Discrete Mathematics
Week 5

Abeyah Calpatura

4.1

Exercises

Abeyah Calpatura
#7, 13, 24, 28

#7 ***Solution:*** There exist real numbers a and b such that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$.

$$a = 16 \text{ and } b = 0$$
$$\sqrt{16+0} = \sqrt{16} + \sqrt{0}$$
$$4 = 4$$

#13 ***Solution:*** For every integer n, if n is odd, then $\frac{n-1}{2}$ is odd.

Negation: There exists an integer n such that n is odd and $\dfrac{n-1}{2}$ is even.
Counterexample: $n = 1$
$\dfrac{1-1}{2} = 0$ which is even.
Conclusion: The statement is false.

#24 ***Solution:*** For every real number x, if $x > 1$, then $x^2 > x$.

Quantification Implicit If x is a real number and $x > 1$, then $x^2 > x$.
First sentence of proof: "Suppose x is a real number greater than 1."
Last sentence of proof: "$x^2 > x$"

#28 ***Solution:***

a. $\forall$ integers m and n, if m and n are odd, then $m + n$ is even,
as $\forall$ odd integers m and n, $m + n$ is even,
and as If m and n are any odd integers, then $m + n$ is even.

b. By **definition of odd,** $m = 2r + 1$ and $n = 2s + 1$ for some integers r and s.
Then
$m + n = (2r + 1) + (2s + 1)$ by **substitution**
$= 2r + 2s + 2$
$= 2(r + s + 1)$ by algebra

c. Let $u = r + s + 1$. Then u is an integer because r, s, and 1 are integers and because **the sum of integers is also an integer**

d. Hence $m + n = 2u$, where u is an integer, and so, by **definition of even**, $m + n$ is even.

4.2

Exercises

Abeyah Calpatura
#2, 5, 17, 20

#2 ***Solution:*** For every integer m, if m is even then $3m + 5$ is odd.

By definition of even, $m = 2k$ for some integer k.
$3m + 5 = 3(2k) + 5$ by substitution
$= 6k + 5$ by algebra
$= 2(3k + 2) + 1$ by algebra
Let $t = 3k + 2$. By subsitution,
$3m + 5 = 2t + 1$

#5 ***Solution:*** If a and b are any odd integers, then $a^2 + b^2$ is even.

Let $a = 2r + 1$ and $b = 2s + 1$
$a^2 + b^2 = (2r + 1)^2 + (2s + 1)^2$
$= 4r^2 + 1 + 4r + 4s^2 + 4s + 1$
$= 2(r^2 + 2r^2 + 2r + 2s + 1)$
Let $t = 2r^2 + 2s^2 + 2r + 2s + 1$
Then $a^2 + b^2 = 2t$

#17 ***Solution:*** This proof assumes what is to be proved.

#20 ***Solution:*** The product of any two odd integers is odd.

$m = 2p + 1$ and $n = 2q + 1$
$mn = (2p + 1)(2q + 1)$
$= 4pq + 2p + 2q + 1$
$= 2(2pq + p + q) + 1$
Let $t = 2pq + p + q$
Then $mn = 2t + 1$
True

4.3

Exercises

Abeyah Calpatura
#5, 15, 21, 36, 38

#5 ***Solution:*** 0.565656565656...

$$
\begin{aligned}
&x = 0.565656565656...\\
&100x = 100(0.56565656...) = 56.56565656\\
&100x - x = 56.56565656... - 0.56565656...\\
&99x = 56\\
&x = \frac{56}{99}
\end{aligned}
$$

#15 ***Solution:*** The product of any two rational numbers ia rational number.

$$
\begin{aligned}
&r = \frac{a}{b}\\
&s = \frac{c}{d}\\
&rs = \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}\\
&\textbf{True } rs = \frac{ac}{bd} \text{ is a rational number since ac and bd are integers.}
\end{aligned}
$$

#21 ***Solution:*** True or false? If m is any even integer and n is any odd integer, then $m^2 + 3n$ is odd. Explain.

$$
\begin{aligned}
&\text{Let } m = 2p \text{ and } n = 2q + 1\\
&m^2 + 3n = (2p)^2 + 3(2q + 1)\\
&= 4p^2 + 6q + 3\\
&= 2(2p^2 + 3q + 1) + 1\\
&\text{Let } t = 2p^2 + 3q + 1\\
&m^2 + 3n = 2t + 1\\
&\textbf{True}
\end{aligned}
$$

#36 ***Solution:***

$$
\begin{aligned}
&\text{Any two rational numbers have a sum that is rational.}\\
&\text{Let } r = \frac{a}{b} \text{ and } s = \frac{c}{d}\\
&r + s = \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}\\
&\textbf{True} \text{ since } ad + bc \text{ and } bd \text{ are integers.}
\end{aligned}
$$

#38 ***Solution:***

"The sum of two fractions is a fraction" has never been proven.
"A rational number is a fraction" is not necessarily true.
The statement is false.

4.4

Exercises

Abeyah Calpatura
#17, 21, 29
#17 ***Solution:*** For all integers a, b, c, and d, if $a|b$ and $c|d$, then $ac|bd$.

By definition of divides, $b = am$ and $d = cn$ for some integers m and n.
$bd = (am)(cn) = ac(mn)$
Let $t = mn$. Then $bd = act$
True

#21 ***Solution:*** The product of any two even integers is a multiple of 4.

Let $m = 2p$ and $n = 2q$
$mn = (2p)(2q) = 4pq$
True since $4pq$ is a multiple of 4.

#29 ***Solution:*** For all integersa and b, if $a|b$ then $a^2|b^2$.

By definition of divides, $b = am$ for some integer m.
$b^2 = (am)^2 = a^2m^2$
Let $t = m^2$. Then $b^2 = a^2t$
True