Limitations of the Monotonic Relative Neighborhood Graph for External Query Points

July 24, 2025

1 Introduction

The *Monotonic Relative Neighborhood Graph* (MRNG) is a sparse proximity graph introduced by Fu *et al.* [1] as an ideal backbone for graphbased approximate nearestneighbour (ANN) search. An MRNG guarantees that from every vertex u there exists a strictly distancedecreasing path to any other vertex v—a property often called *monotone navigability*. Recent work by Zhu and Zhang [2] gives a theoretical account of why such graphs achieve nearlogarithmic greedy search time.

In practice, however, ANN systems must answer queries that are *not* present in the original data set. This note gives a simple planar counterexample showing that the MRNG's monotone property does *not* extend to external query points: a greedy walk can get trapped in a local minimum even when the true nearest neighbour is one hop away.

Figure 1 visualises the construction and the failing search path.

2 MRNG Construction Recap

Given a finite set $P \subset \mathbb{R}^d$, the MRNG is defined as follows [1]:

- **Vertices.** Each data point in P.
- For every ordered pair (u, v), let $L(u, v) = \{x \mid \|x - u\| < \|u - v\| \ \wedge \ \|x - v\| < \|u - v\|\}$ be the *lune*.
- **Edge rule.** Sort $P \setminus \{u\}$ by distance to u. Insert a directed edge $u \to v$ iff $L(u, v)$ contains no point w that already has an outgoing edge from u.

The resulting graph has constant average degree, is strongly connected, and supports distancedecreasing greedy walks between any two data points.

3 Failure for External Queries

Suppose a query point $q \notin P$ is introduced (Figure 1). Starting the greedy walk at p_1 yields the path $p_1 \to p_2 \to p_3$, where the walk terminates because no neighbour of p_3 is closer to q than p_3. Yet the true nearest neighbour is p_5. Thus the MRNG guarantee does not suffice for ANN systems that must serve arbitrary queries.

Implication. Production systems therefore approximate MRNG with supergraphs such as NSG [1] or add multientrypoint and backtracking heuristics [2].

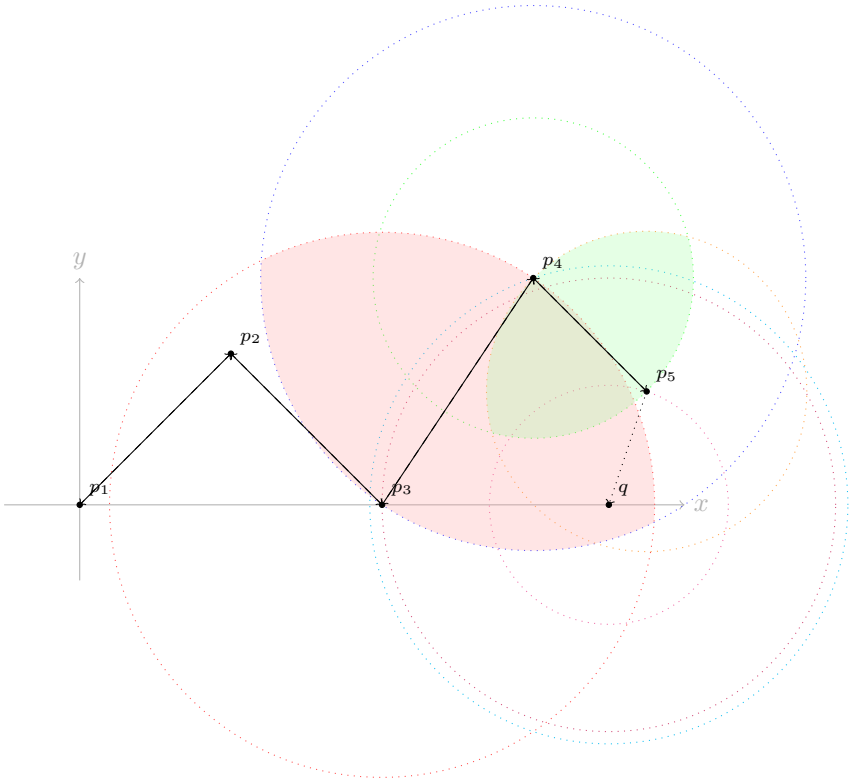

Figure 1: A fivepoint MRNG and a query point q. Greedy search from p_1 stops at p_3, yet p_5 is the true nearest neighbour of q.

4 Conclusion

- The MRNG ensures monotone connectivity for *ingraph* searches.
- External queries can defeat a pure greedy walk.
- Practitioners should augment the graph (NSG, HNSW) or the search procedure (multientry, beam search) to restore recall.

References

[1] Cong Fu, Chao Xiang, Changxu Wang, and Deng Cai.
Fast Approximate Nearest Neighbor Search with the Navigating Spreadingout Graph.
Proceedings of the VLDB Endowment, 12(5):461–474, 2019.

[2] Dantong Zhu and Minjia Zhang.
Understanding and Generalizing Monotonic Proximity Graphs for Approximate Nearest Neighbor Search.
arXiv preprint arXiv:2107.13052, 2021.