Discrete Mathematics
Week 8

Abeyah Calpatura

8.3

Exercises

Abeyah Calpatura
#3,7, 15ab, 16a, 17,36, 37, 38

#3 Solution:

$$A = \{0, 1, 2, 3, 4\}$$
$$R = \{(0,0), (0,4), (1,1), (1,3), (2,2), (3,1), (3,3), (4,0), (4,4)\}$$

equivalence classes: [0], [1], [2], [3]

$[0] = \{x \in A \mid x\ R\ 0\} = \{0, 4\}$
$[1] = \{x \in A \mid x\ R\ 1\} = \{1, 3\}$
$[2] = \{x \in A \mid x\ R\ 2\} = \{2\}$
$[3] = \{x \in A \mid x\ R\ 3\} = \{1, 3\}$
$[4] = \{x \in A \mid x\ R\ 4\} = \{0, 4\}$

The distinct equivalence classes of the relation R are $\{0, 4\}, \{2\}, \{1, 3\}$

#7 Solution: $A = \{(1,3), (2,4), (-4,-8), (3,9), (1,5), (3,6)\}$
R is defined on A as follows: For every (a, b), (c, d) $\in A$,

$$(a, b)\ R\ (c, d) \iff ad = bc$$

Find the distinct equivalence classes of the relation R.

$[(1,3)] = \{(a,b) \in A : (a,b)R(1,3)\}$
$= \{(a,b) \in A : 3a = b\}$
$= \{(1,3), (3,9)\}$

$[(2,4)] = \{(a,b) \in A : (a,b)R(2,4)\}$
$= \{(a,b) \in A : 4a = 2b\}$
$= \{(2,4), (-4,-8), (3,6)\}$

$[(1,5)] = \{(a,b) \in A : (a,b)R(1,5)\}$
$= \{(a,b) \in A : 5a = b\}$
$= \{(1,5)\}$

#15a ***Solution:*** $17 \equiv 2 \pmod 5$

$$17 - 2 = 15 = 3 \cdot 5$$

True

#15b ***Solution:*** $4 \equiv -5 \pmod 7$
s

$$4 - (-5) = 4 + 5 = 9$$

False

#16a ***Solution:*** Let R be the relation of congruence modulo 3. Which of the following equivalence classes are equal?

$$[7], [-4], [-6], [17], [4], [27], [19]$$

$$R = \text{Relation of congruence modulo 3}$$

Equivalence class	a - 7	3 divides a - 7	Equal to equivalence class [7]
[-4]	$-4 - 7 = -11$	No	No
[-6]	$-6 - 7 = -13$	No	No
[17]	$17 - 7 = 10$	No	No
[4]	$4 - 7 = -3$	Yes	Yes
[27]	$27 - 7 = 20$	No	No
[19]	$19 - 7 = 12$	Yes	Yes

Implies: $[7] = [4] = [19]$

Equivalence class	$a - (-4)$	3 divides $a - (-4)$	Equal to equivalence class [-4]
[-6]	$-6 - (-4) = -2$	No	No
[17]	$17 - (-4) = 21$	Yes	Yes
[27]	$27 - (-4) = 31$	No	No

Implies: $[-4] = [17]$

Equivalence class	$a - (-6)$	3 divides $a - (-6)$	Equal to equivalence class [-6]
[27]	$27 - (-6) = -33$	Yes	Yes

Implies: $[-6] = [27]$

s**#17a** ***Solution:*** Prove that all integers m and n, $m \equiv n \pmod 3$ if, and only if *m mod 3 = n mod 3*

Let m and n be integers

Let m mod 3 = n mod 3

Then, m = 3a + b and n = 3c + b, where a, b, c are integers and $0 \leqslant b < 3$

Then, m - n = 3a + b - 3c - b

m - n = 3(a - c)

3 divides m - n

$m \equiv n \pmod 3$

#17b ***Solution:*** Prove that for all integers m and n and any positive integer d, $m \equiv n \pmod{d}$ if, and only if, $m\ mod\ d = n\ mod\ d$

Let m and n be integers
Let m mod d = n mod d
Then, m = da + b and n = dc + b, where a, b, c are integers and 0 ⩽ b < d
Then, m - n = da + b - dc - b
m - n = d(a - c)
d divides m - n
m ≡ n $\pmod{d}$

#36 ***Solution:*** For every a in A, $a \in A$

Let $a \in A$
Since R is an equivalence relation, R is reflexive, symmetric, and transitive
By definition of reflexive: (a, a) $\in R$ or equivalently a R a
a R a is true and since $a \in A$, we note that $a \in [a]$

#37 ***Solution:*** For every a and b in A, if $b \in [a]$ then $a\ R\ b$.

Let a and b be in A
Let $b \in [a]$
By definition of equivalence class: $b \in [a]$ if and only if $b \in A$ and $a\ R\ b$
Since $b \in [a]$, $b \in A$ and $a\ R\ b$
Therefore, if $b \in [a]$ then $a\ R\ b$ by using that R is symmetric and definition of the equivalence class

#38 ***Solution:*** For every a, b, and c in A, if $b\ R\ c$ and $c \in [a]$ then $b \in [a]$.

Let a, b, and c be in A
Let $b\ R\ c$ and $c \in [a]$
By definition of equivalence class: $c \in [a]$ if and only if $c \in A$ and $a\ R\ c$
Since $c \in [a]$, $c \in A$ and $a\ R\ c$
Since $b\ R\ c$, $b\ R\ c$ and $a\ R\ c$
Prove using that R is transitive and the definitino of equivalence class

8.4

Exercises

Abeyah Calpatura
#1, 3, 7, 14, 15,19, 22, 26, 31, 36, 39

#1a ***Solution:*** WHERE SHALL WE MEET

$$\begin{aligned}&23\ 08\ 05\ 18\ 05\ 19\ 08\ 01\ 12\ 12\ 23\ 05\ 13\ 05\ 05\ 20\\&C = (M+3)\\&26\ 11\ 08\ 21\ 08\ 22\ 11\ 04\ 15\ 15\ 26\ 08\ 16\ 08\ 08\ 23\\&\text{ZKHUH VKDOO ZH PHHW}\end{aligned}$$

#1b ***Solution:*** LQ WKH FDIHWHULD

$$\begin{aligned}&12\ 17\ 23\ 11\ 08\ 06\ 04\ 09\ 08\ 23\ 08\ 21\ 12\ 04\\&C = (M-3)\\&09\ 14\ 20\ 20\ 08\ 05\ 03\ 01\ 06\ 05\ 20\ 05\ 18\ 09\ 01\\&\text{IN THE CAFETERIA}\end{aligned}$$

#3 Let a = 25, b = 19, and n = 3
#3a ***Solution:*** Verify that $3 \mid (25-19)$

$$25 - 19 = 6 = 3 \cdot 2$$

#3b ***Solution:*** Explain why $25 \equiv 19 \pmod 3$

Through part a, we determined that 3 divides $25 - 19$

#3c ***Solution:*** What value of k has the proprety that $25 = 19 + 3k$?

$$\begin{aligned}&25 = 19 + 3k\\&6 = 3k\\&k = 2\end{aligned}$$

#3d ***Solution:*** What is the (nonnegative) remainder when 25 is divided by 3? When 19 is divided by 3?

$$\begin{aligned}&25 \div 3 = 8 \text{ remainder } 1\\&19 \div 3 = 6 \text{ remainder } 1\end{aligned}$$

#3e ***Solution:*** Explain why 25 mod 3 = 19 mod 3

The remainder when 25 is divided by 3 is 1
The remainder when 19 is divided by 3 is 1
Both remainders are 1

#7a ***Solution:*** $128 \equiv 2 \pmod 7$ and $61 \equiv 5 \pmod 7$

$$\begin{aligned}&7 \mid (128-2)\\&128 - 2 = 126 = 7 \cdot 18\\&7 \mid (61-5)\\&61 - 5 = 56 = 7 \cdot 8\end{aligned}$$

#7b *Solution:* $(128 + 61) \equiv (2 + 5) \pmod 7$

$$\begin{aligned}
&128 + 61 = 189\\
&2 + 5 = 7\\
&7 \mid ((128 + 61) - (2 + 5))\\
&7 \mid (189 - 7)\\
&7 \mid 182\\
&182 = 7 \cdot 26
\end{aligned}$$

#7c *Solution:* $(128 - 61) \equiv (2 - 5) \pmod 7$

$$\begin{aligned}
&128 - 61 = 67\\
&2 - 5 = -3\\
&7 \mid ((128 - 61) - (2 - 5))\\
&7 \mid (67 + 3)\\
&7 \mid 70\\
&70 = 7 \cdot 10
\end{aligned}$$

#7d *Solution:* $(128 \cdot 61) \equiv (2 \cdot 5) \pmod 7$

$$\begin{aligned}
&128 \cdot 61 = 7808\\
&2 \cdot 5 = 10\\
&7 \mid ((128 \cdot 61) - (2 \cdot 5))\\
&7 \mid (7808 - 10)\\
&7 \mid 7798\\
&7798 = 7 \cdot 1114
\end{aligned}$$

#7e *Solution:* $128^2 \equiv 2^2 \pmod 7$

$$\begin{aligned}
&128^2 = 16384\\
&2^2 = 4\\
&7 \mid (16384 - 4)\\
&7 \mid 16380\\
&16380 = 7 \cdot 2340
\end{aligned}$$

#14 *Solution:*
#15 *Solution:*
#19 *Solution:*
#22 *Solution:*
#26 *Solution:*
#31 *Solution:*
#36 *Solution:*
#39 *Solution:*