Organizer

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Electric Circuit Analysis

Electrical Branch

IIIrd Sem.

Strictly as per new syllabus

Aditya Prakashan

SYLLABUS

Module 1 :

Network Theorems (10 Hours)

Superposition theorem, Thevenin theorem, Norton theorem, Maximum power transfer theorem, Reciprocity theorem, Compensation theorem. Analysis with dependent current and voltage sources. Node and Mesh Analysis. Concept of duality and dual networks.

Module 2 :

Solution of First and Second order networks (8 Hours)

Solution of first and second order differential equations for Series and parallel R-L, R-C, R-L-C circuits , initial and final conditions in network elements, forced and free response, time constants, steady state and transient state response.

Module 3 :

Sinusoidal steady state analysis (8 Hours)

Representation of sine function as rotating phasor, phasor diagrams, impedances and admittances, AC circuit analysis, effective or RMS values, aerage power and complex power. Three-phase circuits. Mutual coupled circuits, Dot convention in coupled circuits, ideal Transformer.

Module 4 :

Electrical Circuit Analysis Using Laplace Transforms (8 Hours) :

Review of Laplace Transform, Analysis of electrical circuits using Laplace Transform for standard inputs, convolution integral, inverse Laplace transform, transformed network with initial conditions. Transfer function representation. Poles and Zeros. Frequency response (magnitude and phase plots), series and parallel resonances.

Module 5 :

Two Port Network and Network Functions (6 Hours)

Two Port Networks, terminal pairs, relationship of two port variables, impedance parameters, admittance parameters, transmission parameters and hybrid parameters, interconnections of two port networks.

CONTENTS

Module 1 :

Network Theorems

Superposition theorem, Thevenin theorem. Norton theorem, Maximum power transfer theorem, Reciprocity theorem, Compensation theorem. Analysis with dependent current and voltage sources. Node and Mesh Analysis. Concept of duality and dual networks.

Module 2 :

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Two Port Network and Network Functions

Two Port Networks, terminal pairs, relationship of two port variables, impedance parameters, admittance parameters, transmission parameters and hybrid parameters, interconnections of two port networks.

1 NETWORK THEOREMS

PART-A
SHORT QUESTIONS WITH SOLUTIONS

Q1. State superposition theorem.

Ans:

In a linear network consisting of several independent sources, the overall response at any point in the network is equal to the vectorial sum of the responses due to each independent source considered separately with all other sources made inoperative.

Q2. State Thevenin's theorem.

Ans:

Any linear network having an active voltage and current sources with two terminals A and B can be replaced by an equivalent voltage source (V_{Th}) and equivalent resistance (R_{Th}) in series combination forming a simple equivalent circuit.

Where, V_{Th} is the open circuit voltage across the terminals A and B and R_{Th} is the equivalent resistance as seen from the terminals A and B when the independent sources are deactivated.

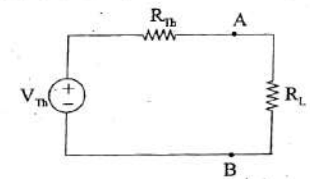


Figure: Thevenin's Equivalent Circuit

Q3. What are the limitations of Thevenin's theorem?

Ans:

The following are the limitations of Thevenin's theorem,

1. Thevenin's theorem is applicable only for linear circuits. But we know that practically no circuit is 100% linear. It is linear only for a specified range of values. Hence, Thevenin's theorem is applicable only for a limited range of values.
2. The power calculated using Thevenin's equivalent circuit is not same as that calculated by taking the original network. This is so because the power of any element is proportional to square of current or square of voltage but not linearly dependent.
3. The Thevenin's equivalent circuit has an equivalent V-I characteristic with respect to load only.

Q4. State Norton's theorem.

Ans:

The Norton's theorem states that any two terminals linear network with current sources, voltage sources and resistances (impedances) can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance (impedance), where the value of the current source is equal to the current passing through the short-circuited terminals and the resistance is equal to the resistance measured between the terminals of the network with all the energy sources replaced by their internal resistance.

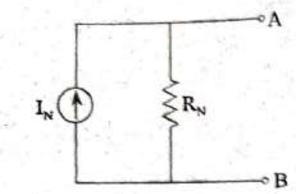


Figure: Norton's Equivalent Circuit

Where, I_N is the current which flows through a short-circuit placed across terminals A and B.

R_N is the circuit resistance looking from the open A-B terminals.

Q5. State the maximum power transfer theorem.

Ans:

Maximum power transfer theorem states that the maximum power can be transferred from source (voltage source, current source) to the load when the resistance (R_L) is equal to the internal resistance of the source (R_s).

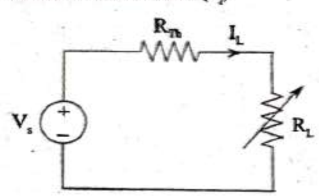


Figure: Maximum Power Transfer Theorem

Q6. State reciprocity theorem.

Ans:

In a linear, passive and bilateral single source network, the ratio of response to the excitation is constant even though the source is interchanged from the input terminals to the output terminals.

Q7. State compensation theorem.

Ans:

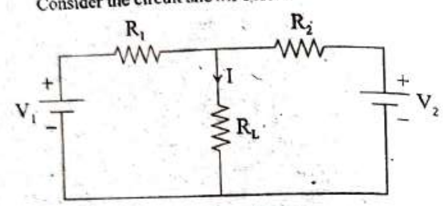
In a linear network, if the current in a branch is 'I' and the impedance is Z, and if the impedance of that branch is changed by ΔZ, then the change in the current in the branches of network can be obtained as the current due to voltage source of value I. ΔZ introduced in that branch in a direction opposing the current I, with all sources in the network reduced to zero.

Q8. Is superposition valid for power? Substantiate your answer.

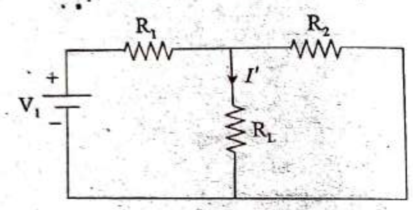
Ans:

Superposition theorem is valid only for linear systems. It cannot be applied for power because the equation for power is non-linear.

Consider the circuit shown below.

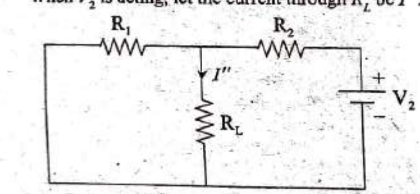


When V_1 is acting, let the current through R_L be I'.



$\therefore$ Power $= (I')^2 R_L$

When V_2 is acting, let the current through R_L be I''.



$\therefore$ Power $= (I'')^2 R_L$.

The total current through R_L by superposition is,

$$I = I' + I''$$

$\therefore$ Power $= I^2 R_L$

But, $(I')^2 R_L + (I'')^2 R_L \neq I^2 R_L$

Since, $I^2 = (I' + I'')^2 = (I')^2 + (I'')^2 + 2I' I''$

Therefore, the superposition theorem is not valid for power.

Q9. Classify all the different types of voltage and current sources.

Ans:

When ideal source and practical source are considered, then the sources are classified as shown in figure (1).

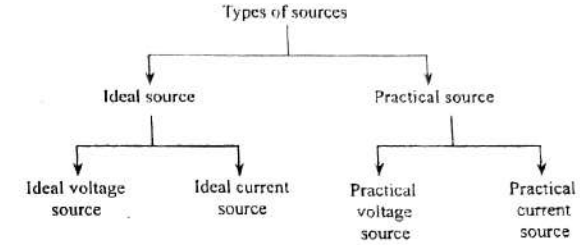


Figure (1)

When independent source and dependent source are considered, then the sources are classified as shown in figure (2).

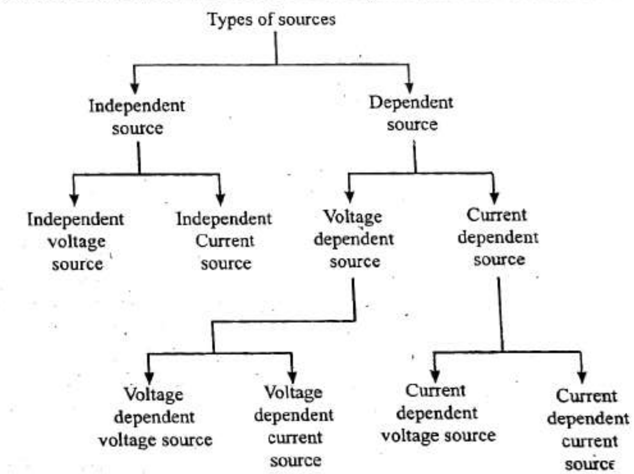


Figure (2)

Q10. Explain the source transformation technique with suitable circuits.

Ans:

Source Transformation

Source transformation is a network reduction technique wherein one form of source can be interchanged with its another equivalent form. Using this technique, a complicated network can be converted into a simple form and hence, calculation can be made easier.

Basically, there are two types of sources i.e., the voltage source and the current source. A practical voltage source always have a resistor in series with it and a practical current source will have a resistor in parallel with it. Using the source transformation technique, a voltage source in series with a resistor can be converted to a current source in parallel with the same resistor.

Two operations can be performed using source transformation technique.

1. Converting a given voltage source to a current source and
2. Converting a given current source to a voltage source.

1. Converting a Given Voltage Source to a Current Source

Consider a given voltage source with magnitude V volts having a resistor 'R' in series as shown in figure (a).

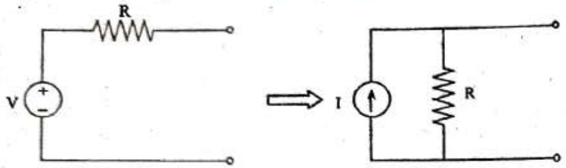


Figure (a) Figure (b)

The voltage source can be converted into a current source in parallel with the same resistor 'R' as shown in figure (b). The value of current source is given by,

$$I = \frac{V}{R} \text{ Amp}$$

2. Converting a Given Current Source to a Voltage Source

Consider a given current source with magnitude I amperes having a resistor 'R' in parallel as shown in figure (c).

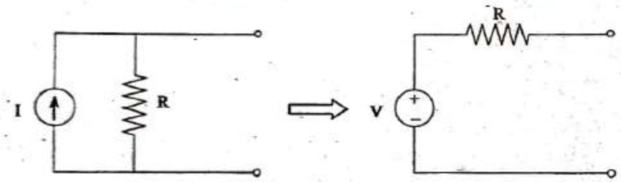


Figure (c) Figure (d)

The current source can be converted into a voltage source in series with the same resistor R as shown in figure (d). The value of voltage source is given by, $V = IR$ Volts.

Q11. What is nodal analysis?

Ans:

Nodal analysis is also called as "node-voltage analysis" or "branch current method". This method is carried out using Kirchhoff's Current Law (KCL).

Nodal analysis is used to find out the potential difference between the nodes in an electrical circuit. KCL is applied at each of the electrical nodes and potential difference between the nodes is determined in terms of branch currents.

In an electrical circuit containing P nodes, one out of the P nodes is chosen as the reference node. Hence, for an electrical circuit containing 'P' nodes a total of $P-1$ node equations and $P-1$ node voltages are obtained.

Q12. What is mesh analysis?

Ans:

Mesh analysis is also called as "loop analysis" or "mesh current method". This method is carried out using Kirchhoff's Voltage Law (KVL). Mesh analysis is applicable only for planar networks.

Mesh analysis is used to determine the voltages and currents of the given electrical circuit. KVL is applied for each of the meshes and unknown voltages and currents are determined.

In an electrical circuit containing P nodes, B branches, the number of independent mesh equations is given by,

$$e = B-(P-1)$$

$$\Rightarrow \quad e = B-P+1$$

The number of mesh currents is equal to the number of mesh equations.

Q13. Explain duality of a network.

OR

What is duality?

Ans:

Duality

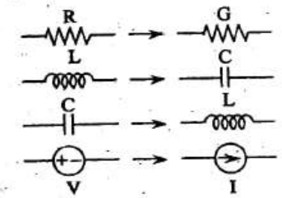
Duality is the graphical representation of the circuit elements that are similar in form. Of course, they are physically different. In electrical circuits, there exists a pair of terms which can be interchanged in order to obtain a new circuit. These pair of terms are called duals which are listed below.

Voltage source (V) ↔ Current source (I)

Resistance (R) ↔ Conductance (G)

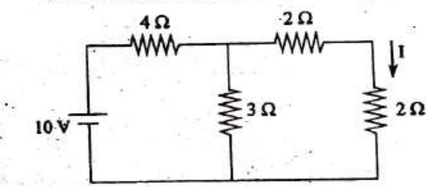
Inductance (L) ↔ Capacitance (C).

Symbolic Representation



Figure

Q14. Verify Reciprocity theorem for the voltage V and current I in the network shown in figure.



Figure

Ans:

The given circuit is as shown in figure (1).

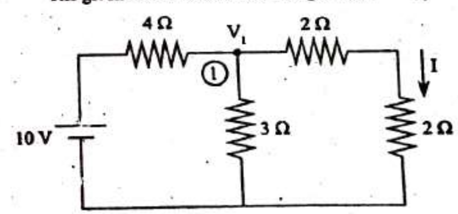


Figure (1)

Let, V_1 be the voltage at node (1).

Applying KCL at node (1), we get,

$$\frac{V_1-10}{4} + \frac{V_1}{3} + \frac{V_1}{4} = 0$$

$$0.833V_1 = 2.5$$

$$V_1 = 3 \text{ V}$$

Current through 2 Ω resistor, $I = \frac{V_1}{(2+2)}$

$$= \frac{3}{4} = 0.75 \text{ A}$$

Ratio of input to response, $\frac{V}{I} = \frac{10}{0.75} = 13.33$

Applying the reciprocity theorem by interchanging the excitation and response as shown in figure (2).

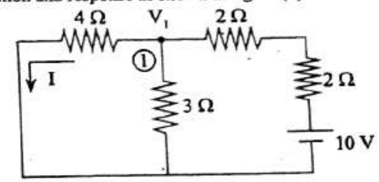


Figure (2)

Applying KCL at node (1), we get,

$$\frac{V_1}{4} + \frac{V_1}{3} + \frac{V_1-10}{4} = 0$$

$$0.833V_1 = 2.5$$

$$V_1 = 3 \text{ V}$$

Current through 4 Ω resistor,

$$I = \frac{3}{4}$$

$$= 0.75 \text{ A}$$

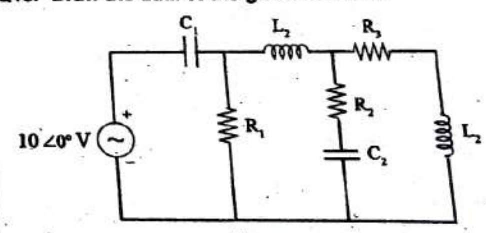
Ratio of input to response,

$$\frac{V}{I} = \frac{10}{0.75}$$

$$= 13.33$$

In both the cases, the ratio of input to response is same. Hence, the reciprocity theorem is verified.

Q15. Draw the dual of the given network.



Figure

Ans:

The given circuit is shown in figure (1).

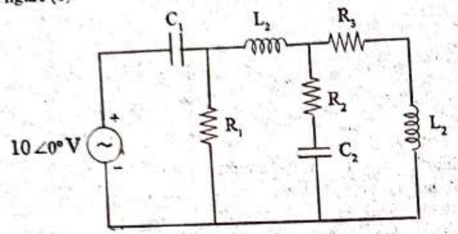


Figure (1)

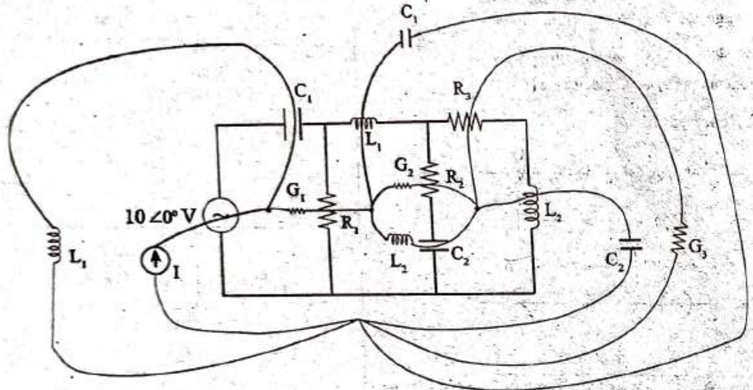


Figure (2)

To draw the dual of given circuit.

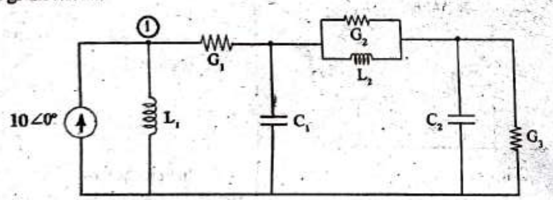
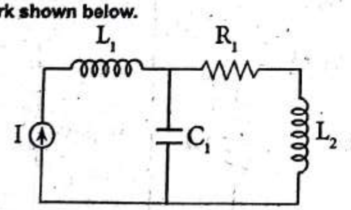


Figure (3): Dual

Q16. Draw the dual of the network shown below.



Figure

Ans:

In order to obtain the dual of a network, we need to follow the following of transformations.

Voltage source (V) ↔ Current source (I)

Resistance (R) ↔ Conductance (G)

Inductance (L) ↔ Capacitance (C)

Series ↔ Parallel

The given network is,

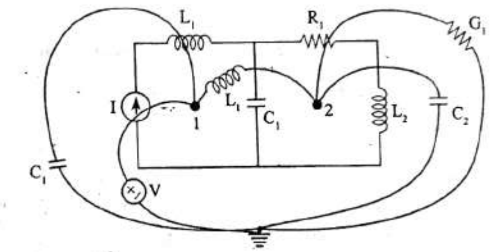


Figure (1)

∴ The obtained dual network is shown in figure (2).

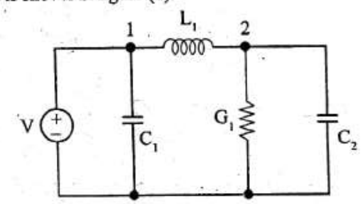


Figure (2)

PART - B
ESSAY QUESTION WITH SOLUTION
1.1 SUPERPOSITION THEOREM

Q17. State and prove the superposition theorem with the help of an example.

Ans:

Statement

"This theorem states that in a linear network comprising of number of independent sources, the total response in any branch of the network is equal to the algebraic sum of individual response acting alone i.e., considering only one source at a time and making all other independent sources to zero". However, the dependent sources must be retained in the network.

Explanation

This theorem is valid only for linear system.

An independent voltage source can be made inoperative by replacing it by a short-circuit and an independent current source can be made inoperative by replacing it by an open-circuit.

This theorem can be better understood with a numerical example. Consider the circuit which has two independent sources as follows,

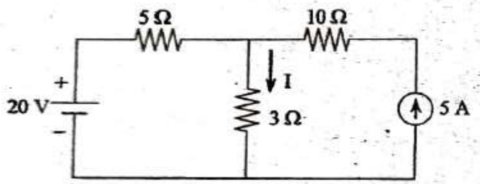


Figure (1)

Steps for Applying Superposition Theorem

Step I

Keep one independent source active and other sources are inoperative. Obtain the branch response (voltage or current).

By superpositions theorem,

Let, only 20 V source is operative and all other sources are inoperative. The circuit now gets modified as,

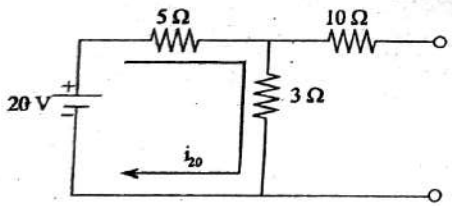


Figure (2)

$$\therefore \quad i_{20} = \frac{20}{5+3} \Rightarrow i_{20} = 2.5 \text{ A}$$

Step II

Repeat the above procedure for each independent source. Let only current source 5 A is operative now and all other sources are inoperative. The circuit is modified as,

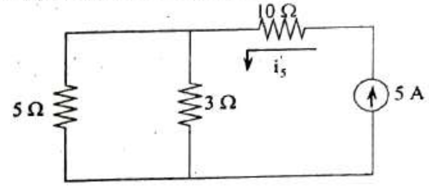


Figure (3)

$$\therefore \quad i_5 = (5) \times \frac{5}{3+5} = 3.125 \text{ A}$$

Step III

To determine the net branch response utilising superposition theorem, just add all the responses obtained in step-I and step-II considering their voltage polarities and current directions.

$\therefore$ The net branch responses in 3 Ω branch is,

$$I = i_{20} + i_5 = 2.5 + 3.125 = 5.625 \text{ A}$$

Q18. State and explain superposition theorem for A.C excitation.

Ans:

Statement for Superposition Theorem

In a linear network consisting of several independent sources, the overall response at any point in the network is equal to the vectorial sum of the responses due to each independent source considered separately with all other sources made inoperative.

Explanation

This theorem is valid only for linear systems. According to this theorem, only one source will be active at a time and the rest of the sources are replaced by their internal impedances. The voltage sources are made inoperative by replacing it with a short-circuit and an independent current source can be made inoperative by replacing it with an open-circuit. This theorem can be better understood with a numerical example. Consider the circuit with two independent sources as shown in figure (1).

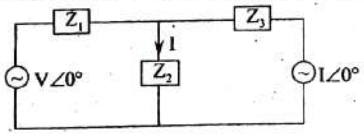


Figure (1)

Now applying superposition theorem for the circuit shown in figure (1) the steps are as follows.

Step 1

Keeping the voltage source $V\angle 0°$ active alone. The current source $I\angle 0°$ will be open circuited. Hence, the figure (1) source can be modified as shown in figure (2).

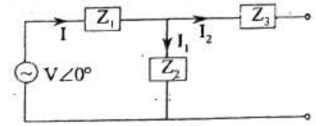


Figure (2)

Now, the current across the load impedance z_2 due to the voltage source $V\angle 0°$ is given by,

$$I_1 = \frac{V\angle 0°}{Z_2}$$

Step 2

When the current source $I\angle 0°$ is acting alone. The voltage source $V\angle 0°$ is replaced by a short circuit line. Hence the figure (1) can be modified as shown in figure (3).

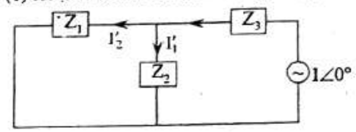


Figure (3)

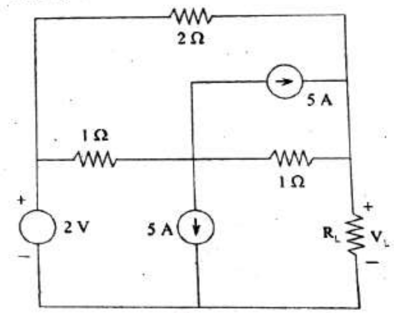
Now the current across the load impedance Z_2 due to the current source $I\angle 0°$ is given by,

$$I_1' = I\angle 0° \times \frac{Z_1}{Z_1 + Z_2} \quad \text{(By current division rule)}$$

Hence, the total current flowing in the circuit is sum of the individual currents due to voltage source and current source, when acting independently i.e ,

$$I = I_1 + I_1'$$

Q19. Find V_L in the circuit shown in figure, using superposition theorem.



Figure

Ans:

The given circuit is as shown in figure (1).

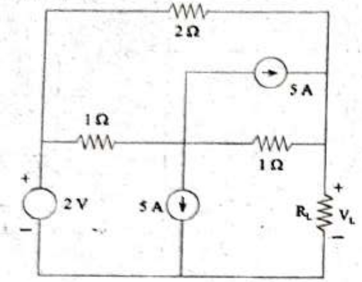


Figure (1)

In the given circuit the value of resistance R_L is not given. Hence, assuming the resistance R_L value to be 1 Ω for simplicity. To determine voltage V_L using superposition theorem.

In superposition theorem only one source is considered at a time. Hence, considering the 2 V voltage source only and short circuiting the current sources. Therefore the circuit with only 2 V voltage source is as shown in figure (2).

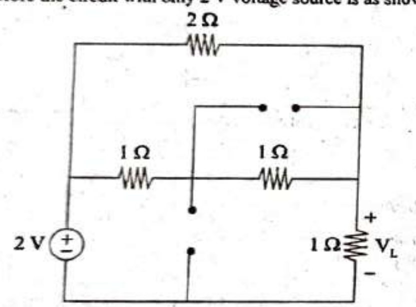


Figure (2)

In the figure (2) 1 Ω is in series with 1 Ω and series combination is in parallel with 2 Ω. Thus the circuit can be further reduced as shown in figure (3).

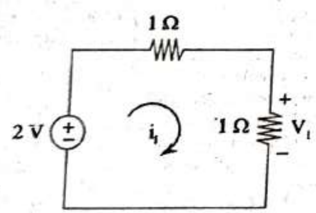


Figure (3)

∴ Current, $i_1 = \frac{V}{R_{eq}} = \frac{2}{1+1} = 1$ Amp

∴ Voltage across 1Ω resistor,

$V_1 = 1 \times i_1$

$V_1 = 1 \times 1 = 1$ V

Now, considering the 5 A current source only. The circuit of figure (1) can now be redrawn as shown in figure (4).

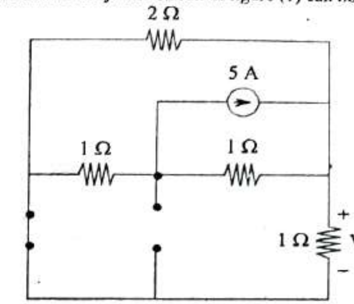


Figure (4)

The circuit can be reduced to the following,

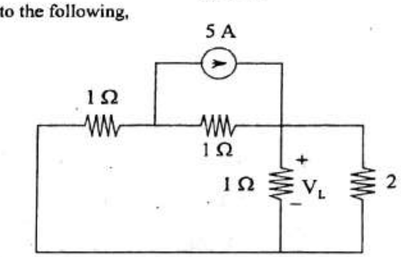


Figure (5)

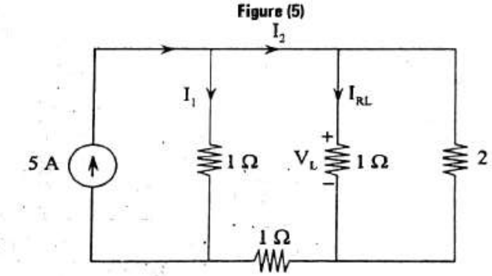


Figure (6)

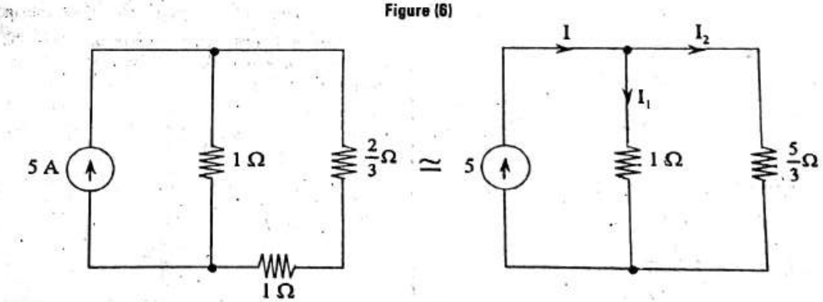


Figure (7)

$$I_1 = 5 \times \frac{\frac{5}{3}}{1+\frac{5}{3}}$$

$$= \frac{\frac{25}{3}}{\frac{8}{3}} = \frac{25}{8} = 3.125 \text{ A}$$

$$I_2 = I - I_1$$
$$= 5 - 3.125$$
$$= 1.875 \text{ A}$$

$$I_{RL} = I_2 \times \frac{2}{2+1} \qquad [\because \text{From figure (6)}]$$
$$= 1.875 \times 0.667$$
$$= 1.25 \text{ A}$$

$\therefore$ The voltage V_L across R_L is,

$$V_2 = R_L \times (-I_{RL})$$
$$= -1.25 \text{ Volts}$$

Here, the negative sign indicates the voltage is with reverse polarity.

Now, considering the other 5 A current source only as shown in figure (8).

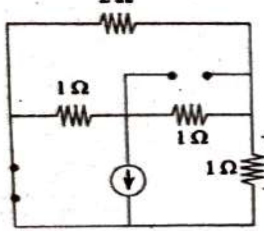


Figure (8)

The circuit can be reduced as shown in figure (9).

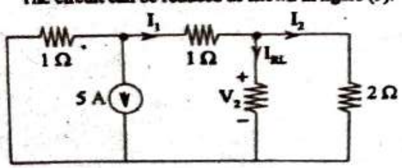


Figure (9)

$$\text{Current, } I_{RL} = -5 \times \frac{1}{1+1+\frac{2}{3}}$$
$$= -1.875 \text{ A}$$

$$\text{Current, } I_{RL} = (-1.875) \times \frac{2}{1+2}$$
$$= -1.25 \text{ A}$$

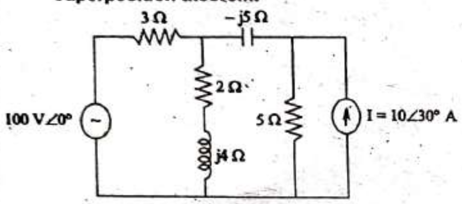
$\therefore$ The voltage across R_L is,

$$V_3 = (-1.25)(1)$$
$$= -1.25 \text{ volts}$$

$\therefore$ According to superposition theorem,

$$V_L = V_1 + V_2 + V_3$$
$$= 1 + (-1.25) + (-1.25)$$
$$= -1.5 \text{ volts}$$

Q20. Find the current through the capacitor of −j5Ω reactance as shown in figure, using superposition theorem.



Figure

Ans: **Model Paper-II, Q3(b)**

The given circuit is shown in figure (1).

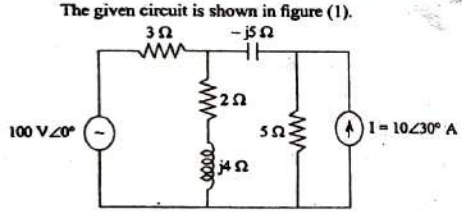


Figure (1)

To determine,

Current flowing through $-j5\ \Omega$ reactance.

Now, deactivating the current source i.e., open circuiting the current source, the circuit shown in figure (1) is modified as shown in figure (2).

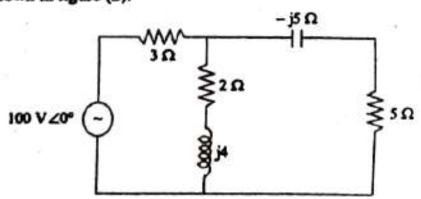


Figure (2)

From figure (2), the equivalent impedance of the circuit is given as,

$$Z_{eq} = 3 + [5 - j5) \,||\, (2 + j4)]$$

$$\Rightarrow \quad Z_{eq} = 3 + \left[\frac{(5-j5)(2+j4)}{(5-j5)+2+j4}\right]$$

$$\Rightarrow \quad Z_{eq} = 3 + \left[\frac{10 + j20 - j10 - j^2.20}{7 - j1}\right]$$

$$\Rightarrow \quad Z_{eq} = 3 + \left[\frac{10 + j10 - (-1).20}{7 - j1}\right] \qquad [\because j^2 = -1]$$

$$\Rightarrow \quad Z_{eq} = 3 + \left[\frac{10 + j10 + 20}{7 - j1}\right]$$

$$\Rightarrow \quad Z_{eq} = 3 + \left[\frac{30 + j10}{7 - j1}\right]$$

$$\Rightarrow \quad Z_{eq} = \frac{3(7 - j1) + (30 + j10)}{7 - j1}$$

$$\Rightarrow \quad Z_{eq} = \frac{21 - j3 + 30 + j10}{7 - j1}$$

$$\Rightarrow \quad Z_{eq} = \frac{51 + j7}{7 - j1}$$

$$\Rightarrow \quad Z_{eq} = \frac{51.4781 \angle 7.8152°}{7.0710 \angle -8.1301}$$

$$\Rightarrow \quad Z_{eq} = 7.2801 \angle 15.9453°$$

Thus, the current flowing through circuit shown in figure (2), is given as,

$$I = \frac{100 \angle 0°}{Z_{eq}}$$

$$\Rightarrow \quad I = \frac{100 \angle 0°}{7.2801 \angle 15.9453°}$$

$$\Rightarrow \quad I = 13.7360 \angle -15.9453° \text{ A}$$

Now,

The current flowing through $-j5\ \Omega$ reactance from the voltage source $100 \angle 0°$ is given as,

$$I'_{-j5} = I \times \frac{2 + j4}{(2 + j4) + (5 - j5)}$$

[$\because$ By current division rule]

$$\Rightarrow \quad I'_{-j5} = I \times \frac{2 + j4}{7 - j1}$$

$$\Rightarrow \quad I'_{-j5} = 13.7360 \angle -15.9453° \times \frac{2 + j4}{7 - j1}$$

$$\Rightarrow \quad I'_{-j5} = \frac{(13.7360 \angle -15.9453°) \times (4.4721 \angle 63.4349°)}{7.0710 \angle -8.1301}$$

$$\Rightarrow \quad I'_{-j5} = \frac{61.4287 \angle 47.4896°}{7.0710 \angle -8.1301°}$$

$$\Rightarrow \quad I'_{-j5} = 8.6874 \angle 55.6197° \text{ A}$$

Deactivating the voltage source, i.e., short circuiting the voltage source, the circuit shown in figure (1) is modified as shown in figure (3).

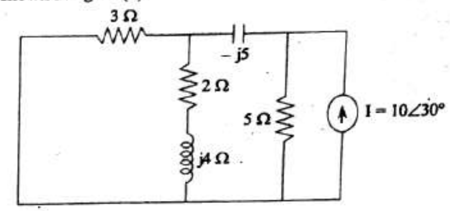


Figure (3)

From figure (3), resistance 3 Ω and a combination of 2 Ω, j4 Ω are in parallel i.e.,

$$Z = \frac{3(2 + j4)}{3 + 2 + j4}$$

$$Z = \frac{6 + j12}{5 + j4}$$

$\therefore$ The circuit in figure (3) now modifies as shown in figure (4).

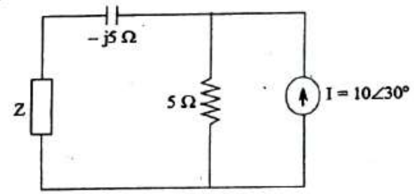


Figure (4)

The current flowing in $-j5\ \Omega$ reactance from the current source $I = 10\angle 30°$ is given as,

$$I''_{-j5\Omega} = I \times \frac{5}{Z - j5 + 5}$$

$$= 10\angle 30° \times \frac{5}{\left(\frac{6 + j12}{5 + j4}\right) - j5 + 5}$$

$$= \frac{10 \angle 30° \times 5}{\left(\frac{(6 + j12) - (j5)(5 + j4) + 5(5 + j4)}{5 + j4}\right)}$$

$$= \frac{50 \angle 30°}{\left(\frac{6 + j12 - j25 - j^2 20 + 25 + j20}{5 + j4}\right)}$$

$$= \frac{50 \angle 30°}{\left(\frac{31 + 20 + j7}{5 + j4}\right)} \qquad (\because j^2 = -1)$$

$$= \frac{50\angle 30^\circ}{\left(\frac{51+j7}{5+j4}\right)}$$

$$= \frac{50\angle 30^\circ}{\left(\frac{51.478\angle 7.815^\circ}{6.403\angle 38.659^\circ}\right)}$$

$$= \frac{50\angle 30^\circ \times 6.403\angle 38.659^\circ}{51.478\angle 7.815^\circ}$$

$$= \frac{320.15\angle 68.659^\circ}{51.478\angle 7.815^\circ}$$

$$= 6.219\angle 60.844^\circ \text{ A}$$

Now, the total current flowing through the $-j5\ \Omega$ reactance is given as,

$$I_{-j5} = I'_{-j5} - I''_{-j5}$$

$$= 8.6874\angle 55.6197^\circ - 6.219\angle 60.844^\circ$$

$$= 4.9056 + j7.1697 - (3.0298 + j5.431)$$

$$= 4.9056 + j7.1697 - 3.0298 - j5.431$$

$$= 1.8758 + j1.7387$$

$$= 2.5576\angle 42.8277^\circ \text{ A}$$

Thevenin's theorem

Q21. State and explain Thevenin's theorem.

Ans:

Thevenin's Theorem

Statement

Any linear network having active voltage and current sources with two terminals A and B can be replaced by an equivalent voltage source (V_{Th}) and equivalent resistance (R_{Th}) in series combination forming a simple equivalent circuit.

Where, V_{Th} is the open circuit voltage across the terminals A and B and R_{Th} is the equivalent resistance as seen from the terminals A and B when the independent sources are deactivated.

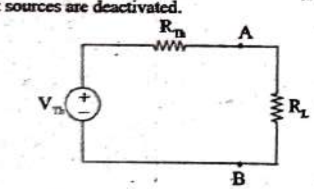


Figure (1): Thevenin's Equivalent Circuit

Procedure to Calculate Thevenin's Equivalent Circuit

Consider a network as shown in figure (2).

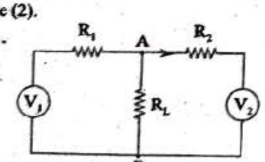


Figure (2)

The circuit consists of two voltage sources V_1 and V_2 and three resistances R_1, R_2 and R_L.

Steps to Calculate Thevenin's Voltage (V_{Th})

In order to obtain Thevenin's voltage, open circuit the resistance (R_L). The circuit becomes as shown in figure (3).

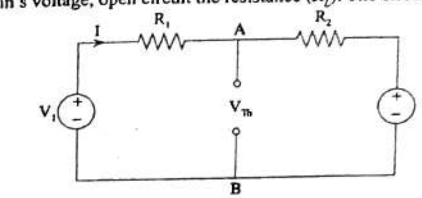


Figure (3)

Thevenin's voltage (V_{Th}) is equal to the voltage across the terminals A and B i.e., V_{AB}.

By applying KVL to the circuit, we get,

$$I(R_1 + R_2) + V_2 - V_1 = 0$$

$$\Rightarrow \quad I = \frac{V_1 - V_2}{R_1 + R_2} \quad \text{... (1)}$$

Since, $V_{Th} = V_{AB} = V_1 - IR_1$ ... (2)

By substituting equation (1) in equation (2) we can calculate the value of V_{Th}.

Steps to Calculate R_{Th}

In order to calculate Thevenin's resistance (R_{Th}), the voltage sources are short circuited and current sources are open circuited.

Therefore, R_{Th} can be found by short circuiting the voltage sources V_1 and V_2. The circuit becomes,

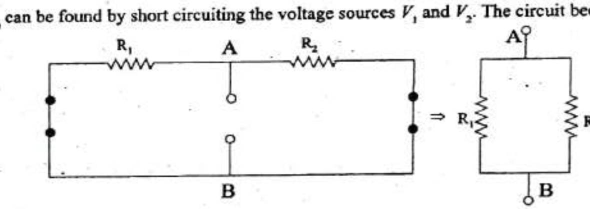


Figure (4)

Thevenins resistance,

$$R_{Th} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

Therefore the Thevenin's equivalent circuit obtained from derived parameters is as follows,

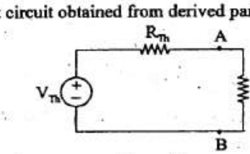


Figure (5)

Q22. State and explain Thevenin's theorem for A.C excitation.

Ans:

Thevenin's Theorem

Statement

Any linear network having active voltage and current sources with two terminals A and B can be replaced by an equivalent voltage source (V_{Th}) and equivalent impedance (Z_{Th}) in series combination forming a simple equivalent circuit.

Where, V_{Th} is the open circuit voltage across the terminals A and B and Z_{Th} is the equivalent impedance as seen from the terminals A and B when the independent sources are deactivated.

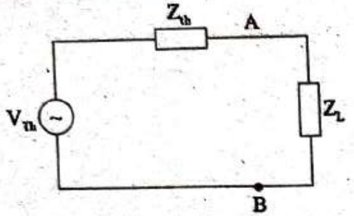


Figure (1): Thevenin's Equivalent Circuit

Procedure to Calculate Thevenin's Equivalent Circuit

Consider a network as shown in figure (2).

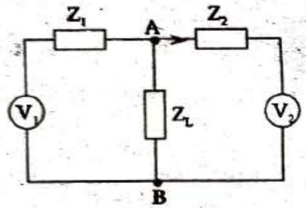


Figure (2)

The circuit consists of two voltage sources V_1 and V_2 and three impedance Z_1, Z_2 and Z_L.

Steps to Calculate Thevenin's Voltage (V_{Th})

In order to obtain thevenin's voltage, open circuit the impedance (Z_L). The circuit is modified as shown in figure (3).

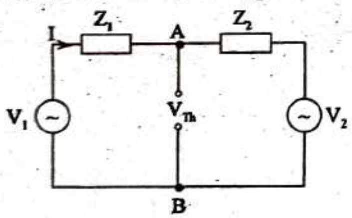


Figure (3)

Thevenin's voltage (V_{Th}) is equal to the voltage across the terminals A and B i.e.,

By applying KVL to the circuit, we get,

$I(Z_1 + Z_2) + V_2 - V_1 = 0$

$$\Rightarrow I = \frac{V_1 - V_2}{Z_1 + Z_2} \quad \text{... (1)}$$

Since, $V_{Th} = V_{AB} = V_1 - IZ_1$ ... (2)

By substituting equation (1) in equation (2) we can calculate the value of V_{Th}.

Steps to Calculate Z_{Th}

In order to calculate Thevenin's impedance (Z_{Th}) the voltage sources are short circuited and current sources are open circuited.

Therefore, Z_{Th} can be found by short circuiting the voltage sources V_1 and V_2. The circuit becomes,

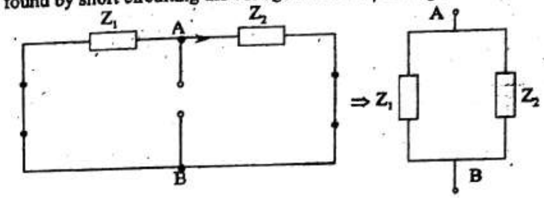


Figure (4)

Thevenin's impedance,

$$Z_{Th} = Z_1 \parallel Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Therefore, the Thevenin's equivalent circuit obtained from derived parameters is as follows.

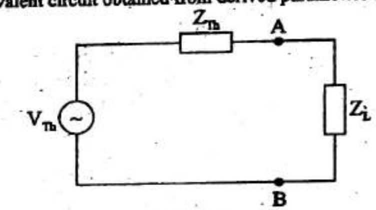
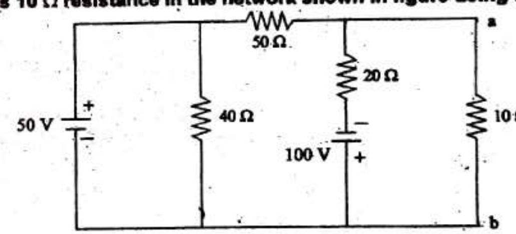


Figure (5)

Q23. Find voltage across 10 Ω resistance in the network shown in figure using the Thevenin's theorem.



Figure

Ans:

The given circuit is shown in figure (1).

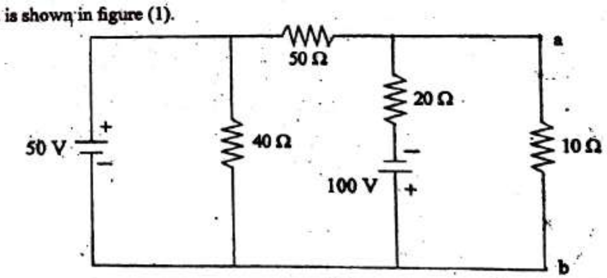


Figure (1)

In order to find the voltage across 10 Ω resistor using Thevenin's theorem, the 10 Ω resistance is removed as shown in figure (2).

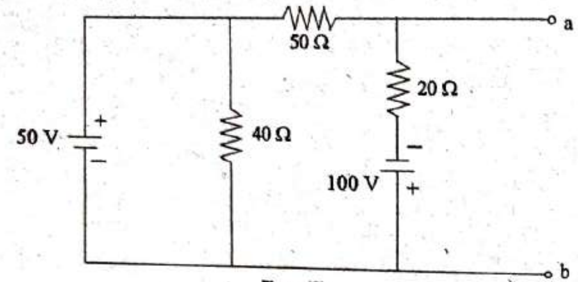


Figure (2)

Let I_1 and I_2 be the currents in loops (1) and (2) as shown in figure(3).

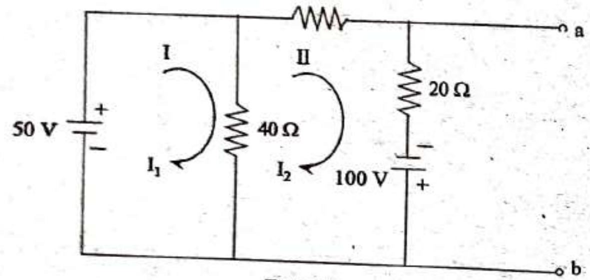


Figure (3)

Applying KVL to loop I, we get,

$$50 = 40\,(I_1 - I_2)$$

$$\Rightarrow \quad I_1 - I_2 = 1.25 \qquad \text{... (1)}$$

Applying KVL to loop II, we get,

$$40\,(I_2 - I_1) + 50\,I_2 + 20\,I_2 = 100$$

$$40\,I_2 - 40\,I_1 + 70\,I_2 = 100$$

$$-40\,I_1 + 110\,I_2 = 100$$

$$\Rightarrow \quad I_1 - 2.75\,I_2 = -2.5 \qquad \text{... (2)}$$

Solving equations (1) and (2), we get,

$$I_1 = 3.393 \text{ A}$$

$$I_2 = 2.143 \text{ A}$$

Voltage across 20 Ω resistance,

$$V_{20\Omega} = 20\,I_2$$

$$= 20 \times 2.143$$

$$= 42.86 \text{ V}$$

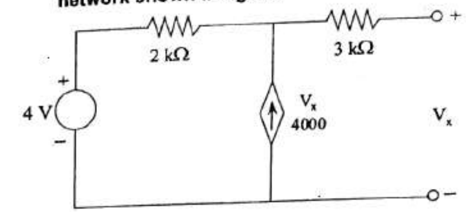
$\therefore$ Voltage across 10Ω resistance,

$$V_{10\Omega} = V_{ab} = V_{20\Omega} - 100$$

$$= 42.86 - 100$$

$$= -57.14 \text{ V}$$

Q24. Obtain Thevenin's equivalent circuit for the network shown in figure.



Figure

Ans:

The given circuit is shown in figure (1).

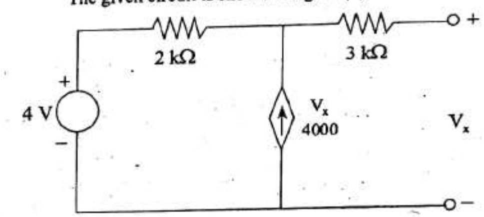


Figure (1)

From figure (1),

Open circuit voltage, $V_x = 4 - \left(\frac{-V_x}{4000}\right)(2 \times 10^3)$

$$\Rightarrow \quad V_x = 4 + 0.5\,V_x$$

$$\Rightarrow \quad V_x - 0.5\,V_x = 4$$

$$\Rightarrow \quad 0.5\,V_x = 4$$

$$\Rightarrow \quad V_x = 8 \text{ V}$$

$\therefore$ Thevenin's voltage, $V_{Th} = V_x = 8\text{V}$

To find Thevenin's resistance short circuiting the source voltage of 4V and applying a voltage source V_x across the open terminals as shown in figure (2).

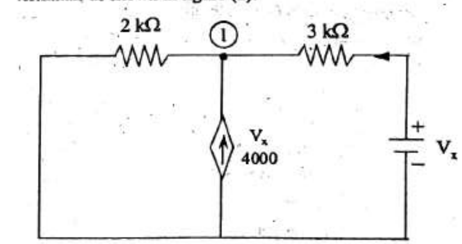


Figure (2)

Let the voltage at node 1 be V_1,

Applying KCL at node, we get,

$$\frac{V_1}{2 \times 10^3} - \frac{V_x}{4000} + \frac{V_1 - V_x}{3 \times 10^3} = 0$$

$$\Rightarrow \quad V_1\left(\frac{1}{2 \times 10^3} + \frac{1}{3 \times 10^3}\right) - V_2\left(\frac{1}{4000} + \frac{1}{3 \times 10^3}\right) = 0$$

$$\Rightarrow \quad (8.3333 \times 10^{-4})\,V_1 - (5.8333 \times 10^{-4})\,V_x = 0$$

$$\Rightarrow \quad (8.3333 \times 10^{-4})\,V_1 = (5.8333 \times 10^{-4})\,V_x$$

$$\Rightarrow \quad V_1 = 0.7\,V_x \qquad \text{...(1).}$$

From figure (2), we have,

$$I = \frac{V_x - V_1}{3 \times 10^3}$$

$$\Rightarrow \quad I = \frac{V_x - 0.7V_x}{3 \times 10^3} \qquad (\because \text{ From equation (1)})$$

$$\Rightarrow \quad I = \frac{0.3}{3000} V_x$$

$$\Rightarrow \quad I = 1 \times 10^{-4}\,V_x$$

$$\Rightarrow \quad \frac{V_x}{I} = \frac{1}{1 \times 10^{-4}}$$

$$\Rightarrow \quad \frac{V_x}{I} = 10{,}000$$

$\therefore$ Thevenin's resistance, $R_{Th} = \frac{V_x}{I} = 10000$

$= 10 \text{ k}\Omega$

Thus, the Thevenin's equivalent circuit for the given network is as shown in figure (3),

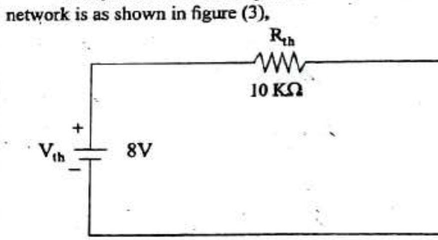
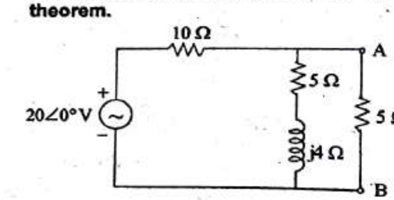


Figure (3)

Q25. Find the current through the branch A-B of the network shown in the figure using Thevenin's theorem.



Figure

Ans:

The given network shown in figure (1)

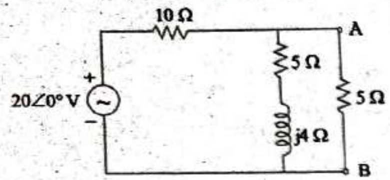


Figure (1)

In order to determine the current through the branch *A-B* using Thevenin's theorem, we have to first remove the 5 Ω branch and then determine V_{TH}, Thevenin's voltage across the open terminals created and R_{TH}, the Thevenin's equivalent resistance seen through the open terminals.

Thevenin's Voltage, V_{Th}

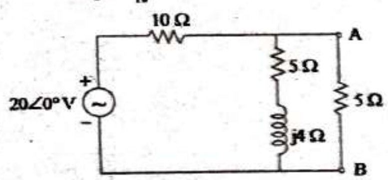


Figure (2)

$V_{TH} = V_{AB}$

V_{AB} = Voltage across $(5 + j4)\ \Omega$

$V_{AB} = I(5 + j4)$

$I = \dfrac{V}{Z_e}$

$Z_e = 10 + 5 + j4$

$= 15 + j4$

$\therefore\ I = \dfrac{20\angle 0°}{15 + j4}$

$= \dfrac{20\angle 0°}{15.524\angle -14.93}$

$I = 1.288\angle -14.93$

$V_{AB} = 1.288\angle -14.93° \times (5 + j4)$

$= 1.288\angle -14.93° \times 6.403\angle 38.66°$

$\therefore\ V_{TH} = V_{AB} = 8.247\angle 23.73°$

Thevenin's Impedance, Z_{TH}

In order to determine Z_{TH}, the voltage source must be replaced with a short-circuit as shown in figure (3).

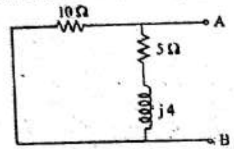


Figure (3)

$\therefore\ Z_{TH} = 10 \parallel (5 + j4)$

$= \dfrac{10(5 + j4)}{10 + 5 + j4}$

$= \dfrac{10 \times 6.403\ \underline{|38.66°}}{15.524\ \underline{|14.93°}}$

$= 4.125\ \underline{|23.73°}$

$= 3.776 + j1.66$

Hence, the Thevenin's equivalent circuit can be drawn as shown in figure (4).

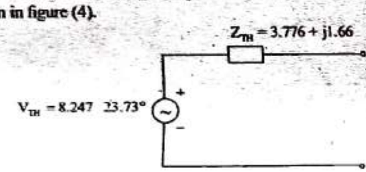


Figure (4)

Current Through Branch A-B

Now, connecting the branch *A-B* with 5 Ω to the Thevenin's equivalent circuit and determining the current flowing through it.

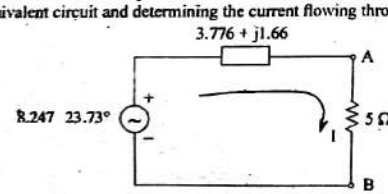


Figure (5)

Let, *I* be the current flowing through the circuit.

Since, the circuit is a series circuit,

$I_{5\Omega} = I$

$I_{5\Omega} = \dfrac{V}{Z_e}$

$= \dfrac{8.247\angle 23.73°}{3.776 + j1.66 + 5}$

$= \dfrac{8.247\angle 23.73°}{8.9316\angle 10.71°}$

$\therefore\ I_{5\Omega} = 0.923\angle 13.02°\ \text{A}$

1.3 NORTON THEOREM

Q26. State and explain Norton's theorem.

Ans:

Statement

The Norton's theorem states that any two terminal linear network with current sources, voltage sources and resistances (impedances) can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance (impedance), where the value of the current source is equal to the current passing through the short-circuited terminals and the resistance is equal to the resistance measured between the terminals of the network with all the energy sources replaced by their internal resistance.

In short, this theorem is used, where it is easier to simplify a network in terms of current instead of voltages. This theorem reduces a normally complicated networks to a simple parallel circuit consisting of,

(a) An ideal current source I_N of infinite internal resistance and

(b) A resistance R_N in parallel with current source as shown in figure (2).

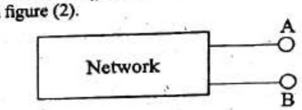


Figure (1): Normally Complicated Circuit

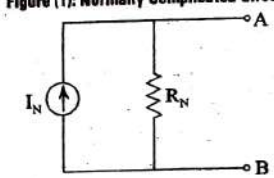


Figure (2): Norton's Equivalent Circuit

Where, I_N is the current which flows through a short-circuit placed across terminals *A* and *B*.

R_N is the circuit resistance looking from the open *A-B* terminals.

Q27. State and explain Norton's theorem for A.C excitation.

Ans:

Statement

The Norton's theorem states that any two terminal linear network with current sources, voltage sources and impedances can be replaced by an equivalent circuit consisting of a current source in parallel with an impedance (Z_N), where the value of the current source is equal to the current passing through the short circuited terminals and the impedance is equal to the impedance measured between the terminals of the network with all the energy sources replaced by their internal impedance.

In short, this theorem is used, where it is easier to simplify a network in terms of current instead of voltages. This theorem reduces a normally complicated networks to a simply parallel circuit consisting of,

(a) An ideal current source I_N of infinite internal impedance.

(b) An impedance 'Z_N' in parallel with current source 'I_N'.

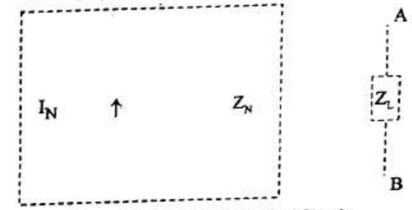


Figure (1): Norton's Equivalent Circuit

Consider the circuit shown in figure (2)

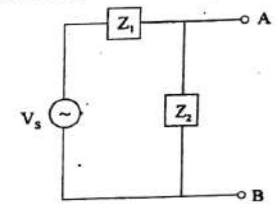


Figure (2).

The Norton's equivalent circuit between the terminals *A* and *B* is calculated as,

The Norton's equivalent current is nothing but the current flowing through the terminals *A* and *B* when it is short circuited.

Norton's current, $I_N = \dfrac{V}{Z_1}$

The equivalent impedance (Z_N) is nothing but the impedance between the two terminals *A* and *B*.

i.e., $Z_2 \parallel Z_1$

$\therefore\ Z_N = \dfrac{Z_1 Z_2}{Z_1 + Z_2}$

The Norton's equivalent circuit is shown in the figure (3).

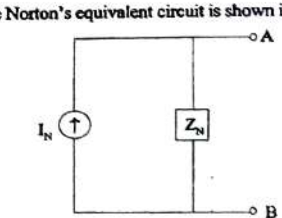


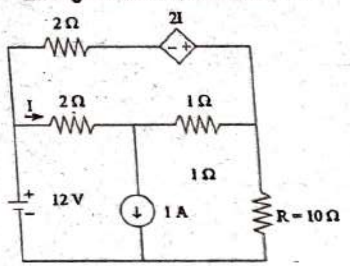
Figure (3)

Q28. For the network shown in figure:

(i) Determine the current through R = 10 ohm resistor using Thevenin's theorem.

(ii) Verify the result using Norton's theorem

(iii) Calculate the maximum power transfer through R and find the value of R.



Figure

Ans:

The given circuit is as shown in figure (1).

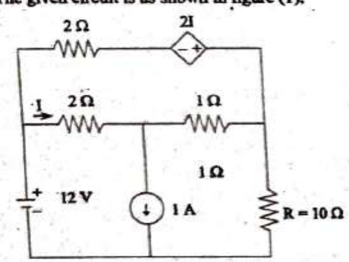


Figure (1)

(i) Thevenin's Theorem

In order to apply the Thevenin's theorem, the 10 Ω resistor has to be removed as shown in figure (2) and the voltage across the open terminal is to be calculated.

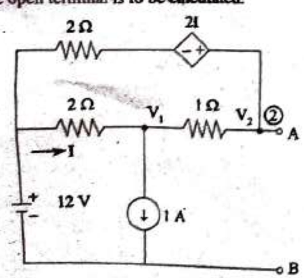


Figure (2)

Applying nodal analysis at node (1), we get,

$$\frac{12-V_1}{2} = \frac{V_1-V_2}{1}+1$$

$$\Rightarrow \frac{12-V_2}{2} - V_1 + V_2 - 1 = 0$$

$$\Rightarrow 12 - V_1 - 2V_1 + 2V_2 - 2 = 0$$

$$\Rightarrow -3V_1 + 2V_2 + 10 = 0$$

$$\Rightarrow -3V_1 + 2V_2 = -10 \quad \text{... (1)}$$

Applying nodal analysis at node (2), we get,

$$\frac{V_1-V_2}{1} + \frac{12+2I-V_2}{2} = 0$$

$$\Rightarrow 2V_1 - 2V_2 + 12 + 2I - V_2 = 0$$

$$\Rightarrow 2V_1 - 3V_2 + 2I + 12 = 0$$

$$\Rightarrow 2V_1 - 3V_2 + 2\left(\frac{12-V_1}{2}\right) + 12 = 0$$

$$\left(\because \text{From figure (2), } I = \frac{12-V_1}{2}\right)$$

$$\Rightarrow 2V_1 - 3V_2 + 12 - V_1 + 12 = 0$$

$$\Rightarrow V_1 - 3V_2 + 24 = 0$$

$$\therefore V_1 - 3V_2 = -24 \quad \text{... (2)}$$

Solving equations (1) and (2), we get,

$$V_1 = \frac{78}{7}\text{ V}$$

$$V_2 = \frac{82}{7}\text{ V}$$

$\therefore$ Thevenin's voltage,

$$V_{th} = V_{oc}$$

$$V_2 = \frac{82}{7}$$

$$= 11.714\text{ V}$$

To find the value of thevenin's resistance R_{th}, short circuiting the terminals $A-B$ as shown in figure(3).

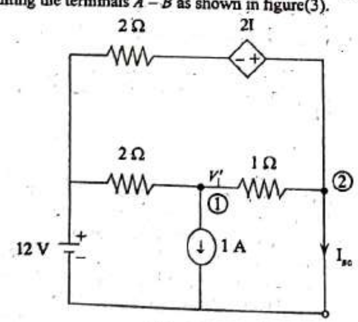


Figure (3)

Now,

Let the voltage at node (1) be V_1' applying nodal analysis at node (1), we get,

$$\frac{12-V_1'}{2} = \frac{V'-0}{1}+1$$

$$\Rightarrow \frac{12-V_1'}{2} - V_1' - 1 = 0$$

$$\Rightarrow 12 - V_1' - 2V_1' - 2 = 0$$

$$\Rightarrow -3V_1' + 10 = 0$$

$$\Rightarrow -3V_1' = -10$$

$$\Rightarrow V_1' = \frac{10}{3}$$

$$\therefore V_1' = 3.33\text{ V}$$

Now, applying nodal analysis at node(2), we get,

$$I_{sc} = \frac{12+2I}{2} + \frac{V_1'-0}{1}$$

$$\Rightarrow I_{sc} = \frac{12+2I+V_1'}{2}$$

$$\Rightarrow 2I_{sc} = 12 + 12 - V_1' + 2V_1'$$

$$\Rightarrow 2I_{sc} = V_1' + 24$$

$$\Rightarrow I_{sc} = \frac{V_1'}{2} + 12$$

Substituting the value of V_1', we get,

$$I_{sc} = \frac{3.33}{2} + 12$$

$$= 13.665\text{ A}$$

$\therefore$ Thevenin's resistance,

$$R_{th} = \frac{V_{oc}}{I_{sc}}$$

$$= \frac{11.714}{13.665}$$

$$= 0.857\ \Omega$$

The thevenin's equivalent circuit is shown in figure (4).

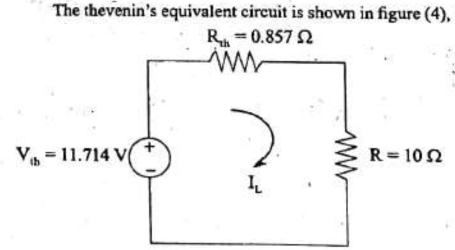


Figure (4)

The current through the load resistance R is,

$$I_L = \frac{V_{th}}{R_{th}+R_L}$$

$$= \frac{11.714}{0.857+10}$$

$$= 1.07\text{ A}$$

(ii) Norton's Theorem

Norton's current,

$$I_N = I_{sc}$$

$$= 13.665\text{ A}$$

Norton's resistance,

$$R_N = R_{th}$$

$$= 0.857\ \Omega$$

$\therefore$ The Norton's equivalent circuit is as shown in figure (5),

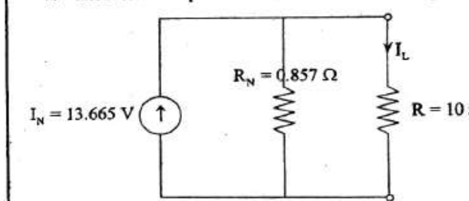


Figure (5)

Now, the current through the load resistance R is,

$$I_L = I_{sc}\frac{R_N}{R_N+R}$$

$$= 13.665 \times \frac{0.857}{0.857+10}$$

$$= 1.07\text{ A}$$

(iii) Maximum Power Transfer Theorem

Maximum power transferred to the load resistance R is,

$$P_{max} = \frac{(V_{th})^2}{4R_{th}}$$

$$= \frac{(11.714)^2}{4\times 0.857}$$

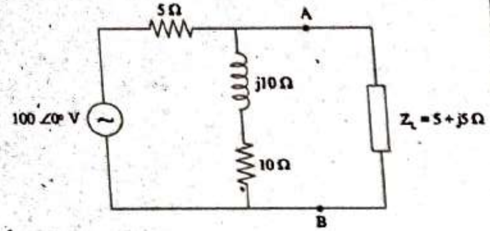
$$= 40.02\text{ W}$$

For maximum power transfer,

$$R = R_{th}$$

$$\therefore R = 0.857\ \Omega$$

Q29. Using Norton's theorem, find the current through the load impedance Z_L as shown in figure.



Figure

Ans:

The circuit is shown in figure (1).

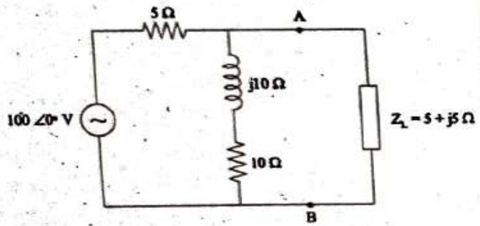


Figure (1)

Current through load impedance, I_L = ?

Finding Norton's Equivalent Circuit by using Norton's Theorem

To find Norton's current,

Short circuiting the load impedance Z_L i.e., terminals *AB*.

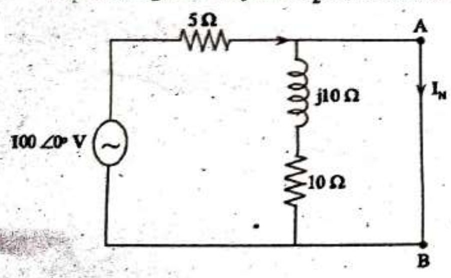


Figure (2)

In figure (2); since $(10 + j10)\Omega$ is in parallel with short circuit, the total current will flow through the short circuit terminals *A-B* and hence $(10 + j10)$ will be inactive. Now, the circuit can be modified as shown in figure (3).

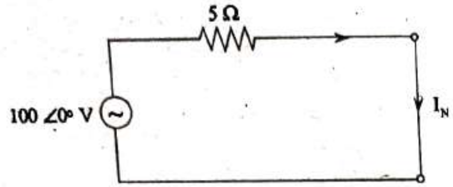


Figure (3)

Norton's current, I_N is the current passing through the terminals *AB*.

$$\therefore I_N = \frac{100\angle 0°}{5} = 20\angle 0° \text{ A}$$

To Find Norton's Impedance

Norton's impedance 'Z_N' is the equivalent impedance seen from terminals *AB* with the voltage source short circuited shown in figure (4).

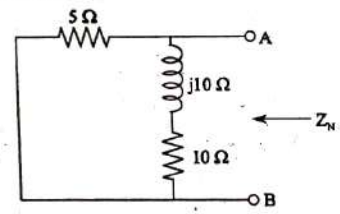


Figure (4)

Norton's impedance,

$$Z_N = \frac{5 \times (10 + j10)}{5 + (10 + j10)}$$

$$= \frac{50 + j50}{15 + j10}$$

$$= 3.922\angle 11.3°\Omega$$

Norton's equivalent circuit is as shown in figure (5).

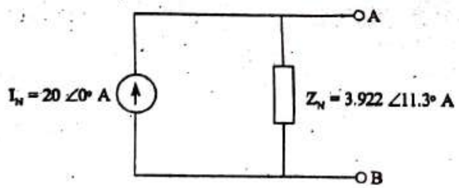


Figure (5)

Connecting the load impedance 'Z_L' between the terminals, *AB* is the Norton's equivalent circuit

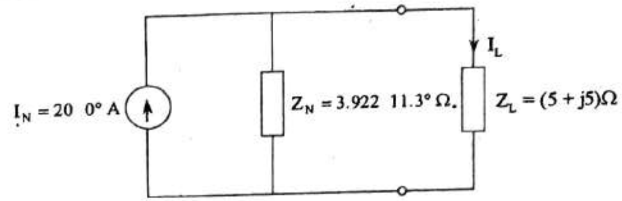


Figure (6)

Load current,

$$I_L = I_N \times \frac{Z_N}{Z_N + Z_L}$$

$$= 20\angle 0° \times \frac{3.922\angle 11.3°}{3.922\angle 11.3° + (5 + j5)}$$

$$= \frac{78.44\angle 11.3°}{10.56\angle 33.1°}$$

$$\therefore I_L = 7.42\angle -21.8° \text{ A}$$

MAXIMUM POWER TRANSFER THEOREM

Q30. State and explain maximum power transfer theorem.

Ans:

Maximum Power Transfer Theorem

Statement

Maximum power transfer theorem states that the maximum power can be transferred from source (voltage source, current source) to the load when the resistance (R_L) is equal to the internal resistance of the source (R_i).

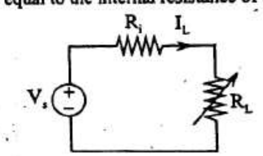


Figure: Maximum Power Transfer Theorem

The figure shown consists of voltage source (V_s), internal resistance (R_i) and variable load resistance (R_L) connected in series.

Power across load is given by,

$$P_L = I_L^2 . R_L \quad \text{... (1)}$$

From the figure, load current can be calculated as follows,

Load current,

$$I_L = \frac{V_s}{(R_i + R_L)} \quad \text{... (2)}$$

Substituting equation (2) in equation (1), we get,

$$P_L = \left(\frac{V_s}{(R_i + R_L)}\right)^2 R_L$$

$$\Rightarrow P_L = \frac{V_s^2}{(R_i + R_L)^2} \times R_L \; I_L^2 \quad \text{... (3)}$$

The condition for maximum power can be obtained by differentiating equation (3) with respect to load resistance, R_L and equating it to zero.

$$\Rightarrow \frac{dP_L}{dR_L} = 0$$

$$\Rightarrow \frac{dP_L}{dR_L} = \frac{d}{dR_L}\left[\frac{V_s^2 R_L}{(R_i + R_L)^2}\right] = 0$$

Applying $\frac{U}{V}$ differentiation rule, we get,

$$\Rightarrow \frac{dP_L}{dR_L} = \frac{(R_i+R_L)^2 \times \frac{d}{dR_L}(V_S^2 R_L) - (V_s^2 R_L)\frac{d}{dR_L}\left[(R_i+R_L)^2\right]}{(R_i+R_L)^4} = 0$$

$$\Rightarrow \frac{(R_i+R_L)^2 \times V_s^2 - (V_s^2 R_L)\frac{d}{dR_L}(R_i^2 + R_L^2 + 2R_iR_L)}{(R_i+R_L)^4} = 0$$

$$\Rightarrow \frac{(R_i^2 + 2R_iR_L + R_L^2)V_s^2 - (V_s^2 R_L)(0 + 2R_L + 2R_i)}{(R_i+R_L)^4} = 0$$

$$\Rightarrow V_s^2[R_i^2 + 2R_iR_L + R_L^2 - 2R_L(R_L + R_i)] = 0$$

$$\Rightarrow R_i^2 + 2R_iR_L + R_L^2 - 2R_L^2 - 2R_iR_L = 0$$

$$\Rightarrow R_i^2 - R_L^2 = 0$$

$$\Rightarrow R_i^2 = R_L^2$$

$$\therefore R_i = R_L$$

Hence, the maximum power will be transferred from source to the load when the internal resistance (R_i) of the source is equal to the load resistance (R_L).

Under this condition, the maximum power transferred to the load will be,

$$P_{L\max} = \frac{V_S^2}{[R_i + R_L]^2} \times R_L \quad \text{[From equation (3)]}$$

$$= \frac{V_S^2}{[2R_L]^2} \times R_L \quad [\because R_i = R_L]$$

$$= \frac{V_S^2}{4R_L}$$

Q31. State and explain the maximum power theorem for A.C excitation.

Ans:

Maximum Power Transfer Theorem

Statement

"Maximum power transfer theorem when applied to complex impedance circuits states that maximum power is transferred from source to load, when load impedance is equal to complex conjugate of source impedance".

$$\Rightarrow Z_L = Z_S^*$$

Consider a network with complex load impedance Z_L as shown in figure (i)

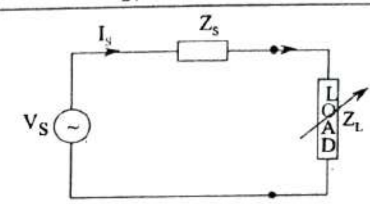


Figure (i)

$$Z_s = R_s + jX_s$$

$$Z_L = R_L + jX_L$$

Current flowing through circuit,

$$I_L = \frac{V_S}{Z_S + Z_L} = \frac{V_S}{(R_S + jX_S) + (R_L + jX_L)}$$

$$= \frac{V_S}{(R_S + R_L) + j(X_S + X_L)}$$

Real power across load is given by,

$$P_L = |I_L|^2 R_L$$

$$\therefore P_L = \left[\frac{V_S}{(R_S + R_L) + j(X_S + X_L)}\right]^2 \times R_L$$

$$= \frac{V_S^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2} \quad \text{... (1)}$$

Now by varying X_L, the condition for maximum power flow is obtained. It is given as,

$$\Rightarrow \frac{dP_L}{dX_L} = 0$$

$$\Rightarrow \frac{d}{dX_L}\left[\frac{V_S^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}\right] = 0$$

$$\Rightarrow V_S^2 R_L \frac{d}{dX_L}([(R_S + R_L)^2 + (X_S + X_L)^2]^{-1}) = 0$$

$$\Rightarrow V_S^2 R_L [(-1)[(R_S + R_L)^2 + (X_S + X_L)^2]^{-1}]^{-1} \frac{d}{dX_L}(X_S + X_L)^2 = 0$$

$$\Rightarrow -V_S^2 R_L \frac{d}{dX_L}[(R_S + R_L)^2 + (X_S + X_L)^2]^{-2} \times 2(X_S + X_L) = 0$$

$$\Rightarrow \frac{-V_S^2 R_L \times 2(X_S + X_L)}{[(R_S + R_L)^2 + (X_S + X_L)^2]^2} = 0$$

$$\Rightarrow 2(X_s + X_L) = 0$$

$$\Rightarrow X_s + X_L = 0$$

$$\therefore X_s = -X_L$$

Now substituting $X_s = -X_L$ in equation (1), we get,

$$P_L = \frac{V_S^2 R_L}{(R_S + R_L)^2} \quad \text{... (2)}$$

The condition for maximum power can be obtain by differentiating equation (2) with respect to load impedance R_L and equating it to zero.

$$\Rightarrow \quad \frac{dP_L}{dR_L} = 0$$

$$\frac{d}{dR_L}\left[\frac{V_S^2 R_L}{(R_S+R_L)^2}\right] = 0$$

Applying $\frac{U}{V}$ differentiation rule, we get,

$$V_S^2\left[\frac{(R_S+R_L)^2 \times 1 - R_L \times 2(R_S+R_L)}{[(R_L+R_S)^2]^2}\right] = 0$$

$$(R_s+R_L)^2 - 2R_L(R_s+R_L) = 0$$

$$R_S^2 + R_L^2 + 2R_SR_L - 2R_LR_S - 2R_L^2 = 0$$

$$R_S^2 - R_L^2 = 0$$

$$\therefore \quad R_S^2 = R_L^2$$

$$\therefore \quad R_s = R_L$$

Hence, maximum power transfer take place when $R_s = R_L$ and $X_s = -X_L$ i.e., $R_s + jX_s = R_L - jX_L$. This means that maximum power will be transferred from source to load when the load impedance (Z_L) is equal to conjugate of source impedance Z_S.

$$\Rightarrow \quad Z_s = Z_L^*$$

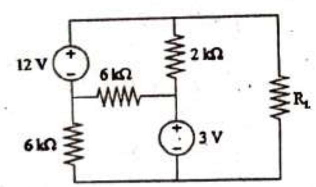
Under this condition, maximum power transferred to the load will be,

$$P_{Lmax} = \frac{V_S^2 R_L}{(R_L+R_S)^2} \quad \text{[From equation (2)]}$$

$$= \frac{V_S^2 R_L}{[2R_L]^2} \quad [\because R_L = R_S]$$

$$P_{Lmax} = \frac{V_S^2}{4R_L}$$

Q32. For the network shown in figure, find the value of R_L forr maximum power transfer. Also find the maximum power transferred to R_L.



Figure

Ans:

The given circuit is as shown in figure (1),

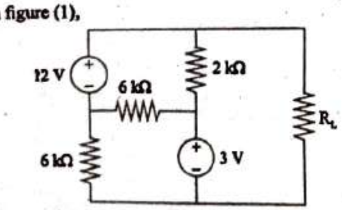


Figure (1)

To find the maximum power Thevenin's equivalent circuit must be detrmined, for which the terminals across the load resistance R_L must be open circuited. The reduced circuit is as shown in figure (2).

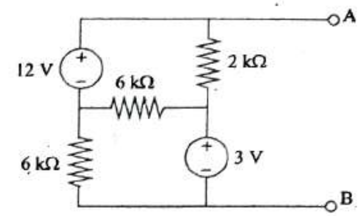


Figure (2)

To find the Thevenin's resistance, short circuiting all the independent voltage sources as shown in figure (3)

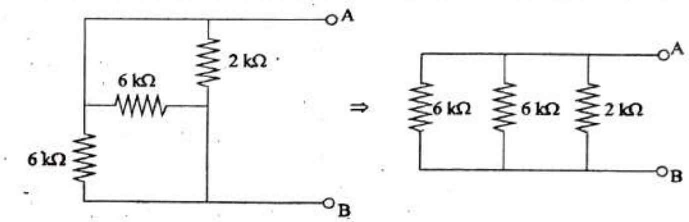


Figure (3)

$\therefore$ Thevenin's resistance,

$$R_{th} = (6 \parallel 6) \parallel 2$$

$$= \left(\frac{6\times 6}{6+6}\right) \parallel 2$$

$$= 3 \parallel 2$$

$$= \frac{3\times 2}{3+2}$$

$$= 1.2 \text{ k}\Omega$$

The circuit shown in figure (2) can be redrawn by representing the node voltages V_1, V_2 and V_3.

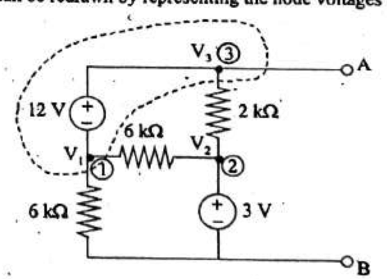


Figure (4)

Here, it is difficult to find the current flowing between nodes (1) and (3), due to the presence of votlage source '12V'. Therefore, considering node (1) and (3) which represented by dotted line as a single super node.

Applying KCL at sup node, we get,

$$\frac{V_1}{6}+\frac{V_1-V_2}{6}+\frac{V_3-V_2}{2}=0$$

$$\Rightarrow V_1+V_1-V_2+3V_3-3V_2=0$$

$$\Rightarrow 2V_1-2V_2+3V_3=0 \quad \text{... (1)}$$

The voltage between nodes (1) and (3) is given by,

$$V_1-V_3=-12$$

$$\Rightarrow V_1=-12+V_3 \quad \text{... (2)}$$

Substituting equation (2) in equation (1), we get,

$$2(-12+V_3)-2V_2+3V_3=0$$

$$\Rightarrow -24+2V_3-2V_2+3V_3=0$$

$$\Rightarrow 5V_3-2V_2-24=0$$

$$\Rightarrow 5V_3-2(3)-24=0 \quad (\because \text{From figure } V_2=3\text{ V})$$

$$\Rightarrow 5V_3-6-24=0$$

$$\Rightarrow 5V_3-30=0$$

$$\Rightarrow 5V_3=30$$

$$\Rightarrow V_3=6\text{ V}$$

Substituting above value in equation (2), we get,

$$V_1=-12+6$$

$$=-6\text{ V}$$

$\therefore$ Thevenin's voltage,

$$V_{th}=V_{AB}=V_3=6\text{ V}$$

The Thevenin's equivalent circuit is as shown in figure (5),

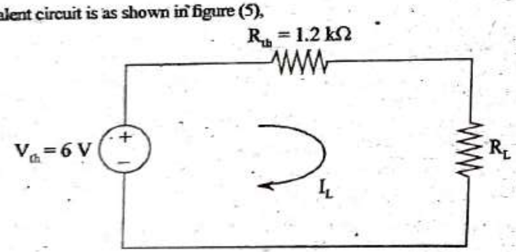


Figure (5)

Maximum power transfer theorem the load resistance R_L from figure (5) is given as,

$$R_L=R_{th}=1.2\text{ k}\Omega$$

$\therefore$ Maximum power transfer to the load resistance R_L,

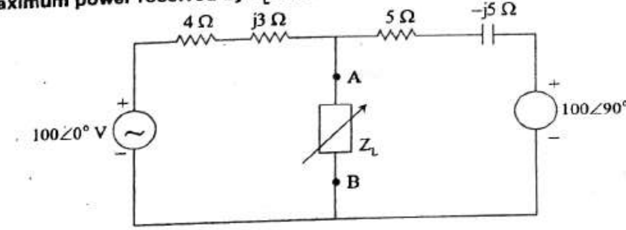
$$P_{max}=\frac{(V_{th})^2}{4R_{th}}$$

$$=\frac{6^2}{4\times(1.2\times10^3)}$$

$$=7.5\times10^{-3}\text{ W}$$

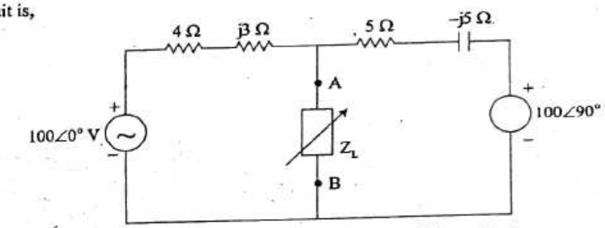
$$=7.5\text{ mW}$$

Q33. Find the maximum power received by Z_L. Also find the maximum power.

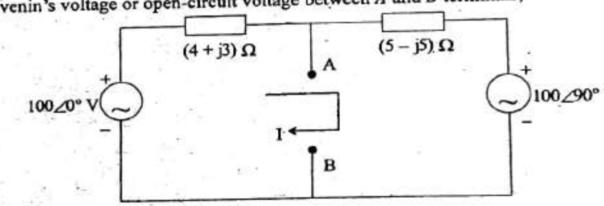


Ans:

The given circuit is,



Finding the Thevenin's voltage or open-circuit voltage between A and B terminals,



Applying KVL to the above circuit,

$$100\angle0°-(4+j3)I-(5-j5)I-100\angle90°=0$$

$$\therefore 100\angle0°-100\angle90°=(4+j3)I+(5-j5)I$$

$$\therefore I(4+j3+5-j5)=100+j0-[0+j100]$$

$$\therefore I(9-j2)=100-j100$$

$$\therefore I=\frac{100-j100}{9-j2}=\frac{141.42\angle-45}{9.219\angle-12.52}$$

$$\therefore I=15.339\angle-32.471°\text{ Amperes}$$

Potential of A with respect to B is

$$V_{AB}=100\angle0°-I(4+3j)$$

$$=100\angle0°-15.339\angle-32.471°\,[5\angle36.86°]$$

$$=100+0j-[76.696\angle4.3985°]$$

$$=100-76.47-j5.88$$

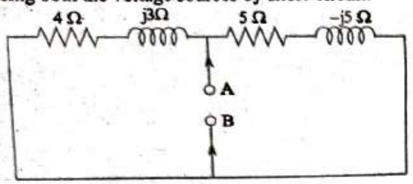
$$=23.53-j5.88$$

$$\therefore V_{AB}=24.25\angle14°\text{V}$$

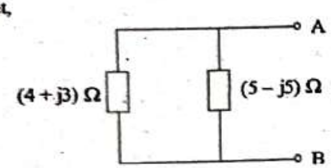
The Thevenin's voltage,

$$V_{Th} = V_{AB} = 24.25\angle 14°\ \text{V}$$

In order to determine Z_{TH}, replacing both the voltage sources by short-circuit.



Redrawing the above circuit, we get,



$$\therefore\ Z_{Th} = \frac{(4+j3)\times(5-j5)}{4+j3+5-j5}$$

$$= \frac{20-j20+j15+15}{9-j2}$$

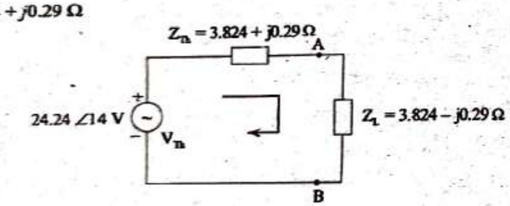
$$= \frac{35-j5}{9-j2} = \frac{35.35\angle -8.13°}{9.219\angle -12.52°}$$

$$\therefore\ Z_{Th} = 3.83\angle 4.39° = 3.824 + j0.29\ \Omega$$

To have maximum power transfer, the load impedance must be the complex conjugate at the Thevenin's impedance.

$$\therefore\ Z_L = 3.824 - j0.29\ \Omega$$

$$Z_{Th} = 3.824 + j0.29\ \Omega$$



Current,

$$I = \frac{V_{Th}}{Z_{Th} + Z_L}$$

$$\therefore\ I = \frac{24.24\angle -14°}{3.824 + j0.29 + 3.824 - j0.29}$$

$$= \frac{24.25\angle -14°}{7.636}$$

$$= 3.17\angle -14°\ \text{Amperes}$$

Maximum power delivered to the load,

$$P_{max} = I^2 R_L = (3.17)^2 \times 3.824$$

$$\therefore\ P_{max} = 38.427\ \text{W}$$

RECIPROCITY THEOREM

Q34. State and explain the reciprocity theorem. Is this theorem valid for network with two sources? Substantiate your answer?

Ans:

Statement

In a linear, passive and bilateral single source network, the ratio of response to the excitation is constant even though the source is the interchanged from the input terminals to the output terminals.

Explanation

Consider the network shown in figure (1).

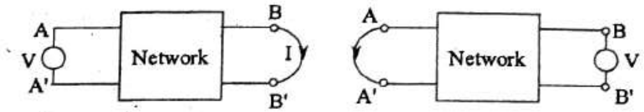


Figure (1)

Suppose application of voltage V across AA' produces current I through BB'. Now, if the positions of source and responses are interchanged, by connecting the voltage source across BB', the resultant current I will flow through terminals AA'. According to reciprocity theorem, the ratio of response to excitation is same in both cases.

i.e., $\frac{I}{V}$ = Constant (In both the cases)

Reciprocity theorem is not valid for a network with two sources. Consider the network shown in figure (2).

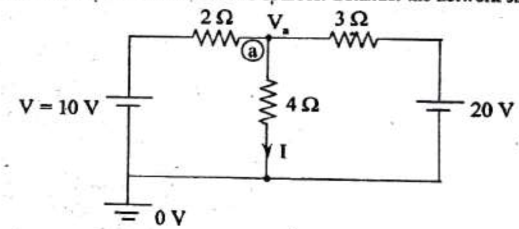


Figure (2)

Let 10 V, voltage source be the excitation (or input) and current, I be the response (or output).

By applying KCL at node 'a', we get,

$$\frac{V_a - 10}{2} + \frac{V_a}{4} + \frac{V_a - 20}{3} = 0$$

$$\Rightarrow\ V_a\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right) - \frac{10}{2} - \frac{20}{3} = 0$$

$$\Rightarrow\ V_a \times \frac{13}{12} = \frac{35}{3}$$

$$\Rightarrow\ V_a = \frac{35}{3} \times \frac{12}{13} = \frac{140}{13}\ \text{V}$$

$$\therefore\ I = \frac{V_a}{4} = \frac{140}{13\times 4} = \frac{35}{13}\ \text{A}$$

The ratio of response to the excitation,

$$= \frac{I}{V} = \frac{35}{13} \times \frac{1}{10} = \frac{7}{26}\ \mho$$

.(1

When the input and output terminals are interchanged, the circuit will be modified as shown in figure (3).

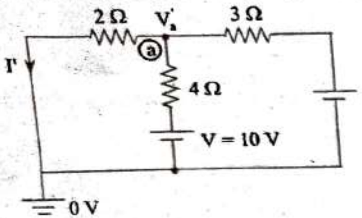


Figure (3)

On applying KCL at node 'a', we get,

$$\frac{V_a'}{2}+\frac{V_a'-10}{4}+\frac{V_a'-20}{3}=0$$

$$\Rightarrow \quad V_a'\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{3}\right)=\frac{10}{4}+\frac{20}{3}$$

$$\Rightarrow \quad V_a'\times\frac{13}{12}=\frac{55}{6}$$

$$\Rightarrow \quad V_a'=\frac{55}{6}\times\frac{12}{13}$$

$$V_a'=\frac{110}{13}\text{ V}$$

$$\therefore \quad I'=\frac{V_a'}{2}=\frac{110}{13\times 2}=\frac{55}{13}\text{ A}$$

The ratio of response to the excitation,

$$=\frac{I'}{V}=\frac{55}{13}\times\frac{1}{10}$$

$$=\frac{11}{26}\,\mho \quad \text{... (2)}$$

As equations (1) and (2) are not equal it can be said that the reciprocity theorem is invalid for network with two or more sources.

Q35. State and explain reciprocity theorem for A.C excitation.

Ans:

Statement for Reciprocity Theorem

In a linear, passive, bilateral single source network containing impedances, the ratio of excitation and response is constant even though the source is interchanged from the input terminals to the output terminals.

Explanation

Consider the network shown in figure (a).

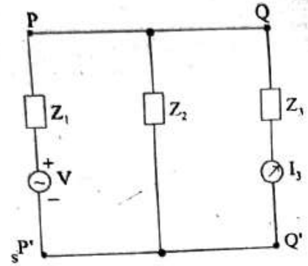


Figure (a)

Suppose that the application of voltage 'V' across PP' produces current I_3 through the branch QQ' having impedance Z_3. Then, the ratio of the response to excitation is given by,

$$\frac{R}{E}=\frac{I_3}{V} \quad \text{... (1)}$$

Now, the positions of source and response are interchanged. The voltage source is connected across the terminals QQ' and it produces current response I_1 through the branch PP' having impedance Z_1 as shown in the figure (b).

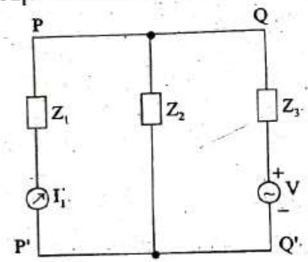


Figure (b)

Now, the ratio of the response to the excitation is given by,

$$\frac{R}{E}=\frac{I_1}{V} \quad \text{... (2)}$$

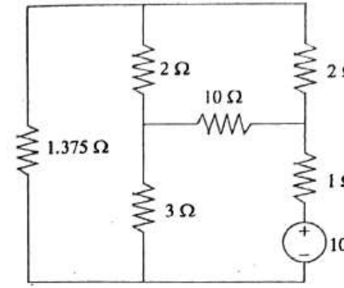
According to reciprocity theorem, we have ratio of response to the excitation constant in both the cases.

Hence,

Equation (1) = Equation (2)

$$\Rightarrow \quad \frac{I_3}{V}=\frac{I_1}{V}=\frac{I}{V}\text{ (Constant).} \qquad [\text{i.e., } I_3=I_1]$$

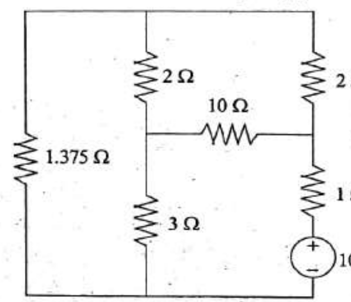
Q36. For the network shown in figure find the current through 1.375 ohms resistor and hence verify reciprocity theorem.



Figure

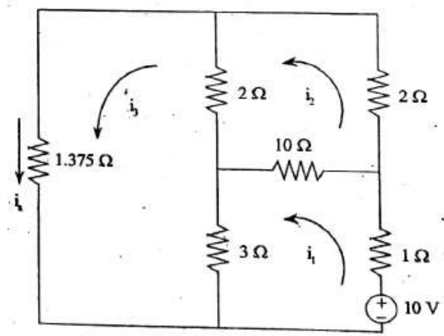
Ans:

Given circuit is as shown in figure (1),



Figure(1)

Using mesh analysis in order to simplify and to obtain the current through 1.375 Ω resistors. Thus we assign 3 mesh currents and then apply KVL around each mesh as shown in figure(2).



Figure(2)

Now applying KVL to mesh (1), we get,

$$10=1i_1+10(i_1-i_2)+3(i_1-i_3)$$

$$=1i_1+10i_1-10i_2-3i_3+3i_1$$

$$\Rightarrow \quad 14i_1-10i_2-3i_3=10 \quad \text{... (1)}$$

Applying KVL to mesh (2), we get,

$$2i_2+2(i_2-i_3)+10(i_2-i_1)=0$$

$$-10i_1+14i_2-2i_3=0 \quad \text{... (2)}$$

Similarly, applying KVL to mesh (3), we get,

$$1.375i_3+3(i_3-i_1)+2(i_3-i_2)=0$$

$$-3i_1-2i_2+6.375i_3=0 \quad \text{... (3)}$$

Now solving equations (1), (2), (3), we get,

i_1 = 2.75 Amps

i_2 = 2.25 Amps

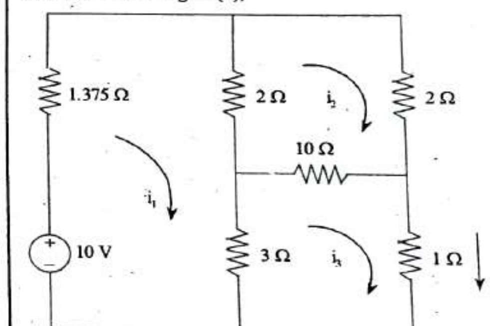
i_3 = 2 Amps

Here, the current flow through 1.375 Ω resistor is nothing but the mesh current i_3,

$$\therefore \quad i_3=i_a=2\text{ Amps} \quad \text{... (4)}$$

Verification with Reciprocity Theorem

The original circuits is redrawn by interchanging the 10 V source as shown in figure (3).



Figure(3)

In figure we have another 3 mesh networks, thus apply KVL around each mesh and 3 mesh currents will be obtained.

Applying KVL in mesh (1), we get,

$$10=1.375i_1+2(i_1-i_2)+3(i_1-i_3)$$

$$\Rightarrow \quad 1.375i_1+2i_1-2i_2+3i_1-3i_3=10$$

$$\Rightarrow \quad 6.375i_1-2i_2-3i_3=10 \quad \text{... (5)}$$

Applying KVL in mesh (2), we get,

$$2i_2 + 10(i_2 - i_3) + 2(i_2 - i_1) = 0$$

$$\Rightarrow \quad 2i_2 + 10i_2 - 10i_3 + 2i_2 - 2i_1 = 0$$

$$\Rightarrow \quad -2i_1 + 14i_2 - 10i_3 = 0 \qquad \text{... (6)}$$

Similarly, applying KVL in mesh (3), we get,

$$1i_3 + 3(i_3 - i_1) + 10(i_3 - i_2) = 0$$

$$-3i_1 - 10i_2 + 14i_3 = 0 \qquad \text{... (7)}$$

On solving equations (5), (6) and (7), we get,

$i_1 = 3.096$ Amps

$i_2 = 1.870$ Amps

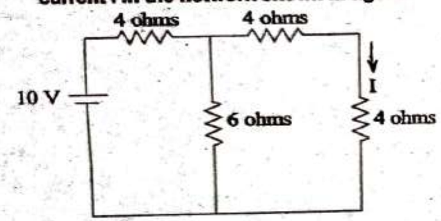
$i_3 = 2$ Amps

$\therefore$ The required current through 1 Ω resistor is equal to mesh current i_3 i.e.,

$\therefore \quad i_3 = i_a = 2$ Amps ... (8)

The response in both cases remains the same (i.e., equation (4) and (8)). Thus reciprocity theorem is verified.

Q37. Verify reciprocity theorem for the voltage V and current I in the network shown in figure.



Figure

Ans:

The given circuit is as shown in figure (1).

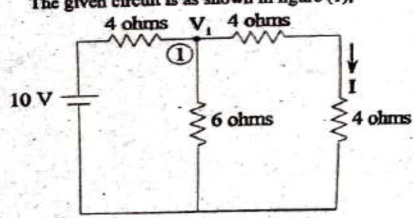


Figure (1)

Let V_1 be the voltage at node (1).

Applying KCL at node (1), we get,

$$\frac{V_1 - 10}{4} + \frac{V_1}{6} + \frac{V_1}{8} = 0$$

$$\frac{6(V_1 - 10) + 4V_1 + 3V_1}{24} = 0$$

$$6V_1 - 60 + 4V_1 + 3V_1 = 0$$

$$13V_1 - 60 = 0$$

$$V_1 = \frac{60}{13}$$

$$V_1 = 4.6 \text{ V}$$

Current through 4 Ω resistor, $I = \frac{V_1}{4+4} = \frac{4.6}{8} = 0.575$ A

Ratio of input to response $= \frac{V}{I} = \frac{10}{0.575} = 17.39\ \Omega$

Applying the reciprocity theorem by interchanging the excitation and response as shown in figure (2).

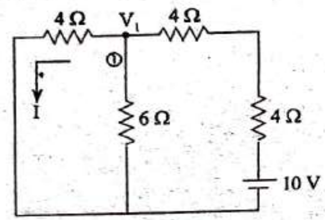


Figure (2)

Applying KCL at node (1), we get,

$$\frac{V_1}{4} + \frac{V_1}{6} + \frac{V_1 - 10}{8} = 0$$

$$\Rightarrow \quad \frac{6V_1 + 4V_1 + 3(V_1 - 10)}{24} = 0$$

$$\Rightarrow \quad 6V_1 + 4V_1 + 3V_1 - 30 = 0$$

$$\Rightarrow \quad 13V_1 = 30$$

$$V_1 = \frac{30}{13}$$

$$\boxed{V_1 = 2.3 \text{ V}}$$

Current through 4 Ω resistor, $I = \frac{2.3}{4}$

$$I = 0.575 \text{ A}$$

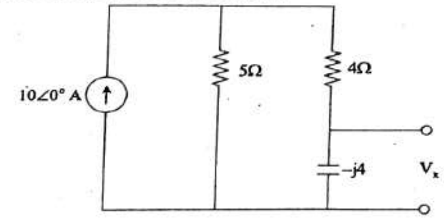
Ratio of input to response $= \frac{V}{I}$

$$= \frac{10}{0.575}$$

$$= 17.39\ \Omega$$

In both the cases, the ratio of input to response is same. Hence, the reciprocity theorem is verified.

Q38. Verify the reciprocity theorem for the network shown.



Figure

Ans:

The given circuit is shown in figure (1),

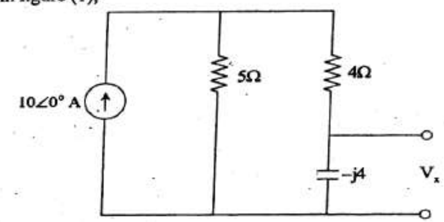


Figure (1)

$$V_x = I_x(-j4)$$

$$V_x = \frac{10 \times 5}{5 + 4 - j4}(-j4) = \frac{50(-j4)}{9 - j4}$$

$$= \frac{-j200}{9 - j4} = \frac{200\angle -90^\circ}{9.848\angle -23.96^\circ}$$

$$\therefore \quad V_x = 20.30\angle -66.04^\circ$$

The ratio of response to excitation i.e.,

$$\frac{\text{Output}}{\text{Input}} = \frac{20.30\angle -66.04^\circ}{10\angle 0^\circ}$$

$$\frac{\text{Output}}{\text{Input}} = 2.03\angle -66.04^\circ$$

Now, interchanging the excitation and response as shown in figure (2).

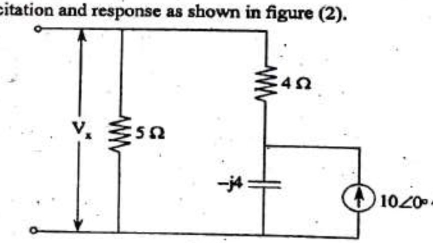


Figure (2)

$V_x = I_2(5\Omega)$

$\therefore \quad V_x = \frac{10\angle 0^\circ(-j4)}{5+4-j4}\times 5 = \frac{-200\,j}{9-j4}$

$= \frac{200\angle -90^\circ}{9.848\angle -23.96^\circ}$

$\therefore \quad V_x = 20.30\angle -66.04^\circ\text{ V}$

The ratio of response to excitation i.e.,

$\frac{\text{Output}}{\text{Input}} = \frac{20.30\angle -66.04^\circ}{10\angle 0^\circ}$

$\therefore \frac{\text{Output}}{\text{Input}} = 2.03\angle -66.04^\circ$

Since, ratio of response to excitation is constant in both the cases.

Reciprocity theorem is verified.

Q39. State and explain compensation theorem.

Ans:

Compensation Theorem

Statement

In a linear network, if the current in a branch is I and the resistance is R, and that branch is changed by ΔR then the changes in the current in the branches of the network can be obtained as the current due to a voltage source of value $I.\Delta R$ introduced in that branch in a direction opposing the current I, with all the sources in the network reduced to zero.

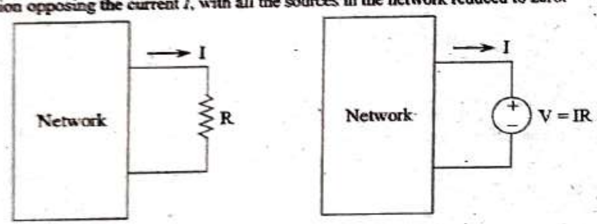


Figure (1)

In figure (1), it can be seen that a voltage source is utilized to replace the element R. The value of the voltage source is equal to the current I passing through R multiplied by R.

Now, consider figure (2) as shown below.

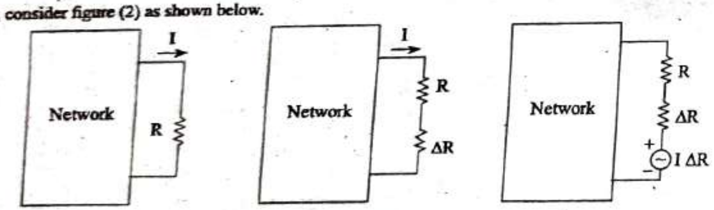


Figure (2)

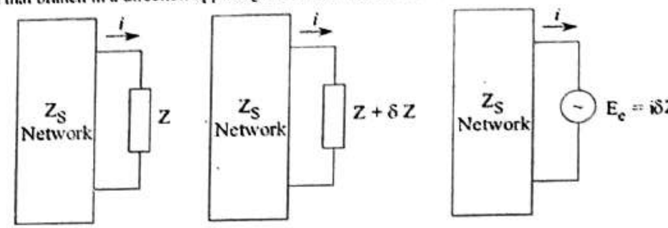
From figure (2), it can be seen that a small change in resistance R i.e., $(R + \Delta R)$ produces a change in current in all branches. This increment in current in other branches is equal to the current produced by the voltage source placed in series with the changed resistance, having a voltage of $I\,\Delta R$.

Q40. State and explain compensation theorem for A.C excitation.

Ans:

Statement

In a linear network, if the current in a branch is 'i' and the impedance is Z, and if the impedance of that branch is changed by ΔZ, then the change in the current in the branches of network can be obtained as the current due to voltage source of value $i.\Delta Z$ introduced in that branch in a direction opposing the current i, with all the sources in the network reduced to zero.

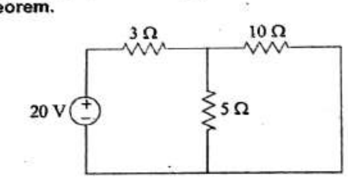


Figure

In the above figure, it can be seen that voltage source is utilized to replace Z. The value of the voltage is equal to current i passing through and multiplied by δZ.

- Due to change in branch impedance, change in current is given by, $\delta i = \frac{i\delta Z}{Z_S + Z + \delta Z}$

Q41. In the circuit shown below, resistor 5 Ω is changed to 5.6 Ω. Determine the change in current in 10 Ω by using compensation theorem.



Ans:

The given circuit is shown in figure (1).

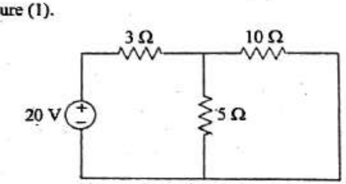


Figure (1)

With 5 Ω in the circuit,

Source current, $I_s = \frac{V}{R_{eq}}$

Equivalent resistance, $R_{eq} = (10\,\Omega \,||\, 5\,\Omega) + 3\,\Omega$

$$= \frac{10 \times 5}{10+5} + 3$$

$$= \frac{50}{15} + 3 = 6.33\,\Omega$$

$$\therefore \quad I_s = \frac{V}{R_{eq}} = \frac{20}{6.333} = 3.158\text{ A}$$

$\therefore$ Current through 5 Ω is, $i_5 = \frac{I_s \times 10}{5+10}$

$$= \frac{3.158 \times 10}{5+10}$$

$$= 2.105\text{ A}$$

Current through 10Ω is, $i_{10} = I_s - i_5$

$$= 3.158 - 2.105$$

$$= 1.052\text{ A}$$

$$\therefore \quad i_{10} = 1.053\text{ A}$$

Now,

Change in resistance, $\Delta R = 5.6 - 5 = 0.6\,\Omega$

$$\therefore \quad \Delta R = 0.6\,\Omega$$

Hence,

Compensation voltage required, $\Delta V = i_s \times \Delta R$

$$= 2.105 \times 0.6$$

$$= 1.263\text{ V}$$

Now,

In order to find the change in current in the 10 Ω resistor, when 5 Ω is changed to 5.6 Ω, using compensation theorem short circuit the voltage source of 20 V and replace 5 Ω resistor with a series branch of 5.6 Ω and ΔV = 1.263 V.

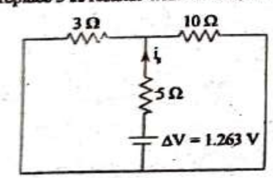


Figure (2)

Now, equivalent resistance of the circuit shown in figure (2) is given by,

$$R'_{eq} = (3||10) + 5$$

$$= \frac{3 \times 10}{3+10} + 5$$

$$= 7.307\,\Omega$$

$\therefore$ Source current, $i_s = \frac{\Delta V}{R'_{eq}}$

$$= \frac{1.263}{7.307}$$

$$= 0.1728\text{ A}$$

Hence,

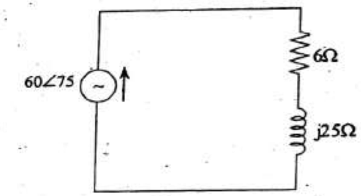
Change in current in 10 Ω resistor,

$$= \frac{i_s \times 3}{10+3}$$

$$= \frac{0.1728 \times 3}{10+3}$$

$$= 0.0398\text{ A}$$

Q42. Find the change in current by using compensation theorem, when reactance has changed to j8Ω.



Figure

Ans:

The given circuit is as shown in figure (1)

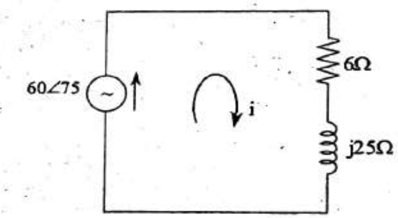


Figure (1)

Change in current, $\delta i = ?$

Circuit current,

$$i = \frac{V}{Z}$$

$$= \frac{60\angle 75^\circ}{6 + j25}$$

$$= 2.33\angle -1.5^\circ\text{ A}$$

Now, when reactance is changed from $j25\Omega$ to $j8\Omega$ the circuit changes as shown in figure (2).

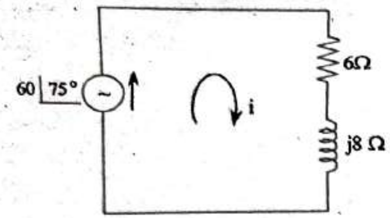


Figure (2)

$\therefore$ Change in impedance, $\delta Z = (j25) - (j8) = j17\Omega$

From compensation theorem, we have,

Change in current,

$$\delta i = \frac{i.\delta Z}{Z_{total}}$$

$$= \frac{2.33\angle -1.5° \times (j17)}{6+j8}$$

$$= \frac{1.04 + j39.6}{6+j8}$$

$$= \frac{39.61\angle 88.49°}{10\angle 53.13°}$$

$$= 3.96\angle 35.36° \text{ A}$$

$\therefore$ Change in current, $\delta i = 3.96\angle 35.36°$ A.

Q43. What are the types of sources? Explain them with suitable diagrams and characteristics.

Ans:

An energy source can be divided into two types as follows,

1. Ideal source
2. Practical source.

Ideal and practical sources are further subdivided into four types. They are as follows,

(i) Ideal voltage source
(ii) Practical voltage source
(iii) Ideal current source
(iv) Practical current source.

(i) Ideal Voltage Source

An ideal voltage source is shown in below figures.

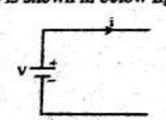


Figure (1)

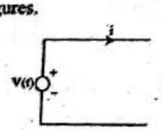


Figure (2)

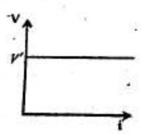


Figure (3)

In an ideal voltage source, the voltage across its terminals is constant always i.e., it is independent of current i.

The characteristics are shown in figure (3). In the voltage source, the voltage may vary depending on the time. This is shown in figure (2) as $V(t)$.

(ii) Practical Voltage Source

In a practical voltage source, the voltage at the terminals depends on the current. The characteristics of practical voltage source are shown in figure (4).

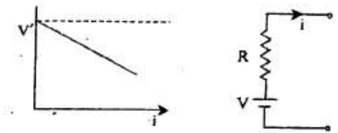


Figure (4)

(iii) Ideal Current Source

In an ideal current source, the current is independent of voltage across terminals.

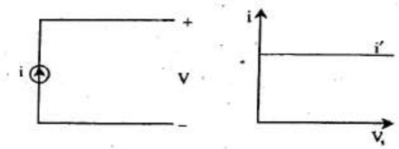


Figure (5)

The current in an ideal current source may vary according to time.

(iv) Practical Current Source

In practical current source, the current depends on the terminal voltage.

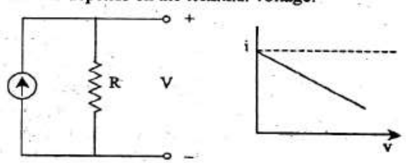


Figure (6)

The magnitude of current decreases as voltage across terminals increases.

Q44. Discuss about Independent and dependent sources. Give their circuit representation.

Ans:

Independent Sources

Independent sources are of two types as follows,

(i) Independent voltage source
(ii) Independent current source.

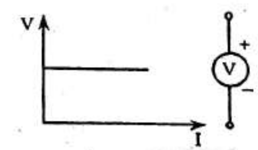


Figure (1)

(i) Independent Voltage Source

A voltage source which maintains a constant voltage that does not change, when changes takes place in an electric network.

(a) In ideal independent voltage source, voltage (V) is independent of current (I) through the source. It should have zero internal resistance.

(b) Ideal independent voltage source does not exist. In practical, independent voltage source have some internal resistance due to which voltage decreases with an increase in current.

(c) Practical voltage source should possess minimum internal resistance.

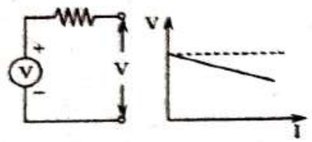


Figure (2)

(ii) Independent Current Source

A current source which gives constant current that does not change when changes takes place in an electric network.

(a) In ideal independent current source, current (I) is independent of voltage (V) across the source. It has infinite internal resistance.

(b) Ideal independent current sources does not exist. In practical, independent current sources have some internal resistance due to which curren' decreases with an increase in voltage.

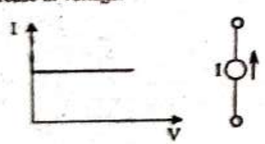


Figure (3)

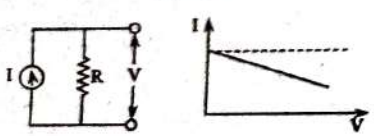


Figure (4)

(c) *In practical current sources internal resistance should be maximum.*

Dependent Sources

In case of dependent sources, the value of the source depends on some parameter existing in the same circuit. The parameter on which the value of source depends can be either a voltage across any element or a current through an element in the same circuit. Hence, based on the type of dependent variable, dependent source are classified as,

(i) Voltage dependent voltage source

(ii) Current dependent voltage source

(iii) Voltage dependent current source

(iv) Current dependent current source.

(i) Voltage Dependent Voltage Source

In this type of dependent source, the value of the source is dependent on the voltage across an element i.e., the value of the voltage source is a function of voltage.

Figure (5) shows a voltage dependent voltage source. V_x is the dependent variable and k_1 is a constant.

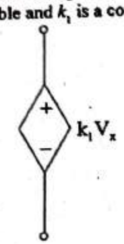


Figure (5)

The variable V_x will be clearly defined in the network. If due to some reason, V_x is zero then the value of dependent source will also be zero and hence the voltage supplied by the source will be zero and it has to be replaced with a short circuit.

(ii) Current Dependent Voltage Source

In this type, the value of the dependent source depends on the current through an element i.e., the value of the voltage source is a function of current.

Figure (6) shows a current dependent voltage source. i_x is the dependent variable and k_2 is a scaling factor having units volts/ampere.

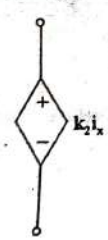


Figure (6)

The variable i_x will be clearly defined in the network. *If due to some reason, i_x is zero, then the value of dependent source will be zero and hence the voltage supplied by it will be zero and it has to be replaced with a short circuit.*

(iii) Voltage Dependent Current Source

In this type, the value of the dependent source depends on the voltage across an element i.e., the current supplied by it will be a function of voltage.

Figure (7) shows a voltage dependent current source. V_x is the dependent variable and k_3 is a scaling constant having units ampere/volts.

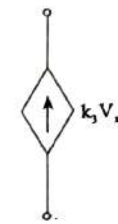


Figure (7)

The variable V_x will be clearly defined in the network. If due to some reason, V_x is zero, then the value of the source will be zero and hence the current supplied by it will be zero and now it has to be replaced with an open circuit.

(iv) Current Dependent Current Source

In this type, the value of the dependent source depends on the current through an element i.e., the value of the current source is a function of current.

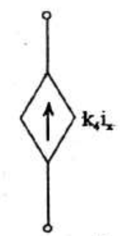


Figure (8)

Figure (8) shows a current dependent current source. i_x is the dependent variable and k_4 is a dimensionless constant. The variable i_x will be clearly defined in the network. If due to some reason, i_x is zero, then the current supplied by the source will be zero and hence it has to be replaced with an open circuit.

Q45. What is nodal analysis? Explain it with an example.

Ans:

Nodal Analysis

For answer refer Unit-I, Q11.

Steps Involved in Nodal Analysis

1. Identify all the nodes (i.e., the points where elements are connected) in a given electrical circuit.
2. From the identified nodes, select one node as the reference node.
3. Assign the variables to the nodes whose voltages are unknown.
4. Apply KCL at each of the unknown node voltages.
5. Solve the system of simultaneous equations which are obtained by the application of KCL at each of the nodes.

Example

Determine the currents in each branch and also write the node voltage equations for the electrical network shown in figure (a).

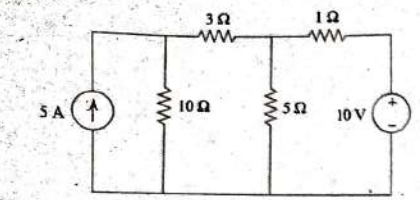


Figure (a)

Step I

In the given electrical network, nodes A, B and C are identified.

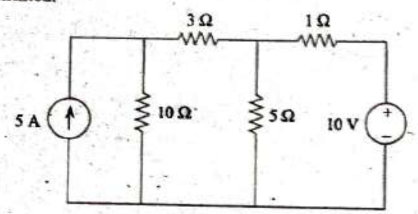


Figure (b)

Step II

Select node C as the reference node.

Step III

Here, the voltages at nodes A and B are unknown. Hence, assign variables V_A and V_B to them.

Step IV

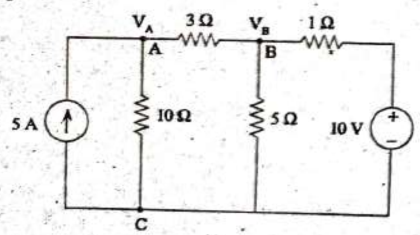


Figure (c)

Applying KCL at node A, we get,

$$\frac{V_A}{10}+\frac{V_A-V_B}{3}=5$$

$$\Rightarrow\ 13V_A-10V_B=150 \qquad \text{... (1)}$$

Applying KCL at node B, we get,

$$\frac{V_B-V_A}{3}+\frac{V_B}{5}+\frac{V_B-10}{1}=0$$

$$\Rightarrow\ -5V_A+23V_B=150 \qquad \text{... (2)}$$

Step V

Solving the system of simultaneous equations (1) and (2), we get,

$$13V_A-10V_B=150$$
$$-5V_A+23V_B=150$$
$$V_A=19.89 \text{ Volts}$$
$$V_B=10.84 \text{ Volts}$$

$$\text{Current in } 10\,\Omega \text{ branch} = \frac{V_A}{10} = \frac{19.89}{10} = 1.989 \text{ Amps}$$

$$\text{Current in } 5\,\Omega \text{ branch} = \frac{V_B}{5} = \frac{10.84}{5} = 2.168 \text{ Amps}$$

$$\text{Current in } 3\,\Omega \text{ branch} = \frac{V_A-V_B}{3} = \frac{19.89-10.84}{3} = 3.01 \text{ Amps}$$

$$\text{Current in } 1\,\Omega \text{ branch} = \frac{V_B-10}{1} = \frac{10.84-10}{1} = 0.84 \text{ Amps}$$

Q46. What is mesh analysis? Explain the steps involved in it with an example.

Model Paper-II, Q2(b)

Ans:

Mesh Analysis

For answer refer Unit-I, Q12.

Steps Involved in Mesh Analysis

1. Identify whether the given electrical circuit is planar (a circuit without having cross-overs) or not.
2. If the given circuit is planar, then identify the number of meshes. (A mesh is a loop or closed path without any other loops within it).
3. Assign mesh currents arbitrarily for the meshes identified.
4. Apply KVL to the meshes identified.
5. Solve the system of simultaneous equations which are obtained by the application of KVL at each of the meshes.

Example

Using mesh analysis write the mesh current equations and determine the currents for the circuit shown in the figure (a).

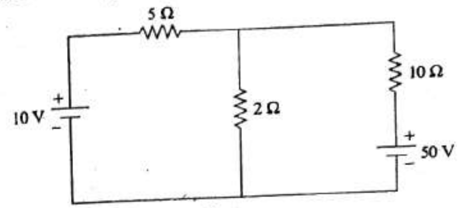


Figure (a)

Step I

The given electrical circuit is planar. Hence, mesh analysis can be applied to it.

Step II

Two meshes M_1, M_2 are present in the given electrical circuit.

Step III

Assign mesh currents I_1 and I_2 for the meshes that are identified.

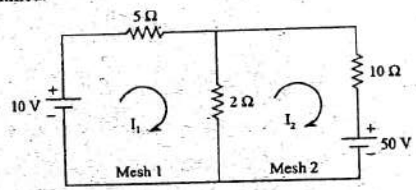


Figure (b)

Step IV

Applying KVL for mesh 1, we get,

$$10=5I_1+2(I_1-I_2)$$

$$\Rightarrow\ 7I_1-2I_2=10 \qquad \text{... (1)}$$

Applying KVL for mesh 2, we get,

$$2(I_2-I_1)+10I_2=-50$$

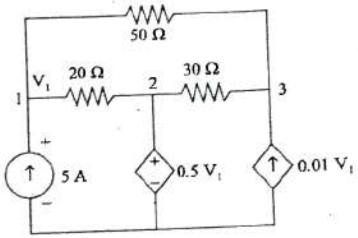
$$\Rightarrow\ -2I_1+12I_2=-50 \qquad \text{... (2)}$$

Step V

Solving the equations (1) and (2), we get,

$$7I_1-2I_2=10$$
$$-2I_1+12I_2=-50$$
$$I_1=0.25 \text{ Amps}$$
$$I_2=-4.125 \text{ Amps}$$

Q47. Use the nodal analysis to determine voltage at node 1 and the power supplied by the dependent current source in the network shown in figure.



Figure

Ans:

The given circuit is as shown in figure (1),

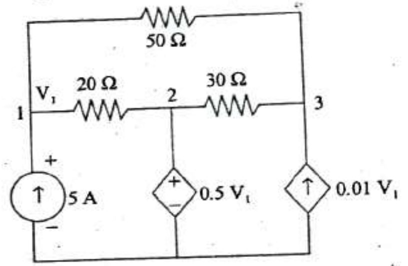


Figure (1)

Let the voltages V_1, V_2 and V_3 are assigned at node, 1, 2 and 3 respectively.

Applying KCL at node 1, We get,

$$\frac{V_1-V_2}{20}+\frac{V_1-V_3}{50}=5$$

$$50V_1-50V_2+20V_1-20V_3=5000$$

$$70V_1-50V_2-20V_3=5000$$

$$7V_1-5V_2-2V_3=500 \qquad \text{... (1)}$$

Applying KCL at node 3, we get,

$$\frac{V_3-V_1}{50}+\frac{V_3-V_2}{30}=0.01\,V_1$$

$$30V_3-30V_1+50V_3-50V_2=15V_1$$

$$-45V_1-50V_2+80V_3=0$$

$$9V_1+10V_2-16V_3=0 \qquad \text{... (2)}$$

And from node-2, we get,

$$V_2=0.5\,V_1 \qquad \text{... (3)}$$

Substituting equation (3) in equation (1) and (2), we get,

$$7V_1 - 5(0.5V_1) - 2V_3 = 500$$
$$7V_1 - 2.5V_1 - 2V_3 = 500$$
$$4.5V_1 - 2V_3 = 500 \quad \text{... (4)}$$
$$9V_1 + 10(0.5V_1) - 16V_3 = 500$$
$$9V_1 + 5V_1 - 16V_3 = 500$$
$$14V_1 - 16V_3 = 500 \quad \text{... (5)}$$

Solving equations (4) and (5), we get,

$$V_1 = 181.81 \text{ V}$$
$$V_3 = 159.09 \text{ V}$$

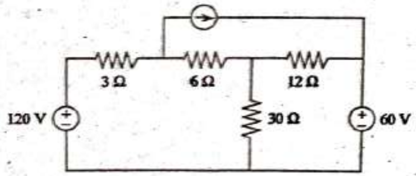
$\therefore$ $V_2 = 0.5\, V_1$ [From equation (3)]

$$= 0.5\,(181.81)$$
$$V_2 = 90.905 \text{ V}$$

The power supplied by the dependent current source is given by,

$$P = VI$$
$$= V_3 \times 0.01\, V_1$$
$$= 159.09 \times 0.01 \times 181.81$$
$$P = 289.24 \text{ Watts}$$

Q48. For the circuit shown in figure, use nodal analysis to determine voltage across 3 Ω and 12 Ω resistance. Compute power absorbed by 6 Ω resistor.



Figure

Ans:

Given circuit is shown in figure (1).

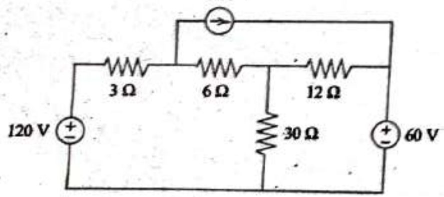


Figure (1)

The circuit can be redrawn as shown in figure (2).

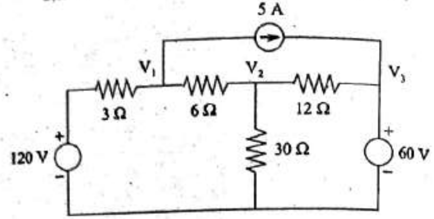


Figure (2)

The voltages V_1, V_2 and V_3 are unknown node voltages.

Applying KCL at node 1, we get,

$$\frac{V_1 - 120}{3} + \frac{V_1 - V_2}{6} = -5$$

$$V_1\left[\frac{1}{3}+\frac{1}{6}\right] - V_2\left[\frac{1}{6}\right] - \frac{120}{3} = -5$$

$$V_1[0.5] - V_2[0.167] = -5 + 40$$
$$0.5V_1 - 0.167V_2 = 35 \quad \text{... (1)}$$

Applying KCL at node 2, we get,

$$\frac{V_2 - V_1}{6} + \frac{V_2}{30} + \frac{V_2 - V_3}{12} = 0$$

$$-V_1\left[\frac{1}{6}\right] + V_2\left[\frac{1}{6}+\frac{1}{30}+\frac{1}{12}\right] - V_3\left[\frac{1}{12}\right] = 0$$

$$-0.167\,V_1 + 0.283\,V_2 - 0.083\,V_3 = 0 \quad \text{... (2)}$$

Applying KCL at node 3, we get,

$$V_3 = 60 \text{ V} \quad \text{... (3)}$$

Substituting equation (3) in equation (2), we get,

$$-0.167\,V_1 + 0.283\,V_2 - 0.083 \times 60 = 0$$
$$-0.167V_1 + 0.283V_2 - 4.98 = 0$$
$$-0.167V_1 + 0.283\,V_2 = 4.98 \quad \text{... (4)}$$

On solving equations (1) and (4), we get,

$$V_1 = 94.5 \text{ V}$$
$$V_2 = 73.36 \text{ V}$$

Voltage across 3 Ω resistor is,

$$= -120 - V_1$$
$$= 120 - 94.5$$
$$= 25.5 \text{ V}$$

Voltage across 12 Ω resistor is,

$$= V_2 - V_3$$
$$= 73.36 - 60$$
$$= 13.36 \text{ V}$$

Current through 6 Ω resistor is,

$$I_{6\Omega} = \frac{V_1 - V_2}{6} = \frac{94.5 - 73.36}{6}$$

$$I_{6\Omega} = 3.523 \text{ Amps}$$

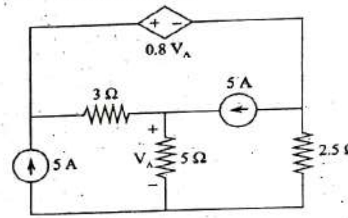
Power dissipated by 6 Ω resistor is,

$$P_{6\Omega} = I_{6\Omega}^2 \times \text{R}$$
$$= 3.523^2 \times 6$$
$$P_{6\Omega} = 74.46\ \Omega$$

Q49. With the help of nodal analysis on the circuit of figure, find,

(i) V_A

(ii) The power dissipated in 2.5 ohms resistor.



Figure

Model Paper-III, Q3(b)

Ans:

Given circuit is shown in figure (1).

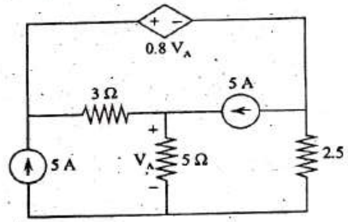


Figure (1)

In this circuit, the nodes 1 and 3 are connected directly through a voltage source without any element. So it becomes a super node.

At super node, we get,

$$\frac{V_1 - V_2}{3} + \frac{V_3}{2.5} = 5 - 5$$

$$2.5V_1 - 2.5V_2 + 3V_3 = 0 \quad \text{... (1)}$$

At node-2, we get,

$$\frac{V_2 - V_1}{3} + \frac{V_2}{5} = 5$$

$$\frac{5V_2 - 5V_1 + 3V_2}{15} = 5$$

$$-5V_1 + 8V_2 = 75 \quad \text{... (2)}$$

Super node equation, $V_1 - V_3 = 0.8V_2$ [$\because V_A = V_2$]

$$V_1 - 0.8V_2 - V_3 = 0 \quad \text{... (3)}$$

Solving equations (1), (2) and (3), we get,

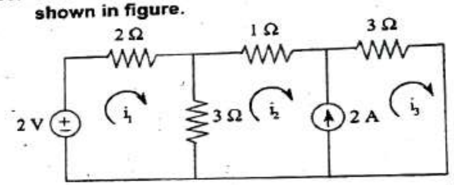
$$V_1 = 18.84 \text{ V},\ V_2 = 21.15 \text{ V},\ V_3 = 1.92 \text{ V}$$

$\therefore$ Voltage across 5 Ω resistor is $V_2 = V_A = 21.15$ V

Power dissipated in 2.5 ohms resistor is,

$$P_{2.5} = \frac{V_3^2}{R}$$
$$= \frac{(1.92)^2}{2.5}$$
$$= 1.474 \text{ W}$$

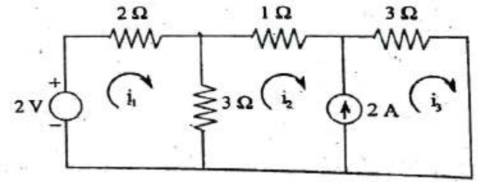
Q50. Calculate the mesh currents in the network shown in figure.



Figure

Ans:

Given circuit is shown in figure,



Figure

The current source is present between the meshes 2 and 3. Thus, a super mesh is formed.

Applying KVL for mesh 1, we get,

$$2i_1 + 3(i_1 - i_2) = 2$$
$$5i_1 - 3i_2 = 2$$
$$5i_1 = 2 + 3i_2$$
$$i_1 = 0.4 + 0.6i_2 \quad \text{... (1)}$$

Applying KVL for meshes 2 and 3, the super mesh equation is,

$$1i_2 + 3i_3 + 3(i_2 - i_1) = 0$$

$$-3i_1 + 4i_2 + 3i_3 = 0 \quad \text{... (2)}$$

Applying KCL for the super mesh present between meshes 2 and 3, we get,

$$i_3 - i_2 = 2$$

$$i_3 = 2 + i_2 \quad \text{... (3)}$$

Substituting equations (1) and (3) in equation (2), we get,

$$-3(0.4 + 0.6i_2) + 4i_2 + 3(2 + i_2) = 0$$

$$-1.2 - 1.8\, i_2 + 4i_2 + 6 + 3i_2 = 0$$

$$4.8 + 5.2i_2 = 0$$

$$5.2i_2 = -4.8$$

$$i_2 = \frac{-4.8}{5.2} = -0.923$$

$$\boxed{i_2 = -0.923 \text{ Amps}}$$

Substituting the value of the i_2 in equations (1) and (3), we get,

$$i_1 = 0.4 + 0.6(-0.923)$$

$$= 0.4 - 0.5538$$

$$\boxed{i_1 = -0.1538 \text{ Amps}}$$

$$i_3 = 2 + (-0.923)$$

$$\boxed{i_3 = 1.077 \text{ Amps}}$$

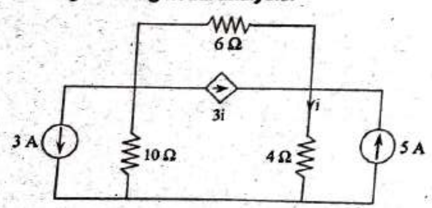
Thus, the mesh currents are,

$i_1 = -0.1538$ A

$i_2 = -0.923$ A

$i_3 = 1.077$ A

Q51. Determine current 'i' in the network shown in figure using nodal analysis.



Figure

Ans:

The given network is shown in figure (1).

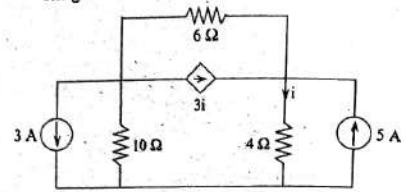


Figure (1)

Let V_1 and V_2 be the voltage at node (1) and node (2) respectively, as shown in figure (2).

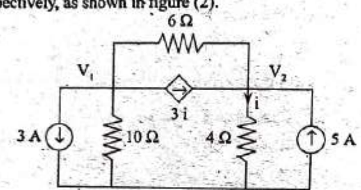


Figure (2)

Applying KCL at node (1), we get,

$$\frac{V_1}{10} + \frac{V_1 - V_2}{6} = -3 - 3i$$

$$V_1\left[\frac{1}{10} + \frac{1}{6}\right] - \frac{V_2}{6} = -3 - 3i$$

$$\frac{4V_1}{15} - \frac{V_2}{6} = -3 - 3i \quad \text{... (1)}$$

Applying KCL at node (2), we get,

$$\frac{V_2 - V_1}{6} + \frac{V_2}{4} = 3i + 5$$

$$\frac{-V_1}{6} + V_2\left[\frac{1}{6} + \frac{1}{4}\right] = 3i + 5$$

$$\frac{-V_1}{6} + \frac{5V_2}{12} = 3i + 5 \quad \text{... (2)}$$

From figure (2), we have,

$$\frac{V_2}{4} = i \quad \text{... (3)}$$

Substituting equation (3) in equation (1), we get,

$$\frac{4V_1}{15} - \frac{V_2}{6} = -3 - 3\left(\frac{V_2}{4}\right)$$

$$\frac{4V_1}{15} - \frac{V_2}{6} = -3 - \frac{3V_2}{4}$$

$$\frac{4V_1}{15} - V_2\left[\frac{1}{6} - \frac{3}{4}\right] = -3$$

$$\frac{4V_1}{15} - \frac{7V_2}{12} = -3$$

$$0.2667\, V_1 - 0.5833\, V_2 = -3 \quad \text{... (4)}$$

Substituting equation (3) in equation (2), we get,

$$\frac{-V_1}{6} + \frac{5V_2}{12} = \frac{3V_2}{4} + 5$$

$$\frac{-V_1}{6} + V_2\left[\frac{5}{12} - \frac{3}{4}\right] = 5$$

$$-0.1667\, V_1 - 0.333\, V_2 = 5 \quad \text{... (5)}$$

On solving equations (4) and (5), we get,

$V_1 = -21.04$ V

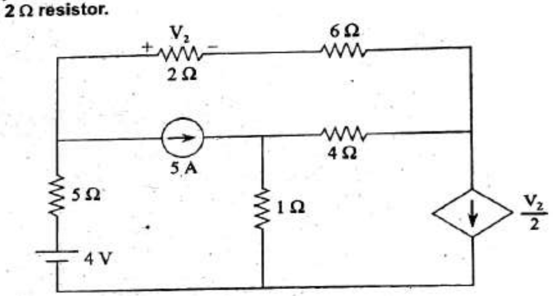
$V_2 = -4.48$ V

Substituting V_2 value in equation (3), we have,

$$i = \frac{-4.48}{4}$$

$$= -1.12 \text{ A}$$

Q52. Determine the power delivered by 4 V source for the electrical circuit shown in figure and also find the voltage across 2 Ω resistor.



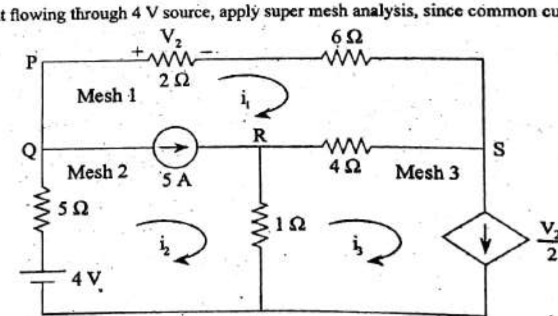
Figure

Ans:

The given problem can be solved by using super mesh analysis.

To find the power delivered by 4 V source, current flowing through it must be known ($\because P = VI$)

To find the current flowing through 4 V source, apply super mesh analysis, since common current sources are present.



Figure

Step I

Meshes 1, 2 and 3 are identified and mesh currents i_1, i_2 and i_3 are assigned. Current sources are present for the branches QR and ST.

Step II

The complete super mesh equation by applying KVL is,

$$2i_1 + 6i_1 + 4(i_1 - i_3) + 1(i_2 - i_3) - 4 + 5i_2 = 0 \quad \text{... (1)}$$

Step III

Applying KCL for the super mesh present between meshes 1 and 3, we get,

$$i_2 - i_1 = 5 \quad \text{... (2)}$$

Applying KVL for the branch DE, we get,

$$i_3 = \frac{V_2}{2} \quad \text{... (3)}$$

Step IV

Solving the equations (1), (2) and (3), we get,

$$\Rightarrow \quad 12i_1 + 6i_2 - 5i_3 = 4 \quad \text{... (4)}$$

From the figure, we have,

$$V_2 = 2i_1 \quad \text{... (5)}$$

Substituting equation (5) in equation (3), we get,

$$i_3 = \frac{2i_1}{2}$$

$$\Rightarrow \quad i_3 = i_1 \quad \text{... (6)}$$

Substituting equation (6) in equation (4), we get,

$$12i_1 + 6i_2 - 5i_1 = 4$$

$$\Rightarrow \quad 7i_1 + 6i_2 = 4 \quad \text{... (7)}$$

Solving equations (2) and (7), we get,

$$-i_1 + i_2 = 5$$

$$7i_1 + 6i_2 = 4$$

$$i_1 = -2 \text{ Amps}$$

$$i_2 = 3 \text{ Amps}$$

So, voltage across 2 Ω resistor, $V_2 = 2i_1$

$$= 2(-2)$$

$$= -4 \text{ V}$$

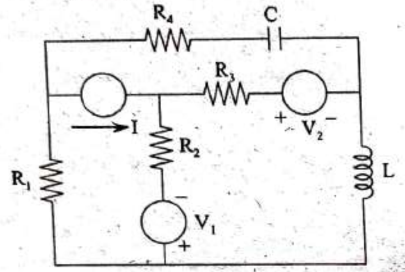
Power delivered by 4 V source $= VI$

$$= 4(i_2)$$

$$= 4(3)$$

$$= 12 \text{ Watts}$$

Q53. What is duality? Write the rules to draw the dual of a network.



Figure

Ans:

Duality

Duality means that the mathematical representation of the circuit elements are similar in form. [Note : Physically different].

Some of the duality pairs are,

(i) Series → Parallel

(ii) Voltage source (V) → Current source (I)

(iii) Resistance (R) → Conductance (G)

(iv) Inductance (L) → Capacitance (C).

Symbolic Representation

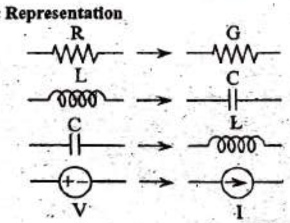


Figure (1)

Duality means that the mathematical representation of the circuit elements are similar in form. Of course, they are physically different. In electrical circuits, there exists a pair of terms which can be interchanged in order to obtain a new circuit. These pair of terms are called duals which are listed below.

Voltage source (V) ↔ Current source (I)

Resistance (R) ↔ Conductance (G)

Inductance (L) ↔ Capacitance (C).

Procedure of Drawing Dual Network

Dot method is the simpler graphical method to draw a dual network without writing network equations. The procedure for dot method is summarized as follows:

Step 1

Inside each mesh of a given network, place a dot (node). Assign numbers to these dots for convenience. These dots serve as nodes for dual network. Place an extra dot, outside the network. This dot is going to be the reference node for the dual network.

Step 2

Draw dotted lines from the node to node through the elements in the original network, traversing only one element at a time. Each element thus traversed in the original network, is now replaced by its dual elements.

Step 3

Continue this process till all the elements in the original network are accounted for.

Step 4

The network constructed in this manner is dual network.

Step 5

Draw the dual network separately.

The given planar network is as shown below,

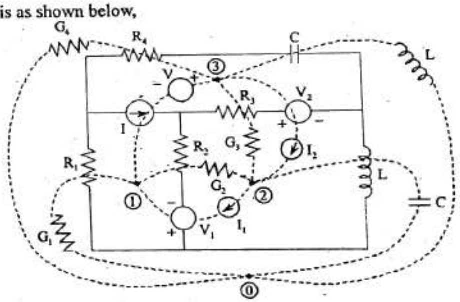


Figure (2)

The dual network is drawn as,

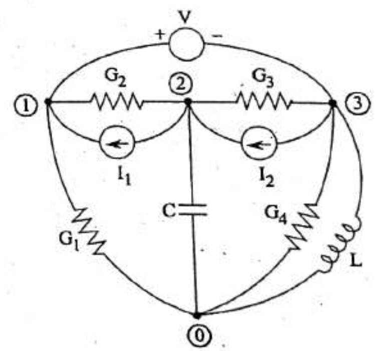


Figure (3)

The dual network can be drawn in a more amenable form as,

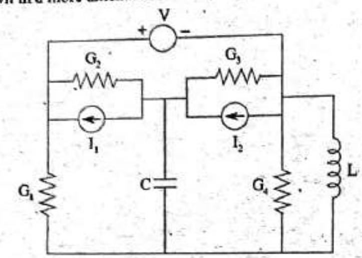
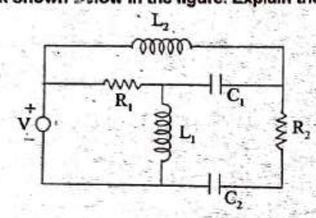


Figure (4)

Q54. Draw the dual of the network shown below in the figure. Explain the procedure employed.



Figure

Ans:

In order to obtain the dual of a network, we need to follow the following transformation,

Voltage, (V) ↔ Current source (I)

Resistance (R) ↔ conductance (G)

Inductance (L) ↔ Capacitance (C)

Series ↔ Parallel.

The given network is,

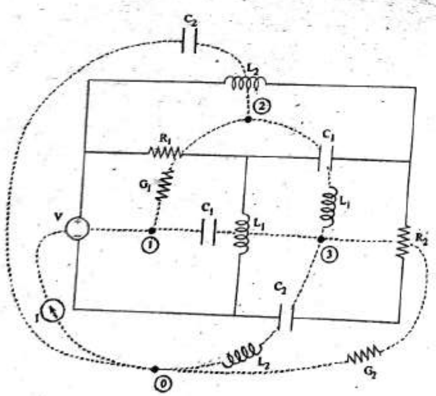


Figure (1)

The dual network is drawn as,

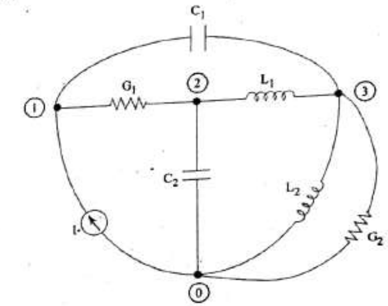


Figure (2)

The dual circuit can be drawn in a more amenable form as,

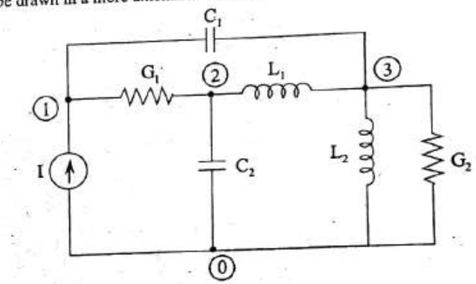
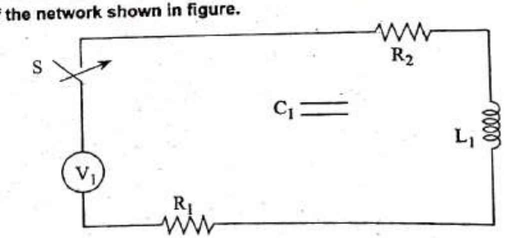


Figure (3)

Q55. Draw the dual of the network shown in figure.



Figure

Ans:

In order to obtain the dual of a network, we need to follow the following transformations,

Voltage source (V) ↔ Current source (I)

Resistance (R) ↔ Conductance (G).

Inductance (L) ↔ Capacitance (C)

Series ↔ Parallel.

∴ The obtained dual network is as shown in figure (2).

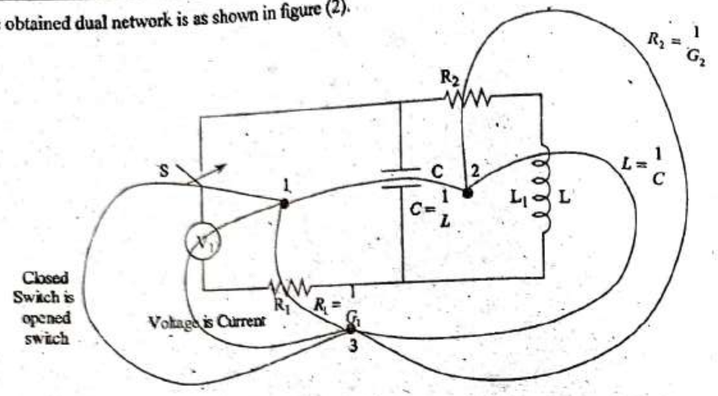


Figure (1)

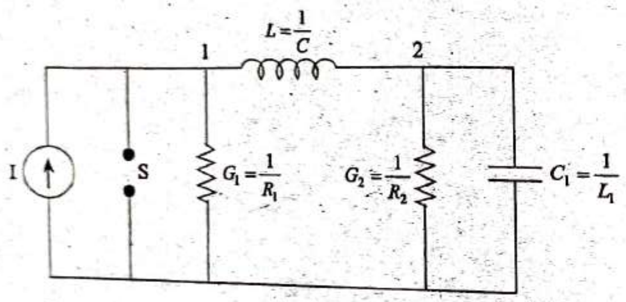
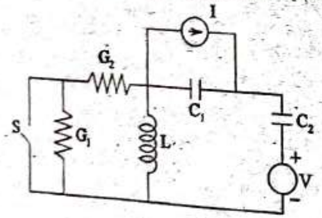


Figure (2)

Q56. Draw the dual of the network shown in below figure 6(b) and explain its procedure.



Figure

Ans:

The circuit of a given network is shown in the figure (1).

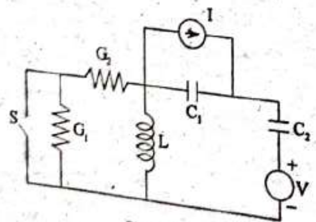


Figure (1)

Procedure for Drawing Dual Network

1. Placing a dot inside each mesh of the given network and assigning a number to these dots for convenience. These dots serve as nodes for the dual network. An extra dot is placed outside the network, which will be the reference node for the dual network as shown in figure (2).

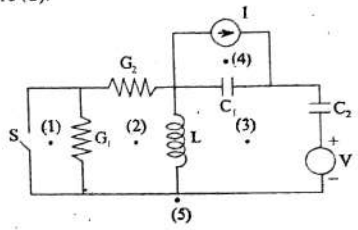


Figure (2)

2. Draw dotted lines from the node to node through the elements in the original network travelling only one element at a time as shown in figure (3).

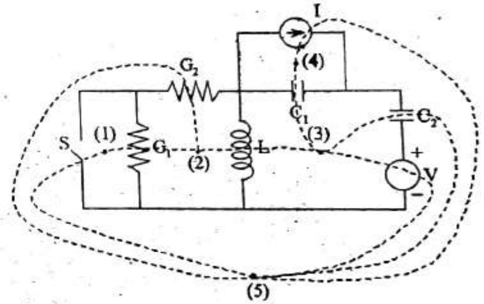


Figure (3)

3. Each element thus travelled in the original network is now replaced by its dual element.
   Voltage source (V) ↔ Current source (I)
   Resistance (R) ↔ Conductance (G)
   Inductance (L) ↔ Capacitance (C)
   Series ↔ Parallel
   Closed switch ↔ Open switch
   Therefore, the dual for the given network is as shown in figure (4).

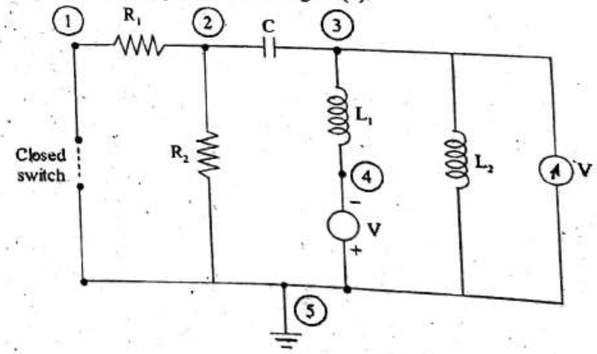
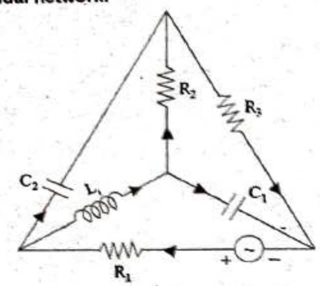


Figure (4)

Q57. For the network shown in figure, formulate its dual network.



Figure

Ans:

In order to obtain the dual of a network, we need to follow the following transformations,

Voltage source (V) ↔ Current Source (I)

Resistance (R) ↔ Conductance (G)

Inductance (L) ↔ Capacitance (C)

Series ↔ Parallel

The given network is,

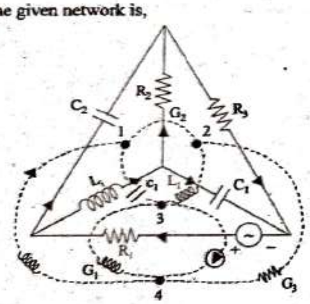


Figure (1)

The dual network is drawn as,

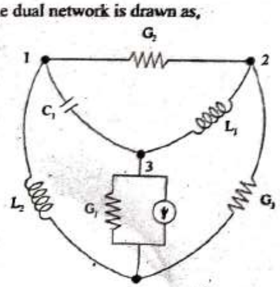


Figure (2)

The dual circuit can be drawn in a more amenable form as,

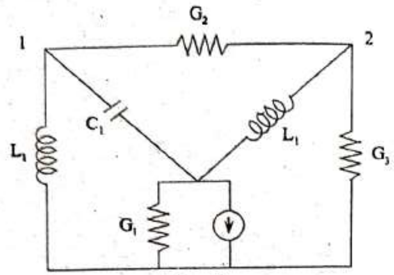


Figure (3)

2

SOLUTION OF FIRST AND SECOND ORDER NETWORKS

PART-A

SHORT QUESTIONS WITH SOLUTIONS

Q1. Define the following,

(i) Resistor

(ii) Inductor

(iii) Capacitor.

Ans:

(i) **Resistor**

Resistor is a passive element that absorbs energy whenever current passes through it. It is denoted by R. The symbolic representation of a resistor is shown in figure (1). The resistance is measured in Ohms.

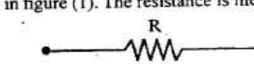


Figure (1)

(ii) **Inductor**

Inductor is a passive element which stores the energy in the form of magnetic field whenever a current is passed through it. It is denoted by 'L' and its symbolic representation is shown in figure (2). The inductance is measured in Henry.

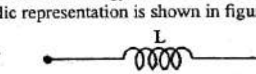


Figure (2)

(iii) **Capacitor**

Capacitor is a passive element which stores the energy in the form of electric field whenever a current is passed through it. It is denoted by 'C' and its symbolic representation is shown in figure (3). The capacitance is measured in Farads.

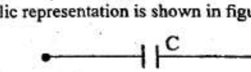


Figure (3)

Q2. What are the different ways of defining time constant?

Ans:

The time constant can be defined in two ways. They are,

1. Time constant is defined as the time required for the current to acquire the steady state value, if the rate of rise of current remains the same at $t = 0$.

2. It can also be defined as the time required to reach 63.2% of the steady state response.

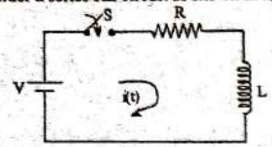
The rate of increase of current is slow when the system has long time constant.

Q3. Define the time constant of series 'RL' circuit.

Ans:

Time Constant of Series RL Circuit

Consider a series RL circuit as shown in figure,



Figure

Time constant τ of the series RL circuit is defined as the interval after which current (or) voltage changes 63.2% of its total charge.

or

It is also defined as the ratio of inductance L to resistance R.

The mathematically expression for time constant is given as,

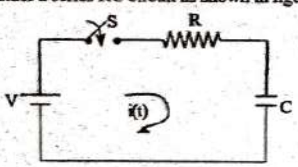
$$\tau = \frac{L}{R} \text{ seconds.}$$

Q4. Define time constant of series RC circuit.

Ans:

Time Constant of Series RC Circuit

Consider a series RC circuit as shown in figure.



Figure

Time constant for RC series circuit is defined as the time by which the capacitor attains 63.2% of steady state voltage. The mathematical expression for the constant.

$$\tau = RC \text{ seconds}$$

In case of RL circuit the time constant is inversely proportional to R. But in RC circuit the time constant τ is directly proportional to R. Hence by changing the values of R and C we can change the time constant.

Q5. Define the rise time of low pass RC circuit.

Ans:

The rise time, t_r is defined as the time taken by the output response i.e., output voltage of a low pass RC circuit excited by a step signal to rise from 0.1 to 0.9 of its final steady state value.

Q6. Define the time constant of low pass RC circuit.

Ans:

The time constant of an RC circuit is defined as the time taken by the output response i.e., output voltage waveform to reach 63.2% of the applied step input signal. It is denoted by τ.

Q7. Illustrate following terms,

(i) Impedance

(ii) Reactance

(iii) Phase difference

(iv) Power factor.

Ans:

(i) Impedance

Impedance is defined as the ratio of phasor voltage (V) to phasor current (I) and is denoted by Z.

(or)

The total opposition offered by an A.C circuit for the flow of current through it is called as impedance.

i.e., $Z = \frac{V}{I}\ \Omega$

(ii) Reactance

When impedance is written in Cartesian form the real part is the resistance (R) and the imaginary part is reactance (X).

(or)

The opposition offered by inductor or capacitor in an A.C circuit for the flow of current through it is called as reactance.

i.e., $Z = R + jX$

Where,

R = Resistive load

X = Reactance.

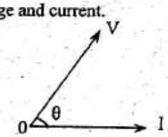
(iii) Phase Angle Difference

The difference between voltage phase angle and current phase angle is called as phase angle difference.

$$\theta = V(\phi) - I(\phi)$$

(iv) Power Factor

Power factor is defined as cosine of the angle of lead or lag between voltage and current.



Figure

$\therefore$ Power factor = $\cos\theta$

Q8. Define steady state and transient responses.

Ans:

Steady State Response

Steady state response is the response of the system as time approaches infinity. In other words, the steady state response of the system is that part of the time response which remains after vanishing the transient response completely. The steady state response of a system will be independent of time.

Transient Response

The transient response of a system is the response, when there is a change in input from one state to another. In other words, the output variation during the time it takes to achieve its final value is called as transient response

or

It is a response of system when time is zero. The transient response of a system is a function of time.

Q9. Discuss briefly about initial conditions.

Ans:

Initial conditions are those conditions that exist in the circuit immediately after switching operation. At $t = 0$, one or more switches are operated which disturbs the equilibrium of the circuit. We assume that, the switch is operated in zero time. At this point, we introduce a notation to distinguish the two states of the network.

We use $t = 0^-$ and $t = 0^+$. The initial condition will depend on the past history of the circuit and the network structure immediately after switching operation ($t = 0^+$).

After switching at $t = 0^+$, the new voltages and currents may appear in the network as the result of the initial capacitor voltages and the initial inductor currents.

Q10. Define time constant and write its significance.

Ans: Model Paper-II, Q1(c)

Time Constant

The time constant can be defined in two ways. They are,

1. Time constant (τ) is defined as the time required for the current to acquire the steady state value, if the rate of rise of current remains the same at $t = 0$.
2. It can also be defined as the time required to reach 63.2% of the steady state response.

The rate of increase of current is slow when the system has long time constant.

Significance of Time Constants

The value of τ plays a significant role in response of the circuit. If τ is small, then circuit response is fast and it takes very less time to reach it steady state. On other hand if τ is large then circuit response is slow and it take very large time to reach its steady state.

Q11. A D.C voltage of 20 V is applied in a R-L circuit where R = 5 Ω and L = 10 H. Find the time constant.

Ans:

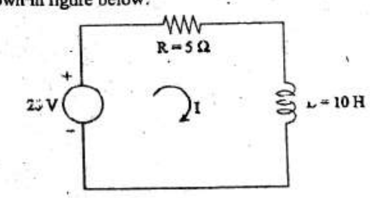
Given that,

D.C voltage, $V = 20$ V

Resistance, $R = 5\ \Omega$

Inductance, $L = 10$ H

Assuming the given R-L circuit as a series R-L circuit as shown in figure below:



Figure

Let I be the steady state current flowing through the circuit:

Time Constant

The time constant T of a series R-L circuit is defined as the ratio of inductance L to resistance R.

Mathematically,

$$\text{Time constant, } T = \frac{L}{R} = \frac{10}{5}$$

$$T = 2 \text{ sec}$$

Q12. If the inductor of figure has a current i_L = 2A at t = 0, find an expression for i(t) valid for t > 0, and its value at t = 200 μs.

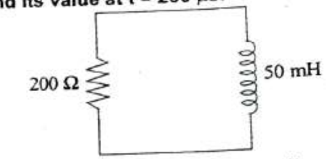


Figure: Simple RL Circuit

Ans:

Given that,

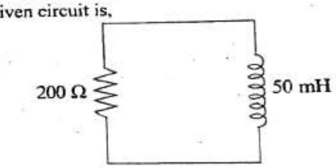
Inductor current at $t = 0$, $i_L = 2$A

$i(0) = 2$A

Inductance, $L = 50$ mH

Resistance, $R = 200\ \Omega$

Given circuit is,



Figure

So, the inductor current at time $t = 200$ μs is given as,

$$i(t) = i(0)\, e^{\frac{-Rt}{L}}$$

$$i(t) = 2\, e^{\frac{-200 \times 200 \times 10^{-6}}{50 \times 10^{-3}}}$$

$$= 2\, e^{-\frac{4}{5}}$$

$$= 2 \times 449.32 \times 10^{-3} \text{ A}$$

$$\therefore \quad i(t) = 898.64 \text{ mA}$$

Q13. Consider a parallel RLC circuit having an inductance of 10 mH and a capacitance of 100 μF. Determine the resistor values that would lead to overdamped and under damped responses.

Ans:

Give that,

Inductance, $L = 10$ mH

Capacitance, $C = 100$ μF

Resistance, $R = ?$

We know that,

The resonant frequency of the circuit is,

$$\omega = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{10\times10^{-3}\times100\times10^{-6}}}$$

$$\therefore \omega = 1000 \text{ rad/sec}$$

The condition of over-damped is $\alpha > \omega_0$ and the condition for under-damped is $\alpha < \omega_0$.

Thus, considering any one,

$$\alpha > \omega_0$$

$$\frac{1}{2RC} > 1000 \qquad \left[\because \alpha = \frac{1}{2RC}\right]$$

$$2RC < \frac{1}{1000}$$

$$R = \frac{1}{2C\times1000}$$

$$R < \frac{1}{2\times100\times10^{-6}\times1000}$$

$$R < 5$$

$R < 5\Omega$ results in overdamped care and $R > 5\Omega$ results in underdamped case.

PART-B
ESSAY QUESTIONS WITH SOLUTIONS

SOLUTION OF FIRST AND SECOND ORDER DIFFERENTIAL EQUATIONS FOR SERIES AND PARALLEL R-L, R-C, R-L-C CIRCUITS, STEADY STATE AND TRANSIENT STATE RESPONSE FOR D.C. AND A.C EXCITATIONS

Q14. Derive the expression for i(t) of a R-L series circuit when D.C. voltage is applied to it at t = 0 by closing the switch. Draw the response curve i(t) Vs t. Define time constant of R-L circuit.

Ans:

Consider the RL circuit shown in figure (1).

Applying KVL,

$$L\frac{di(t)}{dt} + R(i)t = V$$

or $$\frac{di(t)}{dt} + \frac{R}{L}i(t) = \frac{V}{L}$$

$$\Rightarrow \left(D+\frac{R}{L}\right)i = \frac{V}{L}$$

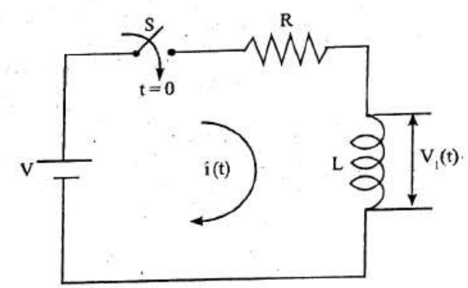


Figure (1)

General solution of this differential equation is given as,

$$i(t) = \frac{V}{R} + Ke^{-(R/L)t}$$

Since, inductor behaves as an open-circuit at switching,

$$\therefore i(0^+) = 0$$

$$0 = \frac{V}{R} + K$$

or

$$K = -\frac{V}{R}$$

Therefore,

$$i(t) = \frac{V}{R} - \frac{V}{R}e^{-(R/L)t}$$

$$i(t) = \frac{V}{R}(1-e^{-(R/L)t})$$

Then the voltage across the resistor and inductor is givenas,

$$V_R(t) = i(t)R = R\frac{V}{R}[1-e^{-(R/L)t}]$$

or $$V_R(t) = V(1-e^{-(R/L)t})$$

$$V_L(t) = L\frac{di(t)}{dt}$$

$$= L\frac{d}{dt}\left[\frac{V}{R}(1-e^{-(R/L)t})\right]$$

$$= L\frac{V}{R}\left[0-\left(\frac{-R}{L}\right)e^{-(R/L)t}\right]$$

$$V_L(t) = Ve^{-(R/L)t}$$

(i) At $t=0$,

$$i(t) = 0 \text{ and } V_L(t) = V$$

[$\because$ L acts as open-circuit at $t = 0$]

(ii) At $t = \infty$,

$$i(t) = \frac{V}{R} \text{ and } V_L(t) = 0$$

[$\because$ L acts as short circuit at $t = \infty$]

$$V_R(t) = 0$$

(iii) At $t = \frac{L}{R} = \tau$

$$i(t) = \frac{V}{R}(1-e^{-1}) = 0.632\ \frac{V}{R}$$

and $V_L(t) = Ve^{-1} = 0.368$ V

$$V_R(t) = 0.632 \text{ V}$$

The values of $V_R(t)$ and $V_L(t)$ are shown in figures (2) and (3).

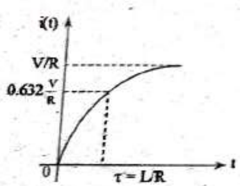


Figure (2): For i(t)

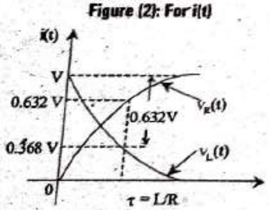


Figure (3): For V_R (t) and V_L (t)

Time Constant

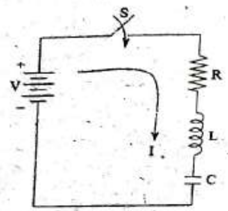
$\tau = \frac{L}{R}$ is known as the time constant of the circuit and is defined as the interval after which current or voltage changes 63.2% of its total change.

Q15. Derive the expression for the transient response in an RLC series circuit excited by a D.C source.

Ans:

Transient Response in an RLC Series Circuit Excited by a D.C Source

A *RLC* series circuit excited by a D.C source is shown in figure (1).



Figure(1)

Let us consider that the capacitor and inductor are initially uncharged. When the switch 'S' is closed at $t = 0$, current 'I' flows through the circuit.

Applying KVL to the circuit, we get,

$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad \text{... (1)}$$

Differentiating equation (1) with respect to t, we get,

$$R\frac{di}{dt} + L\frac{d^2i}{dt} + \frac{i}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)i = 0 \quad \text{... (2)}$$

Therefore, equation (2) represents a second order linear differential equation having only complementary function i.e., the particular solution for this equation is zero.

The characteristic equation for the differential equation of equation (2) can be written as,

$$D^2 + \frac{R}{L}D + \frac{1}{LC} = 0$$

The roots of the characteristic equation are given as,

$$D_1, D_2 = \frac{\frac{-R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 - 4(1)\left(\frac{1}{LC}\right)}}{2.1}$$

$$= \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$= \alpha \pm \beta$$

Where, $\alpha = \frac{-R}{2L}$ and $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$$\Rightarrow \quad D_1 = \alpha + \beta = D_2 = \alpha - \beta$$

Hence, the solution of differential equation (2) can be written as,

$$i = i_c + i_p$$

$$i = [c_1e^{D_1t} + c_2e^{D_2t}] + 0$$

$$i = c_1e^{(\alpha+\beta)t} + c_2e^{(\alpha-\beta)t} \quad \text{... (3)}$$

Where, c_1 and c_2 are constants.

In the above expression β may be positive, negative or zero. Hence, we have to consider the transient response of RLC series circuit excited by D.C source for three cases. They are,

Case 1

When β is positive,

i.e., $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

Therefore, when β is positive, the roots are real and unequal. Now equation (3) can be written as,

$$i = c_1e^{(\alpha+\beta)t} + c_2e^{(\alpha-\beta)t}$$

$$i = e^{\alpha t} + [c_1e^{\beta t} + c_2e^{-\beta t}]$$

For this type of roots we will have over damped response and the current curve for overdamped case is shown in figure (2).

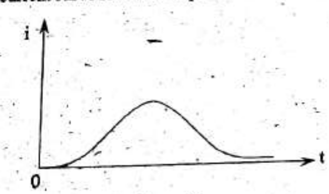


Figure (2)

Case 2

When β is negative.

i.e., $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

Therefore, when β is negative, the roots are complex conjugate. Now equation (3) can be written as,

$$i = c_1e^{(\alpha+j\beta)t} + c_2e^{(\alpha-j\beta)t}$$

$$i = e^{\alpha t}[c_1e^{j\beta t} + c_2e^{(\alpha-j\beta t)}]$$

$$\therefore \quad i = e^{\alpha t}[c_1\cos(\beta t) + c_2\sin(\beta t)]$$

Where,

$$k_1 = c_1 + c_2$$

$$k_2 = j(c_1 + c_2)$$

For this case we will have underdamped response. The curve for the underdamped response is shown in figure (3).

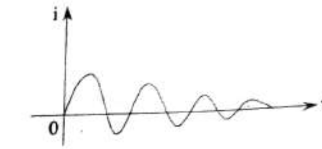


Figure (3)

Case (3)

When β is zero.

i.e., $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

Therefore, when β is zero, the roots are real and equal. Now equation (3) can be written as,

$$i = c_1e^{(\alpha+\beta)t} + c_2e^{(\alpha+\beta)t}$$

$$i = c_1e^{\alpha t} + tc_2e^{\alpha t}$$

$$i = e^{\alpha t}(c_1 + c_2t)$$

For this case the current response will be a critically damped one as shown in figure (4).

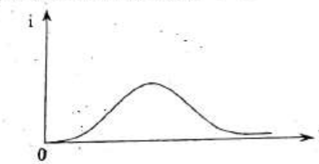


Figure (4)

Q16. Derive the relationship for the current in the series RL circuit with sinusoidal excitation.

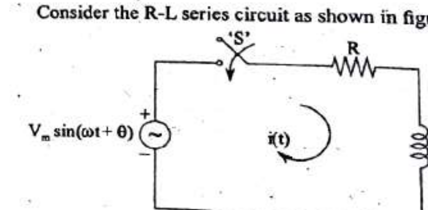
Model Paper-I, Q5(b)

OR

For an RL series circuit, a sinusoidal voltage of V(t) = V_m sin (ωt + ϕ) is applied at t = 0. Find the expression for transient current.

Ans:

Consider the R-L series circuit as shown in figure.



Figure

When the switch is closed, applying KVL to the circuit, we get,

$$L\frac{di}{dt} + Ri = V_m \sin(\omega t + \theta) \quad \text{... (1)}$$

For the above differential equation the particular solution is given as,

$$i_p = c_1 \cos(\omega t + \theta) + c_2 \sin(\omega t + \theta) \quad \text{... (2)}$$

$$\frac{di_p}{dt} = -\omega c_1 \sin(\omega t + \theta) + \omega c_2 \cos(\omega t + \theta)$$

Substituting these values in equation (1), we get,

$$L[-\omega c_1 \sin(\omega t + \theta) + \omega c_2 \cos(\omega t + \theta)]$$
$$+ R[c_1 \cos(\omega t + \theta) + c_2 \sin(\omega t + \theta)]$$
$$= V_m \sin(\omega t + \theta) \quad \text{... (3)}$$

Equating the coefficients of like terms, we get,

$$-L\omega c_1 + Rc_2 = V_m \quad \text{... (4)}$$
$$L\omega c_2 + Rc_1 = 0 \quad \text{... (5)}$$

Solving equations (4) and (5), we get,

$$R \times \text{equation (1)} \Rightarrow -\omega LR\, c_1 + R^2 c_2 = V_m R$$
$$\omega L \times \text{equation (2)} \Rightarrow \omega^2 L^2 c_2 + \omega LR\, c_1 = 0$$
$$\overline{R^2 c_2 + \omega^2 L^2 c_2 = V_m R}$$
$$c_2 = \frac{V_m R}{(R^2 + \omega^2 L^2)}$$

Substituting c_2 in equation (5), we get,

$$L\omega c_2 + Rc_1 = 0$$
$$L\omega \times \left(\frac{V_m R}{R^2 + \omega^2 L^2}\right) + R\,c_1 = 0$$
$$R\,c_1 = -\frac{V_m\, \omega LR}{(R^2 + \omega^2 L^2)}$$
$$c_1 = -\frac{V_m\, \omega L}{R^2 + \omega^2 L^2}$$

Substituting c_1 and c_2 values in equation (2), we get,

$$i_p = \frac{-V_m\, \omega L}{(R^2 + \omega^2 L^2)} \cos(\omega t + \theta)$$
$$+ \frac{V_m R}{(R^2 + \omega^2 L^2)} \sin(\omega t + \theta)$$
$$= \frac{V_m}{(R^2 + \omega^2 L^2)} [R \sin(\omega t + \theta) - L\omega \cos(\omega t + \theta)]$$

Let,

$$L\omega = k \sin\phi$$
$$R = k\cos\phi$$

Thus,

$$i_p = \frac{V_m}{R^2 + \omega^2 L^2} [k\cos\phi \sin(\omega t + \theta) - k \sin\phi \cos(\omega t + \theta)]$$
$$= \frac{V_m\, k}{R^2 + \omega^2 L^2} \sin(\omega t + \theta - \phi) \quad \text{... (6)}$$

Where,

$$k^2 \sin^2\phi + k^2 \cos^2\phi = L^2\omega^2 + R^2$$
$$k^2(\sin^2\phi + \cos^2\phi) = R^2 + L^2\omega^2$$
$$k^2 = R^2 + L^2\omega^2$$
$$k = \sqrt{R^2 + L^2\omega^2}$$

And

$$\frac{k\sin\phi}{k\cos\phi} = \frac{\omega L}{R}$$
$$\Rightarrow \tan\phi = \frac{\omega L}{R}$$
$$\phi = \tan^{-1}\left[\frac{\omega L}{R}\right]$$

Equation (6) can now be written as,

$$i_p = \frac{V_m \times \sqrt{R^2 + L^2\omega^2}}{(R^2 + \omega^2 L^2)} \sin\left(\omega t + \theta - \tan^{-1}\left[\frac{\omega L}{R}\right]\right)$$
$$= \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t + \theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

The complementary function for the circuit is given as,

$$i_c = ce^{-Rt/L}$$

The total solution of i for the R-L series circuit is,

$$i = i_p + i_c$$
$$= \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t + \theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right) + c\,e^{-Rt/L}$$

In the initially relaxed condition, the switch 'S' is closed (i.e., $t = d$) the initial current is '0' because of the inductor.

$$\therefore \; 0 = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(0 + \theta - \tan^{-1}\left(\frac{\omega L}{R}\right) + ce^0\right)$$
$$\Rightarrow ce^0 = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$\therefore$ The transient current is given as,

$$i_c = -\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\theta - \tan^{-1}\frac{\omega L}{R}\right) e^{-Rt/L}$$

Given,

For $t = 0$, transient current in the circuit, $i_c = 0$

$$0 = -\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right) e^{-Rt/L}$$
$$= -\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$
$$= \sin\left(\theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$
$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

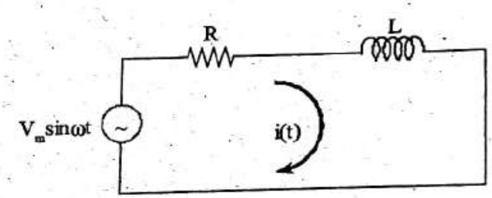
$\therefore$ The value of θ when transient current $i_c = 0$ is $\tan^{-1}\left(\frac{\omega L}{R}\right)$

Q17. Derive an expression for the current response in R-L series circuit with a sinusoidal source.

OR

Derive the expression for i(t) for R-L series circuit when excited by a sinusoidal source.

Ans:



Figure

The above figure represents an R-L series circuit excited by a sinusoidal source.

Applying KVL to the above circuit, we get,

$$V_m \sin \omega t = i(t).R + L.\frac{di(t)}{dt}$$

Applying Laplace transform on both sides, we get,

$$\frac{V_m\omega}{s^2 + \omega^2} = RI(s) + L[s.I(s) - i(0^+)][i(0^+) = i(0^-) = 0]$$
$$I(s)[R + Ls] = \frac{V_m\omega}{s^2 + \omega^2}$$
$$I(s) = \frac{V_m\omega}{(s^2 + \omega^2)(R + Ls)}$$
$$I(s) = \frac{V_m\omega}{L\left(s + \frac{R}{L}\right)(s^2 + \omega^2)}$$
$$\therefore \; I(s) = \frac{V_m\omega}{L\left(s + \frac{R}{L}\right)(s + j\omega)(s - j\omega)}$$

Resolving the above expression into partial fractions, we get,

$$\frac{V_m\omega / L}{\left(s + \frac{R}{L}\right)(s + j\omega)(s - j\omega)} = \frac{A}{s + \frac{R}{L}} + \frac{B}{s + j\omega} + \frac{C}{s - j\omega}$$

Figure

When the switch is closed, applying KVL to the circuit, we get,

$$L\frac{di}{dt} + Ri = V_m \sin(\omega t + \theta) \quad \text{... (1)}$$

For the above differential equation the particular solution is given as,

$$i_p = c_1 \cos(\omega t + \theta) + c_2 \sin(\omega t + \theta) \quad \text{... (2)}$$

$$\frac{di_p}{dt} = -\omega c_1 \sin(\omega t + \theta) + \omega c_2 \cos(\omega t + \theta)$$

Substituting these values in equation (1), we get,

$$L[-\omega c_1 \sin(\omega t + \theta) + \omega c_2 \cos(\omega t + \theta)]$$
$$+ R[c_1 \cos(\omega t + \theta) + c_2 \sin(\omega t + \theta)]$$
$$= V_m \sin(\omega t + \theta) \quad \text{... (3)}$$

Equating the coefficients of like terms, we get,

$$-L\omega c_1 + Rc_2 = V_m \quad \text{... (4)}$$

$$L\omega c_2 + Rc_1 = 0 \quad \text{... (5)}$$

Solving equations (4) and (5), we get,

$$R \times \text{equation (1)} \Rightarrow -\omega LR\, c_1 + R^2 c_2 = V_m R$$

$$\omega L \times \text{equation (2)} \Rightarrow \omega^2 L^2 c_2 + \omega LR\, c_1 = 0$$

$$R^2 c_2 + \omega^2 L^2 c_2 = V_m R$$

$$c_2 = \frac{V_m R}{(R^2 + \omega^2 L^2)}$$

Substituting c_2 in equation (5), we get,

$$L\omega c_2 + Rc_1 = 0$$

$$L\omega \left(\frac{V_m R}{R^2 + \omega^2 L^2}\right) + R c_1 = 0$$

$$R c_1 = -\frac{V_m \omega LR}{(R^2 + \omega^2 L^2)}$$

$$c_1 = -\frac{V_m \omega L}{R^2 + \omega^2 L^2}$$

Substituting c_1 and c_2 values in equation (2), we get,

$$i_p = \frac{-V_m \omega L}{(R^2 + \omega^2 L^2)} \cos(\omega t + \theta)$$

$$+ \frac{V_m R}{(R^2 + \omega^2 L^2)} \sin(\omega t + \theta)$$

$$= \frac{V_m}{(R^2 + \omega^2 L^2)} [R \sin(\omega t + \theta) - L\omega \cos(\omega t + \theta)]$$

Let,

$$L\omega = k \sin \phi$$

$$R = k \cos \phi$$

Thus,

$$i_p = \frac{V_m}{R^2 + \omega^2 L^2} [k \cos \phi \sin(\omega t + \theta)$$
$$- k \sin \phi \cos(\omega t + \theta)]$$

$$= \frac{V_m k}{R^2 + \omega^2 L^2} \sin(\omega t + \theta - \phi) \quad \text{... (6)}$$

Where,

$$k^2 \sin^2\phi + k^2 \cos^2\phi = L^2\omega^2 + R^2$$

$$k^2(\sin^2\phi + \cos^2\phi) = R^2 + L^2\omega^2$$

$$k^2 = R^2 + L^2\omega^2$$

$$k = \sqrt{R^2 + L^2\omega^2}$$

And $$\frac{k \sin \phi}{k \cos \phi} = \frac{\omega L}{R}$$

$$\Rightarrow \quad \tan \phi = \frac{\omega L}{R}$$

$$\phi = \tan^{-1}\left[\frac{\omega L}{R}\right]$$

Equation (6) can now be written as,

$$i_p = \frac{V_m \times \sqrt{R^2 + L^2\omega^2}}{(R^2 + \omega^2 L^2)} \sin\left(\omega t + \theta - \tan^{-1}\left[\frac{\omega L}{R}\right]\right)$$

$$= \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t + \theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

The complementary function for the circuit is given as,

$$i_c = c e^{-Rt/L}$$

The total solution of i for the R-L series circuit is,

$$i = i_p + i_c$$

$$= \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t + \theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right) + c\, e^{-Rt/L}$$

In the initially relaxed condition, the switch 'S' is closed (i.e., t = 0) the initial current is '0' because of the inductor.

$$\therefore \quad 0 = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(0 + \theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right) + c e^0$$

$$\Rightarrow c e^0 = -\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$\therefore$ The transient current is given as,

$$i_c = -\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\theta - \tan^{-1}\frac{\omega L}{R}\right) e^{-Rt/L}$$

Given,

For $t = 0$, transient current in the circuit, $i_c = 0$

$$0 = -\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right) e^{-R0/L}$$

$$= -\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$$= \sin\left(\theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

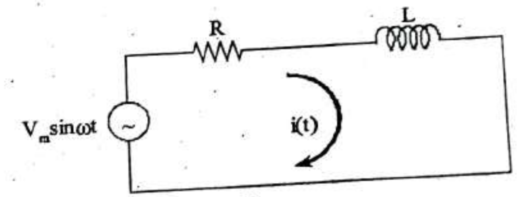
$\therefore$ The value of θ when transient current $i_c = 0$ is $\tan^{-1}\left(\frac{\omega L}{R}\right)$

Q17. Derive an expression for the current response in R-L series circuit with a sinusoidal source.

OR

Derive the expression for i(t) for R-L series circuit when excited by a sinusoidal source.

Ans:



Figure

The above figure represents an R-L series circuit excited by a sinusoidal source.

Applying KVL to the above circuit, we get,

$$V_m \sin \omega t = i(t).R + L\frac{di(t)}{dt}$$

Applying Laplace transform on both sides, we get,

$$\frac{V_m \omega}{s^2 + \omega^2} = RI(s) + L[s.I(s) - i(0^+)][i(0^+) = i(0^-) = 0]$$

$$I(s)[R + Ls] = \frac{V_m \omega}{s^2 + \omega^2}$$

$$I(s) = \frac{V_m \omega}{(s^2 + \omega^2)(R + Ls)}$$

$$I(s) = \frac{V_m \omega}{L\left(s + \frac{R}{L}\right)(s^2 + \omega^2)}$$

$$\therefore \quad I(s) = \frac{V_m \omega}{L\left(s + \frac{R}{L}\right)(s + j\omega)(s - j\omega)}$$

Resolving the above expression into partial fractions, we get,

$$\frac{V_m \omega / L}{\left(s + \frac{R}{L}\right)(s + j\omega)(s - j\omega)} = \frac{A}{s + \frac{R}{L}} + \frac{B}{s + j\omega} + \frac{C}{s - j\omega}$$

$$A(s+j\omega)(s-j\omega)+B\left(s+\frac{R}{L}\right)(s-j\omega)+C\left(s+\frac{R}{L}\right)(s+j\omega)=\frac{V_m}{L}.\omega \quad \text{... (1)}$$

Now substituting $s=\frac{-R}{L}$ in equation (1), we get,

$$A\left[-\frac{R}{L}+j\omega\right]\left[-\frac{R}{L}-j\omega\right]+0+0=\frac{V_m}{L}.\omega$$

$$A\left[\frac{R^2}{L^2}+\omega^2\right]=\frac{V_m}{L}.\omega \quad (\because j^2=-1)$$

$$\therefore \quad A=\frac{\frac{V_m}{L}.\omega}{\frac{R^2+\omega^2L^2}{L^2}}=\frac{V_m\omega L}{R^2+\omega^2L^2}$$

Now substituting $s=-j\omega$ in equation (1), we get,

$$0+B\left[-j\omega+\frac{R}{L}\right][-j\omega-j\omega]+0=\frac{V_m}{L}.\omega$$

$$B\left[-j\omega+\frac{R}{L}\right][-2j\omega]=\frac{V_m}{L}.\omega$$

$$B\left[-j\omega+\frac{R}{L}\right][-2j]=\frac{V_m}{L}$$

$$B\left[-2\omega-\frac{2jR}{L}\right]=\frac{V_m}{L}$$

$$B=\frac{\frac{V_m}{L}}{\frac{-2\omega L-2jR}{L}}=\frac{V_m}{-2(\omega L+jR)}$$

$$B=\frac{V_m}{-2j\left(R+\frac{\omega L}{j}\right)}=\frac{V_m}{-2j(R-j\omega L)} \quad \left(\because \frac{1}{j}=-j\right)$$

$$B=\frac{V_m}{-2j(R-j\omega L)}\times\frac{[R+j\omega L]}{(R+j\omega L)}=\frac{V_m[R+j\omega L]}{-2j(R^2+\omega^2L^2)}$$

Now substituting $s=j\omega$ in equation (1), we get,

$$0+0+C\left[j\omega+\frac{R}{L}\right][j\omega+j\omega]=\frac{V_m}{L}.\omega$$

$$C\left[j\omega+\frac{R}{L}\right][2j\omega]=\frac{V_m}{L}.\omega$$

$$C\left[j\omega+\frac{R}{L}\right][2j]=\frac{V_m}{L}.\omega$$

$$C\left[j\omega+\frac{R}{L}\right][2j]=\frac{V_m}{L}$$

$$C\left[-2\omega+\frac{2jR}{L}\right]=\frac{V_m}{L}$$

$$C=\frac{\frac{V_m}{L}}{\frac{-2\omega L+2jR}{L}}=\frac{V_m}{2j[R+j\omega L]}$$

$$C=\frac{V_m}{2j[R+j\omega L]}\times\frac{[R-j\omega L]}{[R-j\omega L]}=\frac{V_m[R-j\omega L]}{2j[R^2+\omega^2L^2]}$$

$\therefore$ Equation (1) can be written as,

$$I(s)=\frac{\left[\frac{V_m\omega L}{R^2+\omega^2L^2}\right]}{s+\frac{R}{L}}+\frac{\left[\frac{V_m[R+j\omega L]}{-2j[R^2+\omega^2L^2]}\right]}{s+j\omega}+\frac{\left[\frac{V_m[R-j\omega L]}{2j[R^2+\omega^2L^2]}\right]}{s-j\omega} \quad \left(\because \frac{1}{j}=-j\right)$$

$$\therefore \quad I(s)=\frac{V_m.\omega L}{R^2+\omega^2L^2}\left[\frac{1}{\left(s+\frac{R}{L}\right)}\right]+\frac{jV_m[R+j\omega L]}{2[R^2+\omega^2L^2]}\left[\frac{1}{s+j\omega}\right]-\frac{jV_m[R-j\omega L]}{2[R^2+\omega^2L^2]}\left[\frac{1}{s-j\omega}\right]$$

By taking inverse Laplace transform on both sides, the total current $i(t)$ is given by,

$$i(t)=\frac{V_m\omega L}{R^2+\omega^2L^2}.e^{\left(\frac{-R}{L}\right)t}+\frac{jV_m[R+j\omega L]}{2[R^2+\omega^2L^2]}e^{-j\omega t}-\frac{jV_m[R-j\omega L]}{2[R^2+\omega^2L^2]}e^{j\omega t}$$

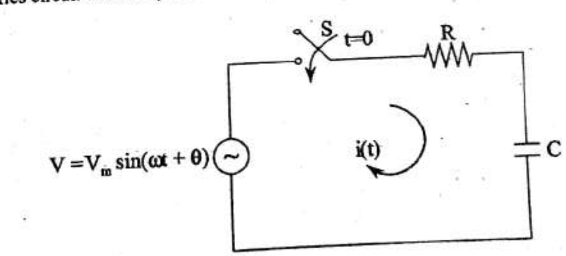
Q18. A series RC circuit has a sinusoidal voltage source $v(t)=V_m\sin(\omega t+\phi)$ applied at time when $\phi=0$. Find the expression for current.

OR

Derive an expression for the current response in R-C series circuit with a sinusoidal source.

Ans:

Consider a R-C series circuit excited by a sinusoidal source as shown in figure.



Figure

When the switch 'S' is closed at $t=0$, by applying KVL to the circuit we get,

$$R.i+\frac{1}{C}\int i\,dt=V$$

$$R.i + \frac{1}{C}\int i dt = V_m \sin(\omega t + \theta) \quad \dots (1)$$

Differentiating equation (1) with respect to time, we get,

$$R\frac{di}{dt} + \frac{i}{C} = \omega V_m \cos(\omega t + \theta)$$

$$\frac{di}{dt} + \frac{i}{RC} = \frac{\omega V_m}{R}\cos(\omega t + \theta)$$

$$\left(D + \frac{1}{RC}\right)i = \frac{\omega V_m}{R}\cos(\omega t + \theta) \qquad \left[\because D = \frac{d}{dt}\right] \quad \dots (2)$$

The current solution of equation (2) is given as,

$$i = i_p + i_c$$

Where,

i_c is the complementary function given as,

$$i_c = c_1 e^{-t/RC} \quad \dots (3)$$

Where,

c_1 is a constant.

And i_p is the particular function and is obtained as,

$$i_p = c_2 \cos(\omega t + \theta) + c_3 \sin(\omega t + \theta) \quad \dots (4)$$

Where,

c_2 and c_3 are constants.

Now, differentiating equation (1), with respect to 't', we get,

$$Di_p = -c_2 \sin(\omega t + \theta) + c_3 \cos(\omega t + \theta) \quad \dots (5)$$

Therefore, substituting the values of i_p and Di_p from equations (4) and (5) in equation (2), we get,

$$[-c_2\,\omega \sin(\omega t + \theta) + c_3\,\omega \cos(\omega t + \theta)] + \frac{1}{RC}\,[c_2 \cos(\omega t + \theta) + c_3 \sin(\omega t + \theta)] = \frac{\omega V_m}{R}\cos(\omega t + \theta)$$

$$\Rightarrow \left[-c_2\omega + \frac{c_3}{RC}\right]\sin(\omega t + \theta) + \left[c_3\omega + \frac{c_2}{RC}\right]\cos(\omega t + \theta) = \frac{\omega V_m}{R}\cos(\omega t + \theta)$$

Equating the coefficients of like terms,

i.e., $-c_3\omega + \frac{c_2}{RC} = \frac{\omega V_m}{R}$

$$\Rightarrow c_3\omega = \frac{\omega V_m C - c_2}{RC}$$

$$\Rightarrow c_3 = \frac{\omega V_m C - c_2}{\omega RC} \quad \dots (6)$$

And $-c_2\omega + \frac{c_3}{RC} = 0$

$$\Rightarrow -c_2\omega + \frac{\omega V_m C - c_2}{\omega R^2 C^2} = 0 \qquad [\because \text{From equation (6)}]$$

$$\Rightarrow \frac{-c_2\omega^2 R^2 C^2 + \omega V_m C - c_2}{\omega R^2 C^2} = 0$$

$$\Rightarrow -c_2(1 + \omega^2 R^2 C^2) + \omega V_m C = 0$$

$$\Rightarrow c_2 = \frac{\omega V_m C}{1 + \omega^2 R^2 C^2} = \frac{V_m}{\omega C\left[\frac{1}{R^2} + R^2\right]}$$

Substituting the value of c_2 in equation (6), we get,

$$c_3 = \frac{\omega V_m C - \dfrac{V_m}{\omega C\left[\frac{1}{\omega^2 C^2} + R^2\right]}}{\omega RC}$$

$$= \frac{\omega V_m C\left[\frac{1}{\omega C} + \omega C R^2\right] - V_m}{\omega RC\left[\frac{1}{\omega C} + \omega C R^2\right]} = \frac{\omega V_m C + (\omega^2 C^2 R^2)\,\omega V_m C - \omega C V_m}{\omega C[R + \omega^2 C^2 R^3]} = \frac{\omega C V_m\,[1 + \omega^2 C^2 R^2 - 1]}{\omega CR[1 + \omega^2 C^2 R^2]}$$

$$= \frac{V_m(\omega^2 C^2 R^2)}{R[1 + \omega^2 C^2 R^2]} = \frac{\omega^2 c^2 V_m R}{\omega^2 C^2\left[\frac{1}{\omega^2 C^2} + R^2\right]} = \frac{V_m R}{\left[\frac{1}{\omega^2 C^2} + R^2\right]}$$

$$\therefore c_3 = \frac{V_m R}{\left[\frac{1}{\omega^2 C^2} + R^2\right]} \quad \dots (8)$$

Now, substituting the values of equations (7) and (8) in equation (4), we get,

$$i_p = \frac{V_m}{\omega C\left[\frac{1}{\omega^2 C^2} + R^2\right]}\cos(\omega t + \theta) + \frac{V_m R}{\left[\frac{1}{\omega^2 C^2} + R^2\right]}\sin(\omega t + \theta)$$

Put, $R = \cos\phi$ and $\frac{1}{\omega C} = \sin\phi$

Where, ϕ being $\tan^{-1}\left[\frac{1}{\omega CR}\right]$

$$\therefore \quad i_p = \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}\left[\frac{1}{\omega C}\cos(\omega t + \theta) + R\sin(\omega t + \theta)\right]$$

$$= \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}[\sin\phi\cos(\omega t + \theta) + \cos\phi\sin(\omega t + \theta)]$$

$$= \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}[\sin(\omega t + \theta + \phi)]$$

$$= \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}\left[\sin\left(\omega t + \theta + \tan^{-1}\left(\frac{1}{\omega CR}\right)\right)\right]$$

$$\therefore \; i = c_1 e^{-t/RC} + \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}\sin\left(\omega t + \theta + \tan^{-1}\frac{1}{\omega CR}\right) \quad \dots (9)$$

$$R.i+\frac{1}{C}\int i\,dt=V_m\sin(\omega t+\theta) \quad \text{... (1)}$$

Differentiating equation (1) with respect to time, we get,

$$R\frac{di}{dt}+\frac{i}{C}=\omega V_m\cos(\omega t+\theta)$$

$$\frac{di}{dt}+\frac{i}{RC}=\frac{\omega V_m}{R}\cos(\omega t+\theta)$$

$$\left(D+\frac{1}{RC}\right)i=\frac{\omega V_m}{R}\cos(\omega t+\theta) \qquad \left[\because D=\frac{d}{dt}\right] \quad \text{... (2)}$$

The current solution of equation (2) is given as,

$$i=i_p+i_c$$

Where,

i_c is the complementary function given as,

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Where,

c_1 is a constant. ... (3)

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$$i_p=c_2\cos(\omega t+\theta)+c_3\sin(\omega t+\theta) \quad \text{... (4)}$$

Where,

c_2 and c_3 are constants.

Now, differentiating equation (1), with respect to 't', we get,

$$Di_p=-c_2\sin(\omega t+\theta)+c_3\cos(\omega t+\theta) \quad \text{... (5)}$$

Therefore, substituting the values of i_p and Di_p from equations (4) and (5) in equation (2), we get,

$$[-c_2\,\omega\sin(\omega t+\theta)+c_3\,\omega\cos(\omega t+\theta)]+\frac{1}{RC}\,[c_2\cos(\omega t+\theta)+c_3\sin(\omega t+\theta)]=\frac{\omega V_m}{R}\cos(\omega t+\theta)$$

$$\Rightarrow \left[-c_2\omega+\frac{c_3}{RC}\right]\sin(\omega t+\theta)+\left[c_3\omega+\frac{c_2}{RC}\right]\cos(\omega t+\theta)=\frac{\omega V_m}{R}\cos(\omega t+\theta)$$

Equating the coefficients of like terms,

i.e., $-c_3\omega+\dfrac{c_2}{RC}=\dfrac{\omega V_m}{R}$

$$\Rightarrow c_3\omega=\frac{\omega V_mC-c_2}{RC}$$

$$\Rightarrow c_3=\frac{\omega V_mC-c_2}{\omega RC} \quad \text{... (6)}$$

And $-c_2\omega+\dfrac{c_3}{RC}=0$

$$\Rightarrow -c_2\omega+\frac{\omega V_mC-c_2}{\omega R^2C^2}=0 \qquad [\because \text{From equation (6)}]$$

$$\Rightarrow \frac{-c_2\omega^2R^2C^2+\omega V_mC-c_2}{\omega R^2C^2}=0$$

$$\Rightarrow -c_2(1+\omega^2R^2C^2)+\omega V_mC=0$$

$$\Rightarrow c_2=\frac{\omega V_mC}{1+\omega^2R^2C^2}=\frac{V_m}{\omega C\left[\frac{1}{R^2}+R^2\right]}$$

Substituting the value of c_2 in equation (6), we get,

$$c_3=\frac{\omega V_mC-\dfrac{V_m}{\omega C\left[\frac{1}{\omega^2C^2}+R^2\right]}}{\omega RC}$$

$$=\frac{\omega V_mC\left[\frac{1}{\omega C}+\omega CR^2\right]-V_m}{\omega RC\left[\frac{1}{\omega C}+\omega CR^2\right]}=\frac{\omega V_mC+(\omega^2C^2R^2)\omega V_mC-\omega CV_m}{\omega C[R+\omega^2C^2R^3]}=\frac{\omega CV_m\,[1+\omega^2C^2R^2-1]}{\omega CR[1+\omega^2C^2R^2]}$$

$$=\frac{V_m\,(\omega^2C^2R^2)}{R[1+\omega^2C^2R^2]}=\frac{\omega^2c^2V_mR}{\omega^2C^2\left[\frac{1}{\omega^2C^2}+R^2\right]}=\frac{V_mR}{\left[\frac{1}{\omega^2C^2}+R^2\right]}$$

$$\therefore c_3=\frac{V_mR}{\left[\frac{1}{\omega^2C^2}+R^2\right]} \quad \text{... (8)}$$

Now, substituting the values of equations (7) and (8) in equation (4), we get,

$$i_p=\frac{V_m}{\omega C\left[\frac{1}{\omega^2C^2}+R^2\right]}\cos(\omega t+\theta)+\frac{V_mR}{\left[\frac{1}{\omega^2C^2}+R^2\right]}\sin(\omega t+\theta)$$

Put, $R=\cos\phi$ and $\dfrac{1}{\omega C}=\sin\phi$

Where, ϕ being $\tan^{-1}\left[\dfrac{1}{\omega CR}\right]$

$$\therefore \quad i_p=\frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\left[\frac{1}{\omega C}\cos(\omega t+\theta)+R\sin(\omega t+\theta)\right]$$

$$=\frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}[\sin\phi\cos(\omega t+\theta)+\cos\phi\sin(\omega t+\theta)]$$

$$=\frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}[\sin(\omega t+\theta+\phi)]$$

$$=\frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\left[\sin\left(\omega t+\theta+\tan^{-1}\left(\frac{1}{\omega CR}\right)\right)\right].$$

$$\therefore\ i=c_1e^{-t/RC}+\frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\sin\left(\omega t+\theta+\tan^{-1}\frac{1}{\omega CR}\right) \quad \text{... (9)}$$

Since capacitor does not allow sudden changes in voltages at $t = 0$, $i = \frac{V_m}{R}\sin\theta$

$$\therefore \frac{V_m}{R}\sin\theta = c_1 e^0 + = \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\sin\left(\theta+\tan^{-1}\frac{1}{\omega CR}\right)$$

$$c_1 = \frac{V_m}{R}\sin\theta - \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\sin\left(\theta+\tan^{-1}\frac{1}{\omega CR}\right)$$

By substituting 'c_1' value in equation (9), we get the complete solution for the current, i.e.,

$$i = e^{-t/RC}\left[\frac{V_m}{R}\sin\theta - \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\sin\left(\theta+\tan^{-1}\frac{1}{\omega CR}\right)\right] + \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\sin\left(\omega t+\theta+\tan^{-1}\frac{1}{\omega CR}\right)$$

Expression for Voltage across the Capacitor in Series RC circuit with Sinusoidal Excitations

The transient voltage across the resistor is,

$$V_R = i(t)\times R$$

$$= \left\{e^{-t/RC}\left[\frac{V_m}{R}\sin\theta - \frac{V_m}{\sqrt{R^2+\frac{1}{(\omega C)^2}}}\sin\left(\theta+\tan^{-1}\frac{1}{\omega CR}\right)\right] + \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\sin\left(\omega t+\theta+\tan^{-1}\frac{1}{\omega CR}\right)\right\}\times R$$

$$V_R = \left\{e^{-t/RC}\left[V_m\sin\theta - \frac{V_m}{\sqrt{1+\left(\frac{1}{\omega CR}\right)^2}}\sin\left(\theta+\tan^{-1}\left(\frac{1}{\omega CR}\right)\right)\right] + \frac{V_m}{\sqrt{1+\left(\frac{1}{\omega CR}\right)^2}}\sin\left(\omega t+\theta+\tan^{-1}\left(\frac{1}{\omega CR}\right)\right)\right\}$$

The voltage across capacitor, $V_C(t)$ is given by,

$$V_C(t) = \text{Input voltage} - V_R$$

$$= V_m\sin(\omega t+\theta) - V_R$$

$$= V_m\sin(\omega t+\theta) - \left\{e^{-t/RC}\left[V_m\sin\theta - \frac{V_m}{\sqrt{1+\left(\frac{1}{\omega CR}\right)^2}}\sin\left(\theta+\tan^{-1}\left(\frac{1}{\omega CR}\right)\right)\right] + \frac{V_m}{\sqrt{1+\left(\frac{1}{\omega CR}\right)^2}}\sin\left(\omega t+\theta+\tan^{-1}\left(\frac{1}{\omega CR}\right)\right)\right\}$$

$$= V_m\left\{\sin(\omega t+\theta) - e^{-t/RC}\left[\sin\theta - \frac{1}{\sqrt{1+\left(\frac{1}{\omega CR}\right)^2}}\sin\left(\theta+\tan^{-1}\left(\frac{1}{\omega CR}\right)\right)\right] + \frac{1}{\sqrt{1+\left(\frac{1}{\omega CR}\right)^2}}\sin\left(\omega t+\theta+\tan^{-1}\left(\frac{1}{\omega CR}\right)\right)\right\}$$

Q19. Derive the expression for transient response of RLC series circuit with unit step input.

Ans:

Consider a series *RLC* circuit shown in figure (1).

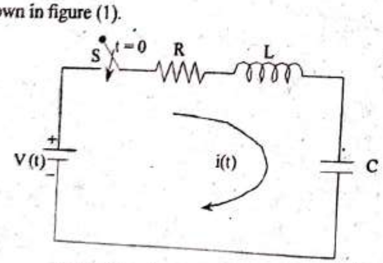


Figure (1): Series RLC Circuit

$V(t)$ is a unit step voltage applied to the series *RLC* circuit.

Let $i(t)$ be the current flowing through the circuit. At $t = 0$, the switch is closed. Let us assume that the initial current through inductor and initial voltage across capacitor is zero i.e.,

$$i(0^+) = i(0^-) = 0 \text{ and } V_c(0^+) = V_c(0^-) = 0$$

Applying KVL to the circuit, we get,

$$V(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt \quad ...(1)$$

Applying Laplace to equation (1), we get,

$$\frac{V}{s} = RI(s) + L[sI(s) - i(0)] + \frac{1}{C}\left[\frac{I(s)}{s}+\frac{q_0}{s}\right]$$

$$\frac{V}{s} = I(s)\left[R+sL+\frac{1}{Cs}\right]$$

[$\because$ Initial conditions are zero]

$$\frac{V}{s} = I(s)\left[\frac{RCs+s^2LC+1}{Cs}\right]$$

$$I(s) = \frac{V_C}{s^2LC+RCs+1}$$

$$= \frac{V/L}{s^2+\frac{R}{L}s+\frac{1}{LC}} \quad ...(2)$$

The roots of the above equation are,

$$s_1, s_2 = \frac{-\frac{R}{L}\pm\sqrt{\left(\frac{R}{L}\right)^2-\frac{4}{LC}}}{2}$$

$$s_1, s_2 = \frac{-R}{2L}\pm\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}$$

Based on the relation between $\left(\frac{R}{2L}\right)^2$ and $\frac{1}{LC}$ we get three types of roots and hence, three types of response.

Case (i): $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

When $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$, the roots will be real and unequal and the system is said to be overdamped.

The response of the system can be determined as,

$$I(s) = \frac{V/L}{(s+s_1)(s+s_2)}$$

$$= \frac{A}{s+s_1}+\frac{B}{s+s_2}$$

$$I(s) = \frac{V/L(s_2-s_1)}{s+s_1}+\frac{V/L(s_1-s_2)}{s+s_2}$$

$$i(t) = \frac{V}{L(s_2-s_1)}[e^{-s_1t}-e^{-s_2t}]$$

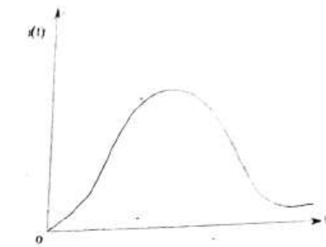


Figure (2)

Figue (2) shows the time response when the system is overdamped.

Case (ii): $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

When, $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ the roots of the system will be real and equal and the system is said to be critically damped.

$$I(s) = \frac{V/L}{(s+s_1)^2}$$

$$i(t) = \frac{V}{L}te^{s_1t}$$

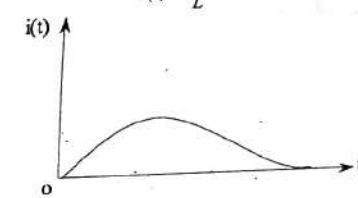


Figure (3)

Figue (3) shows the time response of the critically damped system.

Case (iii): $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

When $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$, the roots of the system will be complex conjugate and the system is said to be underdamped system.

The response of the system can be determined as,

Let the two roots be,

$$s_1 = -A+jB \text{ and } s_2 = -A-jB$$

Where,

$$A = \frac{R}{2L} \text{ and } B = \sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}$$

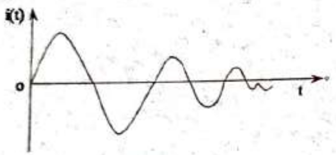
$$\therefore \quad I(s) = \frac{V/L}{(s+A-jB)(s+A+jB)}$$

$$I(s) = \frac{V/j2BL}{s+A-jB} - \frac{V/j2BL}{s+A+jB}$$

$$i(t) = \frac{V}{j2BL}e^{-At}e^{jBt} - \frac{V}{j2BL}e^{-At}e^{-jBt}$$

$$= \frac{Ve^{-At}}{BL}\left[\frac{e^{jBt}-e^{-jBt}}{2j}\right]$$

$$i(t) = \frac{Ve^{-At}}{BL}\sin Bt$$



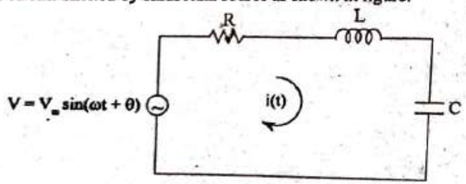
Figue (4)

The response of the underdamped system is shown in figure (4).

Q20. Derive an expression for transient response of R-L-C circuit excited by sinusoidal source use Laplace transform approach.

Ans:

Consider an RLC series circuit excited by sinusoidal source as shown in figure.



Figure

Applying KVL to the circuit, we get,

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = V$$

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = V_m \sin(\omega t+\theta) \quad \text{... (1)}$$

Applying Laplace transform to equation (1), we get,

$$L\left\{Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt\right\} = L\{V_m \sin(\omega t+\theta)\}$$

$$L\{Ri(t)\} + L\left\{L\frac{di(t)}{dt}\right\} + L\left\{\frac{1}{C}\int i(t)dt\right\} = V_m L\{\sin\omega t\cos\theta + \cos\omega t\sin\theta\}$$

Neglecting initial conditions, we have,

$$RI(s) + sLI(s) + \frac{1}{Cs}I(s) = V_m[L\{\sin\omega t\cos\theta\} + L\{\cos\omega t\sin\theta\}]$$

$$I(s)\left[R + sL + \frac{1}{Cs}\right] = V_m\left[\frac{\omega}{s^2+\omega^2}\cos\theta + \frac{s}{s^2+\omega^2}\sin\theta\right]$$

$$I(s)\left[R + sL + \frac{1}{Cs}\right] = \frac{V_m(\omega\cos\theta + s\sin\theta)}{s^2+\omega^2} \quad \text{... (2)}$$

Multiplying equation (2) with '$\frac{s}{L}$', we get.

$$I(s)\times\frac{s}{L}\left(R + sL + \frac{1}{Cs}\right) = \frac{s}{L}\times\frac{V_m(\omega\cos\theta + s\sin\theta)}{(s^2+\omega^2)}$$

$$I(s)\left[\frac{R}{L}s + s^2 + \frac{1}{LC}\right] = \frac{sV_m(s\sin\theta + \omega\cos\theta)}{L(s^2+\omega^2)}$$

$$\Rightarrow \quad I(s) = \frac{sV_m(s\sin\theta+\omega\cos\theta)}{L(s^2+\omega^2)\left(\frac{R}{L}s + s^2 + \frac{1}{LC}\right)}$$

$$= \frac{sV_m(s\sin\theta+\omega\cos\theta)}{L(s^2+\omega^2)\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

$$= \frac{sV_m(s\sin\theta+\omega\cos\theta)}{L(s-s_1)(s-s_2)(s+j\omega)(s-j\omega)} \quad [\because s^2+\omega^2 = (s+j\omega)(s-j\omega)] \quad \text{... (3)}$$

Where,

s_1, s_2 are the roots of quadratic expression,

$$s^2 + \frac{R}{L}s + \frac{1}{LC}$$

$$s_1 = \frac{-R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}, \; s_2 = \frac{-R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

Applying partial fractions to equation (3), we get,

$$\frac{sV_m(s\sin\theta+\omega\cos\theta)}{L(s-s_1)(s-s_2)(s+j\omega)(s-j\omega)} = \frac{A}{(s-s_1)} + \frac{B}{(s-s_2)} + \frac{C}{(s+j\omega)} + \frac{D}{(s-j\omega)}$$

$$A = \frac{s_1(s_1\cos\theta+\omega\cos\theta)}{(s_1+j\omega)(s_1-j\omega)(s_1-s_2)}$$

$$B = \frac{s_2(s_2\sin\theta+\omega\cos\theta)}{(s_2+j\omega)(s_2-j\omega)(s_2-s_1)}$$

$$C = \frac{\omega(\cos\theta - j\sin\theta)}{2(s_1+j\omega)(s_2+j\omega)}$$

$$D = \frac{\omega(\cos\theta + j\sin\theta)}{2(s_1-j\omega)(s_2-j\omega)}$$

The complete response of the circuit is given by,

$$i(t) = \frac{V_m}{L}[Ae^{s_1t} + Be^{s_2t}] + \frac{V_m}{L}[Ce^{-j\omega t} + De^{j\omega t}]$$

= Transient response (i_{trans}) + Steady state response (i_{ss})

$\therefore$ The expression for transient response of RLC series circuit excited by sinusoidal source is,

$$i_{trans} = \frac{V_m}{L}[Ae^{s_1t} + Be^{s_2t}]$$

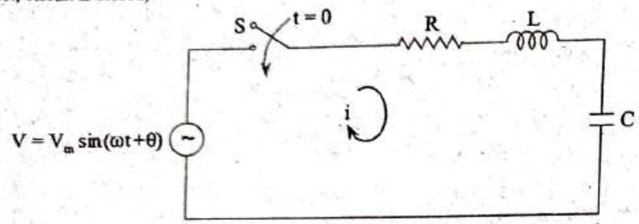
$$= \frac{V_m}{L}\left[\frac{s_1(s_1\sin\theta+\omega\cos\theta)}{(s_1+j\omega)(s_1-j\omega)(s_1-s_2)}e^{s_1t} + \frac{s_2(s_2\sin\theta+\omega\cos\theta)}{(s_2+j\omega)(s_2-j\omega)(s_2-s_1)}e^{s_2t}\right]$$

$$= \frac{V_m}{L}\left[\frac{s_1(s_1\sin\theta+\omega\cos\theta)}{(s_1^2+\omega^2)\sqrt{\frac{R^2}{L^2}-\frac{4}{LC}}}e^{s_1t} + \frac{s_2(s_2\sin\theta+\omega\cos\theta)}{(s_2^2+\omega^2)\sqrt{\frac{R^2}{L^2}-\frac{4}{LC}}}e^{s_2t}\right]$$

Q21. Derive an expression for the current response i(t) in a R-L-C series circuit excited with sinusoidal source.

Ans:

Consider a R-L-C series circuit as shown in figure. When the switch 'S' is closed at time $t = 0$, current i starts flowing through the circuit (i.e., circuit is closed).



Figure

Applying KVL to the circuit, we get,

$$R_i + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V$$

$$R_i + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V_m \sin(\omega t + \theta)$$

Differentiating on both sides with respect to t, we get,

$$R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{i}{C} = \omega\, V_m \cos(\omega t + \theta)$$

Let, $D = \frac{d}{dt}$

$$\therefore \left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)i = \frac{\omega V_m}{L}\cos(\omega t + \theta) \quad \text{... (1)}$$

The total response current of the circuit is given as,

$$i = i_c + i_p \quad \text{... (I)}$$

The particular integral or forced response is obtained as,

$$i_p = A\cos(\omega t + \theta) + B\sin(\omega t + \theta) \quad \text{... (2)}$$

$$i_p' = -A\,\omega\sin(\omega t + \theta) + B\omega\cos(\omega t + \theta) \quad \text{... (3)}$$

$$i_p'' = -A\,\omega^2\cos(\omega t + \theta) - B\omega^2\sin(\omega t + \theta) \quad \text{... (4)}$$

Substituting the values of i_p, i_p' and i_p'' in the equation (1), we get,

$$[-A\omega^2\cos(\omega t + \theta) - B\omega^2\sin(\omega t + \theta)] + \frac{R}{L}[-A\omega\sin(\omega t + \theta) + B\omega\cos(\omega t + \theta)] + \frac{1}{LC}[A\cos(\omega t + \theta) + B\sin(\omega t + \theta)]$$

$$= \frac{\omega V_m}{L}\cos(\omega t + \theta) \quad \text{... (5)}$$

Comparing both sides, separating sine and cosine coefficients of equation (5), we get,

For sine coefficients,

$$-B\omega^2 - A\frac{\omega R}{L} + B\frac{1}{LC} = 0$$

$$\Rightarrow A\left[\frac{\omega R}{L}\right] + B\left[\omega^2 - \frac{1}{LC}\right] = 0 \quad \text{... (6)}$$

$$\Rightarrow A\left(\frac{\omega R}{L}\right) = -B\left[\omega^2 - \frac{1}{LC}\right]$$

$$A = \frac{-B\left[\omega^2 - \frac{1}{LC}\right]}{\frac{\omega R}{L}} = \frac{B\left[\frac{1}{LC} - \omega^2\right]}{\frac{\omega R}{L}} \quad \text{... (7)}$$

For cosine coefficients,

$$-A\omega^2 + B\frac{\omega R}{L} + A\frac{1}{LC} = \frac{\omega V_m}{L}$$

$$A\left[-\omega^2 + \frac{1}{LC}\right] + B\left(\frac{\omega R}{L}\right) = \frac{\omega V_m}{L} \quad \text{... (8)}$$

Now substituting A value in equation (8), we get,

$$\frac{B\left(\frac{1}{LC} - \omega^2\right)}{\frac{\omega R}{L}}\left[-\omega^2 + \frac{1}{LC}\right] + B\left(\frac{\omega R}{L}\right) = \frac{\omega V_m}{L}$$

$$\Rightarrow \frac{B\left(-\omega^2 + \frac{1}{LC}\right)^2}{\frac{\omega R}{L}} + B\left(\frac{\omega R}{L}\right) = \frac{\omega V_m}{L}$$

$$\Rightarrow B\left[\left(-\omega^2 + \frac{1}{LC}\right)^2 + \frac{\omega^2 R^2}{L^2}\right] = \frac{\omega^2 V_m R}{L^2}$$

$$B = \frac{\frac{\omega^2 V_m R}{L^2}}{\left[\left(-\omega^2 + \frac{1}{LC}\right)^2 + \frac{\omega^2 R^2}{L^2}\right]}$$

Substituting 'B' value in equation (8), we get,

$$A = \frac{V_m\,\omega^2 R}{L^2\left[\left(-\omega^2 + \frac{1}{LC}\right)^2 + \frac{\omega^2 R^2}{L^2}\right]} \times \frac{\left(\frac{1}{LC} - \omega^2\right)}{\frac{\omega R}{L}}$$

$$= \frac{V_m\,\omega\left(\frac{1}{LC} - \omega^2\right)}{L\left[\left(-\omega^2 + \frac{1}{LC}\right)^2 + \frac{\omega^2 R^2}{L^2}\right]}$$

Substituting the values of A and B in equation (2), we get,

$$i_p = \frac{V_m\,\omega\left(\frac{1}{LC} - \omega^2\right)}{L\left[\left(-\omega^2 + \frac{1}{LC}\right)^2 + \frac{\omega^2 R^2}{L^2}\right]}\cos(\omega t + \theta) + \frac{\frac{V_m\,\omega^2 R}{L^2}}{\left[\left(-\omega^2 + \frac{1}{LC}\right)^2 + \frac{\omega^2 R^2}{L^2}\right]}\sin(\omega t + \theta)$$

Let, $k\sin\phi = \dfrac{V_m\,\omega\left(\frac{1}{LC}-\omega^2\right)}{L\left[\left(-\omega^2+\frac{1}{LC}\right)^2+\frac{\omega^2R^2}{L^2}\right]}$

And $k\cos\phi = \dfrac{V_m\frac{\omega^2R}{L^2}}{\left[\left(-\omega^2+\frac{1}{LC}\right)^2+\frac{\omega^2R^2}{L^2}\right]}$

$$\frac{k\sin\phi}{k\cos\phi} = \tan\phi$$

$$= \frac{\frac{V_m\omega}{L}\left[-\omega^2+\frac{1}{LC}\right]}{L\left[\left(-\omega^2+\frac{1}{LC}\right)^2+\frac{\omega^2R^2}{L^2}\right]} \times \frac{\left[\left(-\omega^2+\frac{1}{LC}\right)^2+\frac{\omega^2R^2}{L^2}\right]}{V_m\frac{\omega^2R}{L^2}}$$

$$= \frac{L}{\omega R}\left(-\omega^2+\frac{1}{LC}\right) = \frac{1}{R}\left[-\omega L+\frac{1}{\omega C}\right]$$

$$\phi = \tan^{-1}\left[\frac{1}{R}\left(-\omega L+\frac{1}{\omega C}\right)\right]$$

$$\therefore\quad i_p = k\sin\phi\cos(\omega t+\theta) + k\cos\phi\sin(\omega t+\theta)$$
$$= [\sin(\omega t+\theta)\cos\phi + \cos(\omega t+\theta)\sin\phi]k$$
$$= k\sin(\omega t+\theta+\phi)$$

$$\therefore\quad i_p = k\left[\sin\left(\omega t+\theta+\tan^{-1}\frac{1}{R}\left(-\omega L+\frac{1}{\omega C}\right)\right)\right]$$

Now, we can find the value of k as,

$$k = \sqrt{k^2\cos^2\phi + k^2\sin^2\phi} \qquad [\because \sin^2\phi+\cos^2\phi = 1]$$

$$= \frac{V_m}{\sqrt{R^2+\left(\frac{1}{LC}-\omega L\right)^2}}$$

Thus, $i_p = \dfrac{V_m}{\sqrt{R^2+\left(\frac{1}{LC}-\omega L\right)^2}}\sin\left(\omega t+\theta+\tan^{-1}\left[\frac{1}{R}\left(\frac{1}{\omega C}-\omega L\right)\right]\right)$

The complementary function i_c can be obtained by making right hand side of equation (1), equating it to zero. The characteristic equation is,

$$D^2+\frac{R}{L}D+\frac{1}{LC}=0$$

The roots of the characteristic equation are,

$$\alpha_1, \alpha_2 = -\frac{R}{2L}\pm\frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2-\frac{4}{LC}}$$

The complementary function depends on the nature of roots, i.e., i_c

(i) Real and distinct roots $\left(\left(\frac{R^2}{2L}\right) > \frac{1}{LC}\right)$

$$\therefore\quad i_c = (c_1e^{-\beta_1 t}+c_2e^{-\beta_2 t})e^{\alpha t}$$

$\therefore$ The transient current response of RLC circuit for real and distinct roots given as,

$$i(t) = (c_1e^{-\beta_1 t}+c_2e^{-\beta_2 t})e^{\alpha t} + \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega c}-\omega L\right)^2}}\sin\left(\omega t+\theta+\tan^{-1}\left(\frac{1}{\omega c}-\omega L\right)\right)$$

(ii) Real and equal roots

$$\left(\frac{R}{2L}\right)^2 = \left(\frac{1}{LC}\right)$$

$$i_c = (c_1+c_2t)e^{-\alpha t}$$

$\therefore$ The transient current response of RLC circuit for real and equal roots is given as,

$$i(t) = e^{-\alpha t}(c_1+c_2t)e^{-\alpha t} + \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega c}-\omega L\right)^2}}\sin\left(\omega t+\theta+\tan^{-1}\left(\frac{1}{\omega c}-\omega L\right)\right)$$

(iii) Complex conjugate roots $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

$$i_c = e^{-\alpha t}(c_1\cos\beta t + c_2\sin\beta t)$$

$\therefore$ The total response of RLC series circuit is for real and equal roots.

$$i(t) = (c_1+c_2t)\,e^{-\alpha t} + \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega c}-\omega_L\right)^2}}\sin\left(\omega t+\theta+\tan^{-1}\frac{1}{R}\left(\frac{1}{\omega c}-\omega_L\right)\right)$$

Q22. Derive an expression of current in the source free RL circuit.

Ans:

Source Free Circuit

The first order RL and RC circuit can be excited by two ways,

(i) By independent source and

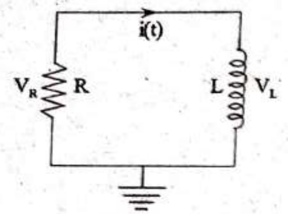
(ii) Without independent sources.

This type of excitation is done by initial condition of circuit storage elements i.e., when the source is disconnected from the circuit, the energy is initially stored in the inductive and capacitive elements due to this energy current flows through the circuit and gets dissipated through the resistor.

Source Free RL Circuit

Consider a series RL circuit shown in figure. Now determining the current $i(t)$ through inductor. At $t = 0$ (initial condition) current through the inductor is i_0. So,

$$i(0) = i_0$$



Figure

On applying KVL, we get,

$$\Rightarrow \quad V_R + V_L = 0$$

$$\Rightarrow \quad iR + L\frac{di}{dt} = 0$$

$$\Rightarrow \quad L\frac{di}{dt} = -iR$$

$$\frac{1}{i}di = \frac{-R}{L}dt$$

Now integrating on both sides by applying limits i_0 and $i(t)$, we get,

$$\Rightarrow \quad \int_0^{i(t)} \frac{1}{i}di = \frac{-R}{L}\int_0^t dt$$

$$[\because i_0 \text{ at } t = 0 \text{ and } i(t) \text{ at } t]$$

$$\Rightarrow \quad [\log_e i]_{i_0}^{i(t)} = \frac{-R}{L}[t]_0^t$$

$$\Rightarrow \quad \log_e i(t) - \log_e i_0 = \frac{-Rt}{L}$$

$$\Rightarrow \quad \log_e \frac{i(t)}{i_0} = \frac{-Rt}{L}$$

$$i(t) = i_0 e^{\frac{-Rt}{L}} \Rightarrow i(t) = i(0)\, e^{\frac{-Rt}{L}}$$

$\therefore$ The current through the inductor is given as,

$$i(t) = i(0)\, e^{\frac{-Rt}{L}}$$

Since $\frac{L}{R} = \tau$ (Time constant) for RL circuit

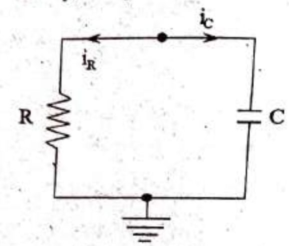
$\therefore \quad i(t) = i(0)\, e^{\frac{-t}{\tau}}$ is the response current of RL circuit.

Q23. Derive an expression of voltage in the source free RC circuit.

Ans:

Consider a series RC circuit, where the capacitor is initially charged, let the voltage across the capacitor be $V(t)$. At $t = 0$ (initial condition) the voltage across the capacitor is,

At $t = 0, \quad V_c(0) = V_0$



Figure

Applying KVL at the nodes, we get,

$$\Rightarrow \quad i_R + i_C = 0$$

$$\Rightarrow \quad \frac{V}{R} + C\frac{dV}{dt} = 0$$

$$\Rightarrow \quad \frac{V}{RC} + \frac{dV}{dt} = 0$$

$$\Rightarrow \quad \frac{dV}{dt} = \frac{-V}{RC}$$

$$\Rightarrow \quad \frac{1}{V}dV = \frac{-1}{RC}dt$$

On applying integration on both sides, we get,

$$\Rightarrow \quad \int \frac{1}{V}dV = \frac{-1}{RC}\int dt$$

$$\Rightarrow \quad \log_e V = \frac{-t}{RC} + \log_e B$$

[Where B is integrating constant]

$$\Rightarrow \quad \log_e V - \log_e B = \frac{-t}{RC}$$

$$\Rightarrow \quad \log_e \frac{V}{B} = e^{\frac{-t}{RC}} \quad [\because \log_e a = b \Rightarrow a = e^b]$$

$$\frac{V}{B} = e^{\frac{-t}{RC}}$$

$$V = Be^{\frac{-t}{RC}}$$

At initial condition we have, $V_c(0) = V_0 = B$. Now above equation becomes,

$$V = V_c(0)\, e^{\frac{-t}{RC}}$$

Since time constant for RC circuit is $\tau = RC$.

$$V = V_c(0)\, e^{\frac{-t}{\tau}}$$

$\therefore$ The voltage response for source free RC circuit is,

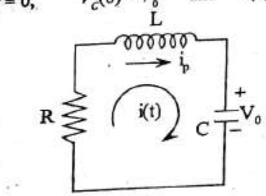
$$V = V(0)\, e^{\frac{-t}{\tau}}$$

Q24. Explain the source free series RLC circuit and its different responses.

Ans:

Consider a source free series RLC circuit, where capacitor and inductor is initially charged. At $t = 0$, the voltage across the capacitor and current in the inductor is given as,

At $t = 0, \quad V_C(0) = V_0 \quad$ and $\quad i(0) = i_0$



Figure

On applying kVL to the circuit shown in figure, we get,

$$V_R + V_L + V_C = 0$$

$$i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(t)dt = 0 \quad \text{... (1)}$$

Differentiating equation (1) with respect to 't', we get,

$$L\frac{d^2 i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{i(t)}{C} = 0$$

$$\frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{i(t)}{LC} = 0 \quad \text{... (2)}$$

At initial condition $t = 0$; equation (1) becomes as,

$$\Rightarrow \quad Ri(0) + L\frac{di(0)}{dt} + V_C(0) = 0$$

$$\Rightarrow \quad L\frac{di(0)}{dt} = -V_C(0) - Ri(0)$$

$$\Rightarrow \quad \frac{di(0)}{dt} = \frac{-1}{L}(V_0 + Ri_0) \quad \text{... (3)}$$

As we have the two initial conditions. Now we can solve the equation (2), we get,

Taking $i(t) = Be^{st}$.

Now substituting $i(t)$ in equation (2), we get,

$$\Rightarrow \quad B\frac{d^2}{dt^2}e^{st} + \frac{BR}{L}\frac{d}{dt}e^{st} + \frac{B}{LC}e^{st} = 0$$

$$\Rightarrow \quad Bs\frac{d}{dt}e^{st} + \frac{BRS}{L}e^{st} + \frac{B}{LC}e^{st} = 0$$

$$\Rightarrow \quad Bs^2e^{st} + Bs.\frac{R}{L}e^{st} + \frac{B}{LC}e^{st} = 0$$

$$\Rightarrow \quad Be^{st}\left(s^2 + \frac{Rs}{L} + \frac{1}{LC}\right) = 0$$

$$\Rightarrow \quad s^2 + \frac{RS}{L} + \frac{1}{LC} = 0 \quad \text{... (4)}$$

Comparing equation (4) with standard form of quadratic equation, $ax^2 + bx + c = 0$, whose roots are,

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where, $a = 1, \; b = \frac{R}{L} \;$ and $C = \frac{1}{LC}$

$$s_1, s_2 = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{2(1)}$$

$$= \frac{-R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{4}{4LC}}$$

$$= \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Let $\frac{R}{2L} = \alpha$ and $\frac{1}{LC} = \omega_0^2$

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$\therefore$ The roots of equation are $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$ and $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$ we get two values of 's', so there exist two solutions for $i(t)$ which are of the form.

$$i_1(t) = B_1e^{s_1t} \quad \text{and} \quad i_2(t) = B_2e^{s_2t}$$

The complete solution can be obtained by adding $i_1(t) \times i_2(t)$ we get,

$$i(t) = B_1 e^{s_1 t} + B_2 e^{s_2 t} \qquad \dots(5)$$

From the initial condition equations of $i(0)$ and $\frac{di(0)}{dt}$, we can determine the values of B_1 and B_2.

From the roots s_1 and s_2, we can get three types of responses they are.

(i) Underdamped response ($\alpha < \omega_0$)

(ii) Critically damped response ($\alpha = \omega_0$) and

(iii) Over damped response ($\alpha > 0$).

(i) Underdamped Response ($\alpha < \omega_0$)

When $\alpha < \omega_0$, the roots s_1 and s_2 are non real complex numbers.

$$s_1, s_2 = -\alpha \pm \sqrt{-(\omega_0^2 - \alpha^2)}$$

$$[\because j^2 = -1 \Rightarrow j = \sqrt{-1}]$$

$$= -\alpha \pm \sqrt{-1}\sqrt{\omega_0^2 - \alpha^2}$$

$$= -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2}$$

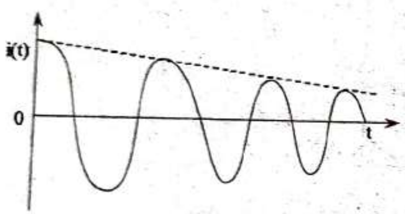


Figure (1): Underdamped Response

Where, $\sqrt{\omega_0^2 - \alpha^2} = \omega_d$ (damping frequency)

$$s_1, s_2 = -\alpha \pm j\omega_d$$

$\therefore$ The roots becomes as $s_1 = -\alpha + j\omega_d$ and $s_2 = -\alpha - j\omega_d$

Now substituting s_1 and s_2 in equation (5), we get,

$$i(t) = B_1 e^{(-\alpha + j\omega_d)t} + B_2 e^{(-\alpha - j\omega_d)t}$$

$$= B_1 e^{-\alpha t} e^{j\omega_d t} + B_2 e^{-\alpha t} e^{-j\omega_d t}$$

$$= e^{-\alpha t}(B_1 e^{j\omega_d t} + B_2 e^{-j\omega_d t}) \qquad [\because e^{j\theta} = \cos\theta + j\sin\theta \text{ and } e^{-j\theta} = \cos\theta - j\sin\theta]$$

$$= e^{-\alpha t}[B_1(\cos\omega_0 t + j\sin\omega_d t) + B_2(\cos\omega_d t - j\sin\omega_d t)]$$

$$= e^{-\alpha t}[\cos\omega_d t(B_1 + B_2) + j\sin\omega_d t(B_1 - B_2)]$$

Taking $B_1 + B_2 = D_1$, $j(B_1 - B_2) = D_2$

$$i(t) = e^{-\alpha t}[D_1 \cos\omega_d t + D_2 \sin\omega_d t]$$

For underdamped response $i(t)$ becomes as $i(t) = e^{-\alpha t}[D_1 \cos\omega_d t + D_2 \sin\omega_d t]$

(ii) Critically Damped Response ($\alpha = \omega_0$)

When $\alpha = \omega_0$, the roots s_1 and s_2 are real and equal.

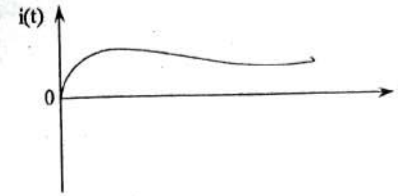


Figure (2): Critically Damped Response

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$s_1, s_2 = -\alpha$$

Where,

$$\alpha = \frac{R}{2L}$$

$\therefore$ $s_1 = -\alpha$ and $s_2 = -\alpha$

Substituting s_1 and s_2 in equation (5), we get,

$$i(t) = B_1 e^{-\alpha t} + B_2 e^{-\alpha t}$$

$$= e^{-\alpha t}(B_1 + B_2)$$

Taking $B_3 = B_1 + B_2$

$$i(t) = B_3 e^{-\alpha t}$$

This condition is not satisfied because the two initial conditions must have their individual constant.

Now taking equation (2).

$$\Rightarrow \frac{d^2 i(t)}{dt} + \frac{R}{L}\frac{di(t)}{dt} + \frac{i(t)}{LC} = 0$$

Where,

$$\alpha = \frac{R}{2L} \text{ and } \omega_0^2 = \frac{1}{LC}$$

$$\Rightarrow \frac{d^2 i(t)}{dt} + 2\alpha\frac{di(t)}{dt} + \omega_0^2 i(t) = 0$$

Where,

$$\alpha = \omega$$

$$\Rightarrow \frac{d^2 i(t)}{dt} + 2\alpha\frac{di(t)}{dt} + \alpha^2 i(t) = 0$$

$$\Rightarrow \frac{d}{dt}\left(\frac{di(t)}{dt} + \alpha i(t)\right) + \alpha\left(\frac{di(t)}{dt} + \alpha i(t)\right) = 0$$

$$\text{Let } y = \frac{di(t)}{dt} + \alpha i(t) \qquad \dots(6)$$

$$\Rightarrow \frac{dy}{dt} + \alpha y = 0$$

$$\Rightarrow \frac{dy}{dt} = -\alpha y$$

$$\Rightarrow \frac{1}{y}dy = -\alpha dt$$

Integrating on both sides, we get,

$$\Rightarrow \int \frac{1}{y}dy = \int -\alpha dt$$

$$\Rightarrow \log_e y = -\alpha t + \log_e B$$

[where, B_1 is integrating constant]

$$\Rightarrow \log_e y - \log_e B_1 = -\alpha t$$

$$\Rightarrow \log_e \frac{y}{B_1} = -\alpha t$$

$$\Rightarrow \frac{y}{B_1} = e^{-\alpha t}$$

$$y = B_1 e^{-\alpha t}$$

Substituting 'y' in equation (6), we get,

$$\Rightarrow \frac{di(t)}{dt} + \alpha i(t) = B_1 e^{-\alpha t}$$

$$\Rightarrow e^{\alpha t}\frac{di(t)}{dt} + e^{\alpha t} i(t) = B_1$$

$$\frac{d}{dt}(i(t).e^{\alpha t}) = B_1$$

$$\left[\because \frac{d}{dx}(U.V) = U\frac{dV}{dx} + V\frac{dU}{dx}\right] \qquad \dots(7)$$

Now integrating equation (7), we get,

$$e^{\alpha t} i(t) = B_1 t + B_2$$

$$i(t) = (B_1 t + B_2)e^{-\alpha t}$$

$\therefore$ For critically damped response $i(t) = (B_1 + B_2)e^{-\alpha t}$

(iii) Overdamped Response ($\alpha > \omega_0$)

When $\alpha > \omega_0$, the roots s_1 and s_2 are unequal and real.

$\therefore$ For the overdampeld response $i(t) = B_1 e^{s_1 t} + B_2 e^{s_2 t}$

Typical overdamped response shown in figure (3).

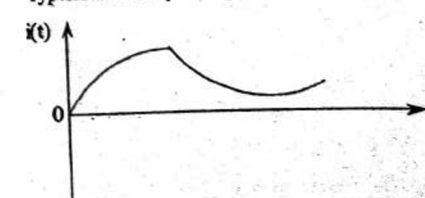


Figure (3): Overdamed Response

Q25. Derive an expression for a source free parallel RLC circuit.

Ans:

Consider parallel RLC circuit as shown in figure.

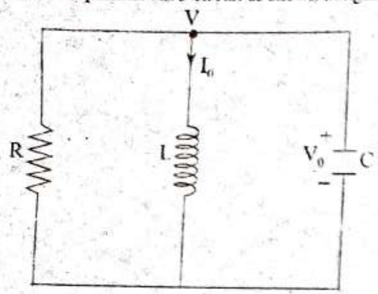


Figure: Source Free Parallel RLC Circuit

Let I_0 be the initial inductor current and V_0 be the initial capacitor voltage.

$$I_0 = i(0) = \frac{1}{L}\int_{\infty}^{0} V(t)dt$$

$$V_0 = V(0)$$

Applying KCL to the circuit, we get,

$$\frac{V}{R} + \frac{1}{L}\int_{-\infty}^{t} Vdt + C\frac{dV}{dt} = 0$$

$$\frac{V}{RC} + \frac{1}{LC}\int_{-\infty}^{t} Vdt + \frac{dV}{dt} = 0$$

Differentiating on both sides, we get,

$$\frac{1}{RC}\frac{dV}{dt} + \frac{1}{LC}V + \frac{d^2V}{dt^2} = 0$$

$$\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{1}{LC}V = 0$$

We know that,

$$\frac{d}{dt} = S$$

$$S^2 + \frac{1}{RC}S + \frac{1}{LC} = 0$$

The above expression is in quadratic form,

$$S_1, S_2 = \frac{\frac{-1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - 4(1)\left(\frac{1}{LC}\right)}}{2(1)}$$

$$= \frac{\frac{-1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}}{2}$$

$$\frac{-1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$S_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$S_2 = \frac{-1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

Let,

$$\alpha = \frac{1}{2RC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The two roots can now be expressed as,

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

The natural response in the exponential form is given by,

$$V(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

Based on the R, L and C values, then three cases are arised.

(i) Over Damped Case

The roots are real and unequal,

$\alpha > \omega_0$ or $\alpha^2 - \omega_0^2 > 0$

i.e., $L > 4R^2C$

$\therefore V(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$

(ii) Critically Damped Case

The roots are real and equal,

$\alpha = \omega_0$ or $z^2 - \omega_0^2 = 0$

i.e., $L = 4R^2C$

$\therefore V(t) = (A_1 + A_2 t)e^{-\alpha t}$

(iii) Under-damped Case

The roots are complex conjugate

$\alpha < \omega_0$ or $\alpha^2 - \omega_0^2 < 0$

i.e., $L < 4R^2C$

$S_1, S_2 = -\alpha \pm j\omega_d$

Where,

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$\therefore V(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$$

The complete response of the circuit is the sum of forced response and natural response as,

$V(t)$ = Forced response + Natural response

$V(t) = V_f(t) + V_n(t)$

Where,

$V_f(t)$ is the constant term or directly taken from the circuit.

$$V_n(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

Q26. What is the significance of time constant of R-L circuit? What are the different ways of defining time constant?

Ans: **Model Paper-II, Q5**

A RL series circuit is shown in figure (1).

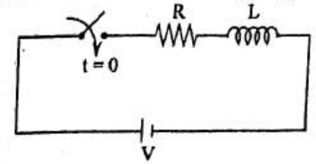


Figure (1)

In the circuit shown in figure (1), the current flowing through the circuit is given by,

$$i = \frac{V}{R}\left(1 - e^{\left(\frac{R}{L}t\right)}\right)$$

The current equation consists of two parts, the steady state part $\frac{V}{R}$ and transient part $\frac{V}{R}e^{\frac{-Rt}{L}}$.

Where, $\frac{L}{R}$ is known as time constant of R-L series circuit. The variation of current with respect to time constant is shown in the figure (2).

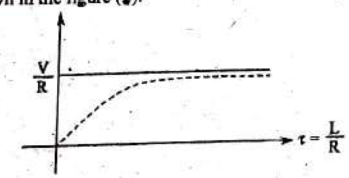


Figure (2)

In the transient part of the solution, the time constant $\frac{L}{R}$ is important in describing the curve since, $\frac{L}{R}$ is the time required for the current to reach from its initial value of zero to the final value of $\frac{V}{R}$.

The time constant of function $\frac{V}{R}e^{-(R/L)t}$ is the time at which the exponent of e is unity, denoted by τ where e is the base of the natural logarithm.

For remaining answer refer Unit-II, Q2.

Q27. Derive the expression for i(t) and voltage across capacitance $V_c(t)$ for series R-C circuit with D.C voltage applied to it at t = 0. Explain about the time constant of R-C circuit.

Ans:

R-C Series Circuit

R-C network with D.C excitation is shown in figure (1).

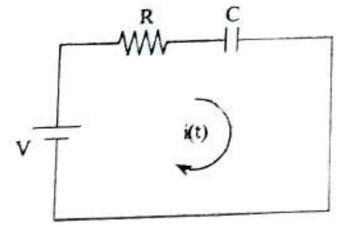


Figure (1): R-C Network with D.C. Excitation

Applying KVL, we get,

$$Ri(t) + \frac{1}{C}\int i(t)dt = V$$

Differentiating with respect to t on both sides, we get,

$$R\frac{di(t)}{dt} + \frac{i(t)}{C} = 0$$

Rearranging and using operator $\frac{di(t)}{dt} = D$, the above equation can be written as $\left(D + \frac{i(t)}{RC}\right) = 0$.

The solution for the above equation is,

$$i(t) = ke^{-t/RC}$$

Where, k is a constant and it can be calculated using initial conditions.

The circuit for calculating $i(t)(0^-)$ is shown in figure (2),

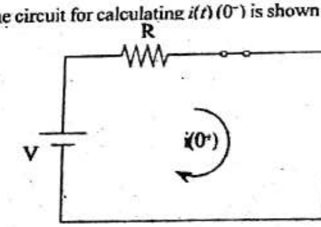


Figure (2)

$$\therefore \quad i(0^+) = \frac{V}{R}$$

At $t = 0^+$

$$i(0^+) = ke^{-0/RC}$$

$$\frac{V}{R} = k$$

Substituting the value of k in equation $i(t)$, we get,

$$i(t) = \frac{V}{R}e^{-t/RC}$$

This type of equation is known as an exponential decay, as shown in figure (3). The plot shows the transition period during which the current adjusts from its initial value of $\frac{V}{R}$ to the final value zero, the steady-state.

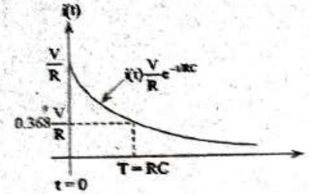


Figure (3)

The transient voltage across the resistor is,

$$V_R = \frac{V}{R}e^{-t/RC} \times R = Ve^{-t/RC}$$

The voltage across the capacitor is,

$$V_C = \text{Input voltage} - V_R$$

$$= V(1 - e^{-t/RC})$$

The resistor voltage transient is an exponential decay with the same time constant as the current, while the voltage across the capacitor is in exponential rise but with the same time constant. The plot of V_R and V_C is shown in figure (4).

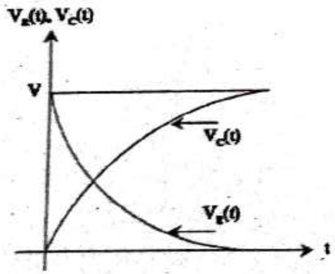


Figure (4)

We know that the time constant is equal to the L/R in R-L circuit. But in the R-C network the time constant τ is proportional to the R. In case of R-L it is inversely proportional to R. Hence, by changing the values of R and C we can change the time constant.

Q28. Explain the D.C response of a second order R-L-C series circuit.

Ans:

Consider a series RLC circuit shown in figure (1).

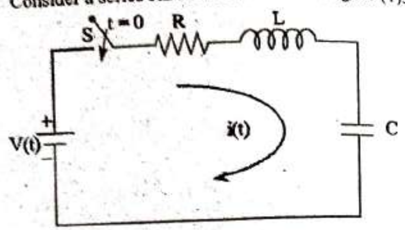


Figure (1): Series RLC Circuit

$V(t)$ is a unit step voltage applied to the series RLC circuit.

Let $i(t)$ be the current flowing through the circuit. At $t = 0$, the switch is closed. Let us assume that the initial current through inductor and initial voltage across capacitor is zero i.e.,

$$i(0^+) = i(0^-) = 0 \text{ and } V_c(0^+) = V_c(0^-) = 0$$

Applying KVL to the circuit, we get,

$$V(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{c}\int i(t)dt \quad \dots (1)$$

Differentiating equation (1) on both sides with respect to 't' we get,

$$\frac{d}{dt}[V(t)] = \frac{d}{dt}\left[Ri(t) + L\frac{d}{dt} + \frac{1}{C}\int i(t).dt\right]$$

$$\Rightarrow \quad R\frac{d}{dt}i(t) + L\frac{d^2}{dt^2}i(t) + \frac{1}{C}i(t) = 0$$

$$\Rightarrow \quad L\frac{d^2}{dt^2}i(t) + R\frac{d}{dt}i(t) + \frac{1}{C}i(t) = 0$$

$$\Rightarrow \quad \frac{d^2}{dt^2}i(t) + \frac{R}{L}\frac{d}{dt}i(t) + \frac{1}{LC}i(t) = 0$$

Let,

$$D = \frac{d}{dt}$$

$$\therefore \quad D^2[i(t)] + \frac{R}{L}D[i(t)] + \frac{1}{LC}[i(t)] = 0$$

$$\Rightarrow \quad \left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)i(t) = 0 \quad \dots (2)$$

Equation (2) is a second order differential equation. Hence, the complete solution is given as,

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right) = 0 \quad \dots (3)$$

The roots of the above equation are,

$$D_1, D_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$D_1, D_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Based on the relation between $\left(\frac{R}{2L}\right)^2$ and $\frac{1}{LC}$ we get three types of roots and hence, three types of response.

Over Damping Case : $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

When $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$, the roots will be real and unequal and the system is said to be over-damped.

The response of the system can be determined as,

$$\left[D - \left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right)\right]\left[D - \left(-\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right)\right]i = 0$$

Hence, the solution in case of over-damped is given as,

$$i(t) = K_1 e^{\left(\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right)t} + K_2 e^{\left(\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right)t}$$

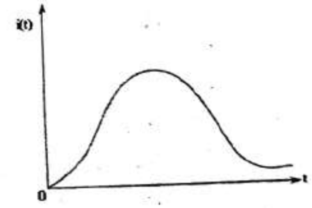


Figure (2)

Figure (2) shows the time response when the system is over-damped.

Critically Damping Case: $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

When $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$, the roots of the system will be real and equal and the system is said to be critically damped.

The response of the system is given as,

$$\left[D - \left(-\frac{R}{2L}\right)\right]\left[D - \left(-\frac{R}{2L}\right)\right]i = 0$$

Hence, the solution in case of critical damping is given as,

$$i = e^{-\left(\frac{R}{2L}\right)t}(K_1 + K_2 t)$$

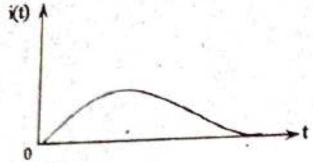


Figure (3)

Figure (3) shows the time response of the critically damped system.

Under Damped Case: $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

When $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$, the roots of the system will be complex conjugate and the system is said to be under-damped system.

The response of the system can be determined as,

$$\left[D-\left(\frac{-R}{2L}+j\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}\right)\right]\left[D-\left(\frac{-R}{2L}-j\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}\right)\right]i=0$$

Hence, the solution for underdamped response is given as,

$$i=e^{-\frac{R}{2L}t}\left[K_1\cos\left[\left(\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}\right)t\right]+K_2\sin\left[\left(\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}\right)t\right]\right]$$

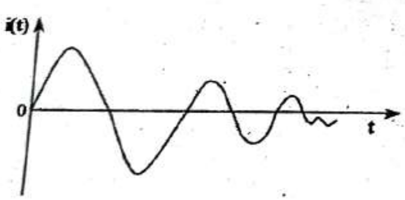


Figure (4)

The response of the under-damped system is shown in figure (4).

Voltage across resistor, R

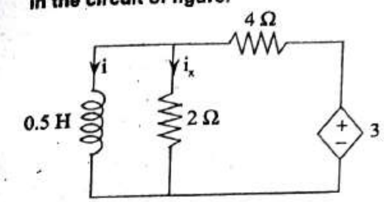
$$V_R = i(t)\times R$$

$$=\left[e^{-\frac{R}{2L}t}\left[K_1\cos\left[\left(\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}\right)t\right]\right]+K_2\sin\left[\left(\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}\right)t\right]\right]\times R$$

$$V_R = Re^{-\frac{R}{2L}t}\left[K_1\cos\left[\left(\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}\right)t\right]\right]+K_2\sin\left[\left(\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}\right)t\right] \quad \text{... (6)}$$

$\therefore$ Voltage across resistor for the given RLC series circuit is given by equations (4), (5) and (6).

Q29. Assuming that I(0) = 10 A, calculate I(t) and I_x(t) in the circuit of figure.



Figure

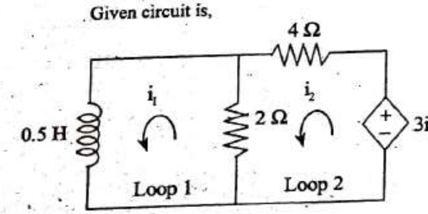
Ans:

Given that,

Cement at $t = 0$, $i(0) = 10$ A

Determine $i(t)$ and $i_x(t)$

Given circuit is,



Figure

Applying KVL to loop 1, we get,

$$0.5\frac{di_1}{dt}+2(i_1-i_2)=0$$

$$\frac{1}{2}\frac{di_1}{dt}+2i_1-2i_2=0$$

$$\frac{di_1}{dt}+4i_1-4i_2=0 \quad \text{... (1)}$$

Applying KVL to loop 2, we get,

$\Rightarrow \quad 2(i_2-i_1)-3i_1+4i_2=0$

$\Rightarrow \quad 2i_2-2i_1-3i_1+4i_2=0$

$\Rightarrow \quad 6i_2-5i_1=0$

$\Rightarrow \quad i_2=\frac{5i_1}{6}$

Substituting 'i_2' in equation (1), we get,

$$\frac{di_1}{dt}+4i_1-4\left(\frac{5i_1}{6}\right)=0$$

$$\frac{di_1}{dt}+4i_1-\frac{10i_1}{3}=0$$

$$\frac{di_1}{dt}+\frac{2i_1-10i_1}{3}=0$$

$$\frac{di_1}{dt}+\frac{2i_1}{3}=0$$

$$\frac{di_1}{dt}=\frac{-2i_1}{3}$$

$$\frac{1}{i_1}di_1=\frac{-2}{3}dt \quad \text{... (2)}$$

Integrating the equation (2), we get,

$\Rightarrow \quad \int_{i(0)}^{i(t)}\frac{1}{i_1}di_1 = \frac{-2}{3}\int_0^t dt$

$\Rightarrow \quad [\log i_1]_{i(0)}^{i(t)} = \frac{-2}{3}[t]_0^t$

$\Rightarrow \quad \log i(t)-\log i(0) = \frac{-2}{3}t$

$$\log_e\frac{i(t)}{i(0)} = \frac{-2t}{3}$$

$$\frac{i(t)}{i(0)} = e^{\frac{-2t}{3}}$$

$$i(t) = i(0)e^{\frac{-2t}{3}}$$

Given $i(0) = 10$ A

$$\boxed{i(t)=10e^{\frac{-2t}{3}}\text{ Amp}} \quad t>0$$

Now, differentiating $i(t)$, we get,

$$\frac{di}{dt}=10\left(\frac{-2}{3}\right)e^{\frac{-2}{3}t}$$

Voltage across the inductor is given as,

$$V=L\frac{di}{dt}=\frac{1}{2}\times10\left(\frac{-2}{3}\right)e^{\frac{-2t}{3}}$$

$$=\frac{-10}{3}e^{\frac{-2t}{3}}=-3.33\,e^{\frac{-2t}{3}}$$

Current across the resistor $i_x(t)$ is given as,

$$i_x(t)=\frac{V}{2}=\frac{-3.33}{2}e^{\frac{-2t}{3}}$$

$$\boxed{i_x(t)=-1.666e^{\frac{-2t}{3}}\text{ Amp}} \quad t>0$$

$\therefore$ Current, $i(t) = 10e^{\frac{-2t}{3}}$ Amps and

$$i_x(t) = -1.666\,e^{\frac{-2t}{3}}\text{ Amps}$$

The 4 Ω and 8 Ω resistor are in series so, 4 + 18 = 12 Ω,

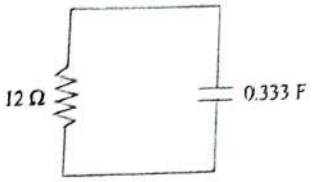


Figure (3)

The voltage across the capacitor is given as,

$$V = V_c(0)\ e^{\frac{-t}{RC}}$$

$$= 30e^{\frac{-t}{12\times0.333}}$$

$$= 30e^{\frac{-t}{3.99}} = 30e^{-0.25t}\ \text{V}$$

$$\boxed{V = V_C = 30e^{-0.25t}}$$

∴ The voltage across capacitor is $V_C = V = 30e^{-0.25t}$ V

Now calculating V_x by applying voltage division rule, we get,

$$V_x = \frac{4}{8+4}V_C$$

$$= \frac{1}{3}\times30e^{-0.25t}$$

$$\boxed{\therefore V_x = 10e^{-0.25t}\ \text{V}}$$

Current,

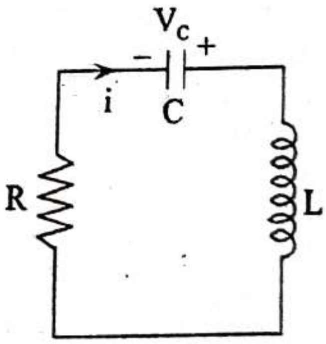
$$i_0 = \frac{V_x - V_C}{R}$$

$$= \frac{10e^{-0.25t} - 30e^{-0.25t}}{8}$$

$$= \frac{-20}{8}e^{-0.25t}$$

$$\boxed{\therefore i_0 = -2.5e^{-0.25t}\ \text{A}}$$

Q32. Given the series RLC circuit shown in figure in which L = 1 H, R = 2 kΩ, C = 1/401 μF, i(0) = 2 mA and $V_c(0)$ = 2 V, find i(t) for t > 0.



Figure

Ans:

Given that,

Source free series RLC circuit,

Resistance, $R = 2$ kΩ

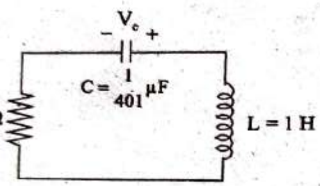
Inductance, $L = 1$ H

Capacitance, $C = \frac{1}{401}\mu F$

Inductor current at $t = 0$, $i(0) = 2$ mA

Capacitor voltage at $t = 0$, $V_c(0) = 2V$

Given circuit is,



Figure

To know the response of the circuit first we have to calculate the values of,

$$\alpha = \frac{R}{2L} \quad \text{and} \quad \omega_o = \frac{1}{\sqrt{LC}}$$

$$\alpha = \frac{2\times1000}{2\times1} \quad \text{and} \quad \omega_o = \frac{1}{\sqrt{1\times\frac{10^{-6}}{401}}}$$

$$\alpha = 1000\ s^{-1} \quad \text{and} \quad \omega_o = 20024.98 \text{ rad/sec}$$

Here $\alpha < \omega_o$, so the response is under-damped.

For under-damped response the $i(t)$ is given as,

$$i(t) = e^{-\alpha t}[D_1 \cos \omega_d t + D_2 \sin \omega_d t]$$

Where,

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{(20024.98)^2 - (1000)^2}$$

$$= 20000 \text{ rad/sec}$$

$$i(t) = e^{-1000t}(D_1 \cos 20000\,t + D_2 \sin 20000t) \text{ A} \quad \text{... (1)}$$

At $t = 0$

$\Rightarrow \quad i(0) = e^{-0}(D_1 \cos(0) + D_2 \sin(0)]$ A

$\Rightarrow \quad 0.002 = 1(D_1 + 0)$ A

$\Rightarrow \quad D_1 = 0.002$ A

Substituting D_1 in equation (1), we get,

$$i(t) = e^{-1000t}(0.002 \cos 2000t + D_2 \sin 20000\,t)\text{A} \quad \text{... (2)}$$

Differentiating equation (2) with respect to 't' we get,

$$\frac{di(t)}{dt} = [e^{-1000t}(-0.002\times20000\sin 20000t + D_2 20000\cos 20000t)$$

$$(-1000)(0.002\cos 20000t + D_2\sin 20000t)e^{-1000t}]$$

$$= e^{-1000t}[-40\sin 20000t + 20000D_2\cos 20000t - 2\cos 20000t - D_2 1000\sin 20000t]$$

Now applying initial condition $t = 0$ to differentiating value, we get,

$$\Rightarrow \frac{di(0)}{dt} = e^{-0}(0 + 20000\,D_2 - 2 - 0)$$

$$\left[\because L\frac{di}{dt} = V_L \Rightarrow \frac{di}{dt} = \frac{V_L}{L}\right]$$

$$\Rightarrow \frac{V_L(0)}{L} = 1(20000\,D_2 - 2)$$

$$\Rightarrow 20000\,D_2 - 2 = \frac{V_C(0) - Ri(0)}{L}$$

$$\Rightarrow 20000\,D_2 - 2 = \frac{2 - 2000\times2\times10^{-3}}{1}$$

$$\Rightarrow 20000\,D_2 - 2 = -2$$

$$\Rightarrow 20000\,D_2 = -2 + 2$$

$$D_2 = 0$$

Substituting $D_2 = 0$ in equation (2), we get,

Therefore, the desired response is,

$$i(t) = e^{-1000t}(0.002 \cos 20000\,t + 0)\text{A}$$

$$i(t) = 0.002\,e^{-1000t} \cos 20000t \text{ A}$$

Q33. A parallel RLC circuit contains a 100 Ω resistor and has the parameter values $\alpha = 1000\ S^{-1}$ and $\omega_o = 800$ rad/s. Find,

(a) C

(b) L

(c) S_1

(d) S_2.

Ans:

Given that,

$R = 100\ \Omega$

$\alpha = 1000\ s^{-1}$

$\omega_o = 800$ rad/sec

(a) We know that,

$$\alpha = \frac{1}{2RC}$$

$$1000 = \frac{1}{2\times100\times C}$$

$$C = \frac{1}{2\times100\times1000}$$

$$\boxed{C = 5\ \mu F}$$

(b) We have,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\sqrt{LC} = \frac{1}{\omega_0}$$

$$\sqrt{L} = \frac{1}{\sqrt{C}\times\omega_0}$$

$$\Rightarrow L = \frac{1}{C\omega_0^2}$$

$$L = \frac{1}{5\times10^{-6}\times800^2}$$

$$\boxed{L = 312.5 \text{ mH}}$$

(c) We have,

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$= -1000 + \sqrt{(1000)^2 - (800)^2}$$

$$S_1 = -400\ s^{-1}$$

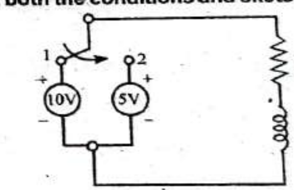
(d) We have,

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$= -1000 - \sqrt{(1000)^2 - (800)^2}$$

$$S_2 = -1600\ s^{-1}$$

Q34. In figure, the switch is close at position 1 at t = 0. At t = 0.5 m sec, the switch is moved to position 2. Find the expression for the current in both the conditions and sketch the transient.



Figure

Ans: The given circuit is,

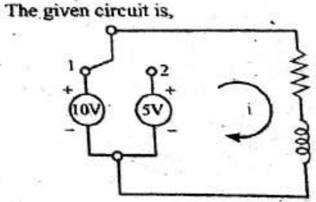


Figure (1)

Let, current i be flowing through the circuit when the switches are close.

(i) When the Switch is at Position 1

$$50i + 0.5\frac{di}{dt} = 10$$

$$\Rightarrow \quad \frac{di}{dt} + 100i = 20$$

$$\Rightarrow \quad (D+100)i = 20 \qquad \text{... (1)}$$

The complete solution of equation (1) includes the complementary function (i_c) and particular solution (i_p).

$$\boxed{\therefore i = i_c + i_p}$$

But, $i_c = c_1 e^{-100t}$ and

$$i_p = e^{-(R/L)t}\int e^{(R/L)t}\left(\frac{V}{L}\right)dt$$

$$i_p = e^{-(R/L)t} \times e^{(R/L)t} \times \frac{L}{R} \times \frac{V}{L}$$

$$\Rightarrow \quad i_p = \frac{V}{R}$$

$$\therefore \quad i = c_1 e^{-100t} + \frac{V}{R}$$

$$\Rightarrow \quad i = c_1 e^{-100t} + \frac{10}{50} \qquad [\text{ Given } V = 10, R = 50]$$

$$\Rightarrow \quad i = ce^{-100t} + 0.2$$

At $t = 0$, $i = 0$,

$$0 = c_1 + 0.2$$

$$c_1 = -0.2$$

$$\therefore \quad i = -0.2e^{-100t} + 0.2$$

$$= 0.2(1 - e^{-100t})$$

At $t = 0.5$ msec,

$$i = -0.2\,e^{-100(0.5\times10^{-3})} + 0.2$$

$$i = 9.754 \times 10^{-3}$$

$$i = 9.754 \text{ mA}$$

(ii) When the Switch is at Position 2

$$50i + 0.5\frac{di}{dt} = 5$$

$$\Rightarrow \quad \frac{di}{dt} + 100 = 10$$

$$\Rightarrow \quad (D+100)i = 10 \qquad \text{... (2)}$$

$$\boxed{\therefore i = i_c + i_p}$$

$$i_c = c_2 e^{-100(t-t_1)} + 0.1$$

$$i_p = \frac{V}{R} = \frac{5}{50} = 0.1$$

$$\Rightarrow \quad i = c_2 e^{-100(t-t_1)} + 0.1$$

At $t = t_1$, $i = 9.75$ mA

$$\Rightarrow \quad 9.75 \times 10^{-3} = c_2 + 0.1$$

$$\Rightarrow \quad c_2 = -0.09$$

$$\therefore \quad i = -0.09\,e^{-100(t-t_1)} + 0.1$$

It is to be noted that when the switch is at position 1, the steady state current is 0.2 A. But, at $t = t_1$ i.e., when the switch is moved to position 2 then the current will move for the final value of 0.1 A.

The transient for this problem is shown in figure below.

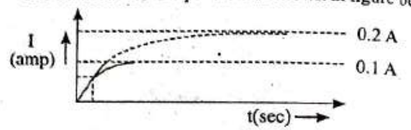


Figure (2)

Q35. A constant voltage of 100 V is applied at t = 0 to a series R-C circuit having R = 5 MΩ and C = 20 µF. Assuming no initial charge to the capacitor, find the expression for i, voltage across R and C.

Ans:

Figure (1) shows the circuit drawn according to the given information.

Let $i(t)$ be the current through the circuit after the switch is closed at $t = 0$.

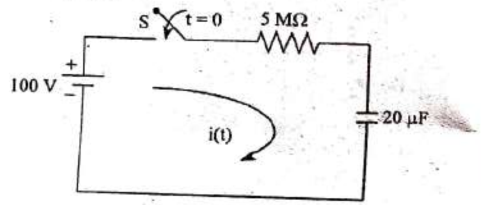


Figure (1)

(i) Expression for Current

Applying KVL to the circuit, we get,

$$100 = 5\times10^6 i(t) + \frac{1}{20\times10^{-6}}\int i(t)dt \qquad \text{... (1)}$$

Differentiating equation (1), with respect to t, we get,

$$0 = 5\times10^6\frac{di}{dt} + \frac{i}{20\times10^{-6}}$$

$$\frac{di}{dt} + 0.01i = 0$$

$$(D + 0.01)i = 0 \qquad \text{... (2)}$$

The solution for equation (2) is given by,

$$i(t) = Ce^{-0.01t} \qquad \text{... (3)}$$

At $t = 0^+$, i.e., immediately after closing the switch, capacitor behaves as short circuit as shown in figure (2),

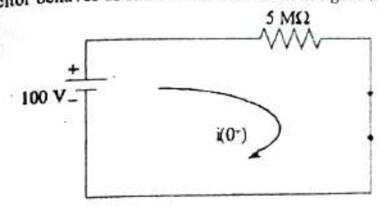


Figure (2)

The current through the circuit is now given as,

$$i(0^+) = \frac{100}{5\times10^6}$$

$$= 20\times10^{-6} \text{ A}$$

Substituting the value of $i(0^+)$ in equation (3), we get,

At $t = 0^+$,

$$i(0^+) = 20\times10^{-6}$$

$$i(0^+) = Ce^0$$

$$C = i(0^+)$$

$$C = 20\times10^{-6}$$

Substituting the value of C in equation (3), we get,

$$i(t) = 20\times10^{-6}e^{-0.01t}$$

(ii) Voltage Across R

$$V_R = i(t)R$$

$$= 20\times10^{-6}e^{-0.01t}\times5\times10^6$$

$$= 100\,e^{-0.01t}$$

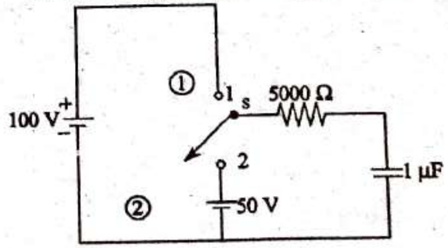
(iii) Voltage Across C

$$V_C = V - V_R$$

$$= 100 - 100e^{-0.01t}$$

$$= 100(1 - e^{-0.01t})$$

Q36. Switch is moved from position 1 to 2 at t = 0. Find the voltages $V_R(t)$ and $V_C(t)$ for t ≥ 0.



Figure

Ans:

When the switch is at position 1, the circuit has been reached to steady state.

The circuit for $t < 0$ is.

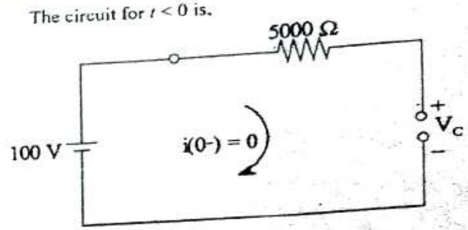


Figure (1)

$$V_C(0^-) = 100 \text{ V}$$

When the switch is at position 2 i.e., at $t = 0^+$ the circuit is.

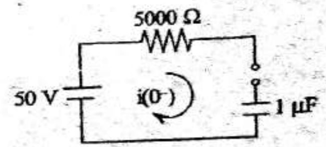


Figure (2)

$$i(0^+) = \frac{50-100}{5000}$$

$$= -0.01 \text{ A} \qquad [\because \text{ Now } V_c = 100 \text{ V}]$$

At $t > 0$ the circuit is shown in figure (3).

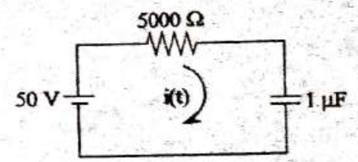


Figure (3)

Applying KVL, we get,

$$50 = 5000\,i(t) + \frac{1}{10^{-6}}\int i(t)dt$$

Differentiating with respect to t on both sides, we get,

$$0 = 5000\frac{di(t)}{dt} + \frac{i}{10^{-6}}$$

Rearranging and using the operator,

$$\frac{di}{dt} = D, \text{ we get,}$$

$$(D + 200)i = 0$$

Solution for this equation is $i(t) = ke^{-200t}$

At $t = 0^+$ $i(0^+) = k.e^{-200\times 0}$

$k = -0.01$

$\therefore \quad i(t) = -0.01\, e^{-200t}$

$V_R(t) = i(t) \times R$

$= -0.01e^{-200t} \times 5000$

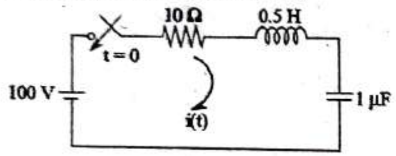
$= -50\, e^{-200t}$ V

$V_C(t) = V(t) - V_R(t)$

$= 50 + 50\, e^{-200t}$

$= 50\,(1 + e^{-200t})$ V

Q37. Obtain the current i(t) for t ≥ 0, using time domain approach.



Figure

Ans:

At, $t = 0^-$

$i(0^+) = 0$ and

$V_c(0^-) = 0$

Applying KVL, we have,

$$100 = 10i + 0.5\frac{di}{dt} + \frac{1}{1\times10^{-6}}\int i dt$$

At $t = 0^-$,

$$100 = 0 + 0.5\frac{di}{dt}(0^-) + 0$$

$$\therefore \quad \frac{di}{dt}(0^+) = \frac{100}{0.5} = 200 \text{ A/sec}$$

$$100 = 0 + 0.5\,\frac{di}{dt} + \frac{1}{10^{-6}}\int i dt$$

Differentiating with respect to t on both sides, we get,

$$\frac{10di}{dt} + \frac{0.5d^2i}{dt^2} + \frac{i}{10^{-6}} = 0$$

$$(D^2 + 20D + 2\times 10^6)i = 0$$

The roots are,

$$D = \frac{-20 \pm \sqrt{20^2 - 4\times2\times10^6\times1}}{2}$$

$$= -10 \pm j1414.178$$

The solution for the above differential equation is,

$\therefore \quad i(t) = e^{-10t}\,[C_1 \cos 1414.178t + C_2 \sin 1414.178t]$ ... (1)

At, $t = 0$, $i(0) = e^{-0}\,[C_1 \cos 0 + C_2 \sin 0]$

$\therefore\ 0 = C_1$

Substituting the value of C_1 in equation (1), we get,

$i(t) = e^{-10t}\, C_2 \sin 1414.178t$ ... (2)

Differentiating equation (2), we get,

$= C_2\times 1414.178 \cos 1414.178t + (-10)\, e^{-10t} \sin 1414.178t]$

At, $t = 0$,

$$t = \frac{di(0)}{dt}$$

$= C_2\,[e^{-0} \times 1414.178 \cos 0^\circ + (-10)\, e^{-0} \sin 0]$

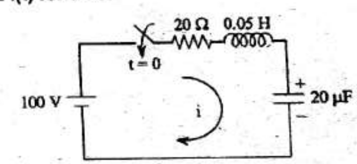
$200 = 1414.178\, C_2$

$$\therefore \quad C_2 = \frac{200}{1414.178} = 0.1414$$

Substituting the value of C_2 in equation (2), we get,

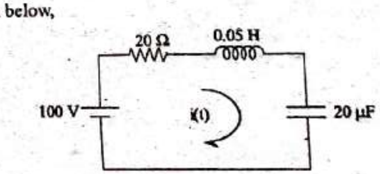
$i(t) = 0.1414\, e^{-10t} \sin 1414.178t$ Amp

Q38. Using classical method find i(t) for t = 0.



Figure

Ans: The circuit after $t > 0$ is shown below,



Figure

Applying KVL to the circuit, we get,

$$100 = 20i + 0.05\,\frac{di}{dt} + \frac{1}{20\times10^{-6}}\int i dt$$

Differentiating with respect to time (t), we get,

$$20\,\frac{di}{dt} + 0.05\,\frac{d^2i}{dt^2} + \frac{i}{20\times10^{-6}} = 0$$

$$(D^2 + 400\,D + 10^6)\,i = 0$$

The roots are,

$$D = \frac{-400 \pm \sqrt{400^2 - 4\times1\times10^6}}{2} \qquad \left[\because \frac{-b\pm\sqrt{b^2-4ac}}{2a}\right]$$

$$D_1 = -200 + j979.79$$

$$D_2 = -200 - j979.79$$

The roots are complex conjugate.

The solution for the above differential equation is,

$$\therefore \quad i(t) = e^{-200t}[C_1 \cos 979.79\,t + C_2 \sin 979.79\,t]$$

At, $t = 0$,

$$i(0) = 0$$

$$0 = e^{-0}[C_1 \cos 0 - C_2 \sin 0]$$

$$\therefore \quad C_1 = 0$$

$$i(t) = e^{-200t} C_2 \sin 979.79\,t$$

Differentiating $i(t)$ with respect to 't' we get,

$$\frac{di(t)}{dt} = C_2\,[[(-200)\,e^{-200t} \sin 979.79t] + e^{-200t} \cos 3979.79t \times 979.79] \qquad \text{... (1)}$$

$\frac{di(t)}{dt}$ at $t = 0$ can be calculated as,

$$100 = 20\,i(t) + 0.05\,\frac{di(t)}{dt} + \frac{1}{20\times10^{-6}}\int i(t)dt$$

At, $t = 0^+$,

$$100 = 20 \times 0 + 0.05\,\frac{di(0^+)}{dt} + V_c(0^-) \qquad (\because i(0^+) = 0)$$

$$100 = 0.05\,\frac{di(0^+)}{dt} + 0 \qquad (\because V_c(0^+) = 0)$$

$$\therefore \quad \frac{di(0^+)}{dt} = \frac{100}{0.05} = 2000 \text{ A/sec}$$

In equation (1), substituting the value of,

$\frac{di(0^+)}{dt} = 2000$, we get,

$$\frac{di(0^+)}{dt} = c_2\,[0 + e^{-0} \cos 0^\circ \times 979.79]$$

$$2000 = 979.79\,c_2$$

$$\therefore \quad c_2 = \frac{2000}{979.79} = 2$$

$$\therefore \quad i(t) = e^{-200t}\,[2 \sin 979.79t]\text{ A}$$

Q39. Find the expression for current of a series RLC circuit fed by constant D.C voltage of 20 V with R = 4 Ω, L = 1 H and C = ¼ F. Assume initial conditions to be zero.

Ans:

Given that,

In a series RLC circuit,

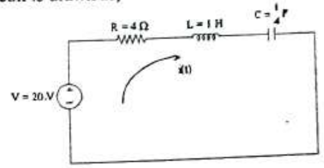
Votlage, $V = 20$ V

Resistance, $R = 4\ \Omega$

Inductance, $L = 1$ H

Capacitance, $C = \frac{1}{4}$ F

According to the given data, the circuit is drawn as,



Figure

Assuming initial conditions to be zero.

i.e., $i_L(0^+) = 0$ and

$V_C(0^+) = 0$

Applying kVL to the circuit, we have,

$$V = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$$

$$20 = 4i(t) + 1\frac{di(t)}{dt} + \frac{1}{\frac{1}{4}}\int i(t)dt$$

$$20 = 4i(t) + \frac{di(t)}{dt} + 4\int i(t)dt \qquad \text{... (1)}$$

Differentiating with respect to t on both sides, we get,

$$0 = 4\frac{di(t)}{dt} + \frac{d^2i(t)}{dt} + 4i(t)$$

$$\frac{d^2i(t)}{dt} + 4\frac{di(t)}{dt} + 4i(t) = 0$$

$$\left(\frac{d^2}{dt} + 4\frac{d}{dt} + 4\right)i(t) = 0$$

$$(D^2 + 4D + 4)i(t) = 0$$

The roots are,

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(4)}}{2(1)}$$

$$D = -2, -2$$

The roots are real and equal. Hence the solution for the differential equation is given by,

$$i(t) = C_1e^{-2t} + tC_2e^{-2t} \quad \text{... (2)}$$

At $t = 0$, $i(0) = 0$

$$\therefore \quad 0 = C_1e^0 + (0)C_2e^0$$

$$\boxed{C_1 = 0}$$

Substituting C_1 in equation (2), we get,

$$i(t) = 0e^{-2t} + tC_2e^{-2t}$$

$$i(t) = tC_2e^{-2t} \quad \text{... (3)}$$

Differentiating equation (3), we get,

$$\frac{di(t)}{dt} = C_2(t.e^{-2t}(-2) + e^{-2t}(1))$$

$$= C_2(-2te^{-2t} + e^{-2t})$$

At $t = 0$,

$$\frac{di(0)}{dt} = C_2(-2(0)e^0 + e^0)$$

$$\frac{di(0)}{dt} = C_2 \quad \text{... (4)}$$

Substituting $t = 0$ in equation (1), we get,

$$20 = 4i(0) + \frac{di(0)}{dt} + 4\int i(0)dt$$

$$20 = 0 + \frac{di(0)}{dt} + 0 \qquad [\because i(0) = 0]$$

$$\frac{di(0)}{dt} = 20$$

Substituting $\frac{di(0)}{dt}$ value in equation (4), we get,

$$\boxed{C_2 = 20}$$

Substituting C_2 value in equation (3), we get,

$$i(t) = 20te^{-2t}\text{ A}$$

3 SINUSOIDAL STEADY STATE ANALYSIS

PART-A

SHORT QUESTIONS WITH SOLUTIONS

Q1. Define waveform and give the properties of alternating quantities.

Ans:

Waveform : The plot of instantaneous values of voltage and current with respect to time is known as waveform. An ideal voltage or current waveform of an alternating quantity is shown in figure (1).

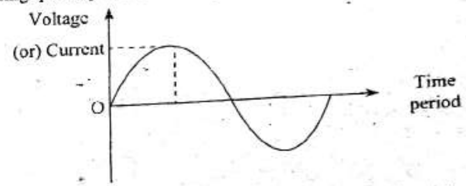


Figure (1): Ideal Sinusoidal Wave of Voltage or Current

Similarly alternating quantities having different waveforms are shown in figure (2) and figure (3)

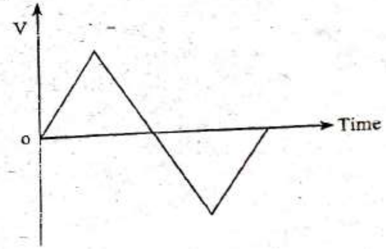


Figure (2): Triangle Waveforms

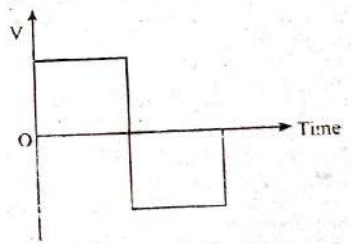


Figure (3): Square Waveforms

Properties of an Alternating Quantity

1. It has variable magnitude.
2. It is bidirectional in nature.
3. It has finite frequency.
4. It is time dependent.

Q2. Define the terms,
(a) Frequency
(b) Form factor.

Ans: Model Paper-I, Q1(e)

(a) Frequency : Frequency is defined as the ratio of number of cycles per second. It is denoted by letter 'f'. Mathematically, it is given as;

$$\text{Frequency, } f = \frac{\text{Number of cycles}}{\text{Second}}$$

(or)

$$f = \frac{1}{\text{Time period}}$$

(or)

$$f = \frac{Np}{120}$$

Where,

p = Number of poles

N = Speed of rotor.

It is represented in Hertz or cycles/ second.

(b) Form Factor

It is defined as the ratio of R.M.S value to average value of wave. Mathematically it is given as,

$$\text{Form factor} = \frac{\text{R. M .S value}}{\text{Average value}}$$

Q3. Define angular velocity and express the value in terms of frequency.

Ans:

Angular velocity is defined as the rate of change of angular displacement, which is a vector quantity specifying angular speed of an object. It is measured in radians per second. On the other hand, the angular frequency is defined as the magnitude of the vector quantity " angular velocity". Hence, from the above, the expression for angular velocity interms of frequency is given as,

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$\boxed{\therefore f = \frac{1}{\text{Time period } (T)}}$$

Q4. Write short notes on phasor representation of alternating quantities.

Ans:

In general, it is tedious and time consuming to perform manipulations of alternating quantities using waveforms and mathematical equations. In order to overcome such problems, a graphical method is developed in which an alternating quantity is represented by a rotating vector. The rotating vector follows the counter clockwise direction. The phasor representation of alternating quantities is shown in figure.

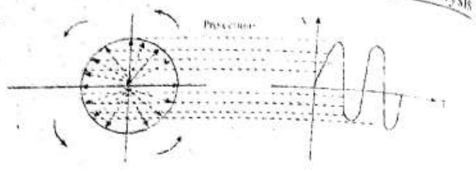


Figure: Phasor Representation of Alternating Quantities

Q5. Define average value and RMS value.

Ans:

Average Value : It is the value of direct current which gives same amount of charge to the network in same time as given by the alternating current to the same electrical network. For any alternating quantity, the average value is given as.

$$\text{Average value} = \frac{1}{T}\int_0^T f(t) \, dt$$

R.M.S Value : It is defined as the square root of the average of the squares of its instantaneous values over one complete cycle. In general, for an alternating quantity, the R. M. S value is given as,

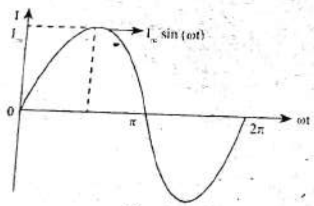
$$\text{R.M.S value} = \sqrt{\frac{1}{T}\int_0^T f^2(t) \, dt}$$

Q6. Give the formula for average value of sinusoidal current.

Ans:

Average Value of Sinusoidal Current

The sinusoidal current waveform is shown in figure below.



Figure

We know that,

The equation of sinusoidal current is given as.

$$I = I_m \sin(\omega t)$$

Now,

The average value of sinusoidal current is given as.

$$I_{avg} = \frac{1}{(\pi - 0)}\int_0^{\pi} i(t) \, . \, d(\omega t)$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin(\omega t) \, . \, d(\omega t) \qquad [\because I = i(t)]$$

$$= \frac{I_m}{\pi}\left[-\cos(\omega t)\right]_0^{\pi} \qquad \left[\because \int \sin\theta \, . \, d\theta = -\cos\theta\right]$$

$$= -\frac{I_m}{\pi}\left[\cos(\omega t)\right]_0^{\pi}$$

$$= \frac{-I_m}{\pi}\left[\cos(\pi) - \cos(0)\right]$$

$$= \frac{-I_m}{\pi}\left[-1-1\right]$$

$$= \frac{-I_m}{\pi}(-2)$$

$$= \frac{2}{\pi}I_m$$

$$= 0.637 \, I_m$$

Average value = 0.637 × Maximum current (I_m).

Q7. Derive the R.M.S value of an alternating quantity.

Ans:

R.M.S Value of Sine Wave (Alternating Quantity)

The sinusoidal alternating quantity is shown in figure below,

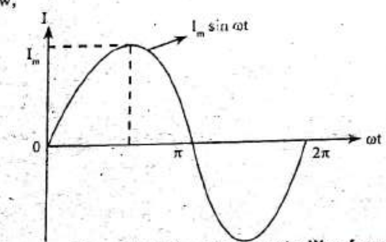


Figure: Sinusoidal Alternating Quantity Waveform

Now,

The R.M.S value of a sinusoidal waveform is given as,

$$\text{R. M. S} = \sqrt{\int_0^{\pi} \frac{i^2(t)}{\pi - 0} \, . \, d(\omega t)}$$

$$= \sqrt{\int_0^{\pi} \frac{I_m^2 \sin^2(\omega t)}{\pi} \, d(\omega t)}$$

$$= I_m \sqrt{\frac{1}{\pi}\int_0^{\pi} \sin^2(\omega t) \, d(\omega t)}$$

$$= I_m \sqrt{\frac{1}{\pi}\left\{\int_0^{\pi}\left[1 - \cos^2(\omega t)\right] d(\omega t)\right\}}$$

$$= I_m \sqrt{\frac{1}{\pi}\left\{\int_0^{\pi}\left[d(\omega t) - \cos^2(\omega t) \, d(\omega t)\right]\right\}}$$

$$= I_m \sqrt{\frac{1}{\pi}\left\{\int_0^{\pi} d(\omega t) - \int_0^{\pi} \cos^2(\omega t) \, d(\omega t)\right\}}$$

$$= I_m \sqrt{\frac{1}{\pi}\left\{[\omega t]_0^{\pi} - \int_0^{\pi} \frac{1 + \cos(2\omega t)}{2} \, . \, (d\omega t)\right\}}$$

$$= I_m \sqrt{\frac{1}{\pi}\left\{(\pi - 0) - \frac{1}{2}\int_0^{\pi}(1 + \cos(2\omega t)) \, d\omega t\right\}}$$

$$= I_m \sqrt{\frac{1}{\pi}\left[\pi - \frac{1}{2}\left[\int_0^{\pi} d(\omega t) + \int_0^{\pi} \cos(2\omega t) \, . \, d\omega t\right]\right]}$$

$$= I_m \sqrt{\frac{\pi}{\pi} - \frac{1}{2\pi}\left[[\omega t]_0^{\pi} + \left(\frac{\sin 2\omega t}{2}\right)_0^{\pi}\right]}$$

$$= I_m \sqrt{1 - \frac{1}{2\pi}\left[(\pi - 0) + \frac{1}{2}(\sin(2\pi) - \sin(0))\right]}$$

$$= I_m \sqrt{1 - \frac{1}{2\pi}\left[\pi + \frac{1}{2}(0 - 0)\right]}$$

$$= I_m \sqrt{1 - \frac{1}{2\pi}\left[\pi + 0\right]}$$

$$= I_m \sqrt{1 - \frac{\pi}{2\pi}}$$

$$= I_m \sqrt{1 - \frac{1}{2}}$$

$$= I_m \sqrt{\frac{2-1}{2}}$$

$$= I_m \sqrt{\frac{1}{2}}$$

$$= \frac{I_m}{\sqrt{2}} = 0.707 \, I_m$$

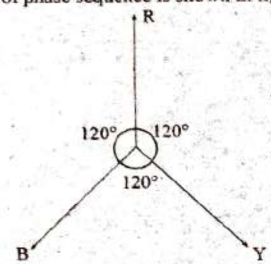
Therefore, the R. M. S value of sinusoidal alternating current waveform is 0.707 I_m. Similarly, for sinusoidal alternating voltage waveform it will be 0.707 V_m.

Q8. Write short notes on the following:
(i) Phase sequence
(ii) Symmetrical system.

Ans: Model Paper-III, Q1(e)

(i) Phase Sequence

The sequence of attaining maximum values by three phase voltage (or) current is known as phase sequence. These are employed to determine the direction of rotation. The graphical representation of phase sequence is shown in figure.



Figure

(ii) **Symmetrical System**

In a three phase system, if magnitudes and frequencies of voltages (or) currents are same and are displaced by an angle of 120° from each other, then such systems are known as symmetrical systems. Further, if they are not same, then such systems are known as un-symmetrical systems.

Q9. What is balanced supply and balanced load?

Ans:

Balanced Supply

If the supply has equal line-to-line voltages which are displaced in phase by 120° with respect to each other then such a supply is known as balanced supply.

$$V_R = V\angle 0° \text{ V}$$

$$V_Y = V\angle -120° \text{ V}$$

$$V_B = V\angle -240° \text{ V (or) } V\angle 120° \text{ V}$$

Balanced Load

If the load has equal magnitudes of impedances and also equal phase angles and possess the same nature then such load is called as balanced load.

i.e., $Z_1\angle\phi_1 = Z_2\angle\phi_2 = Z_3\angle\phi_3 = Z\angle\phi$

Q10. Write short notes on unbalanced systems.

Ans:

A 3-phase system with unequal magnitudes and phase angles of voltages (or) currents is known as un-balanced system. The problems arising due to such system are difficult to handle because of different conditions in phases. Generally, the various types of unbalanced loads are,

(i) Unbalanced delta connected load.

(ii) Unbalanced three-wire star connected load.

(iii) Unbalanced four-wire star connected load.

Q11. What are the advantages of polyphase system over single phase system?

Ans:

The advantages of polyphase system over single phase system are,

1. The single phase motor has a pulsating torque. Power supplied to each phase is pulsating. The total 3-ϕ power supplied to a balanced 3-ϕ circuit is constant at every instant of time. Therefore 3-ϕ have uniform torque.
2. 3-ϕ motors can easily be started when compared with 1-ϕ, so 3-ϕ motors has less vibration.
3. The efficiency of generation and transmission of 3-ϕ is higher compared to 1-ϕ, while transmitting the given amount of power at specified voltage.
4. The output of 3-ϕ machine is more when compared to 1-ϕ.
5. The control equipments of 3-ϕ are smaller, cheaper, lighter in weight and also more efficient.

Q12. Write short notes on unbalanced three wire star connected load.

Ans:

An unbalanced three wire star connected system consists of only three phases. In this type of connection the terminals of source and load are connected to each other and the neutral's of both source and load are isolated. For such connections the potential at load star points and source star point are different making unequal supply of phase voltage with unequal magnitudes and phase angles. These type of connections are employed rarely in practice. Generally, the various types of methods employed for analysis of three wire star connected load systems are,

(i) Loop method (or) Mesh method

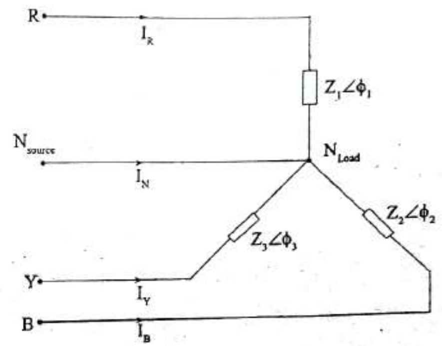
(ii) Star-delta conversion technique

(iii) Application of Millman's method.

Q13. Write short notes on unbalanced four wire star connected load.

Ans:

An unbalanced four wire star connected load consists of three phases and a neutral. In this type of connection, both the star points of source and load are connected to each other making the voltage drop across impedance equal to phase voltage of source and also the phase (or) line currents will be different and will be equal to current flowing through neutral wire. An un-balanced four wire star connected load is represented as shown in figure.



Figure

From figure, we have,

The phase voltages are given as,

$$V_{RN} = V\angle 0°$$

$$V_{BN} = V\angle -120°$$

$$V_{YN} = V\angle -240°$$

And also,

The phase (or) line currents are given as,

$$I_R = \frac{V_{RN}\angle 0°}{Z_1\angle\phi_1}$$

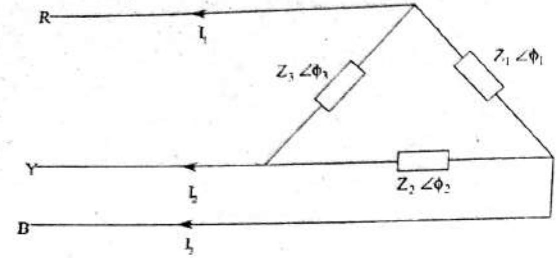
$$I_B = \frac{V_{BN}\angle -120°}{Z_2\angle\phi_2}$$

$$I_Y = \frac{V_{YN}\angle -240°}{Z_3\angle\phi_3}$$

Q14. Write short notes on unbalanced delta connected load.

Ans:

In unbalanced delta connected load, the voltage across the load will be fixed. It is independent of nature of load and is equal to the line voltage of supply and the line current will be phasor difference of the respective phase circuits. The unbalanced delta connected load is represented as shown in figure.



Figure

From figure, we have,

$$V_{RB} = V\angle 0^\circ$$
$$V_{BY} = V\angle -120^\circ$$
$$V_{YR} = V\angle -240^\circ$$

And also,

The phase currents are given as.

$$I_R = \frac{V_{RB}\angle 0^\circ}{Z_1\angle\phi_1}$$

$$I_B = \frac{V_{BY}\angle -120^\circ}{Z_2\angle\phi_2}$$

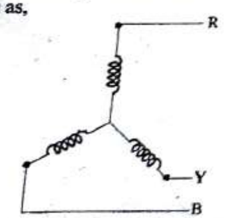
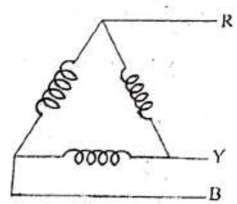
$$I_Y = \frac{V_{YR}\angle -240^\circ}{Z_3\angle\phi_3}$$

Hence, the line currents are given as,

$$I_1 = I_R - I_Y$$
$$I_2 = I_Y - I_B$$
$$I_3 = I_B - I_R$$

Q15. Compare between star and delta connection of 3-ϕ system.

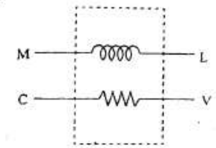
Ans:

	Star Connection		Delta Connection
1.	The pictorial representation of star connection is given as, 	1.	The pictorial representation of delta connection is is given as. 
2.	In star connection the relation between load voltage and phase voltage is given as, $V_L = \sqrt{3}\, V_{ph}$	2.	In delta connection the relation between load voltage and phase voltage is given as, $V_L = V_{ph}$
3.	In star connection, the relation between load current and phase current is given as, $I_L = I_{ph}$	3.	In delta connection the relation between load current and phase current is given as. $I_L = \sqrt{3}\, I_{ph}$

Q16. Write short notes on wattmeter.

Ans:

Wattmeter is a device used to measure voltage, current and power factor to give power in watts. Generally, a wattmeter comprises of a current coil and a voltage coil. The former type of coil is connected in series. This possess large cross-sectional area and small resistance. The later type of coil is connected across the supply terminals. This possess less cross-sectional area and have large resistance. The voltage coil is also known as pressure coil. The physical representation of a wattmeter is shown in figure.



Figure

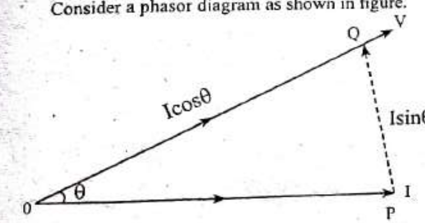
Q17. Write short notes on active and reactive components of current and power.

Ans:

Active and Reactive Components

(a) **Current**

Consider a phasor diagram as shown in figure.



Figure

The current that is inphase with voltage is known active or power component of current. It is represented with $I\cos\phi$

The current that is in quadrature with voltage is known as reactive or wattless component of current. It is represented with $I\sin\phi$.

(b) **Power**

Active Power

The product of voltage, current and cosine of the angle between voltage and current is known as active power. It is also known as true or real power. It is given as,

$$P = VI\cos(\phi_v - \phi_i)$$

Where, ϕ is the angle between voltage and current.

The units of active power is watts.

Reactive Power

Reactive power is measure of the exchange between the source and the reactive part of the load. It is given as,

$$Q = VI\sin\phi$$

The units of reactive power is volt ampere reactive.

Q18. Define self-inductance, mutual inductance and co-efficient of coupling and write the relation between them.

Ans:

(i) **Self Inductance:** The property of a coil that opposes the change in current through it is called self inductances.

It is defined as flux linkage ($N\phi$) per ampere (I). It is denoted by 'L' and measured in Henry (H).

i.e.,

$$L = \frac{N\phi}{I} \text{ Henry}$$

Where, N = Number of turns

ϕ = Flux in Weber

I = Current in a coil.

(ii) **Mutual Inductance:** The phenomenon of production of an induced e.m.f in coil 'Y' by changing current in coil 'X' is called mutual induction. The units of mutual inductance is Henry (H).

(or)

The magnitude of mutually induced e.m.f in one coil is due to rate of change of current in other neighbouring coil. Mutual inductance between two coils is given as,

$$M = \frac{N_2\phi_2}{I_1} \text{ Henry}$$

Where, N_2 = Number of turns of coil Y

ϕ_2 = Flux linking to coil Y

I_1 = Current of coil X.

(iii) **Coefficient of Coupling.** The coefficient of coupling between two coils is defined as the ratio of the actual value of mutual inductance between them to the maximum possible value.

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

Where,

k = Coefficient of coupling

M = Mutual inductance (H)

$\sqrt{L_1 L_2}$ = Maximum value of mutual inductance (H)

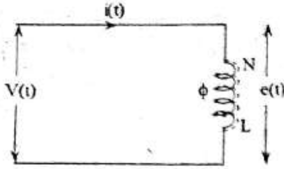
k = 1 for tightly coupled coils

k = 0 for coils which are not magnetically coupled.

Q19. Derive equation of self inductance, $L = \frac{N\phi}{i}$ Henry.

Ans:

Consider a coil L with N number of turns as shown in figure.



Figure

Let $V(t)$ be a alternating voltage applied across it and $i(t)$ be the alternating current flowing through it. As soon as current passes through the coil, a flux $\phi(t)$ is set-up across it. Since, this flux is also alternating (changing) an e.m.f will be induced in it according to Faradays's law of electromagnetic induction. The magnitude of this voltage is given by.

$$e(t) \propto \frac{di(t)}{dt}$$

$$e(t) = L\frac{di(t)}{dt} \quad \text{... (1)}$$

$$e(t) \propto \frac{d\phi(t)}{dt} \quad [\because \phi(t) \propto i(t)]$$

$$e(t) = Nd\frac{\phi(t)}{dt} \quad \text{... (2)}$$

Equating equations (1) and (2), we get,

$$L\frac{di(t)}{dt} = N\frac{d\phi(t)}{dt}$$

$$L\,di = Nd\phi$$

$$L = \frac{Nd\phi}{di}$$

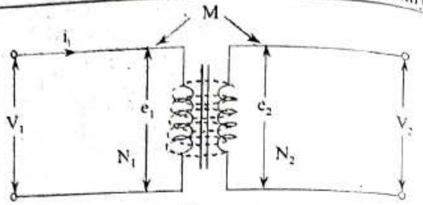
The flux and coil will be linearly related when the coils are linked with air as medium. Therefore the above expression of self inductance can be written as,

$$\therefore \quad L = \frac{N\phi}{i} \text{ Henry}$$

Q20. Derive equation of mutual inductance.

Ans: *Model Paper-III, Q1(f)*

Consider two coils placed near to each other as shown in figure. Let N_1, N_2 be the number of turns of coils 1 and 2 respectively. Let i_1 be the current flowing through coil 1 when an alternating voltage V_1 is applied across the first coil. Let M be the mutual inductance between the two coils.



Figure

As soon as the current start flowing through the coil 1, a flux ϕ_1 is set up in coil 1. Since, the coil is cutting the flux, and e.m.f (e_1) will be induced in coil 1 according to Faraday's law of electromagnetic induction. Since, the second coil is placed in the vicinity of the first coil, a part (ϕ_{12}) of flux ϕ_1 links with the second coil and again e.m.f will also be induced in second coil. The magnitude of the induced e.m.f (e_2) in second coil is given by,

$$e_2 \propto \frac{di_1}{dt}$$

$$e_2 = M\frac{di_1}{dt} \quad \text{... (1)}$$

Also,

$$e_2 \propto \frac{d\phi_{12}}{dt} \quad [\because \phi \propto i]$$

$$e_2 = N_2\frac{d\phi_{12}}{dt} \quad \text{... (2)}$$

Equating equations (1) and (2), we get,

$$M\frac{di_1}{dt} = N_2\frac{d\phi_{12}}{dt}$$

$$\Rightarrow \quad Mdi_1 = N_2\,d\phi_{12}$$

$$\Rightarrow \quad M = \frac{N_2 d\phi_{12}}{di_1}$$

The flux and current will be linearly related when the coils are linked with air as medium. Therefore, the expression for mutual inductance can now be written as,

$$\therefore \quad M = \frac{N_2\phi_{12}}{i_1}$$

Q21. Why the coefficient of coupling in a magnetic circuit is not more than unity?

Ans:

The amount of coupling of magnetic coils is known as coefficient of coupling. It is represented by K and its value is always less than 1 or equal to unity or 1 indicates the maximum value or perfect coupling of magnetic coils. Perfect or 100% coupling means the entire flux of one coil links with the another coil and the value of K will be 1. If half of the flux of one coil links with other coil, then the coefficient of coupling, K is equal to 0.5 or 50%. If two-thirds of the flux of one coil links with other coil, then the coefficient of coupling, K is equal to 0.66 or 66.6%.

Therefore, the coefficient of coupling in magnetic circuits is not more than unity.

Q22. Find the phase of a sinusoidal waveform of 60 Hz when pass through in a period of 5 milliseconds.

Ans:

Given that,

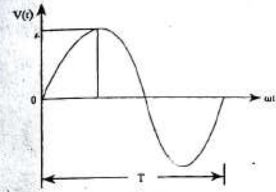
Sinusoidal waveform,

Frequency, $f = 60$ Hz

Time period, $t = 5 \times 10^{-3}$ sec

Phase = ?

According to the above data, the waveform obtained is shown in figure below,



Figure

From figure, it is clear that, the phase of sinusoidal waveform is given as,

$$\phi = \omega t$$

Substituting ω value we get,

$$\phi = 2\pi f \times t \quad [\because \omega = 2\pi f]$$

Substituting f,t values we get,

$$\phi = 2\pi \times 60 \times 5 \times 10^{-3}$$

$$= \frac{600\pi}{1000} = \frac{6\pi}{10} = \frac{3\pi}{5} = 1.8849 \text{ Radians}$$

$$= 1.8849 \times \frac{180}{\pi} \text{ (Degrees)}$$

$$\boxed{\therefore \phi = 107.99°}$$

Q23. Two wattmeters connected to a 3-phase motor indicate the total power input to be 12 kW. The power factor is 0.6. Determine the readings of each wattmeter.

Ans:

Given that,

Total power input, $P = 12$ kW.

Power factor, $\cos\phi = 0.6$

To determine,

Power reading of wattmeter one, $W_1 = ?$

Power reading of wattmeter two, $W_2 = ?$

The total power input measured by two wattmeters is,

$$W_1 + W_2 = 12 \text{ kW} \quad \text{... (1)}$$

We have,

$$\cos\phi = 0.6$$

$$\phi = \cos^{-1}(0.6) = 53.13°$$

$$\therefore \quad \tan\phi = 1.33$$

For a two wattmeter reading we have,

$$\tan\phi = \frac{\sqrt{3}[W_2 - W_1]}{W_1 + W_2} \quad \text{... (2)}$$

Substituting the values in equation (2), we get,

$$1.33 = \frac{\sqrt{3}[W_2 - W_1]}{12}$$

$$\sqrt{3}\,[W_2 - W_1] = 16$$

$$W_2 - W_1 = \frac{16}{\sqrt{3}}$$

$$W_2 - W_1 = 9.237 \text{ kW} \quad \text{... (3)}$$

Adding equations (1) and (3), we get,

$$W_1 + W_2 = 12$$

$$-\,W_1 + W_2 = 9.237$$

$$2W_2 = 21.2376$$

$$\Rightarrow \quad W_2 = 10.618 \text{ kW}$$

Substituting W_2 in equation (1), we get,

$$W_1 + 10.618 = 12$$

$$W_1 = 12 - 10.618$$

$$\Rightarrow \quad W_1 = 1.382 \text{ kW}$$

∴ Power reading of two wattmeters are,

$$W_1 = 1.382 \text{ kW}$$

$$W_2 = 10.618 \text{ kW}$$

Q24. A balanced delta connected load of (2 + j3) Ω per phase is connected to a balanced three-hase 440 V supply. The phase current is 10 A. Find the,

(i) Total active power

(ii) Reactive power and

(iii) Apparent power in the circuit.

Ans:

Given that,

Balanced 3-ϕ delta connected load,

Impedance per phase, $Z = 2 + j3 = 3.6\,\angle 56.3°\,\Omega$

Voltage, $V_L = 440$ V

Current, $I_{ph} = 10$ A

To determine,

(i) Total active power = ?

(ii) Reactive power = ?

(iii) Apparent power in the circuit = ?

We know that,

$$\cos\phi = \frac{R_{ph}}{Z_{ph}} = \frac{2}{3.6} = 0.55$$

$$\sin\phi = \sqrt{1-\cos^2\phi} = 0.83$$

$$\therefore\ I_L = \sqrt{3}\times I_{ph} = \sqrt{3}\times 10 = 17.32\text{ A}$$

(i) Total active power is given as,

$$P = \sqrt{3}\ V_L I_L \cos\phi$$

$$= \sqrt{3}\times 440\times 17.32\times 0.55$$

$$= 7259.78\text{ W}$$

(ii) Reactive power $= \sqrt{3}\ V_L I_L \sin\phi$

$$= \sqrt{3}\times 440\times 17.32\times 0.83$$

$$= 10955.67\text{ VAR}$$

(iii) Apparent power $= \sqrt{3}\ V_L I_L = \sqrt{3}\times 440\times 17.32$

$$= 13199.613\text{ VA}$$

PART-B
ESSAY QUESTIONS WITH SOLUTIONS

REPRESENTATION OF SINE FUNCTION AS ROTATING PHASOR, PHASOR DIAGRAMS, IMPEDANCES AND ADMITANCES, A.C CIRCUIT ANALYSIS, EFFECTIVE OR R.M.S VALUES, AVERAGE POWER AND COMPLEX POWER

Q25. Define the following,

(i) Cycle

(ii) Frequency

(iii) Time period

(iv) Maximum value

(v) Instantaneous value

(vi) Angular velocity.

Ans:

(i) **Cycle:** One complete set of positive and negative values of an alternating quantity is defined as a cycle. The cycle is represented in figure.

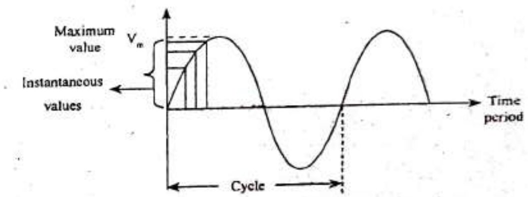


Figure (1)

(ii) **Frequency**

For answer refer Unit-III, Q2, Topic: Frequency.

(iii) **Time Period:** The time required for an alternating quantity to complete one full cycle is known as time period. It is represented in figure below. It is denoted by T.

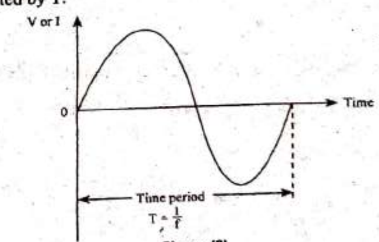


Figure (2)

(iv) **Maximum Value:** It is defined as the maximum instantaneous value reached by the alternating quantity once in every half cycle. It is also known as peak value or amplitude. The maximum value is represented in figure (1).

(v) **Instantaneous Value:** Instantaneous value of an alternating quantity is the magnitude at any particular instant of time. The instantaneous value is shown in figure (1).

(v) **Angular Velocity**

For answer refer Unit-III, Q3.

Q26. Write short notes on following

(i) Phasor representation of alternating quantity

(ii) Phasor diagram.

Ans:

(i) **Phasor Representation of Alternating Quantity**

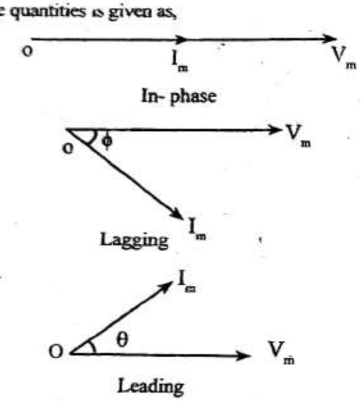
For answer refer Unit- III, Q4.

(ii) **Phasor Diagram :** The diagram that provides exact interrelationship between magnitude and phase angle of two (or) more alternating quantities is known as phasor diagram. Consider an alternating quantity having voltage and current as,

$$V = V_m \sin(\omega t)$$

$$I = I_m \sin(\omega t - \phi)$$

The phasor diagram for the above quantities is given as,



Q27. The instantaneous voltage across each of the four coils connected in series is given by,

$V_1 = 100 \sin 471 t$

$V_2 = 250 \cos 471 t$

$V_3 = 150 \sin (471 t + \pi/6)$

$V_4 = 200 \sin (471 t - \pi/4)$

(a) Determine the total p.d expressed in similar form to those given.

(b) What will be the resultant p.d if V_2 is reversed in signal?

Ans:

Given that,

Instantaneous voltages,

$$V_1 = 100 \sin 471 t$$

$$V_2 = 250 \cos 471t = 250 \sin(471t + 90°)$$

$$V_3 = 150 \sin\left(471t + \frac{\pi}{6}\right) = 150 \sin(471t + 30°)$$

$$V_4 = 200 \sin\left(471t - \frac{\pi}{4}\right) = 200 \sin(471t - 45°)$$

Maximum values of four intantaneous voltages are,

$$V_{max_1} = 100\text{ V}, \quad \theta_1 = 0°$$

$$V_{max_2} = 250\text{ V}, \quad \theta_2 = 90°$$

$$V_{max_3} = 150\text{ V}, \quad \theta_3 = 30°$$

$$V_{max_4} = 200\text{ V}, \quad \theta_4 = -45°$$

Phasors representing maximum values of voltages are shown in figure.

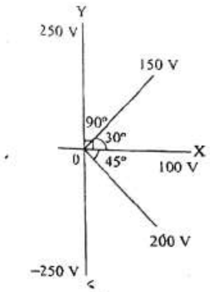


Figure: Phasor Representation

(a) **Total Potential Difference**

Resolving above phasors into horizontal and vertical components, we get,

X-components,

V_x = Sum of all components along *X*-axis

$= 100 \cos 0° + 250 \cos 90° + 150 \cos 30° + 200 \cos (45°)$

$= 100 + 0 + 150 \times \frac{\sqrt{3}}{2} + 200 \times \frac{\sqrt{2}}{2}$

$= 371.32$

Y-components,

V_y = Sum of all components along *Y*-axis

$= 100 \sin 0° + 250 \sin 90° + 150 \sin 30° - 200 \sin 45°$

$= 0 + 250 + 150 \times \frac{1}{2} - 200 \times \frac{\sqrt{2}}{2}$

$= 183.57$

Maximum potential difference,

$$V_{max} = \sqrt{V_x^2 + V_y^2}$$

$$= \sqrt{(371.32)^2 + (183.57)^2} = 414.217\text{ V}$$

Phase angle, $\tan\phi = \frac{V_y}{V_x}$

$$\phi = \tan^{-1}\left(\frac{V_y}{V_x}\right)$$

$$= \tan^{-1}\left(\frac{183.57}{371.32}\right) = 26.306°$$

$\therefore$ **Total potential difference,**

$$V_T = 414.217 \sin(471t + 26.306°)$$

(b) For Reversed Voltage (V_r)

X-components,

$$V_x = 100\cos 0° + 250\cos 90° + 150\cos 30° + 200\cos 45°$$
$$= 371.32$$

Y-components,

$$V_y = 100\sin 0° + (-250\sin 90°) + 150\sin 30° - 200\sin 45°$$
$$= 0 - 250 + 150 \times \frac{1}{2} - 200 \times \frac{\sqrt{2}}{2}$$
$$= -316.421$$

Maximum potential difference, $V_{max} = \sqrt{(371.32)^2 + (-316.421)^2}$
$$= 487.85 \text{ V}$$

Phase angle, $\tan\phi = \frac{V_y}{V_x}$

$$\phi = \tan^{-1}\left(\frac{V_y}{V_x}\right)$$
$$= \tan^{-1}\left(\frac{-316.421}{371.32}\right)$$
$$= -40.436°$$

∴ Resultant potential difference, $V_r = 487.85\sin(471t - 40.436°)$.

Q28. The applied voltage to an A.C circuit is V = 200 sin314t and the current flowing is i = 20 sin314t. Find the following:

(i) Peak values of voltage and current
(ii) Frequency of the voltage and current
(iii) Effective values of voltage and current
(iv) The circuit element and its value
(v) Power factor
(vi) Draw the phasor diagram.

Ans:

Given that,

Applied voltage, $V = 200\sin 314t$

Current, $i = 20\sin 314t$

Comparing the given voltage and current equations with the standard equations,

$$V = V_m \sin\omega t$$
$$I = I_m \sin$$

(i) Peak Values of V... urrent

Peak value of vo... V

Peak value of cu... A

(ii) Frequency of th... and Current

From the given equations, $\omega = 314$

$$2\pi f = 314 \quad [\because \omega = 2\pi f]$$
$$\Rightarrow f = \frac{314}{2\pi}$$
$$= 50 \text{ Hz}$$

∴ Frequency of applied voltage, $f = 50$ Hz

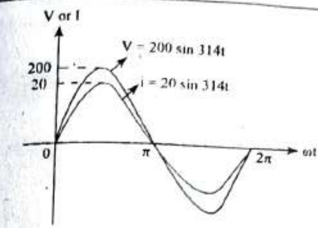


Figure (a)

Frequency of current, $f = 50$ Hz

∴ The applied voltage and current have the same frequency as shown in figure (a).

(iii) Effective Values of Voltage and Current

Effective value of voltage,

$$V_{r.m.s} = \frac{V_m}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141.421 \text{ A}$$

Effective value of current,

$$I_{r.m.s} = \frac{I_m}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14.142 \text{ A}$$

(iv) The Circuit Element and Its Value

Since, the frequency of voltage and current is same and cross their zero position at the same time. The circuit consists of a pure resistive element and is shown in figure (b).

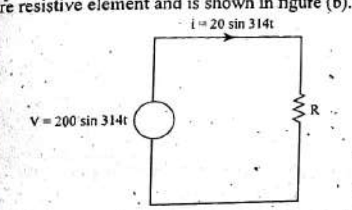


Figure (b): Circuit with Resistive Element

Value of resistive element,

$$R = \frac{V_m}{I_m}$$
$$= \frac{200}{20}$$
$$= 10\ \Omega$$

(v) Power Factor, cos φ

Power factor, $\cos\phi = \frac{R}{Z}$

Where,

$$Z = \text{Impedance of the circuit}$$
$$= \sqrt{R^2}$$
$$= R$$

$$\therefore \cos\phi = \frac{R}{R} = 1$$

Therefore, the power factor of the circuit is unity.

(vi) Phasor Diagram

The applied voltage and current both are in phase and the phasor diagram of circuit is shown in figure (c).

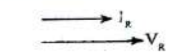
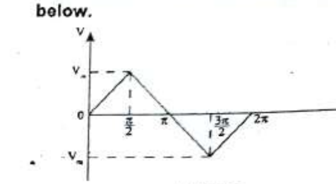


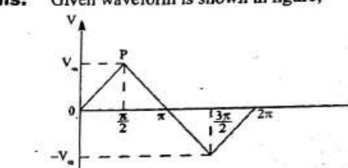
Figure (c): Phasor Diagram of Pure Resistive Circuit

Q29. Find the r.m.s value, average, peak factor and form factor of the waveform shown in figure below.



Figure

Ans: Given waveform is shown in figure,



Figure

From the above waveform only first quarter of cycle upto $\frac{\pi}{2}$ is sufficient to calculate the r.m.s and average values.

1. R.M.S Value ($V_{r.m.s}$)

Equation of instantaneous voltage for first quarter-cycle is,

$$V = mt$$

Where '*m*' is scope of the line $OP = \frac{2V_m}{\pi}$

$$V = \frac{2V_m t}{\pi}$$

$$\therefore \text{R.M.S value, } V_{r.m.s} = \left[\frac{1}{T}\int_0^T V^2\,dt\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{\pi/2}\int_0^{\pi/2}\left(\frac{2V_m t}{\pi}\right)^2 dt\right]^{1/2}$$

$$= \left[\frac{8V_m^2}{\pi^3}\int_0^{\pi/2} t^2\,dt\right]^{1/2}$$

$$= \left[\frac{8V_m^2}{\pi}\left[\frac{t^3}{3}\right]_0^{\pi/2}\right]^{1/2}$$

$$= \left[\frac{8V_m^2}{3\pi^3}\left[\left(\frac{\pi}{2}\right)^3 - 0\right]\right]^{1/2}$$

$$= \left[\frac{8V_m^2}{3\pi^3}\cdot\frac{\pi^3}{8}\right]^{1/2} = \left[\frac{V_m^2}{3}\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{3}} = 0.5773\ V_m$$

2. Average Value (V_{avg})

Average value for first quarter cycle i.e., upto $\frac{\pi}{2}$ is given as,

$$V_{avg} = \frac{1}{T}\int_0^T V\,dt$$

$$= \frac{1}{\pi/2}\int_0^{\pi/2} \frac{2V_m}{\pi} t\,dt$$

$$= \frac{4V_m}{\pi^2}\left[\frac{t^2}{2}\right]_0^{\pi/2}$$

$$= \frac{4V_m}{2\pi^2}\left[\left(\frac{\pi}{2}\right)^2 - (0)^2\right]$$

$$= \frac{4V_m}{2\pi^2}\cdot\frac{\pi^2}{4} = \frac{V_m}{2}$$

$$= 0.5V_m$$

3. Form Factor

$$\text{Form factor, } K_f = \frac{V_{r.m.s}}{V_{avg}} = \frac{0.5773V_m}{0.5V_m} = 1.154$$

4. Peak Factor

$$\text{Peak factor, } K_p = \frac{V_m}{V_{r.m.s}} = \frac{V_m}{0.5773V_m} = 1.732$$

Q30. The equation of an alternating current is V = 400 sin 318t.

Determine,

(i) Maximum value

(ii) Frequency

(iii) R.M.S value

(iv) Average value.

Ans:

Given that,

$V = 400 \sin 318t$ ... (1)

(i) Maximum value $(V_m) = ?$

(ii) Frequency $(f) = ?$

(iii) R.M.S value $(V_{rms}) = ?$

(iv) Average value $(V_{avg}) = ?$

(i) Maximum Value

For an alternating voltage we have,

$V = V_m \sin \omega t$ ... (2)

Comparing equations (1) and (2) we get,

$V_m = 400$ and $\omega = 318$

From the above, we have,

Maximum value, $V_m = 400$ V

(ii) Frequency

$\omega = 318$

But,

$\omega = 2\pi f$

$2\pi f = 318$

$f = \frac{318}{2\pi}$

$f = 50.6112$ Hz

(iii) RMS Value,

The R.M.S value is given as,

$V_{r.m.s} = 0.707\ V_m$

Substituting V_m value, we get,

$V_{r.m.s} = 0.707 \times 400$

$= 282.8$ V

(iv) Average Value

The average value is given as,

$V_{avg} = 0.637\ V_m$

Substituting V_m value, we get,

$V_{avg} = 0.637 \times 400$

$\therefore\ V_{avg} = 254.8$ V

Q31. A sinusoidal current wave is given by i = 100 sin 100πt. Determine,

(i) The average value

(ii) R.M.S value

(iii) Form factor

(iv) Peak factor.

Ans :

Given that,

$i = 100 \sin 100\pi t$ ... (1)

(i) The average value = ?

(ii) R.M.S value = ?

(iii) Form factor = ?

(iv) Peak factor = ?

The sinusoidal alternating current is,

$I = I_m \sin\omega t$ ... (2)

Comparing equations (1) and (2), we get,

$I_m = 100$ and $\omega = 314$

(i) Average Value

The average value for an alternating sinusoidal current is given as,

$I_{avg} = 0.637 I_m$

Substituting I_m value, we get,

$I_{avg} = 0.637 \times 100$

$I_{avg} = 63.7$ Amps

(ii) R.M.S Value

For sinusoidal alternating current, the r.m.s value is given as,

$I_{r.m.s} = 0.707 I_m$

Substituting I_m value, we get,

$= 0.707 \times 100$

$I_{r.m.s} = 70.7$ Amps

(iii) Form Factor

For a sinusoidal alternating current, form factor is given as,

$$\text{Form factor} = \frac{\text{R.M.S value}}{\text{Average value}}$$

$$= \frac{70.7}{63.7}$$

Form factor = 1.109

(iv) Peak Factor

For a sinusoidal alternating current, peak factor is given as,

$$\text{Peak factor} = \frac{\text{Maximum value}}{\text{R.M.S value}}$$

$$= \frac{I_m}{\text{R.M.S value}}$$

$$= \frac{100}{70.7}$$

Peak factor = 1.414

Q32. The equation of an alternating current is i = 200 sin 318 t.

Determine,

(i) Maximum value

(ii) Frequency

(iii) R.M.S value

(iv) Average value.

Ans:

Given that,

$i = 200 \sin 318t$ ... (1)

(i) Maximum value = ?

(ii) Frequency = ?

(iii) R.M.S value = ?

(iv) Average value = ?

The sinusoidal alternating current is,

$I = I_m \sin \omega t$ ... (2)

Comparing equations (1) and (2) we get,

$I_m = 200$ and $\omega = 318$

(i) Maximum Value

Maximum value, $I_m = 200$ Amps

(ii) Frequency

$\omega = 318$

We know that,

$\omega = 2\pi f$

$2\pi f = 318$

$f = \frac{318}{2\pi}$

$f = 50.611$ Hz

(iii) R.M.S Value

For a sinusoidal alternating current, the r.m.s value is given as,

$I_{rms} = 0.707\ I_m$

Substituting I_m value, we get,

$= 0.707 \times 200$

$I_{rms} = 141.4$ Amps

(iv) Average Value

The average value for an alternating sinusoidal current is given as,

$I_{avg} = 0.637\ I_m$

Substituting I_m value, we get,

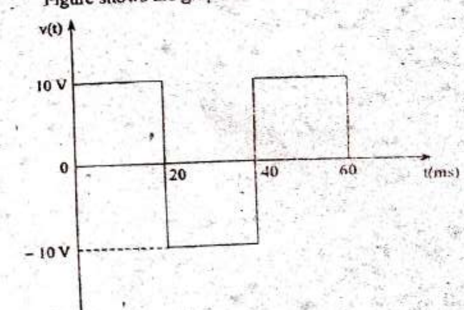
$I_{avg} = 0.637 \times 200$

$I_{avg} = 127.4$ Amps

Q33. Obtain the r.m.s value, average value, form factor and peak factor for a voltage of symmetrical square shape whose amplitude is 10 V and time period is 40 secs.

Ans:

Figure shows the graph drawn according to given data.



Figure

Let $v(t)$ be the instantaneous value of the voltage wave and 't' be the time in msec.

The time period of the given wave is 40 ms. Hence, dividing the time period into two intervals and writing the voltage equations,

$v(t) = 10$ V; $\quad 0 < t < 20$ ms

$v(t) = -10$ V $\quad 20$ ms $< t < 40$ ms

(i) **R.M.S Value :** The r.m.s value of any wave is defined as,

$$V_{r.m.s} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 dt} \quad \text{... (1)}$$

Substituting the value of $v(t)$ in equation (1) for different intervals, we get,

$$V_{r.m.s} = \sqrt{\frac{1}{40\times10^{-3}}\left[\int_0^{20\times10^{-3}} v^2(t)dt + \int_{20\times10^{-3}}^{40\times10^{-3}} v^2(t)dt\right]}$$

$$= \sqrt{\frac{1}{40\times10^{-3}}\left[\int_0^{20\times10^{-3}} (10)^2 dt + \int_{20\times10^{-3}}^{40\times10^{-3}} (-10)^2 dt\right]}$$

$$= \sqrt{\frac{100}{40\times10^{-3}}\left[\int_0^{20\times10^{-3}} dt + \int_{20\times10^{-3}}^{40\times10^{-3}} dt\right]}$$

$$= \sqrt{\frac{100}{40\times10^{-3}}\left[(t)_0^{20\times10^{-3}} + [t]_{20\times10^{-3}}^{40\times10^{-3}}\right]}$$

$$= \sqrt{\frac{100}{40\times10^{-3}}[(20\times10^{-3}-0)+(40\times10^{-3}-20\times10^{-3})]}$$

$$V_{r.m.s} = \sqrt{100} = 10 \text{ V}$$

(ii) **Average Value :** Since, the given wave is symmetrical, average value if calculated for full cycle will be zero. Hence, for symmetrical waves, the average value should be calculated only for half wave.

$$V_{avg} = \frac{1}{T/2}\int_0^{T/2} v(t)dt$$

$$= \frac{1}{20\times10^{-3}}\int_0^{20\times10^{-3}} 10\, dt$$

$$= \frac{10}{20\times10^{-3}}\int_0^{20\times10^{-3}} dt$$

$$= \frac{10}{20\times10^{-3}}[t]_0^{20\times10^{-3}}$$

$$= \frac{10}{20\times10^{-3}}[20\times10^{-3}-0]$$

$$V_{avg} = 10 \text{ V}$$

(iii) **Form Factor :** It is the ratio of r.m.s value to average value of a wave.

$$\text{Form factor} = \frac{V_{r.m.s}}{V_{avg}} = \frac{10}{10} = 1$$

(iv) **Peak Factor :** It is the ratio of peak value to r.m.s value of a wave.

$$\text{Peak factor} = \frac{V_P}{V_{r.m.s}}$$

From figure, $V_P = 10$

$$\therefore \text{Peak factor} = \frac{10}{10} = 1$$

Q34. Calculate the R.M.S value, average value, peak factor and form factor of sinusoidal voltage of maximum value of 200 volts.

Ans:

Given that,

Maximum value of voltage, $V_m = 200$ V

(i) R. M. S Value = ?

(ii) Average value = ?

(iii) Peak factor = ?

(iv) Form factor = ?

(i) **R. M. S Value :** For a sinusoidal voltage waveform, the R.M.S value is given as,

$$V_{r.m.s} = 0.707\, V_m$$

Substituting V_m value, we get,

$$\Rightarrow V_{r.m.s} = 0.707 \times 200$$

$$\boxed{V_{r.m.s} = 141.4 \text{ V}}$$

(ii) **Average Value :** For a sinusoidal voltage waveform, the average value is given as,

$$V_{avg} = 0.637 V_m$$

Substituting V_m value, we get,

$$V_{r.m.s} = 0.637 \times 200$$

$$\boxed{V_{avg} = 127.4 \text{ V}}$$

(iii) **Form Factor :** By the definition of form factor, we have,

$$\text{Form factor} = \frac{\text{R.M.S value}}{\text{Average value}}$$

$$= \frac{141.4}{127.4}$$

$$= 1.1$$

(iv) **Peak Factor :** By the definition of peak factor, we have,

$$\text{Peak factor} = \frac{\text{Maximum value}}{\text{R.M.S value}}$$

$$= \frac{200}{141.4} = 1.414$$

THREE PHASE CIRCUITS

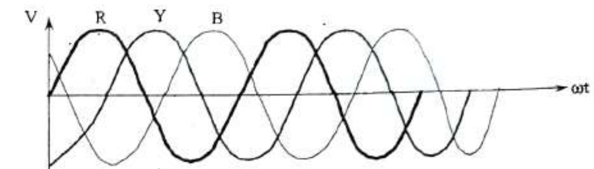
Q35. What is phase sequence? Explain its significance.

Ans:

Phase Sequence

The order in which the three-phase attain their peak or maximum value in a 3-phase A.C system is known as "phase sequence".

The standard 3-ϕ e.m.f phase sequences are illustrated as shown in figure below.



Figure

As shown in the above waveform, the e.m.f attains their maximum values at R, B and Y. All the 3-phases are displaced by 120° phase difference at each other.

The e.m.f equation is given as,

$$V_{RR'} = V_m \sin \omega t$$

$$V_{YY'} = V_m \sin(\omega t - 240°)$$

$$V_{BB'} = V_m \sin(\omega t - 120°)$$

Significance

(i) Phase sequence of the voltage applied to the load is determined by the order in which the 3-phase lines are connected.

(ii) Phase sequence can be reversed by interchanging any pair of lines. In case of induction motor, reversal of sequence results in the reversal direction of motor rotation.

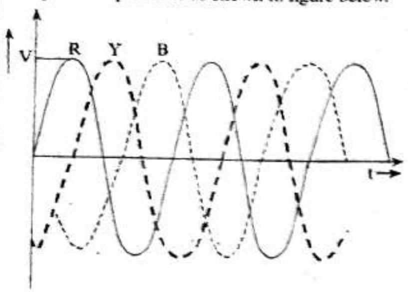
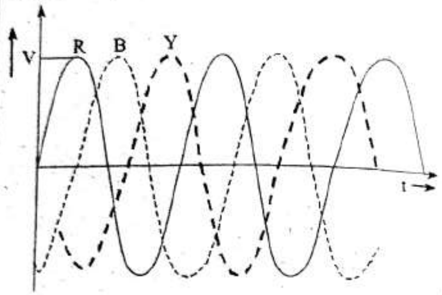
(iii) In case of 3-phase generator, if phase reversal has occurred with it, then it may cause extensive damage to both the machines. Hence, when working on such system, it is essential that phase sequences is to be clearly specified otherwise unnecessary confusion will arise.

Q36. What is the difference between RYB phase sequence and RBY phase sequence?

Ans:

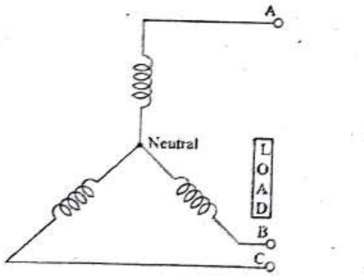
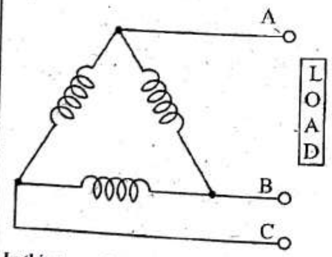
Difference between the RYB phase sequence and RBY phase sequence is as follows.

	RYB Phase Sequence		RBY Phase Sequence
1.	RYB phase sequence is the order in which the induced phase voltages V_R, V_Y and V_B attain their respective maximum peak values.	1.	RBY phase sequence is the order in which the induced phase voltages V_R, V_B and V_Y attain their respective maximum peak values.
2.	The RYB phase sequence is often called as positive phase sequence and is produced by clockwise rotation of alternator.	2.	The RBY phase sequence is often called as negative phase sequence and is produced by anticlockwise rotation of alternator.
3.	The induced phase voltages V_R, V_Y and V_B are displaced at 120 electrical degrees apart from each other.	3.	The induced phase voltages V_R, V_B and V_Y are also displaced at 120 electrical degrees apart from each other.

	RYB Phase Sequence		RBY Phase Sequence
4.	The algebraic sum of the instantaneous values of induced phase voltages is always zero i.e., $V_R + V_Y + V_B = 0$	4.	The algebraic sum of the instantaneous values of induced phase voltages is zero. i.e., $V_R + V_B + V_Y = 0$
5.	The phase sequence is as shown in figure below.  Figure	5.	The phase sequence is as shown in figure below.  Figure

Q37. Compare a three phase star connected system with a delta-connected system. Discuss merits and demerits of the two systems.

Ans:

	Star Connection		Delta Connection
1.	The star connection is obtained by joining the initial ends or terminal ends of three windings together and the remaining ends are connected to a load is referred as three phase star connected system. **Example** 	1.	The delta connection is obtained by joining the terminal end to the initial end of the other winding and continued till all the three windings are formed as a loop. This connection of three windings in a closed loop is referred as delta connected system **Example** 
2.	In this type of system identical ends are connected together.	2.	In this type of system unidentical ends are connected together.
3.	Line voltage (V_L) = $\sqrt{3}$ Phase voltage (V_{ph}).	3.	Line voltage (V_L) = Phase voltage (V_{ph}).
4.	Line current (I_L) = Phase current (I_{ph}).	4.	Line current (I_L) = $\sqrt{3}$ Phase current (I_{ph}).
5.	The carrying of neutral wire to the load can be possible in star connection.	5.	It is impossible to carry neutral wire to load.
6.	Distribution transformers are delta/star connected.	6.	Power transformers are delta/delta connected.

Merits of Star Connected System

1. For a same voltage star connected system requires less number of turns when compared to delta connected system.
2. It also requires less amount of insulation compared to delta connection for the same line voltage.
3. It provides a great advantage by the possibility of earthing the neutral point.
4. Generally 3-phase alternators are star connected.

Demerits of Star Connected System

1. Even if the neutral point is earthed, there is a possibility of distortion in the phase voltage on the secondary side.
2. This connection is rarely used and are suitable only for high voltage small transformers.

Merits of Delta Connected System

1. Delta connected systems are mostly used for rotatory conveyers.
2. Delta connections are more flexible for unbalanced loads because the loads can be added or removed on a single phase, which is cannot possible in star-connected 3 wire load.
3. Generally, 3-phase induction motors are delta connected.

Demerits of Delta Connected System

1. Delta connection is not used for 3-phase 4 wire devices because of the absence of common junction (Neutral point).
2. In a 3-phase transformers of delta connected each transformers has to be provided with full-line voltage.
3. This connection requires more number of turns along with the insulation, which in turn makes the delta connection more expensive.

Q38. Derive the relation between line and phase currents and voltages in star connection of a three phase (balanced and unbalanced) system.

OR

Explain about voltage and current relations in Wye or Star connection of a three-phase circuit.

Ans:

Star Connected System

In a star connected system, one end of the three coils are connected to a common point called neutral point. The wires connected to the other end of the three coils are known as lines.

The voltage measured from neutral wire to any one of the line is known as phase voltage (V_{ph}). The voltage measured between any two lines is known as line voltage (V_L).

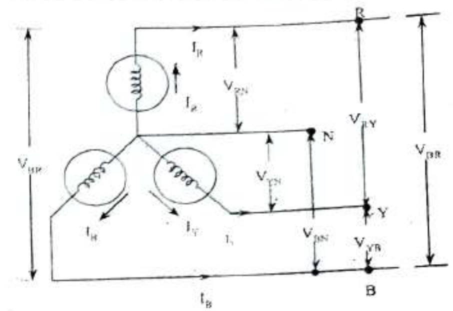


Figure (1)

The star connected system with *RYB* phase sequence is shown in figure (1).

The current flowing in a coil is known as phase current (I_{ph}). The current flowing through a line is known as line current (I_L). The voltage produced by *RYB* windings when they are separated by an angle of 120° and rotated in clockwise are,

$$V_{RN} = V_{ph} \angle 0°$$
$$V_{YN} = V_{ph} \angle -120°$$
$$V_{BN} = V_{ph} \angle -240°$$

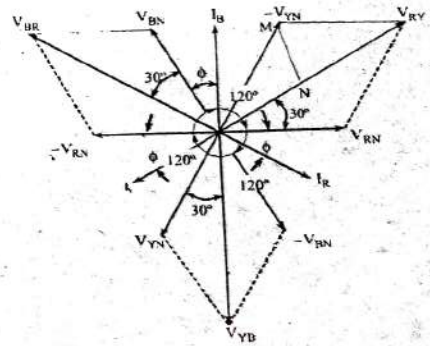


Figure (2)

Line Voltages and Phase Voltages

The line voltage is the vector difference of the phase voltages.

The line voltage V_{RY} is given as,

$$V_{RY} = V_{RN} + V_{NY}$$
$$= V_{RN} - V_{YN}$$

From the phasor diagram, we have,

$$V_{RY} = \sqrt{V_{RN}^2 + V_{YN}^2 + 2V_{RN}V_{YN}\cos\theta}$$

θ is the angle between V_{RN} and V_{YN} i.e., 60°

Also $V_{RN} = V_{YN} = V_{BN} = V_{ph}$ since, the system is a balanced system.

$$\therefore \quad V_{RY} = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}^2 \cos 60^\circ}$$

$$= \sqrt{V_{ph}^2 + V_{ph}^2 + V_{ph}^2} \quad \left[\because \cos 60^\circ = \frac{1}{2}\right]$$

$$V_{RY} = \sqrt{3V_{ph}^2}$$

But, $V_{RY} = V_{YB} = V_{BR} = V_L$

$$V_L = \sqrt{3}V_{ph}$$

$\therefore$ Line voltage = $\sqrt{3}$ times phase voltage (in magnitude).

Figure (2) shows that,

(i) Line voltages are 120° apart.

(ii) Line voltages are 30° ahead of their respective phase voltages.

(iii) The angle between the line currents and the corresponding line voltages is $(30° + \phi)$ with current lagging.

The line voltages are,

$$V_{RY} = \sqrt{3}\, V_{ph} \angle 30^\circ$$

$$V_{YB} = \sqrt{3}\, V_{ph} \angle -90^\circ$$

$$V_{BR} = \sqrt{3}\, V_{ph} \angle -210^\circ$$

Line Currents and Phase Currents

Each line is in series with its individual phase winding. Hence, the line current in each line is the same as the current in the phase winding to which the line is connected.

$$I_L = I_R = I_Y = I_B = I_{ph}$$

$$\therefore \quad I_L = I_{ph}$$

Power

The total active or true power in the circuit is the sum of the three phase powers, since the system is balanced system.

Total power = 3 × Phase power

$$P = 3 \times V_{ph} I_{ph} \cos\phi$$

$$= 3 \times \frac{V_L}{\sqrt{3}} I_{ph} \cos\phi$$

$$P = \sqrt{3}\, V_L\, I_L \cos\phi \quad [\because I_L = I_{ph}]$$

Where 'ϕ' is the angle between phase voltage and phase current.

Q39. Write down the relationship between phase voltage and line voltage and phase current and line current in a delta connected circuit.

OR

Explain the voltage and current relations in three-phase delta connected system.

Ans:

In a delta connected system dissimilar ends of the three coils or phase winding are joined together i.e., the starting end of the one phase is joined to the finishing end of the other phase and so on as shown in figure (1) with phase sequence RYB.

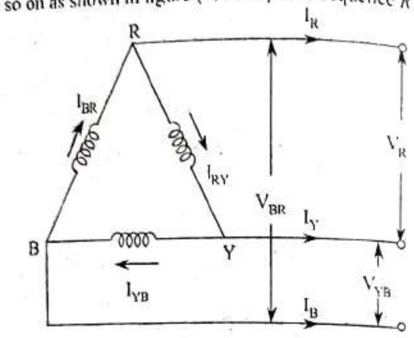


Figure (1)

Line Voltages and Phase Voltages

Figure (1) shows that, there is only one phase winding completely included between any pair of terminals. Hence, in delta connection, the voltage between any pair of lines is equal to the phase voltage of the phase winding connected between the two lines considered. Since, the phase sequence is RYB.

The phase voltage,

V_{RY} = Line voltage between lines R and Y

$$= V_{ph} \angle 0^\circ$$

The phase voltage,

V_{YB} = Line voltage between lines Y and B

$$= V_{ph} \angle -120^\circ$$

The phase voltage,

V_{BR} = Line voltage between lines B and R

$$= V_{ph} \angle -240^\circ$$

$\therefore$ Line voltage = Phase voltage (in magnitude)

$$V_L = V_{ph}$$

Line Currents and Phase Currents

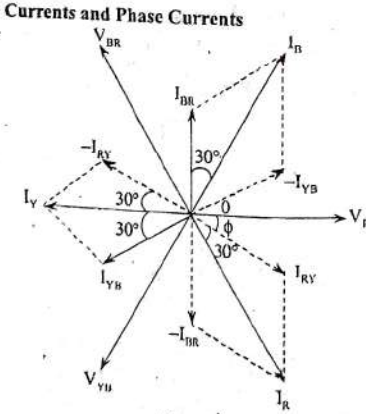


Figure (2)

Figure (2) shows that, current in each line is the vector difference of the two phase currents flowing through that line.

$\therefore$ The line current are,

$$I_R = I_{RY} - I_{BR}$$

$$I_Y = I_{YB} - I_{RY}$$

$$I_B = I_{BR} - I_{YB}$$

The magnitude of I_R, I_Y and I_B are same. The current in line I_R is found by compounding I_{RY} and I_{BR} reversed. From parallelogram law of vector additions, we have,

$$I_R = \sqrt{I_{RY}^2 + I_{BR}^2 + 2I_{RY}I_{BR}\cos\theta}$$

$$I_R = \sqrt{I_{ph}^2 + I_{ph}^2 + I_{ph}^2} \quad [\because I_{RY} = I_{BR} = I_{RY} = I_{ph}, \theta = 60^\circ]$$

$$I_R = \sqrt{3}I_{ph}$$

$$\therefore \quad I_L = I_R = I_Y = I_B = \sqrt{3}I_{ph}$$

Figure (2) shows that,

(i) Line currents are 120° apart.

(ii) Line currents are 30° behind the corresponding phase currents.

(iii) The angle between the line currents and the corresponding line voltages is $(30 + \phi)$ with the current lagging.

Power

Power/phase $= V_{ph} I_{ph} \cos\phi$

Total power $= 3 \times V_{ph} I_{ph} \cos\phi$

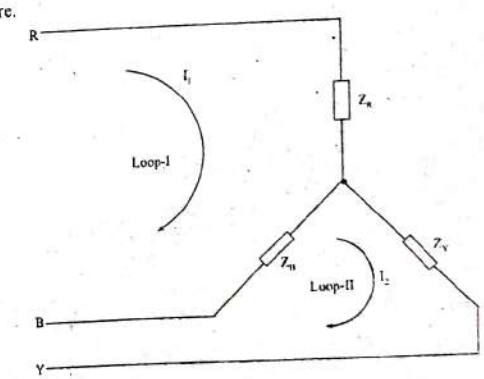
$$= 3V_L \frac{I_L}{\sqrt{3}} \cos\phi \quad \left(\because I_{ph} = \frac{I_L}{\sqrt{3}}, V_{ph} = V_L\right)$$

$$= \sqrt{3}\, V_L\, I_L \cos\phi$$

Q40. Discuss loop method analysis for unbalanced loads.

Ans:

Loop method analysis is employed to solve star connected unbalanced loads. Consider a three wire star connected unbalanced load as shown in figure.



Figure

Applying KVL to figure, we get,

Loop-I

$$I_1 Z_R + (I_1 - I_2)\, Z_B = V_{RB}$$

$$\Rightarrow \quad I_1 Z_R + I_1 Z_B - I_2 Z_B = V_{RB}$$

$$\Rightarrow \quad I_1 (Z_R + Z_B) - I_2 Z_B = V_{RB}$$

Consider an unbalanced star connected load with impedances Z_R, Z_Y and Z_B as shown in figure (2).

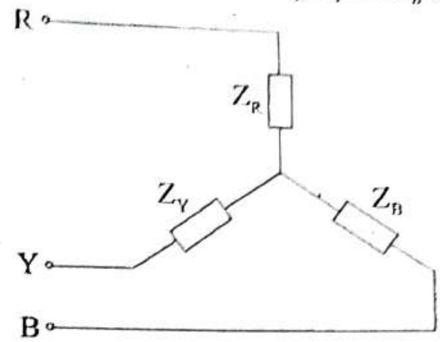


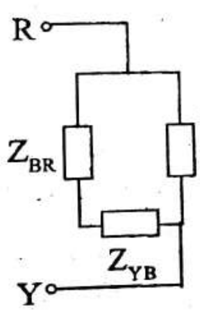
Figure (2)

The equivalent delta connected load is obtained by equating the impedances between the corresponding terminals of the two loads.

On equating the impedances between corresponding pairs of *R* and *Y* terminals of figures (1) and (2), we get,

$$Z_R + Z_Y = \frac{Z_{RY}(Z_{BR}+Z_{YB})}{Z_{RY}+Z_{BR}+Z_{YB}} \qquad \ldots (1)$$

$\because$

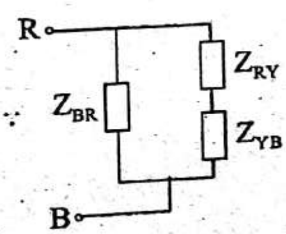


$$\Rightarrow (Z_{BR}+Z_{YB}) \parallel Z_{RY} \Rightarrow \frac{(Z_{BR}+Z_{YB})+Z_{RY}}{Z_{BR}+Z_{YB}+Z_{RY}}$$

On equating the impedances between corresponding pairs of *R* and *B* terminals of figures (1) and (2), we get,

$$Z_R + Z_B = \frac{Z_{BR}(Z_{RY}+Z_{YB})}{Z_{BR}+Z_{RY}+Z_{YB}} \qquad \ldots (2)$$

$\because$

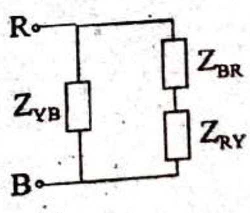


$$\Rightarrow (Z_{BR} \parallel (Z_{RY}+Z_{YB}) \Rightarrow \frac{Z_{BR}(Z_{RY}+Z_{YB})}{Z_{BR}+Z_{YB}+Z_{RY}}$$

On equating the impedances between corresponding pairs of *Y* and *B* terminals of figures (1) and (2), we get,

$$Z_Y + Z_B = \frac{Z_{YB}(Z_{BR}+Z_{RY})}{Z_{YB}+Z_{BR}+Z_{RY}} \qquad \ldots (3)$$

$\because$



$$\Rightarrow (Z_{YB} \parallel (Z_{BR}+Z_{RY}) \Rightarrow \frac{Z_{YB}(Z_{BR}+Z_{RY})}{Z_{YB}+Z_{BR}+Z_{RY}}$$

$$Z_R - Z_B = \frac{Z_{RY}(Z_{BR}+Z_{YB})}{Z_{RY}+Z_{BR}+Z_{YB}} - \frac{Z_{YB}(Z_{BR}+Z_{RY})}{Z_{RY}+Z_{BR}+Z_{YB}}$$

$$\Rightarrow Z_R - Z_B = \frac{Z_{RY}Z_{BR}+Z_{RY}Z_{YB}-Z_{YB}Z_{BR}-Z_{YB}Z_{RY}}{Z_{RY}+Z_{BR}+Z_{YB}}$$

$$\Rightarrow Z_R - Z_B = \frac{Z_{RY}Z_{BR}-Z_{YB}Z_{BR}}{Z_{RY}+Z_{BR}+Z_{YB}}$$

$$Z_R - Z_B = \frac{Z_{BR}(Z_{RY}-Z_{YB})}{Z_{RY}+Z_{BR}+Z_{YB}} \quad \dots (4)$$

Adding equation (2) and equation (4), we get,

$$Z_R + Z_B + Z_R - Z_B = \frac{Z_{BR}(Z_{RY}+Z_{YB})}{Z_{BR}+Z_{RY}+Z_{YB}} + \frac{Z_{BR}(Z_{RY}-Z_{YB})}{Z_{RY}+Z_{BR}+Z_{YB}}$$

$$\Rightarrow 2Z_R = \frac{Z_{BR}Z_{RY}+Z_{BR}Z_{YB}+Z_{BR}Z_{RY}-Z_{BR}Z_{YB}}{Z_{BR}+Z_{RY}+Z_{YB}}$$

$$\Rightarrow 2Z_R = \frac{2Z_{BR}Z_{RY}}{Z_{BR}+Z_{RY}+Z_{YB}}$$

$$\boxed{Z_R = \frac{Z_{BR}Z_{RY}}{Z_{BR}+Z_{RY}+Z_{YB}}}$$

Substracting equation (4) and equation (2), we get,

$$(Z_R + Z_B) - (Z_R - Z_B) = \frac{Z_{BR}(Z_{RY}+Z_{YB})}{Z_{BR}+Z_{RY}+Z_{YB}} - \frac{Z_{BR}(Z_{RY}-Z_{YB})}{Z_{RY}+Z_{BR}+Z_{YB}}$$

$$\Rightarrow 2Z_B = \frac{Z_{BR}Z_{RY}+Z_{BR}Z_{YB}-Z_{BR}Z_{RY}+Z_{BR}Z_{YB}}{Z_{BR}+Z_{RY}+Z_{YB}}$$

$$\Rightarrow 2Z_B = \frac{2Z_{BR}Z_{YB}}{Z_{BR}+Z_{RY}+Z_{YB}}$$

$$\boxed{Z_B = \frac{Z_{BR}Z_{YB}}{Z_{BR}+Z_{RY}+Z_{YB}}}$$

Substituting the value of Z_B in equation (3), we get,

$$Z_Y + \left[\frac{Z_{BR}Z_{YB}}{Z_{BR}+Z_{RY}+Z_{YB}}\right] = \frac{Z_{YB}(Z_{BR}+Z_{RY})}{Z_{YB}+Z_{BR}+Z_{RY}}$$

$$\Rightarrow Z_Y = \frac{Z_{YB}Z_{BR}+Z_{YB}Z_{RY}}{Z_{YB}+Z_{BR}+Z_{RY}} - \frac{Z_{BR}Z_{YB}}{Z_{YB}+Z_{BR}+Z_{RY}}$$

$$\Rightarrow Z_Y = \frac{Z_{YB}Z_{BR}+Z_{YB}Z_{RY}-Z_{BR}Z_{YB}}{Z_{YB}+Z_{BR}+Z_{RY}}$$

$$\boxed{Z_Y = \frac{Z_{YB}Z_{RY}}{Z_{YB}+Z_{BR}+Z_{RY}}}$$

Q42. Explain the conversion of a star connected load with its equivalent delta connected load.

Ans:

The equivalent star connected load of the delta connected load is shown in figure (1).

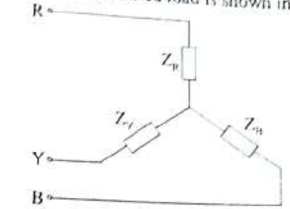


Figure (1)

Consider a delta connected load with impedances Z_{RY}, Z_{BR} and Z_{YB} as shown in figure (2).

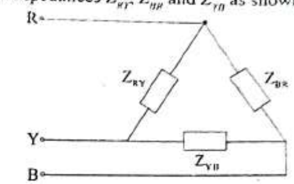


Figure (2)

We know that,

The unbalanced star connected loads, in terms of its equivalent delta connected loads are given by,

$$Z_R = \frac{Z_{BR}Z_{RY}}{Z_{RY}+Z_{YB}+Z_{BR}} \quad \dots ($$

$$Z_Y = \frac{Z_{YB}Z_{RY}}{Z_{RY}+Z_{YB}+Z_{BR}} \quad \dots ($$

$$Z_B = \frac{Z_{BR}Z_{YB}}{Z_{RY}+Z_{YB}+Z_{BR}} \quad \dots ($$

Multiplying equations (1) and (2), we get,

$$Z_R Z_Y = \frac{Z_{BR}Z_{RY}}{Z_{RY}+Z_{YB}+Z_{BR}} \times \frac{Z_{YB}Z_{RY}}{Z_{RY}+Z_{YB}+Z_{BR}}$$

$$\Rightarrow Z_R Z_Y = \frac{Z_{RY}^2 Z_{YB} Z_{BR}}{(Z_{RY}+Z_{YB}+Z_{BR})^2}$$

Multiplying equations (2) and (3), we get,

$$Z_Y Z_B = \frac{Z_{RY}Z_{YB}}{Z_{RY}+Z_{YB}+Z_{BR}} \times \frac{Z_{YB}Z_{BR}}{Z_{RY}+Z_{YB}+Z_{BR}}$$

$$\Rightarrow Z_Y Z_B = \frac{Z_{RY}Z_{YB}^2 Z_{BR}}{(Z_{RY}+Z_{YB}+Z_{BR})^2}$$

Multiplying equations (1) and (3), we get,

$$Z_R Z_B = \frac{Z_{BR}Z_{RY}}{Z_{RY}+Z_{YB}+Z_{BR}} \times \frac{Z_{YB}Z_{BR}}{Z_{RY}+Z_{YB}+Z_{BR}}$$

$$\Rightarrow Z_R Z_B = \frac{Z_{RY}Z_{YB}Z_{BR}^2}{(Z_{RY}+Z_{YB}+Z_{BR})^2}$$

On adding equations (4), (5) and (6), we get,

$$\Rightarrow \quad Z_RZ_Y + Z_YZ_B + Z_RZ_B = \frac{Z_{RY}^2 Z_{YB} Z_{BR}}{(Z_{RY}+Z_{YB}+Z_{BR})^2} + \frac{Z_{RY}Z_{YB}^2 Z_{BR}}{(Z_{RY}+Z_{YB}+Z_{BR})^2} + \frac{Z_{RY}Z_{YB}Z_{BR}^2}{(Z_{RY}+Z_{YB}+Z_{BR})^2}$$

$$\Rightarrow \quad Z_RZ_Y + Z_YZ_B + Z_RZ_B = \frac{Z_{RY}Z_{YB}Z_{BR}(Z_{RY}+Z_{YB}+Z_{BR})}{(Z_{RY}+Z_{YB}+Z_{BR})^2}$$

$$\Rightarrow \quad Z_RZ_Y + Z_YZ_B + Z_RZ_B = \frac{Z_{RY}Z_{YB}Z_{BR}}{Z_{RY}+Z_{YB}+Z_{BR}} \quad \text{... (7)}$$

Now, dividing equation (7) by equation (1), we get,

$$\frac{Z_RZ_Y + Z_YZ_B + Z_RZ_B}{Z_R} = \frac{Z_{RY}Z_{YB}Z_{BR}}{Z_{RY}+Z_{YB}+Z_{BR}} \times \frac{Z_{RY}+Z_{YB}+Z_{BR}}{Z_{BR}Z_{RY}}$$

$$Z_Y + \frac{Z_YZ_B}{Z_R} + Z_B = Z_{YB}$$

$$\boxed{\therefore Z_{YB} = Z_Y + Z_B + \frac{Z_YZ_B}{Z_R}}$$

Dividing equation (7) with equation (2), we get,

$$\frac{Z_RZ_Y + Z_YZ_B + Z_RZ_B}{Z_Y} = \frac{Z_{RY}Z_{YB}Z_{BR}}{Z_{RY}+Z_{YB}+Z_{BR}} \times \frac{Z_{RY}+Z_{YB}+Z_{BR}}{Z_{YB}Z_{RY}}$$

$$Z_R + Z_B + \frac{Z_RZ_B}{Z_Y} = Z_{BR}$$

$$\boxed{\therefore Z_{BR} = Z_R + Z_B + \frac{Z_RZ_B}{Z_Y}}$$

Dividing equation (7) with equation (3), we get,

$$\frac{Z_RZ_Y + Z_YZ_B + Z_RZ_B}{Z_B} = \frac{Z_{RY}Z_{YB}Z_{BR}}{Z_{RY}+Z_{YB}+Z_{BR}} \times \frac{Z_{RY}+Z_{YB}+Z_{BR}}{Z_{YB}Z_{BR}}$$

$$\frac{Z_RZ_Y}{Z_B} + Z_Y + Z_R = Z_{RY}$$

$$\boxed{\therefore Z_{RY} = Z_R + Z_Y + \frac{Z_RZ_Y}{Z_B}}$$

MUTUAL COUPLED CIRCUITS, DOT CONVENTION IN COUPLED CIRCUITS, IDEAL TRANSFORMER

Q43. Explain,

(i) Self inductance

(ii) Mutual inductance.

Model Paper-II, Q7

Ans:

(i) Self Inductance

Consider a solenoid or coil having N turns, through which current of I Amperes is passing. Let the flux produced be ϕ Webers. Then, total magnetic flux linked with coil, due to which current is $N\phi$. Now, this magnetic flux is proportional to the current producing it i.e., $N\phi \propto I$.

or $\quad N\phi = LI \quad$ ... (1)

Where L is a constant of proportionality, called coefficient of self-inductance or simply self induction.

From equation (1), we get,

$$L = \frac{N\phi}{I} \text{ Henry} \quad \text{... (2)}$$

If, $I = 1$ Ampere, then, $L = N\phi$ Weber-turns.

Hence, coefficient of self induction of a coil is equal to the magnetic flux linked with it, when one Ampere current is flowing through it. Alternatively, it is equal to weber-turns per Ampere in the coil.

The other formula for self-inductance is,

$$L = \frac{N\phi}{I} = \frac{N}{I} \times \frac{\text{M.M.F}}{S}$$

$$= \frac{N}{I} \times \frac{\frac{NI}{l}}{\mu_0\mu_r A} = \frac{N^2\mu_0\mu_r A}{l} \text{ Henry}$$

Where N is number of turns, A is area of cross section and l is the length of the solenoid.

$$L = \frac{\text{E.M.F}}{\text{Rate of change of current}} = \frac{e_l}{\frac{dI}{dt}}$$

Where e_l is self induced E.M.F, $\frac{dI}{dt}$ is the rate of change of current.

(ii) Mutual Inductance

The phenomenon of production of an induced E.M.F in one coil by changing the current in another nearby placed coil, is called mutual induction. The unit of mutual induction is Henry (H), which is defined as "the mutual induction produced, when current changes at the rate of one ampere per second in the primary coil, and induces an E.M.F of one volt in the secondary coil".

Let,

ϕ_1 – Flux in the primary coil (A)

I_1 – Current flowing in the primary coil

ϕ_2 – Induced flux in the secondary coil (B) and

I_2 – Current induced in the secondary coil.

Then, $\phi_2 \propto I_1$

or $\phi_2 = MI_1 \quad$ ... (3)

Where M is a constant for the two coils and is called coefficient of mutual-inductance or simply mutual induction. On differentiation of equation (3), we get,

$$\frac{d\phi_2}{dt} = \frac{MdI_1}{dt} \quad \text{(Assuming } M \text{ as constant)}$$

But,

$$\frac{d\phi_2}{dt} = e_2 \quad \text{(Induced E.M.F in coil } B\text{)}$$

$$\therefore e_2 = M\frac{dI_1}{dt} \quad \text{... (4)}$$

$$\text{or} \quad M = \frac{-e_2}{\frac{dI_1}{dt}}$$

If, $\frac{dI}{dt} = 1$ Ampere/second in primary coil (A), then,

$$M = -e_2 \quad \text{... (5)}$$

Hence, coefficient of mutual induction of two circuits is numerically equal to the induced E.M.F in one circuit. When the current changes at a unit rate (i.e., one ampere/second) in the other circuit. Mutual inductance is also defined as,

$$M = N_2\left(\frac{\phi_1}{I_1}\right)$$

$= N_2$ (Flux/Ampere of primary)

But,

$$\phi_1 = \frac{N_1I_1}{\text{Reluctance of primary coil}} = \frac{N_1I_1}{l_1/\mu_0\mu_r A_1}$$

$$M = \frac{\mu_0\mu_r N_1N_2A_1}{l_1} \quad \text{... (6)}$$

The other formula for mutual inductance is,

$$M = \frac{e_m}{\frac{dI}{dt}}$$

Where, e_m is mutually induced E.M.F $\frac{dI}{dt}$ is the rate of change of current.

Q44. What is magnetic coupling? What is its effect? How can you arrange two coils so that they do not have magnetic coupling?

Ans:

Magnetic Coupling

Magnetic coupling is said to exist between two coils when they are placed such that the current flowing through one coil induces e.m.f in the other coil.

The degree of coupling between two coils is measured in terms of coefficient of coupling k. The value of k ranges from 0 to 1. When $k = 1$ the two coils are said to be tightly coupled and the coupling is called as ideal coupling. When $k = 0$, magnetic coupling will not exist between the two coil.

The degree of coupling between the coils depends on the following factors,

(i) Spacing between the coils

(ii) Angle between the coils

(iii) Type of material used for the path of magnetic flux.

Effects of Magnetic Coupling

The effects of magnetic coupling are,

(i) Two coils, without any electrical contact, share common flux.

(ii) An e.m.f will be induced in one coil due to current flowing through the other.

(iii) Electrical energy will be transformed from one circuit to another without any electrical contact between them.

Arrangement of Two Coils for No Magnetic Coupling

The magnetic coupling mainly depends on two factors i.e., spacing between the coils and the angle between the coils. If the spacing between the two coils is made very large such that the lines of flux of one coil do not link with the other, then no e.m.f will be induced and magnetic coupling does not exist between them. Similarly, if the coils are placed such that the angle between them is 90° the magnetic coupling will not exist between them.

Q45. Derive the expression for coefficient of coupling between pair of magnetically coupled coils.

Model Paper-II, Q6

OR

Derive the relation between self inductance, mutual inductance and coefficient of coupling.

Ans:

Coefficient of Coupling

Let N_1, N_2, I_1 and I_2, ϕ_1 and ϕ_2 are the number of turns, current, flux of coil 1 and coil 2 respectively. These two coils are mutually coupled together.

Coefficient of coupling is defined as the ratio of flux linking with one coil to the total flux produced by the other coil. The coefficient of coupling shows the degree of coupling of two coils. The maximum value of k is 1.

$$k = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

Where,

ϕ_{12} – Flux of coil 1 that links coil 2

ϕ_{21} – Flux of coil 2 that links coil 1

ϕ_1 – Total flux of coil 1

ϕ_2 – Total flux of coil 2.

k value lies between 0 and 1.

Let, M be mutual inductance between coil 1 and coil 2.

We know that,

$$M = \frac{N_1 \phi_{21}}{I_2}$$

And $M = \frac{N_2 \phi_{12}}{I_1}$

Multiplying the above equations, we get,

$$M^2 = N_1 N_2 \frac{\phi_{21}}{I_2}\frac{\phi_{12}}{I_1}$$

$$= \frac{N_1}{I_1}\frac{N_2}{I_2} k\phi_2\, k\phi_1$$

$$\Rightarrow M^2 = k^2\left(\frac{N_1\phi_1}{I_1}\right)\left(\frac{N_2\phi_2}{I_2}\right)$$

$$= k^2 L_1 L_2$$

Where,

L_1 and L_2 are self inductance of coil 1 and coil 2 respectively.

$\therefore$ Coefficient of coupling, $k = \frac{M}{\sqrt{L_1 L_2}}$

Q46. Derive the expression of equivalent inductance of two series connected coupled coils.

Ans:

(i) **Coils Joined in Series Aiding**

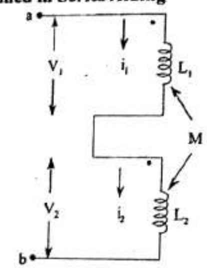


Figure (1)

Let the two inductors be connected in series with self inductances L_1, L_2 and mutual inductance M. The magnetic fluxes of self induction and of mutual induction linking with each element add together.

Let ϕ_1 and ϕ_2 be the flux produced by the coils 1 and 2 respectively. Then the total flux is given by,

$$\phi = \phi_1 + \phi_2$$

We have, $\phi_1 = L_1 i_1 + M i_2$

$\phi_2 = L_2 i_2 + M i_1$

Substituting ϕ_1 and ϕ_2 in the above equation,

$$\phi = L_1 i_1 + M i_2 + L_2 i_2 + M i_1$$

But we have, $\phi = Li$ and $i_1 = i_2 = i$

[$\because$ Circuit is a series circuit]

$$Li = L_1 i + Mi + L_2 i + Mi$$

$$\Rightarrow Li = (L_1 + L_2 + 2M)i$$

$$\boxed{\therefore L_{eq} = L_1 + L_2 + 2M}$$

The equivalent inductance for series aiding coils is

$$L_{eq} = L_1 + L_2 + 2M.$$

(ii) **Coils Joined in Series Opposing**

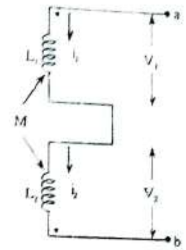


Figure (2)

In series opposing connection, the currents in the two inductors at any instant of time are in opposite direction relative to like terminals. Hence the mutual flux will be negative.

Then the total flux is, $\phi = \phi_1 + \phi_2$

For series opposing,

We have,

$\phi_1 = L_1 i_1 - M i_2$

And $\phi_2 = L_2 i_2 - M i_1$

Substituting ϕ_1 and ϕ_2 in the above equation, we get,

$$\phi = L_1 i_1 - M i_2 + L_2 i_2 - M i_1$$

But, we have,

$\phi = Li$ and $i_1 = i_2 = i$

$$Li = L_1 i - Mi + L_2 i - Mi$$

$$\Rightarrow Li = (L_1 + L_2 - 2M)i$$

$$\boxed{\therefore L_{eq} = L_1 + L_2 - 2M}$$

Therefore, the equivalent impedance for series opposing coil is $L_{eq} = L_1 + L_2 - 2M$.

Q47. Explain the dot convention for mutually coupled coils.

Ans:

Dot convention is a rule used to determine the correct polarity of the mutually induced e.m.f.

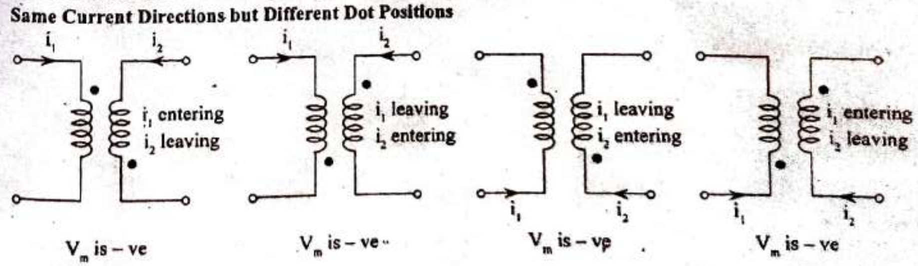
As self induction is a property associated with only one coil hence, self induced e.m.f is always positive. But, mutually induction is a property associated with two or more than two coils. Hence, mutually induced e.m.f may be positive or negative depending on the directions of currents in the coils and the winding sense of the two coils.

In order to determine the polarity of the mutually induced e.m.f, a dot is placed depending on the winding sense near the coils using right hand rule and then dot convention is applied.

According to dot convention, the mutually induced e.m.f is positive if current in both the coils enters or leaves through the dotted terminal else the mutually induced e.m.f will be negative.

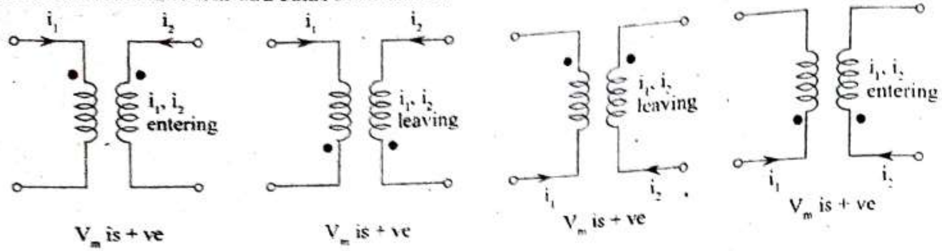
The following figure shows the polarity mutually induced e.m.f in different cases.

(i) **Same Current Directions but Different Dot Positions**



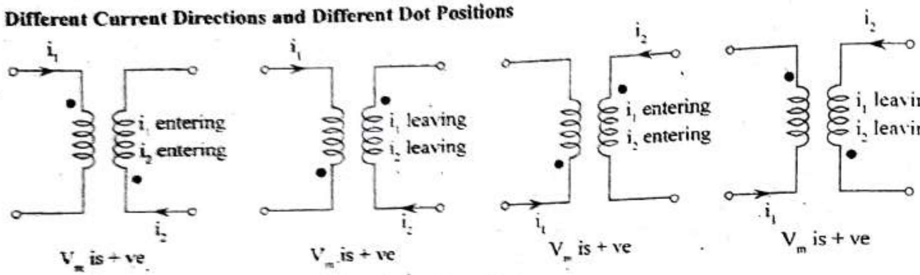
Mutually induced e.m.f in all the above four cases is negative.

(ii) Same Current Directions and Same Dot Positions



Mutually induced e.m.f in all the above four cases is positive.

(iii) Different Current Directions and Different Dot Positions

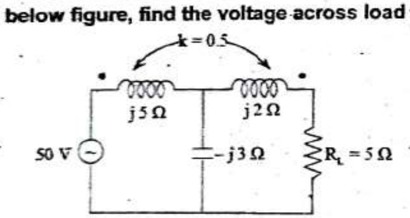


In all the above four cases, mutually induced e.m.f is positive.

When a system contains only two coils, dot(.) is placed near the coils to determine the polarity of mutually induced e.m.f. But when the system contains more than two coils using dots for all will make the calculation confusing. Hence different symbols are used to denote M between the coils.

Symbol	-	M
•	M_{12}	(Between coil 1 and 2)
■	M_{23}	(Between coil 2 and 3)
▲	M_{13}	(Between coil 1 and 3)

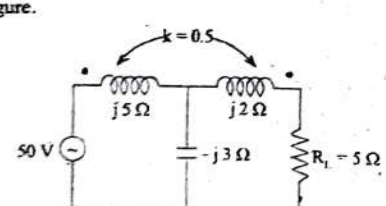
Q48. For the network shown in below figure, find the voltage across load resistance R_L.



Figure

Ans:

The given circuit is shown in figure.



Figure

To determine,

Voltage across the load resistance, $R_L = ?$

From figure, it is clear that,

Inductive reactance of coil-1, $\omega L_1 = j5\ \Omega$

Inductive reactance of coil-2, $\omega L_2 = j2\ \Omega$

Coefficient, $k = 0.5$

Now,

The mutual inductive reactance between the coil-1 and coil-2 is given as,

$$\omega M = k\sqrt{\omega L_1 . \omega L_2}$$

Substituting k, ωL_1 and ωL_2 values, we get,

$$\Rightarrow \quad \omega M = 0.5\sqrt{j5 \times j2}$$
$$= 0.5 \times j\sqrt{10}$$
$$= 0.5 \times j\,3.1622$$
$$= j\,1.5811\ \Omega$$

Let, I_1, I_2 be the loop currents flowing in the circuit.

Now, applying KVL to loop-1 and loop-2, we get,

Loop-1

$$(j5)\,I_1 - j3(I_1 - I_2) - j1.5811\,(I_2) = 50 + j0$$
$$\Rightarrow \quad j5\,I_1 - j3\,I_1 + j3\,I_2 - j1.5811\,I_2 = 50 + j0$$
$$\Rightarrow \quad j2\,I_1 + j\,1.4189\,I_2 = 50 + j0$$

Loop-2

$$(j2).\,I_2 + 5.\,I_2 - j3(I_2 - I_1) - j\,1.5811\,I_1 = 0$$
$$\Rightarrow \quad (j2).\,I_2 + 5\,.\,I_2 - j3.\,I_2 + j3\,.\,I_1 - j\,1.5811\,I_1 = 0$$
$$\Rightarrow \quad 5.\,I_2 - j1\,I_2 + j\,1.4189\,I_1 = 0$$
$$\Rightarrow \quad (5 - j1)\,I_2 + j\,1.4189\,I_1 = 0$$
$$\Rightarrow \quad j\,1.4189\,I_1 + (5 - j1)\,I_2 = 0$$

Representing equations (1) and (2) in matrix form, we get,

$$\begin{bmatrix} j2 & j1.4189 \\ j1.4189 & (5-j1) \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 50+j0 \\ 0 \end{bmatrix}$$

By Cramer's rule of solving, the current (I_2) flowing in loop-2 is given as,

$$I_2 = \frac{\begin{vmatrix} j2 & (50+j0) \\ (j1.4189) & 0 \end{vmatrix}}{\begin{vmatrix} j2 & (j1.4189) \\ (j1.4189) & (5-j1) \end{vmatrix}}$$

$$\Rightarrow \quad I_2 = \frac{(j2)(0) - (50+j0)(j1.4189)}{j2(5-j1) - (j1.4189)^2}$$

$$\Rightarrow \quad I_2 = \frac{0 - (50+j0)(j1.4189)}{j10 - j^2.2 - j^2(1.4189)^2}$$

$$\Rightarrow \quad I_2 = \frac{-(50+j0)(j1.4189)}{j10 - (-1).2 - (-1).(1.4189)^2}$$

$$\Rightarrow \quad I_2 = \frac{-(50+j0)(j1.4189)}{j10+2+(1.4189)^2}$$

$$\Rightarrow \quad I_2 = \frac{-50(j1.4189)}{j10+2+2.0132}$$

$$I_2 = \frac{-j70.945}{4.0132+j10}$$

$$I_2 = \frac{70.945\angle -90^\circ}{10.7752\angle 68.1334^\circ}$$

$$I_2 = 6.5841\angle -158.1334^\circ \text{ A}$$

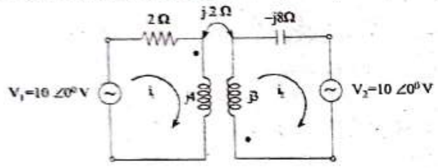
Now, the voltage across the load resistance (R_L) is given as,

$\Rightarrow \quad V_{R_L} = I_2 . R_L$

$\Rightarrow \quad V_{R_L} = (6.5841\angle -158.1334^\circ) \times 5$

$\Rightarrow \quad V_{R_L} = 32.9205\angle -158.1334^\circ$ V

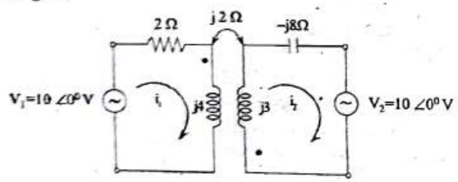
Q49. For the circuit shown in figure determine the currents i_1 and i_2 using loop method of analysis.



Figure

Ans:

Given circuit is as shown in figure,



Figure

Applying KVL to the loops,

Loop 1

$$10\angle 0^\circ = 2i_1 + (j4)i_1 + (j2)i_2 \quad \text{... (1)}$$

Loop 2

$$i_2(j3 - j8) - (j2)i_1 = -10$$

$$-(j2)i_1 - (j5)i_2 = -10$$

$$i_1 = \frac{10-(j5)i_2}{j2}$$

$$i_1 = \frac{10}{j2} - \frac{j5}{j2}i_2$$

$$= -j5 - 2.5\,i_2 \quad \text{... (2)}$$

Substituting equation (2) in equation (1) we get,

$$2(-j5 - 2.5\,i_2) + j4(-j5 - 2.5\,i_2) + (j2)\,i_2 = 10\angle 0^\circ$$

$$-j10 - 5i_2 + 20 - j10i_2 + j2i_2 = 10$$

$$(10 - j10) - (5 + j8)i_2 = 0$$

$$(5 + j8)\,i_2 = 10 - j10$$

$$i_2 = \frac{10 - j10}{5 + j8}$$

$$= \frac{14.142\angle -45^\circ}{9.434\angle 58^\circ}$$

$= 1.5\angle -103^\circ$ Amps

Substituting the value of i_2 in equation (2) we get,

$$i_1 = -j5 - 2.5\,[1.5\angle -103^\circ]$$

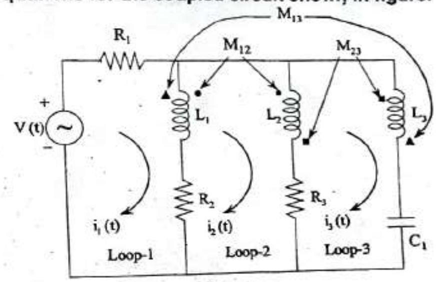
$$= -j5 - 3.75\angle -103^\circ$$

$$= -j5 + 0.843 + j3.653$$

$$= 0.843 - j1.347$$

$= 1.589\angle -57.96^\circ$ Amps

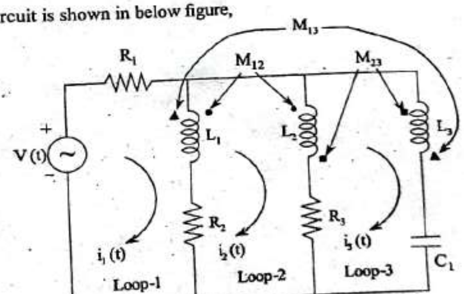
Q50. Write the loop equations for the coupled circuit shown in figure.



Figure

Ans:

Given coupled circuit is shown in below figure,



Figure

To write, the loop equations for the given circuit = ?

From the above figure, we observe that coils L_1, L_2 and L_3 are mutually coupled with each other.

Now, according to dot convention rule,

Applying KVL to loop-1, we get,

$$V(t) = R_1 i_1(t) + R_2[i_1(t) - i_2(t)] + L_1\left[\frac{d}{dt}i_1(t) - i_2(t)\right] + M_{12}\left[\frac{d}{dt}(i_2(t) - i_3(t))\right] - M_{13}\frac{d}{dt}i_3(t)$$

$$V(t) = R_1 i_1(t) + R_2 i_1(t) - R_2 i_2(t) + L_1\frac{d}{dt}i_1(t) - L_1\frac{d}{dt}i_2(t) + M_{12}\frac{d}{dt}i_2(t) - M_{12}\frac{d}{dt}i_3(t) - M_{13}\frac{d}{dt}i_3(t)$$

$$\therefore \quad V(t) = (R_1 + R_2)i_1(t) - R_2 i_2(t) + L_1\frac{d}{dt}i_1(t) - (L_1 - M_{12})\frac{d}{dt}i_2(t) - [M_{12} + M_{13}]\frac{d}{dt}i_3(t) \qquad \ldots (1)$$

Applying KVL to loop-2, we get,

$$L_2\left[\frac{d}{dt}(i_2(t) - i_3(t))\right] + R_3(i_2(t) - i_3(t)) + R_2(i_2(t) - i_1(t)) + L_1\left[\frac{d}{dt}(i_2(t) - i_1(t))\right] - M_{12}\left[\frac{d}{dt}(i_2(t) - i_1(t))\right]$$

$$-M_{12}\left[\frac{d}{dt}(i_2(t) - i_3(t))\right] - M_{23}\frac{d}{dt}i_3(t) + M_{13}\frac{d}{dt}i_3(t) = 0$$

$$\Rightarrow \quad L_2\frac{d}{dt}i_2(t) - L_2\frac{d}{dt}i_3(t) + R_3 i_2(t) - R_3 i_3(t) + R_2 i_2(t) - R_2 i_1(t) + L_1\frac{d}{dt}i_2(t) - L_1\frac{d}{dt}i_1(t) - M_{12}\frac{d}{dt}i_2(t) + M_{12}\frac{d}{dt}i_1(t)$$

$$-M_{23}\frac{d}{dt}i_3(t) - M_{12}\frac{d}{dt}i_2(t) + M_{12}i_3(t) + M_{13}\frac{d}{dt}i_3(t) = 0$$

$$\Rightarrow -R_2 i_1(t) + (R_2 + R_3)i_2(t) - R_3 i_3(t) - (L_1 - M_{12})\frac{d}{dt}i_1(t) + (L_2 + L_1 - 2M_{12})\frac{d}{dt}i_2(t) - (L_2 + M_{23} - M_{12} - M_{13})\frac{d}{dt}i_3(t) = 0$$

$$\therefore \; R_2 i_1(t) - (R_2 + R_3)i_2(t) + R_3 i_3(t) + (L_1 - M_{12})\frac{d}{dt}i_1(t) - (L_2 + L_1 - 2M_{12})\frac{d}{dt}i_2(t) + (L_2 + M_{23} - M_{12} - M_{13})\frac{d}{dt}i_3(t) = 0 \qquad \ldots (2)$$

Applying KVL to loop-3, we get,

$$L_3\frac{d}{dt}i_3(t) + \frac{1}{C_1}\int i_3(t)dt + L_2\left[\frac{d}{dt}(i_3(t) - i_2(t))\right] + R_3(i_3(t) - i_2(t)) + M_{23}\left[\frac{d}{dt}(i_3(t) - i_2(t))\right]$$

$$-M_{13}\left[\frac{d}{dt}[i_1(t) - i_2(t)]\right] - M_{12}\frac{d}{dt}[i_1(t) - i_2(t)] = 0$$

$$\Rightarrow L_3\frac{d}{dt}i_3(t) + \frac{1}{C_1}\int i_3(t)dt + L_2\frac{d}{dt}i_3(t) - L_2\frac{d}{dt}i_2(t) + R_3 i_3(t) - R_3 i_2(t) + M_{23}\frac{d}{dt}i_3(t) - M_{23}\frac{d}{dt}i_2(t) - M_{13}\frac{d}{dt}i_1(t)$$

$$+M_{13}\frac{d}{dt}i_2(t) + M_{23}\frac{d}{dt}i_3(t) - M_{12}\frac{d}{dt}i_1(t) + M_{12}\frac{d}{dt}i_2(t) = 0$$

$$\Rightarrow -R_3 i_2(t) + R_3 i_3(t) + \frac{1}{C_1}\int i_3(t)dt - (M_{13} + M_{12})\frac{d}{dt}i_1(t) - [L_2 + M_{23} - M_{13} - M_{12}]\frac{d}{dt}i_2(t)$$

$$+[L_3 + L_2 + M_{23} + M_{23}]\frac{d}{dt}i_3(t) = 0$$

$$\therefore \quad R_3 i_2(t) - R_3 i_3(t) - \frac{1}{C_1}\int i_3(t)dt + (M_{12} + M_{13})\frac{d}{dt}i_1(t) + [L_2 + M_{23} - M_{13} - M_{12}]\frac{d}{dt}i_2(t)$$

$$+[L_3 + L_2 + 2M_{23}]\frac{d}{dt}i_3(t) = 0 \qquad \ldots (3)$$

Therefore, equations (1), (2) and (3) represents the loop equations of given circuit.

4

ELECTRICAL CIRCUIT ANALYSIS USING LAPLACE TRANSFORMS

PART-A
SHORT QUESTIONS WITH SOLUTIONS

Q1. Define laplace transform and its inverse.

Ans:

Laplace Transform

The laplace transform of a signal $x(t)$ is defined as,

$$L\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t).e^{-st}dt$$

Where,

s – Complex variable and is equal to $\sigma + j\omega$.

Inverse Laplace Transform

Inverse Laplace transform of $X(s)$ is defined as,

$$x(t) = \frac{1}{2\pi j}\int_{c-j\omega}^{c+j\omega} X(s).e^{st}ds$$

or

$$x(t) = \frac{1}{2\pi j}\int_{-\infty}^{\infty} X(s)e^{st}ds$$

Q2. Define unilateral laplace transform.

Ans:

Unilateral Laplace transform of a signal $x(t)$ is defined as,

$$L\{x(t)\} = X(s) = \int_{0}^{\infty} x(t).e^{-st}dt$$

Where,

$s = \sigma + j\omega$ is a complex variable.

Q3. What is the condition to be satisfied for the existence of laplace transform?

Ans:

Laplace transform of signal $x(t)$ is existable if and only if it satisfies the following conditions,

(i) $x(t)$ should be continuous (or) piece wise continuous

(ii) $x(t).e^{-\sigma t}$ must be absolutely integrable, it means that $X(s)$ exists only if,

$$\int_{-\infty}^{\infty}\left|x(t)e^{-\sigma t}\right|dt < \infty$$

or

$$\underset{t\to\infty}{Lt}\; e^{-\sigma t}x(t) = 0$$

Q4. List the properties of Laplace transform.

Ans:

1. **Linearity**

The linearity property states that if $x_1(t) \xleftrightarrow{LT} X_1(s)$ and $x_2(t) \xleftrightarrow{LT} X_2(s)$ then,

$$ax_1(t) + bx_2(t) \xleftrightarrow{LT} aX_1(s) + bX_2(s)$$

2. **Time Shift Property**

The time shift property of Laplace transform is defined as follows,

$$x(t) \xleftrightarrow{LT} X(s)$$

Then, $x(t-t_0) \xleftrightarrow{LT} e^{-st_0}.X(s)$

3. **Time Scaling Property**

The time scaling property of Laplace transform is defined as,

If $x(t) \xleftrightarrow{LT} X(s)$ then,

$$x(at) \xleftrightarrow{LT} \frac{1}{|a|}X\left[\frac{s}{a}\right]$$

4. **Time Reversal**

Time reversal property states that if $x(t) \xleftrightarrow{LT} X(s)$ then $x(-t) \xleftrightarrow{LT} X(-s)$

5. **Differentiation Property: (In Time Domain)**

At zero initial conditions, the differentiation property of Laplace transform is defined as,

$$\frac{d}{dt}x(t) \xleftrightarrow{LT} sX(s)$$

6. **Differentiation Property in s-domain**

The differentiation property of Laplace transform in s-domain is defined as,

$$tx(t) \xleftrightarrow{LT} -\frac{d}{ds}X(s)$$

7. **Frequency Shift**

The frequency shift property of Laplace transform is defined as,

$$e^{-at}x(t) \xleftrightarrow{LT} X(s+a)$$

8. **Time Convolution**

Time convolution property of Laplace transform states that,

$$x_1(t) * x_2(t) \xleftrightarrow{LT} X_1(s).X_2(s).$$

Q5. Define ROC of the Laplace transform.

Ans:

The Laplace transform is the Fourier transform of $x(t)e^{-\sigma t}$. Hence a necessary condition for existence of Laplace transform is absolute integrability of $x(t)e^{-\sigma t}$ i.e., we must have,

$$\int_{-\infty}^{\infty} |x(t)e^{-\sigma t}|\, dt < \infty$$

The range of σ for which the Laplace transform converges is termed 'Region of Convergence' (ROC).

Q6. State initial and final value theorem of Laplace transforms.

Ans:

Initial Value Theorem of Laplace Transform

If Laplace transform of $f(t)$ and its first derivative exists, then its initial value is given by,

$$f(0^+) = \underset{s\to\infty}{Lt}\, [sF(s)]$$

Final Value Theorem

If the Laplace transform of $f(t)$ and its derivative exists, then the final value of $f(t)$ is given by,

$$\underset{s\to 0}{Lt}\, [sF(s)] = f(\infty).$$

Q7. Find the Laplace transform of signal x(t) = e^{-at} u(t).

Ans:

Given that,

$$x(t) = e^{-at}\, u(t) \quad \dots (1)$$

Laplace transform of $x(t)$ is $X(s) = ?$

The Laplace transform of a signal, $x(t)$ is given as,

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st} dt$$

$$= \int_{0}^{\infty} x(t)\, e^{-st}\, dt \quad (\because x(t) \text{ is positive signal}) \quad \dots (2)$$

On substituting the equation (1) in equation (2), we get,

$$X(s) = \int_0^{\infty} e^{-at}u(t)\, e^{-st} dt$$

$$= \int_0^{\infty} e^{-at}e^{-st} dt \qquad \left[\because u(t) = 1 \text{ for } t \geq 0 \atop = 0 \text{ for } t < 0\right]$$

$$= \int_0^{\infty} e^{-(a+s)t}\, dt$$

$$= \int_0^{\infty} e^{-(s+a)t}\, dt$$

$$= \left[\frac{e^{-(s+a)t}}{-(s+a)}\right]_0^{\infty}$$

$$= \left[\frac{e^{-(s+a)\infty}}{-(s+a)}\right] - \left[\frac{e^{-(s+a)(0)}}{-(s+a)}\right] = 0 + \frac{e^0}{s+a}$$

$$= \frac{1}{s+a} \qquad (\because e^{-\infty} = 0, e^0 = 1)$$

$$\boxed{\therefore L[e^{-at}u(t) = X(s) = \frac{1}{s+a}}$$

Q8. Find the Laplace transform of 'sinωt'.

Ans:

Given that,

$$x(t) = \sin\omega t$$

sinωt can be expressed as,

$$\sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

The Laplace transform of $x(t)$ is defined as,

$$L\{x(t)\} = \int_0^{\infty} x(t).e^{-st} dt$$

$$= \int_0^{\infty} \sin\omega t\, e^{-st}\, dt$$

$$= \int_0^{\infty}\left(\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right) e^{-st}\, dt$$

$$= \frac{1}{2j}\int_0^{\infty}\left(e^{j\omega t}.e^{-st} - e^{-j\omega t}.e^{-st}\right)dt$$

$$= \frac{1}{2j}\left[\int_0^{\infty} e^{-(s-j\omega)t} dt - \int_0^{\infty} e^{-(s+j\omega)t} dt\right]$$

$$= \frac{1}{2j}\left[\left[\frac{e^{-(s-j\omega)t}}{-(s-j\omega)}\right]_0^{\infty} - \left[\frac{e^{-(s+j\omega)t}}{-(s+j\omega)}\right]_0^{\infty}\right]$$

$$= \frac{1}{2j}\left[\left(0 + \frac{1}{s-j\omega}\right) + \left(0 - \frac{1}{s+j\omega}\right)\right]$$

$$= \frac{1}{2j}\left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right]$$

$$= \frac{1}{2j}\left[\frac{s+j\omega - s + j\omega}{s^2+\omega^2}\right]$$

$$= \frac{2j\omega}{2j(s^2+\omega^2)}$$

$$\boxed{\therefore L\{\sin\omega t\} = \frac{\omega}{s^2+\omega^2}}$$

Q9. Compare the classical and Laplace transform methods of solution of the network.

Ans:

Laplace transformation is a powerful method of solving differential equations usually encountered in an electric circuits.

The method has a number of advantages over the classical methods. They are as follows.

1. The solutions of differential equations are routine and progress systematically, whereas in classical method the solution of differential equation is random.
2. The laplace method gives the total solution, the particular integral and the complementary function in one operation. In classical method, particular integral and complementary function are separately calculated.
3. Initial conditions are automatically specified in the transformed equation. Further, the initial conditions are incorporated into the problem as the first step unlike the last step in classical method.
4. The laplace transform converts the integral differential equation into an algebraic equation in S (laplace operator). Simple algebraic rules are used to obtain the expression in suitable form. The final solution is obtained by taking the inverse laplace transform.

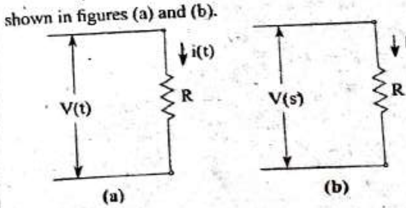
Q10. Obtain the s-domain equivalent for resistor.

Ans:

s-domain Equivalent of Resistor

s-domain equivalent of resistance, R is obtained from a time-domain as follows,

Representation of resistance in time and s-domain are shown in figures (a) and (b).



Figure

V-I relationship for a resistor is,

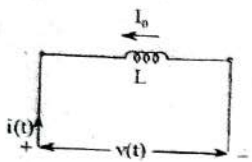
$$V(t) = i(t)\, R$$

Applying Laplace transform, we get,

$$V(s) = I(s)\, R$$

$\therefore$ $I(s)$ R is the equivalent s-domain voltage term of the resistance R.

Q11. Obtain the s-domain equivalent for inductor.



Figure

Ans:

s-domain Equivalent of Inductor

Given that,

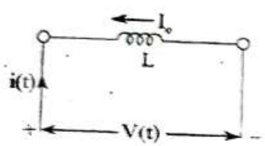


Figure (1)

The V-I relationship of figure (1) is,

$$V(t) = L\frac{di(t)}{dt}$$

Applying Laplace transform, we get,

$$V(s) = LsI(s) - Li(0^-)$$

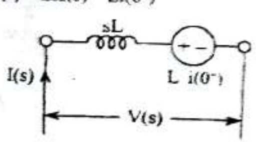
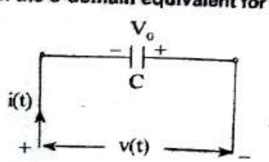


Figure (2)

Figure (2), shows the s-domain equivalent of the inductance shown in figure (1). While the term $sLI(s) - Li(0^-)$ represents s-domain voltage term of the inductance with initial current I_0.

Q12. Obtain the s-domain equivalent for capacitor.



Figure

Ans:

s-domain Equivalent of the Capacitor

Given that,

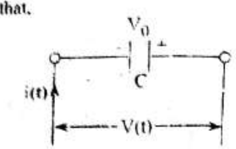


Figure (1)

The V-I relationship of the figure (1) is,

$$i = C\frac{dv}{dt}$$

$$V = \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

Applying Laplace transform, we get,

$$I(s) = Cs(V(s) + V_0)$$

$$= CsV(s) + C\,V_0$$

$$V(s) = \frac{I(s)}{Cs} - \frac{V_0}{s}$$

$\frac{I(s)}{Cs} - \frac{V_0}{s}$ is the equivalent s-domain voltage term of the capacitor shown in figure (1) and its equivalent s-domain representation of capacitors with initial voltage 'V_o' shown in figure (2)

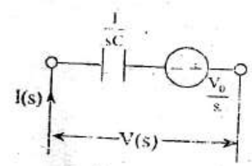


Figure (2)

Q13. Determine the Laplace transform of unit step function.

Ans: Model Paper-II, Q1(g)

Unit Step Function

A unit step function has a constant positive unit magnitude for any time greater than 1 and has a zero magnitude for any time less than zero. The unit step function is denoted by $u(t)$. Mathematically it is expressed as,

$$u(t) = 1 \;\; ; \; t \geq 0$$

$$= 0 \;\; ; \; t < 0$$

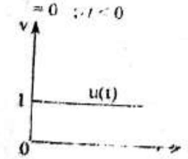


Figure: Unit Step Function

From, the definition of Laplace transform, we have,

$$F(s) = L[f(t)] = \int_0^{\infty} f(t).e^{-st}\,dt \qquad \text{... (1)}$$

When an unit step function is considered, then,

$$f(t) = u(t) = 1 \; ; \; t \geq 0$$

Substitute the value of $f(t)$ in equation (1), we get,

$$F(s) = L[f(t)] = L[u(t)]$$

$$= \int_0^{\infty} u(t).e^{-st}\,dt$$

$$= \int_0^{\infty} 1.e^{-st}\,dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_0^{\infty}$$

$$= \frac{-1}{s}[e^{-st}]_0^{\infty}$$

$$= \frac{-1}{s}[e^{-\infty} - e^{0}]$$

$$= \frac{-1}{s}[0-1] \qquad \left[\because e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0\right]$$

$$= \frac{-1}{s}(-1)$$

$$= \frac{1}{s}$$

Q14. Determine Laplace transform of impulse function.

Ans:

Impulse Function

The function is said to be an impulse function, if the function has only one non-zero value with infinite magnitude i.e., it has zero magnitude for $t > 0$ and has infinite value at t = 0. It is denoted with $\delta(t)$. Mathematically it is expressed as,

$$\delta(t) = 1 \; ; \; t = 0$$

$$0 \; ; \; t > 0$$

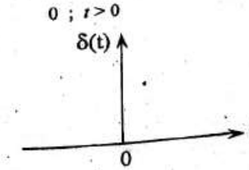


Figure: Impulse Function

The impulse function is obtained by differentiating the unit step function $u(t)$ i.e.,

$$\delta(t) = \frac{d}{dt}[u(t)]$$

Now, Laplace transform of impulse function will be,

$$L[\delta(t)] = L\left[\frac{d}{dt}[u(t)]\right]$$

$$= [sF(s) - f(0+)] \qquad \text{... (1)}$$

Assuming the system to be initially relaxed, we have,

$$f(0+) = 0$$

$$\therefore \; L[\delta(t)] = sF(s) - 0$$

$$= sF(s)$$

$$= s.L[u(t)] \qquad \left(\because \; L[u(t)] = \frac{1}{s}\right)$$

$$= s.\frac{1}{s} = 1$$

Q15. Determine Laplace transform of ramp function.

Ans:

Ramp Function

If any time variant function varies linearly with time then the function is said to be a ramp function. It is denoted by $r(t)$. Mathematically it is expressed as,

$$r(t) = t \; ; \; t > 0$$

$$= 0 \; ; \; t \leq 0$$

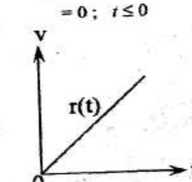


Figure: Ramp Function

The ramp function is obtained by integrating the unit step function, i.e.,

$$r(t) = \int_0^{t} u(t)\,dt$$

Now, the Laplace transform of ramp function is given by,

$$L[r(t)] = L\left[\int_0^{t} u(t)dt\right]$$

$$= \frac{F(s)}{s} - \frac{f'(0)}{s}$$

Assuming the system to be initially relaxed, we have,

$$f(0) = 0$$

$$\therefore \quad L[r(t)] = \frac{F(s)}{s} - \frac{0}{s}$$

$$= \frac{1}{s}.F(s)$$

$$= \frac{1}{s}.L[u(t)]$$

$$= \frac{1}{s} \times \frac{1}{s} \qquad \left(\because L[u(t)] = \frac{1}{s}\right).$$

$$= \frac{1}{s^2}$$

Q16. Show that convolution of any function with unit impulse function is the functions itself.

Ans:

From the convolution theorem of Laplace transforms, we have,

$$L_t[f_1(t) * f_2(t)] = L[f_1(t) * f_2(t)]$$

$$= F_1(s)\, F_2(s).$$

Let us consider a function $f(t)$ whose Laplace transform is $F(s)$.

Now,

$$L[f(t) * \delta(t)] = L[f(t)] \times L[\delta(t)]$$

Laplace transform of a unit impulse function is 1,

i.e., $L[\delta(t)] = 1.$

$$\therefore \quad L[f(t) * \delta(t)] = F(s) \times 1$$

$$= F(s)$$

Taking inverse Laplace Transform on both sides, we get,

$$L^{-1}[L[f(t) * \delta(t)]] = L^{-1}[F(s)]$$

$$f(t) * \delta(t) = f(t)$$

Therefore, convolution of any function with a unit impulse function is the function itself.

Q17. Define driving point impedance.

Ans: Model Paper-III, Q1(g)

Driving point impedance is defined as the ratio of Laplace transform of voltage at any port to the Laplace transform of current at the same port without considering the initial conditions. It is denoted by $Z(s)$.

$$\therefore \quad Z(s) = \frac{V(s)}{I(s)}$$

We know that, driving point impedance, $Z(s) = \frac{V(s)}{I(s)}$

In this case, a pole of $Z(s)$ implies zero current for a finite voltage which means that it is open circuit and when zero is taken into consideration then $Z(s)$ implies zero voltage for a finite current which means a short circuit. Thus, we can conclude that one terminal pair network will have an open circuit for pole frequencies and will have short circuit at zero frequencies.

Q18. What is meant by transformed variable? Give an example.

Ans:

Transformed Variable

The variable which is used in the transformation of elements of a network from time domain to frequency domain is known as transformed variable. It is also called as Laplace transform variable (or) complex variable. It is denoted of a network i.e., R, L and C in terms of complex impedance $Z(s)$ (or) complex admittance $Y(s)$.

Example

The elements R, L and C can be represented in terms of complex impedance and complex admittance as follows,

$$R \rightarrow Z_R(s) = \frac{1}{Y_R(s)} = R$$

$$L \rightarrow Z_L(s) = \frac{1}{Y_L(s)} = Ls$$

$$C \rightarrow Z_C(s) = \frac{1}{Y_C(s)} = \frac{1}{Cs}$$

Q19. Give expressions for following.

(i) Driving point admittance

(ii) Voltage transfer ratio.

Ans:

(i) Driving Point Admittance Function

Driving point admittance function in denoted by $Y(s)$ and is defined as the ratio of Laplace transform of current to Laplace transform of voltage measured at the same port.

We know that,

Driving point admittance,

$$Y(s) = \frac{I(s)}{V(s)}$$

In this case, a pole of $Y(s)$ implies a zero voltage, $V(s)$ for a finite value of driving current resulting in a short circuit. On the other hand, a zero of $Y(s)$ implies a zero current $I(s)$ for a finite value of driving voltage $V(s)$ which means an open circuit.

(ii) Voltage Transfer Ratio

The voltage transfer ratio is given as,

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)}$$

$$\Rightarrow \quad V_2(s) = G_{21}(s)\,.\,V_1(s)$$

In the above expression, the product of the input and the transfer function i.e., $[G_{21}(s) \times V_1(s)]$ can be obtained in the form of a ratio of polynomial in 's' after which by making use of partial fractions a pole of $G_{21}(s)$ or $V_1(s)$ can be obtained without repeating any roots.

Hence from the above, we can conclude that poles are useful in determining the time variation of the response and zeros are useful in determining the magnitude response. Similarly, for any other remaining network functions, poles are useful in determining the time domain behavior and zeros are useful in determining the magnitude of each term of the response.

Q20. What are the poles and zeros? What information do they provide in respect of the network to which they relate?

Ans:

Consider a network functions,

$$H(s) = \frac{C(s)}{R(s)}$$

$$= \frac{m_0 s^p + m_1 s^{p-1} + m_2 s^{p-2} + \ldots + m_p}{n_0 s^q + n_1 s^{q-1} + n_2 s^{q-2} + \ldots + n_q} e^{\theta} \quad \ldots (1)$$

Where, m_0, m_1, m_2,..., m_p are the coefficients of the numerator $C(s)$ and n_0, n_1, n_2 ... n_q are the coefficients of the denominator $R(s)$ in the network function.

The coefficients of numerator and denominator are real and positive for a passive network.

Here, $m_1, m_2, \ldots m_p$ are the p roots of the polynomial $C(s)$ and $n_1, n_2, \ldots n_q$ are the q roots of the polynomial $R(s)$. Then equation (1) can be written as,

$$G(s) = \frac{C(s)}{R(s)}$$

$$= \frac{m_0}{n_0} \frac{(s-m_1)(s-m_2)\ldots(s-m_p)}{(s-n_1)(s-n_2)\ldots(s-n_q)} \quad \ldots (2)$$

The roots of the numerator are called as zero's and the roots of the denominator are called as poles. The ratio $\left(\frac{m_0}{n_0}\right)$ is called the scale factor.

Poles and zeros are useful in describing a network function. Generally, these poles and zeros are plotted on s-plane and such a configuration is termed as pole-zero diagram (or) pole zero plot.

Consider a network function,

$$G(s) = \frac{4s(s+2)}{(s^2+2)(s+3)}$$

The network function the following two zeros and 3 poles, as shown in as figure.

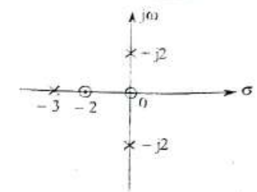


Figure: Pole-zero Plot

At poles i.e., at $s = \pm j2$ or $s = -3$ the network function becomes infinity, while at zeros i.e., at $s = 0$ and at $s = -2$, the network function becomes zeros.

Therefore, poles and zeros gives the information about the network function, where it becomes zero and infinity.

Significance of Poles and Zeros in a Transfer Function

Poles and zeros are critical frequencies of system for which network function becomes infinite at poles and zero at zeros. However, network functions will have a finite non zero value at remaining left over complex frequencies. With the help of poles and zeros, valuable information of the network function can be determined.

Q21. Define frequency response.

Ans: Model Paper-II, Q1(h)

Frequency response is defined as the response of a system to sinusoidal inputs at variable frequency.

Frequency response is usually characterized by variations of magnitude in dB and angle in degrees with variations of frequency in radians.

The stability analysis of a system can be done using frequency domain analysis which is quite easy when compared with time domains analysis.

The frequency domain analysis can be done using the frequency response which is obtained by replacing s with $j\omega$ in the transfer function.

The various techniques available in frequency domain analysis for stability assessment are,

(i) Bode plots

(ii) Nyquist plots

(iii) Root locus

(iv) Polar plots.

Q22. List the advantages of frequency domain analysis.

Ans:

1. We can use the data obtained from the measurements on the physical system without deriving its mathematical model.
2. Frequency response analysis is most powerful in conventional control theory. They are indispensable to robust control theory.

3. Frequency response analysis is very simple and can be made accurate by use of readily available sinusoidal signal generators and precise measurement equipments.
4. By the use of frequency response, a system may be designed so that the effects of undesirable noise are negligible and such analysis and design can be extended to certain non-linear control system.
5. The absolute and relative stability of closed loop system can be estimated from the knowledge of their open loop frequency response.
6. The transfer function of complicated systems can be determined experimentally by the use of frequency tests.

Q23. List the disadvantages of frequency domain analysis.

Ans:

1. The frequency domain analysis gives inaccurate results when applied to non-linear or higher order systems.
2. When compared with the advance methods developed in simulation and modelling by the digital computer's, the frequency domain methods are outdated.
3. In a frequency domain analysis, even the step response of 3rd and 4th order system is assumed to be of the 2nd order and the relationship between the step response and the frequency response is found using Fourier transforms which involves tedious calculations.
4. If the time constants in a system exist for higher periods of time say (hours or days) then these methods fail to apply.
5. To calculate the frequency response practically is a time consuming and cumbersome process.

Q24. List the various frequency domain specifications.

Ans:

The various frequency domain specifications are,

(i) Resonant frequency
(ii) Band width
(iii) Cut-off rate
(iv) Resonant peak
(v) Gain crossover frequency
(vi) Phase crossover frequency
(vii) Gain margin
(viii) Phase margin.

Q25. What is minimum phase function?

Ans:

Minimum phase function is the transfer function which has all the poles and zeroes located in the left half of the s-plane.

Consider the transfer function,

$$G(s) = \frac{(1 + jsT_0)}{(1 + sT_1)(1 + sT_2)}$$

The given transfer function has a zero at $s = \frac{-1}{T_0}$ and poles at $s = \frac{-1}{T_1}$ and $\frac{-1}{T_2}$

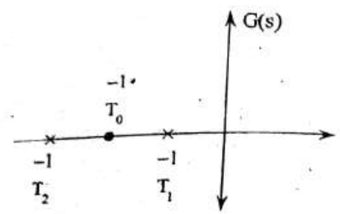


Figure: Pole-zero Pattern of Minimum Phase Function

Q26. What is non-minimum phase function?

Ans:

Non-minimum phase function is the transfer function which has one or more zeroes in right half of the s-plane.

Consider the transfer function,

$$G(s) = \frac{(1 - sT_0)}{(1 + sT_1)(1 + sT_2)}$$

The given transfer function has zero at $s = \frac{1}{T_0}$ and poles at $s = \frac{-1}{T_1}$ and $\frac{-1}{T_2}$

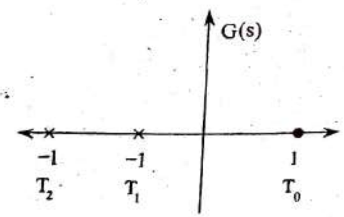


Figure: Pole-zero Pattern of Non-minimum Phase Function

Q27. What is all-pass function?

Ans:

An all pass function is the transfer function having antisymmetric pole-zero pattern about the imaginary axis. Also, the magnitude is unity at all frequencies.

Consider the transfer function,

$$G(s) = \frac{1 - j\omega T_0}{1 + j\omega T_1}$$

The given transfer function has one zero at $s = \frac{1}{T_0}$ and one pole at $s = \frac{-1}{T_1}$ (anti-symmetry pattern).

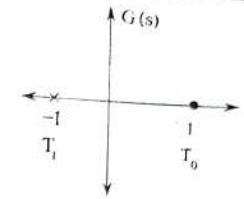


Figure: Pole-zero Pattern of an All Pass Function

Q28. Give the reasons why series resonance circuit is known as voltage resonance circuit and parallel resonance circuit is known as current resonance circuit.

Ans:

In series resonance circuits, the voltages across the inductor and capacitor are much more than the applied voltage. Hence the reason, series resonance circuits are called as voltage resonance circuits.

In parallel resonance circuits, the circulating currents between parallel branches is much more than the line current. Hence the reason, parallel resonance circuits are called as current resonance circuits.

Q29. Write the differences between series and parallel resonance.

Ans:

	Series Resonance		Parallel Resonance
1.	A circuit is said to be in series resonance, when input impedance is minimum or when net reactance is zero, When, $\omega L - \frac{1}{\omega C} = 0$	1.	A circuit is said to be in parallel resonance, when input admittance is minimum (or) input impedance is maximum i.e., when the susceptance part in the admittance becomes zero. $\Rightarrow \omega = \frac{1}{\sqrt{LC}}$
2.	The voltages across capacitance and inductance are equal in magnitude and are 180° out of phase with each other. Hence, they cancel out each other and zero voltage appears across the LC combination at resonance.	2.	The two branch currents are equal in magnitude and are 180° out of phase with each other. Hence, they cancel out each other and the total current in the circuit is zero.
3.	The resonant frequency (f_r) is given by, $f_r = \frac{1}{2\pi\sqrt{LC}}$ Hz	3.	The resonant frequency (f_r) is given by, $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R}{L^2}}$ Hz

Q30. A function is given by $Z(s) = \frac{2s}{s^2 + 16}$. Draw its pole-zero plot.

Ans:

Give that,

$$Z(s) = \frac{2s}{s^2 + 16}$$

$$Z(s) = \frac{2s}{(s^2 + 4^2)}$$

Poles

Equating the denominator of $Z(s)$ to zero, we get,

$$s^2 + 4^2 = 0$$

$$\Rightarrow \quad s^2 = -4^2$$

$$\Rightarrow \quad s = \pm\sqrt{-4^2}$$

$$\Rightarrow \quad s = \pm j4$$

$\therefore$ $s = 4j$ and $s = -4j$ are the poles of given impedance function.

Zeros

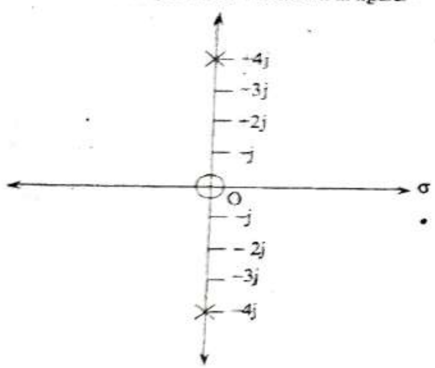
Equating the numerator of $Z(s)$ to zero, we get,

$$2s = 0$$

$$\Rightarrow \quad s = 0$$

$\therefore$ $s = 0$ is the zero of given impedance function.

The Pole-Zero plot of $Z(s)$ is shown in figure.



Figure

Q31. Determine the poles and zeros of the impedance function, $Z(s) = \dfrac{s^2+4s+3}{s^3+6s^2+8s}$ and locate these on the plane.

Ans:

Given that,

$$Z(s) = \frac{s^2+4s+3}{s^3+6s^2+8s}$$

$$= \frac{s^2+3s+1s+3}{s(s^2+6s+8)}$$

$$= \frac{s(s+3)+1(s+3)}{s(s^2+4s+2s+8)}$$

$$= \frac{(s+1)(s+3)}{s(s+4)(s+2)}$$

$\therefore$ From the impedance function the zero are given by,

$$(s+3)(s+1) = 0$$

i.e., $s = -3$ and $s = -1$

And also,

The poles are given as,

$$s(s+2)(s+4) = 0$$

i.e., $s = 0$

$s = -2$ and

$s = -4$

Representing the poles and zeros on s-plane as shown in the following figure.

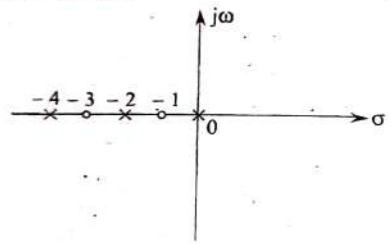


Figure: Pole Zero Diagram

Q32. The dynamic impedance of a parallel resonant circuit is 500 kΩ. The circuit consists of a 250 pF capacitor in parallel with a coil of resistance 10 Ω. Calculate the coil inductance.

Ans:

Given that,

Dynamic impedance, $Z = 500$ kΩ

Capacitor, $C = 250$ pF

Resistance, $R = 10\ \Omega$

To find,

Inductance, $L = ?$

The expression for dynamic impedance is given by,

$$Z = \frac{L}{CR}$$

$$\Rightarrow \quad L = ZCR$$

$$= 500 \times 10^3 \times 250 \times 10^{-12} \times 10$$

$$= 1.25 \times 10^{-3}\ \text{H}$$

$$= 1.25\ \text{mH}$$

$\therefore$ The required coil inductance value is 1.25 mH.

PART-B
ESSAY QUESTIONS WITH SOLUTIONS

REVIEW OF LAPLACE TRANSFORM, ANALYSIS OF ELECTRICAL CIRCUITS USING LAPLACE TRANSFORM FOR STANDARD INPUTS, INVERSE LAPLACE TRANSFORM

Q33. Define Laplace transform. Distinguish between Laplace transform and continuous time Fourier transforms.

Ans:

Laplace Transform

The Laplace transform of a function $x(t)$ is defined as,

$$L[x(t)] = x(s) = \int_0^\infty x(t)e^{-st}\,dt$$

Where 's' is a complex variable and is given as, $s = \sigma + j\omega$.

The difference between Laplace transform and continuous time Fourier transform is as follows.

	Laplace Transform		Continuous Time Fourier Transform
1.	Laplace transform is defined as, $X(s) = \int_0^\infty x(t)e^{-st}\,dt$.	1.	The continuous time Fourier transform is defined as, $X(W) = \int_0^\infty x(t)e^{-j\omega t}\,dt$.
2.	It is a one sided or single sided transform ranges 0 to ∞.	2.	It is a double sided transform that ranges – ∞ to ∞ .
3.	It is applicable for transient analysis.	3.	It is applicable for steady state analysis.
4.	Laplace transform belongs to the entire s-plane.	4.	Fourier transforms belongs to only imaginary i.e., jω axis.

Q34. Explain in detail about Laplace transform and its applications.

Ans:

Laplace transform is a powerful tool for analysis and design of continuous time signals and systems that cannot be analyzed using Fourier transform.

Fourier transform can be applied only to those signals that can be absolutely integrated and have a finite energy. It cannot be applied to signals that cannot be integrated absolutely and have infinite energy. The Fourier transform of signal $f(t)$ is defined as,

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt \quad \text{... (1)}$$

The integral of the infinite energy signal can be made converge or absolute by adding are real part to the '$j\omega$' term in the Fourier transform. That is, by changing the variable '$j\omega$' of the Fourier transform to $s = \sigma + j\omega$ the infinite energy signal can be converged.

If $x(t)$ denotes an infinite energy signal then its transform is obtained as,

$$X(\sigma + j\omega) = \int_{-\infty}^{\infty} x(t)e^{-(\sigma+j\omega)t}\,dt \quad \text{... (2)}$$

Replacing $\sigma + j\omega$ with s, that is, $s = \sigma + j\omega$, we get,

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt \quad \text{... (3)}$$

Equation (3), is the laplace transform of the infinite energy signal $x(t)$. Equation (3), can also be written as,

$$X(s) = \int_{-\infty}^{\infty}[x(t)e^{-\sigma t}]e^{-j\omega t}\,dt$$

Because $|e^{j\omega t}| = 1$, the integral on right-hand side of above equation converges if,

$$\int_{-\infty}^{\infty} \left| x(t) e^{-\sigma t} \right| dt < \infty \quad \dots (4)$$

Hence, Laplace transform exists only if the integral in expression (4) is finite for some value of 'σ'.

Therefore, for a signal $x(t)$, its laplace transform $X(s)$ is defined as,

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \quad \dots (I)$$

And $x(t)$ is said to be inverse laplace trasnform of $X(s)$ if,

$$x(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} X(s) e^{st} ds \quad \dots (II)$$

Where, 'C' is constant chosen to ensure the convergence of the integral in equation (I).

Fourier transform expresses a signal as a superposition of sinusoids, whereas a laplace transform expresses a signals as a superposition of exponentials of the form e^{st}. Laplace transform is used in physics and engineering for analysis of linear time-invariant systems like electrical circuits, harmonic oscillators, optical devices and mechanical systems. In such analyses, the laplace transform is often interpreted as a transformation from time-domain to the frequency domain. Further, the laplace transform is very useful in solving integral and differential equations.

Q35. When a function x(t) is said to be Laplace transformable?

Ans:

A function $f(t)$ is said to be Laplace transformable if and only if it satisfies following condition,

$$\int_{0}^{\infty} |f(t)|\, e^{-\sigma_1 t} dt < \infty \quad \dots (1)$$

For some real positive σ_1, known as the abscissa of absolute convergence.

The Laplace transform $F(s)$ of $f(t)$ is,

$$F(s) = \int_{0}^{\infty} f(t)\, e^{-st} dt = L\{f(t)\} \quad \dots (2)$$

Similarly, given $F(s)$ the inverse Laplace transform $f(t)$ is expressed as,

$$f(t) = L^{-1}\{F(s)\} \quad \dots (3)$$

And defined as,

$$f(t) = \frac{1}{2\pi j} \int_{C-j\infty}^{C+j\infty} F(s)\, e^{st} ds \quad \dots (4)$$

Where, C is a real constant that is greater than the real part of all the singularities of $F(s)$.

Q36. State and prove initial value and final value theorem with respect to Laplace transform.

Ans:

1. Initial Value Theorem of Laplace Transform

If Laplace transform of $f(t)$ and its first derivative exists, then its initial value is given by,

$$f(0^+) = \underset{s\to\infty}{Lt} [sF(s)]$$

Proof

Laplace transform of the derivative of $f(t)$ is given by,

$$sF(s) - f(0^-) = \int_{0^-}^{\infty} \frac{df}{dt} e^{-st} dt$$

$$= \int_{0^-}^{0^+} \frac{df}{dt} e^{-st} dt + \int_{0^+}^{\infty} \frac{df}{dt} e^{-st} dt$$

There may be a jump discontinuity in $f(t)$. So, the derivative of $f(t)$ at $t = 0$ is an impulse of strength equal to the discontinuity.

Thus,

$$\frac{df(0)}{dt} = [f(0^+) - f(0^-)]\, \delta(t);\ e^{-st}(0) = 1$$

$$\therefore\ sF(s) - f(0^-) = \int_{0^-}^{0^+} \left[f(0^+) - f(0^-)\right] \delta(t)\, dt + \int_{0^+}^{\infty} \frac{df}{dt} e^{-st} dt$$

$$\Rightarrow\ sF(s) = f(0^+) - f(0^-) + \int_{0^+}^{\infty} \frac{df}{dt} e^{-st} dt + f(0^-)$$

$$\Rightarrow\ sF(s) = f(0^+) + \int_{0^+}^{\infty} \frac{df}{dt} e^{-st} dt \quad \dots (1)$$

As $s \to \infty$, the second term of R.H.S vanishes.

$$\Rightarrow\ \underset{s\to\infty}{Lt} [sF(s)] = \underset{s\to\infty}{Lt} f(0^+) + \underset{s\to\infty}{Lt} \int_{0^+}^{\infty} \frac{df(t)}{dt} e^{-st} dt$$

$$f(0^+) = \underset{s\to\infty}{Lt} [sF(s)] \quad \left[\because s \to \infty \int_{0}^{\infty} \frac{df(t)}{dt} = 0\right]$$

Note

This theorem exists only when $f(0^+)$ exists.

Final Value Theorem

If the Laplace transform of $f(t)$ and its derivative exist, then the final value of $f(t)$ is given by,

$$\underset{s\to 0}{Lt} [sF(s)] = f(\infty)$$

Proof

Considering equation (1), we get,

$$sF(s) = f(0^+) + \int_{0^+}^{\infty} \frac{df}{dt} e^{-st} dt$$

As $s \to 0$, we get,

$$\underset{s\to 0}{Lt} [sF(s)] = \underset{s\to 0}{Lt} f(0^+) + \underset{s\to 0}{Lt} \int_{0^+}^{\infty} \frac{df}{dt} e^{-st} dt$$

$$= f(0^+) + \int_{0^+}^{\infty} \frac{df}{dt} dt$$

$$= f(0^+) + [f(t)]_{0^+}^{\infty}$$

$$= f(0^+) + f(\infty) - f(0^+)$$

$$\underset{s\to 0}{Lt} [sF(s)] = f(\infty)$$

Q37. List the advantages and limitations of Laplace transform.

Ans: *Model Paper-II, Q9(b)*

The advantages of Laplace transform are,

1. Advantage of using the Laplace transform to solve differential equations is that all initial conditions are automatically included during the process of transformation, so one does not have to find the homogeneous solutions and the particular solution separately.
2. The main advantage of Laplace transform technique is time consuming numerical approaches are not needed. The two parameters are rapidly and easily found via two algebraic fits, one in the time domain followed by another in the Laplace domain (s-domain).
3. By applying the Laplace transform, one can change an ordinary differential equation into an algebraic equation as algebraic equation is generally easier to deal with.

The limitations of Laplace transform are,

(i) Laplace transform is applicable only to linear differential equation

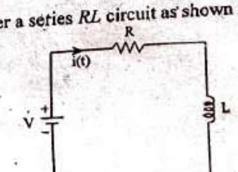
(ii) $s = j\omega$ is applicable only in the analysis of sinusoidal steady state

(iii) Frequency response of a system cannot be calculated.

Q38. Derive an expression for transient response of R-L series circuit excited by D.C source. Use Laplace transform approach.

Ans:

Consider a series RL circuit as shown in figure.



Figure

Applying KVL to figure we have,

$$i(t)R + L\frac{di(t)}{dt} = V$$

Applying Laplace transform on both sides, we get,

$$L\left[i(t)R + L\frac{di(t)}{dt}\right] = L\{V\}$$

$$\Rightarrow\ L\{i(t)R\} + L\left\{L\left[\frac{di(t)}{dt}\right]\right\} = VL\{1\}$$

$$\Rightarrow\ R.L\{i(t)\} + L\left[L\left\{\frac{di(t)}{dt}\right\}\right] = \frac{V}{s}$$

$$\Rightarrow\ R.I(s) + L[sI(s) - I(0^-)] = \frac{V}{s} \quad \dots (1)$$

Assuming the circuit to be initially relaxed, we have,

$$i(0^-) = i(0^-) = 0$$

Therefore, equation (1) becomes,

$$R.I(s) + L(sI(s) - 0] = \frac{V}{s}$$

$$\Rightarrow\ R.I(s) + L[(sI(s)] = \frac{V}{s}$$

$$\Rightarrow\ I(s)\, L\, [R + sL] = \frac{V}{s}$$

$$\Rightarrow\ I(s) = \frac{V}{s(R + sL)}$$

$$\Rightarrow\ I(s) = \frac{V}{Ls\left(\frac{R}{L} + s\right)}$$

$$\Rightarrow\ I(s) = \frac{\left(\frac{V}{L}\right)}{s\left(s + \frac{R}{L}\right)}$$

By partial fraction expansion we have,

$$I(s) = \frac{\left(\frac{V}{L}\right)}{s\left(s + \frac{R}{L}\right)}$$

$$= \frac{A}{s} + \left(\frac{B}{s + \frac{R}{L}}\right) \quad \dots (2)$$

$$\Rightarrow\ \frac{V}{L} = A\left(s + \frac{R}{L}\right) + Bs$$

Now,

Put $s = \frac{-R}{L}$ we get,

$$\Rightarrow\ \frac{V}{L} = A\left(\frac{-R}{L} + \frac{R}{L}\right) + B\left(\frac{-R}{L}\right)$$

$$\Rightarrow\ \frac{V}{L} = A(0) - B\left(\frac{R}{L}\right)$$

$$\Rightarrow\ \frac{V}{L} = -B\left(\frac{R}{L}\right)$$

$$\Rightarrow\ B = \frac{-V}{L} \cdot \frac{L}{R}$$

$$\Rightarrow\ \boxed{B = \frac{-V}{R}}$$

Put $s = 0$, we get,

$\Rightarrow \quad \frac{V}{L} = A\left(0+\frac{R}{L}\right)+0$

$\Rightarrow \quad \frac{V}{L} = A\frac{R}{L}$

$\Rightarrow \quad A = \frac{V}{L}\times\frac{L}{R}$

$\Rightarrow \quad \boxed{A = \frac{V}{R}}$

Substituting A, B values in equation (2), we get,

$$I(s) = \frac{\left(\frac{V}{L}\right)}{s\left(s+\frac{R}{L}\right)}$$

$$= \frac{\frac{V}{R}}{s} + \frac{\left(\frac{-V}{R}\right)}{\left(s+\frac{R}{L}\right)}$$

Applying inverse Laplace, we get,

$$L^{-1}[I(s)] = L^{-1}\left[\frac{\frac{V}{R}}{s} - \frac{\left(\frac{V}{R}\right)}{\left(s+\frac{R}{L}\right)}\right]$$

$\Rightarrow \quad i(t) = L^{-1}\left[\frac{\left(\frac{V}{R}\right)}{s}\right] - L^{-1}\left[\frac{\left(\frac{V}{R}\right)}{\left(s+\frac{R}{L}\right)}\right]$

$\Rightarrow \quad i(t) = \frac{V}{R}L^{-1}\left(\frac{1}{s}\right) - \frac{V}{R}L^{-1}\left(\frac{1}{s+\frac{R}{L}}\right)$

$\Rightarrow \quad i(t) = \frac{V}{R}(1) - \frac{V}{R}e^{-\left(\frac{R}{L}\right)t}$

$\Rightarrow \quad i(t) = \frac{V}{R} - \frac{V}{R}e^{-\left(\frac{R}{L}\right)t}$

$\Rightarrow \quad i(t) = \frac{V}{R}\left(1-e^{-\left(\frac{R}{L}\right)t}\right)$

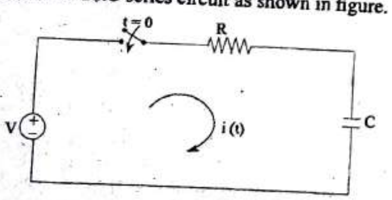
$\therefore$ The expression for transient response of RL series circuit excited by D.C source is,

$$i(t) = \frac{V}{R}\left(1-e^{-\left(\frac{R}{L}\right)t}\right)$$

Q39. Derive the expression for the transient response of RC series circuit excited by a D.C voltage source. Use Laplace technique.

Ans:

Consider a RC series circuit as shown in figure.



Figure

Applying KVL to figure, we have,

$$Ri(t) + \frac{1}{C}\int i(t)\,dt = V$$

Applying Laplace transform on both sides, we get,

$$L\left[R.i(t) + \frac{1}{C}\int i(t)dt\right] = L[V]$$

$\Rightarrow \quad R.L[i(t)] + \frac{1}{C}L\left[\int i(t)dt\right] = VL[1]$

$\Rightarrow \quad R.I(s) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{I(0^+)}{s}\right] = \frac{V}{s}$

Assuming that, the system is initially relaxed, we have,

$$I(0^+) = 0$$

Substituting $I(0^+)$ value, we get,

$\Rightarrow \quad R.I(s) + \frac{1}{C}\left[\frac{I(s)}{s}+0\right] = \frac{V}{s}$

$\Rightarrow \quad R.I(s) + \frac{I(s)}{sC} = \frac{V}{s}$

$\Rightarrow \quad I(s)\left[R+\frac{1}{sC}\right] = \frac{V}{s}$

$\Rightarrow \quad I(s) = \frac{V}{s\left[R+\frac{1}{sC}\right]}$

$\Rightarrow \quad I(s) = \frac{V}{s\left[\frac{R(sC)+1}{sC}\right]}$

$\Rightarrow \quad I(s) = \frac{VsC}{s[R(sC)+1]}$

$\Rightarrow \quad I(s) = \frac{VC}{R(sC)+1}$

$\Rightarrow \quad I(s) = \frac{V.C}{RC\left[s+\frac{1}{RC}\right]}$

$\Rightarrow \quad I(s) = \frac{V}{R\left[s+\frac{1}{RC}\right]}$

Applyng inverse Laplace transform we get,

$\Rightarrow \quad L^{-1}[I(s)] = L^{-1}\frac{V}{R\left[s+\frac{1}{RC}\right]}$

$\Rightarrow \quad i(t) = \frac{V}{R}L^{-1}\left[\frac{1}{s+\frac{1}{RC}}\right]$

$\Rightarrow \quad i(t) = \frac{V}{R}e^{-\left(\frac{1}{RC}\right)t}$

$\Rightarrow \quad i(t) = \frac{V}{R}e^{-\frac{t}{RC}}$

Q40. Derive the expression for transient response of RLC series circuit with unit step input.

OR

Derive expression for RLC series circuit using Laplace transform approach excited by a D.C source.

Ans:

Consider a series RLC circuit shown in figure (1).

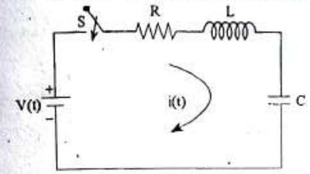


Figure (1): Series RLC Circuit

$V(t)$ is a unit step voltage applied to the series RLC circuit.

Let $i(t)$ be the current flowing through the circuit. At $t = 0$, the switch is closed. Let us assume that the initial current through inductor and initial voltage across capacitor is zero i.e.,

$$i(0^+) = i(0^-) = 0 \text{ and } V_c(0^+) = V_c(0^-) = 0$$

Applying KVL to the circuit, we get,

$$V(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt \quad \text{... (1)}$$

Applying Laplace to equation (1), we get,

$$\frac{V}{s} = RI(s) + L[sI(s) - i(0)] + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{q_0}{s}\right]$$

$$\frac{V}{s} = I(s)\left[R + sL + \frac{1}{Cs}\right]$$

[$\because$ Initial conditions are zero]

$$\frac{V}{s} = I(s)\left[\frac{RCs + s^2LC + 1}{Cs}\right]$$

$$I(s) = \frac{VC}{s^2LC + RCs + 1}$$

$$= \frac{V/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \quad \text{... (2)}$$

The roots of the above equation are,

$$s_1, s_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$s_1, s_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Based on the relation between and $\left(\frac{R}{2L}\right)^2$ and $\frac{1}{LC}$ we get three types of roots and hence, three types of response.

Case (i): $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

When $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$, the roots will be real and unequal and the system is said to be overdamped.

The response of the system can be determined as,

$$I(s) = \frac{V/L}{(s+s_1)(s+s_2)}$$

$$= \frac{A}{s+s_1} + \frac{B}{s+s_2}$$

$$I(s) = \frac{V/L(s_2-s_1)}{s+s_1} + \frac{V/L(s_1-s_2)}{s+s_2}$$

$$i(t) = \frac{V}{L(s_2-s_1)}[e^{-s_1t} - e^{-s_2t}]$$

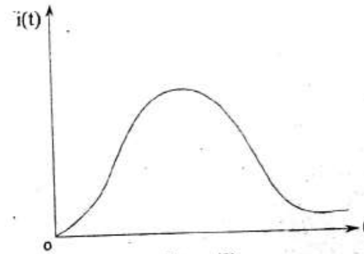


Figure (2)

Figure (2) shows the time response when the system is overdamped.

Case (ii): $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

When, $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ the roots of the system will be real and equal and the system is said to be critically damped.

$$I(s) = \frac{V/L}{(s+s_1)^2}$$

$$i(t) = \frac{V}{L}te^{s_1t}$$

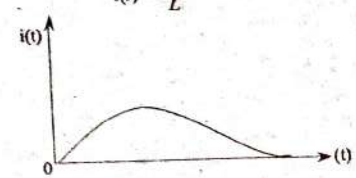


Figure (3)

Figue (3) shows the time response of the critically damped system.

Case (iii): $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

When $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$, the roots of the system will be complex conjugate and the system is said to be underdamped system.

The response of the system can be determined as.

Let the two roots be,

$$s_1 = -A + jB \text{ and } s_2 = -A - jB$$

Where,

$$A = \frac{R}{2L} \text{ and } B = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

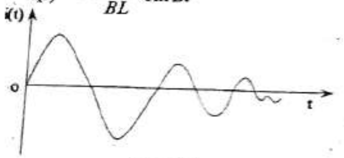
$$\therefore \quad I(s) = \frac{V/L}{(s+A-jB)(s+A+jB)}$$

$$I(s) = \frac{V/j2BL}{s+A-jB} - \frac{V/j2BL}{s+A+jB}$$

$$i(t) = \frac{V}{j2BL}e^{-At}e^{jBt} - \frac{V}{j2BL}e^{-At}e^{-jBt}$$

$$= \frac{Ve^{-At}}{BL}\left[\frac{e^{jBt} - e^{-jBt}}{2j}\right]$$

$$i(t) = \frac{Ve^{-At}}{BL}\sin Bt$$



Figue (4)

The response of the underdamped system is shown in figure (4).

Q41. Derive expression for RL circuit excited by ramp input using Laplace transform method.

Ans: *Model Paper-III, Q9(a)*

Response of Series RL Circuit to Ramp Input

Consider a series RL circuit excited by a ramp input $Vr(t)$, where $r(t)$ is defined as

$$r(t) = t, \quad t > 0$$
$$= 0 \quad t \leq 0$$

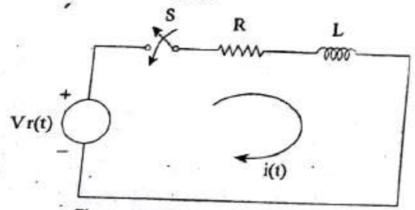


Figure (1) : Ramp Input to Series RL Circuit

Assuming the circuit to be de-energized initially.

Let $i(t)$ be the current through the circuit.

Applying KVL to the circuit, we get,

$$Vr(t) = Ri(t) + \frac{Ldi(t)}{dt} \quad \text{... (1)}$$

Applying Laplace transform to equation (1), we get

$$\frac{V}{s^2} = RI(s) + L[sI(s) - i(0^+)]$$

$i(0^+) = 0$, as the circuit is assumed to be de-energized initially.

$$\frac{V}{s^2} = RI(s) + sLI(s)$$

$$\frac{V}{s^2} = [R + sL]\, I(s)$$

$$I(s) = \frac{V}{s^2(R+sL)}$$

$$I(s) = \frac{V/L}{s^2(s+R/L)} \quad \text{... (2)}$$

Taking partial fractions of equation (2), we get

$$I(s) = \frac{V/L}{s^2(s+R/L)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+R/L}$$

On simplification, we get,

$$A = \frac{-VL}{R^2},\ B = \frac{V}{R},\ C = \frac{VL}{R^2}$$

$\therefore$ Equation (2), becomes,

$$I(s) = \frac{-VL/R^2}{s} + \frac{V/R}{s^2} + \frac{VL/R^2}{s+R/L} \quad \text{... (3)}$$

Applying inverse Laplace to equation (3), we get

$$i(t) = \frac{-VL}{R^2} + \frac{V}{R}t + \frac{VL}{R^2}e^{-\frac{R}{L}t}$$

$$i(t) = \frac{VL}{R^2}\left(e^{-\frac{R}{L}t} - 1\right) + \frac{V}{R}t$$

Response of Parallel RL Circuit to Ramp Input

Consider a parallel RL circuit supplied by a current source $Ir(t)$, where $r(t)$ is a ramp input. Let i_R and i_L be the currents through resistor and inductor respectively.

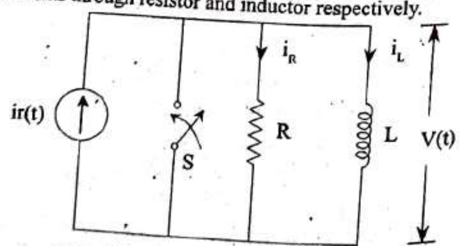


Figure (2) : Ramp Input to Parallel RL Circuit

Let $V(t)$ be the voltage across the parallel combination. Assuming the circuit to be de-energized initially.

According to KCL,

$$Ir(t) = i_R + i_L$$

$$Ir(t) = \frac{V(t)}{R} + \frac{1}{L}\int V(t)\,dt \quad \text{... (4)}$$

Applying Laplace transform to equation (4), we get

$$\frac{I(s)}{s^2} = \frac{V(s)}{R} + \frac{1}{L}\left[\frac{V(s)}{s} - \frac{V(0^-)}{s}\right]$$

$V(0^-) = 0$, as the circuit is initially de-energized

$$\frac{I(s)}{s^2} = V(s)\left[\frac{1}{R} + \frac{1}{sL}\right]$$

$$\frac{I(s)}{s^2} = V(s)\left[\frac{R+sL}{RsL}\right]$$

$$\frac{I(s)}{s} = V(s)\left[\frac{R+sL}{RL}\right]$$

$$V(s) = \frac{I(s)R.L}{s(R+sL)}$$

$$V(s) = \frac{I(s)R}{s(s+R/L)} \quad \text{... (5)}$$

Taking partial fractions of equation (4), we get,

$$V(s) = \frac{A}{s} + \frac{B}{s+R/L}$$

$$A = I(s).L,\ B = -I(s)\,L$$

$$\therefore \quad V(s) = I(s)\left[\frac{L}{s} - \frac{L}{s+R/L}\right] \quad \text{... (6)}$$

Taking inverse Laplace of equation (6), we get,

$$v(t) = i(t)L - i(t)Le^{-R/L\,t}$$

$$v(t) = i(t).L\left(1 - e^{-R/L\,t}\right)$$

Q42. Derive an expression for RL circuit excited by pulse input using Laplace approach.

Ans:

Response of Series RL Circuit to Pulse Input

Consider a series RL circuit excited by a pulse signal as shown in figure (1). Let $i(t)$ be the current through the circuit. Assuming the circuit to be de-energized initially i.e., $i(0^+) = 0$.

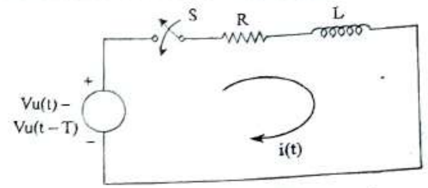


Figure (1) : Pulse Input to Series RL Circuit

Applying KVL to the circuit, we get,

$$Vu(t) - Vu(t-T) = Ri(t) + L\frac{di(t)}{dt} \quad \text{... (1)}$$

Applying Laplace transform to equation (1), we get,

$$\frac{V}{s} - \frac{V}{s}e^{-Ts} = RI(s) + L[sI(s) - i(0^+)]$$

$$\frac{V}{s}\left(1 - e^{-Ts}\right) = I(s)\,[R + sL]$$

$$I(s) = \frac{V\left(1-e^{-Ts}\right)}{s(R+sL)}$$

$$I(s) = \frac{\frac{V}{L}\left(1-e^{-Ts}\right)}{s\left(s+R/L\right)} \quad \text{... (2)}$$

Consider, $\dfrac{1}{s\left(s+\frac{R}{L}\right)}$

Applying partial fraction to above considered part, we get,

$$\frac{1}{s\left(s+\frac{R}{L}\right)} = \frac{A}{s} + \frac{B}{s\ \frac{R}{L}}$$

$$1 = A\left(s + \frac{R}{L}\right) + Bs$$

Put $s = 0$,

$$1 = A\left(0 + \frac{R}{L}\right) + 0$$

$$A = \frac{L}{R}$$

Put $s = \frac{-R}{L}$

$$1 = A\left(-\frac{R}{L} + \frac{R}{L}\right) + B\left(\frac{-R}{L}\right)$$

$$1 = 0 - B\,\frac{R}{L}$$

$$B = \frac{-L}{R}$$

$\therefore$ Equation (2) becomes,

$$I(s) = \frac{V}{L}(1-e^{-Ts})\left[\frac{\frac{L}{R}}{s}+\frac{\frac{-L}{R}}{s+\frac{R}{L}}\right]$$

$$= \frac{V}{R}(1-e^{-Ts})\left[\frac{1}{s}-\frac{1}{s+\frac{R}{L}}\right]$$

$$I(s) = \frac{\frac{V}{R}}{s}-\frac{\frac{V}{R}}{s+\frac{R}{L}}-\frac{\frac{V}{R}}{s}e^{-Ts}+\frac{\frac{V}{R}}{s+\frac{R}{L}}e^{-Ts} \quad \text{... (3)}$$

Taking inverse Laplace of equation (3) we get,

$$i(t) = \frac{V}{R}u(t)-\frac{V}{R}e^{-\frac{R}{L}t}u(t)-\frac{V}{R}u(t-T)+\frac{V}{R}e^{-\frac{R}{L}(t-T)}u(t-T)$$

Response of Parallel RL Circuit to Pulse Input

Consider a parallel RL circuit supplied by a current source, $Iu(t) - Iu(t-T)$ which is a pulse function. Let i_R and i_L be the currents through resistor and inductor respectively. Let $v(t)$ be the voltage across the parallel combination. It is assumed that the circuit is **de-**energized initially.

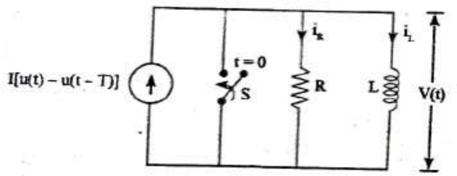


Figure (2): Pulse Input to Parallel RL Circuit

According to KCL,

$$I[u(t)-u(t-T)] = i_R + i_L$$

$$I[u(t)-u(t-T)] = \frac{v(t)}{R}+\frac{1}{L}\int V(t)\,dt \quad \text{... (4)}$$

Applying Laplace transform to equation (4), we get,

$$\frac{I(s)}{s}-\frac{I(s)}{s}e^{-Ts} = \frac{V(s)}{R}+\frac{1}{L}\left[\frac{V(s)}{s}-\frac{V(0^-)}{s}\right]$$

$$\frac{I(s)}{s}\left(1-e^{-Ts}\right) = V(s)\left[\frac{1}{R}+\frac{1}{sL}\right]$$

$$\frac{I(s)\left[1-e^{-Ts}\right]}{s} = \frac{V(s)[R+sL]}{RsL}$$

$$V(s) = \frac{I(s)\left[1-e^{-Ts}\right]}{(R+sL)}\times RL$$

$$V(s) = \frac{I(s)\left[1-e^{-Ts}\right]}{\left(s+R/L\right)}R$$

$$V(s) = \frac{I(s).R}{s+R/L}-\frac{I(s).R.e^{-Ts}}{s+R/L} \quad \text{... (5)}$$

Taking inverse Laplace of equation (5), we get,

$$v(t) = i(t)R\left[e^{\frac{-R}{L}t}u(t)-e^{-\frac{R}{L}(t-T)}u(t-T)\right].$$

Q43. Derive an expression for RL circuit excited by impulse input using Laplace transform approach.

Ans:

Impulse Response of Series RL Circuit

Consider the series RL circuit excited by an impulse input $V\delta(t)$ as shown in figure (1). Let $i(t)$ be the current through the circuit. Let us assume the circuit is initially de-energized.

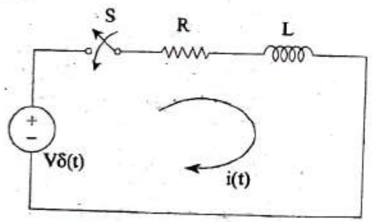


Figure (1): Impulse Input to Series RL Circuit

Applying Kirchhoff's voltage law to the circuit, we get,

$$V\delta(t) = Ri(t)+\frac{Ldi(t)}{dt} \quad \text{... (1)}$$

Applying Laplace transform to equation (1), we get,

$$V = RI(s) + sL\,[I(s) - i(0^-)]$$

$i(0^-) = 0$, as the circuit is assumed to be de-energized initially

$$V = RI(s) + sLI(s)$$

$$V = I(s)\,[R + sL]$$

$$I(s) = \frac{V}{R+sL}$$

$$I(s) = \frac{V/L}{\left(s+R/L\right)} \quad \text{... (2)}$$

Taking inverse Laplace transform of equation (2), we get,

$$i(t) = \frac{V}{R}e^{\frac{R}{L}t}$$

Impulse Response of Parallel RL Circuit

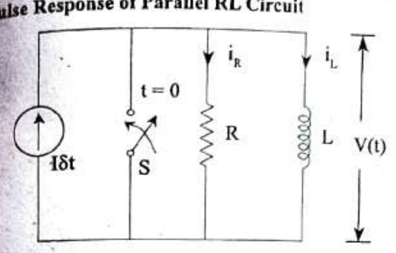


Figure (2) : Impulse Input to Parallel RL Circuit

Consider a parallel RL circuit supplied with a current source $I\delta(t)$, where $\delta(t)$ is an impulse.

Let $v(t)$ be the voltage across the parallel circuit. Assuming the circuit to be de-energized initially.

According to KCL,

$$I\delta(t) = i_R + i_L$$

$$I\delta(t) = \frac{V(t)}{R}+\frac{1}{L}\int V(t)\,dt \quad \text{... (3)}$$

Applying Laplace transform to equation (3), we get,

$$I(s) = \frac{V(s)}{R}+\frac{1}{L}\left[\frac{V(s)}{s}-\frac{V^{-1}(0)}{s}\right]$$

$V^{-1}(0) = 0$, as the circuit is initially de-energized

$$I(s) = \frac{V(s)}{R}+\frac{V(s)}{Ls}$$

$$I(s) = V(s)\left[\frac{R+sL}{RLs}\right]$$

$$V(s) = \frac{I(s)RLs}{R+sL}$$

$$= \frac{I(s)Rs}{s+\frac{R}{L}}$$

$$V(s) = I(s)R\left[1-\frac{R/L}{s+R/L}\right] \quad \text{... (4)}$$

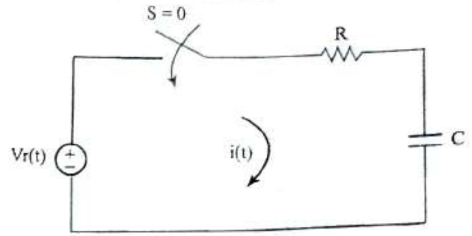
Taking inverse Laplace of equation (4), we get,

$$V(t) = \left[i(t)R\delta(t)-\frac{i(t)R^2}{L}e^{\frac{R}{L}t}\delta(t)\right] \text{ Volts}$$

Q44. Derive expression for RC circuit excited by Ramp input using Laplace approach.

Ans:

Consider a RC series circuit shown in figure (1).



Figure

Now, applying KVL to the ciruit we get,

$$R.i(t) + \frac{1}{C}\int i(t).dt = Vr(t)$$

Applying Laplace transform on both sides, we get,

$$L\,[R.i(t) + \frac{1}{C}\int i(t).dt\,] = L[Vr(t)]$$

$$R.\,I(s) + \frac{1}{C}\left[\frac{I(s)}{s}+\frac{V_0}{s}\right] = \frac{V}{s^2} \quad \text{... (1)}$$

Before, the switch (S) is closed, the initial voltage across capacitor will be zero i.e.,

$$V_0 = 0$$

Substituting V_0 vlaues in equation (1) we get,

$$\Rightarrow \quad R.\,I(s) + \frac{I(s)}{sC}+\frac{0}{s} = \frac{V}{s^2}$$

$$\Rightarrow \quad R.\,I(s) + \frac{I(s)}{sC} = \frac{V}{s^2}$$

$$\Rightarrow \quad \left(R+\frac{1}{sC}\right)I(s) = \frac{V}{s^2}$$

$$\Rightarrow \quad \left[\frac{sRC+1}{sC}\right]I(s) = \frac{V}{s^2}$$

$$\Rightarrow \quad I(s) = \frac{V}{s^2}\times\frac{sC}{RC\left(s+\frac{1}{RC}\right)}$$

$$\Rightarrow \quad I(s) = \frac{V}{s}\frac{C}{RC\left(s+\frac{1}{RC}\right)}$$

$$\Rightarrow \quad I(s) = \frac{V}{R}\frac{1}{s\left(s+\frac{1}{RC}\right)} \quad \text{... (2)}$$

By partial fraction expansion we have,

$$I(s) = \frac{\left(\frac{V}{R}\right)}{s\left(s+\frac{1}{RC}\right)} = \frac{A}{s}+\frac{B}{\left(s+\frac{1}{RC}\right)} \quad \text{... (3)}$$

$$\Rightarrow \quad \frac{V}{R} = A\left(s+\frac{1}{RC}\right)+Bs \quad \text{... (4)}$$

Put $s = 0$ in equation (4), we get,

$$\Rightarrow \left(\frac{V}{R}\right) = A\left(0 + \frac{1}{RC}\right) + B(0)$$

$$\Rightarrow \left(\frac{V}{R}\right) = A\left(\frac{1}{RC}\right) + 0$$

$$\Rightarrow \left(\frac{V}{R}\right) = \frac{A}{RC}$$

$$\Rightarrow A = V.C$$

Put $s = \frac{-1}{RC}$ in equation (4), we get,

$$\Rightarrow \left(\frac{V}{R}\right) = A\left(\frac{-1}{RC} + \frac{1}{RC}\right) + B\left(\frac{-1}{RC}\right)$$

$$\Rightarrow \left(\frac{V}{R}\right) = A(0) + B\left(\frac{-1}{RC}\right)$$

$$\Rightarrow \left(\frac{V}{R}\right) = \frac{-B}{RC}$$

$$\Rightarrow B = VC$$

Substituting A, B values in equation (3) we get,

$$I(s) = \frac{V/(R)}{s\left(s + \frac{1}{RC}\right)} = \frac{VC}{s} - \frac{VC}{\left(s + \frac{1}{RC}\right)}$$

Applying inverse Laplace, we get,

$$\Rightarrow L^{-1}[I(s)] = i(t) = L^{-1}\left[\frac{VC}{s} - \frac{VC}{s + \frac{1}{RC}}\right]$$

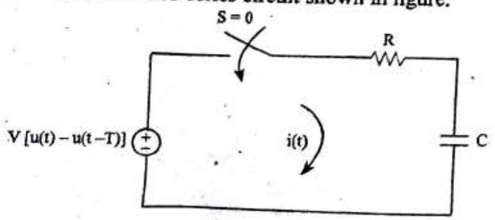
$$\Rightarrow i(t) = C\,VL^{-1}\left[\frac{1}{s} - \frac{1}{\left(s + \frac{1}{RC}\right)}\right]$$

$$\Rightarrow i(t) = C.V\left[1 - e^{-\frac{1}{RC}t}\right]$$

Q45. Derive expression for RC series circuit excited by pulse input using Laplace approach.

Ans:

Consider a RC series circuit shown in figure.



Figure

Applying KVL to circuit we get,

$$R.i(t) + \frac{1}{C}\int i(t).dt = V[u(t) - u(t - T)]$$

Applying Laplace transform on both sides, we get,

$$R.\,I(s) + \frac{I(s)}{sC} + \frac{V_0}{s} = V\left[\frac{1}{s} - \frac{e^{-Ts}}{s}\right] \qquad \ldots (1)$$

Assuming, the circuit to be initially relaxed we have,

$$V_0 = 0$$

Substituting V_0 value in equation (1), we get,

$$R.\,I(s) + \frac{I(s)}{sC} + \frac{0}{s} = V\left[\frac{1}{s} - \frac{e^{-Ts}}{s}\right]$$

$$\Rightarrow R.\,I(s) + \frac{1}{sC}I(s) = V\left[\frac{1}{s} - \frac{e^{-Ts}}{s}\right]$$

$$\Rightarrow \left(R + \frac{1}{sC}\right)I(s) = V\left[\frac{1}{s} - \frac{e^{-Ts}}{s}\right]$$

$$\Rightarrow \left(\frac{sRC + 1}{sC}\right)I(s) = \frac{V}{s}(1 - e^{-Ts})$$

$$\Rightarrow I(s) = \frac{V}{s}(1 - e^{-Ts}) \times \frac{sC}{(sRC + 1)}$$

$$\Rightarrow I(s) = \frac{V}{s}(1 - e^{-Ts}) \times \frac{sC}{RC\left(s + \frac{1}{RC}\right)}$$

$$\Rightarrow I(s) = \frac{V}{R}(1 - e^{-Ts}) \times \frac{1}{\left(s + \frac{1}{RC}\right)}$$

$$\Rightarrow I(s) = \frac{V}{R}\left[\frac{1}{\left(s + \frac{1}{RC}\right)}\right] - \frac{V}{R}\cdot\frac{e^{-Ts}}{\left(s + \frac{1}{RC}\right)}$$

Applying inverse Laplace transform, we get,

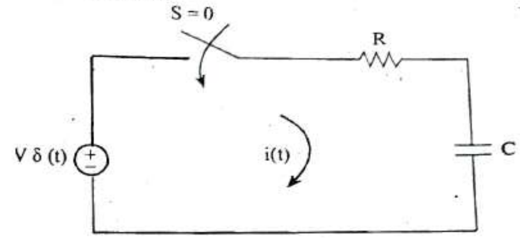
$$L^{-1}[I(s)] = i(t) = L^{-1}\left[\frac{V}{R}\left(\frac{1}{s + \frac{1}{RC}}\right) - \frac{V}{R}\,\frac{e^{-Ts}}{s + \frac{1}{RC}}\right]$$

$$\Rightarrow i(t) = \frac{V}{R}e^{-\frac{1}{RC}t}u(t) - \frac{V}{R}e^{-\frac{1}{RC}(t-T)}u(t - T)$$

Q46. Derive expression for RC series circuit excited by an impulse signal using Laplace approa

Ans:

Consider a RC series circuit as shown in figure (1).



Figure

Applying KVL to cirucit we get,

$$R.i(t) + \frac{1}{C}\int i(t).dt = V\,\delta(t)$$

Applying Laplace transform on both sides, we get,

$$R.\,I(s) + \frac{I(s)}{sC} + \frac{V_0}{s} = V \qquad [\because L[\delta(t) = 1]]$$

Assuming, the circuit to be initially relaxed we have,

$$V_0 = 0$$

Substituting V_0 value in equation (1), we get,

$$R.\,I(s) + \frac{I(s)}{sC} + \frac{0}{s} = V$$

$$\Rightarrow R.\,I(s) + \frac{1}{sC}I(s) = V$$

$$\Rightarrow \left(R + \frac{1}{sC}\right)I(s) = V$$

$$\Rightarrow I(s) = \frac{V}{\left(R + \frac{1}{sC}\right)}$$

$$\Rightarrow I(s) = \frac{V}{\left(\frac{sRC + 1}{sC}\right)}$$

$$\Rightarrow I(s) = \frac{V.sC}{RC\left(s + \frac{1}{RC}\right)}$$

$$\Rightarrow I(s) = \frac{V}{R}\left[\frac{s}{\left(s + \frac{1}{RC}\right)}\right]$$

$$\Rightarrow I(s) = \frac{V}{R}\left[\frac{s + \frac{1}{RC} - \frac{1}{RC}}{s + \frac{1}{RC}}\right]$$

$$\Rightarrow \quad I(s) = \frac{V}{R}\left[\frac{s+\frac{1}{RC}}{s+\frac{1}{RC}} - \frac{1}{RC}\cdot\frac{1}{s+\frac{1}{RC}}\right]$$

$$\Rightarrow \quad I(s) = \frac{V}{R}\left[1-\frac{1}{RC}\cdot\frac{1}{\left(s+\frac{1}{RC}\right)}\right]$$

Applying inverse Laplace transform, we get,

$$L^{-1}[I(s)] = i(t) = L^{-1}\left\{\frac{V}{R}\left[1-\frac{1}{RC}\left(\frac{1}{s+\frac{1}{RC}}\right)\right]\right\}$$

$$\Rightarrow \quad i(t) = \frac{V}{R}[\delta(t) - \frac{1}{RC}e^{-\frac{t}{RC}}u(t)]$$

Q47. Find the Laplace transform for the following functions,

(i) $\sin 2\omega(t-t_o)$

(ii) $t\cos(\omega t+\theta)$.

Ans:

(i) $\sin 2\omega(t-t_o)$

Let, $f(t) = \sin 2\omega(t-t_o) = \sin(2\omega t - 2\omega t_o)$

We have,

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$$f(t) = \sin(2\omega t - 2\omega t_o)$$

$$= \sin(2\omega t)\cos(2\omega t_o) - \cos(2\omega t)\sin(2\omega t_o)$$

Now the Laplace transform is given by,

$$L[f(t)] = L[\sin(2\omega t - 2\omega t_o)]$$

$$= L[\sin(2\omega t)\cos(2\omega t_o) - \cos(2\omega t)\sin(2\omega t_o)]$$

$$= L[\sin(2\omega t)\cos(2\omega t_o)] - L[\cos(2\omega t)\sin(2\omega t_o)]$$

$$= \cos(2\omega t_o)\times L[\sin 2\omega t)] - \sin(2\omega t_o)\times L[\cos(2\omega t)]$$

$$= \cos(2\omega t_o)\times\frac{2\omega}{s^2+(2\omega)^2} - \sin 2\omega t_o\times\frac{s}{s^2+(2\omega)^2} \quad \left[\because L(\sin at) = \frac{a}{s^2+a^2},\ L(\cos at) = \frac{s}{s^2+a^2}\right]$$

$$= \frac{2\omega\cos(2\omega t_o)}{s^2+4\omega^2} - \frac{s.\sin(2\omega t_o)}{s^2+4\omega^2}$$

$$= \frac{2\omega\cos(2\omega t_o) - s.\sin(2\omega t_o)}{s^2+4\omega^2}$$

(ii) $t\cos(\omega t+\theta)$

Let, $f(t) = t\cos(\omega t+\theta)$

We know that,

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\therefore f(t) = t\cos(\omega t+\theta) = t[\cos\omega t\cos\theta - \sin\omega t\sin\theta]$$

$$= t\cos\omega t\cos\theta - t\sin\omega t\sin\theta$$

Now taking Laplace transform on both sides, we get,

$$L[f(t)] = L[t\cos(\omega t+\theta)]$$

$$= L[t\cos\omega t\cos\theta - t\sin\omega t\sin\theta]$$

$$= L[t\cos\omega t\cos\theta] - L[t\sin\omega t\sin\theta]$$

$$= \cos\theta\, L[t\cos\omega t] - \sin\theta\, L[t\sin\omega t]$$

Since we have,

$$L(t\cos at) = \frac{s^2-a^2}{(s^2+a^2)^2} \text{ and}$$

$$L(t\sin at) = \frac{2as}{(s^2+a^2)^2}$$

Therefore,

$$L(t\sin\omega t) = \frac{2\omega s}{(s^2+\omega^2)^2} \text{ and}$$

$$L(t\cos\omega t) = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$$

$$L[f(t)] = L[t\cos(\omega t+\theta)]$$

$$= \cos\theta\times\left[\frac{s^2-\omega^2}{(s^2+\omega^2)^2}\right] - \sin\theta\times\left[\frac{2\omega s}{(s^2+\omega^2)^2}\right]$$

$$= \frac{(s^2-\omega^2)\cos\theta}{(s^2+\omega^2)^2} - \frac{2\omega s\sin\theta}{(s^2+\omega^2)^2}$$

$$= \frac{(s^2-\omega^2)\cos\theta - 2\omega s\sin\theta}{(s^2+\omega^2)^2}$$

Taking inverse Laplace transform of equation (6), we get,

$$L^{-1}[I(s)] = 3\,L^{-1}\left[\frac{e^{-2s}}{s}\right] + L^{-1}\left[\frac{e^{-2s}}{s+4}\right] - 4L^{-1}\left[\frac{e^{-2s}}{s+1}\right]$$

$$i(t) = 3u(t-2) + e^{-4(t-2)}u(t-2) - 4e^{-(t-2)}u(t-2)$$

$$= u(t-2)\,[3 + e^{-4(t-2)} - 4e^{-(t-2)}]$$

4.2 CONVOLUTION INTEGRAL

Q.48 State the shifting theorem and give its expression.

Ans:

Shifting theorem states that the Laplace transform of any function shifted or delayed by a time interval 'a' is e^{-as} times the Laplace transform of the function.

i.e., If $L[f(t)] = F(s)$ then,

$$L[f(t-a)\,u(t-a)] = e^{-as}F(s)$$

From the definition of Laplace transform, we have,

$$L[f(t)] = F(s) = \int_0^\infty f(t)e^{-st}dt$$

$$\therefore L[f(t-a)\,u(t-a)] = \int_0^\infty f(t-a)u(t-a)e^{-st}dt$$

$$= \int_0^\infty f(t-a)e^{-st}dt$$

Let,

$$x = t-a$$
$$t = x+a$$
$$dt = dx$$

$$\therefore \quad L[f(t-a)\,u(t-a)] = \int_0^\infty f(x)e^{-s(x+a)}dx$$

$$= \int_0^\infty f(x)e^{-sx}e^{-as}dx$$

$$= e^{-as}\int_0^\infty f(x)e^{-sx}dx$$

$$= e^{-as}F(s)$$

$$\boxed{\therefore L[f(t-a)u(t-a)] = e^{-as}\cdot F(s)}$$

Q.49 State and prove second shifting theorem.

Ans:

Statement

Let $f(t) = 0;\ t < 0$

If $L[f(t)] = F(s)$ then,

$L[f(t-t_0)] = e^{-st_0}F(s)$ where $t_0 > 0$

Proof

From the definition, the Laplace transform for the function $f(t)$ is given by,

$$L[f(t)] = \int_0^\infty f(t)e^{-st}.dt$$

Similarly,

$$L[f(t-t_0)] = \int_0^\infty f(t-t_0)\_e^{-st}.dt \qquad \text{... (1)}$$

Let, $t - t_0 = x$

$$t = x + t_0$$

Differentiating on both sides, we get,

$$dt = dx$$

Substituting all the values in equation (1), we have,

$$L[f(t-t_0)] = \int_{-t_0}^\infty f(x)e^{-(x+t_0)s}.dx$$

$$= \int_{-t_0}^0 f(x)e^{-(x+t_0)s}dx + \int_0^\infty f(x)e^{-(x+t_0)s}dx$$

But, $f(t) = 0;\ t < 0$

$f(t) = 0;\ x < 0$

$$\therefore \quad L[f(t-t_0)] = \int_{-t_0}^0 0.e^{-(x+t_0)s}.dx + \int_0^\infty f(x)e^{-(x+t_0)s}.dx$$

$$= 0 + \int_0^\infty f(x)e^{-(x+t_0)s}.dx$$

$$= \int_0^\infty f(x)e^{-(x+t_0)s}.dx$$

$$= \int_0^\infty f(x)e^{-sx}.e^{-st_0}dx$$

$$= e^{-st_0}\int_0^\infty f(x)e^{-sx}.dx$$

$$= e^{-st_0}.F(s)$$

$$\therefore \quad L[f(t-t_0)] = e^{-st_0}.F(s)$$

Hence, the second shifting theorem is proved.

Q.50 State and prove the convolution theorem.

Ans:

Convolution Theorem

If $f_1(t)$ and $f_2(t)$ are two functions defined for $t \geq 0$ and $F_1(s)$ and $F_2(s)$ be their Laplace transforms respectively, then convolution theorem states that the Laplace transform of the convolution of $f_1(t)$ and $f_2(t)$ is the product of the individual transforms.

Q.51 Find the expressions for the current i(t) in a series R-L-C circuit, with R = 5 Ω, L = 1 H, C = $\frac{1}{4}$ F, when it is fed by a ramp voltage of 12r(t − 2).

Ans:

Given that,

Resistance, $R = 5\ \Omega$

Inductance, $L = 1$ H

Capacitance, $C = \frac{1}{4}$ F

Input ramp voltage, $v(t) = 12r(t-2)$ V

The series RLC circuit is as shown in figure,

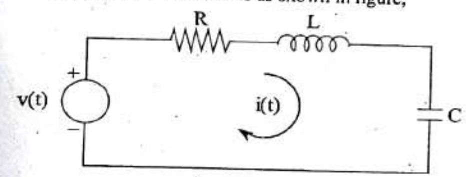


Figure: RLC Series Circuit

Applying Kirchhoff's voltage law to the above circuit.

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = v(t) \qquad \text{... (1)}$$

The input voltage $v(t)$ is a ramp voltage of magnitude $12r(t-2)$

$$\text{i.e., } v(t) = 12r(t-2) \qquad \text{... (2)}$$

Substituting equation (2) in equation (1), we have,

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = 12r(t-2) \qquad \text{... (3)}$$

Applying Laplace transform to the above equation.

$$RI(s) + LsI(s) + \frac{1}{C}\left[\frac{q(0)}{s} + \frac{I(s)}{s}\right] = 12 \times \frac{1}{s^2} \times e^{-2s} \qquad \text{... (4)}$$

Assuming the initial charge on the capacitor to be zero.

$$q(0) = 0$$

Now, equation (4) reduces to,

$$RI(s) + LsI(s) + \frac{1}{C}\left[\frac{I(s)}{s}\right] = \frac{12e^{-2s}}{s^2}$$

$$I(s) = \left[R + Ls + \frac{1}{Cs}\right] = \frac{12e^{-2s}}{s^2}$$

$$I(s) = \frac{12e^{-2s}}{s^2\left[R + Ls + \frac{1}{Cs}\right]} \qquad \text{... (5)}$$

Substituting the given values in equation (5), we get,

$$I(s) = \frac{12e^{-2s}}{s^2\left[5 + 1s + \frac{4}{s}\right]}$$

$$= \frac{12e^{-2s}}{5s^2 + s^3 + 4s}$$

$$= \frac{12e^{-2s}}{s^3 + 5s^2 + 4s}$$

$$= \frac{12e^{-2s}}{s[s^2 + 5s + 4]}$$

$$= 12e^{-2s}\left[\frac{1}{s(s+4)(s+1)}\right]$$

Taking partial fractions, we have,

$$\frac{1}{s(s+4)(s+1)} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+1}$$

$$1 = A(s+4)(s+1) + Bs(s+1) + Cs(s+4)$$

Let, $s = 0$

$$1 = 4A + B(0) + C(0)$$

$$\therefore \quad A = \frac{1}{4}$$

Let, $s = -4$

$$1 = A(-4+4)(-4+1) + B(-4)(-4+1)$$
$$+ C(-4)(-4+4)$$
$$1 = A(0) + 12B + C(0)$$
$$1 = 12B$$

$$\therefore \quad B = \frac{1}{12}$$

Let, $s = -1$

$$1 = A(-1+4)(-1+1) + B(-1)(-1+1) + C(-1)(-1+4)$$
$$1 = A(0) + B(0) + C(-1)(3)$$
$$1 = -3C$$

$$\therefore \quad C = \frac{-1}{3}$$

$$\therefore \quad I(s) = 12e^{-2s}\left[\frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+1}\right]$$

$$= 12e^{-2s}\left[\frac{\frac{1}{4}}{s} + \frac{\frac{1}{12}}{s+4} - \frac{\frac{1}{3}}{s+1}\right]$$

$$= \frac{3e^{-2s}}{s} + \frac{e^{-2s}}{s+4} - 4\frac{e^{-2s}}{s+1} \qquad \text{... (6)}$$

The convolution operation is represented by,

$\therefore\ L[f_1(t)*f_2(t)] = F_1(s).F_2(s)$

Where,

$$[f_1(t)*f_2(t)] = \int_0^t f_1(t-\tau)f_2(\tau)d\tau$$

Proof

From the definition of Laplace transform we have,

$$L[f(t)] = \int_0^\infty f(t)e^{-st}dt$$

$$\therefore\ L[f_1(t)*f_2(t)] = \int_0^\infty\left[\int_0^t f_1(t-\tau)f_2(\tau)d\tau\right]\cdot e^{-st}dt \quad ...(1)$$

Shifted unit step function $u(t-\tau)$, where τ is a dummy variable is defined as,

$$u(t-\tau) = 1\ ;\ \tau \le t$$
$$= 0\ ;\ \tau > t$$

$$\therefore\ \int_0^t f_1(t-\tau)f_2(\tau)d\tau = \int_0^\infty f_1(t-\tau)\,u\,(t-\tau)f_2\,(\tau)\,d\tau \quad ...(2)$$

Substituting equation (2) in equation (1), we have,

$$L[f_1(t)*f_2(t)] = \int_0^\infty\left[\int_0^\infty f_1(t-\tau)u(t-\tau)f_2(\tau)d\tau\right]e^{-st}dt$$

Let, $x = t-\tau$

$t = x+\tau$

$dt = dx$

$$\therefore\ L[f_1(t)*f_2(t)] = \int_0^\infty\int_0^\infty f_1(x)u(x)f_2(\tau)e^{-s(x+\tau)}d\tau dx$$

$$= \int_0^\infty\int_0^\infty f_1(x)\,u(x)f_2(\tau)e^{-s\tau}\cdot e^{-sx}d\tau dx$$

$$= \int_0^\infty f_1(x)\,u(x)e^{-sx}dx\int_0^\infty f_2(\tau)e^{-s\tau}d\tau$$

$$= F_1(x).F_2(x)$$

$\therefore\ L[f_1(t)*f_2(t)] = F_1(s).F_2(s).$

Q.52 **Describe the convolution integral graphically.**

Ans:

Convolution Integral

Convolution integral is defined as integral of two functions, with one function delayed by t sec. Mathematically, it is given as.

$$\int_0^t f_1(\tau)f_2(t-\tau)d\tau$$

(or)

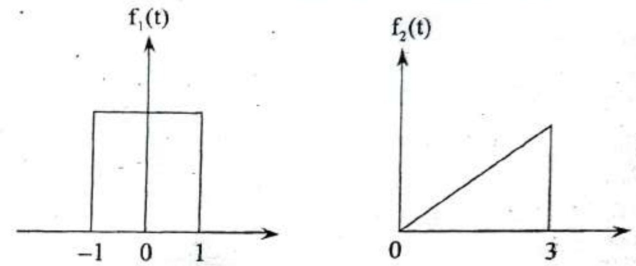
$$\int_0^t f_2(\tau)f_1(t-\tau)d\tau$$

Where,

τ is a dummy variable for t.

The convolution integrals are first studied by Dumhel in 1833, which have various applications in fields such as automatic control and network analysis. These integrals are also known as faulting integrals.

The convolution of a signal can be explained graphically. Let us consider a signal of rectangular pulse and a triangular pulse as shown in the figure, which are represented by the functions $f_1(t)$ and $f_2(t)$ respectively.



(a) Rectangular Pulse **(b) Triangular Pulse**

Figure (1)

Steps Involved in Graphical Interpretation of Convolution

Step1

The given functions (or) the pulses $f_1(t)$ and $f_2(t)$ should be represented on τ-axis as shown in figure (2).

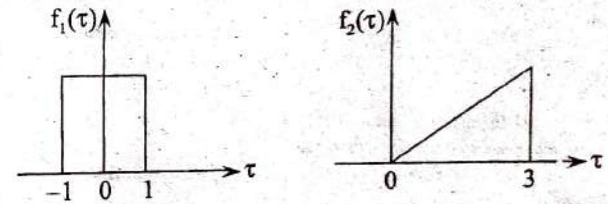


Figure (2)

Step2

Now, the function $f_2(\tau)$ is folded about the vertical axis and is passed through the origin as shown in figure (3) to obtain $f_2(-\tau)$.

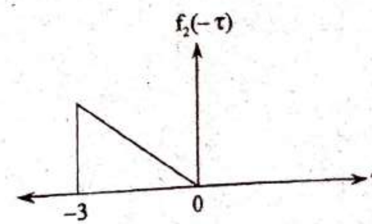


Figure (3)

Step3

Now the function $f_2(-\tau)$ is shifted along the τ axis by t sec as shown in figure (4) to obtain $f_2(t_1 - \tau)$.

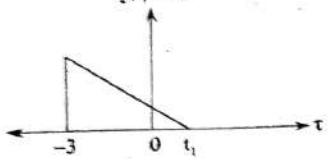


Figure (4)

Step4

Now, the convolution of two functions $f_1(t)$ and $f_2(t)$ is given by,

$$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t_1 - \tau)\, d\tau$$

The area under the product curve of $f_1(\tau)$ and $f_2(t_1 - \tau)$ gives the convolution of the functions $f_1(t)$ and $f_2(t)$ as shown in figure (5).

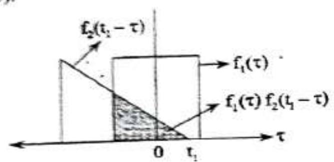


Figure (5)

Step5

The procedure is repeated for different value 't' and shift the function $f_2(-\tau)$ and the area under the new curve can be calculated. The convolution of the two function $f_1(t)$ and $f_2(t)$ can be represented as shown in figure (6).

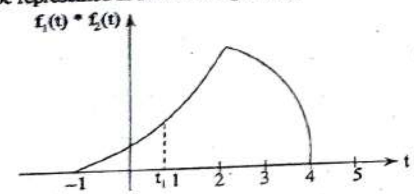


Figure (6)

Q.53 Define the following functions and obtain the Laplace transform of these,

(i) Shifted step function

(ii) Pulse

(iii) Shifted ramp function

(iv) Impulse function.

Ans:

(i) Shifted Step Function

A function is said to be a shifted step function when it is delayed or advanced with time T in order to obtain a zero value till time $t < T$ and a constant value from $t = T$ onwards.

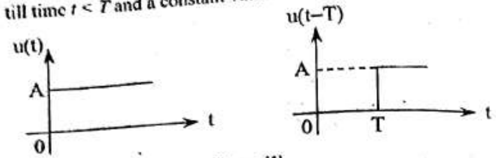


Figure (1)

A unit step function is defined as $u(t)$ and a shifted unit step function as $u(t - T)$.

$$\therefore \quad u(t-T) = A\,;\ t \geq T$$
$$= 0\,;\ t < T$$

From shifting property of Laplace transform we have,

$$L\{f(t-a)\, u(t-a)\} = e^{-as} F(s)$$

$$\therefore \quad L[u(t-T)] = e^{-Ts} \times L[u(t)]$$

$$= e^{-Ts} \times \frac{A}{s} \qquad \left(\because L[u(t)] = \frac{A}{s}\right)$$

$$= \frac{Ae^{-Ts}}{s}$$

(ii) Pulse

A pulse is a function or a signal with only two discontinuities T_1 and T_2, with a constant value A in between these two discontinuities.

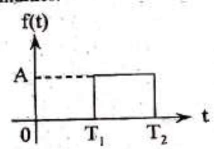


Figure (2)

$$f(t) = A,\ T_1 \leq t \leq T_2$$
$$= 0,\ \text{else}$$

$f(t)$ can also be defined in terms of $u(t)$ as,

$$f(t) = A[u(t - T_1) - u(t - T_2)]$$

$\therefore$ Laplace transform of a pulse function is,

$$F(s) = A \times L[u(t - T_1) - u(t - T_2)]$$

$$= A \times \left[\frac{1}{s} e^{-T_1 s} - \frac{1}{s} e^{-T_2 s}\right]$$

$$= \frac{A}{s}\left[e^{-T_1 s} - e^{-T_2 s}\right]$$

(iii) Shifted Ramp Function

A ramp signal $r(t)$ when delayed or advanced with time T, in order to obtain the signal from time $t = T$ instead of '0' is said to be a shifted ramp function $r(t - T)$.

$$r(t-T) = t - T\ ;\ t \geq T$$
$$= 0\ ;\ t < T$$

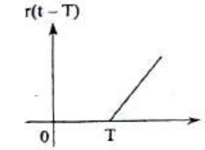


Figure (3)

From shifting property of Laplace transform, we have,

$$L[f(t-a)\, u(t-a)] = a^{-as} F(s)$$

$$L[r(t-T)] = e^{-Ts} \times L[r(t)]$$

$$= e^{-Ts} \times \frac{A}{s^2} \qquad \left(\because L[r(t)] = \frac{A}{s^2}\right)$$

$$= \frac{Ae^{-Ts}}{s^2}$$

(iv) Impulse Function

For answer refer Unit-IV, Q14, Topic: Impulse Function.

Q.54 Find the convolution between $f_1(t) = u(t)$ and $f_2(t) = e^{-t}u(t)$ using,

(a) Graphical convolution

(b) Laplace transform.

Ans:

Given that,

$$f_1(t) = u(t)$$
$$f_2(t) = e^{-t}u(t)$$

To determine,

(a) Graphical convolution between $f_1(t)$ and $f_2(t)$

(b) Convolution of $f_1(t)$ and $f_2(t)$ using Laplace transform.

(a) Graphical Convolution

(i)

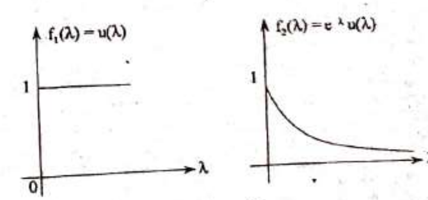


Figure (1)

(ii) Folding $f_2(\lambda)$

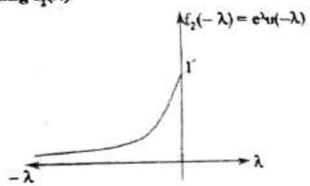


Figure (2)

(iii) Shifting $f_2(-\lambda)$ to $f_2(t-\lambda)$

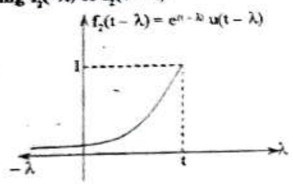


Figure (3)

(iv) Multiplying $f_1(\lambda)$ and $f_2(t-\lambda)$

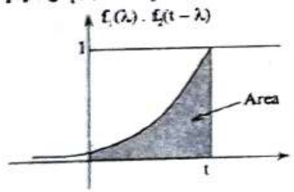


Figure (4)

(v) Integration of the Shaded Area

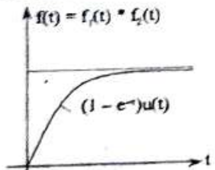


Figure (5)

(b) Laplace Transform

$f_1(t) = u(t)$

$f_2(t) = e^{-t}\, u(t)$

Now,

Laplace transform of $f_1(t)$,

$$L\{f_1(t)\} = L\{u(t)\} = \frac{1}{s} = F_1(s)$$

And also,

$$L\{f_2(t)\} = L\{e^{-t}u(t)\} = \frac{1}{s+1} = F_2(s)$$

We know that,

The convolution of two functions, $f_1(t)$ and $f_2(t)$ using Laplace transforms is given by,

$$f_1(t) * f_2(t) = L^{-1}[F_1(s) \cdot F_2(s)]$$

$$\Rightarrow \quad f_1(t) * f_2(t) = L^{-1}\left[\frac{1}{s} \cdot \frac{1}{(s+1)}\right]$$

$$\Rightarrow \quad f_1(t) * f_2(t) = L^{-1}\left[\frac{1}{s(s+1)}\right] \quad \ldots (1)$$

By partial fraction expansion, we have,

$$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$\Rightarrow \quad 1 = A(s+1) + Bs$$

Put, $s = 0, A = 1$

$s = -1, B = -1$

Therefore, equation (1), becomes,

$$f_1(t) * f_2(t) = L^{-1}\left[\frac{1}{s(s+1)}\right] = L^{-1}\left[\frac{1}{s} - \frac{1}{(s+1)}\right]$$

$$= L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{1}{s+1}\right]$$

$$= u(t) - e^{-t}u(t)$$

$$= u(t)\,[1 - e^{-t}]$$

$$\therefore \quad f_1(t) * f_2(t) = (1 - e^{-t})\, u(t)$$

Q.55. The impulse response of a certain linear system is given by,

$h(t) = e^{-2t}\, u(t) \quad ; \quad t \geq 0$

$= 0 \quad ; \quad t < 0$

Using the convolution integral, determining the response y(t) due to ramp input.

$x(t) = 0 \;;\; t < 0$

$= t \;;\; t \geq 0.$

Ans:

Given that,

Impulse response, $h(t) = e^{-2t}\, u(t) \;;\; t \geq 0$

$= 0 \quad ; t < 0$

Ramp input, $x(t) = 0 \;;\; t < 0$

$= t \;;\; t \geq 0$

Response, $y(t) = ?$

Now, the response $y(t)$ of the system using convolution integral is given by,

$$y(t) = h(t) * x(t)$$

$$= \int_{-t}^{t} h(t-\tau)x(\tau)d\tau$$

$$= \int_{-t}^{0} h(t-\tau)x(\tau)d\tau + \int_{0}^{t} h(t-\tau)x(\tau)d\tau$$

$$= 0 + \int_{0}^{t} h(t-\tau)x(\tau)d\tau$$

$[\because x(\tau) = h(t-\tau) = 0 \text{ for } t < 0]$

$$= \int_{0}^{t} h(t-\tau)x(\tau)d\tau$$

$$= \int_{0}^{t} e^{-2(t-\tau)}u(t-\tau).\tau.d\tau$$

$$= \int_{0}^{t} e^{-2(t-\tau)}.1.\tau.d\tau$$

$[\because u(t-\tau) = 1 \text{ for } t \geq 0]$

$$= \int_{0}^{t} e^{-2(t-\tau)}.\tau d\tau$$

$$= \int_{0}^{t} e^{-2t}.e^{2\tau}.\tau d\tau$$

$$= e^{-2t}\int_{0}^{t} \tau.e^{2\tau}d\tau$$

We know that,

$$\int f(x).g(x) = f(x)\int g(x)dx - \int [f'(x)\int g(x)dx]\, dx$$

$$\therefore \quad y(t) = e^{-2t}\left[\tau\frac{e^{2\tau}}{2} - \int\left[\left(\frac{d}{d\tau}(\tau)\right)\int(e^{2\tau}d\tau)\right]d\tau\right]_0^t$$

$$= e^{-2t}\left[\frac{\tau e^{2\tau}}{2} - \int\frac{e^{2\tau}}{2}d\tau\right]_0^t$$

$$= e^{-2t}\left[\frac{\tau e^{2\tau}}{2} - \frac{1}{2}\times\frac{e^{2\tau}}{2}\right]_0^t$$

$$= \frac{e^{-2t}}{2}\left[\tau.e^{2\tau} - \frac{e^{2\tau}}{2}\right]_0^t$$

$$= \frac{e^{-2t}}{2}\left[\left(te^{2t} - \frac{e^{2t}}{2}\right) - \left(0.e^{0} - \frac{e^{0}}{2}\right)\right]$$

$$= \frac{e^{-2t}}{2}\left[te^{2t} - \frac{e^{2t}}{2} + \frac{1}{2}\right]$$

$$= te^{2t}.\frac{e^{-2t}}{2} - \frac{e^{2t}}{2}.\frac{e^{-2t}}{2} + \frac{e^{-2t}}{2}.\frac{1}{2}$$

$$= t.\frac{e^{0}}{2} - \frac{e^{0}}{4} + \frac{e^{-2t}}{4}$$

$$= \left(\frac{t}{2} - \frac{1}{4} + \frac{e^{-2t}}{4}\right)u(t)$$

Q.56 Find the convolution for the signals,

(i) $x(t) = \begin{cases} 1 & ; \quad 0 < t < T \\ 0 & ; \quad \text{Otherwise} \end{cases}$

(ii) $h(t) = \begin{cases} t & ; \quad 0 < t < 2T \\ 0 & ; \quad \text{Otherwise} \end{cases}$

Ans:

Given that,

$$x(t) = \begin{cases} 1 & ; \quad 0 < t < T \\ 0 & ; \quad \text{Otherwise} \end{cases}$$

$$h(t) = \begin{cases} t & ; \quad 0 < t < 2T \\ 0 & ; \quad \text{Otherwise} \end{cases}$$

Now, assume $f_1(t) = x(t)$ and $f_2(t) = h(t)$.

By the convolution definition, we have,

$$f_1(t) * f_2(t) = \int_0^t f_1(\tau)f_2(t-\tau)d\tau$$

As $x(t)$ valid from $0 < t < T$ and $h(t)$ is valid for $0 < t < 2T$. Hence, taking the limits of integration from 0 to $2T$.

$$\therefore \quad x(t) * h(t) = \int_0^{2T} x(\tau)h(t-\tau)d\tau$$

$$= \int_0^{T} x(\tau)h(t-\tau)d\tau + \int_T^{2T} x(\tau)h(t-\tau)d\tau$$

$$= \int_0^{T} x(\tau)h(t-\tau)d\tau + 0 \; [\because x(\tau) = 0 \text{ for } t > T$$

$$= \int_0^{T} x(\tau)h(t-\tau)d\tau$$

$$= \int_0^T x(\tau)h(t-\tau)d\tau + 0 \quad [\because x(\tau) = 1 \text{ for } 0 < t < T]$$

$$= \int_0^T (t-\tau)d\tau$$

$$= t\int_0^T d\tau - \int_0^T \tau d\tau$$

$$= t[\tau]_0^T - \left[\frac{\tau^2}{2}\right]_0^T$$

$$= t(T-0) - \frac{1}{2}(T^2 - 0)$$

$$= Tt - \frac{T^2}{2}$$

$$= T\left[t - \frac{T}{2}\right]$$

Q.57 Evaluate the following convolution integrals,

(i) $u(t) * e^{-t} u(t)$

(ii) $u(t) * t\, u(t)$.

Ans:

(i) $u(t) * e^{-t}u(t)$

Let,

$f_1(t) = u(t)$ and

$f_2(t) = e^{-t}u(t)$

Now, the convolution of two functions $f_1(t)$ and $f_2(t)$ is given by,

$$f_1(t) * f_2(t) = \int_0^t f_1(t-\tau) f_2(\tau)d\tau \quad \text{... (1)}$$

We know that the unit function is defined as,

$u(t) = 1 \; ; \; t \geq 0$

$= 0 \; ; \; t < 0$

From equation (1), we have,

$$f_1(t) * f_2(t) = u(t) * e^{-t}u(t)$$

$$= \int_{-t}^{t} u(t-\tau)e^{-\tau}u(\tau)d\tau$$

$$= \int_{-t}^{0} u(t-\tau)e^{-\tau}u(\tau)d\tau + \int_0^t u(t-\tau)e^{-\tau}u(\tau)d\tau$$

$$= 0 + \int_0^t u(t-\tau)e^{-\tau}u(\tau)d\tau \quad [\because u(t-\tau) = 0 \text{ for } t < 0]$$

$$= \int_0^t u(t-\tau)e^{-\tau}u(\tau)d\tau$$

$$= \int_0^t 1.e^{-\tau} \cdot 1 \cdot d\tau \quad [\because u(\tau) = u(t-\tau) = 1 \text{ for } t \geq 0]$$

$$= \int_0^t e^{-\tau} d\tau$$

$$= \left[\frac{e^{-\tau}}{-1}\right]_0^t$$

$$= -(e^{-t} - e^{0})$$

$$= -(e^{-t} - 1)$$

$$= (1 - e^{-t})\, u(t)$$

(ii) $u(t) * tu(t)$

Let,

$f_1(t) = u(t)$ and

$f_2(t) = tu(t)$

Now, the convolution is given by,

$$f_1(t) * f_2(t) = u(t) * tu(t)$$

$$= \int_{-t}^{t} u(t-\tau)\tau u(\tau)d\tau$$

$$= \int_{-t}^{0} u(t-\tau)\tau u(\tau)d\tau + \int_0^t u(t-\tau)\tau u(\tau)d\tau$$

$$= 0 + \int_0^t u(t-\tau)\tau u(\tau)d\tau \quad [\because u(\tau) = u(t-\tau) = 0 \text{ for } t < 0]$$

$$= \int_0^t u(t-\tau)\tau u(\tau)d\tau$$

$$= \int_0^t 1.\tau.1d\tau \quad [\because u(t-\tau) = u(\tau) = 1 \text{ for } t \geq 0]$$

$$= \int_0^t \tau.d\tau$$

$$= \left[\frac{\tau^2}{2}\right]_0^t$$

$$= \frac{1}{2}(t^2 - 0^2)$$

$$= \frac{1}{2}t^2.u(t)$$

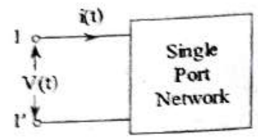
4.3 TRANSFORMED NETWORK WITH INITIAL CONDITION

Q.58. What is a driving point function? Explain the necessary conditions for driving point functions.

Ans:

Driving Point Function

For a single port network, the network functions related to two variables $V(t)$ and $i(t)$ and measured at the same port are called "Driving Point Function".



Figure

There are two types of network functions.

(i) Driving point impedance function

(ii) Driving point admittance function.

(i) Driving Point Impedance Function

For answer refer Unit-IV, Q17.

(ii) Driving Point Admittance Function

For answer refer Unit-IV, Q19(i)

Necessary Conditions for Driving Point Function

1. The coefficient of numerator and denominator of the polynomials present in the network function must be real and positive.
2. The imaginary and complex poles or zeros necessary for the transfer function must occur in conjugate pairs
3. The real part of all the poles and zeros must be negative or zero but for imaginary part the poles and zeros must be simple i.e., non repeated
4. In the numerator and the denominator the coefficients between the lowest and highest powers of 's' should be non-zero i.e., there should no missing terms between the highest and lowest degrees unless all even terms or all odd terms are absent.
5. The highest degree of numerator $y(s)$ and denominator $x(s)$ can maximum differ by one only or must differ by zero.
6. The minimum difference in the degree of numerator and denominator may maximum differ in degree by one.

Q.59. Explain the properties of driving point function.

Ans:

Properties of Driving Point Function

Following are the list of properties of driving point function.

(i) The ratio of transform impedance of element in s is the driving point function,

i.e., $Z(s) = \dfrac{V(s)}{I(s)} = N(s)$

(ii) It provides the information about network function,

(a) Short circuit or

(b) Open circuit.

(iii) Conditions to be fulfilled by poles and zeros,

(a) If any complex pole or zero then they must occur in conjugate pair of it since all the elements of circuit are real and positive.

(b) To have a stable driving point function, all zeros and poles must have negative or zero real part.

(c) It must have simple or distinct, poles and zeros that lie on imaginary axis.

(iv) The polynomials equations must fulfill the following condition,

(a) $C(s)$ and $R(s)$ i.e., numerator and denomi-nator must either have same degree or differ by one.

(b) The lowest degree term in the numerator and the denominator of a driving point function may differ in degree by zero or one only.

(c) The numerator and denominator polynomial must either contain terms without missing any degree term or if in case the degree is missing then all the even or odd degree terms will be absent.

Q.60 Obtain the transform impedance and transform admittance of a resistor. Also draw the transform circuit.

Ans:

The voltage and current relationship in a resistor in time domain using Ohm's law is given as,

$$V_r(t) = R.\, i_r(t) \quad \text{... (1)}$$

From equation (1),

$$i_r(t) = \frac{V_r(t)}{R}$$

$$= G.\, V_r(t) \qquad (\because \text{ Conductance, } G = \frac{1}{R}) \quad \text{... (2)}$$

The transform equations of a resistor are obtained by applying Laplace transformation to equations (1) and (2).

Hence, for equation (1) we get,

$$L[V_r(t)] = L[R.\, i_r(t)]$$

$$\Rightarrow \quad V_r(s) = R.\, I_r(s) \quad \text{... (3)}$$

And for equation (2), we get,

$$L[i_r(t)] = L[G\, V_r(t)]$$

$$\Rightarrow \quad I_r(s) = G.\, V_r(s) \quad \text{... (4)}$$

Thus, equations (3) and (4) represent transform equations of a resistor.

Transform Impedance $Z_r(s)$

Transform impedance of a resistor $Z_r(s)$ is defined as the ratio of transform voltage $V_r(s)$ to that of the transform current $I_r(s)$.

$$\therefore \quad Z_r(s) = \frac{V_r(s)}{I_r(s)} = R \qquad [\because \text{ From equation (3)}]$$

The resistive circuit in time domain is shown in figure (1) and its equivalent transform circuit is shown in figure (2).

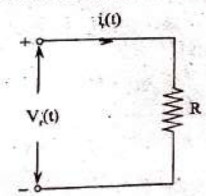


Figure (1)

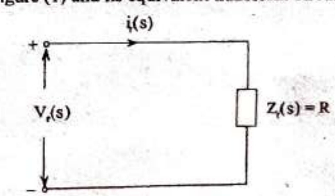


Figure (2)

Transform Admittance $Y_r(s)$

Transform admittance of a resistor $Y_r(s)$ is defined as the ratio of transform current $I_r(s)$ to that of the transform voltage $V_r(s)$.

$$\therefore \quad Y_r(s) = \frac{I_r(s)}{V_r(s)} = \frac{1}{R} = G \qquad [\because \text{ From equation (4)}]$$

Q.61 Obtain the transform impedance and transform admittance of an inductor. Also draw the transform circuit.

Ans:

The voltage and current relationship in an inductor in time domain is given as,

$$V_L(t) = L.\frac{di_L(t)}{dt} \quad \text{... (1)}$$

Integrating both sides of equation (1) from $-\infty$ to t, we get,

$$\int_{-\infty}^{t} V_L(t)dt = L\int_{-\infty}^{t} \frac{di_L(t)}{dt}$$

$$\Rightarrow L.i_L(t) = \int_{-\infty}^{t} V_L(t)$$

$$\Rightarrow i_L(t) = \frac{1}{L}\int_{-\infty}^{t} V_L(t)dt \quad \text{... (2)}$$

The transform equation of equation (1) is obtained by applying Laplace transform to it.

$$\therefore L[V_L(t)] = L.\ L\left[\frac{di_L(t)}{dt}\right]$$

$$\Rightarrow V_L(s) = L.[s.\ i_L(s) - i_L(0^-)]$$

$$\Rightarrow L.s\ i_L(s) = V_L(s) + L.i_L(0^-) \quad \text{... (3)}$$

Where,

$V_L(s)$ – Transform voltage of applied voltage $V(t)$

$L.i_L(0^-)$ – Transform voltage caused by initial current $i(0^-)$ in inductor at $t = 0^-$

Let, the total transform voltage be $V_L'(s)$

Thus, $V_L'(s) = V_L(s) + L.i_L(0^-)$ ... (4)

Substituting equation (4) in equation (3), we get,

$L.s\ i_L(s) = V_L'(s)$ ... (5)

Transform Impedance

The transform impedance of an inductor $Z_L(s)$ is defined as the ratio of total transform voltage $V_L'(s)$ to that of transform current $i_L(s)$.

$$\therefore Z_L(s) = \frac{V_L'(s)}{i_L(s)} = \frac{L.s\,i_L(s)}{i_L(s)} = s.L \quad [\because \text{From equation (5)}]$$

$$\boxed{\therefore Z_L(s) = s.L}$$

A circuit represented by an inductor in time domain is shown in figure (1). Its equivalent transform circuit with transform impedance $Z_L(s)$ is shown in figure (2).

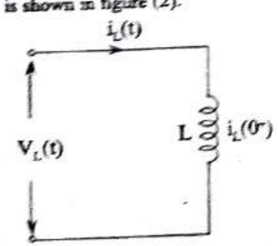


Figure (1)

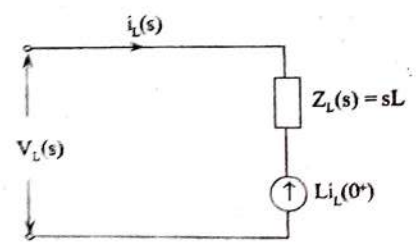


Figure (2)

Transform Admittance

In order to determine transform admittance,

Consider equation (3), we get,

$$L.s\ i_L(s) = V_L(s) + L.i_L(0^+)$$

$$\Rightarrow i_L(s) = \frac{1}{L.s}[V_L(s) + L.i_L(0^+)]$$

$$= \frac{1}{L}\frac{V_L(s)}{s} + \frac{iL(0^+)}{s}$$

$$\Rightarrow \frac{V_L(s)}{Ls} = i_L(s) - \frac{i_L(0^+)}{s} \quad \text{... (6)}$$

In equation (4), the term $\frac{i_L(0^+)}{s}$ represents the transform current caused by the initial current $i_L(0^+)$ in the inductor. Hence, the total transform current is given as,

$$i'_L(s) = i_L(s) - \frac{i_L(0^+)}{s} \quad \text{... (7)}$$

Substituting equation (7) in equation (6), we get,

$$\frac{V_L(s)}{L.s} = i_L'(s) \quad \text{... (8)}$$

Hence, the transform admittance $Y_L(s)$ of an inductor is defined as the ratio of total transform current $i_L'(s)$ to that of transform voltage $V_L(s)$.

$$\therefore Y_L(s) = \frac{i_L'(s)}{V_L(s)} = \frac{i_L'(s)}{L.s\,i_L'(s)} = \frac{1}{Ls} \quad [\because \text{From equation (8)}]$$

A circuit represented by an inductor in time domain and its equivalent transform circuit with transform admittance $Y_L(s)$ are shown in figures (3) and (4) respectively.

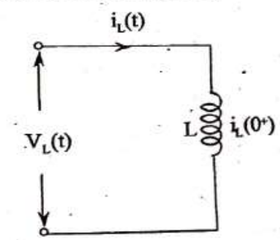


Figure (3)

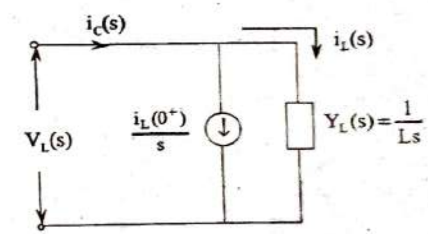
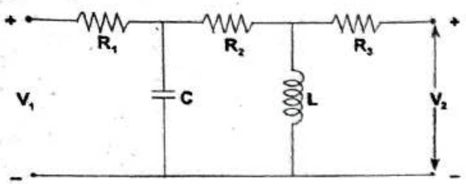


Figure (4)

Q.62. For the two port network shown in figure. Find, $G_{12}\left(\frac{V_2(s)}{V_1(s)}\right)$.



Figure

Ans:

Given two port network is shown in figure (1).

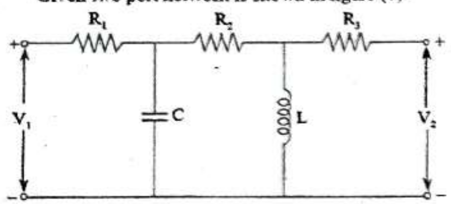


Figure (1)

Transforming the given network in s-domain, the circuit is modified as shown in figure (2).

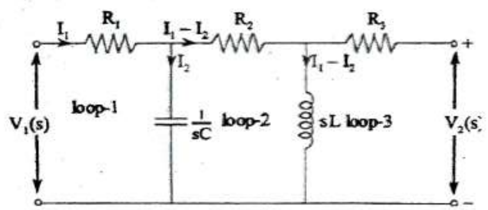


Figure (2)

Applying KVL in loop-1, we get,

$$V_1(s) = I_1(s)R_1 + \frac{1}{sC}I_2(s) \quad \text{... (1)}$$

Applying KVL in loop-2, we get,

$$0 = R_2(I_1(s) - I_2(s)) + sL(I_1(s) - I_2(s)) - \frac{1}{sC}I_2$$

$$0 = R_2I_1(s) - R_2I_2(s) + sL\,I_1(s) - sL\,I_2(s) - \frac{1}{sC}I_2(s)$$

$$0 = I_1(s)(R_2 + sL) - I_2(s)\left(R_2 + sL + \frac{1}{sC}\right)$$

$$0 = I_1(s)(R_2 + sL) - I_2(s)\left(R_2 + sL + \frac{1}{sC}\right)$$

$$I_1(s) = I_2(s)\left(\frac{R_2 + sL + \frac{1}{sC}}{R_2 + sL}\right) \quad \text{... (2)}$$

Applying KVL in loop-3, we get,

$$V_2(s) = sL(I_1(s) - I_2(s))$$

$$V_2(s) = sL\,I_1(s) - sL\,I_2(s) \quad \text{... (3)}$$

Substituting equation (2) in equation (1), we get,

$$V_1(s) = I_2(s)\left(\frac{R_2 + sL + \frac{1}{sC}}{R_2 + sL}\right)R_1 + \frac{1}{sC}I_2(s)$$

$$= I_2(s)\left(\frac{R_1R_2 + R_1sL + \frac{R_1}{sC}}{R_2 + sL} + \frac{1}{sC}\right)$$

$$V_1(s) = I_2(s)\left(\frac{R_1R_2 + R_1sL + \frac{R_1}{sC} + \frac{R_2}{sC} + \frac{L}{C}}{R_2 + sL}\right) \quad \text{... (4)}$$

$$V_2(s) = sLI_2(s)\left(\frac{R_2 + sL + \frac{1}{sC}}{R_2 + sL}\right) - sL\,I_2(s)$$

$$= I_2(s)\left[\frac{R_2sL + s^2L^2 + \frac{L}{C}}{R_2 + sL} - sL\right]$$

$$= I_2(s)\left[\frac{R_2sL + s^2L^2 + \frac{L}{C} - R_2sL - s^2L^2}{R_2 + sL}\right]$$

$$V_2(s) = I_2(s)\left[\frac{\frac{L}{C}}{R_2 + sL}\right] \quad \text{... (5)}$$

Dividing equation (5) by equation (4), we get,

$$\frac{V_2(s)}{V_1(s)} = \frac{I_2(s)\left[\frac{\frac{L}{C}}{R_2 + sL}\right]}{I_2(s)\left[\frac{R_1R_2 + R_1sL + \frac{R_1}{sC} + \frac{R_2}{sC} + \frac{L}{C}}{R_2 + sL}\right]}$$

$$G_{12}\left(\frac{V_2(s)}{V_1(s)}\right) = \frac{L}{C\left(R_1R_2 + R_1sL + \frac{R_1}{sC} + \frac{R_2}{sC} + \frac{L}{C}\right)}$$

$$= \frac{L}{R_1R_1C + R_1sLC + \frac{R_1}{s} + \frac{R_2}{s} + L}$$

$$G_{12}\left(\frac{V_2(s)}{V_1(s)}\right) = \frac{sL}{s^2R_1LC - s(R_1R_2C + L) + R_1 + R_2}$$

4.4 TRANSFER REPRESENTATION, POLES AND ZEROS, FREQUENCY RESPONSE (MAGNITUDE AND PHASE PLOTS)

Q.63. What is a transfer function? What are the properties of a transfer function?

Ans:

Transfer Function

The transfer function is defined as the ratio of Laplace transform of output to the Laplace transform of input function.

$$\underset{\text{Input}}{R(s)} \longrightarrow \boxed{G(s)} \longrightarrow \underset{\text{Output}}{C(s)}$$

$$G(s) = \frac{C(s)}{R(s)}$$

Where,

$G(s)$ is the transfer function, $R(s)$ is input and $C(s)$ is output.

Limitation

(a) Transfer function does not explains the structure of system.

(b) It is applicable only for linear time-invariant systems i.e., input and output characteristics do not change with time.

Properties of Transfer Function

Properties of transfer function are listed below,

(i) Input and output are polynomial in s whose ratio forms transfer function (T.F).

(ii) The coefficient of ratio components T.F i.e., $C(s)$ and $R(s)$ must be real. Hence, when poles and zeros are complex then they will occur in conjugate pairs.

(iii) Poles i.e., roots of denominator must have negative real part and are non-repeating i.e., simple

(iv) All the poles of transfer functions are zero of $Q(s)$, they lie in negative half plane and any zero lying on imaginary $C(j\omega)$ axis must be non-repeating i.e., simple.

(v) Degree of denominator polynomial i.e., $R(s)$ must be higher or equal to the degree of numerator polynomial for given value of $G(s)$.

Q.64. What are the necessary conditions of transfer function?

Ans:

The necessary conditions for the transfer function are as follows,

1. The coefficients $m_0, m_1, m_2 ... m_n$ of the numerator $C(s)$ and the denominator $R(s)$ must be real and positive.
2. The imaginary and complex poles or zeros for the transfer function must occur in conjugate pairs.
3. In the denominator $R(s)$, the coefficients between the lowest and highest powers of 's' should be non-zero i.e., there should be no missing term between the highest and the lowest degree unless all even or all odd term are missing. Whereas the numerator $C(s)$ may have missing terms between the lowest and highest degree.
4. The highest degree of the numerator should be as small as possible and should not depend on the degree of the denominator
5. In case of transfer impedance (Z) and transfer admittance (Y), the highest degree of numerator $C(s)$ should be equal to the highest degree of denominator $R(s)$ plus alone
6. In voltage transfer ratio and current transfer ratio, the highest degree of the numerator $C(s)$ should be equal to the degree of the denominator
7. The real part of the poles and zeros must be negative or zero but for imaginary part the poles and zeros must be simple i.e., non repeated.

Q.65. How can you assess the nature of time domain response from pole-zero plot? Explain.

OR

Explain the time domain behaviour from the pole zero plot.

Ans:

A pole zero plot can be drawn for any given network function and is used for obtaining the information regarding critical frequencies. From this plot time domain response can also be obtained.

Consider a group of poles as shown in figure (a).

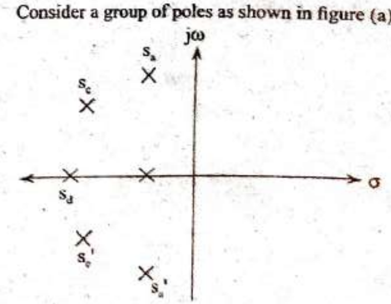


Figure (a) Pole-zero Plot

In figure (a), s_a and s_c are the complex conjugate poles and s_b and s_d are the real poles.

For the real poles, the quadrature function is given by,

$$\Rightarrow \quad s^2 + 2\zeta\,\omega_n s + \omega_n^2 \qquad [\text{For } \zeta > 1] \qquad \dots (1)$$

Where, ζ is the damping ratio,

ω_n is the undamped natural frequency.

On solving the equation (1), we get the roots as,

$$s_b, s_d = \zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} \;:\; \zeta > 1$$

Now, the time domain behaviour is given as,

$$i(t) = M_2 e^{s_b t} + M_4 e^{s_d t}$$

The time domain behaviour of poles s_b and s_d is as shown in figures (b) and (c) respectively.

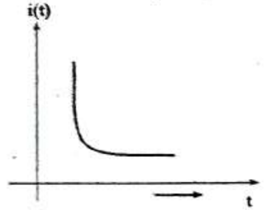


Figure (b): Response due to Poles s_d

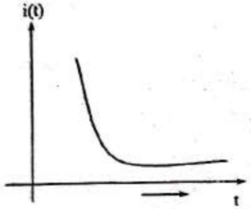


Figure (c): Response due to Pole s_b

The quadratic function if the poles are complex conjugate is given as,

$$s^2 + 2\zeta\,\omega_n s + \omega_n^2 \qquad (\text{for } \zeta < 1) \qquad \dots (2)$$

The roots of the equation (2) are,

$$s_a, s_c = -\zeta\,\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \qquad (\text{for } \zeta < 1)$$

The time domain response for these poles is given as,

$$i(t) = M_1\, e^{-\zeta\omega_n t + j\omega_n\sqrt{1-\zeta^2}\,t} + M_3 e^{-\zeta\omega_n t - j\omega_n\sqrt{1-\zeta^2}\,t}$$

$$= Me^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2})t \qquad \dots (3)$$

$\therefore$ From equation (3), we can say that the response for the conjugate pole is damped sinusoid. Similarly, the behaviour of s_3 and s_d is sinusoidal but it damps faster than that of s.

Let, the input voltage $V(s)$ in applied to a network and $I(s)$ be the transform of a current.

$$\therefore \quad I(s) = \frac{V(s)}{Z(s)} = V(s).\,Y(s)$$

$$= \frac{C(s)}{R(s)}$$

Where, $Y(s)$ is the transfer admittance.

$I(s)$ can be written as,

$$I(s)\ \frac{C(s)}{R(s)} = H\,\frac{(s-s_1)(s-s_2)(s-s_3)\dots(s-s_n)}{(s-s_a)(s-s_b)(s-s_c)\dots(s-s_m)}$$

$$\therefore \quad I(s) = H\,\frac{(s-s_1)(s-s_2)(s-s_3)\dots(s-s_n)}{(s-s_a)(s-s_b)(s-s_c)\dots(s-s_m)} \qquad \dots (4)$$

Now, the time domain behaviour can be evaluated from the zeroes, $s_1, s_2, s_3 \dots s_n$ and poles $s_a, s_b, s_c \dots s_m$ and the scaling factor 'H'.

Applying partial fraction to equation (4), we get,

$$I(s) = \frac{M_1}{(s-s_a)} + \frac{M_2}{(s-s_b)} + \frac{M_3}{(s-s_c)} + \dots \frac{M_n}{(s-s_m)} \qquad \dots (5)$$

Applying inverse Laplace to equation (5) we get, the time domain response as,

$$I(t) = L^{-1}\left[\frac{M_1}{(s-s_a)} + \frac{M_2}{(s-s_b)} + \frac{M_3}{(s-s_c)} + \dots \frac{M_n}{(s-s_m)}\right] \qquad \dots (6)$$

Any coefficients of the equation (6) can be found by using Heavisides method.

The coefficients K_p can be found as,

$$K_p = H\left[\frac{(s-s_1)(s-s_2)(s-s_3)\dots(s-s_n)}{(s-s_a)(s-s_b)(s-s_c)\dots(s-s_m)}\right](s-s_p)\Big|_{s=s_p} \qquad \dots (7)$$

Where, s_p, s_m, s_n are the complex numbers.

$$\Rightarrow K_p = H\left[\frac{(s_p-s_1)(s_p-s_2)(s_p-s_3)\dots(s_p-s_n)}{(s_p-s_a)(s_p-s_b)(s_p-s_c)\dots(s_p-s_m)}\right] \qquad \dots (8)$$

The numerator and denominator is in the from of $(s_p - s_x)$ and $(s_p - s_y)$ which are complex numbers the term $(s_p - s_1)(s_p - s_2)(s_p - s_3)\dots(s_p - s)$ can be expressed as,

$$(s_p - s_x) = N_{px}\, e^{j\alpha_{px}} \qquad \dots (9)$$

Where,

$x = 1, 2, 3, \dots n$

N_{px} is the magnitude

α_{px} is the phase angle

Similarly, denominator terms $(s - s_a)(s - s_b) \dots (s - s_m)$ of equation (8) can be expressed as,

$$(s_p - s_y) = D_{py}\, e^{j\beta_{py}}$$

Where,

$y = a, b, c, \dots m$

D_{py} is the magnitude

β_{py} is the phase angle.

Therefore, equation (8) can be written in magnitude and phase angle as,

$$K_p = H\left[\frac{N_{px}e^{j\alpha_{px}}}{D_{py}e^{j\beta_{py}}}\right] \qquad [\text{Where, } x = 1, 2, 3, \dots n,\ y = a, b, c, \dots m]$$

$$= H\left[\frac{N_{p1}e^{j\alpha_{p1}}.N_{p2}e^{j\alpha_{p2}}.N_{p3}e^{j\alpha_{p3}}\dots N_{pn}e^{j\alpha_{pn}}}{D_{pa}e^{j\beta_{pa}}.D_{pb}e^{j\beta_{pb}}.D_{pc}e^{j\beta_{pc}}\dots D_{pm}e^{j\beta_{pm}}}\right]$$

$$= H\left[\frac{N_{p1}N_{p2}N_{p3}\dots N_{pn}}{D_{pa}D_{pb}D_{pc}\dots D_{pm}} \times \frac{e^{j\alpha_{p1}}e^{j\alpha_{p2}}e^{j\alpha_{p3}}\dots e^{j\alpha_{pn}}}{e^{j\beta_{pa}}e^{j\beta_{pb}}e^{j\beta_{pc}}\dots e^{j\beta_{pm}}}\right]$$

$$= H\left[\frac{N_{p1}N_{p2}N_{p3}\dots N_{pn}}{D_{pa}D_{pb}D_{pc}\dots D_{pm}} \times \frac{e^{j[\alpha_{p1}+\alpha_{p2}+\alpha_{p3}+\dots+\alpha_{pn}]}}{e^{j[\beta_{pa}+\beta_{pb}+\beta_{pc}+\dots+\beta_{pm}]}}\right]$$

$$K_p = H\left[\frac{N_{p1}N_{p2}N_{p3}\dots N_{pn}}{D_{pa}D_{pb}D_{pc}\dots D_{pm}}\right] \times e^{j[\alpha_{p1}+\alpha_{p2}+\alpha_{p3}+\dots+\alpha_{pn}]-[\beta_{pa}+\beta_{pb}+\beta_{pc}+\dots+\beta_{pm}]}$$

Similarly, the other coefficients $K_1, K_2, \dots K_m$ can be evaluated.

Q.67. Explain why it is important to conduct frequency domain analysis of linear control systems.

Ans:

Frequency Response

Steady state output of a system, when sinusoidal signal is given is known as frequency response.

In the time domain where the unit step signal is considered as standard test signal, similarly the standard test signal in frequency domain is unit sinusoidal input signal.

Importance of frequency domain analysis are,

1. A design of a system in the frequency domain provides the designer with bandwidth control of the system. Thus, it became very popular amongst engineers.
2. This frequency domain approach makes designer and researchers to construct models which are realistic and to make appropriate and satisfactory simplifications.
3. Simple models which give enough understandable information about the linear control systems are obtained from frequency domain analysis.
4. With the frequency domain, the transfer function in either s or z is obtained, thus frequency domain analysis is based on evaluating transfer function of s or z domain.
5. In the analysis of stability the frequency domain description of linear, time invariant system are useful.
6. Model properties are expressed directly, in system transfer function.
7. The steady state response to slow and fast oscillatory inputs are characterized easily.
8. The computationally difficult convolution integrals of the time domain are replaced by equivalent multiplications of transfer function in the frequency domain.
9. The data can be obtained from the measurement on the physical system without deriving its mathematical model.
10. Frequency response analysis is most powerful in conventional control theory. They are indispensable to robust control theory.
11. Frequency domain analysis is very simple and can be made accurate by the use of readily available sinusoidal signal generators and precise measurement equipments.
12. By the use of a frequency response, a system may be designed so that the effects of undesirable noise are negligible and such analysis and design can be extended to certain non-linear control systems.
13. The transfer function of such complicated systems can be determined experimentally by the use of frequency test.
14. From the knowledge of the open loop response and frequency response, the estimation of absolute and relative stability of a closed loop system can be done.

Q.68. Derive the expressions for resonant peak and resonant frequency and hence establish the correlation between time response and frequency response.

Ans:

Resonant Peak (M_r)

It is defined as the maximum or peak value of magnitude of closed loop transfer function.

For a second order system, the closed loop transfer function in standard form is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \quad \text{... (1)}$$

Where,

ω_n = Natural frequency undamped

ξ = Damping ratio.

The ratio of actual damping to the critical damping is nothing but, the damping ratio.

Now, substituting $s = j\omega$ in equation (1),

We get,

$$\frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(j\omega)^2+2\xi\omega_n .\omega j+\omega_n^2} \quad \text{... (2)}$$

$$= \frac{\omega_n^2}{-\omega^2+2\xi\omega_n\omega j+\omega_n^2} \quad (\because \; j^2=-1)$$

Now dividing with ω_n^2 to both the numerator and denominator in the above fraction, we get,

$$= \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2+2j\xi\left(\frac{\omega}{\omega_n}\right)+1}$$

Substituting real and imaginary parts in equation (2), we get,

$$= \frac{1}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]+\left(2j\xi\frac{\omega}{\omega_n}\right)}$$

$$= \frac{1}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]+\left[2j\xi\frac{\omega}{\omega_n}\right]} \quad \text{... (3)}$$

The ratio of ω_r and ω_n is called as normalized frequency (u) i.e.,

$$u = \frac{\omega_r}{\omega_n} \quad \text{... (4)}$$

Applying partial fraction to equation (4), we get,

$$I(s) = \frac{M_1}{(s-s_a)}+\frac{M_2}{(s-s_b)}+\frac{M_3}{(s-s_c)}+\dots\frac{M_n}{(s-s_m)} \quad \text{... (5)}$$

Applying inverse Laplace to equation (5) we get, the time domain response as,

$$I(t) = L^{-1}\left[\frac{M_1}{(s-s_a)}+\frac{M_2}{(s-s_b)}+\frac{M_3}{(s-s_c)}+\dots\frac{M_n}{(s-s_m)}\right] \quad \text{... (6)}$$

Any coefficients of the equation (6) can be found by using Heavisides method.

The coefficients K_p can be found as,

$$K_p = H\left[\frac{(s-s_1)(s-s_2)(s-s_3)\dots(s-s_n)}{(s-s_a)(s-s_b)(s-s_c)\dots(s-s_m)}\right](s-s_p)\Big|_{s=s_p} \quad \text{... (7)}$$

Where, s_p, s_m, s_n are the complex numbers.

$$\Rightarrow K_p = H\left[\frac{(s_p-s_1)(s_p-s_2)(s_p-s_3)\dots(s_p-s_n)}{(s_p-s_a)(s_p-s_b)(s_p-s_c)\dots(s_p-s_m)}\right] \quad \text{... (8)}$$

The numerator and denominator is in the from of (s_p-s_x) and (s_p-s_y) which are complex numbers the term $(s_p-s_1)(s_p-s_2)(s_p-s_3)\dots(s_p-s)$ can be expressed as,

$$(s_p-s_x) = N_{px}\,e^{j\alpha_{px}} \quad \text{... (9)}$$

Where,

$x = 1, 2, 3, \dots n$

N_{px} is the magnitude

α_{px} is the phase angle

Similarly, denominator terms $(s-s_a)(s-s_b)\dots(s-s_m)$ of equation (8) can be expressed as,

$$(s_p-s_y) = D_{py}\,e^{j\beta_{py}}$$

Where,

$y = a, b, c, \dots m$

D_{py} is the magnitude

β_{py} is the phase angle.

Therefore, equation (8) can be written in magnitude and phase angle as,

$$K_p = H\left[\frac{N_{px}e^{j\alpha_{px}}}{D_{py}e^{j\beta_{py}}}\right] \quad [\text{Where, } x = 1, 2, 3, \dots n,\; y = a, b, c, \dots m]$$

$$= H\left[\frac{N_{p1}e^{j\alpha_{p1}}.N_{p2}e^{j\alpha_{p2}}.N_{p3}e^{j\alpha_{p3}}\dots N_{pn}e^{j\alpha_{pn}}}{D_{pa}e^{j\beta_{pa}}.D_{pb}e^{j\beta_{pb}}.D_{pc}e^{j\beta_{pc}}\dots D_{pm}e^{j\beta_{pm}}}\right]$$

$$= H\left[\frac{N_{p1}N_{p2}N_{p3}\dots N_{pn}}{D_{pa}D_{pb}D_{pc}\dots D_{pm}}\times\frac{e^{j\alpha_{p1}}e^{j\alpha_{p2}}e^{j\alpha_{p3}}\dots e^{j\alpha_{pn}}}{e^{j\beta_{pa}}e^{j\beta_{pb}}e^{j\beta_{pc}}\dots e^{j\beta_{pm}}}\right]$$

$$= H\left[\frac{N_{p1}N_{p2}N_{p3}\dots N_{pn}}{D_{pa}D_{pb}D_{pc}\dots D_{pm}}\times\frac{e^{j[\alpha_{p1}+\alpha_{p2}+\alpha_{p3}+\dots+\alpha_{pn}]}}{e^{j[\beta_{pa}+\beta_{pb}+\beta_{pc}+\dots+\beta_{pm}]}}\right]$$

$$K_p = H\left[\frac{N_{p1}N_{p2}N_{p3}\dots N_{pn}}{D_{pa}D_{pb}D_{pc}\dots D_{pm}}\right]\times e^{j[\alpha_{p1}+\alpha_{p2}+\alpha_{p3}+\dots+\alpha_{pn}]-[\beta_{pa}+\beta_{pb}+\beta_{pc}+\dots+\beta_{pm}]}$$

Similarly, the other coefficients $K_1, K_2, \dots K_m$ can be evaluated.

Substituting equation (4) in equation (3), we get,

$$\frac{C(j\omega)}{R(j\omega)} = \frac{1}{1-u^2+j2\xi u}$$

Denominator of alone is in rectangular form i.e., $a+b$. Now converting it to polar form i.e., $M\angle\theta$

We have,

$$M = \sqrt{(1-u^2)^2+4\xi^2u^2} \quad \text{... (5)}$$

$$\theta = \tan^{-1}\left(\frac{2\xi u}{1-u^2}\right)$$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{1}{\sqrt{(1-u^2)^2+4\xi^2u^2}\angle\tan^{-1}\left(\frac{2\xi u}{1-u^2}\right)}$$

We know that, mathematically the maximum value of any equation can be obtained by differentiating it will respect to any variable in it and equating it to zero.

Now in order to get the maximum value of magnitude of closed loop transfer function differentiating equation (5) w.r.t u and equating it to zero we get,

$$\frac{dM}{du} = \frac{d}{du}\frac{1}{\left(\sqrt{(1-u^2)^2+4\xi^2u^2}\right)}$$

$$\frac{dM}{du} = \frac{1}{2}\left[(1-u^2)^2+4\xi^2u^2\right]^{-3/2}\left(2(1-u^2)(-2u)+8\xi^2u\right)$$

$$= \frac{-\left[(1-u^2)^2+4\xi^2u^2\right]^{-3/2}}{2}\times\left(-\left(4u(1-u^2)\right)+8\xi^2u\right)$$

$\because$ x^{-n} can be written as $\frac{1}{x^n}$ we get,

$$= \frac{4u(1-u^2)-8\xi^2u}{2[(1-u^2)^2+4\xi^2u^2]^{3/2}}$$

Substituted u_r in place of u and is made to equate it to zero.

Where,

$$u_r = \frac{\omega_r}{\omega_n}$$

$$\Rightarrow \frac{4u_r[1-u_r^2]-8\xi^2u_r}{2((1-u_r^2)^2+4\xi^2u_r^2)^{3/2}} = 0$$

$$\Rightarrow 4u_r(1-u_r^2)-8u_r\xi^2 = 0$$

$$\Rightarrow 4u_r-4u_r^3-8u_r\xi^2 = 0$$

$$\Rightarrow 4u_r = 4u_r^3+8u_r\xi^2$$

$$4u_r = 4u_r[u_r^2+2\xi^2]$$

$$\Rightarrow 1 = u_r^2+2\xi^2$$

$$\Rightarrow u_r^2 = 1-2\xi^2$$

$$\Rightarrow \boxed{u_r = \sqrt{1-2\xi^2}}$$

Replacing $M = M_r$ and substituting 'u_r' in equation (2), we get,

$$M_r = \frac{1}{\sqrt{(1-u_r^2)^2+4\xi^2u_r^2}} = \frac{1}{\sqrt{\left(1-(\sqrt{1-2\xi})^2\right)^2+\left[4\xi^2(\sqrt{1-2\xi^2})^2\right]}} = \frac{1}{\sqrt{(1-1+2\xi^2)^2+4\xi^2(1-2\xi^2)}}$$

$$= \frac{1}{\sqrt{4\xi^4+4\xi^2-8\xi^4}} = \frac{1}{\sqrt{4\xi^2-4\xi^4}} = \frac{1}{\sqrt{4\xi^2(1-\xi^2)}} = \frac{1}{2\xi\sqrt{(1-\xi^2)}}$$

$\therefore$ The value of M_r obtained by substituting u_r in M by equation (2) and reducing M by M_r resonant peak is,

$$\boxed{M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}}$$

Resonant Frequency (ω_n)

It is defined as the frequency at which the magnitude of the closed loop transfer function is maximum.

We know that the value of u_r is,

$$u_r = \sqrt{1-2\xi^2} \quad \text{... (6)}$$

Where, u_r being the normal resonant frequency i.e., the ratio of removal frequency to the undamped natural frequency.

$$u_r = \frac{\omega_r}{\omega_n} \quad \text{.... (7)}$$

Equating equations (3) and (4), we get,

$$\frac{\omega_r}{\omega_n} = \sqrt{1-2\xi^2}$$

$\therefore$ Resonant frequency,

$$\omega_r = \left(\sqrt{1-2\xi^2}\right)\omega_n$$

Correlation between the Time and Frequency Response

The peak overshoot, M_p and the resonant peak, M_r are the time domain and frequency domain specifications of a second order system respectively.

As known earlier

$$\text{Peak overshoot, } M_p = e^{\left[-\pi\xi/\sqrt{1-\xi^2}\right]} \quad \text{... (1)}$$

$$\text{Resonant peak, } M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} \quad \text{... (2)}$$

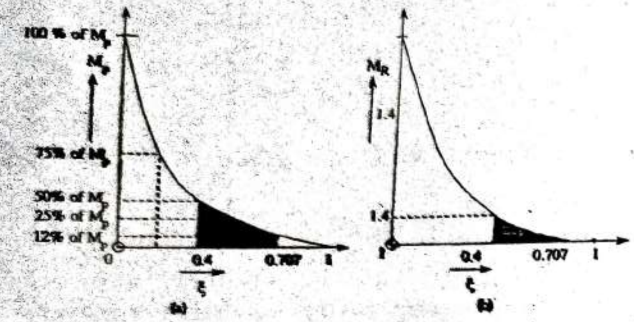
From equations (1) and (2), we can say that the terms peak overshoot, M_p and resonant peak, M_r are related to each other and they both are the functions in 'ξ', which means for a given value of M_r of its frequency response, there should be a corresponding value of M_p existing if the system is subjected to a step input.

From equations (1) and (2), it is clear that the peak overshoot, M_p is exponentially decreasing function is ξ and M_r being inversely related to 'ξ'.

The variation of M_p and M_r can be explained below,

Case 1

When, $\xi = 0$



■ ⟶ The region under which both the quantities M_p and M_R are corelated and beyond which the correlation breakdown's.

Figure

From the figure (a) and figure (b) its clear that they have maximum values at initial point i.e., at $\xi = 0$. At $\xi = 0$ both the quantities M_p and M_r have high values i.e., M_p has a maximum value where as M_r tends. Infinite value, which is not desirable.

Substituting, $\xi = 0$ in M_p and M_r we get,

$$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} = e^{-\pi(0)/\sqrt{1-0^2}} = e^0 = 1$$

Thus M_p achieves 1 which is the 100% overshoot if the maximum value of M_p by substituting $\xi = 0$ in M_r

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{1}{2(0)\sqrt{1-0^2}} = \frac{1}{0} = \infty$$

$\therefore$ M_r is to approach to ∞

Hence, both M_p and M_r are undesirable from the system point of view.

Case 2

When ξ increases beyond zero,

Here in this condition both the M_p and M_r decrease, but M_p decreases at faster rate comparing M_r.

Case 3

When, $\xi = 0.707$ in equation we get,

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{1}{2\times 0.707\times\sqrt{1-(707)^2}} = 1$$

The resonant peak M_r gets minimum and becomes '1' which is the lowest value of it.

Substituting $\xi = 0.707$ in equation (1), we get,

$$M_p = 0.2078 = 20\% \text{ (nearly)}$$

By which its clear that, M_p got reduced to greater extent whereas M_r still holds some small value.

Case 4

When $\xi > 0.707$ with the increase in ξ changed to 0.707, no more resonant peak M_r and exists and the correlation breakdown. At this position of ξ there occurs no correlation between M_p and M_r and the step response oscillations are well damped for this value of ξ hence its not a problem to notice.

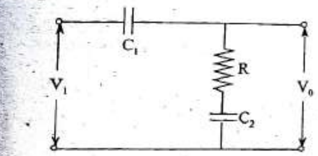
Case 5

When $\xi = 1$ there is no overshoot produced in the system any more as M_p gets decreased to a very low value. There will be no scope of M_r as it gets decreased already to low value at $\xi = 0.707$ itself.

Thus for covering the transient response with frequency response of higher order system. All these alone correlation are useful. Morover a set od specification form time domain can be translated as frequency domain specification from aii thses cases of correlation.

Hence from alone analytic at is dear that the correlation between time domain and frquency domain exists for the range of $0 < \xi < 0.707$

Q.70 Find the transfer function of the network shown in figure.



Figure

Ans:

Given circuit is shown in figure (1).

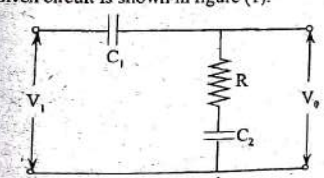


Figure (1)

Now, transforming the given network in s domain, the transformed network is shown in figure (2) as,

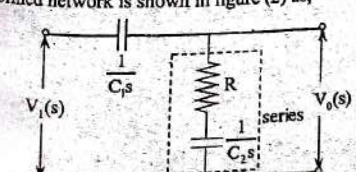


Figure (2)

Here the resistor R and capacitor C_2 are in series. thus we obtained Z_{eq} as,

$$Z_{eq} = R + \frac{1}{sC_2}$$

Now, figure (2) can be modified as,

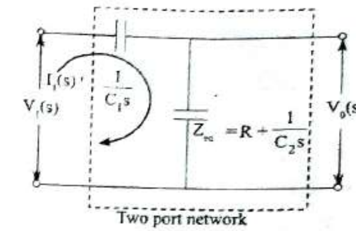


Figure (3)

The voltage transform equations for the above two port network is given by,

$$V_1(s) = I_1(s)\left[\frac{1}{C_1 s} + R + \frac{1}{C_2 s}\right] \quad \text{... (1)}$$

$$V_0(s) = I_1(s)\left[R + \frac{1}{C_2 s}\right] \quad \text{... (2)}$$

Here the transfer function of the network is given by.

$$G(s) = \frac{V_0(s)}{V_1(s)}$$

$$= \frac{I_1(s)\left(R + \frac{1}{C_2 s}\right)}{I_1(s)\left[\frac{1}{C_1 s} + R + \frac{1}{C_2 s}\right]}$$

$$= \frac{R + \frac{1}{C_2 s}}{\frac{1}{C_1 s} + R + \frac{1}{C_2 s}}$$

$$= \frac{\frac{RC_2 s + 1}{C_2 s}}{\frac{C_2 s + RC_1 C_2 s^2 + C_1 s}{C_1 C_2 s^2}}$$

$$= \frac{RC_2 s + 1}{\frac{C_1 s + C_2 s + RC_1 C_2 s^2}{C_1 s}}$$

$$= \frac{(RC_2 s + 1)C_1 s}{C_1 s + C_2 s + RC_1 C_2 s^2}$$

$$= \frac{RC_1C_2s^2 + C_1s}{C_1s + C_2s + RC_1C_2s^2}$$

$$= \frac{s^2 + \frac{1}{RC_2s}}{1 + \frac{1}{RC_1s} + \frac{1}{RC_2s}}$$

Thus the required transfer function of given network shown in figure (1) is,

$$\therefore \quad G(s) = \frac{V_0(s)}{V_1(s)} = \frac{s^2 + \frac{1}{RC_2s}}{1 + \frac{1}{RC_1s} + \frac{1}{RC_2s}}$$

Q.71 Define the following.

(a) Resonance

(b) Q-factor.

Ans:

(a) Resonance

An A.C circuit is said to be in resonance if the applied voltage and resulting current are in phase.

(i) At resonance, the power factor is unity.

(ii) The impedance of A.C circuit consists of only resistance.

(iii) If the inductor and capacitor are in series, the reactance gets cancelled.

(iv) If the inductor and capacitor are in parallel, the susceptance gets cancelled.

Resonance is classified into two types,

1. Series resonance
2. Parallel resonance.

(b) Q-Factor

The quality factor, Q is the ratio of the reactive power in the inductor or capacitor to the true power in the resistance in series with the coil or capacitor.

(or)

The quality factor is also defined as,

$$Q = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}$$

In series RC or LC circuits quality factor,

$$Q = \frac{\omega L}{R} = \frac{1}{\omega CR}$$

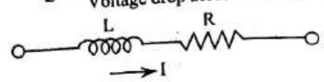
Q.72 Write a short note on quality factor.

Ans:

Quality Factor

The quality factor of an inductor is defined as the ratio of voltage drop across the inductor to the voltage drop across the resistor at resonance condition.

$$Q = \frac{\text{Voltage drop across the inductor}}{\text{Voltage drop across the resistor}}$$



Figure

The quality factor for above figure is given as,

$$Q = \frac{\omega LI}{IR} = \frac{\omega L}{R}$$

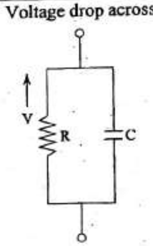
$$= \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} \quad \left[\because \omega = \frac{1}{\sqrt{LC}}\right]$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

Where, I is the current through inductor and resistor.

Similarly, the quality factor of a capacitor is defined as the ratio of voltage drop across the capacitor to the voltage drop across the resistor at resonance condition.

$$Q = \frac{\text{Voltage drop across capacitor}}{\text{Voltage drop across resistor}}$$



Figure

The quality factor for above figure is given as,

$$Q = \frac{\frac{I}{\omega C}}{IR} = \frac{1}{\omega CR}$$

$$= \frac{\sqrt{LC}}{CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$\therefore$ The quality factor of inductor and capacitor are,

$$Q = \frac{\omega L}{R} = \frac{1}{\omega CR}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

The quality factor Q is a frequency dependent function and the value of 'Q' are affected by size, length, diameter and shape of the coil.

Q.74 Explain in detail about the concept of resonance in series RLC circuit. Derive the expression for resonant frequency.

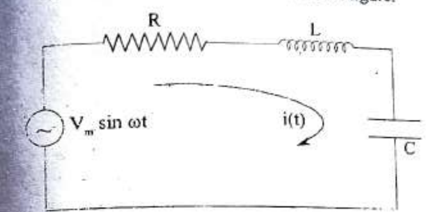
Ans:

Definition

Resonance is a phenomenon at which the net reactance of the circuit is zero or inductive reactance is equal to capacitive reactance.

Explanation

Consider a series RLC circuit shown in figure.



Figure

Let, $V = V_m \sin \omega t$ be the voltage applied across it and $i(t)$ be the current flowing through it.

In case of series RLC circuit, the net impedance is given by,

$$Z = R + j(X_L - X_C) \quad \text{... (1)}$$

Where, X_L – Inductive reactance

X_C – Capacitive reactance.

From equation (1) it is clear that there exist three effects in the circuit i.e., the resistive, inductive and capacitive effect. However, the effect of resistance is inherent hence a series RLC circuit either behaves as inductive or capacitive depending on the relative value of X_L and X_C. If $X_L > X_C$ the circuit behaves as inductive circuit and if $X_C > X_L$ the circuit behaves as capacitive circuit. However, if these two values are equal i.e., $X_L = X_C$ the net reactance will be zero and the circuit is said to be under resonance.

Characteristics of Series Circuit at Resonance

(i) The net impedance of the circuit at resonance is equal to the resistance of the circuit

i.e., $Z = R + j(X_L - X_C)$

$Z = R \quad [\because X_L = X_C]$

(ii) The current is maximum at resonance and is given by,

$$I = \frac{V}{R}$$

(iii) The phase angle between voltage and current is 0.

i.e., $\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0}{R}\right) = 0$

(iv) The power factor of the circuit at resonance is unity.

i.e., p.f $= \cos\theta$

$= \cos 0 = 1$

(v) The voltage across L and C will be equal in magnitude and 180° out of phase with each other. Hence, the net voltage across LC combination is zero.

$V_L = IX_L$

$V_C = -IX_C$

$\therefore \quad X_L = X_C$

$\therefore \quad V_L = IX_L, V_C = -IX_C$

$\therefore \quad V_L = -V_C$ or $V_L + V_C = 0$

(vi) The total voltage appears across resistance.

(vii) Series resonance circuit is also called as selector circuit.

(viii) The voltage across L and C get magnified at resonance. Hence, series circuit at resonance is also called as voltage magnification circuit.

Resonant Frequency

The frequency at which resonance occurs is called as resonant frequency.

The condition for resonance is,

$$X_L = X_C$$

But, we know that,

$$X_L = 2\pi f_r L$$

$$X_C = \frac{1}{2\pi f_r C}$$

Where, f_r is the resonant frequency in Hz.

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad \omega_r = \frac{1}{\sqrt{LC}}$$

Where, ω_r is the resonant frequency in rad/sec.

Q.75 Derive the expression for "bandwidth" in a series R-L-C circuit.

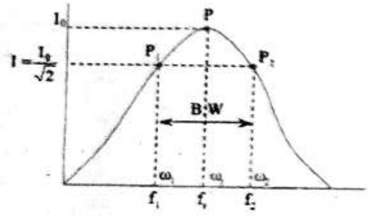
OR

For a series resonance circuit, obtain the expression for bandwidth in terms of resonance frequency and Q-factor.

Ans:

Bandwidth of a series resonant circuit is defined as band of frequencies which lie between two points on either side of the resonant frequency, where power falls to half of its maximum value at resonance (or current falls to 0.707 $\frac{1}{\sqrt{2}}$ of its value at resonance).

Figure shows response of series *RLC* circuit. P_1 and P_2 are the points at which power falls to half or current falls to $\frac{1}{\sqrt{2}}$ = 0.707 of its value at resonance.



Figure

Bandwidth normally abbreviated as *B.W* and represented by Δf in Hz and $\Delta\omega$ in radians per second.

The points P_1 and P_2 are called half power points and frequencies f_1 and f_2 corresponding to these two points are called half power frequencies, f_1 is called lower cut-off frequency and f_2 is called upper cut-off frequency.

Bandwidth, $B.W = f_2 - f_1$

The current at resonance is $I_0 = \frac{V}{R}$

At P_1 current, $I = \frac{I_0}{\sqrt{2}} = \frac{V}{\sqrt{2}R}$

From this the impedance at P_1 is $\sqrt{2}R$

In general, impedance at f_1 is,

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega_1 C} - \omega_1 L\right)^2}$$

Squaring on both sides, we get,

$$(\sqrt{2}R)^2 = R^2 + \left(\frac{1}{\omega_1 C} - \omega_1 L\right)^2 \qquad [\because Z = \sqrt{2}R \text{ at } f_1 \text{ or } P_1]$$

Hence, $\frac{1}{\omega_1 C} - \omega_1 L = R \quad [\because \text{at } \omega_1, X_C >> X_L]$ ... (1)

Similarly,

At P_2,

$\omega_2 L - \frac{1}{\omega_2 C} = R \quad [\because \text{at } \omega_2, X_L >> X_C]$ ... (2)

Equating equations (1) and (2), we get,

$$\frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$$

$$\omega_1 L + \omega_2 L = \frac{1}{\omega_1 C} + \frac{1}{\omega_2 C}$$

$$(\omega_1 + \omega_2)L = \frac{1}{C}\frac{(\omega_1 + \omega_2)}{\omega_1\omega_2}$$

$$\omega_1\omega_2 = \frac{1}{LC}$$

We know that,

Resonant frequency in rad/sec is $\omega_r = \frac{1}{\sqrt{LC}}$ (or)

$\omega_r^2 = \frac{1}{LC}$

$\therefore \omega_r^2 = \omega_1\omega_2$ ... (3)

Adding equations (1) and (2), we get,

$$\frac{1}{\omega_1 C} - \omega_1 L + \omega_2 L - \frac{1}{\omega_2 C} = 2R$$

$$(\omega_2 - \omega_1)L + \frac{1}{C}\frac{(\omega_2 - \omega_1)}{\omega_1\omega_2} = 2R$$

$$(\omega_2 - \omega_1)L + \frac{1}{\omega_r^2 C}(\omega_2 - \omega_1) = 2R$$

[$\because$ From equation (3)]

$L(\omega_2 - \omega_1) + L(\omega_2 - \omega_1) = 2R \qquad [\because L = \frac{1}{\omega_r^2 C}]$

$2L(\omega_2 - \omega_1) = 2R$

$\therefore \quad \omega_2 - \omega_1 = \frac{R}{L}$

$2\pi f_2 - 2\pi f_1 = \frac{R}{L} \qquad [\because \omega = 2\pi f]$

$f_2 - f_1 = \frac{R}{2\pi L}$ ... (4)

$\boxed{\therefore \text{Bandwidth, B.W} = \frac{R}{2\pi L}}$ ... (5)

Now from equation (4), we have,

$$f_2 - f_1 = \frac{R}{2\pi L}$$

But from above equation and from figure, we have,

$(f_r - f_1) = (f_2 - f_r) = \text{Half of } (f_2 - f_1)$

$$= \frac{1}{2} \times \frac{R}{2\pi L} = \frac{R}{4\pi L}$$

$$f_r - f_1 = \frac{R}{4\pi L}$$

$$\text{And } f_2 - f_r = \frac{R}{4\pi L}$$

$\therefore$ Lower frequency limit, $f_1 = f_r - \frac{R}{4\pi L}$ ... (6)

And upper frequency limit, $f_2 = f_r + \frac{R}{4\pi L}$ ... (7)

Q-factor is defined as the ratio of reactance of the coil to its resistance i.e.,

$$Q = \frac{X_L}{R}$$

$Q = \frac{2\pi f_r L}{R}$ ... (8)

Equation (5) can be rearranged as,

$Q = \frac{f_r}{\frac{R}{2\pi L}}$ ... (9)

Substituting equation (4) in equation (6), we get,

$Q = \frac{f_r}{B.W}$ ... (10)

$\therefore \quad B.W = \frac{f_r}{Q}$

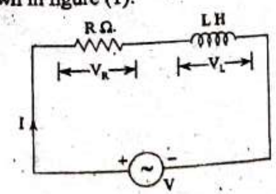
Hence, bandwidth is also defined as the ratio of resonant frequency and *Q*-factor.

Q.76. Explain sinusoidal response of series RL circuit and derive necessary expressions.

Ans:

R-L Series Circuit

Consider an RL series circuit consisting of a resistor of R (Ω) in series with an inductance of L (H) connected to an A.C supply as shown in figure (1).



Applying Kirchhoff's voltage law to the above circuit,

$$V = V_R + V_L$$

Where,

V – Source voltage

V_R – Voltage drop across resistor = IR

V_L – Voltage drop across inductor = IX_L.

The phasor diagram is drawn by taking the current '*I*' as reference phasor as shown in figure (2).

The voltage drop V_R is in phase with current I, since R is a pure resistance and voltage drop V_L leads current I by 90° since L is a pure inductance.

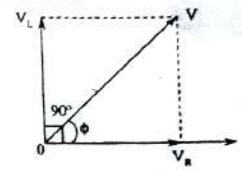


Figure (2): Phasor Diagram of RL Series Circuit

From the phasor diagram, we have,

$$V^2 = V_R^2 + V_L^2$$

$$V = \sqrt{V_R^2 + V_L^2}$$

$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2}$$

$\therefore \quad V = IZ$

Where,

$$Z = \sqrt{R^2 + X_L^2}$$

The phase difference angle "ϕ" can be determined from impedance triangle given in figure (3).

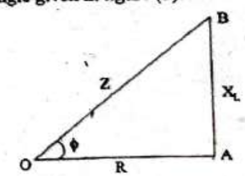


Figure (3)

$$\tan\phi = \frac{AB}{OA}$$

$$\tan\phi = \frac{X_L}{R}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

The power factor of the circuit is given by cosϕ, since i is the cosine function of the phase angle between voltage anc current.

$$\cos\phi = \frac{R}{Z}$$

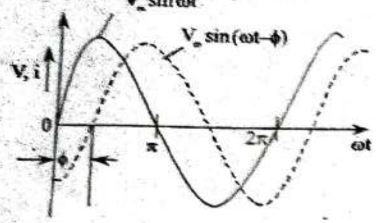


Figure (4)

Let, applied voltage, $v = V_m \sin\omega t$

Then the equation of current will be,

$$i = I_m \sin(\omega t - \phi)$$

Instantaneous power,

$$P = vi$$

$$= (V_m \sin\omega t)\,(I_m \sin(\omega t - \phi))$$

$$= \frac{1}{2} V_m I_m \,[2 \sin\omega t \sin(\omega t - \phi)]$$

$$= \frac{V_m I_m}{2}\,[\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)]$$

$$= \frac{V_m I_m}{2}\,[\cos\phi - \cos(2\omega t - \phi)]$$

$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)$$

In the above expression the first term $\frac{V_m I_m}{2}\cos\phi$ represents the steady power, since V_m, I_m and cosϕ are fixed in magnitude.

The second term $\frac{V_m I_m}{2}\cos(2\omega t - \phi)$ represents the fluctuating power, which is zero over one full cycle.

∴ The average (or) total power,

$$P = \frac{V_m I_m}{2}\cos\phi$$

$$= \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right)\cos\phi$$

$$= VI\cos\phi$$

Where, V and I are R.M.S values of voltage and current.

5 TWO PORT NETWORK AND NETWORK FUNCTIONS

PART-A
SHORT QUESTIONS WITH SOLUTIONS

Q1. What is meant by port?

Ans:

Port: A port is a pair of terminals through which either a source can be connected to the network or the response of the network can be known.

The concept of port can be well understood by the following figure, in which terminals 1-1' constitute a port. A voltage source can be connected to the terminals to activate the circuit or the response of the network can be known by connecting a load.

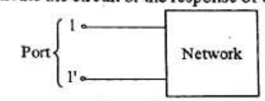


Figure: Port

Q2. Write short notes on two port network.

Ans:

A network with four terminals is known as a two port network. The block diagrams of such network is shown in figure.

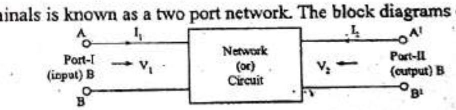


Figure: Two Port Network

Usually, the analysis of two port network is carried out by considering I_1, I_2, V_1 and V_2 variables. Among four variables, two for fixed and the remaining variables are represented in terms of other variables (fixed).

Q3. Explain about the terminal pairs or ports of the network functions.

Ans:

Consider a network consisting of a passive elements and is represented in the form of a rectangular box as shown in the figures (a) and (b).

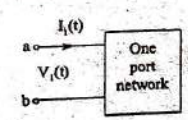


Figure (a)

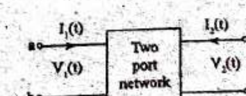


Figure (b)

In figure (a) only a single voltage source and a current exist and therefore it has only a single network function. It has two terminals a and b together called as "port". For a pair of terminals a driving source is connected.

A two port network is as shown in figure (b). It consists of two voltage sources and two currents. The terminals "a-b" constitutes 1 port and the terminal c-d constitutes another port. All together it has two ports. Hence, it is called as "Two Port Network". Here the driving port is connected across the port "a-b" and the load is connected across port "c-d". If the input is connected across the port "c-d". Then, the output is taken from the port "a-b".

Q4. List the formulaes in Z-parameters.

Ans:

The Z-parameters are also known as open circuit parameter.

When output port is open circuited.

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

When input port in open circuited

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Q5. List the formulaes in Y-parameters.

Ans:

The Y-parameters are also known as short circuit parameters.

When output port is short circuited,

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

When input port is short circuited,

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Q6. Write the formulas in ABCD parameters.

Ans:

ABCD parameters are also known as transmission parameters.

When output is open circuited,

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

When input is short circuited,

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

Q7. Write the formulas in h-parameters.

Ans:

The *h* parameters are also known as hybrid parameters. Generally, these are given as,

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Q8. List Z-parameters interms of Y-parameters.

Ans:

Z-parameters interms of Y-parameters are given as,

$$Z_{11} = \frac{Y_{22}}{Y_{11}Y_{22} - Y_{12}Y_{21}}$$

$$Z_{12} = \frac{-Y_{12}}{Y_{11}Y_{22} - Y_{12}Y_{21}}$$

$$Z_{21} = \frac{-Y_{21}}{Y_{11}Y_{22} - Y_{12}Y_{21}}$$

$$Z_{22} = \frac{Y_{11}}{Y_{11}Y_{22} - Y_{12}Y_{21}}$$

Q9. List Z-parameters interms of ABCD parameters.

Ans:

Z-parameters interms of *ABCD* parameters are given as,

$$Z_{11} = \frac{A}{C}$$

$$Z_{12} = \frac{AD - BC}{C}$$

$$Z_{21} = \frac{1}{C}$$

$$Z_{22} = \frac{D}{C}$$

Q10. List Z-parameters interms of h-parameters.

Ans:

Z-parameters interms of *h*-parameters are given as,

$$Z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}$$

$$Z_{12} = \frac{h_{12}}{h_{22}}$$

$$Z_{21} = \frac{-h_{21}}{h_{22}}$$

$$Z_{22} = \frac{1}{h_{22}}$$

Q11. List Y-parameters interms of Z-parameters.

Ans:

Y-parameters interms of Z-parameters are given as,

$$Y_{11} = \frac{Z_{22}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$Y_{12} = \frac{Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$Y_{21} = \frac{-Z_{21}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$Y_{22} = \frac{Z_{11}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

Q12. List Y-parameters interms of ABCD parameters.

Ans:

Y-parameters interms of *ABCD* parameters are given as,

$$Y_{11} = \frac{D}{B}$$

$$Y_{12} = \frac{BC - AD}{B}$$

$$Y_{21} = \frac{-1}{B}$$

$$Y_{22} = \frac{A}{B}$$

Q13. List Y-parameters interms of h-parameters.

Ans:

Y-parameters interms of *h*-parameters are given as,

$$Y_{11} = \frac{1}{h_{11}}$$

$$Y_{12} = \frac{-h_{12}}{h_{11}}$$

$$Y_{21} = \frac{h_{21}}{h_{11}}$$

$$Y_{22} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}$$

Q14. List ABCD parameters interms of Z-parameters.

Ans:

ABCD-parameters interms of Z-parameters are given as,

$$A = \frac{Z_{11}}{Z_{21}}$$

$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$$

$$C = \frac{1}{Z_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}}$$

Q15. List ABCD parameters interms of Y-parameters.

Ans:

ABCD-parameters interms of Y-parameters are given as,

$$A = \frac{-Y_{22}}{Y_{21}}$$

$$B = \frac{-1}{Y_{21}}$$

$$C = \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}$$

$$D = \frac{-Y_{11}}{Y_{21}}$$

Q16. List ABCD parameters interms of h-parameters.

Ans:

ABCD-parameters interms of *h*-parameters are given as,

$$A = \frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}$$

$$B = \frac{-h_{11}}{h_{21}}$$

$$C = \frac{-h_{22}}{h_{21}}$$

$$D = \frac{-1}{h_{21}}$$

Q17. List h-parameters interms of Z-parameters.

Ans:

The *h*-parameters interms of Z-parameters are given as,

$$h_{11} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

$$h_{21} = \frac{-Z_{21}}{Z_{22}}$$

$$h_{22} = \frac{1}{Z_{22}}$$

Q18. List h-parameters interms of Y-parameters.

Ans:

The *h*-parameters interms of Y-parameters are given as,

$$h_{11} = \frac{1}{Y_{11}}$$

$$h_{12} = \frac{-Y_{12}}{Y_{11}}$$

$$h_{21} = \frac{Y_{21}}{Y_{11}}$$

$$h_{22} = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}}$$

Q4. List the formulaes in Z-parameters.

Ans:

The Z-parameters are also known as open circuit parameter.

When output port is open circuited.

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

When input port in open circuited

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Q5. List the formulaes in Y-parameters.

Ans:

The Y-parameters are also known as short circuit parameters.

When output port is short circuited,

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

When input port is short circuited,

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Q6. Write the formulas in ABCD parameters.

Ans:

$ABCD$ parameters are also known as transmission parameters.

When output is open circuited,

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

When input is short circuited,

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

Q7. Write the formulas in h-parameters.

Ans:

The h parameters are also known as hybrid parameters. Generally, these are given as,

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}.$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Q8. List Z-parameters interms of Y-parameters.

Ans:

Z-parameters interms of Y-parameters are given as,

$$Z_{11} = \frac{Y_{22}}{Y_{11}Y_{22} - Y_{12}Y_{21}}$$

$$Z_{12} = \frac{-Y_{12}}{Y_{11}Y_{22} - Y_{12}Y_{21}}$$

$$Z_{21} = \frac{-Y_{21}}{Y_{11}Y_{22} - Y_{12}Y_{21}}$$

$$Z_{22} = \frac{Y_{11}}{Y_{11}Y_{22} - Y_{12}Y_{21}}$$

Q9. List Z-parameters interms of ABCD parameters.

Ans:

Z-parameters interms of $ABCD$ parameters are given as,

$$Z_{11} = \frac{A}{C}$$

$$Z_{12} = \frac{AD - BC}{C}$$

$$Z_{21} = \frac{1}{C}$$

$$Z_{22} = \frac{D}{C}$$

Q10. List Z-parameters interms of h-parameters.

Ans:

Z-parameters interms of h-parameters are given as,

$$Z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}$$

$$Z_{12} = \frac{h_{12}}{h_{22}}$$

$$Z_{21} = \frac{-h_{21}}{h_{22}}$$

$$Z_{22} = \frac{1}{h_{22}}$$

Q11. List Y-parameters interms of Z-parameters.

Ans:

Y-parameters interms of Z-parameters are given as,

$$Y_{11} = \frac{Z_{22}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$Y_{12} = \frac{Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$Y_{21} = \frac{-Z_{21}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$Y_{22} = \frac{Z_{11}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

Q12. List Y-parameters interms of ABCD parameters.

Ans:

Y-parameters interms of $ABCD$ parameters are given as,

$$Y_{11} = \frac{D}{B}$$

$$Y_{12} = \frac{BC - AD}{B}$$

$$Y_{21} = \frac{-1}{B}$$

$$Y_{22} = \frac{A}{B}$$

Q13. List Y-parameters interms of h-parameters.

Ans:

Y-parameters interms of h-parameters are given as,

$$Y_{11} = \frac{1}{h_{11}}$$

$$Y_{12} = \frac{-h_{12}}{h_{11}}$$

$$Y_{21} = \frac{h_{21}}{h_{11}}$$

$$Y_{22} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}$$

Q14. List ABCD parameters interms of Z-parameters.

Ans:

$ABCD$-parameters interms of Z-parameters are given as,

$$A = \frac{Z_{11}}{Z_{21}}$$

$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$$

$$C = \frac{1}{Z_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}}$$

Q15. List ABCD parameters interms of Y-parameters.

Ans:

$ABCD$-parameters interms of Y-parameters are given as.

$$A = \frac{-Y_{22}}{Y_{21}}$$

$$B = \frac{-1}{Y_{21}}$$

$$C = \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}$$

$$D = \frac{-Y_{11}}{Y_{21}}$$

Q16. List ABCD parameters interms of h-parameters.

Ans:

$ABCD$-parameters interms of h-parameters are given as,

$$A = \frac{h_{12}h_{21} - h_{11}h_{22}}{h_{21}}$$

$$B = \frac{-h_{11}}{h_{21}}$$

$$C = \frac{-h_{22}}{h_{21}}$$

$$D = \frac{-1}{h_{21}}$$

Q17. List h-parameters interms of Z-parameters.

Ans:

The h-parameters interms of Z-parameters are given as,

$$h_{11} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

$$h_{21} = \frac{-Z_{21}}{Z_{22}}$$

$$h_{22} = \frac{1}{Z_{22}}$$

Q18. List h-parameters interms of Y-parameters.

Ans:

The h-parameters interms of Y-parameters are given as,

$$h_{11} = \frac{1}{Y_{11}}$$

$$h_{12} = \frac{-Y_{12}}{Y_{11}}$$

$$h_{21} = \frac{Y_{21}}{Y_{11}}$$

$$h_{22} = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}}$$

Q19. List h-parameters interms of ABCD-parameters.

Ans:

The h-parameters interms of $ABCD$-parameters are given as,

$$h_{11} = \frac{B}{D}$$

$$h_{12} = \frac{AD-BC}{D}$$

$$h_{21} = \frac{-1}{D}$$

$$h_{22} = \frac{C}{D}$$

Q20. Write short notes on interconnection of two port networks.

Ans:

Any network is initially designed and constructed in smaller blocks whenever it is possible and then after establishing their properties, these smaller blocks are interconnected with each other to form a complete network. Hence, by this way much more complex networks can also be constructed with great ease. The same procedure holds good for two port networks. The ports of various simpler two port networks are interconnected in certain ways to form the required resultant two port network. The various ways in which the two port networks can be connected are,

1. Cascade connection
2. Series connection
3. Parallel connection
4. Series parallel connection
5. Parallel series connection.

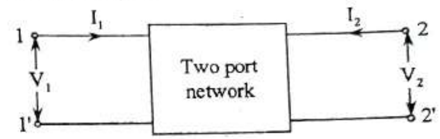
PART-B
ESSAY QUESTIONS WITH SOLUTIONS

TWO PORT NETWORKS, TERMINAL PAIRS, RELATIONSHIP OF TWO PORT VARIABLES

Q21. Explain the relationship between various input and output port variables.

Ans:

Consider a two port network as shown in figure.



Figure

Consider the voltage, current at port 1-1' be V_1, I_1 and the voltage, current at port 2-2' be V_2, I_2 respectively and also assume that the currents I_1 and I_2 are flowing into the network. However, the relationship between the input and output variables can be defined by the linear equations which are equal to the number of ports in a system. Among the variables V_1, I_1, V_2, I_2, two are independent and the other two are dependent. The dependent sources are expressed in terms of independent variables. For a two port network shown in figure, two linear equations are required in terms of four variables. There are six possible ways of selecting these dependent and independent variables and are defined by a set of parameters as shown below.

The relationship between input and output variables for different set of parameters are as follows,

Open Circuit or Z-parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Where, V_1, V_2 are dependent variables and

I_1, I_2 are independent variables

Short Circuit or Y-parameters

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Where I_1, I_2 are dependent variables and

V_1, V_2 are independent variables

Transmission or ABCD Parameters

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Where V_1, I_1 are dependent variables and

V_2, I_2 are independent variables

Hybrid or h-Parameters

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

Where,

V_{11}, I_2 **are dependent variables and** I_1, V_2 **are independent variables.**

IMPEDANCE PARAMETERS

Q22. Why Z-parameters are known as open circuit parameters?

Ans:

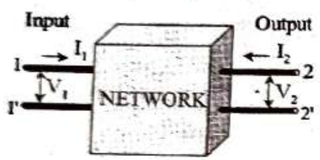
Z-parameters are also known as open circuit parameters because, the input and output voltages V_1 and V_2 of a given two port network can be obtained in terms of input and output currents I_1 and I_2 by simply open circuiting each terminal at a time.

The Z-parameters are defined by the following equations,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{... (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{... (2)}$$

Now, consider an arbitrary two-port network shown in figure. Let V_1, V_2 denote the input and output voltages respectively, whereas I_1 and I_2 denotes the input and output currents respectively.



Figure

The output terminal of the two-port network i.e., 2 – 2' is open-circuited, the current $I_2 = 0$.

∴ From equations (1) and (2), we get,

$$V_1 = Z_{11} I_1 + 0$$

$$\Rightarrow Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \text{Input driving point impedance.}$$

$$V_2 = Z_{21} I_1 + 0$$

$$\Rightarrow Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \text{Forward transfer impedance.}$$

Now, the input terminals 1-1's are open circuited, i.e., $I_1 = 0$

From equations (1) and (2), we get,

$$V_1 = 0 + Z_{12} I_2$$

$$\Rightarrow Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \text{Reverse transfer impedance.}$$

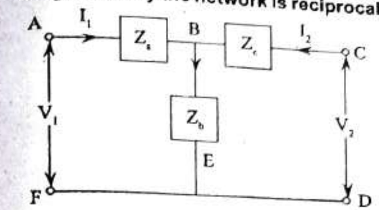
Similarly,

$$V_2 = 0 + Z_{22} I_2$$

$$\Rightarrow Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \text{Output driving point impedance.}$$

Since, the Z-parameters represents the impedances of the two-port network, they are also called as open-circuit impedance parameters.

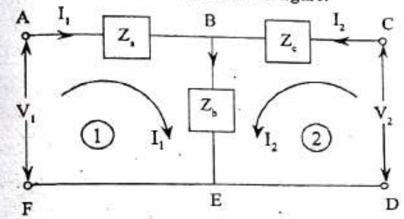
Q23. Find the Z-parameters of the T-network shown in figure. Verify the network is reciprocal or not.



Figure

Ans:

Consider the networks shown in figure.



Figure

Applying KVL to loop (1),

$$V_1 = Z_a I_1 + Z_b(I_1 + I_2)$$

$$V_1 = I_1(Z_a + Z_b) + I_2 Z_b \quad \text{... (1)}$$

Applying KVL to loop (2),

$$V_2 = Z_c I_2 + Z_b(I_1 + I_2)$$

$$V_2 = Z_b I_1 + (Z_b + Z_c)I_2 \quad \text{... (2)}$$

The standard form of Z-parameter equations are given by,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{... (3)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{... (4)}$$

Comparing equations (1) and (2) with equations (3) and (4), we get,

$$Z_{11} = Z_a + Z_b$$
$$Z_{12} = Z_b$$
$$Z_{21} = Z_b$$
$$Z_{22} = Z_b + Z_c$$

Condition for Reciprocity

The condition for the network to be reciprocal in terms of Z-parameters is given by,

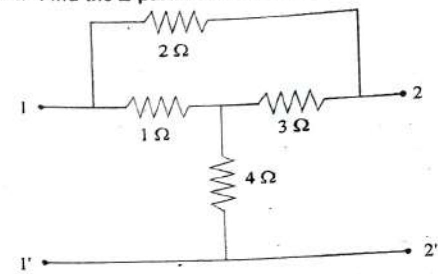
$$Z_{12} = Z_{21}$$

In the given network, $Z_{12} = Z_b$, $Z_{21} = Z_b$

i.e., $Z_{12} = Z_{21} = Z_b$

Hence, the given network is reciprocal.

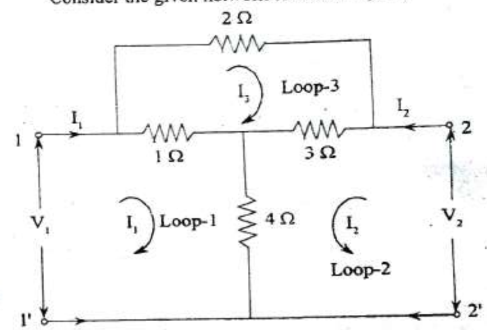
Q24. Find the Z-parameters for the circuit in figure.



Figure

Ans:

Consider the given network shown in figure,



Figure

Let I_1, I_2 and I_3 be the loop currents in loop-1, loop-2 and loop-3 respectively.

The Z-parameters are expressed in terms of I_1 and I_2.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

The individual Z-parameters are given as,

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Applying KVL in loop-1, we get,

$$V_1 = 1(I_1 - I_3) + 4(I_1 + I_2)$$
$$V_1 = 5I_1 + 4I_2 - I_3 \quad \text{... (}$$

Applying KVL in loop-2, we get,

$$V_2 = 3(I_2 + I_3) + 4(I_2 + I_1)$$
$$V_2 = 4I_1 + 7I_2 + 3I_3 \quad \text{... (}$$

Applying KVL in loop-3, we get,

$$0 = 2I_3 + 3(I_3 + I_2) + 1(I_3 - I_1)$$

$$-I_1 + 3I_2 + 6I_3 = 0 \qquad \text{... (3)}$$

For determining Z_{11} and Z_{21} parameters, the output port 2 – 2' should be open circuited.

Then the current I_2 becomes zero.

Substituting $I_2 = 0$ in equation (3), we get,

$$-I_1 + 6I_3 = 0$$

$$I_3 = \frac{I_1}{6} \qquad \text{... (4)}$$

Substituting $I_2 = 0$ and equation (4) in equations (1) and (2), we get,

$$V_1 = 5I_1 + 0 - \frac{I_1}{6}$$

$$V_1 = \frac{29}{6} I_1$$

$$\therefore\ Z_{11} = \frac{V_1}{I_1} = \frac{29}{6}\ \Omega$$

$$Z_{11} = \frac{29}{6}\ \Omega$$

$$V_2 = 4I_1 + 0 + \frac{3I_1}{6}$$

$$V_2 = \frac{9}{2} I_1$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{9}{2}\ \Omega$$

$$\therefore\ Z_{21} = \frac{9}{2}\ \Omega$$

For determining Z_{12} and Z_{22} parameters, the input port 1-1' should be open circuited. Then the current I_1 becomes zero.

Substituting $I_1 = 0$ in equation (3), we get,

$$3I_2 + 6I_3 = 0$$

$$I_3 = \frac{-3I_2}{6} = \frac{-I_2}{2}$$

$$I_3 = \frac{-I_2}{2} \qquad \text{... (5)}$$

Substituting $I_1 = 0$ and equation (5) in equations (1) and (2), we get,

$$V_1 = 0 + 4I_2 - \left(\frac{-I_2}{2}\right)$$

$$V_1 = 4I_2 + \frac{I_2}{2}$$

$$V_1 = \frac{9I_2}{2}$$

$$Z_{12} = \frac{V_1}{I_2} = \frac{9}{2}\ \Omega$$

$$\therefore\ Z_{12} = \frac{9}{2}$$

$$V_2 = 0 + 7I_2 + 3\left(\frac{-I_2}{2}\right)$$

$$V_2 = 7I_2 - \frac{3I_2}{2}$$

$$V_2 = \frac{11I_2}{2}$$

$$Z_{22} = \frac{V_2}{I_2} = \frac{11}{2}\ \Omega$$

$$\therefore\ Z_{22} = \frac{11}{2}\ \Omega$$

Thus the Z-parameters are,

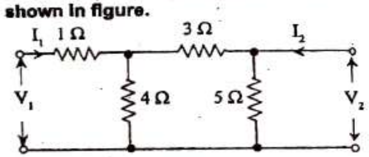
$$Z_{11} = \frac{29}{6}\ \Omega$$

$$Z_{12} = \frac{9}{2}\ \Omega$$

$$Z_{21} = \frac{9}{2}\ \Omega$$

$$Z_{22} = \frac{11}{2}\ \Omega$$

Q25. Determine the Z-parameter of the network shown in figure.



Figure

Ans:

Given circuit shown in figure (1).

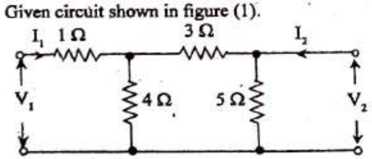


Figure (1)

Now, converting the delta connection of 3 Ω, 4 Ω and 5 Ω into star connection as shown in figure (2).

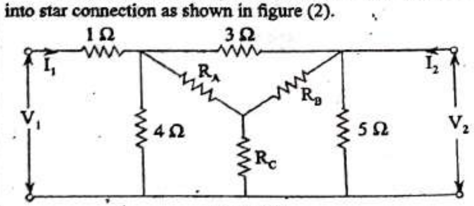


Figure (2)

$$R_A = \frac{3 \times 4}{3+4+5} = 1\ \Omega$$

$$R_B = \frac{3 \times 5}{3+4+5} = 1.25\ \Omega$$

$$R_C = \frac{4 \times 5}{3+4+5} = 1.67\ \Omega$$

Redrawing figure (2),

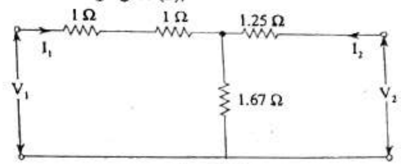


Figure (3)

Two resistors are of 1 Ω each connected in series. Therefore, the circuit in figure (3) gets modified as,

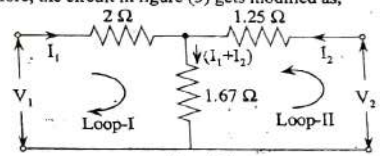


Figure (4)

Now, applying KVL in loop-I, we get,

$$V_1 = 2I_1 + 1.67\,(I_1 + I_2)$$

$$\Rightarrow\quad V_1 = 3.67\,I_1 + 1.67\,I_2 \qquad \text{... (1)}$$

Similarly, applying KVL in loop-II, we get,

$$V_2 = 1.67\,(I_1 + I_2) + 1.25\,I_2$$

$$\Rightarrow\quad V_2 = 1.67\,I_1 + 2.92\,I_2 \qquad \text{... (2)}$$

Open-circuiting terminal-2 i.e., making, $I_2 = 0$.

From equations (1) and (2), we have,

$$V_1 = 3.67\,I_1$$

$$\Rightarrow\quad \frac{V_1}{I_1} = 3.67\ \Omega \text{ and}$$

$$V_2 = 1.67\,I_1$$

$$\Rightarrow\quad \frac{V_2}{I_1} = 1.67\ \Omega$$

Now, open-circuiting terminal-1 i.e., making, $I_1 = 0$

From equations (1) and (2), we have,

$$V_1 = 1.67\,I_2$$

$$\Rightarrow\quad \frac{V_1}{I_2} = 1.67\ \Omega \text{ and}$$

$$V_2 = 2.92\,I_2$$

$$\Rightarrow\quad \frac{V_2}{I_2} = 2.92\ \Omega$$

But, from Z-parameter equations, we have,

$$Z_{11} = \frac{V_1}{I_1} \Rightarrow Z_{11} = 3.67\ \Omega$$

$$Z_{12} = \frac{V_1}{I_2} \Rightarrow Z_{12} = 1.67\ \Omega$$

$$Z_{21} = \frac{V_2}{I_1} \Rightarrow Z_{21} = 1.67\ \Omega \text{ and}$$

$$Z_{22} = \frac{V_2}{I_2} \Rightarrow Z_{22} = 2.92\ \Omega$$

The Z-parameters can also be written in matrix form as,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

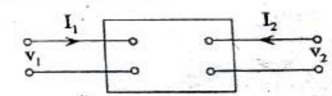
$$\Rightarrow\quad \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3.67 & 1.67 \\ 1.67 & 2.92 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

ADMITTANCE PARAMETERS

Q26. Why Y-parameters are known as short circuit parameters?

Ans:

A general two-port network having I_1, I_2 as input and output currents and V_1, V_2 as input and output voltages respectively as shown in figure.



Figure

From the above figure we can write, input currents I_1, I_2 in terms of V_1 and V_2 as follows,

$$I_1 = Y_{11}\,V_1 + Y_{12}\,V_2 \qquad \text{... (1)}$$

$$I_2 = Y_{21}\,V_1 + Y_{22}\,V_2 \qquad \text{... (2)}$$

Here,

$Y_{11}, Y_{12}, Y_{21}, Y_{22}$ = Admittance parameters

We can obtain the Y-parameters Y_{11} and Y_{21} by shorting the output port as follows.

Thus, when output port is shorted, i.e., $V_2 = 0$

From equations (1) and (2), we have,

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

Similarly, for Y-parameters Y_{12} and Y_{22} can be obtained by shorting the input as follows,

Thus, when input port shorted, $V_1 = 0$

From equations (1) and (2), we get,

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

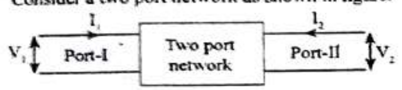
$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Thus, from the above analysis, it is clear that the respective parameters at one port can be determined by short circuiting the remaining parameters at another port. Hence, Y-parameters or admittance parameters are also called as short circuit admittance parameters.

Q27. Derive the condition for a two port network to be reciprocal in terms of admittance parameters.

Ans: Model Paper-I, Q11(a)

Consider a two port network as shown in figure.



Figure

For a two port network, the voltage and current equations in terms of Y-parameters are given by,

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{... (1)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{... (2)}$$

Short circuiting the port - II, we have,

$V_2 = 0$ and $I_2 = -I_2$

Substituting V_2 and I_2 values in equation (2), we get,

$$-I_2 = Y_{21}V_1 + Y_{22}(0)$$

$$\Rightarrow -I_2 = Y_{21}V_1 + 0 \qquad [\because I_2 = -I_2]$$

$$\Rightarrow -I_2 = Y_{21}V_1$$

$$\Rightarrow V_1 = \frac{-I_2}{Y_{21}} \quad \text{... (3)}$$

Now short circuiting the port - I, we have,

$V_1 = 0$ and $I_1 = -I_1$

Substituting V_1 and I_1 values in equation (1) we get,

$$-I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$-I_1 = Y_{11}(0) + Y_{12}V_2$$

$$-I_1 = Y_{12}V_2$$

$$\Rightarrow V_2 = \frac{-I_1}{Y_{12}} \quad \text{... (4)}$$

Assuming the voltages applied on either side of ports to be same, we have,

$$V_1 = V_2$$

Substituting V_1, V_2 values from equations (3) and (4), we have,

$$\Rightarrow \frac{-I_2}{Y_{21}} = \frac{-I_1}{Y_{12}}$$

$$\Rightarrow \frac{I_2}{Y_{21}} = \frac{I_1}{Y_{12}}$$

$$\Rightarrow I_2 Y_{12} = I_1 Y_{21}$$

Assuming same currents are produced on both sides we have,

$$I_1 = I_2$$

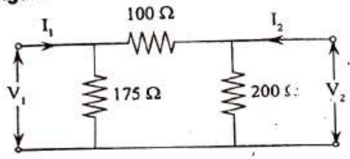
$$\Rightarrow I_2 Y_{12} = I_2 Y_{21}$$

$$\Rightarrow Y_{12} = Y_{21}$$

Thus, the condition for a two port network to be reciprocal in terms of admittance parameters is,

$$Y_{12} = Y_{21}$$

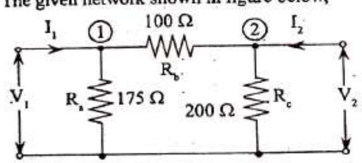
Q28. Find the Y-parameters of the network shown in figure.



Figure

Ans:

The given network shown in figure below,



Figure

Let, the resistances of 175 Ω, 100 Ω and 200 Ω be R_a, R_b and R_c respectively.

Applying KCL at node (1), we get,

$$\Rightarrow -I_1 + \frac{V_1}{R_a} + \frac{V_1 - V_2}{R_b} = 0$$

$$\Rightarrow I_1 = \left(\frac{1}{R_a} + \frac{1}{R_b}\right)V_1 + \left(-\frac{1}{R_b}\right)V_2 \quad \text{... (1)}$$

Applying KCL at node (2), we get,

$$\Rightarrow -I_2 + \frac{V_2}{R_c} + \frac{V_2 - V_1}{R_b} = 0$$

$$\Rightarrow I_2 = \left(\frac{-1}{R_b}\right)V_1 + \left(\frac{1}{R_b} + \frac{1}{R_c}\right)V_2 \quad \text{... (2)}$$

In a two port network, the input and the output currents, I_1 and I_2 can be expressed in terms of input and output voltages, V_1 and V_2 respectively as,

$$[I] = [Y]\,[V]$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow I_1 = Y_{11}V_1 + Y_{12}V_2 \text{ and} \quad \text{... (3)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{... (4)}$$

On comparing equations (1) and (3), we get,

$$Y_{11} = \frac{1}{R_a} + \frac{1}{R_b} = \frac{1}{175} + \frac{1}{100}$$

$$= 15.714 \times 10^{-3}\ \mho \text{ and}$$

$$Y_{12} = \frac{-1}{R_b} = \frac{-1}{100} = -10 \times 10^{-3}\ \mho$$

Similarly, on comparing equations (2) and (4), we get,

$$Y_{21} = \frac{-1}{R_b} = \frac{-1}{100} = -10 \times 10^{-3}\ \mho$$

$$\text{And } Y_{22} = \frac{1}{R_b} + \frac{1}{R_c} = \frac{1}{100} + \frac{1}{200}$$

$$= 15 \times 10^{-3}\ \mho$$

$\therefore$ Admittance matrix,

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

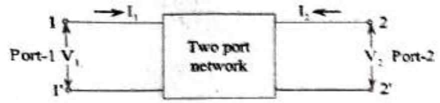
$$= \begin{bmatrix} (15.714\times10^{-3}) & (-10\times10^{-3}) \\ (-10\times10^{-3}) & (15\times10^{-3}) \end{bmatrix}$$

TRANSMISSION PARAMETERS

Q. 29. Irite short notes on ABCD parameters.

Ans: **Model Paper-III, Q10(a)**

Consider a two port network as shown in the figure.



Figure

In the above network out of four variables, two of them are independent and the remaining are dependent variables and is expressed in terms of two independent variables. In the transmission, the voltage and current at port-1 (sending end parameters) are expressed in terms of voltage and current at port-2 (Receiving end parameter). The sending end parameters are given as,

$$V_1 = AV_2 - BI_2 \quad \text{... (1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{... (2)}$$

The constants A, B, C and D in the above equations are called as $ABCD$ parameters or Chain parameters or General circuit or Transmission parameters. The value of I_2 is taken as negative as the reference direction of I_2 is in opposition to that in power transmission parameters. By studying the open circuit and short circuit conditions at port-2 the physical significance of $ABCD$ parameters can be known.

When port-2 is open circuited i.e., $I_2 = 0$ then equation (1) becomes,

$$\frac{1}{A} = \frac{V_2}{V_1}\bigg|_{I_2=0}$$

$$\boxed{A = \frac{V_1}{I_2}\bigg|_{I_2=0}} \quad \text{... (3)}$$

(Open circuit voltage gain)

And also, equation (2) becomes,

$$\frac{1}{C} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

$$\Rightarrow \boxed{C = \frac{I_1}{V_2}\bigg|_{I_2=0}} \quad \text{... (4)}$$

(Open circuited transfer impedance)

When port-2 is short circuited i.e., $V_2 = 0$ then equation (1) becomes,

$$\frac{-1}{B} = \frac{I_2}{V_1}\bigg|_{V_2=0}$$

$$\boxed{B = \frac{-V_1}{I_2}\bigg|_{V_2=0}} \quad \text{... (5)}$$

(Short circuit transfer admittance)

And also, equation (2) becomes,

$$\frac{-1}{D} = \frac{I_2}{I_1}\bigg|_{V_2=0}$$

$$\Rightarrow \boxed{D = \frac{-I_1}{I_2}\bigg|_{V_2=0}} \quad \text{... (6)}$$

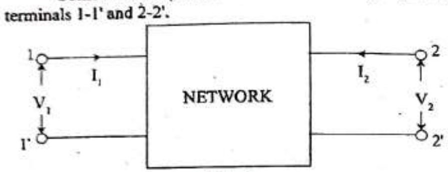
(Short-circuit current gain)

$\therefore$ The equations (3), (4), (5) and (6) defines the $ABCD$ parameters or transmission parameters of a two-port network.

Q. 30. Derive the condition for a two port network to be symmetrical in terms of ABCD parameters.

Ans:

Consider a two-port network as shown in figure, having terminals 1-1' and 2-2'.



Figure

The characteristic equations of the $ABCD$ parameters are given by,

$$V_1 = AV_2 - BI_2 \quad \text{... (1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{... (2)}$$

From equation (2), we have,

$$I_1 = CV_2 - DI_2$$

$$\Rightarrow CV_2 = I_1 + DI_2$$

$$\Rightarrow V_2 = \frac{I_1}{C} + \frac{DI_2}{C} \quad \text{... (3)}$$

Substituting equation (3) in equation (1), we get,

$$V_1 = A\left(\frac{I_1}{C} + \frac{DI_2}{C}\right) - BI_2$$

$$V_1 = \frac{AI_1}{C} + I_2\left(\frac{AD}{C} - B\right) \quad \text{... (4)}$$

The equations relating to Z-parameters are,

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{... (5)}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{... (6)}$$

Comparing equations (3) and (4) with equations (5) and (6), we get,

$$Z_{11} = \frac{A}{C}, \quad Z_{12} = \frac{AD}{C} - B$$

$$Z_{21} = \frac{1}{C}, \quad Z_{22} = \frac{D}{C}$$

The above expressions represent the relation between Z parameters and $ABCD$ parameters (i.e., Z-parameters interms of $ABCD$ parameters).

Condition for Symmetry

From the symmetry of Z-parameter networks, we have,

$$Z_{11} = \frac{A}{C} \quad \text{... (7)}$$

$$Z_{22} = \frac{D}{C} \quad \text{... (8)}$$

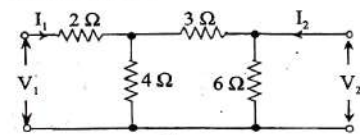
For Z-parameter network, symmetry is checked by the condition,

$$Z_{11} = Z_{22}$$

$$\Rightarrow \frac{A}{C} = \frac{D}{C} \quad [\because \text{From equations (7) and (8)}]$$

$$\boxed{A = D}$$

Q. 31. Determine the ABCD parameters of the network shown in figure.



Figure

Ans:

The given network shown in figure (1).

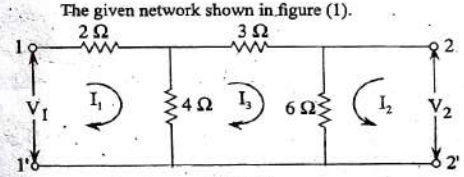


Figure (1)

The characteristic equation of ABCD parameters are given by,

$$V_1 = AV_2 + B(-I_2) \quad \text{... (1)}$$

$$I_1 = CV_2 + D(-I_2) \quad \text{... (2)}$$

In matrix form,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Now, open-circuiting the terminals 2-2', which makes current $I_2 = 0$. The circuit then changes to figure (2),

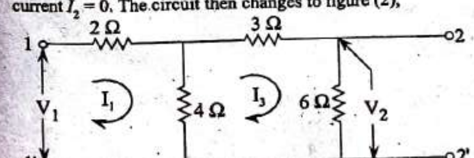


Figure (2)

Applying KVL in loop-1, we get,

$$V_1 = 2I_1 + 4(I_1 - I_3)$$

$$\Rightarrow V_1 = 6I_1 - 4I_3 \quad \text{... (3)}$$

Similarly, applying KVL in loop-2, we get,

$$0 = 3I_3 + 6I_3 + 4(I_3 - I_1)$$

$$\Rightarrow 4I_1 = 13I_3 \Rightarrow I_3 = \frac{4}{13}I_1 \quad \text{... (4)}$$

Substituting equation (4) in equation (3), we get,

$$V_1 = 6I_1 - 4\left(\frac{4}{13}\right)I_1$$

$$\Rightarrow V_1 = 6I_1 - \frac{16}{13}I_1$$

$$\Rightarrow V_1 = \frac{62}{13}I_1 \quad \text{... (5)}$$

And, $V_2 = 6I_3$

$$\Rightarrow V_2 = 6 \times \frac{4}{13}I_1$$

$$\Rightarrow V_2 = \frac{24}{13}I_1 \quad \text{... (6)}$$

We know that,

$$A = \frac{V_1}{V_2} = \frac{62}{13}I_1 \times \frac{13}{24I_1}$$

[$\because$ From equations (5) and (6)]

$$\Rightarrow A = \frac{31}{12}$$

And, $C = \frac{I_1}{V_2} = \frac{13}{24}$ ℧ [$\because$ From equation (6)]

Now, short-circuiting the terminals 2-2', which makes voltage $V_2 = 0$. The circuit, gets modified to figure (3).

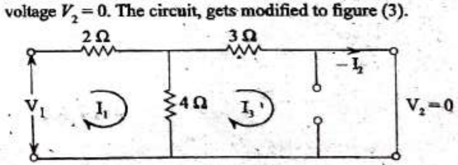


Figure (3)

From the figure (3),

$$I_3 = -I_2$$

Applying KVL in loop-1, we get,

$$V_1 = 2I_1 + 4(I_1 - I_3)$$

$$\Rightarrow V_1 = 6I_1 + 4I_2 \quad [\because I_3 = -I_2] \quad \text{... (7)}$$

Similarly, applying KVL in loop-2, we get,

$$3(I_3) + 4(I_3 - I_1) = 0$$

$$7I_3 = 4I_1$$

$$7(-I_2) = 4I_1 \quad [\because I_3 = -I_2]$$

$$\frac{-I_1}{I_2} = \frac{7}{4} = D$$

$$D = \frac{7}{4}$$

From equation (7), we have,

$$V_1 = 6I_1 + 4I_2$$

$$\Rightarrow \quad V_1 = 6\left(\frac{-7}{4}I_2\right) + 4I_2 \quad \left[\because I_1 = \frac{-7}{4}I_2\right]$$

$$\Rightarrow \quad V_1 = \frac{-42}{4}I_2 + 4I_2$$

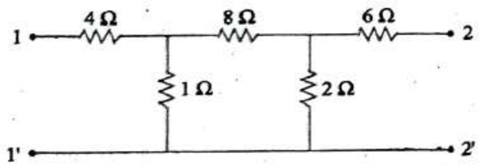
$$\Rightarrow \quad V_1 = \frac{-13}{2}I_2 \Rightarrow \frac{V_1}{I_2} = \frac{-13}{2}$$

$$B = \frac{-V_1}{I_2} = \frac{13}{2}\ \Omega$$

$\therefore$ The ABCD parameters are,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 31/12 & 13/2 \\ 13/34 & 7/4 \end{bmatrix}$$

Q. 32. Find the transmission parameters of the network shown in figure.



Figure

Ans: **Model Paper-II, Q11(b)**

The given circuit is shown in figure (1).

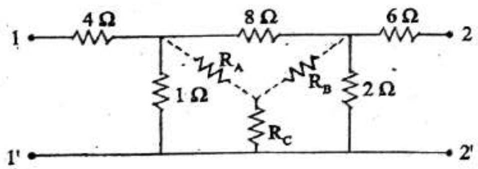


Figure (1)

In figure (1), the resistances 1 Ω, 8 Ω and 2 Ω are delta connected. Converting these resistances into star connection and hence the circuit can be reduced as shown in figure (2). The equivalent star connected resistances can be calculated as follows,

$$R_A = \frac{1\times 8}{1+8+2} = \frac{8}{11} = 0.72\ \Omega$$

$$R_B = \frac{8\times 2}{1+8+2} = \frac{16}{11} = 1.45\ \Omega$$

$$R_C = \frac{2\times 1}{1+8+2} = \frac{2}{11} = 0.18\ \Omega$$

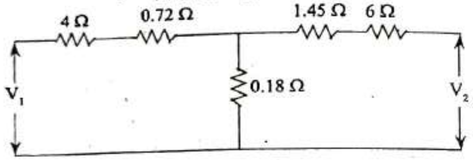


Figure (2)

In figure (2), the resistances 4 Ω, 0.72 Ω and 1.45 Ω, 6 Ω are in series. Hence, the network is reduced as shown in figure (3).

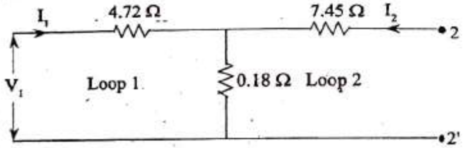


Figure (3)

When the port 2-2' is open circuited in figure (3), the current I_2 will be zero.

Applying KVL to loop-1, we get,

$$4.72\,I_1 + 0.18\,(I_1 + I_2) = V_1$$

But, $I_2 = 0$

$$4.72\,I_1 + 0.18\,(I_1 + 0) = V_1$$

$$4.72\,I_1 + 0.18\,I_1 = V_1$$

$$4.9I_1 = V_1$$

As the port 2-2' is open circuited, the voltage measured at the resistor 0.18 Ω will be the voltage (V_2) at port 2-2', i.e.,

$$V_2 = 0.18I_1$$

Now,

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$= \frac{4.9I_1}{0.18I_1}$$

$$= \frac{4.9}{0.18}$$

$$\therefore A = 27.2$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

$$= \frac{1}{0.18} \quad \left[\because V_2 = 0.18I_1 \Rightarrow \frac{I_1}{V_2} = \frac{1}{0.18}\right]$$

$$\therefore C = 5.5$$

When the port 2-2' is short circuited, then the equivalent circuit is shown in figure (4).

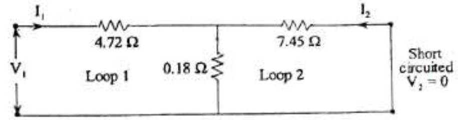


Figure (4)

Applying KVL to the circuit, we get,

Loop 1

$$V_1 = 4.72I_1 + 0.18\,(I_1 + I_2)$$

$$V_1 = (4.72 + 0.18)\,I_1 + 0.18\,I_2$$

$$\Rightarrow \quad V_1 = 4.9\,I_1 + 0.18\,I_2 \quad \ldots (1)$$

Loop 2

$$7.45I_2 + 0.18\,(I_2 + I_1) = 0$$

$$(7.45 + 0.18)\,I_2 + 0.18\,I_1 = 0$$

$$7.63\,I_2 + 0.18\,I_1 = 0 \quad \ldots (2)$$

Solving equations (1) and (2), we get,

$$(V_1 = 4.9\,I_1 + 0.18\,I_2) \times 0.18 \Rightarrow 0.18\,V_1 = \cancel{0.882\,I_1} + 0.0324\,I_2$$

$$(0 = 0.18\,I_1 + 7.63\,I_2) \times 4.9 \Rightarrow 0 = \cancel{0.882\,I_1} + 37.38\,I_2$$

$$(-) \quad (-) \quad (-)$$

$$0.18\,V_1 = -37.34\,I_2$$

$$\Rightarrow \quad I_2 = \frac{-0.18}{37.34}V_1$$

Now,

$$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0}$$

$$B = \frac{-(37.34)}{0.18}$$

$$\therefore B = 207.2$$

From equations (1) and (2), we have,

$$(V_1 = 4.9\,I_1 + 0.18\,I_2) \times 7.63 \Rightarrow 37.38\,I_1 + \cancel{1.37\,I_2} = 7.63\,V_1$$

$$(0 = 0.18\,I_1 + 7.63\,I_2) \times 0.18 \Rightarrow 0.0324\,I_1 + \cancel{1.37\,I_2} = 0$$

$$(-) \quad (-) \quad (-)$$

$$37.34\,I_1 = 7.63\,V_1$$

$$\Rightarrow \quad I_1 = \frac{7.63}{37.34}V_1$$

Now,

$$D = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$$

$$D = \frac{-\left(\frac{7.63}{37.34}\right)V_1}{-\left(\frac{0.18}{37.34}\right)V_1}$$

$$D = \frac{7.63}{37.34} \times \frac{37.34}{0.18} \times \frac{V_1}{V_1}$$

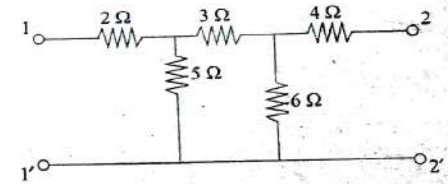
$$D = \frac{7.63}{0.18}$$

$$\boxed{\therefore D = 42.3}$$

Thus, the transmission parameters are given by,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 27.2 & 207.2 \\ 5.5 & 42.3 \end{bmatrix}$$

Q. 33. Find the ABCD parameters for the circuit shown in figure.



Figure

Ans:

The given network is shown in figure(1).

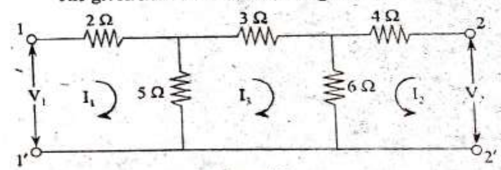


Figure (1)

The characteristic equation of *ABCD* parameters are given by,

$$V_1 = AV_2 + B(-I_2) \quad \ldots (1)$$

$$I_1 = CV_2 + D(-I_2) \quad \ldots (2)$$

In matrix form,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Now open circuiting the terminals 2 – 2', which makes current $I_2 = 0$. The circuit is modified as shown in figure (2),

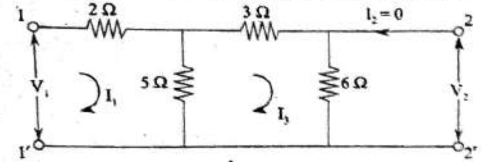


Figure (2)

Applying KVL in loop-1, we get,

$$V_1 = 2I_1 + 5(I_1 - I_3)$$

$$V_1 = 7I_1 - 5I_3 \quad \text{... (3)}$$

Similarly, applying KVL in loop-2, we get,

$$0 = 3I_3 + 6I_3 + 5(I_3 - I_1)$$

$$0 = 14I_3 - 5I_1$$

$$I_3 = \frac{5}{14}I_1 \quad \text{... (4)}$$

Substituting equation (4), in equation (3), we get,

$$V_1 = 7I_1 - 5\left(\frac{5}{14}\right)I_1$$

$$V_1 = 7I_1 - \frac{25}{14}I_1$$

$$V_1 = \frac{73}{14}I_1 \quad \text{... (5)}$$

And, $V_2 = 6I_3$

$$\Rightarrow V_2 = 6\times\frac{5}{14}I_1$$

$$V_2 = \frac{15}{7}I_1 \quad \text{... (6)}$$

We known that,

$$A = \frac{V_1}{V_2} = \frac{\left(\frac{73}{14}\right)I_1}{\left(\frac{15}{7}\right)I_1}$$

$$= \frac{73}{14}I_1 \times \frac{7}{15}I_1$$

[$\because$ From equations (5) and (6)]

$$A = \frac{73}{30}$$

and, $C = \dfrac{I_1}{V_2}$

$$= \frac{I_1}{\frac{15}{7}I_1} \quad [\because \text{From equation (6)}]$$

$$C = \frac{7}{15}$$

Now short circuiting the terminals 2 – 2', which makes voltage $V_2 = 0$. The circuit then changes to figure (3),

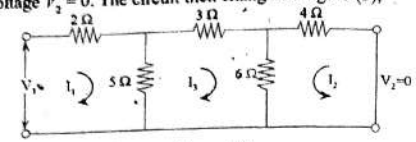


Figure (3)

Applying KVL in loop-1, we get,

$$V_1 = 2I_1 + 5(I_1 - I_3)$$

$$V_1 = 7I_1 - 5I_3 \quad \text{... (7)}$$

Applying KVL in loop-2, we get,

$$3I_3 + 6(I_3 + I_2) + 5(I_3 - I_1) = 0$$

$$14I_3 + 6I_2 - 5I_1 = 0 \quad \text{... (8)}$$

Applying KVL in loop-3, we get,

$$4I_2 + 6(I_2 + I_3) = 0$$

$$10I_2 + 6I_3 = 0$$

$$I_3 = -\frac{10}{6}I_2 \quad \text{... (9)}$$

Substituting equation (9) in equation (8), we get,

$$14\left(\frac{-10}{6}\right)I_2 + 6I_2 - 5I_1 = 0$$

$$-\frac{140}{6}I_2 + 6I_2 - 5I_1 = 0$$

$$-\frac{52}{3}I_2 - 5I_1 = 0$$

$$-5I_1 = \frac{52}{3}I_2$$

$$I_1 = -\frac{52}{15}I_2 \quad \text{... (10)}$$

Substituting equation (9) and (10) in equation (7), we get,

$$V_1 = 7\left(-\frac{52}{15}\right)I_2 - 5\left(-\frac{10}{6}\right)I_2$$

$$V_1 = -\frac{364}{15}I_2 + \frac{50}{6}I_2$$

$$V_1 = -\frac{239}{15}I_2 \Rightarrow \frac{V_1}{-I_2} = \frac{239}{15}$$

We know that,

$$B = \frac{V_1}{-I_2} = \frac{239}{15}$$

$$D = \frac{I_1}{-I_2} = \frac{52}{15} \quad [\because \text{From equation (10)}]$$

HYBRID PARAMETERS

Q. 34. Give a brief note on h-parameters.

Ans:

Hybrid Parameters

The h-parameters are also known as hybrid parameters which represent an impedance, admittance, a voltage gain and a current gain.

The hybrid parameters are exclusively used for modelling of electronic circuits *viz.*, transistors.

Another reason, why these parameters are called hybrid parameters is that, they utilize both the conditions of short-circuit as well as open-circuit.

In hybrid parameter representation, the voltage of the input port and the current of the output port are specified in terms of voltage of the output and current of the input port.

Mathematically,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

From this,

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Where,

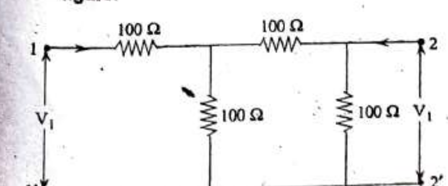
$h_{11} = \left.\dfrac{V_1}{I_1}\right|_{V_2=0}$ = Short-circuit input impedance.

$h_{21} = \left.\dfrac{I_2}{I_1}\right|_{V_2=0}$ = Short-circuit forward current gain.

$h_{12} = \left.\dfrac{V_1}{V_2}\right|_{I_1=0}$ = Open-circuit reverse voltage gain.

$h_{22} = \left.\dfrac{I_2}{V_2}\right|_{I_1=0}$ = Open-circuit output impedance.

Q. 35. Find the h-parameter of the network shown in figure.



Figure

Ans:

The given circuit is shown in figure (1).

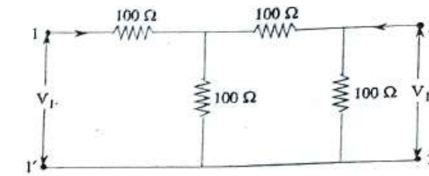


Figure (1)

The hybrid or *h*-parameters are expressed in terms of I_1 and V_2 as,

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad \text{... (1)}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad \text{... (2)}$$

The individual h-parameters are given as,

By short circuiting the port 2-2', V_2 becomes zero.

$$\therefore h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

By open circuiting the port 1-1', I_1 becomes zero.

$$\therefore h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Now, short circuiting the port 2-2', for determining the parameters h_{11} and h_{21}. The voltage V_2 becomes zero as shown in figure (2).

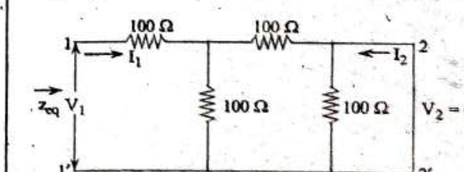


Figure (2)

The modified circuit can be drawn as shown in figure (3).

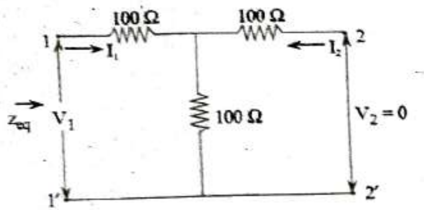


Figure (3)

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

$$V_1 = I_1 Z_{eq}$$

From figure (3), the equivalent impedance is given as,

$$Z_{eq} = 100 + \frac{100\times100}{100+100} = 100 + \frac{10000}{200}$$

$$= 100 + 50 = 150\ \Omega$$

$$\therefore \quad V_1 = I_1(150\ \Omega)$$

$$\frac{V_1}{I_1} = 150\ \Omega$$

$$\therefore \quad h_{11} = \frac{V_1}{I_1} = 150\ \Omega$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$-I_2 = I_1 \times \frac{100}{100+100}$$

$$-I_2 = I_1\left(\frac{100}{200}\right)$$

$$I_2 = -\frac{I_1}{2}$$

$$\frac{I_2}{I_1} = -\frac{1}{2}$$

$$\therefore h_{21} = -\frac{1}{2} = -0.5$$

Now, making the port 1-1' open for determining the parameters h_{12} and h_{22}. The current I_1 becomes zero as shown in figure (4).

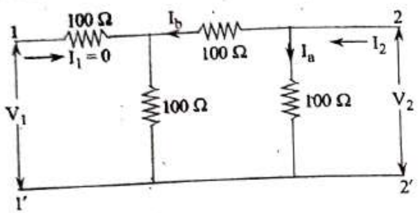


Figure (4)

The modified circuit is shown in figure (5).

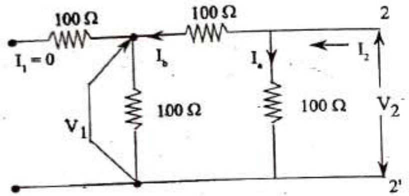


Figure (5)

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$V_1 = I_b \times 100 \qquad \text{... (3)}$$

$$V_2 = I_2 Z_{eq} \qquad \text{... (4)}$$

$$Z_{eq} = (100 + 100) \parallel 100$$

$$= \frac{200\times100}{200+100} = \frac{20000}{300}$$

$$= 66.66\ \Omega$$

$$\therefore \quad V_2 = I_2 Z_{eq}$$

$$V_2 = I_2(66.66) \qquad \text{... (5)}$$

$$I_a = I_2\left[\frac{200}{200+100}\right]$$

$$I_a = I_2\left[\frac{2}{3}\right] \qquad \text{... (6)}$$

$$I_b = I_2 - I_a$$

$$\Rightarrow \quad I_b = I_2 - I_2\left[\frac{2}{3}\right]$$

$$\Rightarrow \quad I_b = I_2\left[1-\frac{2}{3}\right]$$

$$\therefore \quad I_b = I_2\left[\frac{1}{3}\right] \qquad \text{... (7)}$$

Substituting equation (7) in equation (3), we get,

$$\therefore \quad V_1 = \frac{I_2}{3}\times100$$

$$V_1 = \frac{100}{3}I_2 \qquad \text{... (8)}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{100/3}{I_2(66.66)}I_2 = \frac{33.33}{66.66}$$

$$\therefore \quad h_{12} = 0.5$$

From equation (5), we get,

$$\frac{I_2}{V_2} = \frac{1}{66.66} = 0.015$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = 0.015$$

$$h_{22} = 0.015$$

$\therefore$ The h-parameters are expressed as,

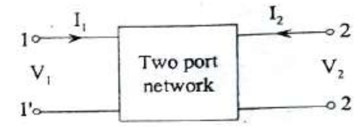
$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 150 & 0.5 \\ -0.5 & 0.015 \end{bmatrix}$$

RELATIONSHIP BETWEEN PARAMETER SETS

Q. 36. Obtain the expressions for Z-parameter in terms of Y-parameters.

Ans:

Consider a two port network as shown in following the figure.



Figure

Let V_1, I_1 be voltage and current at port 1-1' and V_2, I_2 be the voltage and current at port 2-2'.

Z-parameters in terms of Y-parameters

The standard form of equation relating to Z-parameters are,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \qquad \text{... (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \qquad \text{... (2)}$$

Equations (1) and (2) can be written in matrix form as,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[V] = [Z]\,[I] \qquad \text{... (3)}$$

$$\text{But, } [I] = [V]\,[Y] \qquad \text{... (4)}$$

Where $[Y]$ is the admittance matrix given by,

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

Substituting equation (4) in equation (3) we get,

$$[V] = [Z]\,[V]\,[Y]$$

$$1 = [Z]\,[Y]$$

$$[Z] = [Y]^{-1}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{Y_{11}Y_{22}-Y_{12}Y_{21}}\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$= \frac{1}{\Delta Y}\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & -\frac{Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

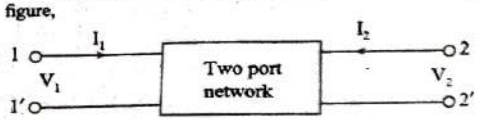
$$\therefore \quad Z_{11} = \frac{Y_{22}}{\Delta Y} \quad Z_{12} = \frac{-Y_{12}}{\Delta Y}$$

$$Z_{21} = \frac{-Y_{21}}{\Delta Y} \quad Z_{22} = \frac{Y_{11}}{\Delta Y}$$

Q. 37. Express z-parameters in terms of h-parameters and ABCD parameters.

Ans:

Consider a two port network as shown in following figure,



Figure

Let V_1, I_1, be voltage and current at port 1-1' and V_2, I_2 be the voltage and current at port 2-2'.

The standard form of equation of Z-parameters are,

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Z-parameters in terms of h-parameters

The h-parameters equations are,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{... (1)}$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{... (2)}$$

From equation (2), we have,

$$h_{22} V_2 = I_2 - h_{21} I_1$$

$$V_2 = \frac{1}{h_{22}} I_2 + \left(-\frac{h_{21}}{h_{22}}\right) I_1$$

$$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2 \quad \text{... (3)}$$

Substituting equation (3) in equation (1), we get,

$$V_1 = h_{11} I_1 + h_{12}\left[\frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right]$$

$$V_1 = \left(\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right) I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_1 = \frac{\Delta h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2 \quad \text{... (4)}$$

Comparing equations (4) and (3) with standard equation of Z-parameter.

$$Z_{11} = \frac{\Delta h}{Z_{22}}; \quad Z_{12} = \frac{h_{12}}{h_{22}}$$

$$Z_{21} = -\frac{h_{12}}{h_{22}}; \quad Z_{22} = \frac{1}{h_{22}}$$

Z-parameters in Terms of ABCD Parameters

Now, the ABCD parameter equations are given by,

$$V_1 = AV_2 - BI_2 \quad \text{... (5)}$$
$$I_1 = CV_2 - DI_2 \quad \text{... (6)}$$

From equation (6), we have,

$$CV_2 = I_1 + DI_2$$

$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2 \quad \text{... (7)}$$

Substituting equation (7) in equation (5), we get,

$$V_1 = A\left(\frac{1}{C} I_1 + \frac{D}{C} I_2\right) - BI_2$$

$$V_1 = \frac{A}{C} I_1 + \left(\frac{AD - BC}{C}\right) I_2 \quad \text{... (8)}$$

Comparing equations (8) and (7) with standard equations of Z-parameter.

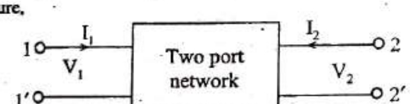
$$Z_{11} = \frac{A}{C}; \quad Z_{12} = \frac{AD - BC}{C}$$

$$Z_{21} = \frac{1}{C}; \quad Z_{22} = \frac{D}{C}$$

Q. 38. Express Y-parameters in terms of Z-parameters and h-parameters

Ans:

Consider a two port network as shown in the following figure,



Let V_1, I_1, be voltage and current at port 1-1' and V_2, I_2 be the voltage and current at port 2-2'.

The standard form of equations relating Y-parameters are,

$$I_1 = V_1 Y_{11} + V_2 Y_{12} \quad \text{... (1)}$$
$$I_2 = V_1 Y_{21} + V_2 Y_{22} \quad \text{... (2)}$$

Y-Parameters in Terms of Z-Parameters

We know,

$$[I] = [Y][V]$$
$$= [Y][Z][I]$$
$$[Y] = [Z]^{-1}$$

$$\therefore \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}$$

$$= \frac{1}{Z_{11}Z_{22} - Z_{12}Z_{21}} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

Let,

$$Z_{11}Z_{22} - Z_{12}Z_{21} = \Delta Z$$

$$= \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

Thus,

$$Y_{11} = \frac{Z_{22}}{\Delta Z} \qquad \boxed{\because \Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$Y_{12} = -\frac{Z_{12}}{\Delta Z}$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z}$$

$$Y_{22} = \frac{Z_{11}}{\Delta Z}$$

Y-Parameters in terms of H-Parameters

The h-parameters equations are,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{... (3)}$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{... (4)}$$

From equation (3), we have,

$$h_{11} I_1 = V_1 - h_{12} V_2$$

$$I_1 = \frac{1}{h_{11}} V_1 + \left(\frac{-h_{12}}{h_{11}}\right) V_2 \quad \text{... (5)}$$

Substituting equation (5) in equation (4), we get,

$$I_2 = h_{21}\left(\frac{1}{h_{11}} V_1 + \left(\frac{-h_{12}}{h_{11}}\right) V_2\right) + h_{22} V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}\right] V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \frac{\Delta h}{h_{11}} V_2 \quad \text{... (6)}$$

Comparing equations (5) and (6) with standard equations of Y-parameters i.e., equations (1) and (2), we get,

$$Y_{11} = \frac{1}{h_{11}}, \qquad Y_{12} = -\frac{h_{12}}{h_{11}}$$

$$Y_{21} = \frac{h_{21}}{h_{11}}, \qquad Y_{22} = \frac{\Delta h}{h_{11}}.$$

Q. 39. Express Y-parameter in terms of ABCD parameters.

OR

Obtain the expression for Y-parameters in terms of transmission parameters.

Ans:

The transmission-parameters (or) $ABCD$ parameters equation are,

$$V_1 = AV_2 - BI_2 \quad \text{... (1)}$$

This equation can be written as,

$$I_2 = -\frac{1}{B} V_1 + \frac{A}{B} V_2 \quad \text{... (2)}$$

And also,

$$I_1 = CV_2 - DI_2 \quad \text{... (3)}$$

Substituting equation (2) in equation (3), we get,

$$I_1 = CV_2 - D\left[-\frac{1}{B} V_1 + \frac{A}{B} V_2\right]$$

$$\therefore \quad I_1 = \frac{D}{B} V_1 - \frac{(AD - BC)}{B} V_2 \quad \text{... (4)}$$

By governing equation of Y-parameters,

$$I_1 = V_1 Y_{11} + V_2 Y_{12} \text{ and} \quad \text{... (5)}$$
$$I_2 = V_1 Y_{21} + V_2 Y_{22} \quad \text{... (6)}$$

Comparing equations (4) and (2) with equations (5) and (6), we get,

$$Y_{11} = \frac{D}{B}$$

$$Y_{12} = -\frac{AD - BC}{B}$$

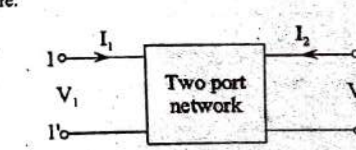
$$Y_{21} = -\frac{1}{B}$$

$$Y_{22} = \frac{A}{B}$$

Q. 40. Derive expression for ABCD parameters in terms on Z, Y parameters.

Ans: Model Paper-I, Q10(a)

Consider a two port network as shown in the following figure.



Let V_1, I_1 be voltage and current at port 1-1' and V_2, I_2 be the voltage and current at port 2-2'.

ABCD parameters or transmission parameters in the standard form is given by,

$$V_1 = AV_2 - BI_2 \quad \text{... (1)}$$
$$I_1 = CV_2 - DI_2 \quad \text{... (2)}$$

(i) Conversion of ABCD Parameters in terms of Z-parameters

The Z-parameters or open circuit parameters in the standard form are given as,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{... (3)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{... (4)}$$

Re-arranging equation (4) in the form of equation (2), we get,

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\Rightarrow Z_{21} I_1 = V_2 - Z_{22} I_2$$

$$I_1 = \frac{V_2 - Z_{22} I_2}{Z_{21}}$$

$$I_1 = \frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2 \quad \text{... (5)}$$

Substituting the value of 'I_1' in equation (3) we get,

$$V_1 = Z_{11}\left[\frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2\right] + Z_{12} I_2$$

$$\Rightarrow \quad V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{Z_{11}Z_{22}}{Z_{21}} I_2 + Z_{12} I_2$$

$$\Rightarrow \quad V_1 = \left(\frac{Z_{11}}{Z_{21}}\right) V_2 - \left(\frac{Z_{11}Z_{22}}{Z_{21}} - Z_{12}\right) I_2$$

$$\Rightarrow \quad V_1 = \left(\frac{Z_{11}}{Z_{21}}\right) V_2 - \left(\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}\right) I_2$$

$$V_1 = \left(\frac{Z_{11}}{Z_{21}}\right) V_2 - \left(\frac{\Delta Z}{Z_{21}}\right) I_2 \quad \text{... (6)}$$

$$(\because \Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21})$$

Comparing equation (6) with equation (1) we get,

$$\boxed{A = \frac{Z_{11}}{Z_{21}}}\ ; \ \boxed{B = \frac{\Delta Z}{Z_{21}}}$$

Now, comparing equation (5) with equation (2) we get,

$$\boxed{C = \frac{1}{Z_{21}}}\ ; \ \boxed{D = \frac{Z_{22}}{Z_{21}}}$$

(ii) Conversion of ABCD Parameters in terms of Y-parameters

The Y-parameters or short circuit parameters in the standard form are given as,

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{... (7)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{... (8)}$$

Rearranging equation (8) in the form of equation (2) we get,

$$Y_{21} V_1 = I_2 - Y_{22} V_2$$

$$V_1 = \frac{I_2 - Y_{22} V_2}{Y_{21}}$$

$$V_1 = \left(\frac{1}{Y_{21}}\right) I_2 - \left(\frac{Y_{22}}{Y_{21}}\right) V_2$$

$$V_1 = -\left(\frac{Y_{22}}{Y_{21}}\right) V_2 - \left(\frac{-1}{Y_{21}}\right) I_2 \quad \text{... (9)}$$

Substituting the value of 'V_1' in equation (7) we get,

$$I_1 = Y_{11}\left[-\left(\frac{Y_{22}}{Y_{21}}\right) V_2 + \left(\frac{1}{Y_{21}}\right) I_2\right] + Y_{12} V_2$$

$$= \left(\frac{-Y_{11}Y_{22}}{Y_{21}}\right) V_2 + \left(\frac{Y_{11}}{Y_{21}}\right) I_2 + Y_{12} V_2$$

$$= \left(\frac{Y_{11}}{Y_{21}}\right) I_2 + \left(\frac{-Y_{11}Y_{22}}{Y_{21}} + Y_{12}\right) V_2$$

$$= \left(\frac{Y_{11}}{Y_{21}}\right) I_2 + \left(\frac{-Y_{11}Y_{22} + Y_{12}Y_{21}}{Y_{21}}\right) V_2$$

$$= \frac{-(Y_{11}Y_{22} - Y_{12}Y_{21})V_2}{Y_{21}} - \left(\frac{-Y_{11}}{Y_{21}}\right) I_2$$

$$I_1 = \left(\frac{-\Delta Y}{Y_{21}}\right) V_2 - \left(\frac{-Y_{11}}{Y_{21}}\right) I_2 \quad \text{... (10)}$$

Comparing equation (9) with equation (1) we get,

$$A = \frac{-Y_{22}}{Y_{21}} \text{ and } B = \frac{-1}{Y_{21}}$$

Comparing equation (10) with equation (2) we get,

$$C = \frac{-\Delta Y}{Y_{21}} \text{ with } D = \frac{-Y_{11}}{Y_{21}}$$

INTERCONNECTION OF TWO PORT NETWORKS

Q. 41. Obtain the Z-parameters used for series connected two port network.

Ans:

Z-parameters are used to describe the parameters of a series connected two-port networks.

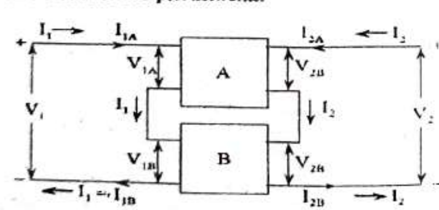


Figure: Series Connection of Two-port Networks

Consider two networks A and B connected in series as shown in figure. Since they are connected in series the current through them remain same, but the voltage gets divided.

$$\therefore \quad I_1 = I_{1A} = I_{1B} \quad \text{... (1)}$$

$$I_2 = I_{2A} = I_{2B} \text{ and}$$

$$V_1 = V_{1A} + V_{1B} \quad \text{... (2)}$$

$$V_2 = V_{2A} + V_{2B}$$

The Z-parameters for two-port network A is,

$$V_{1A} = Z_{11A} I_{1A} + Z_{12A} I_{2A} \quad \text{... (3)}$$

$$V_{2A} = Z_{21A} I_{1A} + Z_{22A} I_{2A}$$

Z-parameters for two-port network B is,

$$V_{1B} = Z_{11B} I_{1B} + Z_{12B} I_{2B} \quad \text{... (4)}$$

$$V_{2B} = Z_{21B} I_{1B} + Z_{22B} I_{2B}$$

Substituting equations (1), (3) and (4) in equation (2), we get,

$$V_1 = Z_{11A} I_1 + Z_{12A} I_2 + Z_{11B} I_1 + Z_{12B} I_2$$

$$= (Z_{11A} + Z_{11B}) I_1 + (Z_{12A} + Z_{12B}) I_2$$

$$\therefore V_1 = Z'_{11} I_1 + Z'_{12} I_2 \quad \text{... (5)}$$

Where,

$$Z'_{11} = Z_{11A} + Z_{11B} \text{ and } Z'_{12} = Z_{12A} + Z_{12B}$$

Similarly,

$$V_2 = Z_{21A} I_1 + Z_{22A} I_2 + Z_{21B} I_1 + Z_{22B} I_2$$

$$= (Z_{21A} + Z_{21B}) I_1 + (Z_{22A} + Z_{22B}) I_2$$

$$\therefore V_2 = Z'_{21} I_1 + Z'_{22} I_2 \quad \text{... (6)}$$

Where,

$$Z'_{21} = Z_{21A} + Z_{21B} \text{ and } Z'_{22} = Z_{22A} + Z_{22B}$$

From the above expressions it is observed that each Z-parameter of the series network is given as the sum of the corresponding parameters of individual networks.

Q. 42. Obtain the Y-parameter used for parallel connected two port network.

Ans:

Y-parameters are used in dealing with the two-port networks connected in parallel.

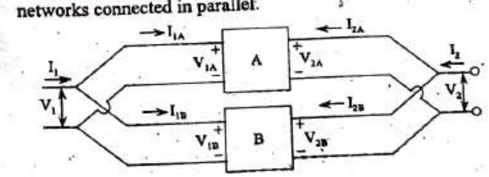


Figure: Parallel Two-port Networks

Figure shows, parallel connection of two ports A and B. Each two-port has a reference node that is common to its input and output port.

Since the two ports are connected in parallel, the voltage across them remains same but the current gets divided.

$$V_1 = V_{1A} = V_{1B} \quad \text{... (1)}$$

$$V_2 = V_{2A} = V_{2B}$$

$$I_1 = I_{1A} + I_{1B}$$

$$I_2 = I_{2A} + I_{2B} \quad \text{... (2)}$$

The admittance parameters of two-port network A are as follows,

$$I_{1A} = Y_{11A} V_{1A} + Y_{12A} V_{2A}$$

$$I_{2A} = Y_{21A} V_{1A} + Y_{22A} V_{2A} \quad \text{... (3)}$$

The Y-parameters of two-port network B is,

$$I_{1B} = Y_{11B} V_{1B} + Y_{12B} V_{2B}$$

$$I_{2B} = Y_{21B} V_{1B} + Y_{22B} V_{2B} \quad \text{... (4)}$$

Substituting equations (1), (3) and (4) in equation (2), we get,

$$I_1 = Y_{11A} V_1 + Y_{12A} V_2 + Y_{11B} V_1 + Y_{12B} V_2$$

$$= (Y_{11A} + Y_{11B}) V_1 + (Y_{12A} + Y_{12B}) V_2$$

$$\therefore \quad I_1 = Y'_{11} V_1 + Y'_{12} V_2 \quad \text{... (5)}$$

Where,

$$Y'_{11} = Y_{11A} + Y_{11B}$$

$$Y'_{12} = Y_{12A} + Y_{12B}$$

Similarly,

$$I_2 = Y_{21A} V_1 + Y_{22A} V_2 + Y_{21B} V_1 + Y_{22B} V_2$$

$$= (Y_{21A} + Y_{21B}) V_1 + (Y_{22A} + Y_{22B}) V_2$$

$$\therefore \quad I_2 = Y'_{21} V_1 + Y'_{22} V_2 \quad \text{... (6)}$$

Where,

$$Y'_{21} = Y_{21A} + Y_{21B}$$

$$Y'_{22} = Y_{22A} + Y_{22B}$$

From the above expressions, it is observed that each Y-parameter of the parallel network is given as the sum of the corresponding parameters of the individual networks.

Q. 43. Explain the cascade connection of the two port network with transmission parameter representation.

Ans:

Transmission parameters are used while dealing with two-ports connected in cascade.

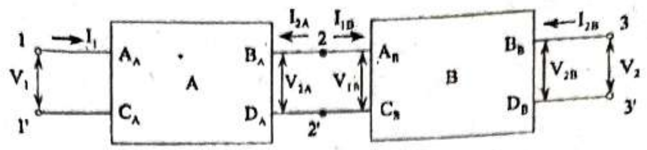


Figure: Cascaded Two-port Network

Consider two two-port networks A and B connected in cascade as shown in the above figure.

The equation relating transmission parameter of network A is given by,

$$V_{1A} = A_A V_{2A} - B_A I_{2A} \quad \text{... (1)}$$

$$I_{1A} = C_A V_{2A} - D_A I_{2A} \quad \text{... (2)}$$

Equations (1) and (2) can be written in matrix form as,

$$\begin{bmatrix} V_{1A} \\ I_{1B} \end{bmatrix} = \begin{bmatrix} A_A & B_A \\ C_A & D_A \end{bmatrix} \begin{bmatrix} V_{2A} \\ -I_{2A} \end{bmatrix}$$

The equation relating transmission parameter of network B is given by,

$$V_{1B} = A_B V_{2B} - B_B I_{2B} \quad \text{... (3)}$$

$$I_{1B} = C_B V_{2B} - D_B I_{2B} \quad \text{... (4)}$$

Equations (3) and (4) can be written in matrix form as,

$$\begin{bmatrix} V_{1B} \\ I_{1B} \end{bmatrix} = \begin{bmatrix} A_B & B_B \\ C_B & D_B \end{bmatrix} \begin{bmatrix} V_{2B} \\ -I_{2B} \end{bmatrix} \quad \text{... (5)}$$

From the above figure, we observe that at 2-2' port,

$$V_1 = V_{1A},\ V_{2A} = V_{1B},\ V_2 = V_{2B}$$

$$I_1 = I_{1A},\ I_{2A} = -I_{1B},\ I_2 = I_{2B} \quad \text{... (6)}$$

$$\begin{bmatrix} V_{1A} \\ I_{1A} \end{bmatrix} = \begin{bmatrix} A_A & B_A \\ C_A & D_A \end{bmatrix} \begin{bmatrix} V_{2A} \\ -I_{2A} \end{bmatrix},\ \begin{bmatrix} V_{1B} \\ I_{1B} \end{bmatrix}$$

$$= \begin{bmatrix} A_B & B_B \\ C_B & D_B \end{bmatrix} \begin{bmatrix} V_{2B} \\ -I_{2B} \end{bmatrix} \quad \text{... (7)}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_A & B_A \\ C_A & D_A \end{bmatrix} \begin{bmatrix} V_{1B} \\ I_{1B} \end{bmatrix} \quad \text{... (8)}$$

Substituting equation (7) in equation (8), we get,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_A & B_A \\ C_A & D_A \end{bmatrix} \begin{bmatrix} A_B & B_B \\ C_B & D_B \end{bmatrix} \begin{bmatrix} V_{2B} \\ -I_{2B} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_A & B_A \\ C_A & D_A \end{bmatrix} \cdot \begin{bmatrix} A_B & B_B \\ C_B & D_B \end{bmatrix} \begin{bmatrix} V_{2B} \\ -I_{2B} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

i.e., $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_A & B_A \\ C_A & D_A \end{bmatrix} \begin{bmatrix} A_B & B_B \\ C_B & D_B \end{bmatrix}$

Where, $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is the transmission parameters matrix for the cascaded network.

Hence, the equivalent transmission parameter of two cascaded networks is equal to the product of individual transmission parameter of each network.