

# Discrete Mathematics

## Week 5

Abeyah Calpatura

## 4.1

### Exercises

Abeyah Calpatura

#7, 13, 24, 28

**#7 Solution:** There exist real numbers  $a$  and  $b$  such that  $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ .

$$a = 16 \text{ and } b = 0$$

$$\sqrt{16+0} = \sqrt{16} + \sqrt{0}$$

$$4 = 4$$

**#13 Solution:** For every integer  $n$ , if  $n$  is odd, then  $\frac{n-1}{2}$  is odd.

**Negation:** There exists an integer  $n$  such that  $n$  is odd and  $\frac{n-1}{2}$  is even.

**Counterexample:**  $n = 1$

$$\frac{1-1}{2} = 0 \text{ which is even.}$$

**Conclusion:** The statement is false.

**#24 Solution:** For every real number  $x$ , if  $x > 1$ , then  $x^2 > x$ .

**Quantification Implicit** If  $x$  is a real number and  $x > 1$ , then  $x^2 > x$ .

**First sentence of proof:** "Suppose  $x$  is a real number greater than 1."

**Last sentence of proof:** " $x^2 > x$ "

**#28 Solution:**

- a.  $\forall$  integers  $m$  and  $n$ , if  $m$  and  $n$  are odd, then  $m+n$  is even,  
as  $\forall$  odd integers  $m$  and  $n$ ,  $m+n$  is even,  
and as If  $m$  and  $n$  are any odd integers, then  $m+n$  is even.

- b. By **definition of odd**,  $m = 2r + 1$  and  $n = 2s + 1$  for some integers  $r$  and  $s$ .

Then

$$m+n = (2r+1) + (2s+1) \text{ by } \mathbf{substitution}$$

$$= 2r + 2s + 2$$

$$= 2(r+s+1) \text{ by algebra}$$

Let  $u = r+s+1$ . Then  $u$  is an integer because  $r$ ,  $s$ , and 1 are integers and because **definition of**

## 4.2

### Exercises

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#2, 5, 17, 20

**#2 Solution:** question

*answer*

**#5 Solution:** question

*answer*

**#17 Solution:** question

*answer*

**#20 Solution:** question

*answer*