# **Supporting Information**

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SI Text

Dynamical Matching. An algorithm similar to that used to solve the problem of the stereometric matching was used to perform the dynamical matching (or tracking), i.e., to associate to each bird's coordinates at time t its corresponding coordinates at time t + dt. In our case the time interval between two consecutive reconstructions was dt = 0.1 s. The algorithm is based on a pattern recognition principle. First we subtract from the t + 1 reconstruction the movement of the center of mass of the flock. Then, we compare the translated coordinates at time t + 1 with the ones at time t. Because each bird moves approximately as the whole group, after the translation the coordinates of the same individual will differ only by small fluctuations. Thus, given a bird iat time t, we consider as candidates for the corresponding bird at time t + 1 only individuals who are close enough to the reference one. Then, to discriminate among these candidates, we check which one has the most similar neighbors pattern to i. The idea is that an individual and its neighbors move in a similar manner, thus preserving mutual relationships in distance. We then consider the pattern formed by i and its nearest neighbors and compare it to the pattern formed by a candidate and its neighbors. If the candidate is the correct match of i, also their neighbors identify corresponding birds, and the two patterns are similar. We assign a similarity score (quantifying congruency of neighbors) to the match. We then select the correct dynamical matching by optimizing the global score. From this dynamical match one can work out the individual velocities as the ratio between displacement and time. The efficiency of our dynamical matching algorithm, defined as the ratio between the number of matched birds and the original number of birds, is 0.77 on average.

What Is Information Transfer? Our statement that scale-free correlation is necessary to transmit undamped information across the system seems to contradict the ordinary physical fact that waves can coherently travel for a length that is generally much larger than the correlation length of the system. Sound propagation in the air (where the correlation length is very short) is the most obvious example of such a phenomenon. One may thus object that orientation or density waves propagating across the flock could transport information even in the absence of scale-free correlations.

It is certainly true that a *perturbation* can propagate across the flock even with a short correlation length. However, this does not automatically mean that information is transferred. To transform perturbation into information a nontrivial process of encoding and decoding is needed: For example, information may be encoded in speech, which is propagated as sound, and decoded again into information by the recipient. If all is well, the state of the recipient is then changed in a useful and permanent way as a result of such information transfer. One may suggest that, similarly, a panic wave triggered by a predator at one end of the flock may travel to birds at the other end and change their cognitive state in a useful way ("switch to panic"). However, even though some information is actually transferred ("panic"), this is absolutely insufficient to make the flock respond collectively: The cognitive change of state of the birds is not telling them in what direction to turn, nor how to keep cohesion in the group, nor how to respond coherently. Traveling waves do not encode all this essential information.

A far more effective way to transmit information consists in transferring a perturbation that changes the state of the individuals in a *directly* useful and *permanent* way, without the need

of any process of decoding: For example, a bird changes direction of motion as a result of an attack, heading away from the predator. This change is transferred to all correlated birds, so that they all change their heading in a similar and permanent way. There is no need to interpret the transferred perturbation, as it is directly useful to each recipient to get away from the predator. In this case perturbation and information are the same thing. Unlike a passing wave, which in the absence of decoding leaves unaltered the final state of the bird, information transferred through correlation makes a permanent change in the dynamical state of the individual, which is essential to achieve a collective response.

In physical terms, we may say that, in the absence of decoding, what is needed to transfer permanent information is a finite response at zero frequency, namely a finite static susceptibility, the span of which is regulated by the static correlation length. For example, oscillatory speed waves (nonzero frequency) could propagate through the flock, but they would leave unaltered the dynamical state of the birds after they have passed. To change the speed of the entire flock *permanently* (and thus usefully) one needs a finite response at zero frequency of the speed and hence a scale-free correlation length of the modulus. This is exactly what we found in starling flocks.

We therefore conclude that the only way to transfer directly useful information in a permanent way to *all* individuals in the flock, with no need of a complex neural process of encoding/ decoding, is indeed to have scale-free correlation of the entire dynamical states of the birds.

Comparison with Theoretical Models. The fact that correlations in flocks are scale free seems to be consistent with some theoretical and numerical studies performed in self-propelled particle systems, where the correlation function was found to be a power law (1, 2). The comparison, however, is not straightforward. First, the theoretical studies of refs. 1 and 2 were performed in the presence of periodic boundary conditions, which is clearly not the experimental situation. Second, in these studies there were no bounded flocks, as "birds" were sparse across the entire periodic system. Hence, all of the features we have found for the correlation function, which are crucially related to the presence of flocks of various finite sizes, cannot be compared with the theoretical predictions. Finally, in most models the modulus of the velocity is a constant and therefore by construction there cannot be any correlation of the modulus fluctuations, in contrast with what we found here.

Collective Response, Criticality, and Neural Systems. There is an intriguing similarity between our results in bird flocks and some recent speculations in neurosciences. Some experiments (3–5) and mathematical models (6–8) suggest that assemblies of neurons develop long-range correlations and large response as the result of criticality. Also in this case, as in flocks, being critical is a way for the system to be always ready to optimally respond to an external perturbation, be it a sensory stimulus as in the case of neural assemblies or a predator attack as in the case of flocks. Commonalities between animal groups and neural systems have already been noted and discussed in the literature (9–10). Even though the analogy remains highly speculative at this time, and would require more robust biological and mathematical foundations, it suggests a stimulating perspective to the role of criticality in biological systems.

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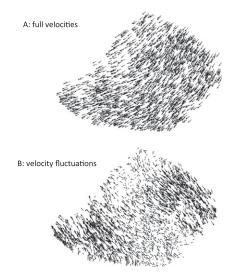


Fig. S1. (A) The 2D projection of the velocities of the individual birds within a starling flock (event 57-03; 3,242 birds, vectors scaled for clarity). (B) This is the 2D projection of the individual velocity fluctuations in the same flock at the same instant of time as in A (vectors scaled for clarity). Large domains of strongly correlated birds are clearly visible. We recall that the modulus of the fluctuations is in fact much smaller than that of the full velocities (see Fig.1C in main text).

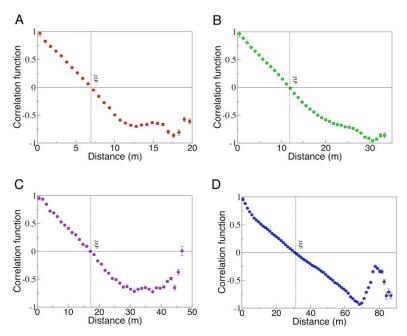
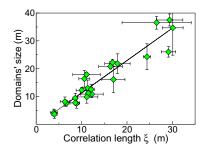


Fig. S2. The correlation function C(r) in four other flocks: event 49-05 (A), event 25-10 (B), event 69-13 (C), and event 57-03 (D). The behavior of the correlation function is quite stable across different flocks; only for very large values of the distance r boundary effects take over and the specific shape of the flock dominates the behavior of the correlation function.

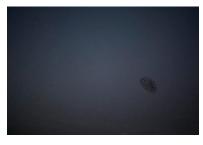


**Fig. S3.** Size of the domains calculated from the covariance matrix vs.  $\xi$ , i.e., the zero of the correlation function, for all analyzed flocks. There is a clear linear correlation (Pearson's correlation test: n = 24, r = 0.95,  $P < 10^{-11}$ ). The angular coefficient is 1.16. This result shows that the correlation length  $\xi$  is indeed an excellent estimator of the domains' size.

Table S1. Polarization and other features in the analyzed flocks

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Event	No. of birds	Polarization	Velocity, m/s	Flock's size L, m
16-05	2,941	0.962	15.2	79.2
17-06	552	0.935	9.4	51.8
21-06	717	0.973	11.8	32.1
25-08	1,571	0.962	12.1	59.8
25-10	1,047	0.991	12.5	33.5
25-11	1,176	0.959	10.2	43.3
28-10	1,246	0.982	11.1	36.5
29-03	440	0.963	10.4	37.1
31-01	2,126	0.844	6.8	76.8
32-06	809	0.981	9.8	22.2
42-03	431	0.979	10.4	29.9
48-17	871	0.886	11.2	31.1
49-05	797	0.995	13.9	19.2
54-08	4,268	0.966	19.1	78.7
57-03	3,242	0.978	14.1	85.7
58-06	442	0.984	10.1	23.1
58-07	554	0.977	10.5	19.1
63-05	890	0.978	9.9	52.9
69-09	239	0.985	11.8	17.1
69-10	1,129	0.987	11.9	47.3
69-13	1,947	0.937	9.6	44.8
69-19	803	0.975	13.8	26.4
72-02	122	0.992	13.2	10.6
77-07	186	0.978	9.3	9.1

Flocking events are labeled according to session number and to the position within the session they belong to. The polarization is defined in Eq. 1 of the main text. The linear size  $\boldsymbol{L}$  of the flock is defined as the maximum distance between two birds belonging to the flock. The velocity is that of the center of mass, i.e., the mean velocity of the group. All values are averaged over several instants of time during the event.



**Movie S1.** Video of flocking event 32-06. The video is built with consecutive images shot with one of the cameras of the stereoscopic apparatus. The resolution of individual frames has been reduced (original images are  $3,504 \times 2,336$  pixels). The 3D reconstructions used in data analysis correspond to instants of time where the flock is entirely included in the field of view of all of the cameras.

Movie S1 (AVI)



Movie S2. Video of flocking event 49-05. The reconstructed flock considered in data analysis is the one in the upper east corner.

#### Movie S2 (AVI)



Movie S3. Video of flocking event 54-08.

#### Movie S3 (AVI)



**Movie S4.** Dynamical evolutions of a starling flock close to the roost. The flock is disturbed by several seagulls. The attacks trigger several changes in the dynamical state of the birds, which spread to the entire flock without being damped. The video was shot with a video camera at the same roosting site as the stereoscopic experiment (original resolution has been reduced).

## Movie S4 (MOV)

# Other Supporting Information Files

**Dataset S1.** 3D coordinates of two consecutive instants of time, for event 28-10. The data correspond to the velocity fields of Fig. 1 and to the correlation functions of Fig. 2 A and B. These data allow computing velocity fields and correlation functions at one single instant of time. Note, however, that to obtain statistically significant results it is often convenient (and sometimes necessary) to average over several instants of time, to (i) reduce the effect of statistical fluctuations typical of finite systems and (ii) describe the representative behavior of a flock in time. For example, in Fig. 2 C and D, each value of the correlation length is in fact an average over several instants of time in a single flocking event, as specified in the figure legend.

### Dataset S1 (RTF)