

**Q.4 Ans:**

Initially begin with purely exponential or polynomial functions.

The best way to solve these kinds of problems is first let's start with exponential and polynomial functions.

Here, the function  $g_3(n) = n(\log n)^3$  Which can be written as  $O(n^{4/3})$ .

So the growth rate of this function is  $n(\log n)^3 = O(n^{4/3})$ . Which means  $g_3$  will follow  $g_4$ .

Now let's consider polynomial functions,  $g_2$  and  $g_7$  both are polynomial functions. And the order it follows is  $g_2$  and  $g_7$ .

Here  $2^n = O(2^{n^2})$  and

$2^{n^2} = O(2^{2n})$ .

Let's look at the functions which have involved both exponential and logarithms.

Comparing upon the  $g_3$  and  $g_4$  functions it is evident that the  $g_3$  will have less growth rate than  $g_4$ .

Let's say the base algorithm for all the logs here in the question is 2 and the function  $g_1$  returns the same value as  $n^{1/2}$  which becomes the regular equation. So now this can be written in according to growth order is  $2^{\sqrt{\log n}} = O(2^{\log n})$ .

So we can conclude that  $\sqrt{\log n}$  is followed by  $\log n$ .

$n^{4/3} < O(n^{\log n})$  this is because the  $n$  power is always some part of  $n(\text{power})\log n$ .

Other remaining functions are  $g_6, g_7, g_2$ . Let's compare  $g_5$  with these three functions.

$G_5$  which is  $n(\log n)$  and  $g_6, g_7$  and  $g_2$  are exponential function of 2, which follows the below order.

$N(\log n)$  will have less growth rate,  $2(n)$  and  $2(n)^2$  will follow one by one. At the end the  $g_7$  will have highest growth order.

$2^{\sqrt{\log n}}$  and  $(n(\log n)) = 2^{\sqrt{\log n}} = 2^{(\log n)^{1/2}}$  which can be written as  $O(2^n)$

Growth rate for exponential will be always more so that these functions fall at last in the sequence when comparing with the growth rate.

The following is the ascending order of the growth rate functions:

$$g_1 < g_3 < g_4 < g_5 < g_2 < g_7 < g_6.$$