

Project #4

SAT Solver and Implicit State Enumeration

ECE 582

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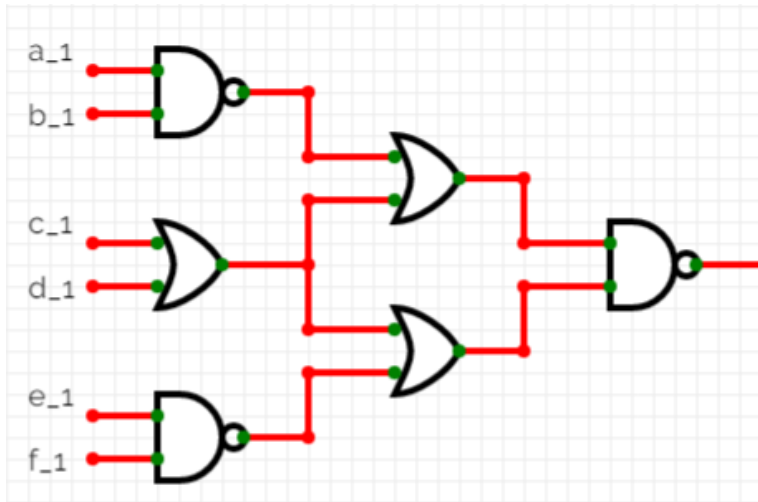
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Problem 1:

Task 1

Circuit C1

$$a_1b_1\bar{c}_1\bar{d}_1 + \bar{c}_1\bar{d}_1e_1f_1$$

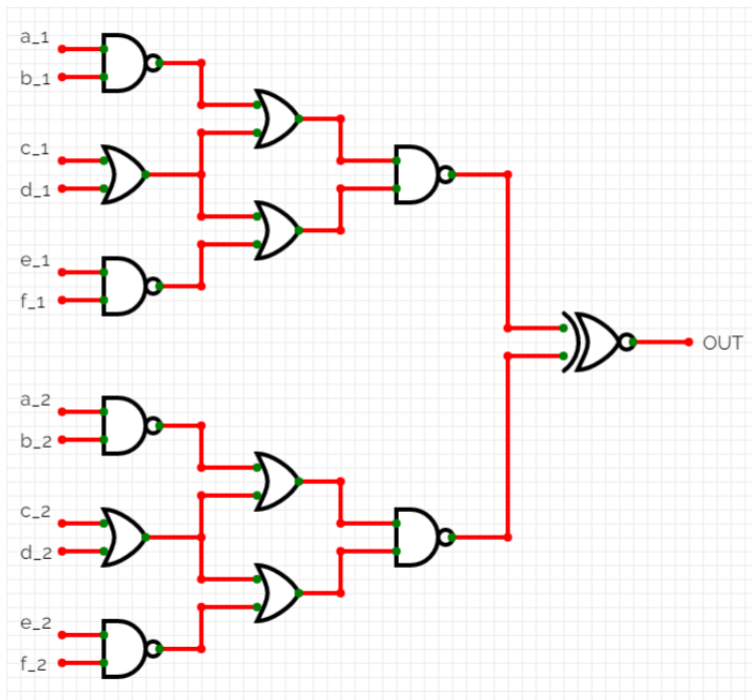


C1 and C2 Comparison

The equation for the equivalence digital circuit, C1 and C2 XORd using, ('A+B)(A+B)

$$(a_1b_1\bar{c}_1\bar{d}_1 + \bar{c}_1\bar{d}_1e_1f_1 + a_2b_2\bar{c}_2\bar{d}_2 + \bar{c}_2\bar{d}_2e_2f_2)$$

$$[(\bar{a}_1 + \bar{b}_1 + c_1 + d_1)(c_1 + d_1 + \bar{e}_1 + \bar{f}_1) + (\bar{a}_2 + \bar{b}_2 + c_2 + d_2)(c_2 + d_2 + \bar{e}_2 + \bar{f}_2)]$$



CNF

CNF for circuit C1

```
p cnf 6 2|
1 2 -3 -4 0
-3 -4 5 6 0
```

Solve

SAT

Comparing C1 and C2 where $C1 = C2$ for equivalence where the inputs are equal to each other. a, b, c, d, e, f

```
p cnf 6 2
1 2 -3 -4 -3 -4 5 6 1 2 -3 -4 -3 -4 5 6 0
-1 -2 3 4 3 4 -5 -6 -1 -2 3 4 3 4 -5 -6 0
```

Solve

SAT

Comparing C1 and C2 where $C1 = C2$ for equivalence where the inputs are equivalent but unique: a_1, b_1, c_1, d_1, e_1, f_1 and a_2, b_2, c_2, d_2, e_2, f_2

```
p cnf 12 2
1 2 -3 -4 -3 -4 5 6 7 8 -9 -10 -9 -10 11 12 0
-1 -2 3 4 3 4 -5 -6 -7 -8 9 10 9 10 -11 -12 0
```

Solve

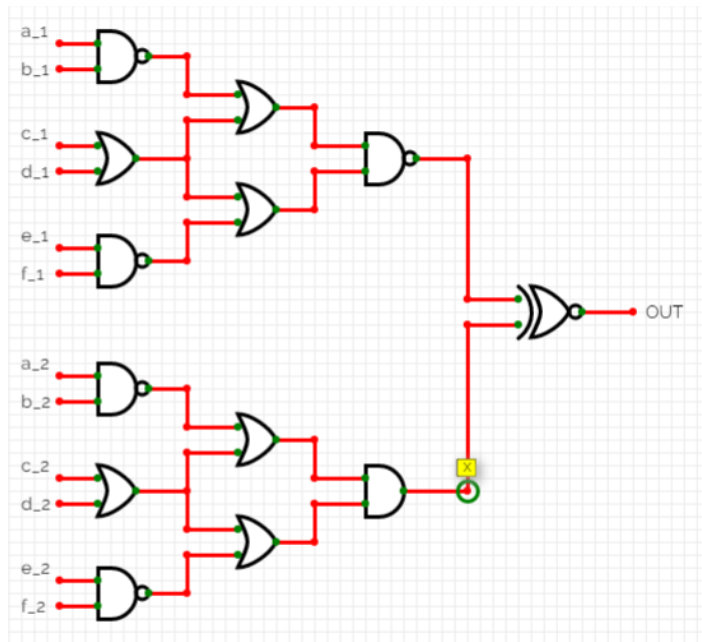
SAT

Task 2

Circuit C3

$$[(\overline{a_2} + \overline{b_2} + c_2 + d_2)(c_2 + d_2 + \overline{e_2} + \overline{f_2})]$$

Comparing circuit C1 and C3



$$A + B = [(a_1 b_1 \overline{c_1} \overline{d_1} + \overline{c_1} \overline{d_1} e_1 f_1) + (\overline{a_2} + \overline{b_2} + c_2 + d_2)(c_2 + d_2 + \overline{e_2} + \overline{f_2})]$$

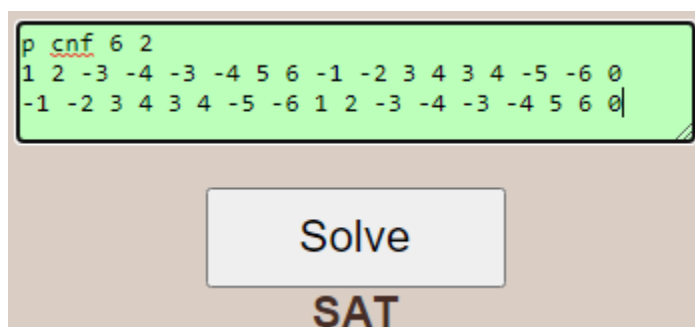
$$\overline{A} + \overline{B} = [(\overline{a_1} + \overline{b_1} + c_1 + d_1)(c_1 + d_1 + \overline{e_1} + \overline{f_1})] + (a_2 b_2 \overline{c_2} \overline{d_2} + \overline{c_2} \overline{d_2} e_2 f_2)$$

The equivalence is determined by:

$$(A + B)(\overline{A} + \overline{B})$$

CNF

Using shared inputs for a, b, c, d, e, f for both C1 and C3 circuits is satisfiable.



Using 12 unique variables for C1 and C3 circuits is satisfiable.

```
p cnf 12 2
1 2 -3 -4 -3 -4 5 6 -7 -8 9 10 9 10 -11 -12 0
-1 -2 3 4 3 4 -5 -6 7 8 -9 -10 -9 -10 11 12 0
```

Solve

SAT

Problem 2

From project #1, we have the following circuit to check for equivalence:

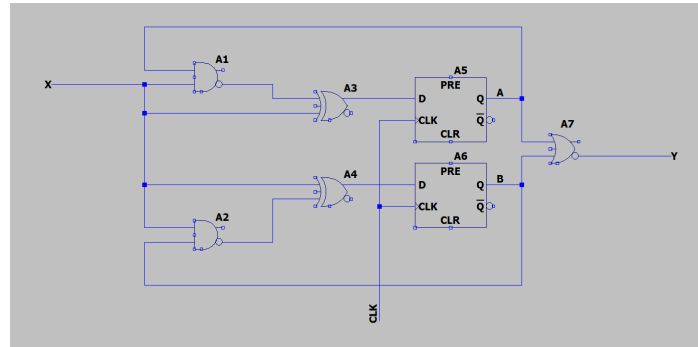


Figure 1: Sequential circuit S1

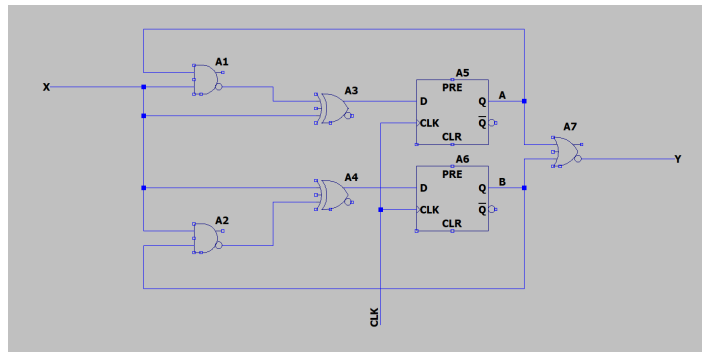


Figure 2: Sequential circuit S2

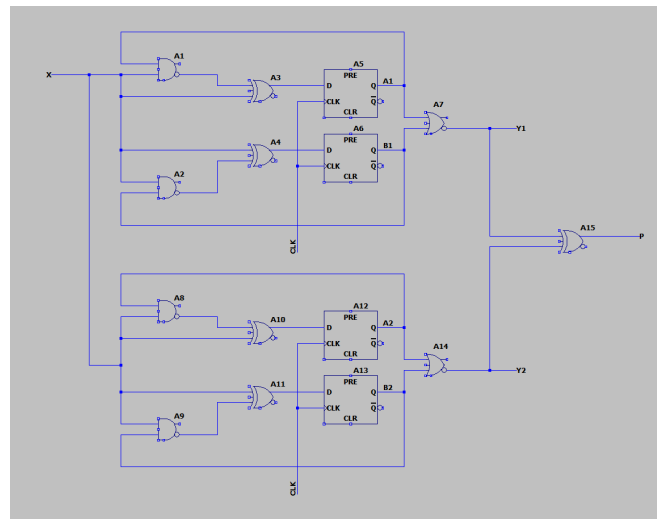


Figure 3: Product machine with the shared input

Implicit Equivalence Checking

$a1, b1, a2, b2$ – present state

$a1', b1', a2', b2'$ – next state

$$S0 = !a1!b1!a2!b2$$

$$R1 = !x!a1!b1a1'b1' + !x!a1b1a1'b1' + !xa1!b1a1'b1' + !xa1b1a1'b1' \\ + x!a1!b1!a1'b1' + x!a1b1!a1'b1' + xa1!b1a1'b1' + xa1b1a1'b1'$$

$$R2 = !x!a2!b2a2'b2' + !x!a2b2a2'b2' + !xa2!b2a2'b2' + !xa2b2a2'b2' \\ + x!a2!b2!a2'b2' + x!a2b2!a2'b2' + xa2!b2a2'b2' + xa2b2a2'b2'$$

$$\begin{aligned} \square x \square a1 \square b1 \square a2 \square b2 (S0 \wedge R1 \wedge R2) \\ = \square x \square a1 \square b1 \square a2 \square b2 ((!a1!b1!a2!b2) \\ \wedge (!x!a1!b1a1'b1' + !x!a1b1a1'b1' + !xa1!b1a1'b1' + !xa1b1a1'b1' \\ + x!a1!b1!a1'b1' + x!a1b1!a1'b1' + xa1!b1a1'b1' + xa1b1a1'b1') \\ \wedge (!x!a2!b2a2'b2' + !x!a2b2a2'b2' + !xa2!b2a2'b2' + !xa2b2a2'b2' \\ + x!a2!b2!a2'b2' + x!a2b2!a2'b2' + xa2!b2a2'b2' + xa2b2a2'b2')) \\ = !a1'b1'a2'b2' + a1'b1'a2'b2' \end{aligned}$$

$$B = \text{Rename} (\square x \square a1 \square b1 \square a2 \square b2 (S0 \wedge R1 \wedge R2)) = !a1!b1!a2!b2 + a1b1a2b2$$

$$\text{Output: } P = Y1 \oplus Y2 = !(a1+b1) \oplus !(a2+b2)$$

$$P \wedge B = (!(a1+b1) \oplus !(a2+b2)) (!a1!b1!a2!b2 + a1b1a2b2) = 0 \rightarrow \text{equivalence proved}$$