

# Discrete Mathematics

## Week 5

Abeyah Calpatura

## 4.1

### Exercises

Abeyah Calpatura

#7, 13, 24, 28

**#7 Solution:** There exist real numbers  $a$  and  $b$  such that  $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ .

$$a = 16 \text{ and } b = 0$$

$$\sqrt{16+0} = \sqrt{16} + \sqrt{0}$$

$$4 = 4$$

**#13 Solution:** For every integer  $n$ , if  $n$  is odd, then  $\frac{n-1}{2}$  is odd.

**Negation:** There exists an integer  $n$  such that  $n$  is odd and  $\frac{n-1}{2}$  is even.

**Counterexample:**  $n = 1$

$$\frac{1-1}{2} = 0 \text{ which is even.}$$

**Conclusion:** The statement is false.

**#24 Solution:** For every real number  $x$ , if  $x > 1$ , then  $x^2 > x$ .

**Quantification Implicit** If  $x$  is a real number and  $x > 1$ , then  $x^2 > x$ .

**First sentence of proof:** "Suppose  $x$  is a real number greater than 1."

**Last sentence of proof:** " $x^2 > x$ "

**#28 Solution:**

- a.  $\forall$  integers  $m$  and  $n$ , if  $m$  and  $n$  are odd, then  $m+n$  is even,  
as  $\forall$  odd integers  $m$  and  $n$ ,  $m+n$  is even,  
and as If  $m$  and  $n$  are any odd integers, then  $m+n$  is even.

- b. By **definition of odd**,  $m = 2r + 1$  and  $n = 2s + 1$  for some integers  $r$  and  $s$ .

Then

$$m+n = (2r+1) + (2s+1) \text{ by } \mathbf{substitution}$$

$$= 2r + 2s + 2$$

$$= 2(r+s+1) \text{ by algebra}$$

Let  $u = r+s+1$ . Then  $u$  is an integer because  $r$ ,  $s$ , and 1 are integers and because **definition of**

## 4.2

### Exercises

Abeyah Calpatura

#2, 5, 17, 20

**#2 Solution:** For every integer  $m$ , if  $m$  is even then  $3m + 5$  is odd.

By definition of even,  $m = 2k$  for some integer  $k$ .

$3m + 5 = 3(2k) + 5$  by substitution

$= 6k + 5$  by algebra

$= 2(3k + 2) + 1$  by algebra

Let  $t = 3k + 2$ . By substitution,

$3m + 5 = 2t + 1$

**#5 Solution:** If  $a$  and  $b$  are any odd integers, then  $a^2 + b^2$  is even.

Let  $a = 2r + 1$  and  $b = 2s + 1$

$a^2 + b^2 = (2r + 1)^2 + (2s + 1)^2$

$= 4r^2 + 1 + 4r + 4s^2 + 4s + 1$

$= 2(r^2 + 2r^2 + 2r + 2s + 1)$

Let  $t = 2r^2 + 2s^2 + 2r + 2s + 1$

Then  $a^2 + b^2 = 2t$

**#17 Solution:** This proof assumes what is to be proved.

**#20 Solution:** The product of any two odd integers is odd.

$m = 2p + 1$  and  $n = 2q + 1$

$mn = (2p + 1)(2q + 1)$

$= 4pq + 2p + 2q + 1$

$= 2(2pq + p + q) + 1$

Let  $t = 2pq + p + q$

Then  $mn = 2t + 1$

**True**

## 4.3

### Exercises

Abeyah Calpatura

#5, 15, 21, 36, 38

**#5 Solution:**  $0.565656565656\dots$

$$x = 0.565656565656\dots$$

$$100x = 100(0.56565656\dots) = 56.56565656$$

$$100x - x = 56.56565656\dots - 0.56565656\dots$$

$$99x = 56$$

$$x = \frac{56}{99}$$

**#15 Solution:** The product of any two rational numbers is a rational number.

**#21 Solution:**

**#36 Solution:**

**#38 Solution:**