ACCQ 206 - Entanglement

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Recap

Quantum states

n-qubit state $|\psi\rangle = \begin{pmatrix} \alpha_0 \\ \cdots \\ \alpha_{2^n} \end{pmatrix} \in \mathbb{C}^{2^n}$

$\sum_i |\alpha_i|^2 = 1$

Evolution

Unitary matrices

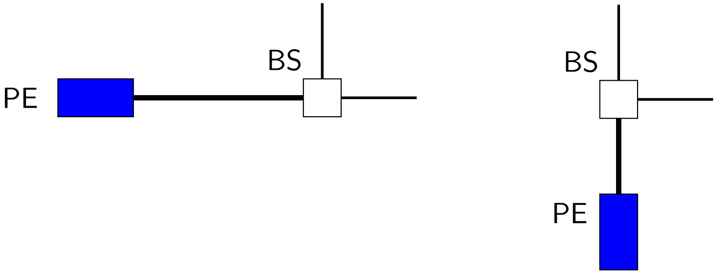
$U : UU^\dagger = U(U^*)^T = I$

Measurement in comp. basis

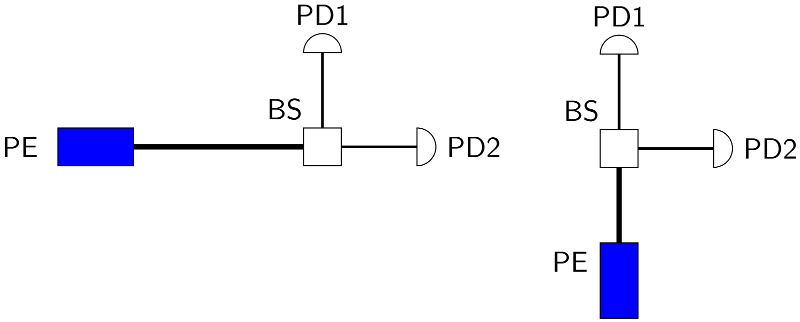
output i w.p. $|\alpha_i|^2$

state collapses to $|i\rangle$

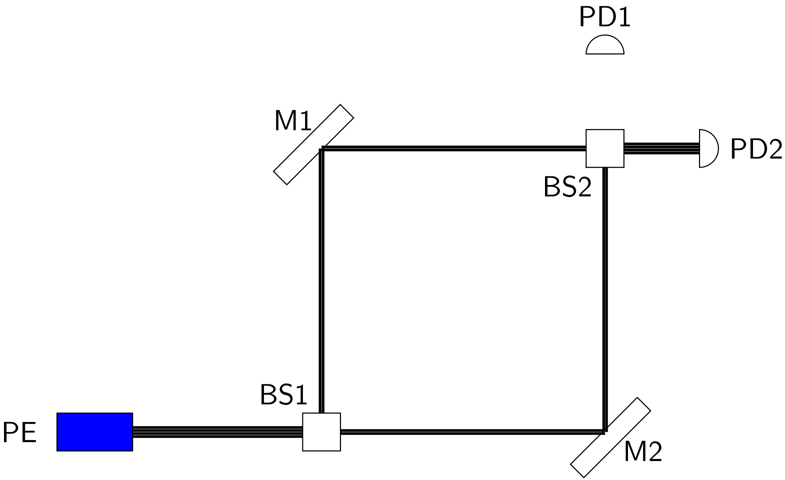
Beam splitter

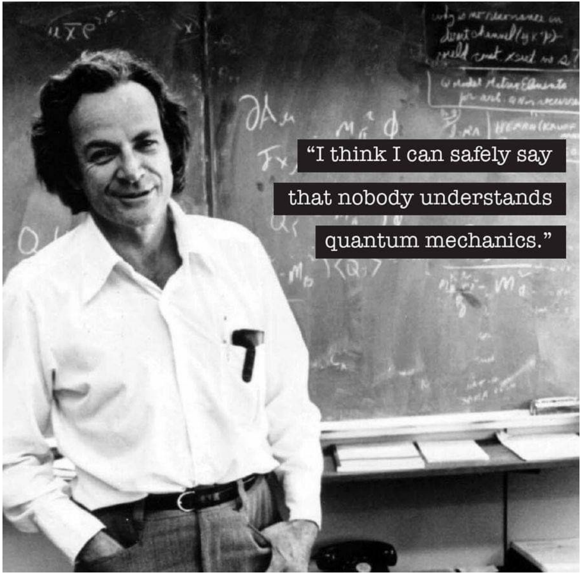


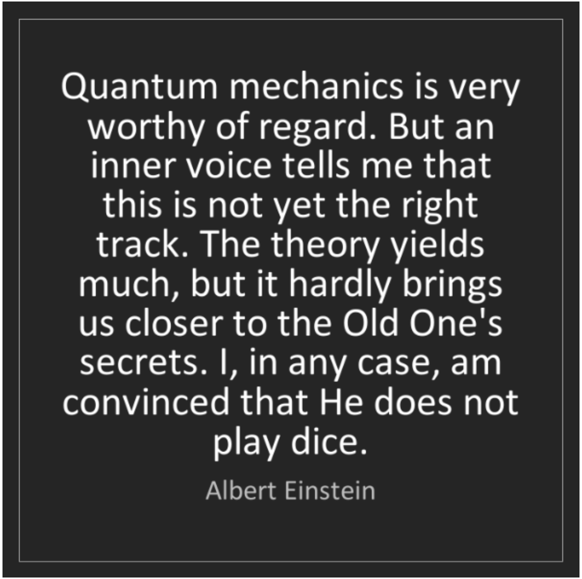
Beam splitter



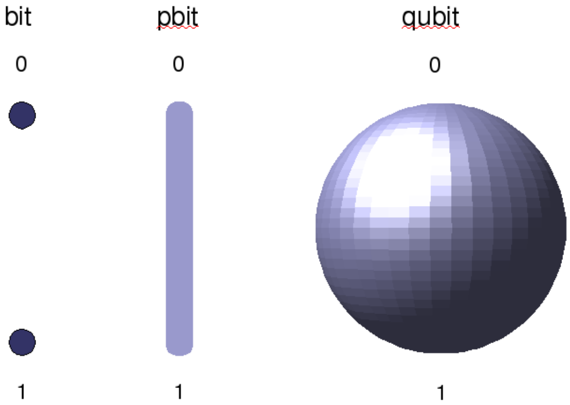
Interferometer




"I think I can safely say
that nobody understands
quantum mechanics."


Quantum mechanics is very worthy of regard. But an inner voice tells me that this is not yet the right track. The theory yields much, but it hardly brings us closer to the Old One's secrets. I, in any case, am convinced that He does not play dice.
Albert Einstein

Bit vs. pbit vs qubit



Observables

- Hermitian operator $A = A^\dagger$
  - Real values $a_1, \ldots, a_k$ and corresponding projectors, $P_1, \ldots, P_k$
  - We have $A = \sum_{i \in [2^n]} a_i P_i$
  - Output a_i with probability $q_i := \langle\psi| P_i |\phi_i\rangle$
  - State collapses to

$$\frac{P_i |\psi\rangle}{\sqrt{q_i}}$$

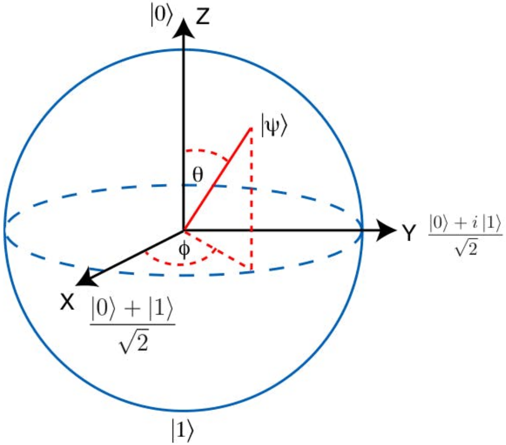
- Average value of the outcome is

$$\langle A\rangle = \langle\psi| A |\psi\rangle = \sum_i \langle\psi| P_i |\psi\rangle$$

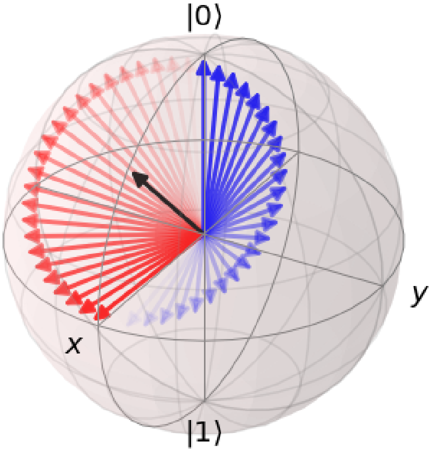
Global phases: $|\psi\rangle$ vs. $e^{i\theta} \cdot |\psi\rangle$

Bloch sphere

Geometrical representation of 1 qubit



Bloch sphere



Product states

A two qubit state $|\psi\rangle$ is said to be a product state if there exists $|\psi_1\rangle$ and $|\psi_2\rangle$ such that

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle = |\psi_1\rangle |\psi_2\rangle$$

Not all states are product

Entangled states

Entangled states: quantum states that are not product

Examples

$\mathrm{GHZ}_n = \frac{1}{\sqrt{2}}(|0...0\rangle + |1...1\rangle)$

$\mathrm{W}_n = \frac{1}{\sqrt{n}}(|10...0\rangle + |01...0\rangle + |00...1\rangle)$

Bell basis

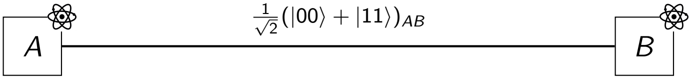
Canonical basis

$$\left\{ \begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \right\}$$

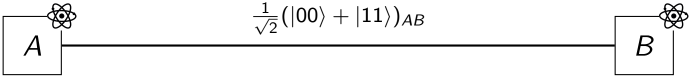
Bell basis

$$\left\{ \begin{array}{c} \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \\ \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \\ \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \\ \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \end{array} \right\}$$

Source of quantum weirdness



Source of quantum weirdness

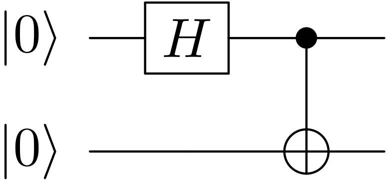


Violation of relativity theory?

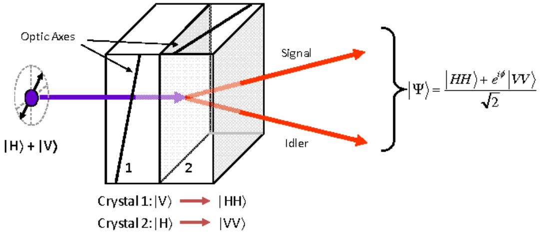
"I cannot seriously believe in it [quantum theory] because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance" Albert Einstein

Generating entangled states

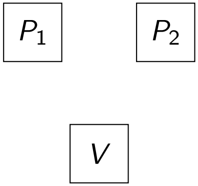
Theory



Practice



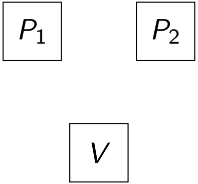
Non-local games



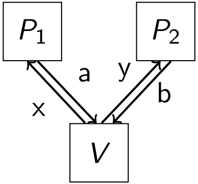
Classical value $\omega(G)$

Quantum value $\omega^*(G)$

CHSH game



CHSH game - classical strategies

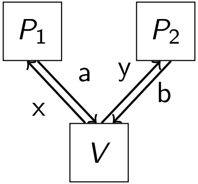


$x, y \in_R \{0, 1\}$
$x \cdot y = a \oplus b$
$a \neq b$ iff $x = y = 1$

Example

x	y	a	b	Win?
0	0	0	0	
0	1	0	0	
1	0	0	0	
1	1	0	0	

CHSH game - quantum strategies

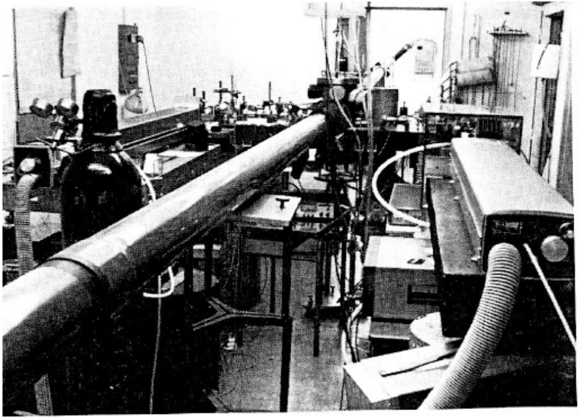


$$x, y \in_R \{0, 1\}$$
$$x \cdot y = a \oplus b$$
$$a \neq b \text{ iff } x = y = 1$$

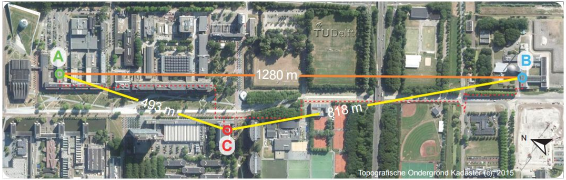
Bell inequalities

Bell inequalities give us a way for testing if Nature is not classical $\Rightarrow$ implement the quantum experiments in the lab and if the acceptance probability is strictly larger tha

Alain Aspect (1982, Institut d'Optique)



QuTech group (2015, TU Delft)



Further notions

We have seen a quantum strategy that achieves $\omega^*(CHSH) = \cos^2 \frac{\pi}{8}$

Is this strategy optimal?

Is this optimal strategy unique?

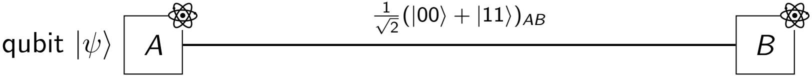
Quantum teleportation

Problem: Alice wants to send a qubit $|\psi\rangle$ to Bob.
If they have a quantum channel to communicate:

If they have a classical channel to communicate:

If they have a classical channel to communicate + pre-shared quantum state:

Quantum teleportation



Super-dense coding

Alice receives two random bits a and b and she wants Bob to learn both of them.

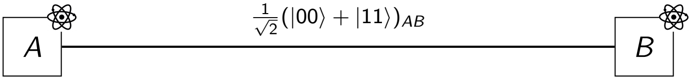
If Alice sends a single classical bit:

If they have shared randomness and Alice sends a single classical bit:

If they have shared quantum state and Alice sends a single classical bit:

If they have shared quantum state and Alice sends a single qubit:

Super-dense coding



Mixed states

- Mixed states: probabilistic distribution of quantum states

Examples

$((\frac{1}{2}, |0\rangle), (\frac{1}{2}, |+\rangle))$, $((\frac{1}{3}, |0\rangle), (\frac{2}{3}, |1\rangle))$

- Density matrices: mathematical representation of mixed states
  $(1, |\psi\rangle)$

  $((p_1, |\psi_1\rangle), (p_2, |\psi_2\rangle), (p_k, |\psi_k\rangle))$

- Properties of density matrices
  - Its trace is 1 (The trace of a square matrix is $\sum_i a_{i,i}$)
  - Positive semi-definitive (all its eigenvalues are non-negative)
- Definition of evolution and measurements can be extended to density matrices

“Parts” of quantum states

- Trace-out: ignore qubits of a larger quantum state