Discrete Mathematics
Week 5

Abeyah Calpatura

4.1

Exercises

Abeyah Calpatura
#7, 13, 24, 28

#7 ***Solution:*** There exist real numbers a and b such that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$.

$$a = 16 \text{ and } b = 0$$
$$\sqrt{16+0} = \sqrt{16} + \sqrt{0}$$
$$4 = 4$$

#13 ***Solution:*** For every integer n, if n is odd, then $\frac{n-1}{2}$ is odd.

Negation: There exists an integer n such that n is odd and $\dfrac{n-1}{2}$ is even.
Counterexample: $n = 1$
$\dfrac{1-1}{2} = 0$ which is even.
Conclusion: The statement is false.

#24 ***Solution:*** For every real number x, if $x > 1$, then $x^2 > x$.

Quantification Implicit If x is a real number and $x > 1$, then $x^2 > x$.
First sentence of proof: "Suppose x is a real number greater than 1."
Last sentence of proof: "$x^2 > x$"

#28 ***Solution:***

a. $\forall$ integers m and n, if m and n are odd, then $m + n$ is even,
as $\forall$ odd integers m and n, $m + n$ is even,
and as If m and n are any odd integers, then $m + n$ is even.

b. By **definition of odd,** $m = 2r + 1$ and $n = 2s + 1$ for some integers r and s.
Then
$m + n = (2r + 1) + (2s + 1)$ by **substitution**
$= 2r + 2s + 2$
$= 2(r + s + 1)$ by algebra
Let $u = r + s + 1$. Then u is an integer because r, s, and 1 are integers and because **definition of**

4.2

Exercises

Abeyah Calpatura
#2, 5, 17, 20

#2 ***Solution:*** For every integer m, if m is even then $3m + 5$ is odd.

By definition of even, $m = 2k$ for some integer k.
$3m + 5 = 3(2k) + 5$ by substitution
$= 6k + 5$ by algebra
$= 2(3k + 2) + 1$ by algebra
Let $t = 3k + 2$. By subsitution,
$3m + 5 = 2t + 1$

#5 ***Solution:*** If a and b are any odd integers, then $a^2 + b^2$ is even.

Let $a = 2r + 1$ and $b = 2s + 1$
$a^2 + b^2 = (2r + 1)^2 + (2s + 1)^2$
$= 4r^2 + 1 + 4r + 4s^2 + 4s + 1$
$= 2(r^2 + 2r^2 + 2r + 2s + 1)$
Let $t = 2r^2 + 2s^2 + 2r + 2s + 1$
Then $a^2 + b^2 = 2t$

#17 ***Solution:*** This proof assumes what is to be proved.

#20 ***Solution:*** The product of any two odd integers is odd.

$m = 2p + 1$ and $n = 2q + 1$
$mn = (2p + 1)(2q + 1)$
$= 4pq + 2p + 2q + 1$
$= 2(2pq + p + q) + 1$
Let $t = 2pq + p + q$
Then $mn = 2t + 1$
True

4.3

Exercises

Abeyah Calpatura
#5, 15, 21, 36, 38

#5 ***Solution:*** 0.565656565656...

$$x = 0.565656565656...$$
$$100x = 100(0.56565656...) = 56.56565656$$
$$100x - x = 56.56565656... - 0.56565656...$$
$$99x = 56$$
$$x = \frac{56}{99}$$

#15 ***Solution:*** The product of any two rational numbers ia rational number.

#21 ***Solution:***

#36 ***Solution:***

#38 ***Solution:***