Discrete Mathematics
Week 6

Abeyah Calpatura

4.5

Exercises

Abeyah Calpatura
#1, 2, 7, 17, 22

#1 ***Solution:***

$n = 70$ and $d = 9$
$70 = 9q + r$
$q = 7$ and $r = 7$
Since $70 = 9(7) + 7$

#2 ***Solution:***

$n = 62$ and $d = 7$
$62 = 7q + r$
$q = 8$ and $r = 6$
Since $62 = 7(8) + 6$

#7 ***Solution:***

a. 43 div 9: 4
b. 43 mod 9: 7

#17 ***Solution:*** Prove directly from definitions that for every integer n, $n^2 - n + 3$ is odd. Use division into two cases: n is even and n is odd.

Case 1: n is even
$n = 2k$ for some integer k
$n^2 - n + 3 = (2k)^2 - 2k + 3$
$n^2 - n + 3 = 4k^2 - 2k + 3$
$n^2 - n + 3 = 2(2k^2 - k + 1) + 1$
$n^2 - n + 3 = 2q + 1$
where $q = 2k^2 - k + 1$
Case 2: n is odd
$n = 2k + 1$ for some integer k
$n^2 - n + 3 = (2k + 1)^2 - (2k + 1) + 3$
$n^2 - n + 3 = 4k^2 + 4k + 1 - 2k - 1 + 3$
$n^2 - n + 3 = 4k^2 + 2k + 3$
$n^2 - n + 3 = 2(2k^2 + k + 1) + 1$
$n^2 - n + 3 = 2q + 1$
where $q = 2k^2 + k + 1$
Therefore, $n^2 - n + 3$ is odd for every integer n

#22 Solution: Suppose c is any integer. If $c \ mod \ 15 = 3$, what is $10c \ mod \ 15$? In other words, if division of c by 15 gives a remainder of 3, what is the remainder when 10c is divided by 15? Your solution should show that you obatin the same answer no matter what integer you start with.

$$
\begin{aligned}
&c \ mod \ 15 = 3 \\
&c = 15q + 3 \\
&10c = 10(15q + 3) \\
&10c = 150q + 30 \\
&10c = 15(10q + 2) \\
&10c \ mod \ 15 = 0
\end{aligned}
$$

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Exercises

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#2, 4, 6, 7, 10a

#2 Solution:

$$\lceil 17/4 \rceil = \lceil 4.25 \rceil = 5$$
$$\lfloor 17/4 \rfloor = \lfloor 4.25 \rfloor = 4$$

#4 Solution:

$$\lceil -32/5 \rceil = \lceil -6.4 \rceil = -6$$
$$\lfloor -32/5 \rfloor = \lfloor -6.4 \rfloor = -7$$

#6 Solution: If k is an integer, what is $\lceil k \rceil$? Why?

By deifintion of ceiling, k is an integer and the ceiling of an integer is itself since:
$k - 1 < k \leqslant k$
Therefore, $\lceil k \rceil = k$

#7 Solution: If k is an integer, what is $\lceil k + \frac{1}{2} \rceil$? Why?

By definition of ceiling, k is an integer and the ceiling of an integer is itself since:
$k < k + \dfrac{1}{2} \leqslant k + 1$
Therefore, $\lceil k + \dfrac{1}{2} \rceil = k + 1$

#10a *Solution:*

i. n = 2050

$$= \left(2050 + \left\lfloor \frac{2050-1}{4} \right\rfloor - \left\lfloor \frac{2050-1}{100} \right\rfloor - \left\lfloor \frac{2050-1}{400} \right\rfloor\right)\ mod\ 7$$

$= (2050 + 512 - 20 + 5)\ mod\ 7$

$= (2547)\ mod\ 7$

$= 6$

Corresponds to **Saturday**

ii. n = 2100

$$= \left(2100 + \left\lfloor \frac{2100-1}{4} \right\rfloor - \left\lfloor \frac{2100-1}{100} \right\rfloor - \left\lfloor \frac{2100-1}{400} \right\rfloor\right)$$

$= (2100 + 524 - 20 + 5)\ mod\ 7$

$= (2609)\ mod\ 7$

$= 5$

Corresponds to **Friday**

iii. n = 2004

$$= \left(2004 + \left\lfloor \frac{2004-1}{4} \right\rfloor - \left\lfloor \frac{2004-1}{100} \right\rfloor - \left\lfloor \frac{2004-1}{400} \right\rfloor\right)$$

$= (2004 + 500 - 20 + 5)\ mod\ 7$

$= (2609)\ mod\ 7$

$= 4$

Corresponds to **Thursday**

4.7

Exercises

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#2, 4, 9b

#2 ***Solution:*** Is $\frac{1}{0}$ an irrational number? Explain.

$\dfrac{1}{0}$ is not an irrational number, because division by 0 is not defined.
Thus, the number of $\dfrac{1}{0}$ does not exist, which implies that the number is not an irrational number.

#4 ***Solution:*** Use proof by contradiction to show that for every integer m, $7m + 4$ is not divisible by 7.

By the definition of divisble, there exists an integer k such that:

$$7m + 4 = 7k$$
$$4 = 7k - 7m$$
$$4 = 7(k - m)$$

Since $k - m$ is an integer, 4 is divisible by 7.
This is a contradiction, since 4 is not divisible by 7.

#9b ***Solution:*** Prove that the difference of any irrational number and any rational number is irrational.

Let us assume that x is an irrational number and y is a rational number such that their difference $x - y$ is rational.
By the definition of rational, there exist integers a, b, c, and d with $b \neq 0$ and $d \neq 0$ such that

$$x = \frac{a}{b}$$
$$y = \frac{c}{d}$$
$$x - y = \frac{a}{b} - \frac{c}{d}$$
$$x - y = \frac{ad - bc}{bd}$$

Since $ad - bc$ and bd are integers, $x - y$ is rational.
This is a contradiction, since $x - y$ is irrational.