

AI Homework 2

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Problem 1)

1) All cats are animal.

$\forall x (\text{cat}(x) \Rightarrow \text{animal}(x))$

2) Everyone who owns a car also has a bicycle.

$\forall x (\text{own_car}(x) \Rightarrow \text{has_bicycle}(x))$

3) There is a student registered to AI class who talks to other students that are registered to AI class.

$\exists x \exists y (\text{Registered}(x, \text{AI}) \wedge \text{Talks}(x, y) \wedge \text{Registered}(y, \text{AI}))$

Problem 2)

- English, French, Russian and Turkish belong to language category (Same as "English, French, Russian and Turkish are languages")

$\forall x \text{ English}(x) \Rightarrow \text{Language}(x)$

$\forall x \text{ French}(x) \Rightarrow \text{Language}(x)$

$\forall x \text{ Russian}(x) \Rightarrow \text{Language}(x)$

$\forall x \text{ Turkish}(x) \Rightarrow \text{Language}(x)$

- Arda, Cihan and Gamze all are students in the same university.

$\text{Student}(\text{Arda})$

$\text{Student}(\text{Cihan})$

$\text{Student}(\text{Gamze})$

$\exists x (\text{University}(x) \wedge \text{Enrolled}(\text{Arda}, x) \wedge \text{Enrolled}(\text{Cihan}, x) \wedge \text{Enrolled}(\text{Gamze}, x))$

- Each student in the university speaks at least two languages.

$\forall x \exists y \exists z (\text{Student}(x) \wedge \text{University}(x) \Rightarrow \text{Speak}(x, \text{Language}(y)) \wedge \text{Speak}(x, \text{Language}(z)) \wedge (y \neq z))$

- Fish and hamburger belong to food category.

$\forall x \text{ Fish}(x) \Rightarrow \text{Food}(x)$

$\forall x \text{ Hamburger}(x) \Rightarrow \text{Food}(x)$

- Classic, jazz and rock belong to music category.

$\forall x \text{ Classic}(x) \Rightarrow \text{Music}(x)$

$\forall x \text{ Jazz}(x) \Rightarrow \text{Music}(x)$

$\forall x \text{ Rock}(x) \Rightarrow \text{Music}(x)$

- Students who speak French like jazz music and dislike rock music.

$\forall x (\text{Student}(x) \wedge \text{Speaks}(x, \text{French}) \Rightarrow \text{Like}(x, \text{Jazz}) \wedge \neg \text{Like}(x, \text{Rock}))$

- All students who speak Russian like rock music.

$\forall x (\text{Student}(x) \wedge \text{Speaks}(x, \text{Russian}) \Rightarrow \text{Like}(x, \text{Rock}))$

- All students who like hamburger speak English but do not speak Turkish.

$\forall x (\text{Student}(x) \wedge \text{Like}(x, \text{Hamburger}) \Rightarrow \text{Speak}(x, \text{English}) \wedge \neg \text{Speak}(x, \text{Turkish}))$

- All students who like fish and classic music speak Turkish.

- $\forall x (\text{Student}(x) \wedge \text{Like}(x, \text{Fish}) \wedge \text{Like}(x, \text{Classic}) \Rightarrow \text{Speak}(x, \text{Turkish}))$
- Arda likes jazz music, hamburger, fish but dislikes classic music, rock music.

$\text{Like}(\text{Arda}, \text{Jazz}) \wedge \text{Like}(\text{Arda}, \text{Hamburger}) \wedge \text{Like}(\text{Arda}, \text{Fish}) \wedge \neg \text{Like}(\text{Arda}, \text{Classic}) \wedge \neg \text{Like}(\text{Arda}, \text{Rock})$

- Cihan dislikes whatever music Arda likes, and he likes whatever music Arda dislikes. He likes hamburger and dislikes fish.

$\forall x (\text{Music}(x) \wedge \text{Like}(\text{Arda}, x) \Rightarrow \neg \text{Like}(\text{Cihan}, x))$
 $\forall x (\text{Music}(x) \wedge \neg \text{Like}(\text{Arda}, x) \Rightarrow \text{Like}(\text{Cihan}, x))$
 $\text{Like}(\text{Cihan}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Cihan}, \text{Fish})$

- Gamze likes fish, classic music but dislikes hamburger, jazz music and rock music.

$\text{Like}(\text{Gamze}, \text{Fish}) \wedge \text{Like}(\text{Gamze}, \text{Classic}) \wedge \neg \text{Like}(\text{Gamze}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Gamze}, \text{Jazz}) \wedge \neg \text{Like}(\text{Gamze}, \text{Rock})$

a) Knowledge Base

Non CNF knowledge base:

$\forall x \text{ English}(x) \Rightarrow \text{Language}(x)$
 $\forall x \text{ French}(x) \Rightarrow \text{Language}(x)$
 $\forall x \text{ Russian}(x) \Rightarrow \text{Language}(x)$
 $\forall x \text{ Turkish}(x) \Rightarrow \text{Language}(x)$
 $\text{Student}(\text{Arda})$
 $\text{Student}(\text{Cihan})$
 $\text{Student}(\text{Gamze})$
 $\exists x (\text{University}(x) \wedge \text{Enrolled}(\text{Arda}, x) \wedge \text{Enrolled}(\text{Cihan}, x) \wedge \text{Enrolled}(\text{Gamze}, x))$
 $\forall x \exists y \exists z (\text{Student}(x) \wedge \text{University}(x) \Rightarrow \text{Speak}(x, \text{Language}(y)) \wedge \text{Speak}(x, \text{Language}(z)) \wedge (y \neq z))$
 $\forall x \text{ Fish}(x) \Rightarrow \text{Food}(x)$
 $\forall x \text{ Hamburger}(x) \Rightarrow \text{Food}(x)$
 $\forall x \text{ Classic}(x) \Rightarrow \text{Music}(x)$
 $\forall x \text{ Jazz}(x) \Rightarrow \text{Music}(x)$
 $\forall x \text{ Rock}(x) \Rightarrow \text{Music}(x)$
 $\forall x (\text{Student}(x) \wedge \text{Speaks}(x, \text{French}) \Rightarrow \text{Like}(x, \text{Jazz}) \wedge \neg \text{Like}(x, \text{Rock}))$
 $\forall x (\text{Student}(x) \wedge \text{Speaks}(x, \text{Russian}) \Rightarrow \text{Like}(x, \text{Rock}))$
 $\forall x (\text{Student}(x) \wedge \text{Like}(x, \text{Hamburger}) \Rightarrow \text{Speak}(x, \text{English}) \wedge \neg \text{Speak}(x, \text{Turkish}))$
 $\forall x (\text{Student}(x) \wedge \text{Like}(x, \text{Fish}) \wedge \text{Like}(x, \text{Classic}) \Rightarrow \text{Speak}(x, \text{Turkish}))$
 $\text{Like}(\text{Arda}, \text{Jazz}) \wedge \text{Like}(\text{Arda}, \text{Hamburger}) \wedge \text{Like}(\text{Arda}, \text{Fish}) \wedge \neg \text{Like}(\text{Arda}, \text{Classic}) \wedge \neg \text{Like}(\text{Arda}, \text{Rock})$
 $\forall x (\text{Music}(x) \wedge \text{Like}(\text{Arda}, x) \Rightarrow \neg \text{Like}(\text{Cihan}, x))$
 $\forall x (\text{Music}(x) \wedge \neg \text{Like}(\text{Arda}, x) \Rightarrow \text{Like}(\text{Cihan}, x))$
 $\text{Like}(\text{Cihan}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Cihan}, \text{Fish})$
 $\text{Like}(\text{Gamze}, \text{Fish}) \wedge \text{Like}(\text{Gamze}, \text{Classic}) \wedge \neg \text{Like}(\text{Gamze}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Gamze}, \text{Jazz}) \wedge \neg \text{Like}(\text{Gamze}, \text{Rock})$

Convert knowledge base to CNF:

- 1) $\neg \text{English}(x) \vee \text{Language}(x)$
- 2) $\neg \text{French}(x) \vee \text{Language}(x)$
- 3) $\neg \text{Russian}(x) \vee \text{Language}(x)$
- 4) $\neg \text{Turkish}(x) \vee \text{Language}(x)$
- 5) $\text{Student}(\text{Arda})$
- 6) $\text{Student}(\text{Cihan})$
- 7) $\text{Student}(\text{Gamze})$
- 8) $\text{University}(G(x)) \wedge \text{Enrolled}(\text{Arda}, G(x)) \wedge \text{Enrolled}(\text{Cihan}, G(x)) \wedge \text{Enrolled}(\text{Gamze}, G(x))$
- 9) $\neg \text{Student}(x) \vee \neg \text{University}(x) \vee (\text{Speak}(x, \text{Language}(G_1(x))) \wedge \text{Speak}(x, \text{Language}(G_2(x)))) \wedge (y \neq z)$
- 10) $\neg \text{Fish}(x) \vee \text{Food}(x)$
- 11) $\neg \text{Hamburger}(x) \vee \text{Food}(x)$
- 12) $\neg \text{Classic}(x) \vee \text{Music}(x)$
- 13) $\neg \text{Jazz}(x) \vee \text{Music}(x)$
- 14) $\neg \text{Rock}(x) \vee \text{Music}(x)$
- 15) $[\neg \text{Student}(x) \vee \neg \text{Speaks}(x, \text{French}) \vee (\text{Like}(x, \text{Jazz}))] \wedge [\neg \text{Student}(x) \vee \neg \text{Speaks}(x, \text{French}) \vee \neg \text{Like}(x, \text{Rock})]$
- 16) $\neg \text{Student}(x) \vee \neg \text{Speaks}(x, \text{Russian}) \vee (\text{Like}(x, \text{Rock}))$
- 17) $[\neg \text{Student}(x) \vee \neg \text{Like}(x, \text{Hamburger}) \vee \text{Speak}(x, \text{English})] \wedge [\neg \text{Student}(x) \vee \neg \text{Like}(x, \text{Hamburger}) \vee \neg \text{Speak}(x, \text{Turkish})]$
- 18) $\neg \text{Student}(x) \vee \neg \text{Like}(x, \text{Fish}) \vee \neg \text{Like}(x, \text{Classic}) \vee (\text{Speak}(x, \text{Turkish}))$
- 19) $\text{Like}(\text{Arda}, \text{Jazz}) \wedge \text{Like}(\text{Arda}, \text{Hamburger}) \wedge \text{Like}(\text{Arda}, \text{Fish}) \wedge \neg \text{Like}(\text{Arda}, \text{Classic}) \wedge \neg \text{Like}(\text{Arda}, \text{Rock})$
- 20) $\neg \text{Music}(x) \vee \neg \text{Like}(\text{Arda}, x) \vee (\neg \text{Like}(\text{Cihan}, x))$
- 21) $\neg \text{Music}(x) \vee \text{Like}(\text{Arda}, x) \vee (\text{Like}(\text{Cihan}, x))$
- 22) $\text{Like}(\text{Cihan}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Cihan}, \text{Fish})$
- 23) $\text{Like}(\text{Gamze}, \text{Fish}) \wedge \text{Like}(\text{Gamze}, \text{Classic}) \wedge \neg \text{Like}(\text{Gamze}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Gamze}, \text{Jazz}) \wedge \neg \text{Like}(\text{Gamze}, \text{Rock})$

b) Use resolution to answer the following queries:

1) Which student(s) speak(s) French

With the resolution algorithm we can only determine whether a query is True or False. Therefore for this query, (which expects a group of students as an answer) we have to determine whether each student can speak French separately.

Students = {Arda, Cihan, Gamze}

$\forall x (\text{Student}(x) \wedge \text{Speak}(x, \text{French}))$ Which contains the queries:

$\text{Speak}(\text{Arda}, \text{French}), \text{Speak}(\text{Cihan}, \text{French}), \text{Speak}(\text{Gamze}, \text{French})$

Using Proof by Contradiction for query **Speak(Arda, French)**

- 1) $\neg \text{English}(x) \vee \text{Language}(x)$
- 2) $\neg \text{French}(x) \vee \text{Language}(x)$
- 3) $\neg \text{Russian}(x) \vee \text{Language}(x)$
- 4) $\neg \text{Turkish}(x) \vee \text{Language}(x)$
- 5) $\text{Student}(\text{Arda})$
- 6) $\text{Student}(\text{Cihan})$
- 7) $\text{Student}(\text{Gamze})$
- 8) $\text{University}(G(x)) \wedge \text{Enrolled}(\text{Arda}, G(x)) \wedge \text{Enrolled}(\text{Cihan}, G(x)) \wedge \text{Enrolled}(\text{Gamze}, G(x))$
- 9) $\neg \text{Student}(x) \vee \neg \text{University}(x) \vee (\text{Speak}(x, \text{Language}(G_1(x))) \wedge \text{Speak}(x, \text{Language}(G_2(x))) \wedge (G_1(x) \neq G_2(x)))$
- 10) $\neg \text{Fish}(x) \vee \text{Food}(x)$
- 11) $\neg \text{Hamburger}(x) \vee \text{Food}(x)$
- 12) $\neg \text{Classic}(x) \vee \text{Music}(x)$
- 13) $\neg \text{Jazz}(x) \vee \text{Music}(x)$
- 14) $\neg \text{Rock}(x) \vee \text{Music}(x)$
- 15) $[\neg \text{Student}(x) \vee \neg \text{Speaks}(x, \text{French}) \vee (\text{Like}(x, \text{Jazz}))] \wedge [\neg \text{Student}(x) \vee \neg \text{Speaks}(x, \text{French}) \vee \neg \text{Like}(x, \text{Rock})]$
- 16) $\neg \text{Student}(x) \vee \neg \text{Speaks}(x, \text{Russian}) \vee (\text{Like}(x, \text{Rock}))$
- 17) $[\neg \text{Student}(x) \vee \neg \text{Like}(x, \text{Hamburger}) \vee \text{Speak}(x, \text{English})] \wedge [\neg \text{Student}(x) \vee \neg \text{Like}(x, \text{Hamburger}) \vee \neg \text{Speak}(x, \text{Turkish})]$
- 18) $\neg \text{Student}(x) \vee \neg \text{Like}(x, \text{Fish}) \vee \neg \text{Like}(x, \text{Classic}) \vee (\text{Speak}(x, \text{Turkish}))$
- 19) $\text{Like}(\text{Arda}, \text{Jazz}) \wedge \text{Like}(\text{Arda}, \text{Hamburger}) \wedge \text{Like}(\text{Arda}, \text{Fish}) \wedge \neg \text{Like}(\text{Arda}, \text{Classic}) \wedge \neg \text{Like}(\text{Arda}, \text{Rock})$
- 20) $\neg \text{Music}(x) \vee \neg \text{Like}(\text{Arda}, x) \vee (\neg \text{Like}(\text{Cihan}, x))$
- 21) $\neg \text{Music}(x) \vee \text{Like}(\text{Arda}, x) \vee (\text{Like}(\text{Cihan}, x))$
- 22) $\text{Like}(\text{Cihan}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Cihan}, \text{Fish})$
- 23) $\text{Like}(\text{Gamze}, \text{Fish}) \wedge \text{Like}(\text{Gamze}, \text{Classic}) \wedge \neg \text{Like}(\text{Gamze}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Gamze}, \text{Jazz}) \wedge \neg \text{Like}(\text{Gamze}, \text{Rock})$
- 24) $\neg \alpha$: $\neg \text{Speak}(\text{Arda}, \text{French})$**
- 25)** $\text{Like}(\text{Arda}, \text{Jazz}) \wedge \text{Like}(\text{Arda}, \text{Hamburger}) \wedge \text{Like}(\text{Arda}, \text{Fish}) \wedge \neg \text{Like}(\text{Arda}, \text{Classic}) \wedge \neg \text{Like}(\text{Arda}, \text{Rock}) \wedge \text{Student}(\text{Arda})$ by **using 19 and 5** (x: Arda)
- 26)** $\text{Like}(\text{Arda}, \text{Jazz}) \wedge \text{Like}(\text{Arda}, \text{Hamburger}) \wedge \text{Like}(\text{Arda}, \text{Fish}) \wedge \neg \text{Like}(\text{Arda}, \text{Classic})] \wedge \text{Student}(\text{Arda}) \wedge \neg \text{Speaks}(\text{Arda}, \text{Russian})$ by **using 26 and 16** (x: Arda)
- 27)** $\text{Like}(\text{Arda}, \text{Jazz}) \wedge \text{Like}(\text{Arda}, \text{Hamburger}) \wedge \text{Like}(\text{Arda}, \text{Fish}) \wedge \neg \text{Like}(\text{Arda}, \text{Classic})] \wedge \neg \text{Speaks}(\text{Arda}, \text{Russian}) \wedge [\text{Speak}(\text{Arda}, \text{English})] \wedge [\neg \text{Speak}(\text{Arda}, \text{Turkish})]$ by **using 26 and 17** (x: Arda)
- 28)** $\text{Like}(\text{Arda}, \text{Jazz}) \wedge \text{Like}(\text{Arda}, \text{Hamburger}) \wedge \text{Like}(\text{Arda}, \text{Fish}) \wedge \neg \text{Like}(\text{Arda}, \text{Classic})] \wedge \neg \text{Speaks}(\text{Arda}, \text{Russian}) \wedge [\text{Speak}(\text{Arda}, \text{English})] \wedge [\neg \text{Speak}(\text{Arda}, \text{Turkish})] \wedge \neg \text{Speak}(\text{Arda}, \text{French})$ by **using 27 and 24** (x: Arda)
- 29) {}** by **using 28 and 9** (x: Arda) $\rightarrow \text{KB} \wedge \neg \alpha$ is unsatisfiable therefore α is True (Arda speaks French)

Using Proof by Contradiction for query **Speak(Cihan, French)**

- 1) $\neg \text{English}(x) \vee \text{Language}(x)$
- 2) $\neg \text{French}(x) \vee \text{Language}(x)$
- 3) $\neg \text{Russian}(x) \vee \text{Language}(x)$
- 4) $\neg \text{Turkish}(x) \vee \text{Language}(x)$
- 5) $\text{Student}(\text{Arda})$
- 6) $\text{Student}(\text{Cihan})$
- 7) $\text{Student}(\text{Gamze})$
- 8) $\text{University}(G(x)) \wedge \text{Enrolled}(\text{Arda}, G(x)) \wedge \text{Enrolled}(\text{Cihan}, G(x)) \wedge \text{Enrolled}(\text{Gamze}, G(x))$
- 9) $\neg \text{Student}(x) \vee \neg \text{University}(x) \vee (\text{Speak}(x, \text{Language}(G_1(x))) \wedge \text{Speak}(x, \text{Language}(G_2(x))) \wedge (G_1(x) \neq G_2(x)))$
- 10) $\neg \text{Fish}(x) \vee \text{Food}(x)$
- 11) $\neg \text{Hamburger}(x) \vee \text{Food}(x)$
- 12) $\neg \text{Classic}(x) \vee \text{Music}(x)$
- 13) $\neg \text{Jazz}(x) \vee \text{Music}(x)$
- 14) $\neg \text{Rock}(x) \vee \text{Music}(x)$
- 15) $[\neg \text{Student}(x) \vee \neg \text{Speaks}(x, \text{French}) \vee (\text{Like}(x, \text{Jazz})] \wedge [\neg \text{Student}(x) \vee \neg \text{Speaks}(x, \text{French}) \vee \neg \text{Like}(x, \text{Rock})]$
- 16) $\neg \text{Student}(x) \vee \neg \text{Speaks}(x, \text{Russian}) \vee (\text{Like}(x, \text{Rock}))$
- 17) $[\neg \text{Student}(x) \vee \neg \text{Like}(x, \text{Hamburger}) \vee \text{Speak}(x, \text{English})] \wedge [\neg \text{Student}(x) \vee \neg \text{Like}(x, \text{Hamburger}) \vee \neg \text{Speak}(x, \text{Turkish})]$
- 18) $\neg \text{Student}(x) \vee \neg \text{Like}(x, \text{Fish}) \vee \neg \text{Like}(x, \text{Classic}) \vee (\text{Speak}(x, \text{Turkish}))$
- 19) $\text{Like}(\text{Arda}, \text{Jazz}) \wedge \text{Like}(\text{Arda}, \text{Hamburger}) \wedge \text{Like}(\text{Arda}, \text{Fish}) \wedge \neg \text{Like}(\text{Arda}, \text{Classic}) \wedge \neg \text{Like}(\text{Arda}, \text{Rock})$
- 20) $\neg \text{Music}(x) \vee \neg \text{Like}(\text{Arda}, x) \vee (\neg \text{Like}(\text{Cihan}, x))$
- 21) $\neg \text{Music}(x) \vee \text{Like}(\text{Arda}, x) \vee (\text{Like}(\text{Cihan}, x))$
- 22) $\text{Like}(\text{Cihan}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Cihan}, \text{Fish})$
- 23) $\text{Like}(\text{Gamze}, \text{Fish}) \wedge \text{Like}(\text{Gamze}, \text{Classic}) \wedge \neg \text{Like}(\text{Gamze}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Gamze}, \text{Jazz}) \wedge \neg \text{Like}(\text{Gamze}, \text{Rock})$
- 24) $\neg \alpha$: $\neg \text{Speak}(\text{Cihan}, \text{French})$**
- 25) $[\text{Like}(\text{Arda}, \text{Rock}) \vee (\text{Like}(\text{Cihan}, \text{Rock}))] \wedge \text{Student}(\text{Cihan})$ by combining 6 and 21 (x: Rock)**
- 26) $[\text{Like}(\text{Cihan}, \text{Rock})] \wedge \text{Student}(\text{Cihan})$ by combining 25 and 19**
- 27) $[\neg \text{Speaks}(\text{Cihan}, \text{French}) \vee (\text{Like}(\text{Cihan}, \text{Jazz})] \wedge [\neg \text{Speaks}(\text{Cihan}, \text{French})] \wedge [\text{Like}(\text{Cihan}, \text{Rock})] \wedge \text{Student}(\text{Cihan})$ by combining 26 and 15 (x: Cihan) \rightarrow we obtained our query $\neg \text{Speaks}(\text{Cihan}, \text{French})$ by combining other expressions in our knowledge base therefore the query is false (Cihan does not speak French)**

Using Proof by Contradiction for query **Speak(Gamze, French)**

- 1) $\neg \text{English}(x) \vee \text{Language}(x)$
- 2) $\neg \text{French}(x) \vee \text{Language}(x)$
- 3) $\neg \text{Russian}(x) \vee \text{Language}(x)$
- 4) $\neg \text{Turkish}(x) \vee \text{Language}(x)$
- 5) $\text{Student}(\text{Arda})$
- 6) $\text{Student}(\text{Cihan})$
- 7) $\text{Student}(\text{Gamze})$
- 8) $\text{University}(G(x)) \wedge \text{Enrolled}(\text{Arda}, G(x)) \wedge \text{Enrolled}(\text{Cihan}, G(x)) \wedge \text{Enrolled}(\text{Gamze}, G(x))$
- 9) $\neg \text{Student}(x) \vee \neg \text{University}(x) \vee (\text{Speak}(x, \text{Language}(G_1(x))) \wedge \text{Speak}(x, \text{Language}(G_2(x))) \wedge (G_1(x) \neq G_2(x)))$
- 10) $\neg \text{Fish}(x) \vee \text{Food}(x)$
- 11) $\neg \text{Hamburger}(x) \vee \text{Food}(x)$
- 12) $\neg \text{Classic}(x) \vee \text{Music}(x)$
- 13) $\neg \text{Jazz}(x) \vee \text{Music}(x)$
- 14) $\neg \text{Rock}(x) \vee \text{Music}(x)$
- 15) $[\neg \text{Student}(x) \vee \neg \text{Speaks}(x, \text{French}) \vee (\text{Like}(x, \text{Jazz})] \wedge [\neg \text{Student}(x) \vee \neg \text{Speaks}(x, \text{French}) \vee \neg \text{Like}(x, \text{Rock})]$
- 16) $\neg \text{Student}(x) \vee \neg \text{Speaks}(x, \text{Russian}) \vee (\text{Like}(x, \text{Rock}))$
- 17) $[\neg \text{Student}(x) \vee \neg \text{Like}(x, \text{Hamburger}) \vee \text{Speak}(x, \text{English})] \wedge [\neg \text{Student}(x) \vee \neg \text{Like}(x, \text{Hamburger}) \vee \neg \text{Speak}(x, \text{Turkish})]$
- 18) $\neg \text{Student}(x) \vee \neg \text{Like}(x, \text{Fish}) \vee \neg \text{Like}(x, \text{Classic}) \vee (\text{Speak}(x, \text{Turkish}))$
- 19) $\text{Like}(\text{Arda}, \text{Jazz}) \wedge \text{Like}(\text{Arda}, \text{Hamburger}) \wedge \text{Like}(\text{Arda}, \text{Fish}) \wedge \neg \text{Like}(\text{Arda}, \text{Classic}) \wedge \neg \text{Like}(\text{Arda}, \text{Rock})$
- 20) $\neg \text{Music}(x) \vee \neg \text{Like}(\text{Arda}, x) \vee (\neg \text{Like}(\text{Cihan}, x))$
- 21) $\neg \text{Music}(x) \vee \text{Like}(\text{Arda}, x) \vee (\text{Like}(\text{Cihan}, x))$
- 22) $\text{Like}(\text{Cihan}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Cihan}, \text{Fish})$
- 23) $\text{Like}(\text{Gamze}, \text{Fish}) \wedge \text{Like}(\text{Gamze}, \text{Classic}) \wedge \neg \text{Like}(\text{Gamze}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Gamze}, \text{Jazz}) \wedge \neg \text{Like}(\text{Gamze}, \text{Rock})$
- 24) $\neg \alpha: \neg \text{Speak}(\text{Gamze}, \text{French})$**
- 25) $\text{Like}(\text{Gamze}, \text{Fish}) \wedge \text{Like}(\text{Gamze}, \text{Classic}) \wedge \neg \text{Like}(\text{Gamze}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Gamze}, \text{Jazz}) \wedge \neg \text{Like}(\text{Gamze}, \text{Rock}) \wedge \text{Student}(\text{Gamze})$ by combining 23 and 7**
- 26) $\text{Like}(\text{Gamze}, \text{Fish}) \wedge \text{Like}(\text{Gamze}, \text{Classic}) \wedge \neg \text{Like}(\text{Gamze}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Gamze}, \text{Jazz}) \wedge \neg \text{Like}(\text{Gamze}, \text{Rock}) \wedge \text{Student}(\text{Gamze}) \wedge [\neg \text{Speaks}(\text{Gamze}, \text{French})] \wedge [\neg \text{Speaks}(\text{Gamze}, \text{French}) \vee \neg \text{Like}(\text{Gamze}, \text{Rock})]$ by combining 25 and 15 (x: Gamze) \rightarrow We obtained our contradicted query $\neg \text{Speak}(\text{Gamze}, \text{French})$ by combining other expressions in our knowledge base. Therefore **$\text{Speaks}(\text{Gamze}, \text{French})$ is False (Gamze does not speak French)****

Results: $\forall x (\text{Student}(x) \wedge \text{Speak}(x, \text{French})) \equiv$ Which student(s) speak(s) French $\equiv \{\text{Arda}\}$

2) Which student(s) speak(s) both English and Turkish

With the resolution algorithm we can only determine whether a query is True or False. Therefore for this query, (which expects a group of students as an answer) we have to determine whether each student can speak both English and Turkish separately.

Students = {Arda, Cihan, Gamze}

$\forall x (\text{Student}(x) \wedge \text{Speak}(x, \text{English}) \wedge \text{Speak}(x, \text{Turkish}))$ Which contains the queries:

$\text{Speak}(\text{Arda}, \text{Turkish}) \wedge \text{Speak}(\text{Arda}, \text{English}), \text{Speak}(\text{Cihan}, \text{Turkish}) \wedge \text{Speak}(\text{Cihan}, \text{English}), \text{Speak}(\text{Gamze}, \text{Turkish}) \wedge \text{Speak}(\text{Gamze}, \text{English})$

Using Proof by Contradiction for query: $\text{Speak}(\text{Cihan}, \text{English}) \wedge \text{Speak}(\text{Cihan}, \text{Turkish})$

- 1) $\neg \text{English}(x) \vee \text{Language}(x)$
- 2) $\neg \text{French}(x) \vee \text{Language}(x)$
- 3) $\neg \text{Russian}(x) \vee \text{Language}(x)$
- 4) $\neg \text{Turkish}(x) \vee \text{Language}(x)$
- 5) $\text{Student}(\text{Arda})$
- 6) $\text{Student}(\text{Cihan})$
- 7) $\text{Student}(\text{Gamze})$
- 8) $\text{University}(G(x)) \wedge \text{Enrolled}(\text{Arda}, G(x)) \wedge \text{Enrolled}(\text{Cihan}, G(x)) \wedge \text{Enrolled}(\text{Gamze}, G(x))$
- 9) $\neg \text{Student}(x) \vee \neg \text{University}(x) \vee (\text{Speak}(x, \text{Language}(G_1(x))) \wedge \text{Speak}(x, \text{Language}(G_2(x))) \wedge (G_1(x) \neq G_2(x)))$
- 10) $\neg \text{Fish}(x) \vee \text{Food}(x)$
- 11) $\neg \text{Hamburger}(x) \vee \text{Food}(x)$
- 12) $\neg \text{Classic}(x) \vee \text{Music}(x)$
- 13) $\neg \text{Jazz}(x) \vee \text{Music}(x)$
- 14) $\neg \text{Rock}(x) \vee \text{Music}(x)$
- 15) $[\neg \text{Student}(x) \vee \neg \text{Speaks}(x, \text{French}) \vee (\text{Like}(x, \text{Jazz})] \wedge [\neg \text{Student}(x) \vee \neg \text{Speaks}(x, \text{French}) \vee \neg \text{Like}(x, \text{Rock})]$
- 16) $\neg \text{Student}(x) \vee \neg \text{Speaks}(x, \text{Russian}) \vee (\text{Like}(x, \text{Rock}))$
- 17) $[\neg \text{Student}(x) \vee \neg \text{Like}(x, \text{Hamburger}) \vee \text{Speak}(x, \text{English})] \wedge [\neg \text{Student}(x) \vee \neg \text{Like}(x, \text{Hamburger}) \vee \neg \text{Speak}(x, \text{Turkish})]$
- 18) $\neg \text{Student}(x) \vee \neg \text{Like}(x, \text{Fish}) \vee \neg \text{Like}(x, \text{Classic}) \vee (\text{Speak}(x, \text{Turkish}))$
- 19) $\text{Like}(\text{Arda}, \text{Jazz}) \wedge \text{Like}(\text{Arda}, \text{Hamburger}) \wedge \text{Like}(\text{Arda}, \text{Fish}) \wedge \neg \text{Like}(\text{Arda}, \text{Classic}) \wedge \neg \text{Like}(\text{Arda}, \text{Rock})$
- 20) $\neg \text{Music}(x) \vee \neg \text{Like}(\text{Arda}, x) \vee (\neg \text{Like}(\text{Cihan}, x))$
- 21) $\neg \text{Music}(x) \vee \text{Like}(\text{Arda}, x) \vee (\text{Like}(\text{Cihan}, x))$
- 22) $\text{Like}(\text{Cihan}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Cihan}, \text{Fish})$
- 23) $\text{Like}(\text{Gamze}, \text{Fish}) \wedge \text{Like}(\text{Gamze}, \text{Classic}) \wedge \neg \text{Like}(\text{Gamze}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Gamze}, \text{Jazz}) \wedge \neg \text{Like}(\text{Gamze}, \text{Rock})$
- 24) $\neg \alpha: \neg \text{Speak}(\text{Cihan}, \text{Turkish}) \vee \neg \text{Speak}(\text{Cihan}, \text{English})$**
- 25) $\text{Student}(\text{Cihan}) \wedge \text{Like}(\text{Cihan}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Cihan}, \text{Fish})$ by combining 6 and 22**
- 26) $[\text{Speak}(\text{Cihan}, \text{English})] \wedge [\neg \text{Speak}(\text{Cihan}, \text{Turkish})] \wedge \text{Student}(\text{Cihan}) \wedge \text{Like}(\text{Cihan}, \text{Hamburger}) \wedge \neg \text{Like}(\text{Cihan}, \text{Fish})$ by combining 25 and 17 (x:Cihan)**
- 27) $[\neg \text{Speak}(\text{Cihan}, \text{Turkish})] \wedge \neg [\text{Speak}(\text{Cihan}, \text{English})] \wedge [\neg \text{Speak}(\text{Cihan}, \text{English}) \vee \neg \text{Speak}(\text{Cihan}, \text{Turkish})] \wedge \text{Student}(\text{Cihan}) \wedge$**

Like(Cihan, Hamburger) \wedge \neg Like(Cihan,Fish) **by combining 26 and 24 \rightarrow we obtained our query $\neg\alpha$ by combining other expressions. Therefore α is False (Cihan does not speak both English and Turkish)**

Using Proof by Contradiction for query: Speak(Arda, English) \wedge Speak(Arda, Turkish)

- 1) \neg English(x) \vee Language(x)
- 2) \neg French(x) \vee Language(x)
- 3) \neg Russian(x) \vee Language(x)
- 4) \neg Turkish(x) \vee Language(x)
- 5) Student(Arda)
- 6) Student(Cihan)
- 7) Student(Gamze)
- 8) University(G(x)) \wedge Enrolled(Arda,G(x)) \wedge Enrolled(Cihan,G(x)) \wedge Enrolled(Gamze,G(x))
- 9) \neg Student(x) \vee \neg University(x) \vee (Speak(x, Language(G₁(x))) \wedge Speak(x, Language(G₂(x))) \wedge (G₁(x) \neq G₂(x)))
- 10) \neg Fish(x) \vee Food(x)
- 11) \neg Hamburger(x) \vee Food(x)
- 12) \neg Classic(x) \vee Music(x)
- 13) \neg Jazz(x) \vee Music(x)
- 14) \neg Rock(x) \vee Music(x)
- 15) [\neg Student(x) \vee \neg Speaks(x, French) \vee (Like(x, Jazz))] \wedge [\neg Student(x) \vee \neg Speaks(x, French) \vee \neg Like(x, Rock)]
- 16) \neg Student(x) \vee \neg Speaks(x, Russian) \vee (Like(x, Rock))
- 17) [\neg Student(x) \vee \neg Like(x, Hamburger) \vee Speak(x,English)] \wedge [\neg Student(x) \vee \neg Like(x, Hamburger) \vee \neg Speak(x,Turkish)]
- 18) \neg Student(x) \vee \neg Like(x, Fish) \vee \neg Like(x, Classic) \vee (Speak(x, Turkish))
- 19) Like(Arda, Jazz) \wedge Like(Arda, Hamburger) \wedge Like(Arda, Fish) \wedge \neg Like(Arda,Classic) \wedge \neg Like(Arda, Rock)
- 20) \neg Music(x) \vee \neg Like(Arda,x) \vee (\neg Like(Cihan, x))
- 21) \neg Music(x) \vee Like(Arda,x) \vee (Like(Cihan, x))
- 22) Like(Cihan, Hamburger) \wedge \neg Like(Cihan,Fish)
- 23) Like(Gamze, Fish) \wedge Like(Gamze, Classic) \wedge \neg Like(Gamze, Hamburger) \wedge \neg Like(Gamze, Jazz) \wedge \neg Like(Gamze, Rock)
- 24) $\neg\alpha$: \neg Speak(Arda, Turkish) \vee \neg Speak(Arda, English)**
- 25) Student(Arda) \wedge Like(Arda, Jazz) \wedge Like(Arda, Hamburger) \wedge Like(Arda, Fish) \wedge \neg Like(Arda,Classic) \wedge \neg Like(Arda, Rock) by combining 5 and 19**
- 26) [Speak(Arda,English)] \wedge [\neg Speak(Arda,Turkish)] \wedge Student(Arda) \wedge Like(Arda, Jazz) \wedge Like(Arda, Hamburger) \wedge Like(Arda, Fish) \wedge \neg Like(Arda,Classic) \wedge \neg Like(Arda, Rock) by combining 25 and 17 (x:Arda)**
- 27) [\neg Speak(Arda, Turkish)] \wedge [\neg Speak(Arda, English)] \wedge [\neg Speak(Arda,English) \vee \neg Speak(Arda,Turkish)] \wedge Student(Arda) \wedge Like(Arda, Jazz) \wedge Like(Arda, Hamburger) \wedge Like(Arda, Fish) \wedge \neg Like(Arda,Classic) \wedge \neg Like(Arda, Rock) by combining 26 and 24 \rightarrow we obtained our query $\neg\alpha$ by combining other expressions. Therefore α is False (Arda does not speak both English and Turkish)**

Problem 3)

Here I implemented the Minimax algorithm with and without using alpha-beta pruning.

To run the program, enter the code directory and run the command:
`python ./main.py <minimax or alphabeta> <filename>`
to run the program.

The class (node) used, which is called DotsAndBoxes, is located in the folder `dots_and_boxes.py`.

The minimax without pruning algorithm is located in the folder `dots_and_boxes_ai.py` with the function name `minimax()`.

The minimax with alpha beta pruning algorithm is located in the folder `dots_and_boxes_ai.py` with the function name `alphabeta()`.

The runtime of the 1x1 and 1x2 boards finish almost instantly when using both methods.

The runtime of the 2x2 board takes 8-10 seconds to finish using alpha-beta pruning method.

The runtime of the 2x2 board takes over 15 minutes (I did not wait for it to finish) to finish using minimax without pruning method.