Discrete Mathematics
Week 6

Abeyah Calpatura

4.5

Exercises

Abeyah Calpatura
#1, 2, 7, 17, 22

#1 ***Solution:***

$n = 70$ and $d = 9$
$70 = 9q + r$
$q = 7$ and $r = 7$
Since $70 = 9(7) + 7$

#2 ***Solution:***

$n = 62$ and $d = 7$
$62 = 7q + r$
$q = 8$ and $r = 6$
Since $62 = 7(8) + 6$

#7 ***Solution:***

a. 43 div 9: 4
b. 43 mod 9: 7

#17 ***Solution:*** Prove directly from definitions that for every integer n, $n^2 - n + 3$ is odd. Use division into two cases: n is even and n is odd.

Case 1: n is even
$n = 2k$ for some integer k
$n^2 - n + 3 = (2k)^2 - 2k + 3$
$n^2 - n + 3 = 4k^2 - 2k + 3$
$n^2 - n + 3 = 2(2k^2 - k + 1) + 1$
$n^2 - n + 3 = 2q + 1$
where $q = 2k^2 - k + 1$
Case 2: n is odd
$n = 2k + 1$ for some integer k
$n^2 - n + 3 = (2k + 1)^2 - (2k + 1) + 3$
$n^2 - n + 3 = 4k^2 + 4k + 1 - 2k - 1 + 3$
$n^2 - n + 3 = 4k^2 + 2k + 3$
$n^2 - n + 3 = 2(2k^2 + k + 1) + 1$
$n^2 - n + 3 = 2q + 1$
where $q = 2k^2 + k + 1$
Therefore, $n^2 - n + 3$ is odd for every integer n

#22 Solution: Suppose c is any integer. If c *mod* $15 = 3$, what is $10c$ *mod* 15? In other words, if division of c by 15 gives a remainder of 3, what is the remainder when 10c is divided by 15? Your solution should show that you obatin the same answer no matter what integer you start with.

$$
\begin{aligned}
&c \ \mathit{mod}\ 15 = 3 \\
&c = 15q + 3 \\
&10c = 10(15q + 3) \\
&10c = 150q + 30 \\
&10c = 15(10q + 2) \\
&10c \ \mathit{mod}\ 15 = 0
\end{aligned}
$$

4.6

Exercises

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#2, 4, 6, 7, 10a