The reason for this approximation is that the rate of vaporisation of GaAs under laser irradiation conditions is not known. Thus it is impossible to allow for vaporisation in our computed values of $k(g)$.

From the foregoing results we have established that Cr redistributes in GaAs:Cr during the short melt time resulting from the laser irradiation at energy densities of 0·2 and 1·4 J/cm^2. It is possible that the observed surface pile-up of Cr is partly responsible for the failure of the laser annealing technique to activate low dose implants in GaAs:Cr.[1,2] The melt depths extracted from the s.i.m.s. results presented in Fig. 1 are the first published results for laser annealed GaAs. A comparison of these results with theoretical calculations will be presented elsewhere.

Acknowledgments: The authors wish to thank I. G. Gale (PRL) for carrying out the atomic absorption measurements and Prof. K. G. Stephens and J. M. Shannon (PRL) for valuable discussions.

M. H. BADAWI
B. J. SEALY

Department of Electronic & Electrical Engineering
University of Surrey, Guildford, Surrey, England

J. B. CLEGG

Solid State Electronics Division
Philips Research Laboratories
Cross Oak Lane, Redhill, Surrey, England

30th May 1980

References

1 FERRIS, S. D., LEAMY, H. J., and POATE, J. M. (EDS.): 'Laser-solid interactions and laser processing—1978', *AIP Conf. Proc.*, **50**, American Institute of Physics, 1979
2 WHITE, C. W., and PEERCY, P. S. (EDS.): 'Laser and electron beam processing of materials—1979' (Academic Press, to be published)
3 TUCK, B., ADEGBOYEGA, G. A., JAY, P. R., and CARDWELL, M. J.: 'Out-diffusion of chromium from GaAs substrates', *Inst. Phys. Conf. Ser.*, 1979, **45**, p. 114
4 HUBER, A. M., MORILLOT, G., LINH, N. T., FAVENNEC, P. N., DEVEAUD, B., and TOULOUSE, B.: 'Chromium profiles in semi-insulating GaAs after annealing with a Si_3N_4 encapsulant', *Appl. Phys. Lett.*, 1979, **34**, p. 858
5 EVANS, C. A., JR., DELINE, V. R., SIGMON, T. W., and LIDOW, A.: 'Redistribution of Cr during annealing of ^{80}Se-implanted GaAs', *ibid.*, 1979, **35**, p. 291
6 FAVENNEC, P. N., and L'HARIDON, H.: 'Implantation of shallow impurities in Cr-doped semi-insulatimg GaAs', *ibid.*, 1979, **35**, p. 699
7 CLEGG, J. B.: 'Quantitative measurement of impurities in GaAs layers by secondary ion mass spectrometry', *Surface & Interface Analysis*, (in press)
8 BADAWI, M. H., SEALY, B. J., and STEPHENS, K. G.: 'Vaporisation of GaAs during laser annealing', *Electron. Lett.*, 1979, **15**, p. 786
9 AUSTON, D. H., GOLOVCHENKO, J. A., SIMONS, A. L., SLUSHER, R. E., SMITH, P. R., SURKO, C. M., and VENKATESAN, T. N. C.: 'Dynamics of laser annealing', *AIP Conf. Proc.*, 1979, **50**, p. 11
10 HAISTY, R. W., and CRONIN, G. R.: 'A comparison of doping effects of transition elements in GaAs'. Physics of Semiconductors, Proceedings of the 7th int. conf., Academic Press, Paris 1964, p. 1161
11 PAMPLIN, B. R.: 'Crystal growth' (Pergamon Press, 1975), p. 116
12 WANG, J. C., WOOD, R. F., WHITE, C. W., APPLETON, B. R., PRONKO, P. P., WILSON, S. R., and CHRISTIE, W. H.: 'Dopant profile changes induced by laser irradiation of Si: comparison of theory and experiment', *AIP Conf. Proc.*, 1979, **50**, p. 123

0013-5194/80/140554-03$1.50/0

ALGORITHMS FOR COMPUTATION OF RELATIVE NEIGHBOURHOOD GRAPH

Indexing terms: Pattern recognition, Graph theory

The definition of the relative neighbourhood graph (r.n.g.) of a set of data points is considered from an alternative viewpoint. This leads to properties of the r.n.g. which can be exploited to give faster algorithms, one of which is restricted to use with two-dimensional data and Euclidean distance, and the other being more general.

Introduction: In a recent paper,[1] Toussaint defined the relative neighbourhood graph (r.n.g.) of a set of n points. This graph is potentially the basis of a new method for graph theoretical clustering (in a similar way to the minimum spanning tree (m.s.t.)[2]). Furthermore Toussaint suggested that the r.n.g. extracts a perceptually meaningful structure from a set of dots in two dimensions, and showed that the r.n.g. is a subset of the Delaunay triangulation (d.t.) and a superset of the m.s.t. Two algorithms for computing the r.n.g. are presented, one being a 'naive' algorithm running in $O(n^3)$ time and the other running in $O(n^2)$ time but being restricted to two dimensions and Euclidean distance. Toussaint raised a number of open questions on the r.n.g. including whether or not faster running algorithms are possible.

In this letter, modified algorithms are presented which result in improved running times.

Theory (1): Let (p_i, p_j) denote the line connecting two data points p_i, p_j, and let $d(p_i, p_j)$ denote their distance apart. Then the r.n.g. is constructed by connecting two data points p_i and p_j $(i \neq j)$ if and only if they are 'relatively close' (this is essentially from Lankford's definition[3]), i.e.

$$(p_i, p_j) \in \text{r.n.g.} \quad \text{iff} \quad d(p_i, p_j) \leq \max [d(p_i, p_k), d(p_j, p_k)] \quad (1)$$

for all $k = 1, 2, \ldots n, k \neq i, k \neq j$ (see Fig. 1). A logically equivalent expression is

$$(p_i, p_j) \notin \text{r.n.g.} \quad \text{iff} \quad d(p_i, p_j) > \max [d(p_i, p_k), d(p_j, p_k)] \quad (2)$$

for any $k = 1, 2, \ldots n, k \neq i, k \neq j$. In other words, when considering any three points in a data set, the longest side (if one exists) of the triangle connecting them satisfies eqn. 2 and so cannot be an edge of the r.n.g. So when testing points for inclusion into the r.n.g. (according to eqn. 1) it is possible to simultaneously reject any line segments satisfying eqn. 2. This leads to the r.n.g. algorithm below, which can generalise to d dimensions and any distance measure.

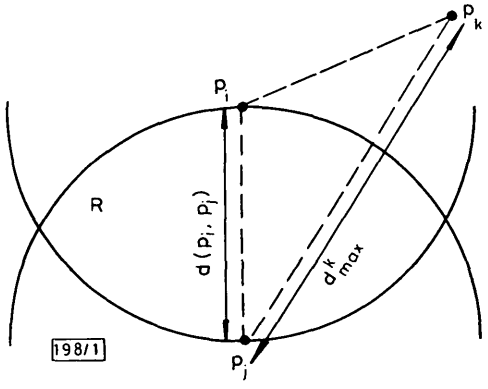

Fig. 1 *Illustration of r.n.g. definition; p_i and p_j will be connected as r.n.g. edge only if no other point p_k lies in R, where R denotes the interior of the intersection of two circles with centres at p_i and p_j*

Algorithm (1)

(i) Compute the distance between all pairs of points $d(p_i, p_j)$ $i, j = 1, 2, \ldots n$ $(i > j)$.
(ii) For each pair of points (p_i, p_j) that has not been rejected as an edge of the r.n.g. by a previous application of this stage, compute

$$d^k_{max} = \max [d(p_i, p_k), d(p_j, p_k)]$$

for each point p_k $(k = 1, 2, \ldots n, k \neq i, k \neq j)$.
If $d(p_i, p_j) > d^k_{max}$ then $(p_i, p_j) \notin$ r.n.g. (eqn. 2).
If $d(p_i, p_j) < d^k_{max}$ then the pair of points separated by $d^k_{max} \notin$ r.n.g. (eqn. 2).
If $d(p_i, p_j) \leq d^k_{max}$ for every p_k $(i \neq k, j \neq k)$ then $(p_i, p_j) \in$ r.n.g. (eqn. 1).

Step (i) of this algorithm runs in $O(n^2)$ time and is computed once. Step (ii) runs in $O(n)$ time and is computed at least m times where m is the number of edges in the r.n.g.; it has been shown that m is $O(n)$.[1] Thus the complexity of step (ii) is at least $O(n^2)$, so that the algorithm runs in at least $O(n^2)$ time. This algorithm is suitable for d-dimensional data and any distance measure.

Theory (2): A triangulation T is a maximal subset of the total number of line segments whose endpoints are in P, the set of all points in a plane, such that the line segments of T only intersect at their endpoints. Thus, in two dimensions, an edge of T will be a side of at most two triangles in T (Fig. 2); and so a given pair of points p_i, p_j forming an edge will be connected to at most two other points p_a and p_b.

Therefore, since the r.n.g. is a subset of the d.t., it is clear that the d.t. can be converted into the r.n.g. by elimination of certain edges according to eqn. 2. In fact, the longest edges (if they exist) of each triangle in the d.t. will be removed. Furthermore, in two dimensions a given edge of the d.t. (p_i, p_j) need only be compared with edges connecting it to p_a and p_b. Thus an improved algorithm is proposed which is restricted to two dimensions and Euclidean distance.

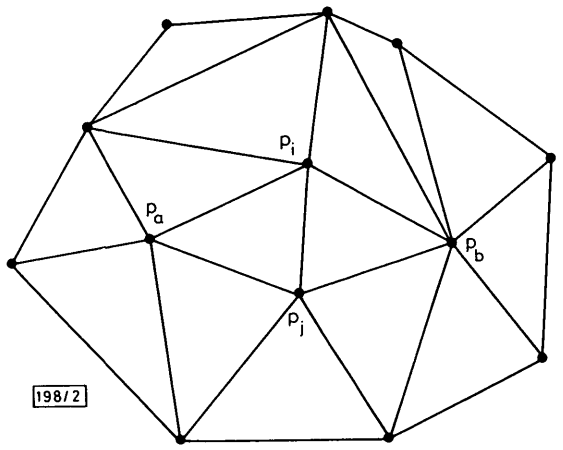

Fig. 2 *Triangulation of a series of dots. Note that the pair of points p_i, p_j is connected only to p_a and p_b*

Algorithm (2)

(i) Compute the Voronoi diagram for the set of points.

(ii) Obtain the Delaunay triangulation from the Voronoi diagram.
(iii) For each pair of points p_i, p_j associated with an edge of the d.t. that has not already been previously eliminated by this step, compute $d^k_{max} = \max [d(p_i, p_k), d(p_j, p_k)]$ for each of the points p_k connected to p_i, p_j by the d.t. (there will be at most two).

If $d(p_i, p_j) > d^k_{max}$ then $(p_i, p_j) \notin$ r.n.g. (eqn. 2).
If $d(p_i, p_j) < d^k_{max}$ then the line segment of length $d^k_{max} \notin$ r.n.g. (eqn. 2).
If $d(p_i, p_j) \leq d^k_{max}$ for each of the points then $(p_i, p_j) \in$ r.n.g. (eqn. 1).

Step (i) runs in $O(n \log n)$ time, step (ii) runs in $O(n)$. Step (iii) is repeated $O(n)$ times since there are $O(n)$ edges to the d.t.,[1] and involves at most four comparisons. Therefore the computation is dominated by step (i) making the algorithm run in $O(n \log n)$ time.

It is hoped that these improved algorithms will enable the r.n.g. to be a more attractive tool in pattern recognition, as a result of improved computation time.

Acknowledgments: The author would like to acknowledge J. E. S. Macleod and A. Dowie for useful discussions during the writing of this letter.

R. B. URQUHART *20th May 1980*

Department of Electronics & Electrical Engineering
University of Glasgow, Glasgow G12 8QQ, Scotland

References

1 TOUSSAINT, G. T.: 'The relative neighbourhood graph of a finite planar set'. Technical Report SOCS 79.7, McGill University, Montreal; also presented at BPRA Conference, Oxford, 1980
2 ZAHN, C.: 'Graph-theoretical methods for detecting and describing Gestalt clusters', *IEEE Trans.*, 1971, C-20
3 LANKFORD, P. M.: 'Regionalization: theory and alternative algorithms', *Geographical Analysis*, 1969, 1

0013-5194/80/140556-02$1.50/0

COMPARISON OF PULSE RESPONSES OF STEP-INDEX FIBRES

Indexing terms: Optical fibres, Waveguides

Pulse responses of step-index fibres with V parameters lying between 2 and 40 are presented. The results are based on computations using the normal WKB, a modified WKB, the 'weakly guiding' and the 'exact' eigenvalue equations.

Introduction: The normal WKB approximation[1] appears to be a popular method for investigating the modal behaviour of optical fibres, due both to the relative simplicity of its eigenvalue equation and to its applicability to most refractive-index profiles of interest. However, the assumptions behind the method (e.g. a slowly varying phase function) cannot always be satisfied, particularly for discontinuous refractive-index profiles such as those found in typical step-index fibres.

The step-index profile is also convenient for comparison purposes, because it is both one of the worst-case tests for the WKB method, and its fields can be exactly described.[2] Moreover, the comparison can be enlarged to include the 'weakly guiding' approximation[3] and an interesting modification suggested by Hartog and Adams.[4]

The propagation constants of step-index fibres have been extensively investigated using the WKB and weakly guiding approximations. However, computed pulse responses using different approaches have not, to our knowledge, been published, even though their shapes are both more useful from the viewpoint of optical communications, and are a more revealing estimate of rather small propagation constant variations.

In this letter, we present the results of pulse response computations based on the eigenvalue equations of the above-mentioned methods for fibre V parameters between 2 and 39. We show that for core/cladding refractive-index differences of less than about 0·5%, the exact and weakly guiding temporal responses begin to agree rather well, but that the WKB solutions are noticeably different in all cases. In contrast, with the use of the phase factors proposed by Hartog and Adams,[4] the WKB responses become surprisingly good.

Computational procedure: The computations are based on the calculation of the axial propagation constants $\beta_{\mu\nu}$, followed by an evaluation of the modal delays $\tau_{\mu\nu}$. The pulse response has then been obtained by incoherent summation over all propagation modes, under the assumption of uniform excitation. The pulse shape of each mode is also assumed to be the convolution of the input pulse with the impulse response due to material dispersion.

The 'exact' solution has been obtained by numerical evaluation of the corresponding eigenvalue equation.[2] The weakly guiding eigenvalue[2,3] equation has also been numerically eval-