16 ***
## education         0.0722416  0.0061959  11.660  < 2e-16 ***
## experience        0.0892966  0.0112131   7.964 4.55e-15 ***
## experienceSquare -0.0026714  0.0005312  -5.029 5.86e-07 ***
## raceColor        -0.2345897  0.0305852  -7.670 4.09e-14 ***
## rural            -0.0963238  0.0263220  -3.659 0.000266 ***
## city              0.1677263  0.0269991   6.212 7.67e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3761 on 993 degrees of freedom
## Multiple R-squared:  0.2977, Adjusted R-squared:  0.2935
## F-statistic: 70.16 on 6 and 993 DF,  p-value: < 2.2e-16
```