Discrete Mathematics
Week 6

Abeyah Calpatura

4.5

Exercises

Abeyah Calpatura
#1, 2, 7, 17, 22

#1 ***Solution:***

$$n = 70 \text{ and } d = 9$$
$$70 = 9q + r$$
$$q = 7 \text{ and } r = 7$$
$$\text{Since } 70 = 9(7) + 7$$

#2 ***Solution:***

$$n = 62 \text{ and } d = 7$$
$$62 = 7q + r$$
$$q = 8 \text{ and } r = 6$$
$$\text{Since } 62 = 7(8) + 6$$

#7 ***Solution:***

a. 43 div 9: 4
b. 43 mod 9: 7

#17 ***Solution:*** Prove directly from definitions that for every integer n, $n^2 - n + 3$ is odd. Use division into two cases: n is even and n is odd.

Case 1: n is even
$n = 2k$ for some integer k
$n^2 - n + 3 = (2k)^2 - 2k + 3$
$n^2 - n + 3 = 4k^2 - 2k + 3$
$n^2 - n + 3 = 2(2k^2 - k + 1) + 1$
$n^2 - n + 3 = 2q + 1$
where $q = 2k^2 - k + 1$
Case 2: n is odd
$n = 2k + 1$ for some integer k
$n^2 - n + 3 = (2k + 1)^2 - (2k + 1) + 3$
$n^2 - n + 3 = 4k^2 + 4k + 1 - 2k - 1 + 3$
$n^2 - n + 3 = 4k^2 + 2k + 3$
$n^2 - n + 3 = 2(2k^2 + k + 1) + 1$
$n^2 - n + 3 = 2q + 1$
where $q = 2k^2 + k + 1$
Therefore, $n^2 - n + 3$ is odd for every integer n

#22 *Solution:* Suppose c is any integer. If c *mod* $15 = 3$, what is $10c$ *mod* 15? In other words, if division of c by 15 gives a remainder of 3, what is the remainder when 10c is divided by 15? Your solution should show that you obatin the same answer no matter what integer you start with.

$$
\begin{aligned}
&c \ \mathit{mod}\ 15 = 3 \\
&c = 15q + 3 \\
&10c = 10(15q + 3) \\
&10c = 150q + 30 \\
&10c = 15(10q + 2) \\
&10c \ \mathit{mod}\ 15 = 0
\end{aligned}
$$

4.6

Exercises

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#2, 4, 6, 7, 10a

#2 Solution:

$$\lceil 17/4 \rceil = \lceil 4.25 \rceil = 5$$
$$\lfloor 17/4 \rfloor = \lfloor 4.25 \rfloor = 4$$

#4 Solution:

$$\lceil -32/5 \rceil = \lceil -6.4 \rceil = -6$$
$$\lfloor -32/5 \rfloor = \lfloor -6.4 \rfloor = -7$$

#6 Solution: If k is an integer, what is $\lceil k \rceil$? Why?

By deifintion of ceiling, k is an integer and the ceiling of an integer is itself since:
$k - 1 < k \leqslant k$
Therefore, $\lceil k \rceil = k$

#7 Solution: If k is an integer, what is $\lceil k + \frac{1}{2} \rceil$? Why?

By definition of ceiling, k is an integer and the ceiling of an integer is itself since:
$k < k + \dfrac{1}{2} \leqslant k + 1$
Therefore, $\lceil k + \dfrac{1}{2} \rceil = k + 1$

#10a ***Solution:***

i. n = 2050

$$= \left(2050 + \left\lfloor \frac{2050-1}{4} \right\rfloor - \left\lfloor \frac{2050-1}{100} \right\rfloor - \left\lfloor \frac{2050-1}{400} \right\rfloor\right) \ mod\ 7$$

$$= (2050 + 512 - 20 + 5) \ mod\ 7$$

$$= (2547) \ mod\ 7$$

$$= 6$$

Corresponds to **Saturday**

ii. n = 2100

$$= \left(2100 + \left\lfloor \frac{2100-1}{4} \right\rfloor - \left\lfloor \frac{2100-1}{100} \right\rfloor - \left\lfloor \frac{2100-1}{400} \right\rfloor\right)$$

$$= (2100 + 524 - 20 + 5) \ mod\ 7$$

$$= (2609) \ mod\ 7$$

$$= 5$$

Corresponds to **Friday**

iii. n = 2004

$$= \left(2004 + \left\lfloor \frac{2004-1}{4} \right\rfloor - \left\lfloor \frac{2004-1}{100} \right\rfloor - \left\lfloor \frac{2004-1}{400} \right\rfloor\right)$$

$$= (2004 + 500 - 20 + 5) \ mod\ 7$$

$$= (2609) \ mod\ 7$$

$$= 4$$

Corresponds to **Thursday**

4.7

Exercises

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#2, 4, 9b

#2 Solution: Is $\frac{1}{0}$ an irrational number? Explain.

$\dfrac{1}{0}$ is not an irrational number, because division by 0 is not defined.
Thus, the number of $\dfrac{1}{0}$ does not exist, which implies that the number is not an irrational number.

#4 Solution: Use proof by contradiction to show that for every integer m, $7m + 4$ is not divisible by 7.

By the definition of divisble, there exists an integer k such that:

$$7m + 4 = 7k$$
$$4 = 7k - 7m$$
$$4 = 7(k - m)$$

Since $k - m$ is an integer, 4 is divisible by 7.
This is a contradiction, since 4 is not divisible by 7.

#9b Solution: Prove that the difference of any irrational number and any rational number is irrational.

Let us assume that x is an irrational number and y is a rational number such that their difference $x - y$ is rational. By the definition of rational, there exist integers a, b, c, and d with $b \neq 0$ and $d \neq 0$ such that

$$x = \frac{a}{b}$$
$$y = \frac{c}{d}$$
$$x - y = \frac{a}{b} - \frac{c}{d}$$
$$x - y = \frac{ad - bc}{bd}$$

Since $ad - bc$ and bd are integers, $x - y$ is rational.
This is a contradiction, since $x - y$ is irrational.

4.8

Exercises

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#5, 7

#5 ***Solution:***

The actual negation that should've been used is:

Suppose there exists an irrational number such that its cube root is rational.

The mistake is the negation that was used in the proof.

#7 ***Solution:*** $3\sqrt{2} - 7$ is rational.

Suppose that $3\sqrt{2} - 7$ is rational.

By definition of rational, there exist integers a and b with $b \neq 0$ such that $3\sqrt{2} - 7 = \frac{a}{b}$

$$3\sqrt{2} = \frac{a}{b} + 7$$
$$3\sqrt{2} = \frac{a + 7b}{b}$$
$$\sqrt{2} = \frac{a + 7b}{3b}$$

Since $\frac{a + 7b}{3b}$ is rational, this is a contradiction since $\sqrt{2}$ is irrational which implies that $3\sqrt{2} - 7$ is irrational.

4.9

Exercises

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#2, 3, 7, 8, 14

#2 Solution:

$$
\begin{aligned}
&deg(v_1) = 1\\
&deg(v_2) = 5\\
&deg(v_3) = 4\\
&deg(v_4) = 4\\
&deg(v_5) = 1\\
&deg(v_6) = 3\\
&\text{Total degree} = 18\\
&9 = \frac{1}{2} \cdot 18
\end{aligned}
$$

#3 Solution:

$$
\begin{aligned}
&deg(v_1) = 0\\
&deg(v_2) = 2\\
&deg(v_3) = 2\\
&deg(v_4) = 3\\
&deg(v_5) = 9\\
&\text{Total Degree} = \sum_{i=1}^{5} deg(v_i)\\
&\text{Total Degree} = 0 + 2 + 2 + 3 + 9 = 16\\
&m = \frac{\text{Total Degree}}{2}\\
&m = \frac{16}{2} = 8\\
&\text{Graph has 8 edges}
\end{aligned}
$$

#7 Solution:

$deg(v_1) = 1$
$deg(v_1) = 1$
$deg(v_1) = 1$
$deg(v_1) = 4$

$$\text{Total Degree} = \sum_{i=1}^{4} deg(v_i)$$

Total Degree $= 1 + 1 + 1 + 4 = 7$

Does not exist since 7 is odd. Due to the Handshaking Theorem, the total degree must be even.

#8 *Solution:*

$deg(v_1) = 1$
$deg(v_1) = 2$
$deg(v_1) = 3$
$deg(v_1) = 4$

Total Degree $= \sum_{i=1}^{4} deg(v_i)$

Total Degree $= 1 + 2 + 3 + 4 = 10$

The total degree is even and since there are an even number of odd degrees, such a graph will exist.

#14 *Solution:*

a. total degree of the graph $= 2 \cdot 1 + 5 \cdot 2 + x \cdot 3$
$= 2 + 10 + 3x$
$= 12 + 3x$
Total degree of the graph $= 2 \cdot 15 = 30$. Since the graph has 15 edges
$12 + 3x = 30$
$3x = 18$
$x = 6$
The graph has 6 vertices of degree 3
6 people at the party knew three other people at the party.

b. the number of people at the party $= 2 + 5 + 6 = 13$

4.10

Exercises

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#2, 5, 12, 15, 16

#2 Solution:

$$z = 2$$

#5 Solution:

#12 Solution:

#15 Solution:

#16 Solution: