

# Homework 3

*Rohan Thakur, Charles Kekeh and Megan Jasek*

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```
# Load the dataframe
load("twoyear.RData")
desc
```

```
##      variable                                label
## 1   female                                =1 if female
## 2   phsrank  % high school rank; 100 = best
## 3     BA                                =1 if Bachelor's degree
## 4     AA                                =1 if Associate's degree
## 5   black                                =1 if African-American
## 6  hispanic                                =1 if Hispanic
## 7     id                                ID Number
## 8   exper  total (actual) work experience
## 9     jc                                total 2-year credits
## 10   univ                                total 4-year credits
## 11   lwage                                log hourly wage
## 12  stotal  total standardized test score
## 13  smcity                                =1 if small city, 1972
## 14  medcity                                =1 if med. city, 1972
## 15  submed  =1 if suburb med. city, 1972
## 16  lgcity                                =1 if large city, 1972
## 17  sublg   =1 if suburb large city, 1972
## 18  vlgcity =1 if very large city, 1972
## 19  subvlg  =1 if sub. very lge. city, 1972
## 20     ne                                =1 if northeast
## 21     nc                                =1 if north central
## 22   south                                =1 if south
## 23  totcoll                                jc + univ
```

## Question 1

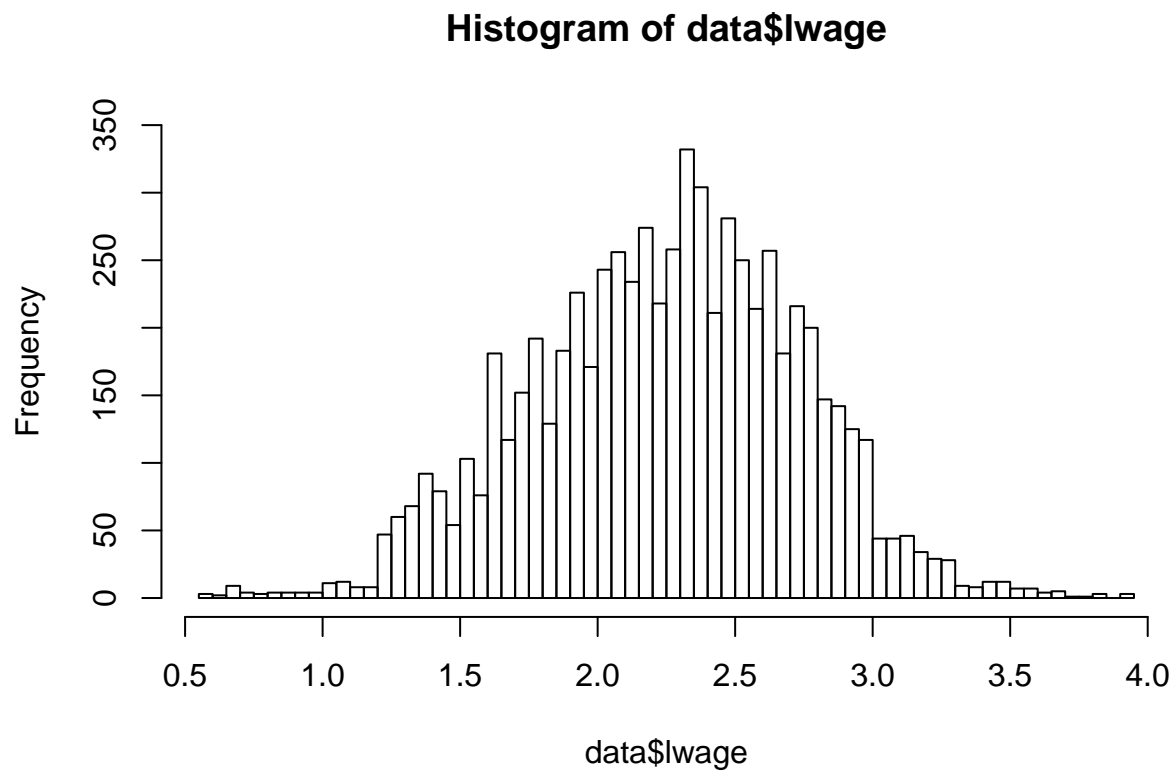
```
summary(data$lwage)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.5555  1.9250  2.2760  2.2480  2.5970  3.9120
```

```
print(quantile(data$lwage, probs = c(0.01, 0.05, 0.1,
  0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##      1%      5%      10%      25%      50%      75%      90%      95%
## 1.148702 1.398129 1.609438 1.925291 2.276300 2.596916 2.851921 2.995732
##      99%     100%
## 3.325316 3.911953
```

```
hist(data$lwage, 50, ylim = c(0, 350))
```



```
summary(data$jc)
```

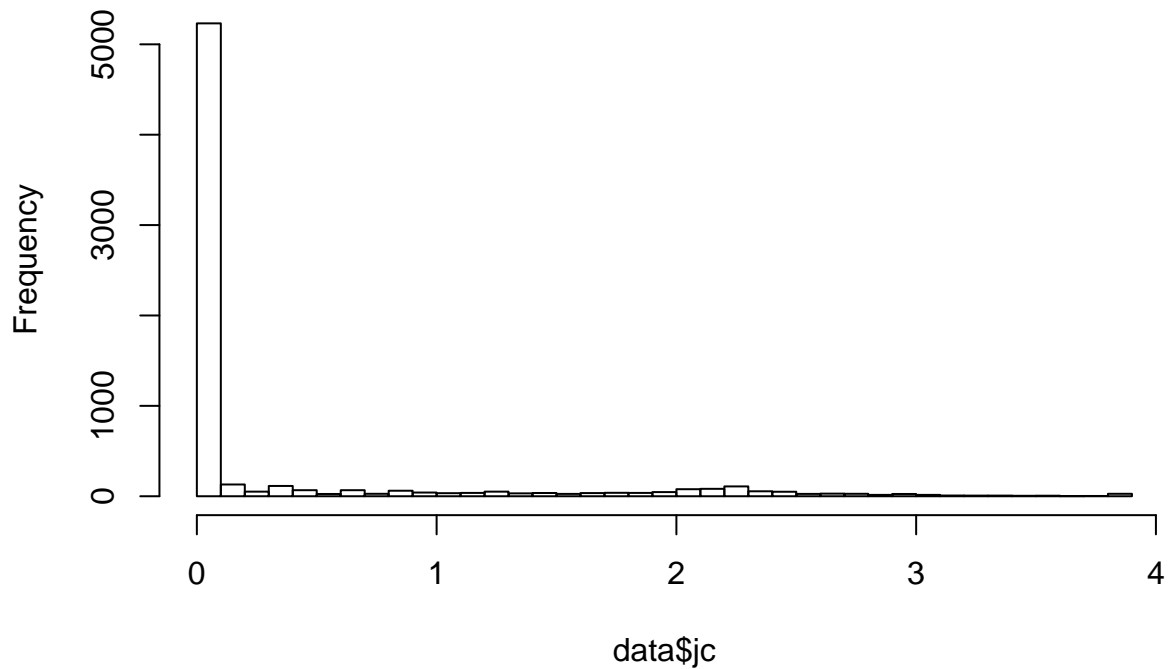
```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.00000 0.00000 0.00000 0.3389 0.00000 3.8330
```

```
print(quantile(data$jc, probs = c(0.01, 0.05, 0.1,
  0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##      1%      5%     10%     25%     50%     75%     90%     95%
## 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.766667 2.266667
##      99%     100%
## 3.089665 3.833333
```

```
hist(data$jc, 50)
```

## Histogram of data\$jc



```
summary(data$univ)
```

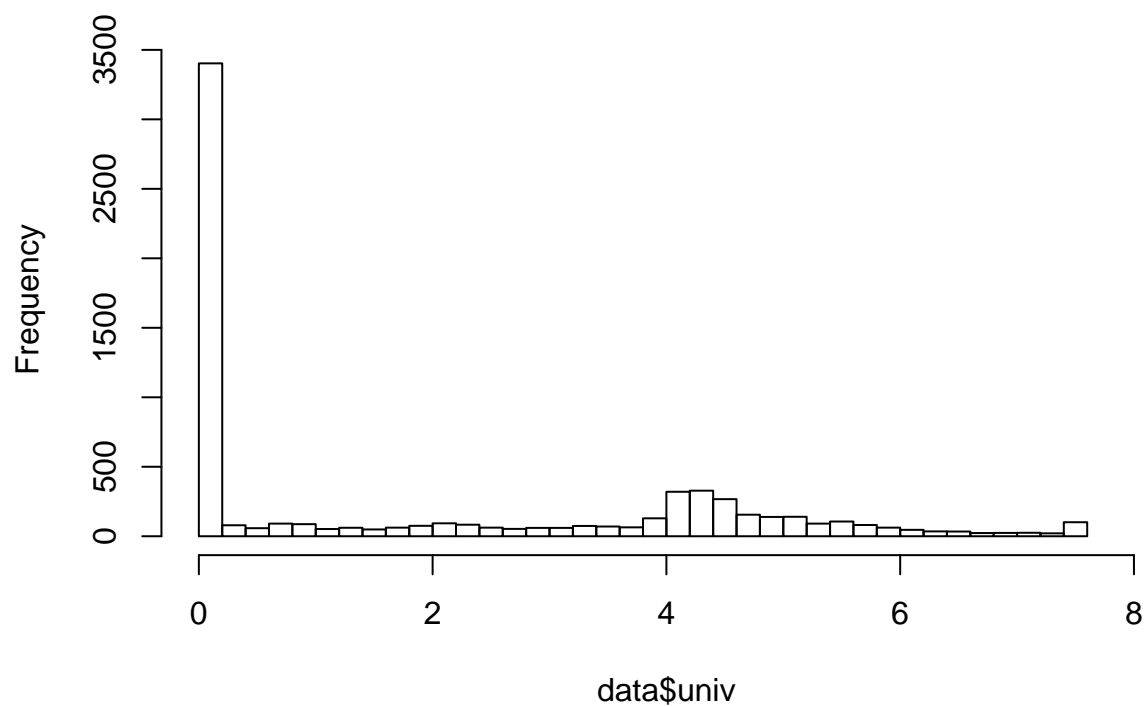
```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.000  0.000   0.200   1.926  4.200   7.500
```

```
print(quantile(data$univ, probs = c(0.01, 0.05, 0.1,
    0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##           1%           5%           10%           25%           50%           75%           90%
## 0.0000000 0.0000000 0.0000000 0.0000000 0.1999997 4.1999998 5.1777687
##           95%           99%          100%
## 5.9099934 7.5000000 7.5000000
```

```
hist(data$univ, 50, xlim = c(0, 8))
```

**Histogram of data\$univ**



```
summary(data$exper)
```

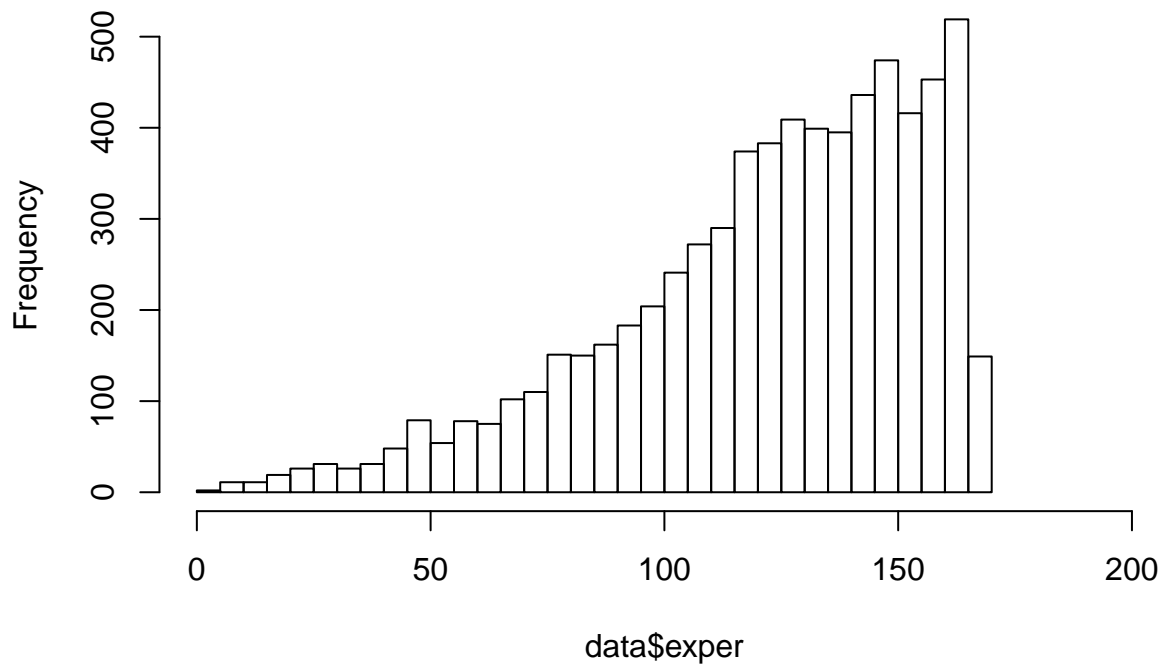
```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##       3.0   104.0   129.0   122.4   149.0   166.0
```

```
print(quantile(data$exper, probs = c(0.01, 0.05, 0.1,
    0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##      1%    5%   10%   25%   50%   75%   90%   95%   99%  100%
##      25    56    74   104   129   149   160   163   166   166
```

```
hist(data$exper, 50, xlim = c(0, 200))
```

## Histogram of data\$exper



```
summary(data$black)
```

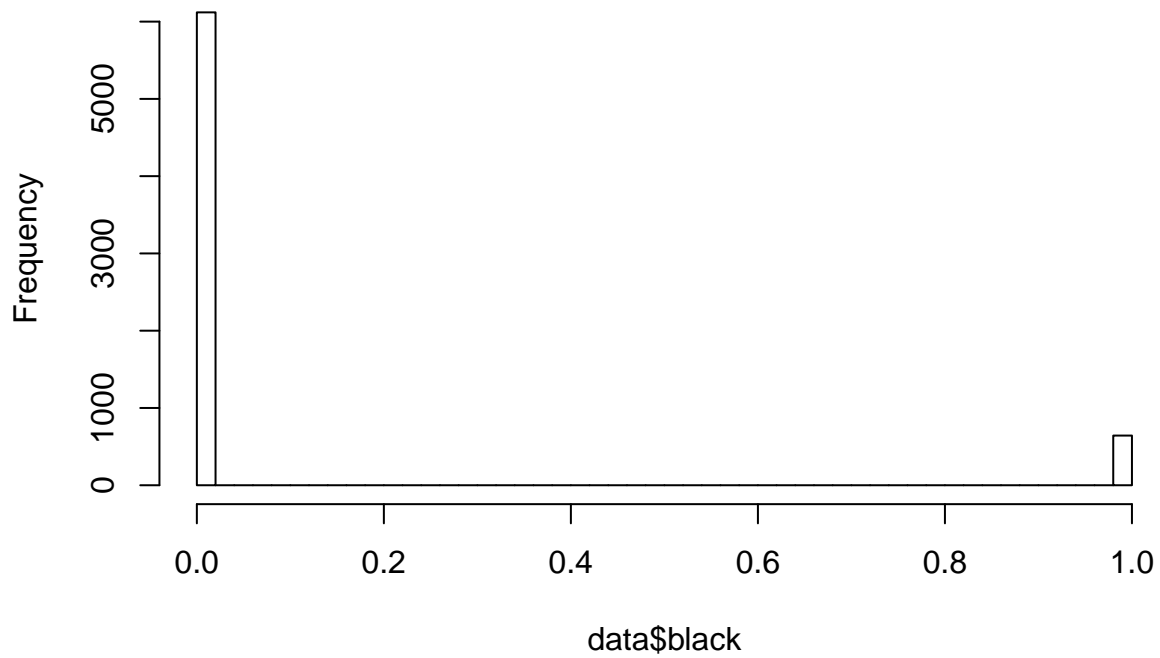
```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.00000 0.00000 0.00000 0.09508 0.00000 1.00000
```

```
print(quantile(data$black, probs = c(0.01, 0.05, 0.1,
    0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##      1%      5%     10%    25%    50%    75%    90%    95%    99%   100%
##       0       0       0       0       0       0       0       1       1       1
```

```
hist(data$black, 50)
```

## Histogram of data\$black



```
summary(data$hispanic)
```

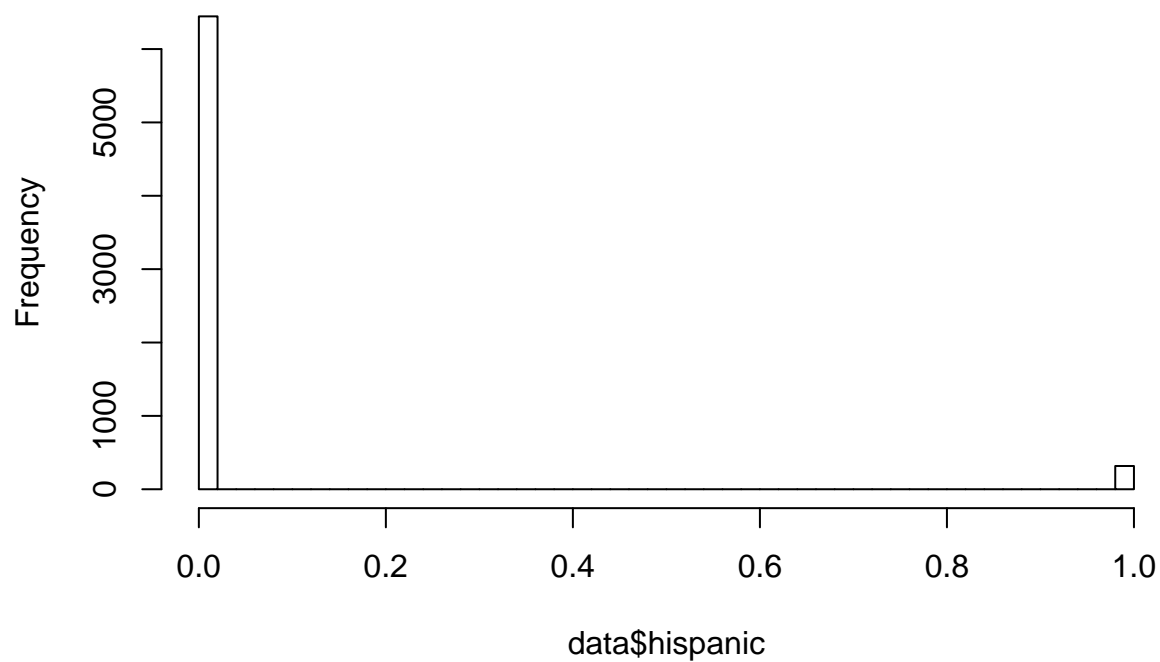
```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.00000 0.00000 0.00000 0.04687 0.00000 1.00000
```

```
print(quantile(data$hispanic, probs = c(0.01, 0.05,
    0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##      1%    5%   10%   25%   50%   75%   90%   95%   99%  100%
##       0     0     0     0     0     0     0     0     1     1
```

```
hist(data$hispanic, 50)
```

## Histogram of data\$hispanic



```
summary(data$AA)
```

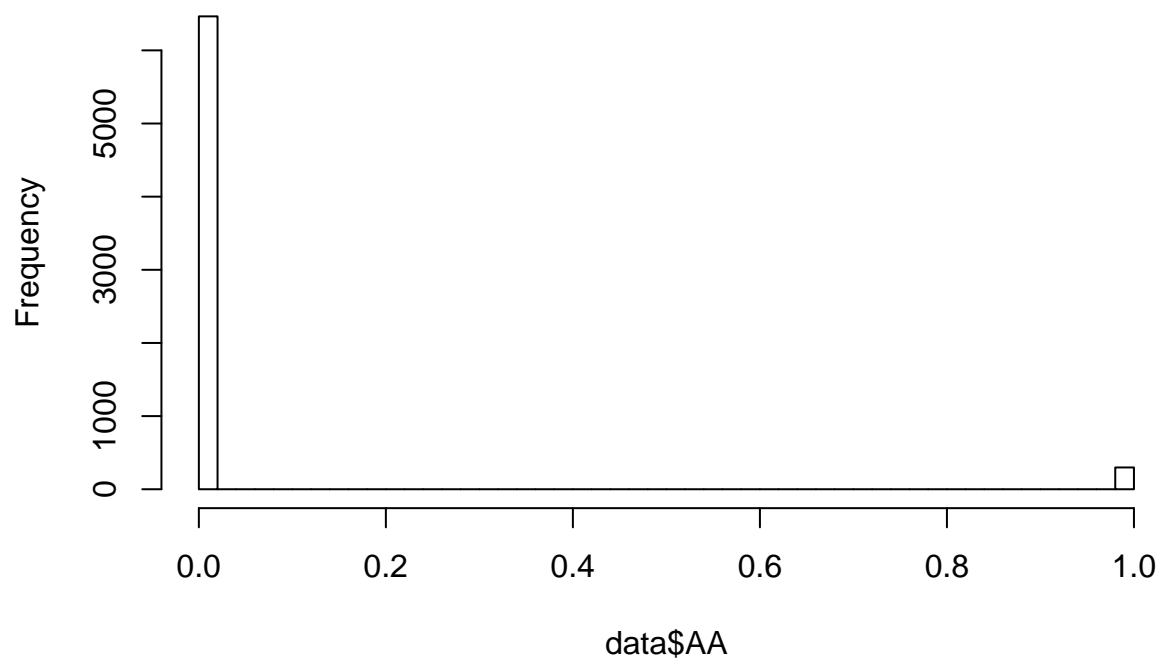
```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.00000 0.00000 0.00000 0.04406 0.00000 1.00000
```

```
print(quantile(data$AA, probs = c(0.01, 0.05, 0.1,
                                     0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##      1%   5%  10%  25%  50%  75%  90%  95%  99% 100%
##       0    0    0    0    0    0    0    0    1    1
```

```
hist(data$AA, 50)
```

## Histogram of data\$AA



```
summary(data$BA)
```

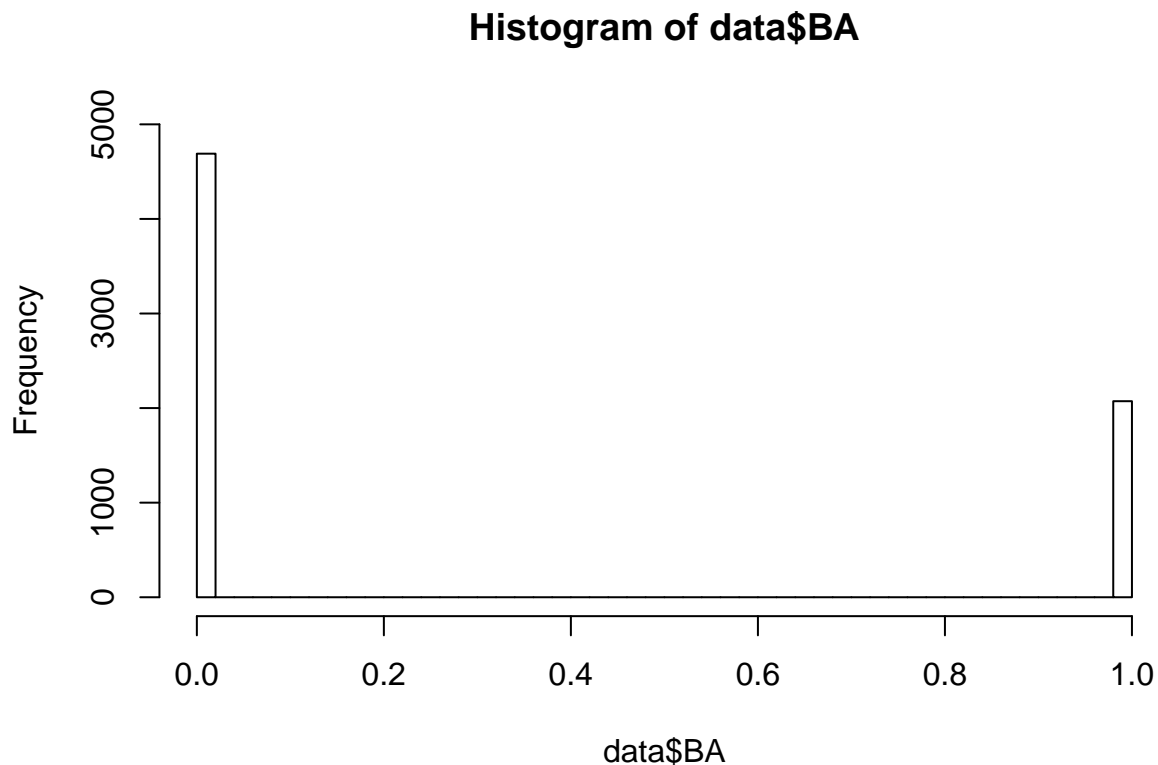
```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.0000 0.0000 0.0000 0.3065 1.0000 1.0000
```

```
print(quantile(data$BA, probs = c(0.01, 0.05, 0.1,
                                     0.25, 0.5, 0.75, 0.9, 0.95, 0.99, 1)))
```

```
##      1%   5%  10%  25%  50%  75%  90%  95%  99% 100%
##      0    0   0   0   0   1   1   1   1   1
```

```
hist(data$BA, 50, ylim = c(0, 5000))
```





## Basic structure of the data

There are no missing values in the data.

**lwage** variable has a normal-like distribution.

**jc** variable has values from 0 to about 4 and is heavily positively skewed with a majority of values at or near 0.

**univ** variable has values from 0 to 7.5 and is heavily positively skewed with a majority of values at or near 0.  
**exper** variable has values from 0 to 166 and is negatively skewed with a hill-climb distribution from 0 to about 500.

**black**, **hispanic**, **AA**, **BA** variables are binary with values of 0 or 1.

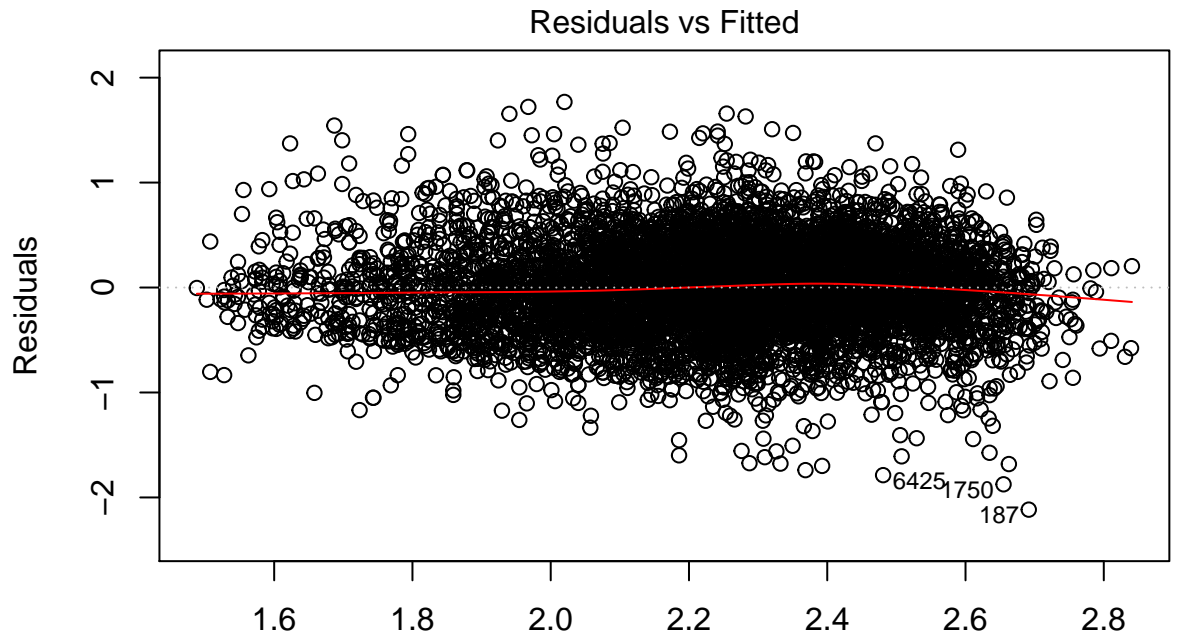
## Question 2

```
# Create the experXblack variable by multiplying
# the exper and black variables.
data$experXblack = data$exper * data$black

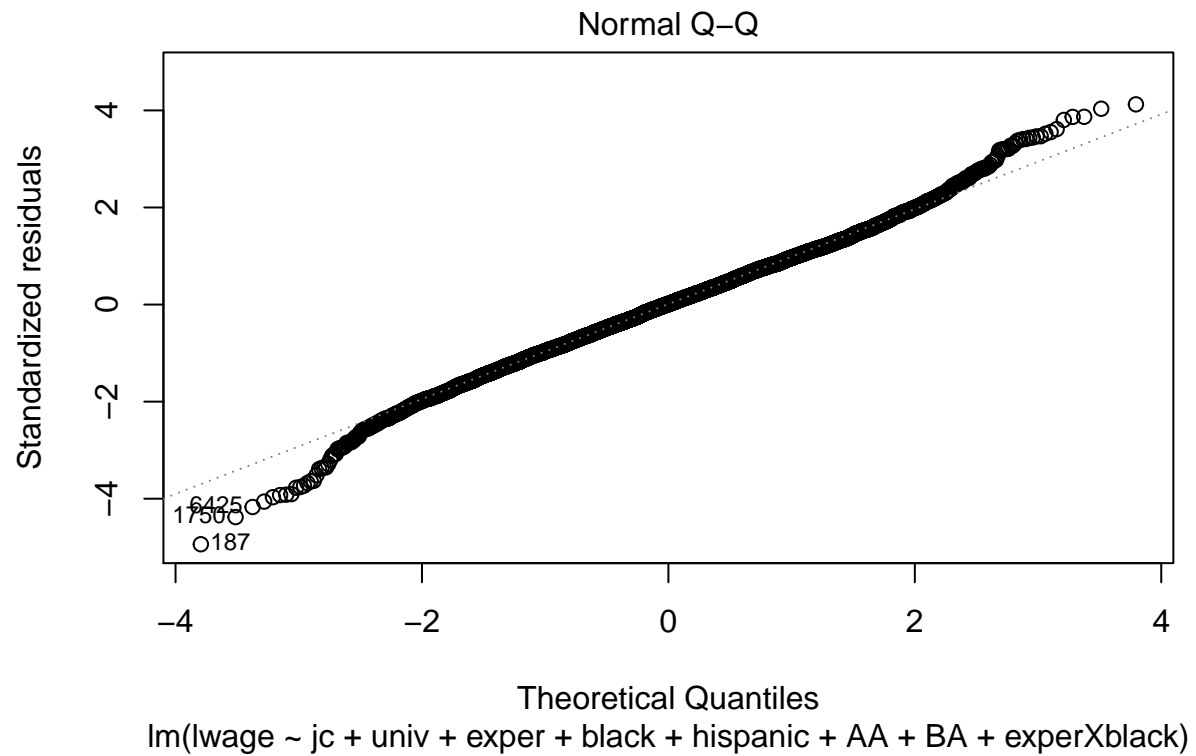
# Run the requested OLS regression.
ols.lwage.8ind = lm(lwage ~ jc + univ + exper + black +
  hispanic + AA + BA + experXblack, data = data)
summary(ols.lwage.8ind)
```

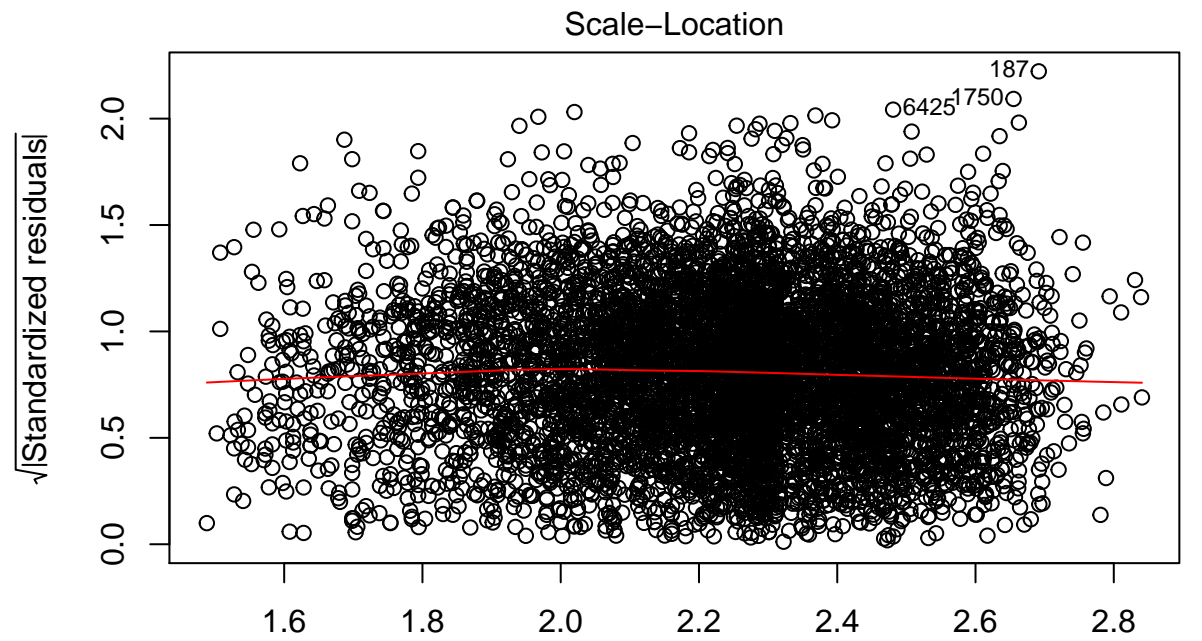
```
##
## Call:
## lm(formula = lwage ~ jc + univ + exper + black + hispanic + AA +
##      BA + experXblack, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.11612 -0.27836  0.00432  0.28676  1.76811
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.4773315  0.0223780  66.017 < 2e-16 ***
## jc           0.0637926  0.0079034   8.072 8.15e-16 ***
## univ         0.0732806  0.0031486  23.274 < 2e-16 ***
## exper        0.0050234  0.0001667  30.141 < 2e-16 ***
## black        0.0331709  0.0613984   0.540  0.5890
## hispanic     -0.0193629  0.0248914  -0.778  0.4367
## AA           -0.0077759  0.0295497  -0.263  0.7924
## BA            0.0176735  0.0156553   1.129  0.2590
## experXblack -0.0012679  0.0004991  -2.541  0.0111 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4287 on 6754 degrees of freedom
## Multiple R-squared:  0.2282, Adjusted R-squared:  0.2272
## F-statistic: 249.6 on 8 and 6754 DF,  p-value: < 2.2e-16

# Print the diagnostic plots
plot(ols.lwage.8ind)
```

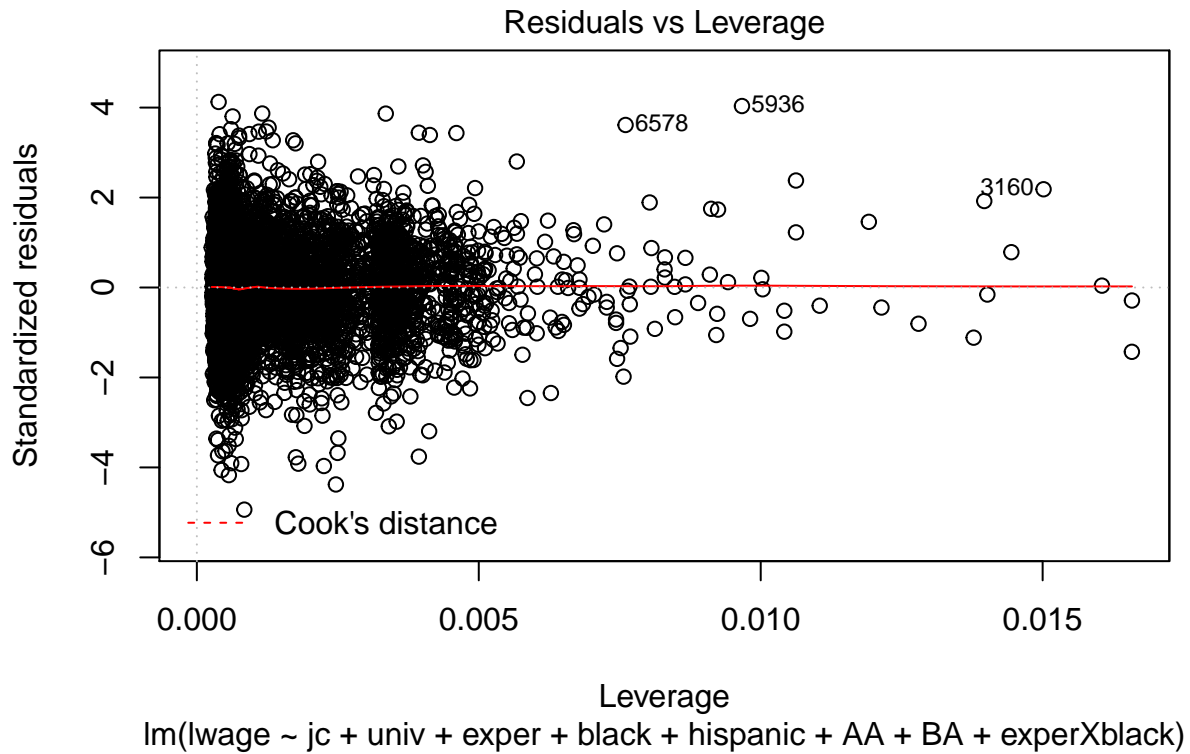


Fitted values  
 $\text{lm}(\text{lwage} \sim \text{jc} + \text{univ} + \text{exper} + \text{black} + \text{hispanic} + \text{AA} + \text{BA} + \text{experXblack})$





Fitted values  
 $\text{lm}(\text{lwage} \sim \text{jc} + \text{univ} + \text{exper} + \text{black} + \text{hispanic} + \text{AA} + \text{BA} + \text{experXblack})$



```
# Print the B_hat4 and B_hat8 coefficients
print(ols.lwage.8ind$coefficients[5])
```

```
##      black
## 0.03317088
```

```
print(ols.lwage.8ind$coefficients[9])
```

```
##  experXblack
## -0.001267898
```

### Interpret the coefficients $\hat{\beta}_4$ and $\hat{\beta}_8$

$\hat{\beta}_4$  is the estimate for the black variable coefficient.

$\hat{\beta}_8$  is the estimate for the experXblack variable.

Do we talk about:

zero-conditional mean seems to be met

homoskedasticity seems to be met

assuming random sample

assuming linear relationship

### Question 3

```
# Show the summary of the model again
```

```
summary(ols.lwage.8ind)
```

```
##
## Call:
## lm(formula = lwage ~ jc + univ + exper + black + hispanic + AA +
##     BA + experXblack, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.11612 -0.27836  0.00432  0.28676  1.76811
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.4773315   0.0223780   66.017 < 2e-16 ***
## jc           0.0637926   0.0079034    8.072 8.15e-16 ***
## univ         0.0732806   0.0031486   23.274 < 2e-16 ***
## exper        0.0050234   0.0001667   30.141 < 2e-16 ***
## black        0.0331709   0.0613984    0.540  0.5890
## hispanic     -0.0193629   0.0248914   -0.778  0.4367
## AA           -0.0077759   0.0295497   -0.263  0.7924
## BA           0.0176735   0.0156553    1.129  0.2590
## experXblack -0.0012679   0.0004991   -2.541  0.0111 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4287 on 6754 degrees of freedom
## Multiple R-squared:  0.2282, Adjusted R-squared:  0.2272
## F-statistic: 249.6 on 8 and 6754 DF,  p-value: < 2.2e-16
```

```
# Print the univ coefficient
```

```
print(ols.lwage.8ind$coefficients[3])
```

```
##      univ
## 0.07328063
```

```
(0.0733 - 0.07)/(0.0031)
```

```
## [1] 1.064516
```

```
2 * (1 - 0.8554)
```

```
## [1] 0.2892
```

Test that the return to university education is 7%.

Null Hypothesis:  $H_0: \beta_2 = 0.07$ .

Alternate Hypothesis:  $H_1: \beta_2 \neq 0.07$ .

Formula for t-statistic =  $(\beta_2 - H_0)/(se) = (.0733 - .07)/(.0031) = 1.064516$

p-value =  $2 * (1 - .8554) = 0.2892$

Based on the p-value, the test is not significant at the 0.05% significance level. Therefore, we can't reject the null hypothesis that the return to university education is 7%.

## Question 4

Test that the return to junior college education is equal for black and non-black

## Question 5

To answer this question, two new variables need to be created. Since the question asks about the return from 1 year of working experience and the current experience variable (exper) is in months a new variable called experYr was created that converted the current exper variable to years by dividing it by 12. Also, to do this problem a variable that represents the sum of the univ and experYr variables needs to be created.

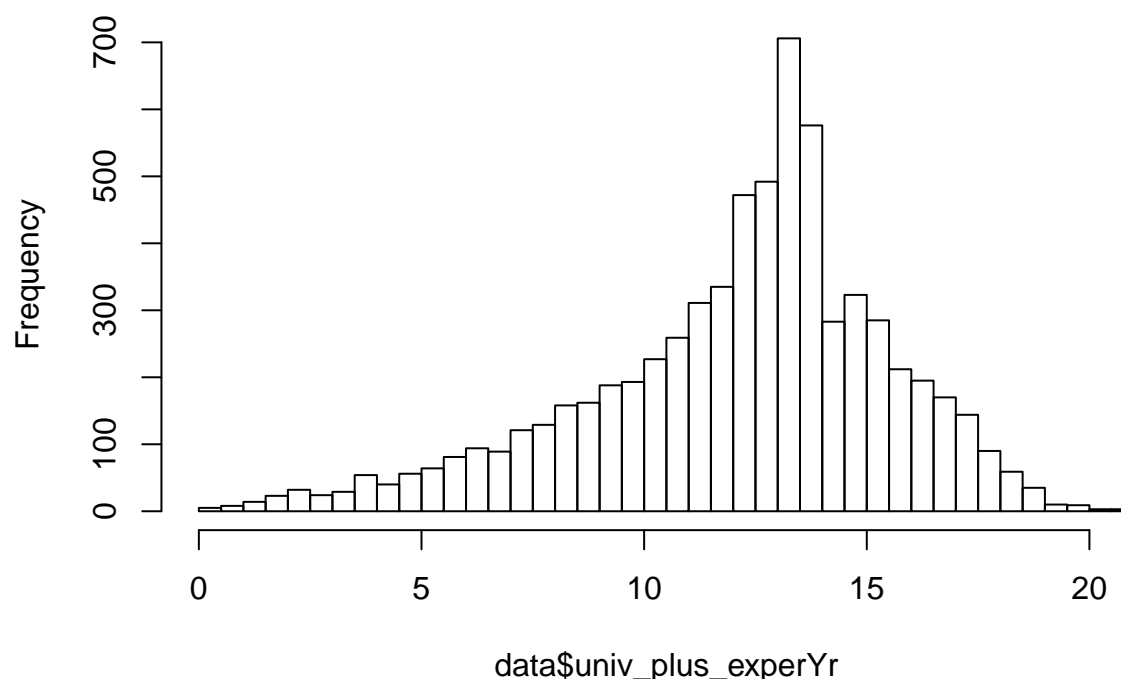
```
# Convert the exper variable from months to years  
# by dividing it by 12.  
data$experYr = data$exper/12  
# Create a variable that is the sum of the univ and  
# experYr variables  
data$univ_plus_experYr = data$univ + data$experYr  
# Analyze the new variable with the summary and  
# hist commands  
summary(data$univ_plus_experYr)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.  
## 0.4167 10.3300 12.7500 12.1200 14.2200 21.0000
```

```
hist(data$univ_plus_experYr, breaks = 50)
```



## Histogram of data\$univ\_plus\_experYr



```
# Rerun the regression by taking out the exper
# variable and replacing it with the
# univ_plus_experYr variable.
ols.lwage.univ.experYr = lm(lwage ~ jc + univ + univ_plus_experYr +
  black + hispanic + AA + BA + experXblack, data = data)
# Display a summary of the new model
summary(ols.lwage.univ.experYr)
```

```
##
## Call:
## lm(formula = lwage ~ jc + univ + univ_plus_experYr + black +
##   hispanic + AA + BA + experXblack, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.11612 -0.27836  0.00432  0.28676  1.76811
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    1.4773315   0.0223780   66.017 < 2e-16 ***
## jc              0.0637926   0.0079034    8.072 8.15e-16 ***
## univ            0.0129997   0.0035721    3.639 0.000276 ***
## univ_plus_experYr 0.0602810   0.0020000   30.141 < 2e-16 ***
## black           0.0331709   0.0613984    0.540 0.589038
## hispanic       -0.0193629   0.0248914   -0.778 0.436659
## AA              -0.0077759   0.0295497   -0.263 0.792446
```

```
## BA          0.0176735  0.0156553   1.129 0.258972
## experXblack -0.0012679  0.0004991  -2.541 0.011088 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4287 on 6754 degrees of freedom
## Multiple R-squared:  0.2282, Adjusted R-squared:  0.2272
## F-statistic: 249.6 on 8 and 6754 DF,  p-value: < 2.2e-16
```

## Test whether the return to university education is equal to the return to 1 year of working experience.

By replacing the exper variable in the model with the univ\_plus\_experYr, now the univ variable represents the difference between the univ and experYr variables and to answer the question we want to know if that difference is 0, so we run the following test.

$$lwage = \beta_0 + \beta_1 jc + \beta_2 univ + \beta_3 univPlusexperYr + \beta_4 black + \beta_5 hispanic + \beta_6 AA + \beta_7 BA + \beta_8 experXblack$$

Null Hypothesis:  $H_0: \beta_2 = 0$ .

Alternate Hypothesis:  $H_1: \beta_2 \neq 0$ .

Based on the very low p-value for  $\beta_2$ , the test is significant at the 0.05% significance level. And even though the value of  $\beta_2$  is close to 0 at 0.0129997, we can reject the null hypothesis that  $\beta_2 = 0$ .

## Question 6

```
print(sqrt(0.2282))
```

```
## [1] 0.4777028
```

## Test the overall significance of this regression.

Here is the output from the summary of our model.

Residual standard error: 0.4287 on 6754 degrees of freedom

Multiple R-squared: 0.2282, Adjusted R-squared: 0.2272

F-statistic: 249.6 on 8 and 6754 DF, p-value: < 2.2e-16

1. Our model null hypothesis is that there is no relationship among any of the independent variables and lwage variable. We are able to reject the null hypothesis since our p-value of the f-statistic of the model is significant at < 2.2e-16.
2. Practical significance: we have an R-squared value of 0.2282, indicating that 22.82% of the variation in lwage is explained by our model. An R value of 0.478 indicates a ?? effect size.  
??which regression model are we supposed to be using here, the one with univPlusexperYr or the first one??

## Question 7

```
data$experXexper = data$exper * data$exper
ols.lwage.9ind = lm(lwage ~ jc + univ + exper + black +
  hispanic + AA + BA + experXblack + experXexper,
  data = data)
summary(ols.lwage.9ind)

##
## Call:
## lm(formula = lwage ~ jc + univ + exper + black + hispanic + AA +
##     BA + experXblack + experXexper, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.11982 -0.27743  0.00475  0.28741  1.77397
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.510e+00  4.427e-02  34.108 < 2e-16 ***
##      jc       6.417e-02  7.916e-03   8.106 6.14e-16 ***
##     univ     7.382e-02  3.211e-03  22.992 < 2e-16 ***
##     exper    4.301e-03  8.588e-04   5.008 5.64e-07 ***
##     black    2.994e-02  6.152e-02   0.487  0.6265
##   hispanic  -1.932e-02  2.489e-02  -0.776  0.4378
##      AA     -7.539e-03  2.955e-02  -0.255  0.7986
##      BA     1.797e-02  1.566e-02   1.147  0.2513
##   experXblack -1.239e-03  5.002e-04  -2.477  0.0133 *
##   experXexper  3.379e-06  3.939e-06   0.858  0.3911
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4287 on 6753 degrees of freedom
## Multiple R-squared:  0.2282, Adjusted R-squared:  0.2272
## F-statistic: 221.9 on 9 and 6753 DF,  p-value: < 2.2e-16
```

### Estimated return to work experience in this model

$$lwage = \beta_0 + \beta_1 jc + \beta_2 univ + \beta_3 exper + \beta_4 black + \beta_5 hispanic + \beta_6 AA + \beta_7 BA + \beta_8 experXblack + \beta_9 experXexper$$

$$\Delta lwage / \Delta exper = \beta_3 + \beta_8 black + 2\beta_9 exper$$

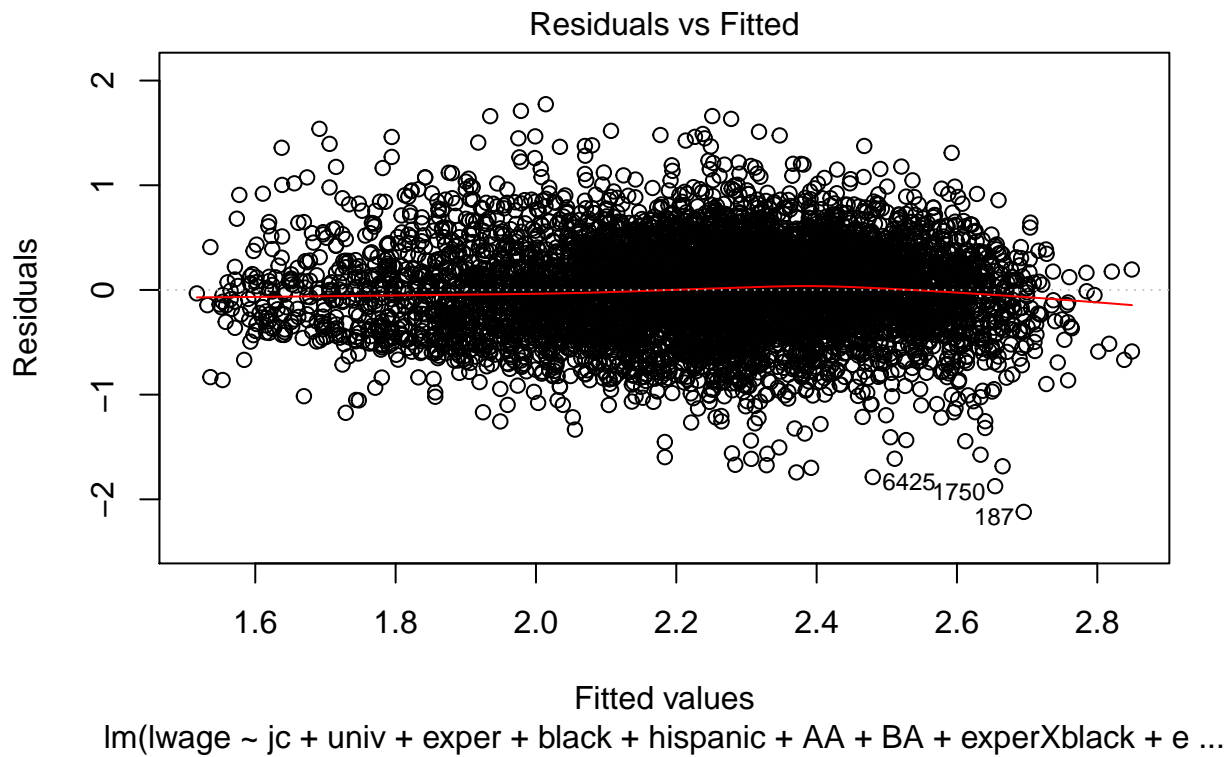
$$= (.004301 - .001239 * black + 2 * .000003379 * exper)$$

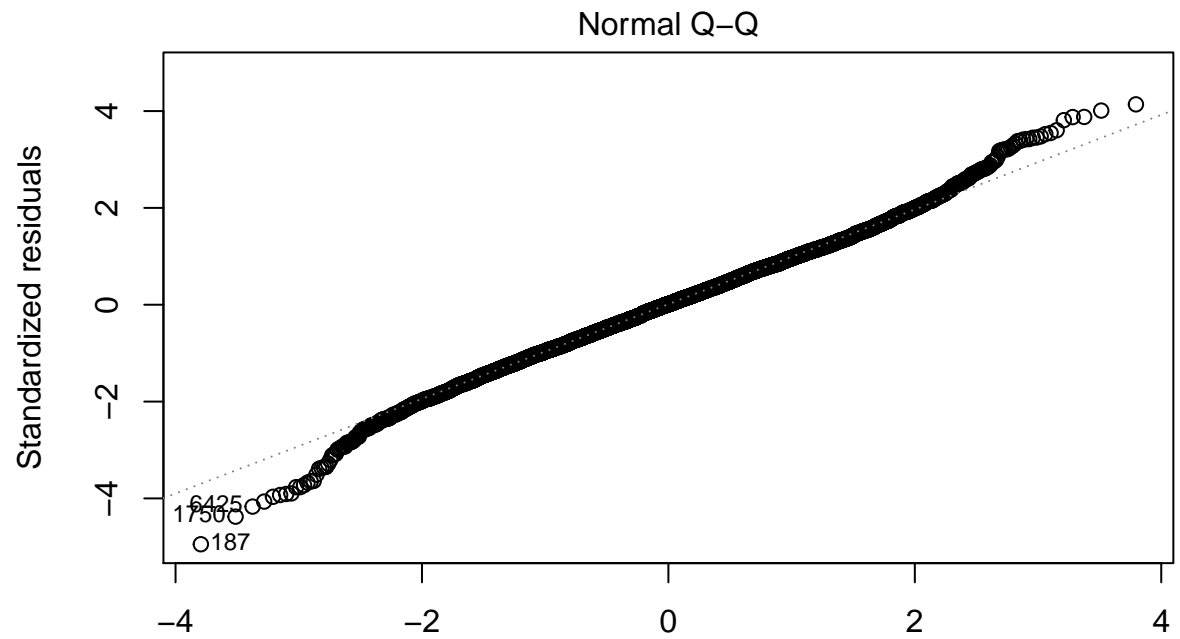
Now convert the log wage back to wage by exponentiating. This gives us a return to work experience:

$$= e^{(.004301 - .001239 * black + 2 * .000003379 * exper)}$$

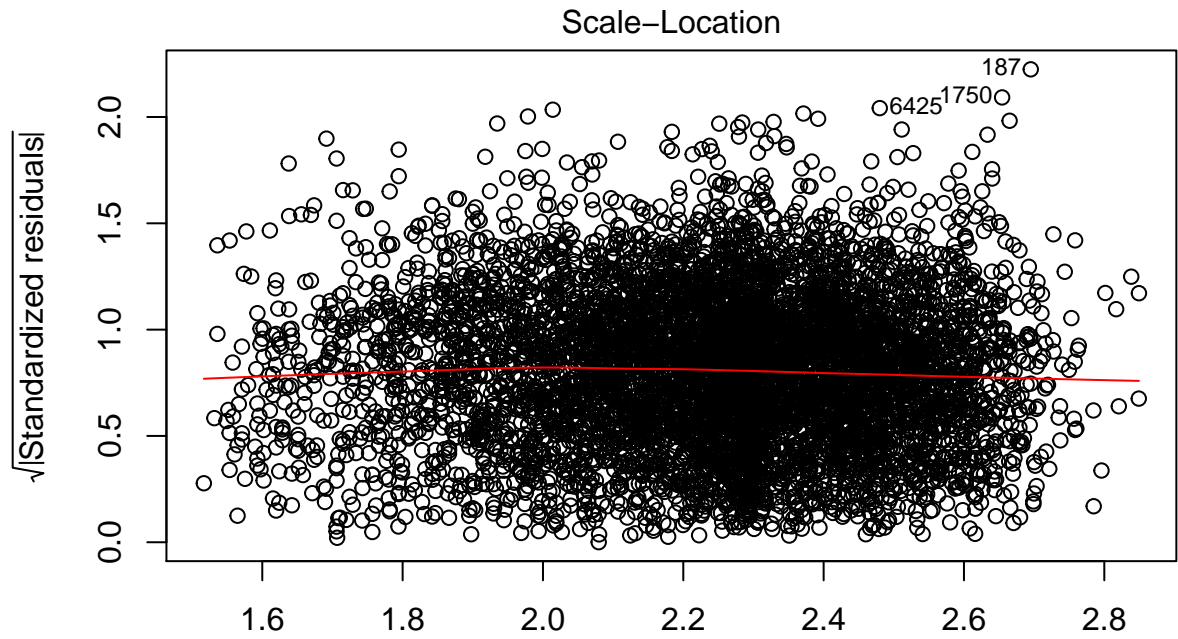
## Question 8

```
# Based on the violation of homoskedasticity, we  
# must run robust standard errors. coeftest(model,  
# vcov=vcovHC) waldtest(model, vcov=vcovHC)  
plot(ols.lwage.9ind)
```

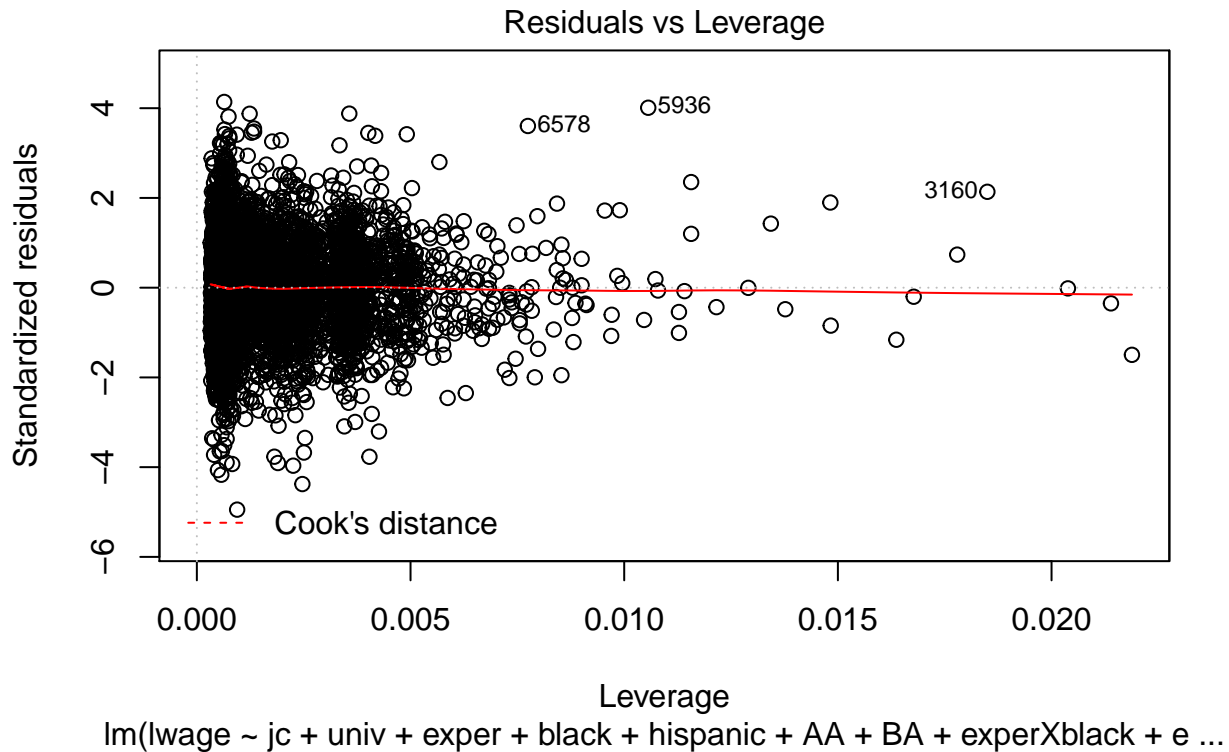




Im(lwage ~ jc + univ + exper + black + hispanic + AA + BA + experXblack + e ...



Fitted values  
 $\text{lm}(\text{lwage} \sim \text{jc} + \text{univ} + \text{exper} + \text{black} + \text{hispanic} + \text{AA} + \text{BA} + \text{experXblack} + \text{e} \dots)$



### Homoskedasticity analysis:

The assumption of homoskedasticity holds:

- 1 - We can see from the residuals vs fitted plot that the variance band is about the same as we move to higher fitted values.
- 2 - The same story is told by the scale-location plot where we see that the smoothing line is almost completely horizontal, which is what we get if homoskedasticity is met.
- 3 - We do not look at the Breusch Pagan test since we have a large number of observations, therefore we know almost certainly that we will obtain significance.

The implication of homoskedasticity in the data is that the standard error of the univ coefficient ( $\beta_2$ ) is unbiased. Unbiased standard errors will not impact the outcomes of statistical tests. Therefore, it does not affect the testing of no effect of university education on salary change.