Contents

1 Network Design Model

This section covers the mathematical optimization modeling of the case study. Firstly, the variable sets, decision variables and cost are defined. Afterwards, the forecast of the demand is calculated based on the information provided in the case study given by the course. Next, and taking this information into account, the mathematical optimization modeling is applied using Julia programming and the results are presented at the end of the section.

The first step taken in order to solve the network design problem described in the case study for the period between 2007 and 2011, was creating a mathematical optimization model taking into consideration all the information given. The sets, decision variables, costs and capacity and demand were defined as shown in Tables 2, 3, 4 & 5.

Sets

$Customers:$ i $\in I = \{Northwest, Southwest, UpperMidwest, LowerMidwest, Northeast, Southeast\}$

$Periods : p \in P = \{2007, 2008, 2009, 2010, 2011\}$

$Warehouses : j \in J = \{Seattle-S, Denver-S, St.Louis-S, Atlanta-S, Philadelphia-S, Seattle-L, Denver-L, St.Louis-L, Atlanta-L, Philadelphia-L\}$
(S - Small Warehouse; L - Large Warehouse)

Set	Size	Description
I	6	Customer/Zone
J	10	Warehouse
P	5	Year

Table 2: Sets Description

Decision Variables

Decision Variables	Type	Description
x_{jp}	Binary: {0,1}	1 if warehouse j is open in year p, 0 otherwise
y_{ijp}	Positive integer: Z+	Number of units shipped from the warehouse j to customers i in year p
α_{jp}	Binary: {0,1}	Lease coefficient, 1 if lease of warehouse j starts in year p, 0 otherwise

Table 3: Decision Variables

Coefficients of the model

Costs	**Variable**	**Description**
Fixed costs	f_j	Fixed yearly cost for leasing warehouse j
Variable costs	b_j	Variable cost for warehouse j per unit flow
Transportation costs	$c_{ji} - 3$	Transportation cost of shipping 4 units from warehouse j to customer i
Inventory costs	$475000 \cdot x_{jp} + 0.165 \cdot \sum_{i=1}^{I} y_{ijp}$	Inventory cost per warehouse j for the total unit flow per year

Table 4: Costs

Name	**Variable**	**Description**
Annual demand	h_{ip}	Demand of customer i in year p
Capacity	v_j	Capacity of warehouse j

Table 5: Demand & Capacity

Mathematical Model

The final mathematical model is then as follows:

$$Min[\sum_{p=1}^{P}\sum_{j=1}^{J}(f_j \cdot x_{jp} + 475000 \cdot x_{jp} + 0.165\sum_{i=1}^{I} y_{ijp}) + \sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{i=1}^{I}(b_j \cdot y_{ijp} + \frac{c_{ji} - 3}{4} \cdot y_{ijp})] \tag{1}$$

s.t.

$$\sum_{j=1}^{J} y_{ijp} = h_{ip} \qquad \forall\, i \in I,\ p \in P \tag{2}$$

$$\sum_{i=1}^{I} y_{ijp} \leq v_j \cdot x_{jp} \qquad \forall\, j \in J,\ p \in P \tag{3}$$

$$\sum_{p=p}^{p+2} a_{jp} \leq 1 \qquad \forall\, j \in J,\ p = 1:3 \tag{4}$$

$$\sum_{p=p}^{p+1} a_{jp} \leq 1 \qquad \forall\, j \in J,\ p = 4 \tag{5}$$

$$3 \cdot a_{jp} \leq \sum_{p=p}^{p+2} x_{jp} \qquad \forall\, j \in J,\ p = 1:3 \tag{6}$$

$$2 \cdot a_{jp} \leq \sum_{p=p}^{p+1} x_{jp} \qquad \forall\, j \in J,\ p = 4 \tag{7}$$

$$\sum_{q=p-2}^{p} a_{jp} \geq x_{jp} \qquad \forall\, j \in J,\ p = 3:5 \tag{8}$$

$$\sum_{q=p-1}^{p} a_{jp} \geq x_{jp} \qquad \forall\, j \in J,\ p = 2 \tag{9}$$

$$a_{jp} \geq x_{jp} \qquad \forall\, j \in J,\ p = 1 \tag{10}$$

$$x_{jp} + x_{j+5p} \leq 1 \qquad \forall\, p \in P,\ j = 1:5 \tag{11}$$

$$x_{jp} \in \{0, 1\} \qquad \forall\, p \in P,\ j \in J \tag{12}$$

$$y_{ijp} \in Z^{+} \qquad \forall\, p \in P,\ j \in J,\ i \in I \tag{13}$$

$$a_{jp} \in \{0, 1\} \qquad \forall\, p \in P,\ j \in J \tag{14}$$

The objective function 1 computes the total fixed, variable, inventory and transportation cost, and the goal is to minimize the total sum of all these costs.

Constraints 2 require the total amount of units y_{ijp} shipped from warehouse j to customer i to be equal to customer's demand in period p. Therefore this constraint ensures that demand is satisfied at all periods.

Constraints 3 require the total amount of units y_{ijp} shipped to all customers $i \in I$ from warehouse j in a period p to be less or equal to the capacity v_j of warehouse j. It also ensures that if a warehouse is not being leased one period p, then we cannot use it to satisfy part of the demand (basically that if $x_{jp} = 0$, then $y_{ijp} = 0$ too).

The next set of constraints, from 4 to 10 all help ensure that the lease constraint holds and the lease coefficient is well defined:

For warehouse j, constraints 4 tells that the lease coefficient α_{jp} can only be 1 once in 3 consecutive periods. For warehouse j if the lease starts at p=4, constraint 5 restricts the lease coefficient α_{jp} to be 1 only once in 2 consecutive periods instead of 3, since we are only considering 5 periods in total.

Constraints 6 require that warehouse j must be open at least 3 consecutive years, if it has been leased in periods p=1:3. Constraints 7 require that warehouse j must be open at least 2 consecutive years, if it has been leased in period p=4. Therefore these constraints help to ensure that the lease agreement holds.

Constraints 8,9,10 require that a warehouse j can be open in period p if it has been leased on the same period or in previous periods p. This constraints as a whole help ensure that the variables a_{jp} are well defined, in the sense that they must take value 1 if some x_{jp} takes value 1.

Constraint 11 ensures that for a certain city a small warehouse and a large warehouse cannot be open or leased at the same time.

Finally, constraints 12,13 and 14 are in the model to make sure the decision variables are well defined in the range values that they should.

The respective code can be found in the file - Project_1-Part 1.jl

Results

Objective value/Total cost = $14.661263 MM

Warehouse-Customer Matrix: (See table in the next page)

As can be observed in table 6, in the first two years only a large warehouse in Denver was needed to be in operation in order to fully cover the customers demand. In 2009, the total demand of customers was covered, but this time using both a large warehouse in Denver and a small one in Philadelphia. The small warehouse in Philadelphia served the Northeast and Southeast customers, while all the rest were serviced by the large in Denver. In the next two years, the total demand was covered using two large warehouses in Seattle and in

Atlanta and a small one in in Philadelphia again. In these two last years however Northeast customers were serviced by the small warehouse almost entirely, with only a 2.02% of the demand being covered by the large warehouse in Atlanta.

Year (p)	Warehouse (j)	Customer (i)	Units (y_{ijp})
2007	Denver - L	Northwest	320,000
		Southwest	200,000
		Upper Midwest	160,000
		Lower Midwest	220,000
		Northeast	250,000
		Southeast	175,000
2008	Denver - L	Northwest	576,000
		Southwest	360,000
		Upper Midwest	288,000
		Lower Midwest	396,000
		Northeast	630,000
		Southeast	315,000
2009	Philadelphia - S	Northeast	1,134,000
		Southeast	567,000
	Denver - L	Northwest	1,036,800
		Southwest	648,000
		Upper Midwest	518,400
		Lower Midwest	712,800
2010	Philadelphia - S	Northeast	2,000,000
	Seattle - L	Northwest	1,866,240
		Southwest	1,166,400
	Atlanta - L	Upper Midwest	933,120
		Lower Midwest	1,283,040
		Northeast	41,200
		Southeast	1,020,600
2011	Philadelphia - S	Northeast	2,000,000
	Seattle - L	Northwest	1,866,240
		Southwest	1,166,400
	Atlanta - L	Upper Midwest	933,120
		Lower Midwest	1,283,040
		Northeast	41,200
		Southeast	1,020,600

Table 6: Results

2 Flexibility

The flexibility section of the case study evaluates what happens with the previous model if the lease agreement is valid only for a year instead of three, and the leasing cost only covers the part of the warehouse in use. Therefore, there is no need to lease an entire warehouse, but only the part of it that is being used.

<u>**Sets**</u>

We are using in this case the same sets that were defined in the previous section.

Set	Size	Description
I	6	Customer/Zone
J	10	Warehouse
P	5	Year

Table 7: Sets Description

<u>**Decision Variables**</u>

Decision Variables	Type	Description
x_{jp}	Positive real $\in [0,1]$	Fraction of the warehouse j that is utilized in year p
y_{ijp}	Potive integer: Z^+	Number of units shipped from the warehouse j to customers i in year p

Table 8: Decision Variables

As seen in the table above, the decision variables in this new model have been adjusted. The variables α_{jp} were not included anymore, since there isn't a lease constraint. Also now the variables x_jp instead of indicating if a warehouse is open or not, they indicate the proportion of the warehouse j that is utilized. Therefore, they are not binary anymore, but instead they are defined as a positive real number that can take values between 0 and 1.

Coefficients of the model

The costs and other coefficients needed in the model have remained the same.

Costs	Variable	Description
Fixed costs	f_j	Fixed cost for warehouse j
Variable costs	b_j	Variable cost for warehouse j per unit
Transportation costs	$c_{ji} - 3$	Transportation cost of shipping 4 units from warehouse j to customer i
Inventory costs	$475000 \cdot x_{jp} + 0.165 \cdot \sum_{i=1}^{I} y_{ijp}$	Inventory cost per warehouse j for the total unit flow per year

Table 9: Costs

Name	Variable	Description
Annual demand	h_{ip}	Demand of customer i in year p
Capacity	v_j	Capacity of warehouse j

Table 10: Demand & Capacity

Mathematical Model

The resulting mathematical model is as follows:

$$Min[\sum_{p=1}^{P}\sum_{j=1}^{J}(f_j \cdot x_{jp} + 475000 \cdot x_{jp} + 0.165\sum_{i=1}^{I} y_{ijp}) + \sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{i=1}^{I}(b_j \cdot y_{ijp} + \frac{c_{ji} - 3}{4} \cdot y_{ijp})] \tag{1}$$

$$\text{s.t.} \quad \sum_{j=1}^{J} y_{ijp} = h_{ip} \qquad \forall\, i \in I,\, p \in P \tag{2}$$

$$\sum_{i=1}^{I} y_{ijp} \leq v_j \cdot x_{jp} \qquad \forall\, j \in J,\, p \in P \tag{3}$$

$$x_{jp} \cdot x_{j+5p} = 0 \qquad \forall\, p \in P,\, j = 1:5 \tag{4}$$

$$x_{jp} \in [0,1] \qquad \forall\, p \in P,\, j \in J \tag{5}$$

$$y_{ijp} \in Z^{+} \qquad \forall\, p \in P,\, j \in J,\, i \in I \tag{6}$$

Firstly it can be seen that now there is much simpler model, in terms of the number of constraints needed.

The objective function 1 which has not changed, computes the total fixed, variable, inventory and transportation cost, as in the previous part of the case.

Constraints 2 require the total amount of units y_{ijp} shipped from warehouse j to customer i to be equal to customer's demand in period p as in the previous part.

Constraints 3 require the total amount of units y_{ijp} shipped from warehouse $j \in J$ in a period p to be less or equal to the capacity v_j of the warehouse as in the previous part too.

Constraints 4 ensures that for a certain city a small warehouse and a large warehouse cannot be open or leased at the same time. It cannot be used the same constraint that we did before in order to ensure this, because the type of the variables x_{jp} has changed. Now instead it is required to use a non-linear constraint, and consequently a non-linear solving method was implemented.

Finally, the last set of constraints 5 and 6 inforce that the decision variables are well defined.

The respective code can be found in the file - Project_1-Part 2.jl
Result

Objective value/Total cost = $10.188652 MM

Warehouse-Customer Matrix: (See table in the next page)

As observed in table 11, only large warehouses are leased in all the periods. This result is due to the lower cost per unit of the large warehouses as compared to the small ones. This lower cost is possible only because of the fraction of the fixed cost directly proportional to the fraction of warehouse leased that is taken into account. Due to this flexibility, the total cost of the network is now reduced in comparison with the obtained in the first section. As seen in the table, in the first year 2007, the capacity of each warehouse needed to service all customers is low with less than 10% utilized for each. The capacities start to increase the following year and afterwards, reaching higher fractions of around 50% in 2010 and 2011.

Year (p)	Warehouse (j)	Cap. % (x_{jp})	Customer (i)	Units (y_{ijp})
2007	Seattle - L	8%	Northwest	320,000
	Denver - L	5%	Southwest	200,000
	St. Louis - L	9.5%	Upper Midwest	160,000
			Lower Midwest	220,000
	Atlanta - L	4.37%	Southeast	175,000
	Philadelphia - L	8.75%	Northeast	350,000
2008	Seattle - L	14.39%	Northwest	576,000
	Denver - L	9%	Southwest	360,000
	St. Louis - L	17.09%	Upper Midwest	288,000
			Lower Midwest	396,000
	Atlanta - L	7.87%	Southeast	315,000
	Philadelphia - L	15.75%	Northeast	630,000
2009	Seattle - L	25.91%	Northwest	1,036,800
	Denver - L	16.2%	Southwest	648,000
	St. Louis - L	30.77%	Upper Midwest	518,400
			Lower Midwest	712,800
	Atlanta - L	14.17%	Southeast	567,000
	Philadelphia - L	28.35%	Northeast	1,134,000
2010	Seattle - L	46.65%	Northwest	1,866,240
	Denver - L	29.15%	Southwest	1,166,400
	St. Louis - L	55.4%	Upper Midwest	933,210
			Lower Midwest	1,283,040
	Atlanta - L	25.51%	Southeast	1,020,600
	Philadelphia - L	51.03%	Northeast	2,041,200
2011	Seattle - L	46.65%	Northwest	1,866,240
	Denver - L	29.15%	Southwest	1,166,400
	St. Louis - L	55.4%	Upper Midwest	933,210
			Lower Midwest	1,283,040
	Atlanta - L	25.51%	Southeast	1,020,600
	Philadelphia - L	51.03%	Northeast	2,041,200

Table 11: Results

3 Resilience

This section evaluates the model's response to significant deviations, testing the potential reactions under uncertainty. For this purpose, a scenario based approach was selected, using the stochastic optimization. Firstly, a set of scenarios was defined using a defined deviation from the base case description, afterwards each scenario was assigned a matching probability and, finally, the optimization model was applied using these values. In this section of the case study it was required to evaluate two different deviations, first demand and then warehouse capacity which are described in the corresponding subsections.

3.1 Demand deviation

One of the situations where the behaviour of the model can be tested is in case the demand increases more than expected. Therefore 6 scenarios were selected, 5 considering a rather small deviation of 10% in each year respectively from the base scenario, and the last one considering an extreme demand deviation of 60% for all years, compared to the base scenario.

Sets

$Customers:$ i $\in I = \{Northwest, Southwest, UpperMidwest, LowerMidwest, Northeast, Southeast\}$

$Periods : p \in P = \{2007, 2008, 2009, 2010, 2011\}$

$Warehouses : j \in J = \{Seattle - S, Denver - S, St.Louis - S, Atlanta - S, Philadelphia - S, Seattle - L, Denver - L, St.Louis - L, Atlanta - L, Philadelphia - L\}$

$Scenarios : s \in S = \{S1, S2, S3, S4, S5, S6\}$

Set	Size	Description
I	6	Customer/Zone
J	10	Warehouse
P	5	Year
S	6	Scenarios

Table 12: Sets Description

Decision Variables

Decision Variables	Type	Description
x_{jp}	Binary: {0,1}	1 if warehouse j is open in year p, 0 otherwise
y_{ijps}	Positive integer: Z+	Number of units shipped from the warehouse j to customers i in year p in scenario s
α_{jp}	Binary: {0,1}	Lease coefficient, 1 if lease of warehouse j starts in year p, 0 otherwise

Table 13: Decision Variables

Notice that the main and only difference with the original definition of the variables is that now, the variables y_{ijps} depend on the scenario that they are in.

Coefficients of the model

Costs	Variable	Description
Fixed costs	f_j	Fixed cost for warehouse j
Variable costs	b_j	Variable cost for warehouse j per unit
Transportation costs	$c_{ji} - 3$	Transportation cost of shipping 4 units from warehouse j to customer i
Inventory costs	$475000 \cdot x_{jp} + 0.165 \cdot \sum_{i=1}^{I} y_{ijps}$	Inventory cost per warehouse j for the total unit flow per year

Table 14: Costs

Name	Variable	Description
Annual demand	h_{ips}	Demand of customer i in year p in scenario s
Capacity	v_j	Capacity of warehouse j
Probability	q_s	Probability of occurrence of scenario s

Table 15: Demand & Capacity

Notice that now the demand depends on the scenario that they are in, and also, it is necessary add the coefficients q_s in order to be able to apply the stochastic optimization method.

Scenarios

The following table shows the assumptions made for each scenario, both in the description of it and also in the probability that it was decided to grant it. It was decided that the base scenario should be the most probable one, and the last scenario, which is the most extreme one, should be the least probable. The rest of the scenarios were set to have same probabilities of happening. According to that, the split of the probabilities for each is shown in the table.

Scenario (s)	Description	Probability (q_s)
S1	Base/Same as Part 1	0.3
S2	10% increase in 2008	0.15
S3	10% increase in 2009	0.15
S4	10% increase in 2010	0.15
S5	10% increase in 2011	0.15
S6	60% increase in 2007-2011	0.1

Table 16: Scenarios

Mathematical Model

The final mathematical model that was used can be expressed as follows:

$$Min[\sum_{p=1}^{P}\sum_{j=1}^{J}(f_j x_{jp} + 475000 x_{jp} + 0.165\sum_{s=1}^{S}\sum_{i=1}^{I} q_s y_{ijps}) + \sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{i=1}^{I}(b_j q_s y_{ijps} + \frac{c_{ji}-3}{4} q_s y_{ijps})] \tag{1}$$

$$\text{s.t.} \quad \sum_{j=1}^{J} y_{ijps} = h_{ips} \qquad \forall\, i \in I,\ p \in P,\ s \in S \tag{2}$$

$$\sum_{i=1}^{I} y_{ijps} \leq v_j \cdot x_{jp} \qquad \forall\, j \in J,\ p \in P,\ s \in S \tag{3}$$

$$\sum_{p=p}^{p+2} a_{jp} \leq 1 \qquad \forall\, j \in J,\ p = 1:3 \tag{4}$$

$$\sum_{p=p}^{p+1} a_{jp} \leq 1 \qquad \forall\, j \in J,\ p = 4 \tag{5}$$

$$3 \cdot a_{jp} \leq \sum_{p=p}^{p+2} x_{jp} \qquad \forall\, j \in J,\ p = 1:3 \tag{6}$$

$$2 \cdot a_{jp} \leq \sum_{p=p}^{p+1} x_{jp} \qquad \forall\, j \in J,\ p = 4 \tag{7}$$

$$\sum_{q=p-2}^{p} a_{jp} \geq x_{jp} \qquad \forall\, j \in J,\ p = 3:5 \tag{8}$$

$$\sum_{q=p-1}^{p} a_{jp} \geq x_{jp} \qquad \forall\, j \in J,\ p = 2 \tag{9}$$

$$a_{jp} \geq x_{jp} \qquad \forall\, j \in J,\ p = 1 \tag{10}$$

$$x_{jp} + x_{j+5p} \leq 1 \qquad \forall\, p \in P,\ j = 1:5 \tag{11}$$

$$x_{jp} \in \{0,1\} \qquad \forall\, p \in P,\ j \in J \tag{12}$$

$$y_{ijps} \in Z^{+} \qquad \forall\, p \in P,\ j \in J,\ i \in I,\ s \in S \tag{13}$$

$$a_{jp} \in \{0,1\} \qquad \forall\, p \in P,\ j \in J \tag{14}$$

The objective function 1 computes the total fixed, variable, inventory and transportation cost, although the last three are dependent on the scenario, and therefore are weighted on the probability of it happening.

Constraints 2 require the total amount of units y_{ijps} shipped from all warehouses j to customer i to be equal to customer's demand in period p in each scenario s. That is, it is needed to ensure that no matter what scenario occurs, the demand will always be satisfied.

Constraints 3 require the total amount of units y_{ijps} shipped to all customers $i \in I$ from a warehouse j in a period p in any possible scenario s to be less or equal to the capacity v_j of the warehouse j. This is to ensure that no matter what the scenario is, it will never use more capacity than it has. Also, as it is mentioned in the first part, this constraints also make sure that they are only sending units to customers from warehouses that are open.

The remaining sets of constraints, 4 - 14 are all exactly the same as mentioned in the first section.

The respective code can be found in the file - Project_1-Part 3.1.jl

Results

Objective value/Total cost = $17.130196 MM

First - Stage Decision

Before any scenario occurs, we decide on which warehouse (j) should be open. This is what we call a first stage decision.

The results that we obtained for these decisions are shown in the table below.

Warehouse (j)	2007	2008	2009	2010	2011
Seattle - S				X	X
Denver - S					
St. Louis - S					
Atlanta - S					
Philadelphia - S					
Seattle - L					
Denver - L		X	X	X	X
St. Louis - L					
Atlanta - L	X	X	X	X	X
Philadelphia - L				X	X

Table 17: First-Stage Decision

Second - Stage Decision

Later, for each scenario, the model decided how to best supply the customers, given the warehouses that have been opened.

The solutions obtained can be seen in the following tables for each scenario:

S1

Year (p)	Warehouse (j)	Customer (i)	Units (y_{ijp})
2007	Atlanta - L	Northwest	320,000
		Southwest	200,000
		Upper Midwest	160,000
		Lower Midwest	220,000
		Northeast	350,000
		Southeast	175,000
2008	Denver - L	Northwest	576,000
		Southwest	360,000
		Upper Midwest	288,000
	Atlanta - L	Lower Midwest	396,000
		Northeast	630,000
		Southeast	315,000
2009	Denver - L	Northwest	1,036,800
		Southwest	648,000
		Upper Midwest	518,400
	Atlanta - L	Lower Midwest	712,800
		Northeast	1,134,000
		Southeast	567,000
2010	Seattle - S	Northwest	1,969,920
	Denver - L	Southwest	1,166,400
		Upper Midwest	933,120
	Atlanta - L	Lower Midwest	1,283,040
		Southeast	1,020,600
	Philadelphia - L	Northeast	2,041,200
2011	Seattle - S	Northwest	1,866,240
	Denver - L	Southwest	1,166,400
		Upper Midwest	933,120
	Atlanta - L	Lower Midwest	1,283,040
		Southeast	1,020,600
	Philadelphia - L	Northeast	2,041,200

Table 18: Results - S1

S2, S3, S4 & S5

The following Warehouse-Customer Matrix show the results obtained for the different scenarios. As explained at the beginning of this subsection, it was considered a deviation of 10% per year in each Scenario. Meaning that S2 represents a 10% deviation for 2008, S3 for 2009, ... The results showed similar values regarding the customers satisfied in each scenario.

However, the demand for S2 changed in year 2009 and remained similar to the base scenario (S1) for the rest of the modeled period. S3, S4 S5 showed similar behaviours. Therefore, for the purpose of representing the results in this report, the Warehouse-Customer Matrix combines the results of the the Scenarios in the corresponding years that changed compared to S1.

Year (p)	Warehouse(j)	Customer (i)	Units (y_{ijp})
2007	Atlanta-L	Northwest	320,000
		Southwest	200,000
		Upper Midwest	160,000
		Lower Midwest	220,000
		Northeast	350,000
		Southeast	175,000
2008	Denver-L	Northwest,	633,600
		Southwest	396,000
		Upper Midwest	316,800
	Atlanta-L	Lower Midwest	435,600
		Northeast	693,000
		Southeast	346,500
2009	Denver-L	Northwest,	1,140,480
		Southwest	712,800
		Upper Midwest	570,240
	Atlanta-L	Lower Midwest	784,080
		Northeast	1,247,400
		Southeast	623,700
2010	Seattle-S	Northwest,	2,000,000
	Denver-L	Northwest	52,864
		Southwest	1,283,040
		Upper Midwest	1,026,432
	Atlanta-L	Lower Midwest	1,411,344
		Southeast	1,122,660
	Philadelphia-L	Northeast	2,245,320
2011	Seattle-S	Northwest,	2,000,000
	Denver-L	Northwest	52,864
		Southwest	1,283,040
		Upper Midwest	1,026,432
	Atlanta-L	Lower Midwest	1,411,344
		Southeast	1,122,660
	Philadelphia-L	Northeast	2,245,320

Table 19: Results - S2,S3,S4,S5

S6

Year (p)	Warehouse(j)	Customer (i)	Units (y_{ijp})
2007	Atlanta-L	Northwest	512,000
		Southwest	320,000
		Upper Midwest	256,000
		Lower Midwest	352,000
		Northeast	560,000
		Southeast	280,000
2008	Denver-L	Northwest	921,600
		Southwest	576,000
		Upper Midwest	460,800
	Atlanta-L	Northwest	921,600
		Southwest	576,000
		Upper Midwest	460,800
2009	Denver-L	Northwest	1,658,880
		Southwest	1,036,800
		Upper Midwest	829,440
	Atlanta-L	Lower Midwest	1,140,480
		Northeast	1,814,400
		Southeast	907,200
2010	Seattle-S	Northwest	2,000,000
	Denver-L	Northwest	985,984
		Southwest	1,866,240
		Upper Midwest	1,147,776
	Atlanta-L	Lower Midwest	2,052,864
		Southeast	1,632,960
	Philadelphia-L	Upper Midwest	345,216
		Northeast	3,265,920
2011	Seattle-S	Northwest	2,000,000
	Denver-L	Northwest	985,984
		Southwest	1,866,240
		Upper Midwest	1,147,776
	Atlanta-L	Lower Midwest	2,052,864
		Southeast	1,632,960
	Philadelphia-L	Upper Midwest	345,216
		Northeast	3,265,920

Table 20: Results - S6

3.2 Capacity deviation

Another situation where the behaviour of the model can be tested is in the case the warehouse capacity is much lesser than expected. Therefore 5 scenarios were selected, 4 considering a rather small/no deviation and the last one considering an extreme reduction from the base scenario.

Sets

$Customers:$ i $\in I = \{Northwest, Southwest, UpperMidwest, LowerMidwest, Northeast, Southeast\}$

$Periods: p \in P = \{2007, 2008, 2009, 2010, 2011\}$

$Warehouses: j \in J = \{Seattle-S, Denver-S, St.Louis-S, Atlanta-S, Philadelphia-S, Seattle-L, Denver-L, St.Louis-L, Atlanta-L, Philadelphia-L\}$

$Scenarios: s \in S = \{S1, S2, S3, S4, S5\}$

Set	Size	Description
I	6	Customer/Zone
J	10	Warehouse
P	5	Year
S	5	Scenarios

Table 21: Sets Description

Decision Variables

Decision Variables	Type	Description
x_{jp}	Binary: {0,1}	1 if warehouse j is open in year p, 0 otherwise
y_{ijps}	Positive integer: Z+	Number of units shipped from the warehouse j to customers i in year p in scenario s
α_{jp}	Binary: {0,1}	Lease coefficient, 1 if lease of warehouse j starts in year p, 0 otherwise

Table 22: Decision Variables

Coefficients of the model

Costs	**Variable**	**Description**
Fixed costs	f_j	Fixed cost for warehouse j
Variable costs	b_j	Variable cost for warehouse j per unit
Transportation costs	$c_{ji} - 3$	Transportation cost of shipping 4 units from warehouse j to customer i
Inventory costs	$475000 \cdot x_{jp} + 0.165 \cdot \sum_{i=1}^{I} y_{ijps}$	Inventory cost per warehouse j for the total unit flow per year

Table 23: Costs

Name	**Variable**	**Description**
Annual demand	h_{ip}	Demand of customer i in year p
Capacity	v_{js}	Capacity of warehouse j in scenario s
Probability	q_s	Probability of occurrence of scenario s

Table 24: Demand & Capacity

Notice that now the difference is that the capacity varies depending on the scenario, instead of the demand, as he have just previously studied.

Scenarios

The following table shows the assumptions made for each scenario, both in the description of it and also in the probability that it was decided to grant it. It was decided that the base scenario should be the most probable one, and the others scenarios have the same probability of happening.

Scenario (s)	Description	Probability (q_s)
S1	Base/Same as Part 1	0.4
S2	20% decrease	0.15
S3	25% decrease	0.15
S4	30% decrease	0.15
S5	50% decrease	0.15

Table 25: Scenarios

Mathematical Model

$$Min[\sum_{p=1}^{P}\sum_{j=1}^{J}(f_j x_{jp} + 475000 x_{jp} + 0.165\sum_{s=1}^{S}\sum_{i=1}^{I} q_s y_{ijps}) + \sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{i=1}^{I}(b_j q_s y_{ijps} + \frac{c_{ji}-3}{4} q_s y_{ijps})] \tag{1}$$

$$\text{s.t.} \quad \sum_{j=1}^{J} y_{ijps} = h_{ip} \qquad \forall\, i \in I,\ p \in P,\ s \in S \tag{2}$$

$$\sum_{i=1}^{I} y_{ijps} \leq v_{js} \cdot x_{jp} \qquad \forall\, j \in J,\ p \in P,\ s \in S \tag{3}$$

$$\sum_{p=p}^{p+2} a_{jp} \leq 1 \qquad \forall\, j \in J,\ p = 1:3 \tag{4}$$

$$\sum_{p=p}^{p+1} a_{jp} \leq 1 \qquad \forall\, j \in J,\ p = 4 \tag{5}$$

$$3 \cdot a_{jp} \leq \sum_{p=p}^{p+2} x_{jp} \qquad \forall\, j \in J,\ p = 1:3 \tag{6}$$

$$2 \cdot a_{jp} \leq \sum_{p=p}^{p+1} x_{jp} \qquad \forall\, j \in J,\ p = 4 \tag{7}$$

$$\sum_{q=p-2}^{p} a_{jp} \geq x_{jp} \qquad \forall\, j \in J,\ p = 3:5 \tag{8}$$

$$\sum_{q=p-1}^{p} a_{jp} \geq x_{jp} \qquad \forall\, j \in J,\ p = 2 \tag{9}$$

$$a_{jp} \geq x_{jp} \qquad \forall\, j \in J,\ p = 1 \tag{10}$$

$$x_{jp} + x_{j+5p} \leq 1 \qquad \forall\, p \in P,\ j = 1:5 \tag{11}$$

$$x_{jp} \in \{0,1\} \qquad \forall\, p \in P,\ j \in J \tag{12}$$

$$y_{ijps} \in Z^{+} \qquad \forall\, p \in P,\ j \in J,\ i \in I,\ s \in S \tag{13}$$

$$a_{jp} \in \{0,1\} \qquad \forall\, p \in P,\ j \in J \tag{14}$$

Notice that the main change in this model is in the constraint 3, since now the capacities depend on the scenarios. Basically, it still helps to ensure that the capacity of each warehouse is never exceeded, no matter the scenario.

The rest of constraints in the model have already been previously explained.

The respective code can be found in the file - Project_1-Part 3.2.jl

Results

Objective value/Total cost = $18.787476663 MM

First - Stage Decision

As before, the first stage decision is to decide on which warehouse j to open, before any scenario occurs. For this case, the solution obtained can be seen in the table below.

Warehouse (j)	2007	2008	2009	2010	2011
Seattle - S		X	X	X	X
Denver - S					
St. Louis - S					
Atlanta - S					
Philadelphia - S					
Seattle - L					
Denver - L				X	X
St. Louis - L				X	X
Atlanta - L	X	X	X	X	X
Philadelphia - L			X	X	X

Table 26: First - Stage Decision

Second - Stage Decision

Again, the second stage decision reflects how the customers are supplied by the warehouses in each scenario. The optimal solution found can be seen in the following tables for each scenario.

S1

Year (p)	Warehouse (j)	Customer (i)	Units (y_{ijp})
2007	Atlanta - L	Lower Midwest	220,000
		Northeast	350,000
		Northwest	320,000
		Southeast	175,000
		Southwest	200,000
		Upper Midwest	160,000
2008	Atlanta-L	Lower Midwest	396,000
		Northeast	630,000
		Southeast	315,000
		Upper Midwest	288,000
	Seattle-S	Northwest	576,000
		Southwest	360,000
2009	Atlanta-L	Lower Midwest	712,800
		Southeast	567,000
	Philadelphia-L	Northeast	1,134,000
		Upper Midwest	518,400
	Seattle-S	Northwest	1,036,800
2010	Atlanta-L	Lower Midwest	1,283,040
		Southeast	1,020,600
	Denver-L	Southwest	1,166,400
		Upper Midwest	933,120
	Philadelphia-L	Northeast	2,041,200
	Seattle-S	Northwest	1,866,240
2011	Atlanta-L	Lower Midwest	1,283,040
		Southeast	1,020,600
	Denver-L	Southwest	1,166,400
		Upper Midwest	933,120
	Philadelphia-L	Northeast	2,041,200
	Seattle-S	Northwest	1,866,240

Table 27: Results - S1

S2

Year (p)	Warehouse (j)	Customer (i)	Units (y_{ijp})
2007	Atlanta - L	Lower Midwest	220,000
		Northeast	350,000
		Northwest	320,000
		Southeast	175,000
		Southwest	200,000
		Upper Midwest	160,000
2008	Atlanta - L	Lower Midwest	396,000
		Northeast	630,000
		Southeast	315,000
		Upper Midwest	288,000
	Seattle - S	Northwest	576,000
		Southwest	360,000
2009	Atlanta - L	Lower Midwest	712,800
		Southeast	567,000
		Southwest	84,800
	Philadelphia - L	Northeast	1,134,000
		Upper Midwest	518,400
	Seattle - S	Northwest	1,036,800
		Southwest	563,200
2010	Atlanta-L	Lower Midwest	1,283,040
		Southeast	1,020,600
	Denver-L	Northwest	266,240
		Southwest	1,166,400
		Upper Midwest	933,120
	Philadelphia-L	Northeast	2,041,200
	Seattle-S	Northwest	1,600,000
2011	Atlanta-L	Lower Midwest	1,283,040
		Southeast	1,020,600
	Denver-L	Northwest	266,240
		Southwest	1,166,400
	Philadelphia-L	Northeast	2,041,200
	Seattle-S	Northwest	1,600,000
	St.Louis-L	Upper Midwest	933,120

Table 28: Results - S2

S3

Year (p)	Warehouse (j)	Customer (i)	Units (y_{ijp})
2007	Atlanta - L	Lower Midwest	220,000
		Northeast	350,000
		Northwest	320,000
		Southeast	175,000
		Southwest	200,000
		Upper Midwest	160,000
2008	Atlanta - L	Lower Midwest	396,000
		Northeast	630,000
		Southeast	315,000
		Upper Midwest	288,000
	Seattle - S	Northwest	576,000
		Southwest	360,000
2009	Atlanta - L	Lower Midwest	712,800
		Southeast	567,000
		Southwest	184,800
	Philadelphia - L	Northeast	1,134,000
		Upper Midwest	518,400
	Seattle - S	Northwest	1,036,800
		Southwest	463,200
2010	Atlanta-L	Lower Midwest	1,283,040
		Southeast	1,020,600
	Denver-L	Northwest	366,240
		Southwest	1,166,400
		Upper Midwest	933,120
	Philadelphia-L	Northeast	2,041,200
	Seattle-S	Northwest	1,500,000
2011	Atlanta-L	Lower Midwest	1,283,040
		Southeast	1,020,600
	Denver-L	Northwest	366,240
		Southwest	1,166,400
	Philadelphia-L	Northeast	2,041,200
	Seattle-S	Northwest	1,500,000
	St.Louis-L	Upper Midwest	933,120

Table 29: Results - S3

S4

Year (p)	Warehouse (j)	Customer (i)	Units (y_{ijp})
2007	Atlanta - L	Lower Midwest	220,000
		Northeast	350,000
		Northwest	320,000
		Southeast	175,000
		Southwest	200,000
		Upper Midwest	160,000
2008	Atlanta - L	Lower Midwest	396,000
		Northeast	630,000
		Southeast	315,000
		Upper Midwest	288,000
	Seattle - S	Northwest	576,000
		Southwest	360,000
2009	Atlanta - L	Lower Midwest	712,800
		Southeast	567,000
		Southwest	284,800
	Philadelphia - L	Northeast	1,134,000
		Upper Midwest	518,400
	Seattle - S	Northwest	1,036,800
		Southwest	363,200
2010	Atlanta-L	Lower Midwest	1,283,040
		Southeast	1,020,600
	Denver-L	Northwest	466,240
		Southwest	1,166,400
		Upper Midwest	933,120
	Philadelphia-L	Northeast	2,041,200
	Seattle-S	Northwest	1,400,000
2011	Atlanta-L	Lower Midwest	1,283,040
		Southeast	1,020,600
	Denver-L	Northwest	466,240
		Southwest	1,166,400
	Philadelphia-L	Northeast	2,041,200
	Seattle-S	Northwest	1,400,000
	St.Louis-L	Upper Midwest	933,120

Table 30: Results - S4

<u>S5</u>

Year (p)	Warehouse (j)	Customer (i)	Units (y_{ijp})
2007	Atlanta - L	Lower Midwest	220,000
		Northeast	350,000
		Northwest	320,000
		Southeast	175,000
		Southwest	200,000
		Upper Midwest	160,000
2008	Atlanta - L	Lower Midwest	396,000
		Northeast	630,000
		Southeast	315,000
		Upper Midwest	288,000
	Seattle - S	Northwest	576,000
		Southwest	360,000
2009	Atlanta - L	Lower Midwest	712,800
		Northwest	36,800
		Southeast	567,000
		Southwest	284,800
	Philadelphia - L	Northeast	1,134,000
		Upper Midwest	518,400
	Seattle - S	Northwest	1,000,000
2010	Atlanta-L	Lower Midwest	290,000
		Southeast	1,020,600
	Denver-L	Northwest	833,600
		Southwest	1,166,400
	Philadelphia-L	Northeast	2,000,000
	Seattle-S	Northwest	1,000,000
	St. Louis-L	Lower Midwest	993,040
		Northeast	41,200
		Northwest	32,640
		Upper Midwest	933,120
2011	Atlanta-L	Lower Midwest	979,400
		Southeast	1,020,600
	Denver-L	Northwest	833,600
		Southwest	1,166,400
	Philadelphia-L	Northeast	2,000,000
	Seattle-S	Northwest	1,000,000
	St.Louis-L	Lower Midwest	303,640
		Northeast	41,200
		Northwest	32,640
		Upper Midwest	933,120

Table 31: Results - S5

Comments on the results

Comparing the results found in this section with the ones obtained in Part 1, it can be seen that in order to make the decisions more resilient to the uncertainty caused by a possible increase of demand and also of a reduction in warehouse capacity, the final objective value is increased in both cases. However, it is noticeable that there is a bigger increase in the total costs when it has to assume the possible reduction in the capacity of the warehouses than when it has to assume a bigger demand. It can be seen that, for example, in the case of a reduction in the capacity, the results obtained show that for the last two years it is actually required to open a warehouse in all possible locations. Also in the same case with capacity reduction, it is interesting to see that the large warehouse in St.Louis is leased and used in years 2010 and 2011 just to cover the probability of scenario 5 happening.

It is important to remember though that the fact there is a bigger increase in the total cost when it is considered the capacity reduction might be because it was set a more extreme scenario. However, by viewing the results obtained, it is concluded that it is more important to have a specific knowledge on the capacities of the warehouses, than to forecast well demand, since it leads to higher total cost.

4 Flexibility+Resilience

In this last section both the flexibility and resilience of the mathematical model were tested. This means, a combination of the two previous sections of this report. The utilization of the warehouses was not considered a binary variable, being able to take values between 0 and 1, and the mathematical model was analysed in case of an increase in demand and a decrease in the warehouses' capacities. The same scenarios as the previous sections were applied for this modeling.

For this section, all the sets, decision variables and coefficients of the models used have been previously defined. For this reason, it will only be shown the formulation of both optimization models, for each of the resilience approaches.

4.1 Demand deviation

<u>**Mathematical Model**</u>

$$Min[\sum_{p=1}^{P}\sum_{j=1}^{J}(f_j x_{jp} + 475000 x_{jp} + 0.165\sum_{s=1}^{S}\sum_{i=1}^{I} q_s y_{ijps}) + \sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{i=1}^{I}(b_j q_s y_{ijps} + \frac{c_{ji}-3}{4} q_s y_{ijps})] \tag{1}$$

$$\text{s.t.} \quad \sum_{j=1}^{J} y_{ijps} = h_{ips} \qquad \forall\, i \in I,\ p \in P,\ s \in S \tag{2}$$

$$\sum_{i=1}^{I} y_{ijps} \le v_j \cdot x_{jp} \qquad \forall\, j \in J,\ p \in P,\ s \in S \tag{3}$$

$$x_{jp} \cdot x_{j+5p} = 0 \qquad \forall\, p \in P,\ j = 1:5 \tag{4}$$

$$x_{jp} \in [0,1] \qquad \forall\, p \in P,\ j \in J \tag{5}$$

$$y_{ijps} \in Z^{+} \qquad \forall\, p \in P,\ j \in J,\ i \in I,\ s \in S \tag{6}$$

The objective function 1 computes the total fixed, variable, inventory and transportation cost, although the last three are dependent on the scenario, and therefore are weighted on the probability of it happening.

Constraints 2 require customer's i demand to be always satisfied, in every period p and for each scenario s.

Constraints 3 make sure the capacity v_j of warehouse j for each scenario s is never exceeded, in any period.

Constraints 4 ensure that for a certain city a small warehouse and a large warehouse cannot be open or leased at the same time. Again, it is non-linear due to the variable types, and because of this a non-linear solving method has been implemented.

Finally, the sets of constraints 5 and 6 make sure the type of the variables is well defined.

The respective code can be found in the file - Project_1-Part 4.1.jl

Results

Objective value/Total cost = $13.8761542 MM

First - Stage Decision

Before any scenario occurs, we decide on which warehouse (j) should be open. This is what it is called a first stage decision.

The results that was obtained for these decisions are shown in the table below.

Warehouse (j)	2007	2008	2009	2010	2011
Seattle - S					
Denver - S					
St. Louis - S					
Atlanta - S					
Philadelphia - S					
Seattle - L	X	X	X	X	X
Denver - L	X	X	X	X	X
St. Louis - L	X	X	X	X	X
Atlanta - L	X	X	X	X	X
Philadelphia - L	X	X	X	X	X

Table 32: First-Stage Decision

Second - Stage Decision

Again, the second stage decision reflects how the customers are supplied by the warehouses in each scenario. The optimal solution found can be seen in the following tables for each scenario.

Year (p)	Warehouse (j)	Cap. % (x_{jp})	Customer (i)	Units (y_{ijp})
2007	Seattle - L	8%	Northwest	320,000
	Denver - L	12.8%	Southwest	200,000
			Upper Midwest	160,000
	Atlanta - L	9.9%	Lower Midwest	220,000
			Southeast	175,000
	Philadelphia - L	14%	Northeast	350,000
2008	Seattle - L	15.8%	Northwest	576,000
	Denver - L	21.6%	Southwest	360,000
	St.Louis - L	11.5%	Upper Midwest	288,000
	Atlanta - L	28.4%	Lower Midwest	396,000
			Southeast	315,000
	Philadelphia - L	25.2%	Northeast	630,000
2009	Seattle - L	28.5%	Northwest	1,036,800
	Denver - L	38.9%	Southwest	648,000
	St.Louis - L	20.7%	Upper Midwest	518,400
	Atlanta - L	51.2%	Lower Midwest	712,800
			Southeast	567,000
	Philadelphia - L	45.4%	Northeast	1,134,000
2010	Seattle - L	51.3%	Northwest	1,866,240
			Southwest	1,866,240
	Denver - L	70%	Southwest	979,776
			Upper Midwest	933,120
	Atlanta - L	92.1%	Lower Midwest	1,283,040
			Southeast	1,020,600
	Philadelphia - L	81.7%	Northeast	2,041,200
2011	Seattle - L	51.3%	Northwest	1,866,240
	Denver - L	70%	Southwest	1,166,400
			Upper Midwest	933,120
	St.Louis - L	60.9%	Lower Midwest	1,283,040
	Atlanta - L	68.5%	Southeast	1,020,600
	Philadelphia - L	81.7%	Northeast	2,041,200

Table 33: Results - S1

Year (p)	Warehouse (j)	Cap. % (x_{jp})	Customer (i)	Units (y_{ijp})
2007	Seattle - L	8%	Northwest	320,000
	Denver - L	12.8%	Southwest	200,000
	St.Louis - L	12.3%	Upper Midwest	160,000
			Lower Midwest	220,000
	Atlanta - L	9.9%	Southeast	175,000
	Philadelphia - L	14%	Northeast	350,000
2008	Seattle - L	15.8%	Northwest	633,600
	Denver - L	21.6%	Southwest	396,000
			Upper Midwest	316,800
	Atlanta - L	28.4%	Lower Midwest	435,600
			Southeast	346,500
	Philadelphia - L	25.2%	Northeast	693,000
2009	Seattle - L	28.5%	Northwest	1,036,800
	Denver - L	38.9%	Southwest	648,000
			Upper Midwest	518,400
	St.Louis - L	20.7%	Lower Midwest	712,800
	Atlanta - L	51.2%	Southeast	576,000
	Philadelphia - L	45.4%	Northeast	1,134,000
2010	Seattle - L	51.3%	Northwest	1,866,240
	Denver - L	70%	Southwest	1,166,400
			Upper Midwest	933,120
	Atlanta - L	92.1%	Lower Midwest	1,283,040
			Southeast	1,020,600
	Philadelphia - L	81.7%	Northeast	2,041,200
2011	Seattle - L	51.3%	Northwest	1,866,240
			Southwest	186,624
	Denver - L	70%	Southwest	979,776
			Upper Midwest	933,120
	Atlanta - L	68.5%	Lower Midwest	1,283,040
			Southeast	1,020,600
	Philadelphia - L	81.7%	Northeast	2,041,200

Table 34: Results - S2

Year (p)	Warehouse (j)	Cap. % (x_{jp})	Customer (i)	Units (y_{ijp})
2007	Seattle - L	8%	Northwest	320,000
	Denver - L	12.8%	Southwest	200,000
	St.Louis - L	12.3%	Upper Midwest	160,000
			Lower Midwest	220,000
	Atlanta - L	9.9%	Lower Midwest	220,000
			Southeast	175,000
	Philadelphia - L	14%	Northeast	350,000
2008	Seattle - L	15.8%	Northwest	576,000
			Southwest	57,600
	Denver - L	21.6%	Southwest	302,400
			Upper Midwest	288,000
	St.Louis - L	11.5%	Lower Midwest	396,000
	Atlanta - L	28.4%	Southeast	315,000
	Philadelphia - L	25.2%	Northeast	630,000
2009	Seattle - L	28.5%	Northwest	140,480
	Denver - L	38.9%	Southwest	712,800
			Upper Midwest	570,240
	Atlanta - L	51.2%	Lower Midwest	784,080
			Southeast	623,700
	Philadelphia - L	45.4%	Northeast	1,247,400
2010	Seattle - L	51.3%	Northwest	1,866,240
	Denver - L	70%	Southwest	1,166,400
	St.Louis - L	37.3%	Upper Midwest	933,120
	Atlanta - L	92.1%	Lower Midwest	1,283,040
			Southeast	1,020,600
	Philadelphia - L	81.7%	Northeast	2,041,200
2011	Seattle - L	51.3%	Northwest	1,866,240
	Denver - L	70%	Southwest	1,166,400
			Upper Midwest	933,120
	Atlanta - L	68.5%	Lower Midwest	1,283,040
			Southeast	1,020,600
	Philadelphia - L	81.7%	Northeast	2,041,200

Table 35: Results - S3

Year (p)	Warehouse (j)	Cap. % (x_{jp})	Customer (i)	Units (y_{ijp})
2007	Seattle - L	8%	Northwest	320,000
	Denver - L	12.8%	Southwest	200,000
			Upper Midwest	160,000
	St.Louis - L	12.3%	Lower Midwest	220,000
	Atlanta - L	9.9%	Southeast	175,000
	Philadelphia - L	14%	Northeast	350,000
2008	Seattle - L	15.8%	Northwest	576,000
	Denver - L	21.6%	Southwest	360,000
	St.Louis - L	11.5%	Upper Midwest	288,000
	Atlanta - L	28.4%	Lower Midwest	396,000
			Southeast	315,000
	Philadelphia - L	25.2%	Northeast	630,000
2009	Seattle - L	28.5%	Northwest	1,036,800
	Denver - L	38.9%	Southwest	648,000
			Upper Midwest	518,400
	Atlanta - L	51.2%	Lower Midwest	712,800
			Southeast	567,000
	Philadelphia - L	45.4%	Northeast	1,134,000
2010	Seattle - L	51.3%	Northwest	2,052,864
	Denver - L	70%	Southwest	1,283,040
			Upper Midwest	1,0264,320
	Atlanta - L	92.1%	Lower Midwest	1,411,344
			Southeast	1,122,660
	Philadelphia - L	81.7%	Northeast	2,245,320
2011	Seattle - L	51.3%	Northwest	1,866,240
	Denver - L	70%	Southwest	1,166,400
			Upper Midwest	933,120
	St.Louis - L	60.9%	Lower Midwest	1,283,040
	Atlanta - L	68.5%	Southeast	1,020,600
	Philadelphia - L	81.7%	Northeast	2,041,200

Table 36: Results - S4

Year (p)	Warehouse (j)	Cap. % (x_{jp})	Customer (i)	Units (y_{ijp})
2007	Seattle - L	8%	Northwest	320,000
	Denver - L	12.8%	Southwest	200,000
	St.Louis - L	12.3%	Upper Midwest	160,000
	Atlanta - L	9.9%	Lower Midwest	220,000
			Southeast	175,000
	Philadelphia - L	14%	Northeast	350,000
2008	Seattle - L	15.8%	Northwest	576,000
			Southwest	57,600
	Denver - L	21.6%	Southwest	302,400
			Upper Midwest	288,000
	Atlanta - L	28.4%	Lower Midwest	396,000
			Southeast	315,000
	Philadelphia - L	25.2%	Northeast	630,000
2009	Seattle - L	28.5%	Northwest	1,036,800
	Denver - L	38.9%	Southwest	648,000
	St.Louis - L	20.7%	Upper Midwest	518,400
	Atlanta - L	51.2%	Lower Midwest	712,800
			Southeast	567,000
	Philadelphia - L	45.4%	Northeast	1,134,000
2010	Seattle - L	51.3%	Northwest	1,866,240
	Denver - L	70%	Southwest	1,166,400
			Upper Midwest	933,120
	Atlanta - L	92.1%	Lower Midwest	1,283,040
			Southeast	1,020,600
	Philadelphia - L	81.7%	Northeast	2,041,200
2011	Seattle - L	51.3%	Northwest	2,052,864
	Denver - L	70%	Southwest	1,283,040
	St.Louis - L	60.9%	Upper Midwest	1,026,432
			Lower Midwest	1,411,344
	Atlanta - L	68.5%	Southeast	1,122,660
	Philadelphia - L	81.7%	Northeast	2,245,320

Table 37: Results - S5

Year (p)	Warehouse (j)	Cap. % (x_{jp})	Customer (i)	Units (y_{ijp})
2007	Seattle - L	8%	Northwest	320,000
	Denver - L	12.8%	Northwest	192,000
			Southwest	320,000
	St.Louis - L	12.3%	Upper Midwest	256,000
			Lower Midwest	237,000
	Atlanta - L	9.9%	Lower Midwest	115,000
			Southeast	280,000
	Philadelphia - L	14%	Northeast	560,000
2008	Seattle - L	15.8%	Northwest	633,600
	Denver - L	21.6%	Northwest	288,000
			Southwest	576,000
	St.Louis - L	11.5%	Upper Midwest	460,800
	Atlanta - L	28.4%	Lower Midwest	633,600
			Southeast	504,000
	Philadelphia - L	25.2%	Northeast	1,008,000
2009	Seattle - L	28.5%	Northwest	1,140,480
	Denver - L	38.9%	Northwest	518,400
			Southwest	1,036,800
	St.Louis - L	20.7%	Upper Midwest	829,440
	Atlanta - L	51.2%	Lower Midwest	1,140,480
			Southeast	907,200
	Philadelphia - L	45.4%	Northeast	1,814,400
2010	Seattle - L	51.3%	Northwest	2,052,864
	Denver - L	70%	Northwest	933,120
			Southwest	1,866,240
	St.Louis - L	37.3%	Upper Midwest	1,492,992
	Atlanta - L	92.1%	Lower Midwest	2,052,864
			Southeast	1,632,960
	Philadelphia - L	81.7%	Northeast	3,265,920
2011	Seattle - L	51.3%	Northwest	2,052,864
	Denver - L	70%	Northwest	933,120
			Southwest	1,866,240
	St.Louis - L	60.9%	Upper Midwest	1,492,992
			Lower Midwest	944,784
	Atlanta - L	68.5%	Lower Midwest	1,108,080
			Southeast	1,632,960
	Philadelphia - L	81.7%	Northeast	3,265,920

Table 38: Results - S6

4.2 Capacity deviation

Mathematical Model

$$Min[\sum_{p=1}^{P}\sum_{j=1}^{J}(f_j x_{jp} + 475000 x_{jp} + 0.165\sum_{s=1}^{S}\sum_{i=1}^{I} q_s y_{ijps}) + \sum_{p=1}^{P}\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{i=1}^{I}(b_j q_s y_{ijps} + \frac{c_{ji}-3}{4} q_s y_{ijps})] \tag{1}$$

$$\text{s.t.} \quad \sum_{j=1}^{J} y_{ijps} = h_{ip} \qquad \forall\, i \in I,\ p \in P,\ s \in S \tag{2}$$

$$\sum_{i=1}^{I} y_{ijps} \leq v_{js} \cdot x_{jp} \qquad \forall\, j \in J,\ p \in P,\ s \in S \tag{3}$$

$$x_{jp} \cdot x_{j+5p} = 0 \qquad \forall\, p \in P,\ j = 1:5 \tag{4}$$

$$x_{jp} \in [0,1] \qquad \forall\, p \in P,\ j \in J \tag{5}$$

$$y_{ijps} \in Z^{+} \qquad \forall\, p \in P,\ j \in J,\ i \in I,\ s \in S \tag{6}$$

Now the only difference is in constraints 3, which state that the capacity of every warehouse j can never be exceeded for all different scenarios.

The respective code can be found in the file - Project_1-Part 4.2.jl

Results

Objective value/Total cost = $15.77290969 MM

First - Stage Decision

Before any scenario occurs, we decide on which warehouse (j) should be open. This is what we call a first stage decision.

The results that we obtained for these decisions are shown in the table below.

Warehouse (j)	2007	2008	2009	2010	2011
Seattle - S					
Denver - S					
St. Louis - S					
Atlanta - S					
Philadelphia - S					
Seattle - L	X	X	X	X	X
Denver - L	X	X	X	X	X
St. Louis - L	X	X	X	X	X
Atlanta - L	X	X	X	X	X
Philadelphia - L	X	X	X	X	X

Table 39: First-Stage Decision

Second - Stage Decision

Again, the second stage decision reflects how the customers are supplied by the warehouses in each scenario. The optimal solution found can be seen in the following tables for each scenario.

Year (p)	Warehouse (j)	Cap. % (x_{jp})	Customer (i)	Units (y_{ijp})
2007	Seattle - L	11.4%	Northwest	320,000
	Denver - L	14.6%	Southwest	200,000
			Upper Midwest	160,000
	Atlanta - L	14.1%	Lower Midwest	220,000
			Southeast	175,000
	Philadelphia - L	17.5%	Northeast	350,000
2008	Seattle - L	20.6%	Northwest	576,000
	Denver - L	26.2%	Southwest	360,000
			Upper Midwest	288,000
	Atlanta - L	35.6%	Lower Midwest	396,000
			Southeast	315,000
	Philadelphia - L	31.5%	Northeast	630,000
2009	Seattle - L	37.0%	Northwest	1,036,800
	Denver - L	47.2%	Southwest	648,000
			Upper Midwest	518,400
	Atlanta - L	45.7%	Lower Midwest	712,800
			Southeast	567,000
	Philadelphia - L	56.7%	Northeast	1,134,000
2010	Seattle - L	66.7%	Northwest	1,866,240
	Denver - L	85%	Southwest	1,166,400
			Upper Midwest	933,120
	Atlanta - L	100%	Lower Midwest	1,283,040
			Southeast	1,020,600
	Philadelphia - L	100%	Northeast	2,041,200
2011	Seattle - L	66.7%	Northwest	1,866,240
	Denver - L	85%	Southwest	1,166,400
			Upper Midwest	933,120
	Atlanta - L	100%	Lower Midwest	1,283,040
			Southeast	1,020,600
	Philadelphia - L	100%	Northeast	2,041,200

Table 40: Results - S1

Year (p)	Warehouse (j)	Cap. % (x_{jp})	Customer (i)	Units (y_{ijp})
2007	Seattle - L	11.4%	Northwest	320,000
	Denver - L	14.6%	Southwest	200,000
			Upper Midwest	160,000
	Atlanta - L	14.1%	Lower Midwest	220,000
			Southeast	175,000
	Philadelphia - L	17.5%	Northeast	350,000
2008	Seattle - L	20.6%	Northwest	576,000
	Denver - L	26.2%	Southwest	360,000
			Upper Midwest	288,000
	Atlanta - L	35.6%	Lower Midwest	396,000
			Southeast	315,000
	Philadelphia - L	31.5%	Northeast	630,000
2009	Seattle - L	37.0%	Northwest	1,036,800
			Southwest	1,481,120
	Denver - L	47.2%	Southwest	499,888
			Upper Midwest	518,400
	Atlanta - L	45.7%	Lower Midwest	712,800
			Southeast	567,000
	Philadelphia - L	56.7%	Northeast	1,134,000
2010	Seattle - L	66.7%	Northwest	1,866,240
	Denver - L	85%	Southwest	1,166,400
	St.Louis - L	63.9%	Upper Midwest	933,120
	Atlanta - L	100%	Lower Midwest	1,283,040
			Southeast	1,020,600
	Philadelphia - L	100%	Northeast	2,041,200
2011	Seattle - L	66.7%	Northwest	1,866,240
	Denver - L	85%	Southwest	1,166,400
	St.Louis - L	64%	Upper Midwest	933,120
	Atlanta - L	100%	Lower Midwest	1,283,040
			Southeast	1,020,600
	Philadelphia - L	100%	Northeast	2,041,200

Table 41: Results - S2

Year (p)	Warehouse (j)	Cap. % (x_{jp})	Customer (i)	Units (y_{ijp})
2007	Seattle - L	11.4%	Northwest	320,000
	Denver - L	14.6%	Southwest	200,000
			Upper Midwest	160,000
	Atlanta - L	14.1%	Lower Midwest	220,000
			Southeast	175,000
	Philadelphia - L	17.5%	Northeast	350,000
2008	Seattle - L	20.6%	Northwest	576,000
	Denver - L	26.2%	Southwest	360,000
			Upper Midwest	288,000
	Atlanta - L	35.6%	Lower Midwest	396,000
			Southeast	315,000
	Philadelphia - L	31.5%	Northeast	630,000
2009	Seattle - L	37.0%	Northwest	1,036,800
	Denver - L	47.2%	Southwest	648,000
			Upper Midwest	518,400
	Atlanta - L	45.7%	Lower Midwest	712,800
			Southeast	567,000
	Philadelphia - L	56.7%	Northeast	1,134,000
2010	Seattle - L	66.7%	Northwest	1,866,240
	Denver - L	85%	Southwest	1,166,400
	St.Louis - L	63.9%	Upper Midwest	933,120
	Atlanta - L	100%	Lower Midwest	1,283,040
			Southeast	1,020,600
	Philadelphia - L	100%	Northeast	2,041,200
2011	Seattle - L	66.7%	Northwest	1,866,240
	Denver - L	85%	Southwest	1,166,400
	St.Louis - L	64%	Upper Midwest	933,120
	Atlanta - L	100%	Lower Midwest	1,283,040
			Southeast	1,020,600
	Philadelphia - L	100%	Northeast	2,041,200

Table 42: Results - S3

Year (p)	Warehouse (j)	Cap. % (x_{jp})	Customer (i)	Units (y_{ijp})
2007	Seattle - L	11.4%	Northwest	319,998
	Denver - L	14.6%	Northwest	2
			Southwest	200,000
			Upper Midwest	160,000
	Atlanta - L	14.1%	Lower Midwest	220,000
			Southeast	175,000
	Philadelphia - L	17.5%	Northeast	350,000
2008	Seattle - L	20.6%	Northwest	575,999
	Denver - L	26.2%	Northwest	1
			Southwest	360,000
			Upper Midwest	288,000
	Atlanta - L	35.6%	Lower Midwest	396,000
			Southeast	315,000
	Philadelphia - L	31.5%	Northeast	630,000
2009	Seattle - L	37.0%	Northwest	1,036,798
	Denver - L	47.2%	Northwest	2
			Southwest	648,000
			Upper Midwest	518,400
	Atlanta - L	45.7%	Lower Midwest	712,800
			Southeast	567,000
	Philadelphia - L	56.7%	Northeast	1,134,000
2010	Seattle - L	66.7%	Northwest	1,866,239
	Denver - L	85%	Northwest	1
			Southwest	1,166,400
			Upper Midwest	933,120
	Atlanta - L	100%	Lower Midwest	1,283,040
			Southeast	1,020,600
	Philadelphia - L	100%	Northeast	2,041,200
2011	Seattle - L	66.7%	Northwest	1,866,240
	Denver - L	85%	Southwest	1,166,400
	St.Louis - L	64%	Upper Midwest	933,120
	Atlanta - L	100%	Lower Midwest	1,283,040
			Southeast	1,020,600
	Philadelphia - L	100%	Northeast	2,041,200

Table 43: Results - S4

Year (p)	Warehouse (j)	Cap. % (x_{jp})	Customer (i)	Units (y_{ijp})
2007	Seattle - L	11.4%	Northwest	228,570
	Denver - L	14.6%	Northwest	91,430
			Southwest	200,000
	St.Louis - L	13.6%	Upper Midwest	160,000
			Lower Midwest	112,857
	Atlanta - L	14.1%	Lower Midwest	107,143
			Southeast	175,000
	Philadelphia - L	17.5%	Northeast	350,000
2008	Seattle - L	20.6%	Northwest	411,428
	Denver - L	26.2%	Northwest	164,572
			Southwest	360,000
	St.Louis - L	14.4%	Upper Midwest	288,000
	Atlanta - L	35.6%	Lower Midwest	396,000
			Southeast	315,000
	Philadelphia - L	31.5%	Northeast	630,000
2009	Seattle - L	37.0%	Northwest	740,570
	Denver - L	47.2%	Northwest	296,230
			Southwest	648,000
	St.Louis - L	44.2%	Upper Midwest	518,400
			Lower Midwest	365,657
	Atlanta - L	45.7%	Lower Midwest	347,143
			Southeast	567,000
	Philadelphia - L	56.7%	Northeast	1,134,000
2010	Seattle - L	66.7%	Northwest	1,333,028
	Denver - L	85%	Northwest	533,212
			Southwest	1,166,400
	St.Louis - L	63.9%	Upper Midwest	933,120
			Lower Midwest	303,640
			Northeast	41,200
	Atlanta - L	100%	Lower Midwest	979,400
			Southeast	1,020,600
	Philadelphia - L	100%	Northeast	2,000,000
2011	Seattle - L	66.7%	Northwest	1,333,029
	Denver - L	85%	Northwest	533,211
			Southwest	1,166,400
	St.Louis - L	64%	Upper Midwest	933,120
			Lower Midwest	344,840
	Atlanta - L	100%	Lower Midwest	938,200
			Northeast	41,200
			Southeast	1,020,600
	Philadelphia - L	100%	Northeast	2,000,000

Table 44: Results - S5

Comments on the results

For the case of demand deviation, it can be observed that the objective optimal value is bigger than it was on Part 2, but smaller to the value of the corresponding in Part 3. This is reasonable since in Part 2, flexibility was already considered, while resilience of the model was not and therefore by adding it now, it should be expected to have a bigger total cost. On the other hand, in Part 3 resilience was already being considered while flexibility was not, and that's why in Part 4 there is a lower total cost in comparison.

It can seen that now all big warehouses are being leased, as in Part 2, because of the same logic: the cost per unit in these warehouses is smaller. However now a greater proportion of them is being leased.

It can be seen as well that in Part 3 a fewer number of warehouses were being leased, possibly due to the fact that the leasing constraints were in place. However, thanks to the flexibility that is being added, now all can be open, but use only portions of them and still the total cost is smaller.

For the case of capacity deviation, the same effect is experienced. It has a larger total cost than it had in Part 2, but a smaller than the corresponding of Part 3. The reasoning behind it is the same.

In this case, it is also needed to lease all the big warehouses all years, but at an even higher capacity, to make up for the possible reduction.

It is obtained again also that it is much more expensive to have uncertainty in the capacity of the warehouses than it is to not be able to forecast well the demand. The total cost difference is even much bigger in this case between the two cases.