

Discrete Mathematics

Week 6

Abeyah Calpatura

4.5

Exercises

Abeyah Calpatura

#1, 2, 7, 17, 22

#1 Solution:

$$n = 70 \text{ and } d = 9$$

$$70 = 9q + r$$

$$q = 7 \text{ and } r = 7$$

$$\text{Since } 70 = 9(7) + 7$$

#2 Solution:

$$n = 62 \text{ and } d = 7$$

$$62 = 7q + r$$

$$q = 8 \text{ and } r = 6$$

$$\text{Since } 62 = 7(8) + 6$$

#7 Solution:

$$\text{a. } 43 \text{ div } 9: 4$$

$$\text{b. } 43 \text{ mod } 9: 7$$

#17 Solution: Prove directly from definitions that for every integer n , $n^2 - n + 3$ is odd. Use division into two cases: n is even and n is odd.

Case 1: n is even

$$n = 2k \text{ for some integer } k$$

$$n^2 - n + 3 = (2k)^2 - 2k + 3$$

$$n^2 - n + 3 = 4k^2 - 2k + 3$$

$$n^2 - n + 3 = 2(2k^2 - k + 1) + 1$$

$$n^2 - n + 3 = 2q + 1$$

$$\text{where } q = 2k^2 - k + 1$$

Case 2: n is odd

$$n = 2k + 1 \text{ for some integer } k$$

$$n^2 - n + 3 = (2k + 1)^2 - (2k + 1) + 3$$

$$n^2 - n + 3 = 4k^2 + 4k + 1 - 2k - 1 + 3$$

$$n^2 - n + 3 = 4k^2 + 2k + 3$$

$$n^2 - n + 3 = 2(2k^2 + k + 1) + 1$$

$$n^2 - n + 3 = 2q + 1$$

$$\text{where } q = 2k^2 + k + 1$$

Therefore, $n^2 - n + 3$ is odd for every integer n

#22 Solution: Suppose c is any integer. If $c \bmod 15 = 3$, what is $10c \bmod 15$? In other words, if division of c by 15 gives a remainder of 3, what is the remainder when $10c$ is divided by 15? Your solution should show that you obtain the same answer no matter what integer you start with.

$$c \bmod 15 = 3$$

$$c = 15q + 3$$

$$10c = 10(15q + 3)$$

$$10c = 150q + 30$$

$$10c = 15(10q + 2)$$

$$10c \bmod 15 = 0$$

4.6

Exercises

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#2, 4, 6, 7, 10a

#2 Solution:

$$\lceil 17/4 \rceil = \lceil 4.25 \rceil = 5$$

$$\lfloor 17/4 \rfloor = \lfloor 4.25 \rfloor = 4$$

#4 Solution:

$$\lceil -32/5 \rceil = \lceil -6.4 \rceil = -6$$

$$\lfloor -32/5 \rfloor = \lfloor -6.4 \rfloor = -7$$

#6 Solution: If k is an integer, what is $\lceil k \rceil$? Why?

By definition of ceiling, k is an integer and the ceiling of an integer is itself since:

$$k - 1 < k \leq k$$

Therefore, $\lceil k \rceil = k$

#7 Solution: If k is an integer, what is $\lceil k + \frac{1}{2} \rceil$? Why?

By definition of ceiling, k is an integer and the ceiling of an integer is itself since:

$$k < k + \frac{1}{2} \leq k + 1$$

Therefore, $\lceil k + \frac{1}{2} \rceil = k + 1$

#10a Solution:

i. $n = 2050$

$$\begin{aligned} &= \left(2050 + \left\lfloor \frac{2050-1}{4} \right\rfloor - \left\lfloor \frac{2050-1}{100} \right\rfloor - \left\lfloor \frac{2050-1}{400} \right\rfloor \right) \text{ mod } 7 \\ &= (2050 + 512 - 20 + 5) \text{ mod } 7 \\ &= (2547) \text{ mod } 7 \\ &= 6 \end{aligned}$$

Corresponds to **Saturday**

ii. $n = 2100$

$$\begin{aligned} &= \left(2100 + \left\lfloor \frac{2100-1}{4} \right\rfloor - \left\lfloor \frac{2100-1}{100} \right\rfloor - \left\lfloor \frac{2100-1}{400} \right\rfloor \right) \\ &= (2100 + 524 - 20 + 5) \text{ mod } 7 \\ &= (2609) \text{ mod } 7 \\ &= 5 \end{aligned}$$

Corresponds to **Friday**

iii. $n = 2004$

$$\begin{aligned} &= \left(2004 + \left\lfloor \frac{2004-1}{4} \right\rfloor - \left\lfloor \frac{2004-1}{100} \right\rfloor - \left\lfloor \frac{2004-1}{400} \right\rfloor \right) \\ &= (2004 + 500 - 20 + 5) \text{ mod } 7 \\ &= (2609) \text{ mod } 7 \\ &= 4 \end{aligned}$$

Corresponds to **Thursday**