Discrete Mathematics
Week 6

Abeyah Calpatura

4.5

Exercises

Abeyah Calpatura
#1, 2, 7, 17, 22

#1 *Solution:*

$$n = 70 \text{ and } d = 9$$
$$70 = 9q + r$$
$$q = 7 \text{ and } r = 7$$
$$\text{Since } 70 = 9(7) + 7$$

#2 *Solution:*

$$n = 62 \text{ and } d = 7$$
$$62 = 7q + r$$
$$q = 8 \text{ and } r = 6$$
$$\text{Since } 62 = 7(8) + 6$$

#7 *Solution:*

a. 43 div 9: 4

b. 43 mod 9: 7

#17 *Solution:* Prove directly from definitions that for every integer n, $n^2 - n + 3$ is odd. Use division into two cases: n is even and n is odd.

Case 1: n is even

$n = 2k$ for some integer k

$n^2 - n + 3 = (2k)^2 - 2k + 3$

$n^2 - n + 3 = 4k^2 - 2k + 3$

$n^2 - n + 3 = 2(2k^2 - k + 1) + 1$

$n^2 - n + 3 = 2q + 1$

where $q = 2k^2 - k + 1$

Case 2: n is odd

$n = 2k + 1$ for some integer k

$n^2 - n + 3 = (2k + 1)^2 - (2k + 1) + 3$

$n^2 - n + 3 = 4k^2 + 4k + 1 - 2k - 1 + 3$

$n^2 - n + 3 = 4k^2 + 2k + 3$

$n^2 - n + 3 = 2(2k^2 + k + 1) + 1$

$n^2 - n + 3 = 2q + 1$

where $q = 2k^2 + k + 1$

Therefore, $n^2 - n + 3$ is odd for every integer n

#22 Solution: Suppose c is any integer. If $c \bmod 15 = 3$, what is $10c \bmod 15$? In other words, if division of c by 15 gives a remainder of 3, what is the remainder when 10c is divided by 15? Your solution should show that you obatin the same answer no matter what integer you start with.

$$c \bmod 15 = 3$$
$$c = 15q + 3$$
$$10c = 10(15q + 3)$$
$$10c = 150q + 30$$
$$10c = 15(10q + 2)$$
$$10c \bmod 15 = 0$$

4.6

Exercises

Abeyah Calpatura
#2, 4, 6, 7, 10a

#2 *Solution:*

$$\lceil 17/4 \rceil = \lceil 4.25 \rceil = 5$$
$$\lfloor 17/4 \rfloor = \lfloor 4.25 \rfloor = 4$$

#4 *Solution:*

$$\lceil -32/5 \rceil = \lceil -6.4 \rceil = -6$$
$$\lfloor -32/5 \rfloor = \lfloor -6.4 \rfloor = -7$$

#6 *Solution:* If k is an integer, what is $\lceil k \rceil$? Why?

By deifintion of ceiling, k is an integer and the ceiling of an integer is itself since:
$k - 1 < k \leqslant k$
Therefore, $\lceil k \rceil = k$

#7 *Solution:* If k is an integer, what is $\lceil k + \frac{1}{2} \rceil$? Why?

By definition of ceiling, k is an integer and the ceiling of an integer is itself since:
$k < k + \dfrac{1}{2} \leqslant k + 1$
Therefore, $\lceil k + \dfrac{1}{2} \rceil = k + 1$

#10a *Solution:*

    i. n = 2050

$$= \left(2050 + \left\lfloor \frac{2050 - 1}{4} \right\rfloor - \left\lfloor \frac{2050 - 1}{100} \right\rfloor - \left\lfloor \frac{2050 - 1}{400} \right\rfloor \right) \ mod \ 7$$

$$= (2050 + 512 - 20 + 5) \ mod \ 7$$

$$= (2547) \ mod \ 7$$

$$= 6$$

Corresponds to **Saturday**

    ii. n = 2100

$$= \left(2100 + \left\lfloor \frac{2100 - 1}{4} \right\rfloor - \left\lfloor \frac{2100 - 1}{100} \right\rfloor - \left\lfloor \frac{2100 - 1}{400} \right\rfloor \right)$$

$$= (2100 + 524 - 20 + 5) \ mod \ 7$$

$$= (2609) \ mod \ 7$$

$$= 5$$

Corresponds to **Friday**

    iii. n = 2004

$$= \left(2004 + \left\lfloor \frac{2004 - 1}{4} \right\rfloor - \left\lfloor \frac{2004 - 1}{100} \right\rfloor - \left\lfloor \frac{2004 - 1}{400} \right\rfloor \right)$$

$$= (2004 + 500 - 20 + 5) \ mod \ 7$$

$$= (2609) \ mod \ 7$$

$$= 4$$

Corresponds to **Thursday**