Contents

1 Demand Analysis

This section contains the calculation for the demand distribution for salmon at each distribution center (DC) on each of the eight days in the planning horizon. For this purpose, it is necessary to calculate the demand's mean and standard deviation. The mean and standard deviation were calculated using Excel. Firstly, for the mean calculation, the forecasted quantities of each product, for each day and distribution center were added using the pivot table function. Afterwards, and considering that each product contains 100 gm of salmon, the forecasted sum was adjusted accordingly to portray the average salmon quantity needed, for each day and for each distribution center. The results are shown in the table:

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
DC 01	75.90	75.90	75.90	75.90	75.90	85.70	85.70	75.90
DC 02	62.90	94.80	94.80	94.80	94.80	68.65	36.75	62.90
DC 03	67.20	67.20	67.20	67.20	67.20	67.15	67.15	67.20
DC 04	102.30	156.00	156.00	156.00	156.00	119.05	65.35	102.30
DC 05	107.40	130.10	130.10	130.10	130.10	65.15	42.45	107.40

Table 2: Mean (kg)

Regarding the standard deviation, a similar procedure was implemented. Given the sum of the variance for each product on each forecasted day in the planning horizon for each distribution center, the square root of this value multiplied by the amount of salmon each product contains results in the desired value.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
DC 01	11.87	11.87	11.87	11.87	11.87	14.69	14.69	11.87
DC 02	10.91	13.89	13.89	13.89	13.89	14.14	11.22	10.91
DC 03	11.27	11.27	11.27	11.27	11.27	12.86	12.86	11.27
DC 04	13.19	16.03	16.03	16.03	16.03	15.05	11.98	13.19
DC 05	13.04	14.00	14.00	14.00	14.00	11.08	9.84	13.04

Table 3: Standard Deviation (kg)

The benefit of having salmon as the base ingredient in each of the three products would evade the decline in demand for these products and increase the flexibility, as we saw in the lectures, it is a postponement strategy. Inversely, having different fishes for each product could complicate the supply chain eliminating the possibility of using one to substitute another. Another possible downside for using different fishes is the need to use different suppliers, therefore having different lead times which would restrict the supply chain.

The respective calculations can be found in the file - Project_1-Part 1.1.xlsx

2 Solving the Problem

The objective of the problem is to build and solve an inventory routing model that optimize the routes for the trucks and the inventory levels for each DC. The model is a combination of both vehicle routing and inventory management at the same time as the DC want to minimize holding costs and the supplier want to minimize transportation costs.

2.1 Model

Sets

Nodes: i,j $\in N = \{Supplier, DC1, DC2, DC3, DC4, DC5\}$

$Days : t \in T = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$Vehicles : k \in K = \{Truck1, Truck2, Truck3\}$

Set	Size	Description
N	6	Nodes (Supplier = Node 1)
K	3	Vehicles
T	8	Days

Table 4: Sets Description

Decision Variables

Decision Variables	Type	Description
x_{ijkt}	Binary: {0,1}	1 If truck k leaves DC i and goes to DC j
y_{ikt}	Binary: {0,1}	1 if truck k arrives at DC i
q_{ikt}	R^+	Quantity delivered to DC i by truck k at the start of period t
z_{ijk}	R^+	Load of truck k when arriving at j

Table 5: Decision Variables

Parameters

Parameter	Description
I_{it}	Inventory level at node i at the end of period t
I_{i0}	Inventory level at node i at the start of the planning horizon
C_i	Inventory holding capacity of node i
D_k	Capacity of truck k
h	Holding cost per unit of inventory at node i (0.025 kr.)
d_{it}	Demand at node i at period t

Table 6: Parameters

Mathematical Model

The resulting mathematical model is as follows:

$$Min[\sum_{i=2}^{N}\sum_{t=1}^{T} h \cdot I_{it} + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{K}\sum_{t=1}^{T} c_{ij} \cdot x_{ijkt}] \tag{1}$$

$$\text{s.t.} \qquad I_{i,t-1} + \sum_{k=1}^{K} q_{ikt} = I_{it} + d_{it} \qquad \forall\, i = 2:N,\ t \in T \tag{2}$$

$$I_{i,t-1} + \sum_{k=1}^{K} q_{ikt} \leq C_i \qquad \forall\, i = 2:N,\ t \in T \tag{3}$$

$$q_{ikt} \leq C_i \cdot y_{ikt} \qquad \forall\, i = 2:N,\ t \in T,\ k \in K \tag{4}$$

$$\sum_{i=2}^{N} q_{ikt} \leq D_k \cdot y_{1kt} \qquad \forall\, t \in T,\ k \in K \tag{5}$$

$$\sum_{j=2}^{N} x_{ijkt} = \sum_{j=2}^{N} x_{jikt} = y_{ikt} \qquad \forall\, i \in N,\ t \in T,\ k \in K \tag{6}$$

$$z_{ikt} - d_{it} \geq z_{jkt} - (1 - x_{ijkt}) \cdot \sum_{i=2}^{N} d_{it} \qquad \forall\, i, j = 2:N,\ t \in T,\ k \in K \tag{7}$$

The objective function 1 will minimize the costs associated with holding costs of the inventory and transport costs to the different DCs.

Constraint 2 is a balance constraint for DC i to ensure that each DC receives the amount of salmon as the demand needed at each period t.

Constraint 3 is the capacity constraint which ensures that each DC i doesn't receive more than the capacity at each period t.

Constraint 4 ensures, that when a DC i receives salmon, it also registers that a truck k has arrived at the DC.

Constraint 5 ensures that when a truck k is sent out, it doesn't exceed the 900 kg capacity.

Constraint 6 ensures that when a truck k arrives at a DC i, it will also leave, implying that the inflow is equal to the outflow.

Constraint 7 ensures subtour elimination with the load constraint logic.

The respective code can be found in the file - Project_2-Part 2.1.ipynb

Result

The mathematical model is optimized using Julia v1.7.2 (Programming Language) and Gurobi (Solver) in Jupyter Notebook.

Objective value/Total cost = 4481.57 kr.

Optimal Route:

Truck 1: 1->DC5->1 (Day 1) ; 1->DC3->DC4->DC5->1 (Day 3)
Truck 2: 1->DC3->DC4->DC2->DC1->1(Day 1);1->DC1->DC2->DC4->DC3->1(Day 6)

Route-Truck Matrix:

Route	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
1->DC5	526.30							
DC5->1	0							
1->DC3			201.60					
DC3->DC4			467.60					
DC4->DC5			230.80					
DC5->1			0					

Table 7: Quantity delivered (kg) by Truck 1

Route	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
1->DC3	92							
DC3->DC4	235.30							
DC4->DC2	292							
DC2->DC1	280.70							
DC1->1	0							
1->DC1						247.30		
DC1->DC2						164.50		
DC2->DC4						286.70		
DC4->DC3						201.50		
DC3->1						0		

Table 8: Quantity delivered (kg) by Truck 2

Inventory Level

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
DC 1	303.60	227.70	151.80	75.90	0	161.60	75.90	0
DC 2	383	288.20	193.40	98.60	3.8	99.65	62.90	0
DC 3	67.20	0	134.40	67.20	0	134.35	67.20	0
DC 4	156.40	0	312	156	0	167.65	102.30	0
DC 5	504.60	374.50	475.20	345.10	215	149.85	107.40	0

Table 9: Inventory Level (kg)

2.2 Simulation

The problem now is to build a simulation that helps estimate if the fill rate target of DC i, which is stated as 0.95, can be met with a probability of 0.99 on each day t. The fill rates for the different DC's over the different days should be reported. Fill rate can be calculated as:

$$FillRate = \frac{max(d_{it} + min(I_{it}, 0), 0)}{d_{it}} \tag{8}$$

A simulation running 10000 iterations was built, in which the demand was stochastic (d_{it} = mean and std_{it} = standard deviation). During the simulations, for each value of the demand, the fill rate was updated, and a counter variable was set up, so that it saved the information on the total number of iterations for which the fill rate is higher than our critical value of 0.95.

As it can be noticed in the table, the days for which each DC i does not satisfy the fill rate of 0.95 with a probability of at least 0.99 are colored in red. This shows that the

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
DC 1	1	1	1	0.9997	0.5613	1	0.9923	0.5434
DC 2	1	1	1	1	0.6151	0.9988	0.9683	0.5354
DC 3	1	0.5799	1	0.9992	0.5545	1	0.9885	0.5421
DC 4	1	0.6570	1	1	0.5939	1	0.9953	0.5532
DC 5	1	1	1	1	1	1	0.9995	0.5575

Table 10: Probability of Fill Rate > 0.95

simulation shows that the fill rate target is not met.

The respective code can be found in the file - Project_2-Part 2.2.ipynb

2.3 Adapted Model with Safety Factor

Since the simulation has proved that the fill rate target cannot be met, an adaptation to the inventory routing model was made so that it includes the safety stock. The safety stock added to the demand is equal to the standard deviation (std_{it}) multiplied by a safety factor (z_α), which was considered to be common for all demands.

$$d_{new,it} = d_{it} + z_\alpha \cdot std_{it} \tag{9}$$

Looking at the literature, the safety factor can be expressed as the α^{th} quantile of the standard normal distribution, where α is a value to be determined.

Result

For this case, α was determined by trying different possible values until it could be ensured that the fill rate target would be met with a probability of 0.99. The approach was to take a random value of α (0.95, in our case). It turned out that it satisfies the fill rate target. Therefore then $\alpha = 0.94$ was taken which did not meet the fill rate target. The smallest value of α that was found to satisfy this condition was 0.948, which corresponds to a value of the safety factor of 1.64.

a	z_α	Total Cost	Fill Rate Target
0.95	1.64	5682.80	Met
0.945	1.60	5681.22	Not Met
0.947	1.62	5681.84	Not Met
0.948	1.63	5682.15	Met

Objective value/Total cost = 5682.15 kr.

As it was expected, increasing demand by adding the safety stock corresponds to an increase in total cost. The total cost increased by 1200.58 kr.

The respective code can be found in the file - Project_2-Part 2.3.ipynb

3 Capacity Analysis

This section evaluates the capacity utilization of the model developed in Part 2 and analyses the model behavior in different scenarios trying to identify the model's bottleneck.

3.1 Capacity Utilization

Firstly, the DC i and truck k capacity utilization were evaluated on each day and the bottleneck was identified. Using the results from Part 2, the capacity utilization were calculated.

$$CapacityUtilization_{k/Truck} = \frac{\sum_{i=2}^{N} \cdot q_{ikt}}{D_k} \qquad \forall\, t \in T,\ k \in K \tag{10}$$

$$CapacityUtilization_{i/DC} = \frac{I_{it}}{C_i} \qquad \forall\, t \in T,\ i \in 2:N \tag{11}$$

Result

Objective value/Total cost = 5682.15 kr.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
DC 1	0.7%	63.3%	44.3%	25.4%	62.6%	40.8%	19%	0%
DC 2	83.5%	59.4%	35.4%	11.3%	46.6%	27.8%	16.5%	0%
DC 3	38.9%	0%	61.1%	22.3%	40%	0%	38.9%	0%
DC 4	37.5%	0%	62.5%	25.1%	29.5%	0%	25.5%	0%
DC 5	1.3%	46.9%	52.4%	31.8%	11.2%	0%	17.3%	0%

Table 11: Capacity Utilization of DCs

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
Truck 1	0%	0%	0%	0%	0%	0%	63.2%	0%
Truck 2	0%	100%	100%	0%	0%	0%	0%	0%
Truck 3	88.7%	0%	0%	0%	100%	0%	0%	0%

Table 12: Capacity Utilization of Trucks

As it can be observed, the capacity of the trucks is almost fully utilized (when operated), while the DCs are not utilized at full capacity. Therefore, it appears that the truck capacity is the bottleneck.
The respective code can be found in the file - Project_2-Part 3.1.ipynb

3.2 Capacity Scenarios

Safety Stock is calculated using the formula:

$$SafetyStock_{it} = z_\alpha \cdot std_{it} \qquad \forall\, i = 2 : N,\; t \in T \tag{12}$$

3.2.1 Truck Capacity = 10x

In this scenario, the truck capacity is increased 10 times. Here we take $\alpha = 0.921$ or $z_\alpha = 1.412$ to ensure that the fill rate target is met.

<u>**Result**</u>

Objective value/Total cost = 5358.94 kr.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
DC 1	16.758	16.758	16.758	16.758	16.758	20.740	20.740	16.758
DC 2	15.403	19.610	19.610	19.610	19.610	19.963	15.841	15.403
DC 3	15.911	15.911	15.911	15.911	15.911	18.156	18.156	15.911
DC 4	18.622	22.632	22.632	22.632	22.632	21.248	16.914	18.622
DC 5	18.410	19.766	19.766	19.766	19.766	15.643	13.892	18.410

Table 13: Safety Stock (Truck Capacity = 10x)

The respective code can be found in the file - Project_2-Part 3.2a.ipynb

3.2.2 Storage Capacity = 10x

In this scenario, the storage capacity is increased 10 times. Here we take $\alpha = 0.913$ or $z_\alpha = 1.359$ to ensure that the fill rate target is met.

<u>**Result**</u>

Objective value/Total cost = 3561.17 kr.

The respective code can be found in the file - Project_2-Part 3.2b.ipynb

3.2.3 Truck & Storage Capacity = 10x

In this scenario, the truck and storage capacity is increased 10 times at the same time. Here we take $\alpha = 0.790$ or $z_\alpha = 0.8064$ to ensure that the fill rate target is met.

<u>**Result**</u>

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
DC 1	16.137	16.137	16.137	16.137	16.137	19.971	19.971	16.137
DC 2	14.832	18.883	18.883	18.883	18.883	19.223	15.253	14.832
DC 3	15.321	15.321	15.321	15.321	15.321	17.483	17.483	15.321
DC 4	17.931	21.792	21.792	21.792	21.792	20.460	16.286	17.931
DC 5	17.727	19.032	19.032	19.032	19.032	15.063	13.377	17.727

Table 14: Safety Stock (Storage Capacity = 10x)

Objective value/Total cost = 1919.26 kr.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
DC 1	9.572	9.572	9.572	9.572	9.572	11.846	11.846	9.572
DC 2	8.798	11.201	11.201	11.201	11.201	11.403	9.048	8.798
DC 3	9.088	9.088	9.088	9.088	9.088	10.371	10.371	9.088
DC 4	10.637	12.927	12.927	12.927	12.927	12.137	9.661	10.637
DC 5	10.516	11.29	11.29	11.29	11.29	8.935	7.935	10.516

Table 15: Safety Stock (Truck & Storage Capacity = 10x)

The respective code can be found in the file - Project_2-Part 3.2c.ipynb

3.2.4 Comparison of Results

When comparing the results shown above, a decreasing tendency in both costs and safety stocks can be observed. Increasing the truck capacity results in a reduction of 5.69% in the total costs, however, when the DCs storage capacity is increased, the reduction on the total costs is 37.3%. Taking this into account, it appears that the key restrictive factor is the DCs storage capacity instead of the truck capacity.

	Total Cost	z_α	Savings %
Base	5682.15	0.948	-
Truck Capacity = 10x	5358.94	0.921	5.69%
Storage Capacity = 10x	3561.17	0.913	37.3%
Truck & Storage Capacity = 10x	1919.26	0.8064	66.23%

Table 16: Comparison

3.3 Analysis of Truck & Storage Capacity = 10x

For this section, it is asked to reflect on the effect that increasing both DCs and truck capacities has over the safety factor used to comply with the fill rate. When analyzing the routes obtained in the optimal solutions for the base scenario and the increased capacities ones, it can be observed that the latter complies with the demand for the planning horizon using just one transportation cycle on the first day of the horizon. During this transportation cycle, a single truck visits every node once. Meanwhile, the base scenario needs several transportation cycles using three trucks to fulfill the demand during the planning horizon. Increasing the DCs storage capacity increases the cycle stock, which in consequence, decreases the safety stock needed to act as a buffer against uncertainty. Increasing the cycle stock and being able to fulfill the demand using a single transportation cycle impacts on both holding costs and transportation costs, decreasing the value of the objective function results.

4 Flexibility Analysis

In this section, it is considered that, instead of providing the orders on the previous evening, the customers provide the orders a day earlier. This condition increases the *safety lead time* (pg 556, 'Fundamentals of supply chain theory', 2019) which it is inversely proportional to the safety stock. Therefore, instead of needing a high amount of safety stock to satisfy the fill rate, it is possible to decrease the safety stock and consequently decrease the holding costs.

This condition also adds more flexibility to the model, allowing to evaluate and plan the transporting routes with more time in advance. The inventory routing can be updated, to comply with the needed demand every two days, as long as the truck and storage capacities allow this, and therefore decreasing the transportation costs.

In general, there is more flexibility in the model because there is one more day to supply the DCs and meet the requested demand.Therefore, less safety stock is needed because the demand is less uncertain. All of above, should help decrease costs in the end.