Inferential Models Applied to the Poisson

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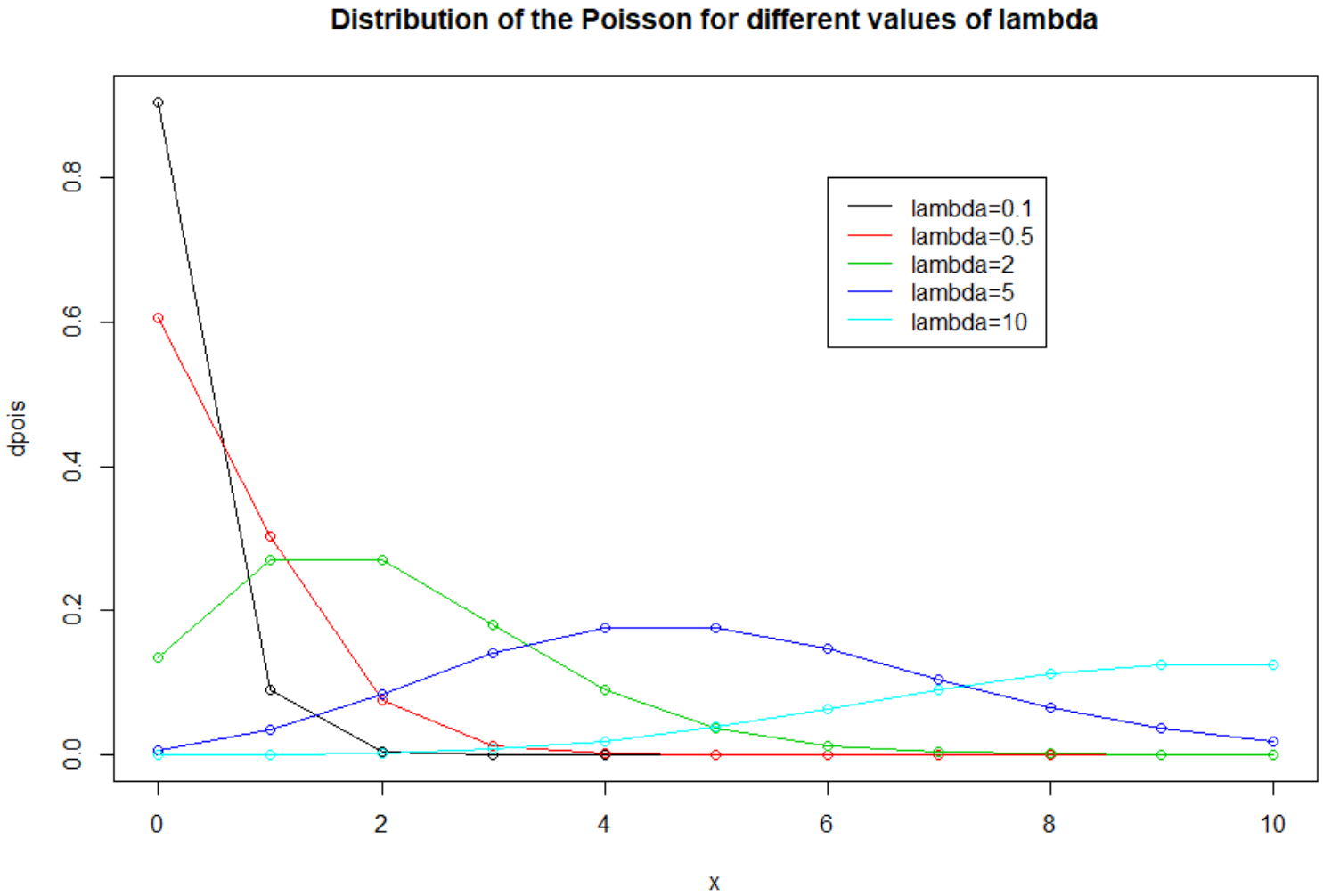
Overview

Poisson

- $P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}, \qquad k = 0, 1, 2, ...$



Interesting Applications of the Poisson

- IBM conducted a study where it utilized the Poisson model combined with Bayesian statistics to understand employees previous experience and compared to success within the company.
- In physics, this distribution is used to model several topics, such as scattering of alpha particles. This experiment observes a small number of particles, and is best modeled by the Poisson distribution for its discreteness and counting characteristics.

Inference

- Data: $X_1, ..., X_n \overset{iid}{\sim} N(\theta, \sigma^2)$
- θ is unknown, and the parameter we want to make inference on.
- *Belief*, written as $bel_x(A)$, has some differences in properties than a probability measure, but for simplicity we can think of them similarly.
- plausibility: $pl_x(a) = 1 - bel_x(A^c)$

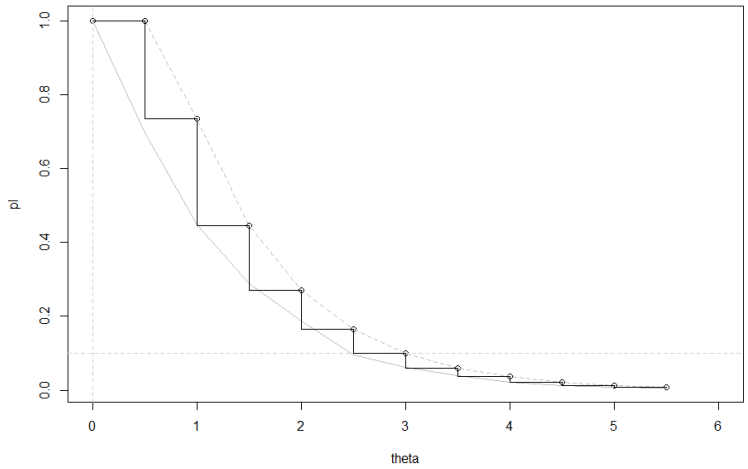
Framework for Inferential Models

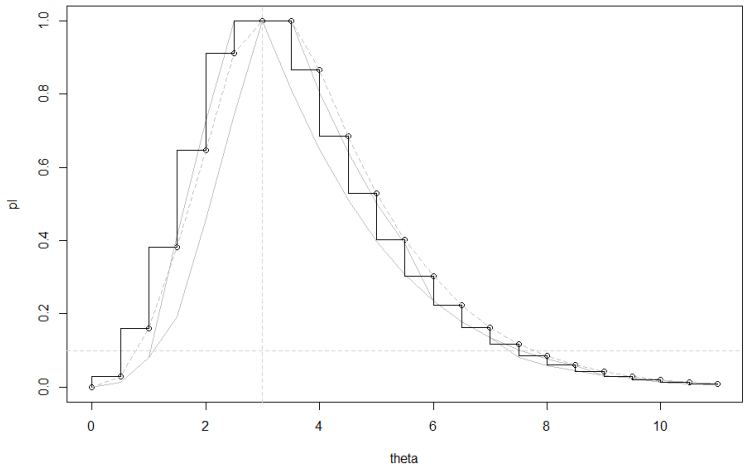
3 Steps to define an Inferential Model:

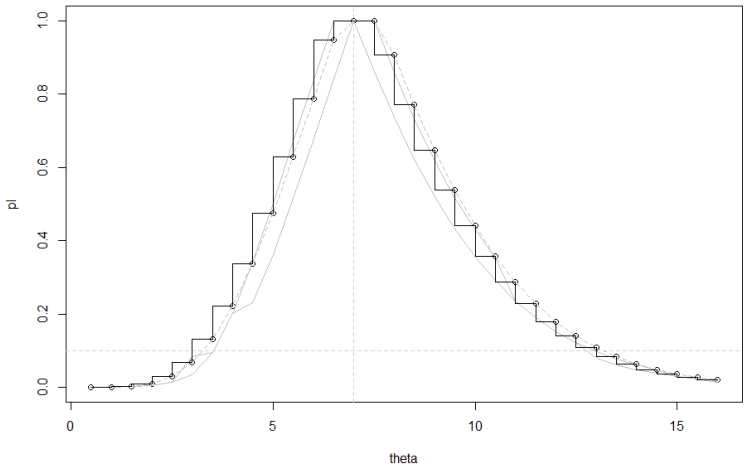
- A-step. Associate X, θ, and $U \sim P_U$ in a way consistent with the sampling distribution $X \sim P_{X|\theta}$ such that for all $x \in \mathbb{X}$ and all $u \in \mathbb{U}$, it defines a unique subset $\Theta_x(u) \subseteq \Theta$, possibly empty, containing all possible candidate values of θ given (x, u)
- P-step. Predict the unobserved value u^* of U associated with the observed data by an admissible predictive random set $\mathcal{S}$.
- C-step. Combine $\mathcal{S}$ and the association $\Theta_x(u)$ specified in the A-step to obtain

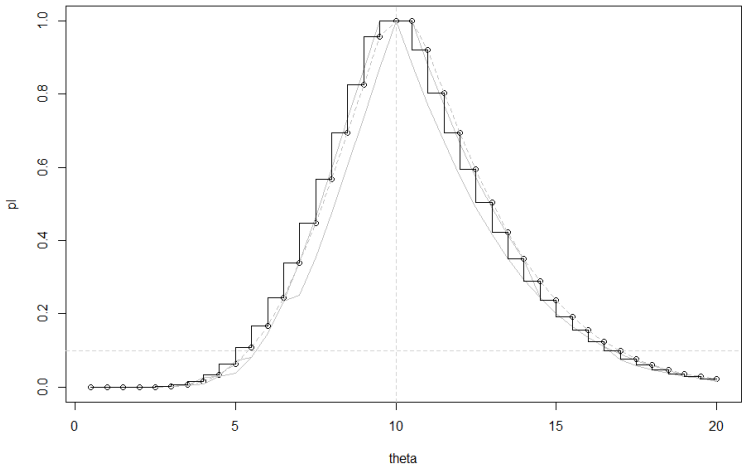
$$\Theta_x(\mathcal{S}) = \bigcup_{u \in \mathcal{S}} \Theta_x(u) \tag{1}$$

Plots Replicated









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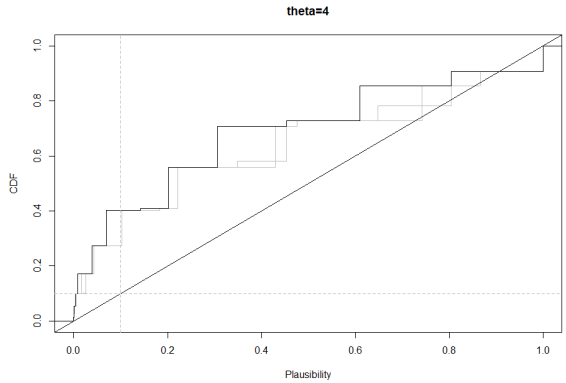

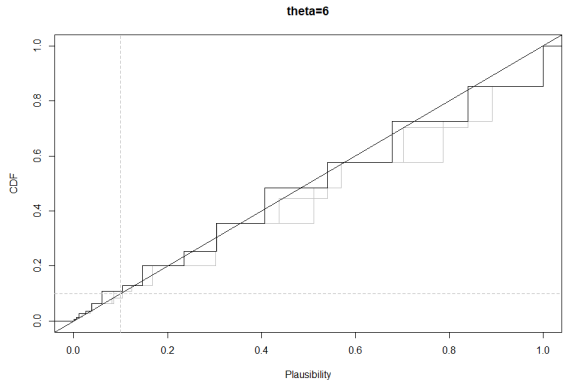

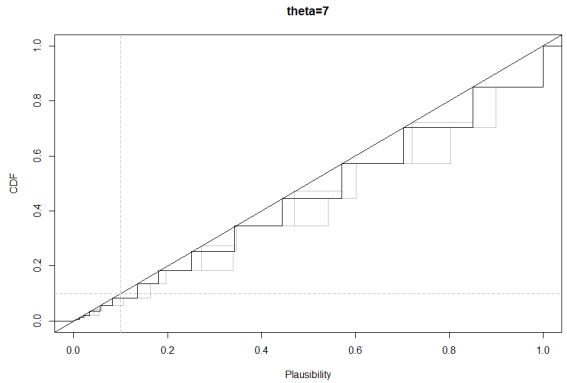

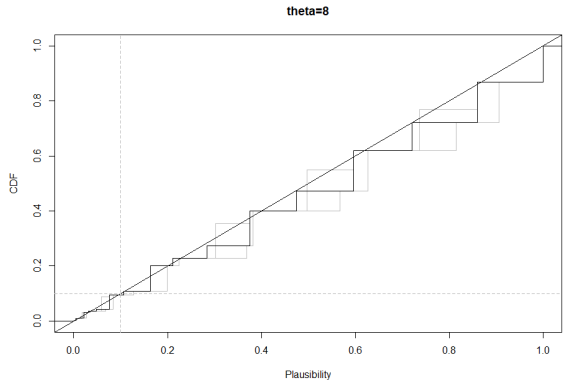

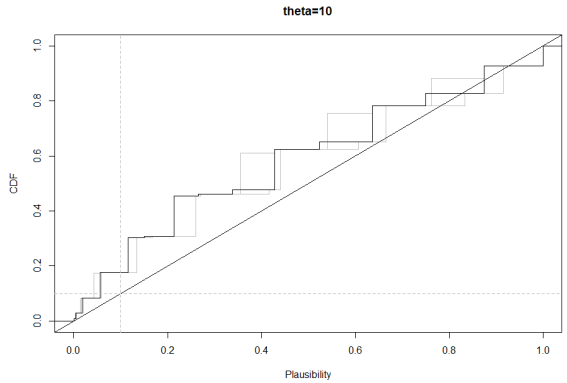

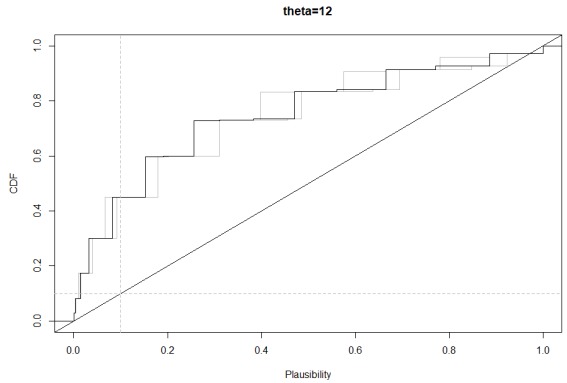

Citations

- J. Rice. *Mathematical Statistics and Data Analysis*. Thomson Higher Education. 2007.
- Ryan Martin. *Inferential Models*. CRC Press. 2016.
- Ryan Martin et al. *Optimal Inferential Models for a Poisson Mean*. p. 20 (2012).