Discrete Mathematics
Week 6

Abeyah Calpatura

4.5

Exercises

Abeyah Calpatura
#1, 2, 7, 17, 22

#1 *Solution:*

$n = 70$ and $d = 9$
$70 = 9q + r$
$q = 7$ and $r = 7$
Since $70 = 9(7) + 7$

#2 *Solution:*

$n = 62$ and $d = 7$
$62 = 7q + r$
$q = 8$ and $r = 6$
Since $62 = 7(8) + 6$

#7 *Solution:*

a. 43 div 9: 4
b. 43 mod 9: 7

#17 *Solution:* Prove directly from definitions that for every integer n, $n^2 - n + 3$ is odd. Use division into two cases: n is even and n is odd.

Case 1: n is even
$n = 2k$ for some integer k
$n^2 - n + 3 = (2k)^2 - 2k + 3$
$n^2 - n + 3 = 4k^2 - 2k + 3$
$n^2 - n + 3 = 2(2k^2 - k + 1) + 1$
$n^2 - n + 3 = 2q + 1$
where $q = 2k^2 - k + 1$
Case 2: n is odd
$n = 2k + 1$ for some integer k
$n^2 - n + 3 = (2k + 1)^2 - (2k + 1) + 3$
$n^2 - n + 3 = 4k^2 + 4k + 1 - 2k - 1 + 3$
$n^2 - n + 3 = 4k^2 + 2k + 3$
$n^2 - n + 3 = 2(2k^2 + k + 1) + 1$
$n^2 - n + 3 = 2q + 1$
where $q = 2k^2 + k + 1$
Therefore, $n^2 - n + 3$ is odd for every integer n

#22 Solution: Suppose c is any integer. If c mod $15 = 3$, what is $10c$ mod 15? In other words, if division of c by 15 gives a remainder of 3, what is the remainder when 10c is divided by 15? Your solution should show that you obatin the same answer no matter what integer you start with.

$$c \ mod \ 15 = 3$$
$$c = 15q + 3$$
$$10c = 10(15q + 3)$$
$$10c = 150q + 30$$
$$10c = 15(10q + 2)$$
$$10c \ mod \ 15 = 0$$

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Exercises

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#2, 4, 6, 7, 10a

#2 Solution:

$$\lceil 17/4 \rceil = \lceil 4.25 \rceil = 5$$
$$\lfloor 17/4 \rfloor = \lfloor 4.25 \rfloor = 4$$

#4 Solution:

$$\lceil -32/5 \rceil = \lceil -6.4 \rceil = -6$$
$$\lfloor -32/5 \rfloor = \lfloor -6.4 \rfloor = -7$$

#6 Solution: If k is an integer, what is $\lceil k \rceil$? Why?

By deifintion of ceiling, k is an integer and the ceiling of an integer is itself since:
$k - 1 < k \leqslant k$
Therefore, $\lceil k \rceil = k$

#7 Solution: If k is an integer, what is $\lceil k + \frac{1}{2} \rceil$? Why?

By definition of ceiling, k is an integer and the ceiling of an integer is itself since:
$k < k + \dfrac{1}{2} \leqslant k + 1$
Therefore, $\lceil k + \dfrac{1}{2} \rceil = k + 1$

#10a ***Solution:***

i. n = 2050

$$= \left(2050 + \left\lfloor \frac{2050-1}{4} \right\rfloor - \left\lfloor \frac{2050-1}{100} \right\rfloor - \left\lfloor \frac{2050-1}{400} \right\rfloor\right) \ mod\ 7$$
$$= (2050 + 512 - 20 + 5) \ mod\ 7$$
$$= (2547) \ mod\ 7$$
$$= 6$$

Corresponds to **Saturday**

ii. n = 2100

$$= \left(2100 + \left\lfloor \frac{2100-1}{4} \right\rfloor - \left\lfloor \frac{2100-1}{100} \right\rfloor - \left\lfloor \frac{2100-1}{400} \right\rfloor\right)$$
$$= (2100 + 524 - 20 + 5) \ mod\ 7$$
$$= (2609) \ mod\ 7$$
$$= 5$$

Corresponds to **Friday**

iii. n = 2004

$$= \left(2004 + \left\lfloor \frac{2004-1}{4} \right\rfloor - \left\lfloor \frac{2004-1}{100} \right\rfloor - \left\lfloor \frac{2004-1}{400} \right\rfloor\right)$$
$$= (2004 + 500 - 20 + 5) \ mod\ 7$$
$$= (2609) \ mod\ 7$$
$$= 4$$

Corresponds to **Thursday**

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Exercises

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#2, 4, 9b

#2 *Solution:* Is $\frac{1}{0}$ an irrational number? Explain.

$\dfrac{1}{0}$ is not an irrational number, because division by 0 is not defined.
Thus, the number of $\dfrac{1}{0}$ does not exist, which implies that the number is not an irrational number.

#4 *Solution:* Use proof by contradiction to show that for every integer m, $7m + 4$ is not divisible by 7.

By the definition of divisble, there exists an integer k such that:

$$7m + 4 = 7k$$
$$4 = 7k - 7m$$
$$4 = 7(k - m)$$

Since $k - m$ is an integer, 4 is divisible by 7.
This is a contradiction, since 4 is not divisible by 7.

#9b *Solution:* Prove that the difference of any irrational number and any rational number is irrational.

Let us assume that x is an irrational number and y is a rational number such that their difference $x - y$ is rational.
By the definition of rational, there exist integers a, b, c, and d with $b \neq 0$ and $d \neq 0$ such that

$$x = \frac{a}{b}$$
$$y = \frac{c}{d}$$
$$x - y = \frac{a}{b} - \frac{c}{d}$$
$$x - y = \frac{ad - bc}{bd}$$

Since $ad - bc$ and bd are integers, $x - y$ is rational.
This is a contradiction, since $x - y$ is irrational.

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Exercises

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#5, 7

#5 *Solution:*

The actual negation that should've been used is:

Suppose there exists an irrational number such that its cube root is rational.

The mistake is the negation that was used in the proof.

#7 *Solution:* $3\sqrt{2} - 7$ is rational.

Suppose that $3\sqrt{2} - 7$ is rational.

By definition of rational, there exist integers a and b with $b \neq 0$ such that $3\sqrt{2} - 7 = \frac{a}{b}$

$$3\sqrt{2} = \frac{a}{b} + 7$$
$$3\sqrt{2} = \frac{a + 7b}{b}$$
$$\sqrt{2} = \frac{a + 7b}{3b}$$

Since $\frac{a + 7b}{3b}$ is rational, this is a contradiction since $\sqrt{2}$ is irrational which implies that $3\sqrt{2} - 7$ is irrational.

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Exercises

Abeyah Calpatura
#2, 3, 7, 8, 14

#2 Solution:

$$
\begin{aligned}
&deg(v_1) = 1\\
&deg(v_2) = 5\\
&deg(v_3) = 4\\
&deg(v_4) = 4\\
&deg(v_5) = 1\\
&deg(v_6) = 3\\
&\text{Total degree} = 18\\
&9 = \frac{1}{2} \cdot 18
\end{aligned}
$$

#3 Solution:

$$
\begin{aligned}
&deg(v_1) = 0\\
&deg(v_2) = 2\\
&deg(v_3) = 2\\
&deg(v_4) = 3\\
&deg(v_5) = 9\\
&\text{Total Degree} = \sum_{i=1}^{5} deg(v_i)\\
&\text{Total Degree} = 0 + 2 + 2 + 3 + 9 = 16\\
&m = \frac{\text{Total Degree}}{2}\\
&m = \frac{16}{2} = 8\\
&\text{Graph has 8 edges}
\end{aligned}
$$

#7 Solution:

$deg(v_1) = 1$
$deg(v_1) = 1$
$deg(v_1) = 1$
$deg(v_1) = 4$

$$\text{Total Degree} = \sum_{i=1}^{4} deg(v_i)$$

Total Degree $= 1 + 1 + 1 + 4 = 7$
Does not exist since 7 is odd. Due to the Handshaking Theorem, the total degree must be even.

#8 *Solution:*

$deg(v_1) = 1$
$deg(v_1) = 2$
$deg(v_1) = 3$
$deg(v_1) = 4$

Total Degree $= \sum_{i=1}^{4} deg(v_i)$

Total Degree $= 1 + 2 + 3 + 4 = 10$

The total degree is even and since there are an even number of odd degrees, such a graph will exist.

#14 *Solution:*

a. total degree of the graph $= 2 \cdot 1 + 5 \cdot 2 + x \cdot 3$
$= 2 + 10 + 3x$
$= 12 + 3x$
Total degree of the graph $= 2 \cdot 15 = 30$. Since the graph has 15 edges
$12 + 3x = 30$
$3x = 18$
$x = 6$
The graph has 6 vertices of degree 3
6 people at the party knew three other people at the party.

b. the number of people at the party $= 2 + 5 + 6 = 13$

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Exercises

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#2, 5, 12, 15, 16

#2 ***Solution:***

$$z = 2$$

#5 ***Solution:***

$$\begin{aligned}
&\textbf{1st loop: } f = 2 * 1 = 2 \text{ and } e = 0 + \frac{1}{2} = \frac{1}{2}\\
&\textbf{2nd loop: } f = 2 * 2 = 4 \text{ and } e = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}\\
&\textbf{3rd loop: } f = 4 * 3 = 12 \text{ and } e = \frac{3}{4} + \frac{1}{12} = \frac{5}{6}\\
&\textbf{4th loop: } f = 12 * 4 = 48 \text{ and } e = \frac{5}{6} + \frac{1}{48} = \frac{41}{48}\\
&e = \frac{41}{48}
\end{aligned}$$

#12 ***Solution:*** Find the prime factorizations of 48 and 54.

$$\begin{aligned}
&48 = 2^4 \cdot 3 \; 54 = 2 \cdot 3^3\\
&\gcd(48{,}54) = 2 \cdot 6\\
&\gcd(48{,}54) = 6
\end{aligned}$$

#15 ***Solution:*** 10933 and 832

$$\begin{array}{r|l} & \;\;13 \\ 832 & 10933 \\ & \;\;832 \\ \hline & \;\;2613 \\ & \;\;2496 \\ \hline & \;\;\;117 \end{array}$$

gcd(10933, 832) = gcd(832, 117)

$$\begin{array}{r|l} & \;\;7 \\ 117 & 832 \\ & 819 \\ \hline & \;\;13 \end{array}$$

gcd(832, 117) = gcd(117, 13)

$$\begin{array}{r|l} & \;\;9 \\ 13 & 117 \\ & 117 \\ \hline & \;\;\;0 \end{array}$$

gcd(117, 13) = gcd(13, 0)

gcd(10933, 832) = 13

#16 *Solution:* 4131 and 2431

$$\begin{array}{rr} & 1 \\ 2431 & \overline{)4131} \\ & 2431 \\ & \overline{1700} \end{array}$$

gcd(4131, 2431) = gcd(2431, 1700)

$$\begin{array}{rr} & 1 \\ 1700 & \overline{)2431} \\ & 1700 \\ & \overline{731} \end{array}$$

gcd(2431, 1700) = gcd(1700, 731)

$$\begin{array}{rr} & 2 \\ 731 & \overline{)1700} \\ & 1462 \\ & \overline{238} \end{array}$$

gcd(1700, 731) = gcd(731, 238)

$$\begin{array}{rr} & 3 \\ 238 & \overline{)731} \\ & 714 \\ & \overline{17} \end{array}$$

gcd(731, 238) = gcd(238, 17)

$$\begin{array}{rl} & \ \ 14 \\ 17 & \overline{)238} \\ & \underline{17} \\ & \ \ 68 \\ & \ \ \underline{68} \\ & \ \ \ \ 0 \end{array}$$

gcd(238,17) = gcd(17, 0)

gcd(4131, 2431) = 17