

For 3rd Semester

ELECTRICAL MACHINE-I

Strictly according to the
NEW SYLLABUS



 **ORGANIZER**

SYLLABUS

Module 1 : Magnetic fields and magnetic circuits (6 Hours)

Review of magnetic circuits – MMF flux, reluctance, inductance; review of Ampere Law and Biot Savart Law; Visualization of magnetic fields produced by a bar magnet and a current carrying coil through air and through a combination of iron and air; influence of highly permeable materials on the magnetic flux lines.

Module 2 : Electromagnetic force and torque (9 Hours)

B-H curve of magnetic materials; flux-linkage vs current characteristic of magnetic circuits; linear and nonlinear magnetic circuits; energy stored in the magnetic circuit; force as a partial derivative of stored energy with respect to position of a moving element; torque as a partial derivative of stored energy with respect to angular position of a rotating element. Examples : galvanometer coil, relay contact, lifting magnet, rotating element with eccentricity or saliency.

Module 3 : Transformers (12 Hours)

Principle, construction and operation of single-phase transformers, equivalent circuit, phasor diagram, voltage regulation, losses and efficiency Testing - open circuit and short circuit tests, polarity test, back-to-back test, separation of hysteresis and eddy current losses Three-phase transformer - construction, types of connection and their comparative features, Parallel operation of single-phase and three-phase transformers, Autotransformers - construction, principle, applications and comparison with two winding transformer, Magnetizing current, effect of nonlinear B-H curve of magnetic core material, harmonics in magnetization current, Phase conversion - Scott connection, three-phase to six-phase conversion, Tap-changing transformers - No load and on-load tap- changing of transformers, Three winding transformers. Cooling of transformers.

Module 4 : DC machines (8 Hours)

Basic construction of a DC machine, magnetic structure - stator yoke, stator poles, pole-faces or shoes, air gap and armature core, visualization of magnetic field produced by the field winding excitation with armature winding open, air gap flux density distribution, flux per pole, induced EMF in an armature coil, armature winding and commutation - Elementary armature coil and commutator, lap and wave windings, construction of commutator, linear commutation Derivation of back EMF equation, armature MMF wave, derivation of torque equation, armature reaction, air gap flux density distribution with armature reaction.

Module 5 : DC machine - motoring and generation (7 Hours)

Armature circuit equation for motoring and generation, Types of field excitations - separately excited, shunt and series. Open circuit characteristic of separately excited DC generator, back EMF with armature reaction, voltage build-up in a shunt generator, critical field resistance and critical speed. V-I characteristics and torque-speed characteristics of separately excited, shunt and series motors. Speed control through armature voltage. Losses, load testing and back-to-back testing of DC machines.



Magnetic Fields and Magnetic Circuits

This chapter covers

- * **Magnetic Field and Magnetic Circuits :** Review of magnetic circuits – MMF, flux, reluctance, inductance; review of Ampere Law and Biot Savart Law; Visualization of magnetic fields produced by a bar magnet and a current carrying coil - through air and through a combination of iron and air; influence of highly permeable materials on the magnetic flux lines.

INTRODUCTION.

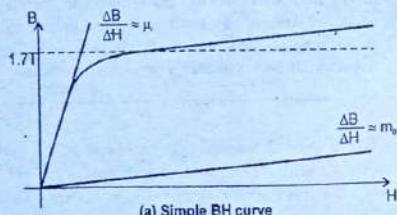
In 1820, Han Chritian Oersted observed that a compass needle gets deflected, when placed near a wire carrying electrical current. Moving charges or current in a wire (or coil) produces magnetic lines of forces (called magnetic flux) in the surrounding space causing deflection of magnetic needle, is termed as magnetic filed. Magnetic flux cannot be completely directed to follow a given path in a magnetic material; but if the permeability of the magnetic material is very high as compared to material surrounding it (such as free space), then most of the flux flows through highly permeable material and almost negligible flux conducts through surrounding low permeability medium. This flow of magnetic flux is very similar to the flow of electrical current through a conductor. Therefore, the path followed by the flux in a magnetic material is called magnetic circuit.

For example, Magnetic field can be obtained from a permanent magnet or by using an electromagnet. Electromagnet consists of windings carrying electrical currents to obtain magnetic field by replacing permanent magnets by coils carrying a relatively small electrical current. For an electromagnet magnetic circuit is made of ferromagnetic material (iron) to conduct the magnetic flux as electrical current conducts through copper conductor. As the copper conductor provides a low resistance path to electrical current, so does the magnetic circuit provide a path to magnetic flux with low magnetic resistance.

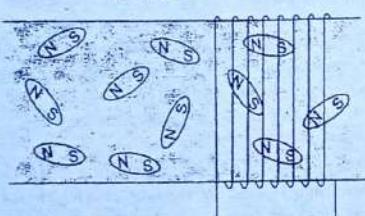
Q. 1. Define Magnetomotive force ? Also give its unit.

Ans. Review of Magnetic Circuits :

Magnetomotive force (mmf) is defined as the product of current and number of turns of the coil and given as F_{mmf} [i.e. $F_{mmf} = NI$]. It can also be defined as the strength of a magnetic field in a coil or the flux producing ability of electric current in a magnetic circuit. It depends on the magnitude of current flowing in the turns of coil i.e. more is the magnitude of current, stronger is the magnetic field; and more is the number of turns of the wire, more concentrated lines of force is obtained. Unit of mmf is ampere-turn (A-t). In some text books unit of mmf is given as Ampere (A), as the number of turns is not a necessary condition for production of mmf in a wire.



(a) Simple BH curve



(b) Molecular representation of magnetic material

Fig. 1.1

Magnetomotive force (mmf) is equal to line integral of field H through closed path passing through magnetic circuit

$$F_{mmf} = \int H dl = NI \text{ (ampere-turns)} \quad \dots(1.1)$$

F_{mmf} is a scalar quantity.

Magnetomotive force (mmf) is analogous to electromotive force, emf (i.e. difference in electric potential, or voltage between the terminals of a source of electricity, e.g., a battery). Since it causes magnetic flux in a magnetic circuit, mmf (F_{mmf}) can also be given as,

$$F_{mmf} = NI \quad \dots(1.2)$$

where

N is the number of turns in the coil,
 I is the electric current through the circuit.

- $F_{mmf} = \phi \mathcal{R}$... (1.3)
- where ϕ is the magnetic flux and \mathcal{R} is the reluctance.
- $F_{mmf} = HI$

H is the magnetizing force (the strength of the magnetizing field),
 I is the mean length of solenoid or the circumference of a toroid.

Q. 2. Define Reluctance of Magnetic circuit.

Ans. Reluctance of Magnetic Circuit :

In fact, reluctance is the opposition offered to the magnetic flux by the magnetic path. The lower is the reluctance, the higher will be the magnetic flux and vice-versa.

Reluctance (\mathcal{R}) is represented in magnetic circuit as resistance (R) in case of electrical circuit. A magnetic material presents reluctance (\mathcal{R}) for the flow of magnetic flux, when an mmf (F_{mmf}) is applied to the magnetic circuit.

i.e. $F_{mmf} = \mathcal{R} \cdot \phi$... (1.4)
mmf = Reluctance \times flux

For a uniform field

$$F_{mmf} = NI = Hl, \\ \text{and } \phi = B.A. = \mu H.A, \text{ where } \mu = \mu_0 \mu_r \\ \text{Therefore, } HI = \mathcal{R} \mu H.A.$$

and magnetic reluctance $\mathcal{R} = \frac{l}{\mu A}$... (1.5)

where, l is mean length of magnetic circuit and A is cross-section of magnetic circuit.
Unit of magnetic reluctance is Henry⁻¹ (H^{-1}) or ampere -turn per weber (A-t/Wb).

Magnetic permeance (\mathcal{P}) is inverse of magnetic reluctance and given as

$$\mathcal{P} = \frac{1}{\mathcal{R}} = \frac{\mu A}{l} \text{ (Henry)} \quad \dots(1.6)$$

i.e., permeance is the ease with which magnetic flux can be setup in a material. It is similar to conductance in an electric circuit.

Q. 3. State Ampere's circuital law ?

Or, Derive Ampere circuit law ?

Ans. Ampere's Circuital Law :

Ampere's circuital law deals with current flow through any path. It is also called as Ampere's work law.

Statement : It states that the line integral of magnetic field intensity (\vec{H}) around any closed path is exactly equal to the constant dc current enclosed by that path.

$$\oint \bar{H} \cdot d\bar{L} = I$$

where

\bar{H} → magnetic field intensity

$d\bar{L}$ → small differential length

I → current enclosed by the path

Proof: Consider a long-straight conductor carrying

Current I as shown in the fig.1.2

Shown in fig.1.2, the magnetic flux density is along the circular path. Now at a distance 'R' from this conductor, the magnitude of magnetic flux density is given by

$$B = \frac{\mu I}{2\pi R}$$

Here

μ → Permeability of medium

I → Current passing through the conductor

R → Radius of circular path

Now, we will integrate \bar{B} around a path of radius R enclosing the wire

$$\oint \bar{B} \cdot d\bar{L} = \frac{\mu I}{2\pi R} \oint dL$$

$$\text{But } \oint dL = 2\pi R$$

$$\therefore \oint \bar{B} \cdot d\bar{L} = \frac{\mu I}{2\pi R} \cdot 2\pi R = \mu I$$

$$\therefore \oint \frac{\bar{B}}{\mu} \cdot d\bar{L} = I \quad [\because \bar{B} = \mu \bar{H}]$$

$$\therefore \oint \bar{H} \cdot d\bar{L} = I$$

Hence proved Ampere's circuital law.

Q. 4. State Biot-Savart law ?

Or, Derive Biot-Savart law ?

Ans. Biot-Savart Law :

Biot-Savart Law deals with the magnetic field and current carrying element/conductor.

When current flows throughs a conductor, magnetic field is produced.

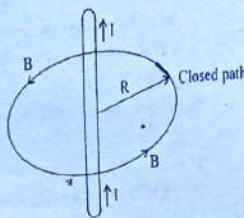


Fig. 1.2 : Conduction Carrying Current I

Statement : Magnetic field intensity produced at a point P due to a differential current element IdL is proportional to the product IdL and the sine of the angle "α" between the element and the line joining point "P" to the element and is inversely proportional to the square of the distance "R" between point P and the element.

From the fig. 1.3 :

From the statement of Biot-Savart Law, we have,

$$(i) d\bar{H} \propto IdL$$

$$(ii) d\bar{H} \propto \sin \alpha$$

$$(iii) d\bar{H} \propto \frac{1}{R^2}$$

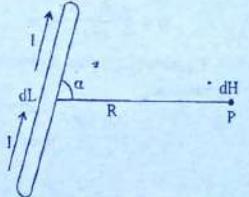


Fig. 1.3 : dH at Point "P" Due to IdL

Putting together, we can write

$$d\bar{H} \propto \frac{IdL \sin \alpha}{R^2} \quad \dots(1.7)$$

$$\text{or, } d\bar{H} = \frac{k IdL \sin \alpha^2}{R^2} \quad \dots(1.8)$$

$$\text{But in S.I. units } k = \frac{1}{4\pi}$$

$$d\bar{H} = \frac{IdL \sin \alpha}{4\pi R^2} \quad \dots(1.9)$$

Let, dL = Magnitude of vector length dL

\hat{a}_R = Unit vector in the direction from differential current element at Point P

Then, $dL \times \hat{a}_R = DL |\hat{a}_R| \sin \theta = dL \sin \theta$

Thus Biot-Savart law in vector form is

$$d\bar{H} = \frac{IdL \times \hat{a}_R}{4\pi R^2} A/m \quad \dots(1.10)$$

Equation (1.10) gives the differential magnetic field at point P. Total magnetic field intensity can be obtained by integrating equation (1.10).

$$\bar{H} = \oint d\bar{H}$$

[1-6] Magnetic Fields and Magnetic Circuits

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$$\vec{H} = \oint \frac{IdL \times \hat{R}}{4\pi R^2} = \oint \frac{IdL}{4\pi R^2} \times \frac{\vec{R}}{|R|} \quad \dots(1.11)$$

$$\vec{H} = \oint \frac{IdL \times \vec{R}}{4\pi R^3} A/m \quad \dots(1.12)$$

Equation (1.12) gives the integral form of Biot-Savart Law. Since we can have different current distribution as

$$\vec{H} = \int_L \frac{IdL \times \vec{R}}{4\pi R^3} \text{ (line current)}$$

$$\vec{H} = \int_S \frac{k dS \times \vec{R}}{4\pi R^3} \text{ (surface current)}$$

where, k = surface current density (amp/m)

$$\vec{H} = \int_V \frac{j dV \times \vec{R}}{4\pi R^3} \text{ (Volume current)}$$

where, j = Volume current (Amp/m²)

Visualization of Magnetic fields produces by a bar magnet and a current carrying coil through a combination of iron and air :

Q. 5. Explain Magnetic Circuit of Toroid ?

Ans. Magnetic Circuit in Form of Toroid : Considering a rectangular cross-section of toroid wound with closely spaced helical winding having Number of turns, as shown in Fig.

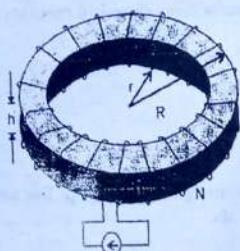


Fig. 1.4 : Uniform Wound Toroidal Winding

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[1-7] Magnetic Fields and Magnetic Circuits

The inner and outer radii of toroid are r and R respectively. The rectangular cross-section has height h . Considering magnetic field intensity tube in ϕ direction inside the core, as show in fig. (1.4) can be given by Amper's law as

$$\oint \vec{H} \cdot d\vec{l} = \oint j \cdot d\vec{s} = 2\pi r' H_\phi = I_{\text{enclosed}} \quad \dots(1.13)$$

where, $r \leq r' \leq R$

H = Magnetic field intensity

j = Current density

H_ϕ = Magnetic field flux intensity

Total current enclosed by closed path is NI . For analysis of magnetic current, the total enclosed current is referred in terms of applied magneto motive force (mmf). Since units of turns in SI unit is dimensionless, we use ampere turn (A-t) as the unit of mmf to differentiate it from the basic unit of current.

Thus from Amper's law we have

$$H_\phi = \frac{NI}{2\pi r'} \quad \dots(1.14)$$

when r' is any radius such that $r \leq r' \leq R$

The magnetic flux density within the core is given as

$$B_\phi = \mu H_\phi = \frac{\mu NI}{2\pi r'} \quad \dots(1.15)$$

where μ = permeability = $\frac{B}{H}$

The total flux in the core is

$$\phi = \oint \vec{B} \cdot d\vec{s} = \frac{\mu NI}{2\pi} \int_r^R \frac{1}{r'} dr' \int_0^h dz = \frac{\mu NI}{2\pi} h \log \left(\frac{R}{r} \right) \quad \dots(1.16)$$

When the toroidal core is made of a very high permeable magnetic material having winding concentrated over a small portion, as shown in fig. (1.5), major portion of magnetic flux circulates through the core. A fraction of the total flux produced by the coil complete path through magnetic core and gets associated with the coil surrounding the flux through magnetic core is termed as leakage flux, as shown in fig. (1.5). To have maximum flux linkage, design of magnetic circuit should be so as to contain minimum leakage flux as far as technically & economically possible. Therefore, for magnetic circuit analysis, leakage flux should be minimised.

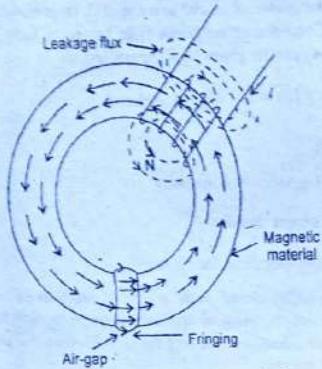


Fig. 1.5 : A Magnetic Circuit with an Air Gap

Magnetic flux density is maximum at the inner radius of the toroid and minimum at the outer radius. In magnetic circuit analysis, the magnetic flux density is considered as uniform within the magnetic material, and its magnitude equal to the magnetic flux density at the mean radius.

The toroid is generally closed magnetic path but in some applications such as rotating machines, the closed path is often broken by an air gap, as shown in fig. (1.5). In such case, highly permeable magnetic material is considered to be in series with air gap. Since, the circuit is series circuit, the magnetic flux in magnetic material is equal to the magnetic flux in the air gap. The spreading of the magnetic flux in the air gap is termed as fringing. For very small air gap as compared to other dimensions, most of the flux lines are well confined between opposite surfaces of the magnetic core at the air gap and the fringing effect is negligible.

Thus magnetic circuit analysis, we consider following assumptions:

- (i) The magnetic flux is restricted to flow through the magnetic material with no leakage.
- (ii) There is no spreading or fringing of the magnetic flux in the air gap regions.
- (iii) The magnetic flux density is uniform within the magnetic material.

Example 1.1 : An electromagnet of square cross-section similar to the one shown in fig. 1.3 has a tightly wound coil with 1500 turns. The inner & outer radii of the magnetic core are 10 cm and 12 cm respectively. The length of the air gap is 1 cm. If the current in the coil is 4 A and relative permeability of magnetic material is 1200, determine flux density of magnetic circuit.

Solution : Since permeability of magnetic material is constant, we can use the reluctance method to determine flux density in the core.

The mean radius is 11 cm, and mean length of magnetic path is
 $l_m = (2\pi \times 11) - 1 = 68.12 \text{ cm} = 68.12 \times 10^{-2} \text{ m}$
 Neglecting fringing effect, cross-sectional area of magnetic path is same as the air-gap, i.e.
 $A_m = A_g = 2 \times 2 = 4 \text{ cm}^2 = 4 \times 10^{-4} \text{ m}^2$

The reluctance of each region is

$$\mathcal{R}_m = \frac{68.12 \times 10^{-2}}{1200 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 1.129 \times 10^5 \text{ A-t/Wb}$$

$$\mathcal{R}_g = \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 19.894 \times 10^5 \text{ A-t/Wb}$$

Total reluctance is series circuit is

$$\mathcal{R} = \mathcal{R}_m + \mathcal{R}_g = 21.023 \times 10^5 \text{ A-t/Wb}$$

Thus, the flux in the magnetic circuit is

$$\phi = \frac{1500 \times 4}{21.032 \times 10^5} = 285.402 \times 10^{-6} \text{ Wb}$$

Finally, the flux density in either the air-gap or the magnetic region is

$$B_m = B_g = \frac{285.402 \times 10^{-6}}{4 \times 10^{-4}} = 0.714 \text{ T}$$

Q. 6. Explain Magnetic Circuit of Rectangular core ?

Ans. Magnetic Circuit with Rectangular Core :

In fig. (1.6) magnetic circuit (core) is made of iron, a ferromagnetic material with permeability $\mu = B/H$, when, $\mu > \mu_0$.

The magnetic flux ϕ crossing the surface s is the surface integral of the normal component of B .

$$\text{i.e. } \phi = \int_s B_n ds$$

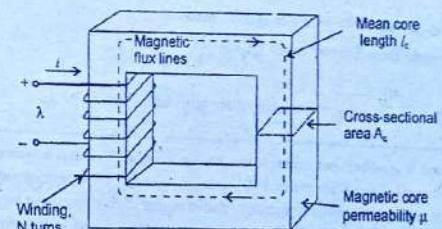


Fig. 1.6 : A Simple Rectangular Magnetic Circuit

[1-10] Magnetic Fields and Magnetic Circuits

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For uniform magnetic flux density across the core cross-section, we have

$$\phi_c = B_c A_c$$

where,

ϕ_c = flux is core

B_c = flux density in core

A_c = Cross-section area of core

For coil having N turns carrying current I, applied mmf in NI and given as

$$F_{mm} = NI = \oint \vec{H} \cdot d\vec{l}$$

Since the permeability of magnetic core is much higher, the magnetic flux remains confined to core and will be considerably lower in the air gap. The core dimensions are such that the path followed by flux lines is almost same as mean core length l_c . Therefore, line integral of H can be given as scalar product $H l_c$

$$\text{i.e. } F_{mm} = NI = H l_c$$

When H_c is the average magnitude of magnetic flux intensity H in the core.

Note: The direction of H_c in the core can be found by right-hand rule, which can be stated in two equivalent ways.

- (i) Imagine a current carrying conductor held in the right hand with the thumb pointing in the direction of current flow; the fingers then point in the direction of magnetic field created by current
- (ii) Equivalently, if a coil is grasped in the right hand with fingers pointing the direction of current, the thumb will point in the direction of the magnetic field.

Magnetic flux density in the magnetic material is

$$B = \mu H_c = \frac{\mu NI}{l_c}$$

Which μ is permeability of the magnetic material.

and magnetic flux density in air gap is

$$B_g = \mu H_g = \frac{\mu_0 NI}{l_g}, \text{ where } l_g \text{ is size of air gap such that } l_c \gg l_g$$

Note : In SI unit B in webers per square meter, known as teslas (T) and μ is webers per ampere-turn-meter or equivalently henrys per meter. In SI unit $\mu_0 = 4\pi \times 10^{-7}$. The permeability of ferromagnetic material can be given as μ_r , and permeability relative to that of free space can be given as $\mu = \mu_r \mu_0$. Typical value of μ_r ranges from 2000 to 80,000 for materials used in transformers and rotating machines; while it is 1 for non-magnetic materials.

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[1-11] Magnetic Fields and Magnetic Circuits

Example 1.2 : Figure 1.5(a) shows rectangular magnetic core with an air-gap. Find the exciting current needed to cause a flux density of $B_g = 1.2 T$ in the air-gap. Given $N = 400$ turns and μ_r (iron) = 4000.

Solution : Equivalent circuit of Fig. 1.7(a) as series magnetic circuit with its analogous has been shown in Fig. 1.7(b).

$$\text{Core length, } l_c = [(20-4)+(16-4)] - 0.2 = 55.8 \text{ cm} = 55.8 \times 10^{-2} \text{ m}$$

$$\text{Cross-sectional area of core, } A_c = 16 \text{ cm}^2 = 16 \times 10^{-4} \text{ m}^2$$

$$\text{Core reluctance, } R_c = \frac{l_c}{\mu A_c} = \frac{l_c}{\mu_0 \mu_r A_c} = \frac{55.8 \times 10^{-2}}{400 \times 4\pi \times 10^{-7} \times 16 \times 10^{-4}} \\ = 0.694 \times 10^5 \text{ AT/Wb}$$

$$\text{Air-gap length, } l_g = 0.2 \text{ cm} = 0.2 \times 10^{-2} \text{ m}$$

$$\text{Area of air-gap, } A_g = 16 \text{ cm}^2 = 16 \times 10^{-4} \text{ m}^2$$

$$\text{Air-gap reluctance, } R_g = \frac{0.2 \times 10^{-2}}{4\pi \times 10^{-7} \times 16 \times 10^{-4}} \\ = 9.95 \times 10^5 \text{ AT/Wb}$$

$$\text{Total reluctance, } R = R_c + R_g \\ = 0.694 \times 10^5 + 9.95 \times 10^5 = 10.64 \times 10^5 \text{ AT/Wb}$$

$$\text{Flux in the magnetic circuit, } \phi = BA = 1.2 \times 16 \times 10^{-4} = 1.92 \text{ mWb}$$

$$\text{Now } NI = R_g \phi \\ = 1.92 \times 10^{-3} \times 10.64 \times 10^5 \\ = 2043 \text{ AT}$$

$$\therefore \text{Exciting current, } I = \frac{2043}{400} = 5.11 \text{ A}$$

The ratio of reluctance is given as

$$R_g / R_c = 14.34$$

Therefore for simplicity of computation, R_c (magnetic core reluctance) may be altogether neglected. Then

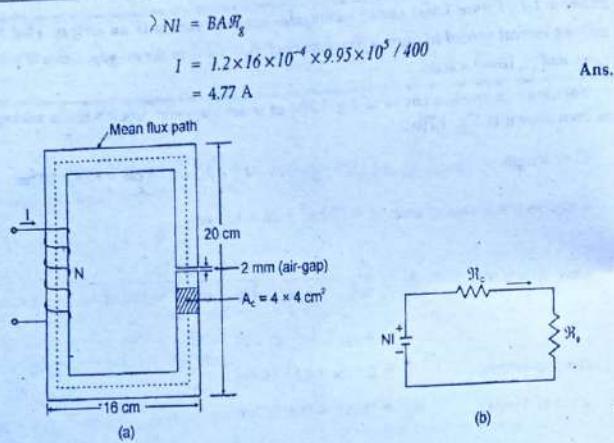


Fig. 1.7

Example 1.3 : The magnetic circuit of Fig. 1.8 has a cast steel core with dimensions as shown. It is required to establish a flux of 0.8 mWb in the air-gap of the central limb. Determine the mmf of the exciting coil, if for the core material $\mu_r = \infty$. Neglect fringing.

Solution : Since $\mu_r = \infty$, no mmf drops in any member of the core. The analogous electrical circuit is drawn in Fig. 1.9(a). It can be reduced to the circuit of Fig. 1.9(b) by parallel combination for \mathcal{R}_{g2} and \mathcal{R}_{g3} . Various gap reluctances are :

$$\mathcal{R}_{g1} = \frac{0.02 \times 10^{-2}}{4\pi \times 10^{-7} \times 2 \times 1 \times 10^{-4}} = 0.796 \times 10^6 \text{ AT/Wb}$$

$$\mathcal{R}_{g2} = \frac{0.02 \times 10^{-2}}{4\pi \times 10^{-7} \times 1 \times 1 \times 10^{-4}} = 1.592 \times 10^6 \text{ AT/Wb}$$

$$\mathcal{R}_{g3} = \frac{0.025 \times 10^{-2}}{4\pi \times 10^{-7} \times 1 \times 1 \times 10^{-4}} = 1.989 \times 10^6 \text{ AT/Wb}$$

$$\mathcal{R}_{g2} \parallel \mathcal{R}_{g3} = \frac{1.592 \times 1.989}{1.592 + 1.989} \times 10^6 = 0.884 \times 10^6 \text{ AT/Wb}$$

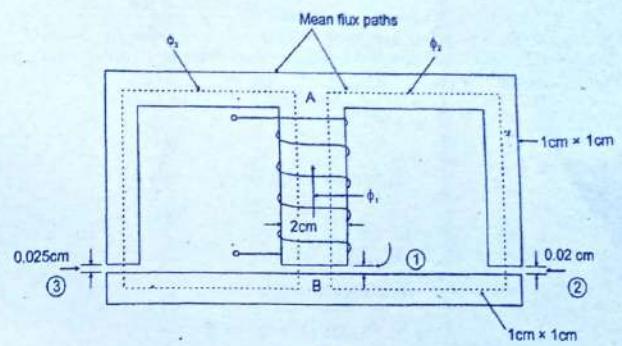


Fig. 1.8

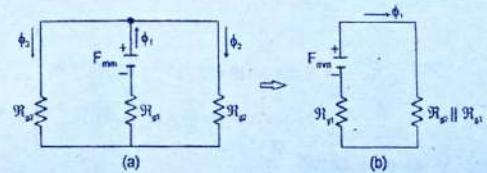


Fig. 1.9

$$\begin{aligned} \text{Exciting coil mmf, } F_{mm} &= \phi_1 (\mathcal{R}_{g1} + \mathcal{R}_{g2} \parallel \mathcal{R}_{g3}) \\ &= 0.8 \times 10^{-3} (0.796 + 0.884) \times 10^6 \\ &= 1344 \text{ AT} \end{aligned}$$

Ans.

Example 1.4 : A coil with an axial length of 25 cm and diameter of 20 cm has 200 turns. It is placed in a uniform radial flux of 0.002 wb/m². (i) If the coil is rotated at 25 rev. per second, find the voltage induced in the coil. (ii) What will be the force on each conductor and torque acting on the coil, if it carries a current of 10 A.

Solution : (i) Given $t = 25 \text{ cm}$
 $d = 20 \text{ cm}$
 $N = 200 \text{ turn}$
 $\phi = 0.002 \text{ wb/m}^2$

$$\nu = 25 \text{ rev. per sec.}$$

$$e = T$$

$$e = Btv$$

$$B = \frac{\phi}{A}$$

$$A = \frac{\pi}{4} (d^2) = \frac{\pi}{4} \times (20 \times 10^{-2})^2 = 0.0314 \text{ m}^2$$

$$B = \frac{0.002}{0.0314} T$$

$$= 0.06369 \text{ T}$$

$$e = 0.06363 \times (25 \times 10^{-2}) \times 25$$

$$= 0.398$$

$$\approx 0.4 \text{ V}$$

Ans.

$$(ii) F_{mm} = BIL$$

Ans.

$$F = 0.06369 \times 10 \times 25 \times 10^{-2} = 0.159225 \text{ N}$$

$$= 2 \times 10 \times 0.06369 \times \frac{20}{2} \times 10^{-2} \times 25 \times 10^{-2} = 0.031845 \text{ N} - \text{m} \text{ Ans.}$$

Analysis of Magnetic Circuit

Magnetic circuit consists of magnetic field created by one or several current circuits or permanent magnets. The laws applicable for analysis of magnetic circuit are.

⇒ Flux conservation law

⇒ Generalised form of ampere's law

⇒ Constitutive relation $B(H)$ which describes magnetic material.

Q. 7. Explain influence of Highly Permeable material on magnetic flux lines.

Ans. Influence of Highly Permeable Materials on the Magnetic Flux Lines : Permanent magnets have found wide application in designing dc and synchronous machines. This has caused development of new permanent magnetic materials. Some of the permanent magnetic material includes Alnico, M-5 electrical steel, samarium based rareearth cobalt, ceramic etc. For the machines employing permanent magnets to setup required magnetic field for permanent magnets should have large retentivity (residual flux density) to obtain sufficient mechanical strength and large coercivity to obtain non-demagnetisation property due to stray magnetic fields. These properties ensure the presence of magnetic flux in the magnetic circuit even in the absence of external excitation or cause magnetic field as obtained in coils carrying current.

Permanent Magnet Materials

An ideal permanent magnet should have flat-topped and wide hysteresis curve so that residual magnetism is sufficiently high in absence of external excitation. In other words, area enclosed by hysteresis curve is very large. It can be well illustrated with alnico-5 and M-5 electrical steel. Demagnetisation characteristics can be seen in fig. 1.10.

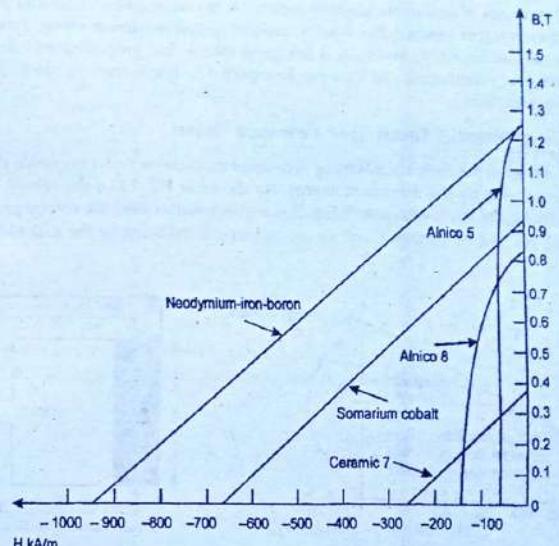


Fig. 1.10 : DC Magnetization Curves for Common Permanent Magnet Materials.

Alnico 5 is widely used version by alloy of iron, nickel, aluminum, and cobalt. It has relatively large residual flux density to that of Alnico 8, where as higher coercivity than Alnico-5. Therefore for Alnico-8 is less subjected to demagnetization than Alnico-5. Disadvantages of Alnico materials are their low coercivity and their mechanical brittleness.

Ceramic permanent magnet material (also known as ferrite magnets) are made for iron oxide and barium or strontium carbonate powders and have lower residual flux densities than Alnico material but significantly higher coercivities. As a result, they are much less prone to demagnetization. One such material, ceramic 7, is shown in fig. 1.10, where its dc magnetization characteristic is almost a straight line. Ceramic magnets have good mechanical characteristics and are inexpensive to manufacture; as a result, they are the most widely used of permanent magnet materials.

Samarium cobalt represents a significant advance in permanent magnet technology which begins in the 1960 with the discovery of rare-earth permanent magnet material. From Fig. 1.10 it can be seen to have a high residual flux density such as is found with the Alnico material, while at the same time having a much higher coercively and maximum energy product.

The newest of rare-earth magnetic material is the neodymium iron-boron material. It features even larger residual flux density, coercively, and maximum energy product than does samarium cobalt. In addition, it has good mechanical properties and is relatively inexpensive to manufacture and thus can be expected to find increasing use in permanent magnet applications.

Series Magnetic Circuit Using Permanent Magnet

In design of magnetic circuits using permanent magnets we prefer to operate the magnet where it can supply the maximum energy. As shown in Fig. 1.11, the energy density is simply the area of hysteresis loop ($B \times H$). This area is usually called the energy product. The operating point at its maximum energy product level is indicated by the dashed line in the Fig. 1.11.

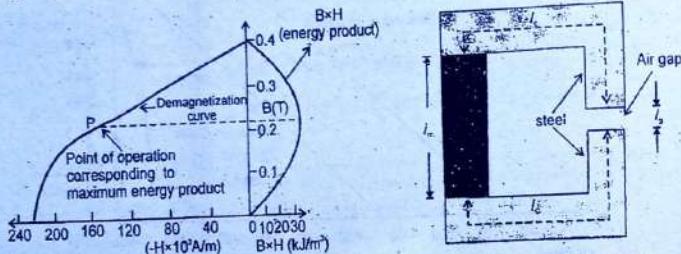


Fig. 1.11 : Typical Demagnetization and Fig. Energy Product Curve of a Permanent Magnetic

Fig. 1.12 shows a series magnetic circuit that employs a permanent to setup the necessary flux in the air gap-region. The L shaped section are usually made of a highly permeable magnetic material and are needed to channel the flux towards the air-gap the application of Ampere's law to the closed magnetic circuit yields,

$$H_m l_m + H_c l_c + H_g l_g + H_{c'} l_{c'} = 0 \quad (1.19)$$

where, the subscripts m, c, and g refer to permanent magnet, steel core, and air gap region respectively.

In this magnetic circuit, the magnetic flux is same in magnitude and continuous through the entire circuit. Therefore,

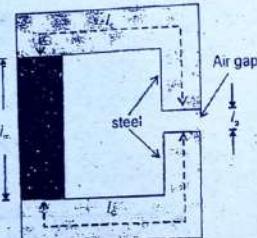


Fig. 1.12 : Series Magnetic Circuit using a Permanent Magnet

$$\phi = B_m A_m = B_g A_g = B_c A_c \quad (1.20)$$

Since, $B = \mu H$,

$$So, \quad H_g = \frac{B_m A_m}{\mu_0 A_g} \text{ and } H_c = \frac{B_m A_m}{\mu_c A_c} \quad (1.21)$$

Equation (1.19) can now be given as

$$H_m = - \left[\frac{l_g A_m}{\mu_0 l_m A_g} + \frac{2l_c A_m}{\mu_c l_m A_c} \right] B_m \quad (1.22)$$

Equation (1.22) is called operating line and its intersection with demagnetization curve yields the operating point.

If the permeability of the steel used for two L-sections is very high, equation (1.22) can be approximated as

$$H_m \approx - \frac{l_g A_m}{\mu_0 l_m A_g} B_m \quad (1.23)$$

H_m is negative in second quadrant of hysteresis loop. So we can drop negative sign in equation (1.23) and H_m as magnitude of coercive force corresponding to the flux density B_m in the magnet.

If the area of the air gap is same as that of the magnet, i.e. $A_g = A_m$, then the flux density in the magnet ($B_g = B_m$). Thus from equation (1.23), we have

$$B_g = B_m = \frac{\mu_0 l_m}{l_g} H_m$$

$$\Rightarrow B_g^2 = \mu_0 \frac{l_m A_m}{l_g A_g} H_m B_m$$

$$\Rightarrow B_g (\mu_0 H_g) = \mu_0 \frac{l_m A_m}{l_g A_g} H_m B_m$$

$$\Rightarrow B_g H_g V_g = B_m H_m V_m \quad (1.24)$$

Where V_g and V_m are volume of Air gap and magnet respectively. The above equation emphasizes the fact when the point of operation compounds to maximum energy product of magnet.

[1-18] Magnetic Fields and Magnetic Circuits

Machine-1

Example 1.5 : Considering a magnetic circuit consisting of a core of high permeability having a air gap and a section of permanent magnet to obtain necessary flux linkage, as shown in fig. 1.13.

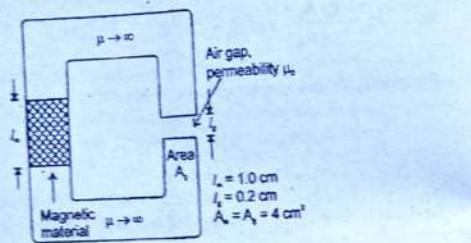


Fig. 1.13 : Magnetic Circuit

Solution : The mmf of the magnetic arrangement can be given $F_{mm} = H_m l_m + H_g l_g$ for every high permeability core material, H_g becomes negligible and net mmf approaches 3000. In ratio of mmf drop across magnet & air gap is -1, when the subscript m, s & g refer to permanent magnet, steel & air gap respectively.

$$\text{i.e. } F_{mm} = 0 = H_m l_m + H_g l_g \quad \dots(A)$$

$$\Rightarrow H_g = -\frac{l_m}{l_g} H_m$$

$$\text{i.e., } H_g l_g = -1$$

In a series magnetic circuit, magnetic flux is same in magnitude and continuous through the magnetic circuit i.e. $\phi = B_m A_m = B_g A_g = B_s A_s$

$$\Rightarrow B_g = \frac{A_m}{A_g} B_m \quad \dots(B)$$

$$\Rightarrow H_g = \frac{A_m}{\mu_0 A_g} B_m \quad (\because B_g = \mu_0 H_g)$$

from eqn. A & B we get

$$-\frac{l_m}{l_g} H_m = \frac{A_m}{\mu_0 A_g} B_m$$

$$\Rightarrow B_m = -\mu_0 \left(\frac{l_m}{l_g} \right) \left(\frac{A_g}{A_m} \right) H_m = -5 \mu_0 H_m = 6.28 \times 10^{-6} H_m$$

if $A_g = A_m$, then $B_g = B_m$

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[1-19] Magnetic Fields and Magnetic Circuits

$$\text{i.e. } B_m = B_g = -\mu_0 \left(\frac{l_m}{l_g} \right) H_m \Rightarrow B_g^2 = \mu_0 \left(\frac{l_m A_m}{l_g A_g} \right) (-H_m B_m)$$

$$= \mu_0 \frac{V_m}{V_g} (-H_m B_m)$$

$$\text{and } B_g H_g V_g = B_m H_m V_m$$

Here, V_g & V_m refer to volume of air gap and permanent magnet respectively.

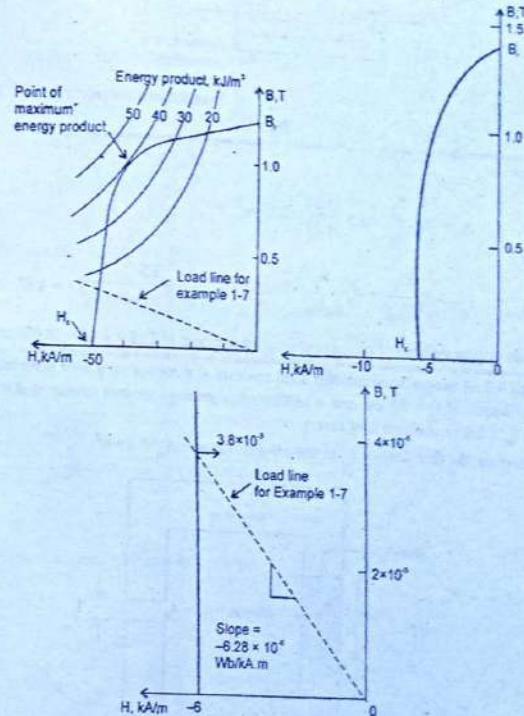


Fig. 1.14 : (a) Second Quadrant for Hysteresis Loop for Alnico (b) Second Quadrant Operation for M-5 Electrical Steel (c) Hysteresis Loop for M-5 Electrical Steel Expanded for Small B.

Example 1.6 : In a magnetic circuit having high permeability core with a air gap at 1.5 cm² and a part of permanent magnet. What is the minimum volume of magnet required to obtain air gap flux density of 1.5 tesla. Given, $B_m = 1$ Tesla & $H_m = -40$ kA/m $I_g = 0.15$ cm

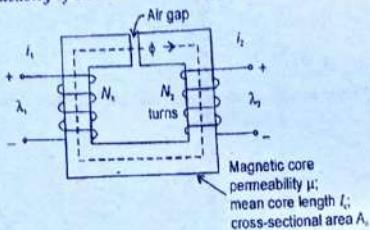


Fig. 1.15

Solution:

$$A_m = A_g \frac{B_g}{B_m} = 1.5 \left(\frac{1.5}{1.0} \right) = 2.25 \text{ cm}^2$$

$$\text{and } i_m = -I_g \frac{H_g}{H_m} = -I_g \frac{B_g}{\mu_0 H_m} = -0.15 \times \frac{1.5}{4\pi \times 10^{-7} \times (-40 \times 10^3)} = 4.47 \text{ cm}$$

so, minimum volume of permanent magnet required is $2.25 \times 4.47 = 10.057 \text{ cm}^3$

Example 1.7 : A magnetic circuit (fig. 1.16) consists of a core of very high permeability, an air-gap length of $l_g = 0.4 \text{ cm}$ and a section of permanent magnet (made of Alnico 5) of length $l_m = 2.4 \text{ cm}$. Assume μ of core = ∞ .

Calculate the flux density B_g in the air-gap. Given : $A_m = 4 \text{ mm}^2$.

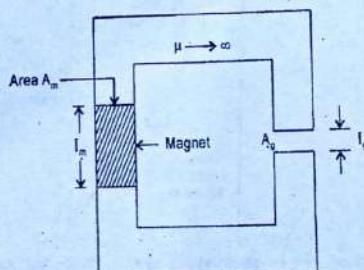


Fig. 1.16 : A magnetic circuit with a PM

Solution :

$$B_{core} = \infty \Rightarrow H_{core} = 0$$

From Ampere's circuital law

$$H_m l_m + H_g l_g = 0 = F \quad \dots(i)$$

$$\text{or } H_g = -\left(\frac{l_m}{l_g}\right) H_m \quad \dots(ii)$$

where H_g and H_m are the magnetic field intensities in the air-gap and the PM respectively. Thus the existence of an air-gap is equivalent to the application of a negative field to the PM material.

As the flux must be continuous around the path

$$\phi = B_m A_m = B_g A_g \quad \dots(iii)$$

$$\text{Also } B_g = \mu_0 H_g$$

We obtain from equations (i) and (ii)

$$B_m = -\mu_0 \left(\frac{A_g}{A_m} \right) \left(\frac{l_m}{l_g} \right) H_m \quad \dots(iv)$$

Substituting values we get

$$B_m = -6 \mu_0 H_m = -7.54 \times 10^{-5} H_m \quad \dots(v)$$

Thus is a straight line (also called load line) shown in fig. (i), where its intersection with the demagnetization curve at point 'a' gives the solution for B_m .

Thus

$$B_g = B_m = 0.33 \text{ T}$$

Note : If we repeat the above problem for M-5 electrical steel, it is easy to find the answer since the load line is the same as given by equation (v). It can be shown that $B_m = 4 \times 10^{-5} \text{ T}$. This is much less than the value of B_m for Alnico 5.

From equation (ii) we can get the expression for B_g as

$$B_g = \mu_0 H_g = -\mu_0 \left(\frac{l_m}{l_g} \right) H_m \quad \dots(vi)$$

From equations (iii) and (vi), we get

$$B_g^2 = \mu_0 \left(\frac{l_m A_m}{l_g A_g} \right) (-H_m B_m) \\ = \mu_0 \left(\frac{Vol_m}{Vol_g} \right) (-H_m B_m) \quad \dots(vii)$$

$$Vol_m = \frac{B_g Vol_g}{\mu_0 (-H_m B_m)} ; \text{ a positive value as } H_m \text{ is negative (fig. (i))} \quad \dots(viii)$$

Thus, to produce a flux density B_g in an air-gap of volume Vol_g , minimum volume of magnet material would be required if the material is operated in the state represented by the maximum value of the product $B_m H_m$.

From equation (viii) it may appear that one can get an arbitrarily large air-gap flux density just by reducing the air-gap volume. However, in practice this cannot be achieved because the on increasing flux density in the magnetic circuit beyond a given point, the magnetic core gets saturated and the assumption of infinite core permeability becomes invalid.

It may be noted that in fig. (1.16) a set of constant BH product curves (hyperbolas) is also plotted.

Q. 8. What are the analogies between electric and magnetic circuit :

Ans. Analogy Between Electric and Magnetic Circuit :

Magnetic field density and flux ϕ in magnetic circuit depends on current flowing through the winding wound on magnetic core. The strength of magnetic field intensity $H = NI$. The net flux ϕ established in magnetic circuit depends on NI . Therefore, the ratio NI/ϕ is termed as magnetic resistance of circuit. A circuit having smaller magnetic resistance (reluctance) will reach the given flux with smaller currents.

A magnetic circuit can have many parts including ferromagnetic material, permanent magnets, non-magnetic materials, or air. Air filled parts of magnetic circuits are also called air gaps or fringe. Energy conversion devices, which includes moving elements, must have air gap in their magnetic circuit. A magnetic circuit with air gap is shown in fig. (1.17).

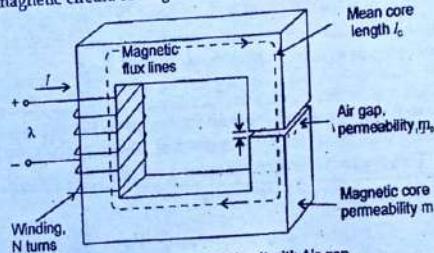


Fig. 1.17 : Magnetic Circuit with Air-gap

For very small air gap l_g as compared to dimensions of adjacent core faces, most of the flux lines are well confined between opposite surfaces of core in air gap and is continuous through out the magnetic circuit.

In fig. (1.17) the flux density is uniform and the cross-sectional area is A_c .

$$\text{Therefore for core, } B_c = \frac{\phi}{A_c}$$

$$\text{and in air gap } B_g = \frac{\phi}{A_g}$$

where ϕ is the flux in magnetic circuit.

The effect of air gap field or fringing field is to increase the effective cross section area A_g of the air gap as shown in Fig. 1.18. The fringing field is usually corrected in small air gaps by adding the gap length to each of the two dimensions making up its cross sectional area. So, for convenience fringing is minimized to obtain

$$A_C = A_g$$

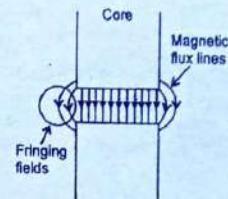


Fig. 1.18 : Fringing in Air-gap

$$\text{and hence } B_g = B_C = \frac{\phi}{A_C}$$

$$\text{But, we have } F_{mn} = NI = H_c l_c \quad \text{and} \quad \mu H_c = B$$

$$\text{Therefore, } F_{mn} = NI = H_c l_c + H_g l_g = \frac{B_C}{\mu} l_c + \frac{B_g}{\mu_0} l_g \quad \dots(1.26)$$

from eqn. (1.25) & (1.26), we have

$$F_{mn} = \phi \frac{l_c}{\mu A_c} + \phi \frac{l_g}{\mu_0 A_c} \quad \dots(1.27)$$

Here, the term $l_c/\mu A_c$ is called core reluctance R_c and $l_g/\mu_0 A_c$ is called air gap reluctance R_g . The other name of reluctance is magnetic resistance.

$$F_{mn} = \phi(R_c + R_g)$$

$$\phi = \frac{F_{mn}}{R_c + R_g} \quad \dots(1.28)$$

This equation is analogous to voltage current equation in an electric circuit having series resistances R_1 & R_2 connected to voltage source v . Here we have F_{mn} (analogous to voltage

In electric circuit, flux ϕ (analogous to current in electric circuit) and \mathcal{H}_c & \mathcal{H}_g (analogous to series connected electrical resistance). The analogous circuit can be given as shown fig. (1.19)

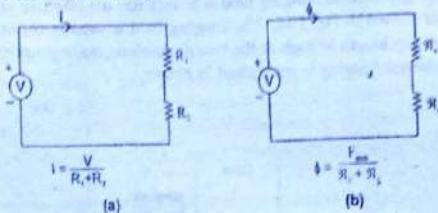


Fig. 1.19: Analogous Relation Between (a) Electric and (b) Magnetic Circuit

In the analogy, we observe that flux of the magnetic circuit is inversely proportional to its reluctance, since permeability of the core increases with reduction in reluctance and the reluctance of the core is much lesser as compared to reluctance of the air gap. i.e. $\mu \gg \mu_0$, $\mathcal{H}_c \ll \mathcal{H}_g$ and $\mathcal{H}_c + \mathcal{H}_g \approx \mathcal{H}_g$

So, in magnetic circuit

$$\phi = \frac{F_{mmf}}{\mathcal{H}_g} = F_{mmf} \frac{\mu_0 A_c}{l_g} = NI \frac{\mu_0 A_c}{l_g} \quad \dots(1.29)$$

and permeance (\mathcal{P}) (term which multiplies mmf) is

$$\mathcal{P}_g = \frac{1}{\mathcal{H}_g} = \frac{\mu_0 A_c}{l_g} \quad \dots(1.30)$$

Ohm's law Analogy

If $v = IR$, then $F_{mmf} = \phi R$

Electrical Circuit		Magnetic Circuit	
Current	I (Amps)	Flux	ϕ (Wb)
Emf	V (Volts)	mmf	F_{mmf} (A - t)
Resistance	R (ohm's)	Reluctance	\mathcal{R} (A-t/wb)
Corductivity	σ (s/m) (siemens/meter)	Permeability	μ (H/m)
Electric field	E (volt/m)	Magnetic field	H (Amps/m)
Current density	J (Amps/m²)	Flux density	B (Wb/m² or Tesla)

Q. 9. Define Eddy current loss ?

Ans. When a.c. voltage is applied to a coil wound on a magnetic core, the current, not only induced voltage within the coil, it induces voltage in the magnetic core, which causes current to circulate in itself. These currents look like whirlpools or eddies and hence are called eddy currents. Due to the resistance of the material to those currents, power losses occurs and is called eddy current loss.

It is difficult to determine the exact eddy current loss in the core either by experimental or analytical means.

The following empirical formula is used to find eddy current loss:

$$P_e = \frac{i^2 f^2 B_m^2}{J}$$

where, i is the thickness of material in meters.

f is the frequency in Hz.

J is the specific resistance of the magnetic material in $\Omega\text{-m}$.

The eddy current losses can be reduced by using laminated sheets of small thickness. It can also be reduced by increasing the cross-sectional resistance of the core. The sum of hysteresis and eddy current loss is known as core losses or iron-loss.

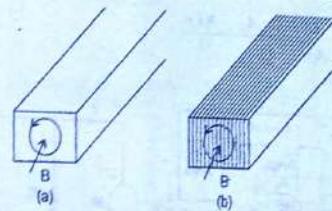


Fig. 1.20 : Eddy Current Loss in Core (a) Solid Core (b) Laminated Core

Q. 10. Explain Series and Parallel Magnetic Circuits.

Ans. Series and Parallel Magnetic Circuits :

Since for $\mu \gg \mu_0$, $\mathcal{H}_c \ll \mathcal{H}_g$ and the net reluctance is almost same as air gap reluctance. So for convenience, core reluctance has been neglected.

(1) Series circuit

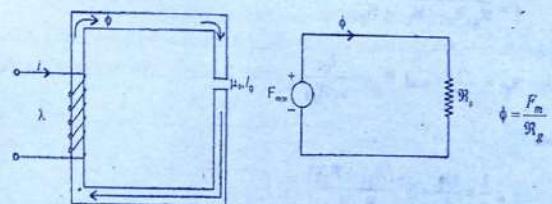


Fig. 1.21 : Series Magnetic Circuit

For magnetic circuit having linear relationship between ϕ & I due to constant permeability of the material or dominating air gap, the λ - I relation for inductance L is given as.

$$\begin{aligned} L &= \frac{\lambda}{I} \\ &= \frac{N\phi}{I} \\ &= \frac{N}{I} \cdot \frac{NI\mu_0 A_c}{l_g} \\ &= \frac{N^2 \mu_0 A_c}{l_g} \end{aligned} \quad \dots(1.32)$$

$$B_c = B_g = \frac{\phi}{A_c} = \frac{1}{A_c} \cdot \frac{NI\mu_0 A_c}{l_g} = \frac{NI\mu_0}{l_g} \quad \dots(1.33)$$

(2) Parallel circuit

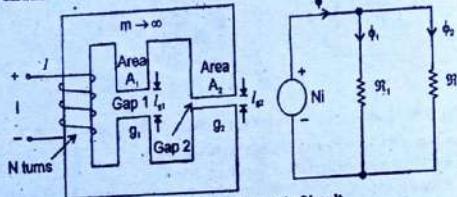


Fig. 1.22 : Parallel Magnetic Circuit

$$\phi = \frac{NI}{R_{g1}R_{g2}/R_{g1} + R_{g2}} \quad \dots(1.34)$$

$$\text{where } R_{g1} = \frac{l_{g1}}{\mu_0 A_{g1}} \text{ and } R_{g2} = \frac{l_{g2}}{\mu_0 A_{g2}}$$

Inductance

$$\begin{aligned} L &= \frac{\lambda}{I} = \frac{N\phi}{I} = N^2 \frac{(R_{g1} + R_{g2})}{R_{g1}R_{g2}} \\ &= \mu_0 N^2 \left(\frac{A_{g1}}{l_{g1}} + \frac{A_{g2}}{l_{g2}} \right) \end{aligned} \quad \dots(1.35)$$

for, 1st circuit

$$\phi_1 = \frac{NI}{R_{g1}} = \frac{\mu_0 A_{g1} NI}{l_{g1}} \quad \dots(1.36)$$

$$\text{and } B_1 = \frac{\phi_1}{A_{g1}} = \frac{\mu_0 NI}{l_{g1}} \quad \dots(1.37)$$

Similarly expression for B_2 can be given.

Series parallel magnetic circuit

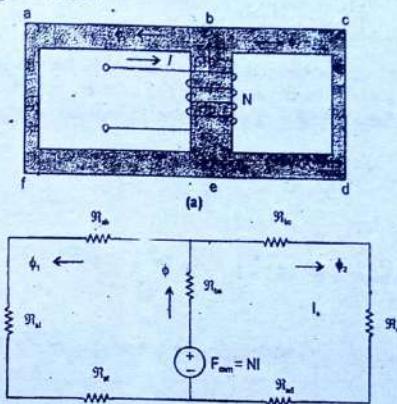


Fig. 1.23 : Series Parallel Magnetic Circuit

Flux ϕ_1 , inductance (L) & magnetic field (B) can be obtained as obtained in above two cases.
ENERGY STORED IN MAGNETIC COIL

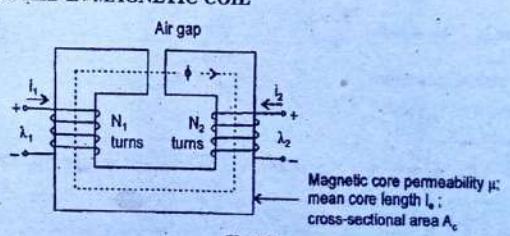


Fig. 1.24

[1-28] Magnetic Fields and Magnetic Circuits

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In the shown fig 1.24 total mmf is given as ... (1.38)

$$F_{\text{mm}} = NI = N_1 I_1 + N_2 I_2$$

and the core flux ϕ is given as

$$\phi = (N_1 I_1 + N_2 I_2) \frac{\mu_0 A_C}{l_s}$$

Neglecting reluctance of the core, we have

$$\phi = (N_1 I_1 + N_2 I_2) \frac{\mu_0 A_C}{l_s} \quad \dots(1.39)$$

In eqn. (1.39) ϕ is the resultant core flux produced by simultaneous action of both mmfs. It is this resultant ϕ which determines the operating point of the core material. If equation (1.39) is splitted in terms of individual currents, the resultant flux linkage of coil 1 is given as

$$\lambda_1 = N_1 \phi = N_1^2 \frac{\mu_0 A_C}{l_s} I_1 + N_1 N_2 \frac{\mu_0 A_C}{l_s} I_2 \quad \dots(1.40)$$

i.e. $\lambda_1 = L_{11} I_1 + L_{12} I_2$

$$\text{where self inductance of coil 1 is } L_{11} = N_1^2 \frac{\mu_0 A_C}{l_s}$$

$$\text{and mutual inductance of coil 1 with coil 2 is } L_{12} = N_1 N_2 \frac{\mu_0 A_C}{l_s}$$

$$\text{Similarly } \lambda_2 = N_2 \phi = N_1 N_2 \frac{\mu_0 A_C}{l_s} I_1 + N_2^2 \frac{\mu_0 A_C}{l_s} I_2$$

i.e. $\lambda_2 = L_{21} I_1 + L_{22} I_2$

$$\text{where } L_{12} = L_{21} = N_1 N_2 \frac{\mu_0 A_C}{l_s} I_1$$

and mutual inductance

$$L_{22} = N_2^2 \frac{\mu_0 A_C}{l_s}$$

For a static magnetic circuit inductance L is constant and voltage drop across the coil is given as

$$e = L \frac{di}{dt}$$

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[1-29] Magnetic Fields and Magnetic Circuits

for time varying inductance, like in electromechanical energy conversion devices is given as differential of $\lambda = LI$

$$e = \frac{d}{dt}(LI) = L \frac{di}{dt} + i \frac{dL}{dt}$$

power stored in the coil is given as

$$P = Le = i \frac{dL}{dt} \text{ watts or joules per second}$$

thus, change in magnetic stored energy ΔW in magnetic circuit in time interval t_1 to t_2 is given as

$$\Delta W = \int_{t_1}^{t_2} P dt = \int_{t_1}^{t_2} i d\lambda \quad \dots(1.41)$$

For single winding system of constant inductance, the change in magnetic stored energy can be given as

$$\Delta W = \int_{t_1}^{t_2} i d\lambda = \int_{t_1}^{t_2} \frac{\lambda}{L} d\lambda = \frac{1}{2L} (\lambda_2^2 - \lambda_1^2) \quad \dots(1.42)$$

The total magnetic energy stored at given value of λ can be given for $\lambda_1 = 0$

$$\text{i.e. } W = \frac{1}{2L} \lambda^2 = \frac{1}{2} i^2 \quad \dots(1.43)$$

Table : Symbols and equations of magnetic circuits

Terms Name	Symbol (Unit)	Equation
Magneto-motive force mmf	F_{mm} (Ampere-Turns) or (A-t)	$F_{\text{mm}} = NI$
Magnetic field Intensity	H (A-t/meter)	$H = NI/L$ (L = Path length in meters)
Flux density	B (Tesla) [$1 \text{ tesla} = 1 \text{ wb/m}^2$]	$B = \mu H$
Permeability	μ (H/meter)	$\mu = \mu_r \mu_0$, when $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$
Flux	ϕ (Weber wb)	$\phi = BA$ (A is cross section area in square meter) $\phi = BA = \mu_0 \mu_r H A = \mu_0 \mu_r A N I / L = NI/R$
Flux Linkage	λ (Weber turns)	$\lambda = N\phi$
Inductance	L (Henries)	$L = \lambda/I = N^2/R$
Reluctance	R (A-t/wb)	$R = 1/\mu_0 \mu_r A$

SOLVED EXAMPLES

Example 1 : A singly excited system has a linear relationship between flux linkage and current. the inductance varies as $L_1 + L_2 \cos 2\theta$. Derive an expression for torque and find its average value for direct current.

Solution :

$$\tau_e = \frac{1}{2} i^2 \frac{dL}{d\theta}$$

$$\tau_e = \frac{1}{2} i^2 \frac{dL}{d\theta} (L_1 + L_2 \cos 2\theta)$$

$$\tau_e = -i^2 L_2 \sin 2\theta$$

With steady current, the torque oscillates sinusoidally, but the total torque is zero over one revolution.

With a sinusoidally varying current $i = I_m (\cos \omega t + \delta)$

Since $\theta = \omega t$, $i = I_m \cos(\theta + \delta)$

and $\tau_e = -i^2 L_2 \sin 2\theta$

$$= -[I_m \cos(\theta + \delta)]^2 L_2 \sin 2\theta$$

The average torque over a cycle

$$\tau_e (\text{average}) = \frac{1}{2\pi} \int_0^{2\pi} \tau_e d\theta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} -[I_m \cos(\theta + \delta)]^2 L_2 \sin 2\theta d\theta$$

$$= \frac{-I_m^2}{2\pi} \int_0^{2\pi} \left[\frac{1 + \cos 2(\theta + \delta)}{2} \right] \sin 2\theta d\theta$$

$$= \frac{I_m^2 L_2}{4\pi} \int_0^{2\pi} \left[\sin 2\theta + \frac{\sin 2(2\theta + \delta) - \sin 2\delta}{2} \right] d\theta$$

$$= \frac{I_m^2 L_2}{4\pi} \int_0^{2\pi} \left[-\sin 2\theta - \frac{\sin 2(2\theta + \delta) - \sin 2\delta}{2} \right] d\theta$$

$$= \frac{I_m^2 L_2}{4\pi} \left[\frac{\cos 2\theta}{2} + \frac{\cos 2(2\theta + \delta)}{2} + \frac{\theta \sin 2\delta}{2} \right]_0^{2\pi}$$

$$= \frac{I_m^2 L_2}{4\pi} \cdot \frac{2\pi}{2} \sin 2\delta$$

$$\tau_e (\text{average}) = 0.25 I_m^2 L_2 \sin 2\delta \text{ Nm.}$$

Useful torque is developed when the angular velocity is equal to the angular frequency of the supply current. Single phase reluctance motors operate on this principle.

Example 2 : A circuit coil of 500 turns with a mean diameter of 50 cms is rotated about a vertical axis in the earth's field at 40 revolutions per second. Find the instantaneous value of emf induced in the coil when its plane is :

(i) parallel and (ii) inclined at 30° degree to the magnetic meridian. Take value of H as 14.3 AT/m.

Solution : When a coil rotates in a magnetic field, the instantaneous value of induced emf is

$$e = N \omega \phi \sin \theta$$

Where

N = number of turns

ω = angular speed, rad/sec.

ϕ = flux, Wb

θ = angle between field and direction of rotation

(i) When plane of the coil is parallel to the field, the rotation will be perpendicular to the field i.e., $\theta = 90^\circ$

$$\phi = B \times \text{area} = \mu_0 H \times \text{area}$$

$$\phi = (4\pi \times 10^{-7}) (14.3) \times \pi \times (0.25)^2$$

$$\phi = 35.28 \times 10^{-7} \text{ wb}$$

$$\omega = 40 \times 2\pi \text{ rad/sec.}$$

$$e = 500 \times 40 \times 2\pi \times 35.28 \times 10^{-7} = 0.443 \text{ V}$$

(ii) For inclination of 30°, $\theta = 60^\circ$

$$e = 0.443 \sin 60^\circ = 0.3836 \text{ V}$$

Example 3 : (i) A coil with an axial length of 25 cm and diameter of 20 cm has 200 turns. It is placed in a uniform radial flux of 0.002 Web/m². If the coil is rotated at 25 rev. per second, find the voltage induced in the coil

(ii) What will be the force on each conductor and torque acting on the coil, if it carries a current of 10 Amps?

Solution :(i) $I = 25 \text{ cm}$

$$d = 20 \text{ cm}$$

$$N = 200 \text{ turn}$$

$$\phi = 0.002 \text{ Wb/m}^2$$

$$v = 25 \text{ rev. per sec.}$$

$$e = T$$

$$e = Blv$$

$$B = \frac{\phi}{a}$$

$$a = \frac{\pi}{4}(d^2) = \frac{\pi}{4} \times (20 \times 10^{-2})^2 = 0.0314 \text{ m}^2$$

$$B = \frac{0.002}{0.0314} \text{ T}$$

$$B = 0.06369 \text{ T}$$

$$e = 0.06363 \times (25 \times 10^{-2}) \times 25$$

$$e = 0.398$$

$$e = 0.4 \text{ volts}$$

$$f = Blv$$

$$F = 0.06369 \times 10 \times 25 \times 10^{-2}$$

$$= 0.159225 \text{ newtons}$$

(ii) $T = 2 IBR/I$

$$= 2 \times 10 \times 0.06369 \times \frac{20}{2} \times 10^{-2} \times 25 \times 10^{-2}$$

$$= 0.03184 \text{ N-m}$$



Electromagnetic Force and Torque

This chapter covers

* Electromagnetic force and torque : B-H curve of magnetic materials; flux-linkage v/s current characteristic of magnetic circuits; linear and non linear magnetic circuits; energy stored in the magnetic circuit; force as a partial derivative of stored energy with respect to position of a moving element; torque as a partial derivative of stored energy with respect to angular position of a rotating element. Examples : galvanometer coil, relay contact, lifting magnet, rotating element with eccentricity or saliency.

Q. 1. Define (i) flux density, (ii) Relative permeability.**Or, Explain B-H curve of magnetic material ?****Ans. Magnetic Hysteresis :**

The lag or delay of a magnetic material known commonly as Magnetic Hysteresis, relates to the magnetisation properties of a material by which it firstly becomes magnetised and then de-magnetised. We know that the magnetic flux generated by an electromagnetic coil is the amount of magnetic field or lines or force produced within a given area is commonly called "Flux Density".

Magnetic strength of an electromagnet depends upon the number of turns of the coil or the current flowing through the coil or the type of core material being used. If, either the current or the number of turns is increased, there will be increase in the magnetic field strength (H).

The relative permeability, μ is defined as the product of the permeability of medium μ_r and the permeability of free space μ_0 (a vacuum). However, the relationship between the flux density, B and the magnetic field strength, H can be defined by the fact that the relative permeability, μ is not a constant but a function of the magnetic field intensity thereby giving magnetic flux density as: $B = \mu H$. Then the magnetic flux density in the material will be increased by a larger factor as a result of its relative permeability for the material compared to the magnetic flux density in vacuum, $\mu_0 H$ and for an air-cored coil this relationship is given as:

$$B = \frac{\Phi}{A} \text{ and } \frac{B}{H} = \mu_0$$

So for ferromagnetic materials the ratio of flux density to field strength (B/H) is not constant but varies with flux density. However, for air cored coils or any non-magnetic medium core such as woods or plastics, this ratio can be considered as a constant and this constant is known as μ_0 , the permeability of free space, ($\mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}$).

By plotting values of flux density, (B) against the field strength, (H) we can produce a set of curves called Magnetisation Curves, Magnetic Hysteresis Curves or more commonly B-H Curves for each type of core material used as shown below.

Q. 2. Define magnetic Saturation ?

Or, Draw the magnetisation of B-H curve ?

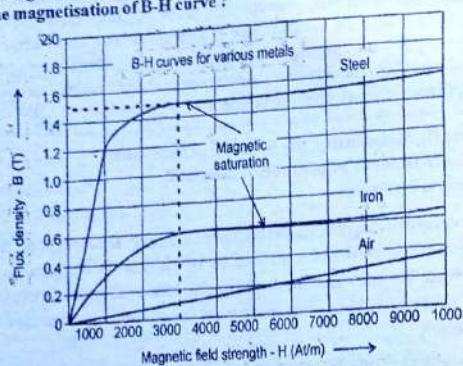


Fig. 2.1

The set of magnetisation curves, in fig. 2.1 represents an example of the relationship between B and H for soft-iron and steel cores but every type of core material will have its own set of magnetic hysteresis curves. It may be noticed that the flux density increases in proportion to the field strength until it reaches a certain value where it cannot increase any more becoming almost level and constant as the field strength continues to increase.

This is because there is a limit to the amount of flux density that can be generated by the core as all the domains in the iron are perfectly aligned. Any further increase will have no effect on the value of magnetisation (M) and the point on the graph where the flux density reaches its limit is called Magnetic Saturation also known as Saturation of the Core as shown in fig. 2.1 where, the saturation point of the steel curve begins at about 3000 ampere-turns per metre.

As the magnetic field strength, (H) increases these molecular magnets become more

and more aligned until they reach perfect alignment producing maximum flux density and any increase in the magnetic field strength due to an increase in the electrical current flowing through the coil will have little or no effect.

Q. 3. Explain Hysteresis loss ?

Ans. Hysteresis Loops :

Through out the complete cycle of magnetization the flux density lags behind the magnetic field intensity, this lagging phenomenon in the magnetic core is known as hysteresis.

When current i is varied over one cycle, during some part of the cycles, the coil stores energy from the source and during same part it returns energy back to source. However energy flowing into the coil is greater than the energy returned back to the source hence during one cycle of variation of current i (Hence H), there is a net energy flow from the source to the coil core assembly, the energy difference goes to heat the core, the loss of power in the core due to hysteresis effect is known as hysteresis loss.

The area of hysteresis loops is proportional to the hysteresis loss.

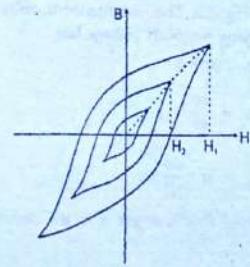


Fig. 2.2 : Three Hysteresis Loops—magnetizations Characteristics of the Magnetic Core

Example 2.1 : The area of the hysteresis loop of a given magnetic material is 50 cm^2 with the two axes scaled as $1 \text{ cm} = 20 \text{ AT}$ and $1 \text{ cm} = 50 \text{ mWb}$, for 50Hz frequency find the total hysteresis loss.

Solution :

Given

$$f = 50 \text{ Hz}$$

$$x = 20, (1 \text{ cm} = 20 \text{ AT})$$

$$y = 50 \text{ mWb} = 0.05 \text{ Wb}$$

$$\text{So hysteresis loss} = xy \times \text{Area of (B-H) loop Joules/m}^3/\text{cycle}$$

$$= 20 \times 0.05 \times 50$$

$$= 50 \text{ Joules/m}^3/\text{cycle}$$

$$\text{Total hysteresis loss}$$

$$= 50 \times 50$$

$$= 2500 \text{ Watt}$$

Ans.

Q. 4. Derive the magnetic energy stored in a single Excited Magnetic System ?

Ans. Flux-Linkage V/S Current Characteristic of Magnetic Circuits :

1. Single Excited Magnetic System

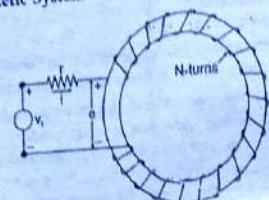


Fig. 2.3 : Toroidal Core Excited from Single Source

- (i) **Electric energy input:** Consider a simple magnetic system of a toroid, excited by a single coil, as shown in Fig. 2.3. The instantaneous voltage equation for the electric circuit is written by applying Kirchoff's voltage law.

$$v_t = ir + e$$

$$e = \frac{d\psi}{dt}$$

$$\text{and } v_t = ir + \frac{d\psi}{dt} idt \quad \dots (2.1)$$

Here ψ is the instantaneous flux linkages with the circuit. Multiplying both sides of equation 2.1 by i , we get

$$v_t idt = r i^2 dt + i d\psi$$

$$\text{or } (v_t - ir) idt = i d\psi$$

$$\text{or } e idt = i d\psi$$

$$\text{Since } dW_{elec} = e idt = i \cdot d\psi \quad \dots (2.2)$$

The flux linkages ψ are equal to $N\phi$ Wb-turns. Therefore, from equation (2.2)

$$dW_{elec} = i \cdot d\psi = iN d\phi = F d\phi \quad \dots (2.3)$$

In equation (2.3), ϕ is the instantaneous value of the coil flux and $F = iN$ is the instantaneous coil m.m.f.

Imp : The flow of charges or current against the reaction emf (e) causes the extraction of energy from electrical system.

- (ii) **Magnetic field energy stored:** Consider a simple magnetic relay of Fig. 2.4(a). Initially the armature is in the open position. When switch S is closed, current i is established in

the N -turn coil. The flux set up depends on m.m.f. Ni and the reluctance of the magnetic path. The magnetic field thus produced, creates north and south poles as shown in Fig. 2.4(a) and as a result of it, there is established a magnetic force tending to shorten the air-gap. If the armature is not allowed to move, the mechanical work done, dW_{mech} is zero. According to equation (2.3) therefore,

$$dW_{elec} = 0 + dW_{fld}$$

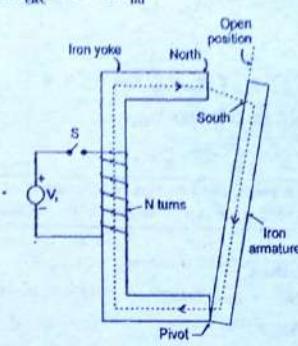


Fig. 2.4(a) : Simple Magnetic Relay

Imp : This shows that when the movable part of any physical system is kept fixed, the entire electrical energy input is stored in the magnetic field.

$$dW_{fld} = dW_{elec}$$

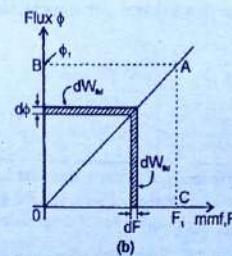
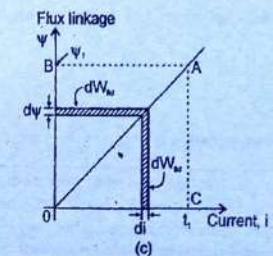


Fig. 2.4 (b) and (c) : Pertaining to Field Energy and Co-energy for a Linear Magnetic Circuit

From equation 2.3 $dW_{fld} = dW_{elec} = i \cdot d\psi = F \cdot d\phi \quad \dots (2.4)$

If the initial flux is zero, then the magnetic field energy stored W_{fld} in establishing a flux ϕ_i or flux linkage ψ_i , is given by



$$W_{fd} = \int_0^{\psi_1} i \cdot d\psi = \int_0^{\psi_1} F \cdot d\phi \quad \dots (2.5)$$

In equation 2.5, i and F must be expressed in terms of ψ and ϕ respectively.

$$\text{For Fig. 2.4(b), } W_{fd} = \int_0^{\psi_1} dV_{fd} = \int_0^{\psi_1} F \cdot d\phi = \text{Area OABO} \quad \dots (2.6)$$

$$\text{For Fig. 2.4(c), } W_{fd} = \int_0^{\psi_1} dW_{fd} = \int_0^{\psi_1} F \cdot d\psi = \text{Area OABO} \quad \dots (2.7)$$

In Fig. 2.4(b) and (c)

$$\text{Area OACO} = \int dW_{fd} = \int_0^{\psi_1} \phi \cdot dF = \int_0^{\psi_1} \psi \cdot di$$

This area OACO is called the co-energy W_{fd} .

$$W_{fd} = \int_0^{\psi_1} \phi \cdot dF = \int_0^{\psi_1} \psi \cdot di \quad \dots (2.8)$$

Imp: In equation 2.12, ϕ and ψ must be expressed in terms of F and i respectively. Co-energy has no physical significance, it is however useful in calculating the magnetic forces.

magnetic forces.

With no magnetic saturation.

$$\text{Area OABO} = \text{Area OACO}$$

$$\text{Or } W_{fd} = W_{fd'}$$

$$\text{And } W_{fd} + W_{fd'} = \text{Area OCABO} = \phi_1 F_{max} = \psi_1 i_1$$

In general for a linear magnetic circuit,

$$W_{fd} = W_{fd'} = \frac{1}{2} \psi i = \frac{1}{2} F_{max} \phi \quad \dots (2.9)$$

The self-inductance L is defined as the magnetic flux-linkages per ampere, i.e.,

$$L = \frac{\psi}{i} \quad \dots (2.10)$$

$$\therefore \text{From equation 2.9, } W_{fd} = W_{fd'} = \frac{1}{2} Li^2 = \frac{1}{2} \frac{\psi^2}{L} \quad \dots (2.11)$$

Imp: This fact that field energy can be expressed in terms of circuit parameter L , clears the way for electric circuit approach to the analysis of electrical machines. i.e., the generalized theory of electrical machines. Thus the field-energy approach serves as the physical basis for the generalized theory of electrical machines.

(iii) Mechanical Work Done

Consider the simple magnetic relay of Fig. 2.5 (a) again

Now in Fig. 2.5 (a)

Case 1 : The armature is assumed to be held in open position.

Now when the switch is closed, the current area starts from zero to $i_1 = v_f / r$ and the flux linkage increases from zero to ψ_1 .

So $W_{fd} = \text{Area OABO}$

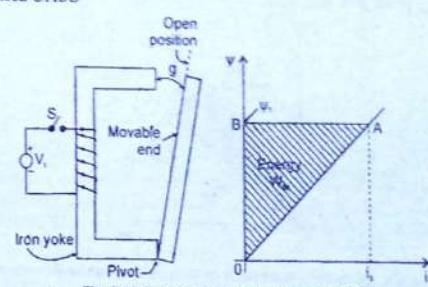


Fig. 2.5 (a) : Armature Held in Open Position

Case 2 : The armature is assumed to be close held in closed position (Fig. 2.5 (b))

Now when the switch is closed, the current rises from zero to $i_1 = v_f / r$, and the flux linkage rises for zero to ψ_2 .

Remember that ψ_2 is greater than ψ_1 because during the armature movement reluctance decreases.

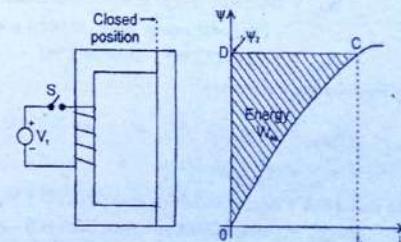


Fig. 2.5 (b) : Armature Held in Closed Position

Imp: These increments in flux linkages induces a counter emf in the coil, which opposes the flow of exciting current i , i.e.,

$$i = \frac{v - \text{emf}}{2} \quad \dots (2.12)$$

The magnitude of counter emf induced in the exciting coil depends on how fast the armature moves, three cases are possible.

(1) Slow Movement:

With the armature in the open position, the exciting current is i_1 , the flux linkages are ψ_1 and the operating point is A (Fig. 2.6).

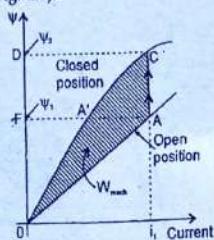


Fig. 2.6 : Mechanical Work Done (a) With Slow Armature Movement

In the closed armature position, the flux linkages are ψ_2 , current is i_1 and the operating point is C.

Imp : Now movement causes negligible amount at counter emf so the current remains constant.

Now change in the stored energy of magnetic field W_{fld} during the time armature moves from open (point A) to the closed position (Point C) is given by,

$$\begin{aligned} W_{\text{fld}} &= (\text{Magnetic energy stored in the closed position}) \\ &\quad - (\text{Magnetic energy stored in open position}) \end{aligned}$$

Or

$$W_{\text{fld}} = \text{Area OA'CDFO} - \text{Area OAA'FO}$$

Electric energy input during this change is

$$W_{\text{elec}} = \int_{\psi_1}^{\psi_2} i_1 d\psi = i_1 (\psi_2 - \psi_1) = \text{Area ACDFA'A} \quad \dots(2.13)$$

But

$$W_{\text{elec}} = W_{\text{fld}} + W_{\text{mech}}$$

$$\text{So Area ACDFA'A} = \text{Area OA'CDFO} - \text{Area OAA'FO} + W_{\text{mech}}$$

$$W_{\text{mech}} = (\text{Area ACDFA'A} + \text{Area OAA'FO}) - \text{Area OA'CDFO} \quad \dots(2.14)$$

$$W_{\text{mech}} = \text{Area OACDFO} - \text{Area OA'CDFO}$$

$$W_{\text{mech}} = \text{Area OACA'O} \quad \dots(2.14)$$

Equation 2.14 shows that the mechanical work done is equal to the area enclosed between the two magnetization curves at open and closed positions and the vertical $\psi - i$ locus during the slow armature movement. This is shown by cross-hatched area in Fig. 2.7.

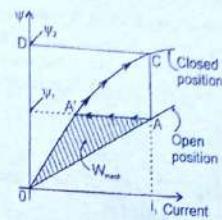


Fig. 2.7 : With Instantaneous Armature Movement

(2) Instantaneous Movement:

Here the armature is assumed to move from the open to closed position instantaneously. According to the constant flux linkage theorem, the flux linkages with an inductive circuit can't change suddenly. So here also, during the fast movement of the armature, the flux linkages don't change and remain constant at ψ_1 .

The operating point, therefore, travels horizontally from A to A'. After the armature has closed, the operating point travels from A' to C along the closed-position magnetization curve Fig. 2.7, since the final operating point has to be C.

During the time instantaneous movement of the armature occurs from open (point A) to closed position (point A'), we have

Change in the magnetic stored energy,

$$W_{\text{fld}} = \text{area OA'FO} - \text{area OAA'FO}$$

$$W_{\text{elec}} = \int_{\psi_2}^{\psi_1} i d\psi = 0 \quad (\text{constant flux linkage})$$

But

$$W_{\text{elec}} = W_{\text{fld}} + W_{\text{mech}}$$

$$\therefore O = \text{Area OA'FO} - \text{Area OAA'FO} + W_{\text{mech}}$$

or

$$W_{\text{mech}} = \text{Area OAA'FO} \quad \dots(2.15)$$

Equation 2.15 shows that the mechanical work done is equal to the area enclosed between the two magnetization curves at open and closed positions and the horizontal $\psi - i$ locus during the instantaneous movement of armature. This is indicated by cross-hatched area in Fig. 2.7.

During fast armature movement.

(a) There is no electrical energy input.

(b) Mechanical energy output = Reduction in the magnetic stored energy.

(3) Transient Movement:

The armature movement will neither be too slow nor too fast, but will lie

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[2-10] Electromagnetic Force and Torque

between the two extreme limits discussed above. Initially the armature movement is slow and as it is nearing the closed position, its movement becomes fast. The $\psi - i$ locus will, therefore, be AC'C as illustrated in Fig. 2.8. The operating point A reaches C' during the time armature moves from open to closed position. Since the final operating point has to be C, time armature moves from open to closed position.

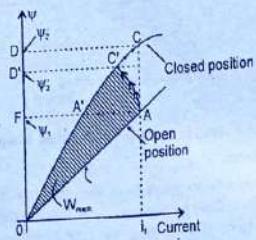


Fig. 2.8 : Flux linkage-current Locus During Transient Movement of Armature

During the time armature moves from open (point A) to closed position (point C'), we have : Change in the magnetic stored energy,

$$W_{\text{fld}} = \text{Energy stored in the closed position} - \text{Energy stored in the open position}$$

$$= \text{Area } OA'C'D'FO - \text{Area } OAA'FO$$

$$W_{\text{elec}} = \int_{V_1}^{\psi_2} id\psi = \text{Area } AC'D'FA'A$$

But

$$W_{\text{elec}} = W_{\text{fld}} + W_{\text{mech}}$$

$$\text{Area } AC'D'FA'A = \text{Area } OA'C'D'FO - \text{Area } OAA'FO + W_{\text{mech}}$$

or

$$W_{\text{mech}} = (\text{Area } OAA'FO + \text{Area } AC'D'FA'A) - \text{Area } OA'C'D'FO$$

$$= \text{Area } OAC'A'O \quad \dots(2.16)$$

Equation 2.16 again shows that the energy converted to mechanical (or mechanical work done) is equal to the area enclosed between the two magnetization curves at open and closed positions and the $\psi - i$ locus during the transient movement of armature.

Q. 5. Explain how energy is stored in the magnetic circuit.

Or, Derive energy stored in a linear case ?

Energy can be stored or retrieved from a magnetic system by means of an exciting coil connected to an electric source. Consider, for example the magnetic system of an attracted armature relay of fig. 2.9. The resistance of the coil is shown by a series lumping outside the coil which is regarded as an ideal loss-less coil. The coil current causes magnetic flux to be established in the magnetic circuit. It is assumed that all the flux ϕ is confined to the iron core and therefore links all the N turns creating the coil flux linkages of

Machine-1

[2-11]

Electromagnetic Force and Torque

$$\lambda = N\phi \quad \dots(2.17)$$

The flux linkage causes a reaction emf of

$$e = \frac{d\lambda}{dt} \quad \dots(2.18)$$

to appear at the coil terminals with polarity (as per Lenz's law) shown in the fig. 2.9. The associated circuit equation is

$$v = iR + e$$

$$= iR + \frac{d\lambda}{dt} \quad \dots(2.19)$$

The electric energy input into the ideal coil due to the flow of current i in time dt is

$$dW_e = ei dt \quad \dots(2.20)$$

Assuming for the time being that the armature is held fixed at position x , all the input energy is stored in the magnetic field. Thus

$$dW_f = ei dt = dW_f \quad \dots(2.21)$$

where dW_f is the change in field energy in time dt . When the expression for e in equation (2.21) is substituted in equation (2.20), we have

$$dW_e = id\lambda = F df = dW_f \quad \dots(2.22)$$

where $F = Ni$, the magnetomotive force (mmf).

The relationship $i - \lambda$ or $F - \lambda$ is a functional one corresponding to the magnetic circuit which in general is nonlinear (and is also history-dependent, i.e. it exhibits hysteresis). The energy absorbed by the field for finite change in flux linkages for flux is obtained from equation (2.22) as

$$\Delta W_f = \int_{\lambda_1}^{\lambda_2} i(\lambda) d\lambda = \int_{\phi_1}^{\phi_2} F(\phi) d\phi \quad \dots(2.23)$$

As the flux in the magnetic circuit undergoes a cycle $\phi_1 \rightarrow \phi_2 \rightarrow \phi_1$, an irrecoverable loss in energy takes place due to hysteresis and eddy-currents in the iron, assuming here that these losses are separated out and are supplied directly by the electric source. This assumption renders the ideal coil and the magnetic circuit as a conservative system with energy interchange between themselves so that the net energy is conserved.

The energy absorbed by the magnetic system to establish flux ϕ (or flux linkages λ) from initial zero flux is

$$W_f = \int_0^{\lambda} i(\lambda) d\lambda = \int_0^{\phi} F(\phi) d\phi \quad \dots(2.24)$$

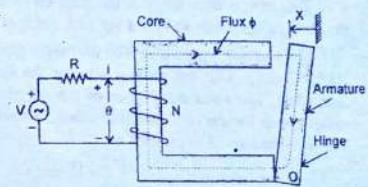


Fig. 2.9 : Attracted Armature Relay

This then is the energy of the magnetic field with given mechanical configuration when its state corresponds to flux ϕ (or flux linkages λ).

The $i-\lambda$ relationship is indeed the magnetization curve which varies with the configuration variable x (fig. 2.9) : the air-gap between the armature and core varies with position x of the armature. The total reluctance of the magnetic path decreases as x increases. The $i-\lambda$ relationship for various values of x is indicated in fig. 2.10. It immediately follows that this relationship can be expressed as

$$i = i(\lambda, x)$$

if λ is the independent variable as

$$\lambda = \lambda(i, x)$$

if i is the independent variable.

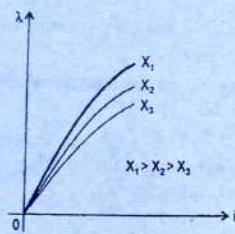


Fig. 2.10 : $i-\lambda$ Relationship with Variable x

Therefore, the field energy (equation (2.24)) is in general a function of two variables, i.e.,

$$W_f = W_f(\lambda, x) \quad \dots(2.25)$$

or

$$W_f = W_f(i, x) \quad \dots(2.26)$$

According to equations (2.25) and (2.26) field energy is determined by the instantaneous values of the system states (λ, x) or (i, x) and is independent of the path followed by these states to reach the present values. This means that the field energy at any instant is history independent.

A change in λ with fixed x causes electric-magnetic energy interchange governed by the circuit equation (2.25) and the energy equation (2.26). Similarly, if x is allowed to change with fixed λ , energy with interchange between the magnetic circuit and the mechanical system.

As per equation (2.25) the field energy is the area between the λ -axis and $i-\lambda$ curve as shown in fig. 2.11. A new term, co-energy is now defined as

$$W_f'(i, x) = i\lambda - W_f(\lambda, x) \quad \dots(2.27)$$

wherein by expression λ as $\lambda(i, x)$, the independent variables of W_f' become i and x . The co-energy on fig. 2.11 is shown to be the complementary area of the $i-\lambda$ curve.

$$W_f' = \int_0^i \lambda di \quad \dots(2.28)$$

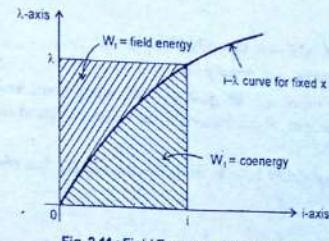


Fig. 2.11 : Field Energy and Coenergy

Linear Case :

Electromechanical energy conversion devices are built with air-gaps in the magnetic circuit which serve to separate the stationary and moving members. As a result the $i-\lambda$ relationship of the magnetic circuit is almost linear; also the losses of magnetic origin are separately accounted for by semi-empirical methods. With the linearity assumption the analysis is greatly simplified. Losses and certain nonlinear effects may then be incorporated at a later-stage.

Assuming linearity,

$$W_f = \frac{1}{2} i \lambda = \frac{1}{2} F_{mm} \phi = \frac{1}{2} \mathcal{R} \phi^2 \quad \dots(2.29)$$

where, it is known, $\mathcal{R} = F_{mm}/\phi$ = reluctance of the magnetic circuit. Since the coil inductance

$$L = \lambda/i$$

the field energy can be expressed as

$$W_f = \frac{1}{2} \frac{\lambda^2}{L} \quad \dots(2.30)$$

In the linear case the inductance L is independent of i but is a function of configuration x . Thus the field energy is a special function of two independent variables λ and x , i.e.,

$$W_f(\lambda, x) = \frac{1}{2} \frac{\lambda^2}{L(x)} \quad \dots(2.31)$$

The field energy is distributed throughout the space occupied by the field. Assuming no losses and constant permeability, the energy density of the field is

$$W_f = \int_0^B H dB = \frac{1}{2} HB = \frac{1}{2} \frac{B^2}{\mu} J/m^3 \quad \dots(2.32)$$

where,

H = magnetic field intensity (AT/m)

B = magnetic flux density (T)

The energy density expression of equation (2.32) is important from the point of view of design wherein the capability of the material is to be fully utilized in arriving at the gross dimensions of the device.

For the linear case it easily follows from equation (2.29) that coenergy is numerically equal to energy, i.e.,

$$W_f' = W_f = \frac{1}{2} \lambda i = \frac{1}{2} F_{mn} \phi \quad \dots(2.33)$$

Also in terms of the coil inductance

$$W_f' = \int_0^l (\lambda = Li) di = \frac{1}{2} Li^2$$

or in general

$$W_f'(i, x) = \frac{1}{2} L(x)i^2 \quad \dots(2.24)$$

If $A(\omega^2)$ and $l(m)$ are the area and length dimensions of the field, then from equation (2.24)

$$W_f' = \frac{W_f'}{Al} = \int_0^l i N d \left(\frac{\lambda}{NA} \right) = \int_0^B H dB \quad \dots(2.34)$$

The expression for coenergy density is

$$W_f' = \int_0^H B dH \quad \dots(2.35)$$

which for the linear case becomes

$$W_f' = \frac{1}{2} \mu H^2 = \frac{1}{2} \frac{B^2}{\mu} \quad \dots(2.36)$$

FORCE AND TORQUE AS A PARTIAL DERIVATIVE OF STORED ENERGY WITH RESPECT TO ANGULAR POSITION OF A ROTATING ELEMENT

The magnetic force tending to shorten the air gap increases as the gap length decreases

$$f_e(\text{average}) = \frac{\text{Mechanical work done}}{\text{Distance travelled}} \quad \dots(2.37)$$

In order to obtain a suitable expression for it, the movable part is allowed a virtual displacement dx (or ds) in the direction of magnetic force f_e (or torque T_e), then its effect on energy balance equation is investigated in order to obtain the magnitude and direction of magnetic force f_e or magnetic torque T_e .

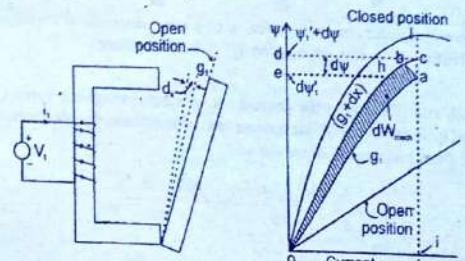


Fig. 2.12 : Mechanical Work Done for Differential Movement of Armature

Assume the armature to be at a distance g_1 from the open position, then a virtual displacement are in the direction of magnetic force f_e is considered.

- (a) 'a' is the operating point at position g_1 , and ψ_1' and i_1 are the corresponding values.
- (b) 'c' is the operating point at position $(g + dx)$, and $(\psi_1 + d\psi)$ and i_1 are the corresponding values. The mechanical work done in the virtual displacement dx is $f_e dx$ over the virtual displacement dx may be taken as instantaneous.

Now

$$dW_{ele} = \int_{\psi_1}^{\psi_2} id\psi \quad \dots(2.38)$$

Putting these values in energy balance equation.

$$0 = f_e dx + dW_{fd}$$

$$\text{of constant } \psi$$

$$f_e dx = dW_{fd}$$

$$\text{at constant } \psi$$

Imp : Electrical energy flow during virtual displacement dx is zero.
The mechanical work $f_e dx$ is done at the expense of field energy stored, so it is indicated by negative sign.

Or

$$f_e = - \left(\frac{dW_{fd}}{dx} \right)_{\psi=\text{constant}}$$

Also

$$f_e = - \left(\frac{dW_{fd}}{dx} \right)_{\phi=\text{constant}} \quad \dots (2.39)$$

Note that W_{fd} must be expressed in terms of ψ and x or ϕ and x . In view of this equation 2.39 leads to the parametric equations for magnetic force as

$$f_e = - \frac{\partial W_{fd}(\psi, x)}{\partial x} = - \frac{\partial W_{fd}(\phi, x)}{\partial x} \quad \dots (2.40)$$

In the above expression for magnetic force, ψ or ϕ are independent variables. As voltage is equal to the derivative of ψ , this expression gives f_e for a *voltage-controlled system*.

Equations (2.40) and (2.41) give the magnitude of electro-magnetic force f_e because the armature movement is linear. For angular movements of armature, the electromagnetic torque T_e can be obtained from parametric equation as

$$T_e = - \frac{\partial W_{fd}(\psi, \theta)}{\partial \theta} = - \frac{\partial W_{fd}(\phi, \theta)}{\partial \theta} \quad \dots (2.41)$$

Also

$$W_{fd} = \frac{1}{2} \psi^2$$

Note

$$f_e = \frac{1}{2} \psi^2 \frac{d}{dx} \left[\frac{1}{L} \right]$$

Similarly for electromagnetic torque T_e

$$T_e = \frac{1}{2} i^2 \frac{dL}{d\theta} = - \frac{1}{2} \psi \frac{\partial i}{\partial \theta} (\psi, \theta) = \frac{1}{2} i \frac{\partial \psi}{\partial \theta} (i, \theta) \quad \dots (2.42)$$

Example 2.2 : For a certain relay, the magnetization curves for open and closed positions of the armature are linear. If the armature of the relay moves from open to closed position at constant current (i.e. very slowly), show that the electrical energy input is shared equally between field energy stored and the mechanical work done.

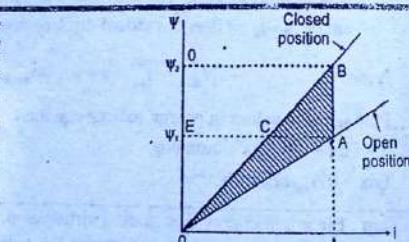


Fig. 2.13 : Magnetization Curve of Open and Close Position

Solution : With the relay in the open position, assume the operating point to be A, so that the current and flux linkage are i_1 and ψ_1 respectively, (Fig. 2.13).

In the closed position, the operating point is B, so that the current is i_2 and the flux linkage is ψ_2 .

The armature movement from open to closed position is at constant current, therefore, the ψ - i locus is along the vertical line AB.

During the movement from open to closed position, the electrical energy input is

$$W_{elec} = \int_{\psi_1}^{\psi_2} i_1 d\psi = i_1 (\psi_2 - \psi_1)$$

The mechanical work done W_{mech} is given by the cross-hatched area OABC.

$$W_{mech} = \text{Triangular area OABC}$$

$$= \frac{1}{2} (\psi_2 - \psi_1) i_1$$

Magnetic stored energy

$$\begin{aligned} W_{fd} &= \text{Triangular area OCBDEO} \\ &\quad - \text{Triangular area OACEO} \\ &= \frac{1}{2} \psi_2 i_1 - \frac{1}{2} \psi_1 i_1 = \frac{1}{2} (\psi_2 - \psi_1) i_1 \end{aligned}$$

$$\text{It is seen from above that } W_{fd} = W_{mech} = \frac{1}{2} W_{elec} \quad \dots (2.43)$$

This proves the required result.

Example 2.3 : For the simple magnetic relay the variation of flux linkage ψ in terms of current i and displacement x from the open position is given by the relation $\psi = ix^{1/2}$.

Obtain an expression for the magnetic force.

Solution : The electromagnetic force f_e can be obtained by taking the partial derivative of either the field energy function W_{fd} or co-energy function W_{fd} .

From magnetic stored energy considerations, the magnetic force is

$$f_e = - \frac{\partial W_{fd}(\psi, x)}{\partial x}$$

$$\text{From equation (2.4) } W_{fd}(\psi, x) = \int_0^\psi id\psi = \int_0^\psi \frac{\psi}{x^{1/2}} d\psi = \frac{1}{x^{1/2}} \cdot \frac{\psi^2}{2}$$

$$f_e = - \frac{\partial}{\partial x} \left[\frac{1}{x^{1/2}} \cdot \frac{\psi^2}{2} \right] = - \frac{\psi^2}{2} \left(-\frac{1}{2} \right) x^{-3/2}$$

$$= \frac{\psi^2}{4} \cdot \frac{1}{x^{3/2}}$$

From the co-energy considerations

$$f_e = \frac{\partial W_{fd}}{\partial x}(i, x)$$

But from equation 2.11,

$$W_{fd}(i, x) = \int_0^\psi id\psi = \int_0^\psi ix^{1/2} di = x^{1/2} \cdot \frac{i^2}{2}$$

$$f_e = \frac{\partial}{\partial x} \left[x^{1/2} \cdot \frac{i^2}{2} \right] = \frac{i^2}{2} \cdot \frac{1}{2} \cdot x^{-1/2}$$

$$= \frac{i^2}{4} \cdot \frac{1}{x^{1/2}}$$

Example 2.4: In an electromagnetic relay, functional relation between the current i in the exciting coil, the position of armature x and the flux linkages ψ is given by

$$i = 2\psi^3 + 3\psi(1-x+x^2), x > 0.5$$

Find the force on the armature as a function of ψ .

Solution :

$$i = 2\psi^3 + 3\psi(1-x+x^2)$$

Field energy stored, $W_{fd}(\psi, x) = \int_0^\psi id\psi = \int_0^\psi [2\psi^3 + 3\psi(1-x+x^2)] d\psi$

$$= \frac{2\psi^4}{4} + 3 \cdot \frac{\psi^2}{2} (1-x+x^2)$$

Magnetic force is given by equation 2.48

$$\begin{aligned} f_e &= -\frac{\partial W_{fd}(\psi, x)}{\partial x} = -\frac{\partial}{\partial x} \left[\frac{\psi^4}{2} + \frac{3\psi^2}{2} (1-x+x^2) \right] \\ &= -\left[0 + \frac{3\psi^2}{2} (0-1+2x) \right] = \frac{3\psi^2}{2} (1-2x) \quad \text{Ans.} \end{aligned}$$

For $x > 0.5$, f_e is negative, therefore f_e acts to decrease the field energy stored at constant flux linkages.

Example 2.5 : The simple magnetic relay gave the following $\psi-i$ characteristics :

(i) Open position, $\psi = 0.04 i$ Wb-turn for all values of current i ,

(ii) Closed position, $\psi = 0.06 i$ $0 \leq i \leq 20$

$$= 1.2 + 0.3(i-20) \quad i > 20$$

For an armature movement from open to closed position, find the magnitude of average magnetic force. The air-gap length is 2 cm and the current during armature movement is assumed to remain constant at 40 A.

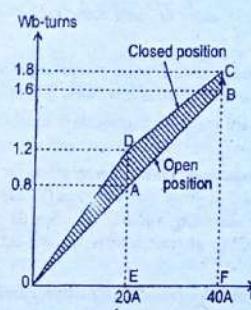


Fig. 2.14 : $\psi-i$ characteristics

Solution : The variation of flux linkage with current are sketched in Fig. 2.14, both for open and closed position of the armature. B is the operating point for open position and C is the operating point for the closed position. The $\psi-i$ locus during the armature movement is along the vertical line BC, since current remains constant at 40 A.

The mechanical work done during the armature movement

$$\begin{aligned} &= \text{Area OABCDO} = \text{area ODCFEO} - \text{area OABFO} \\ &= \left[\frac{1}{2} (20)(1.2) + \frac{1}{2} (1.2 + 1.8)(20) \right] - \left[\frac{1}{2} (40)(1.6) \right] \\ &= 42 - 32 = 10 \text{ joules} \end{aligned}$$

Average electromagnetic force,

$$f_{e(av)} = \frac{10}{2 \times 10^{-2}} = 500 \text{ N}$$

Ans.

Q. 6. Give application of Electro-Mechanical Machines and devices.

Or, Give application of Acoustic Transducers.

Ans. Galvandmeter coil, Relay contact, lifting magnet, rotating element with eccentricity or saliency :

Permanent magnets are used in many applications including large industries to common human activity. Some common applications are as here under :

1. Electro-Mechanical Machines and Devices

(a) Electric Motors :

Types - DC (commutator and brush less), synchronous, induction start/synchronous run, hysteresis; rotary and linear, continuous, servo, torque, or stepping operation. Geometries - permanent magnet stator (conventional and iron less armatures), permanent magnet rotor; inner or outer rotor; radial or axial field (disc) motors.

(b) Generators :

Types - Magnets, ignition or other pulse generators, tachometers, auxiliary exciters, alternators, multiphase synchronous machines, homopolar DC machines. Geometries - permanent magnet rotor; radial or axial field; stator winding with or without iron.

(c) Electro-Mechanical Actuators : Linear - Force motors for valves, etc.; printer hammer mechanism; computer disc-drive head actuators (VCM); laser focusing and tracking mechanism; (optic/magneto-optic recording: audio CDs, video, data); recorder pen positioners. Rotary-Disc drive VCMs; aircraft control surface actuators; materials handling robots.

(d) Measuring Instruments : Moving-coil (d'Arsonval and long scale geometries) and moving-magnet meters for many functions.

(e) Electric Current Control Circuit breakers, reed switches, miniature biased relays, thermostats, automotive ignition, eddy current motor over speed switch, arc blow-out magnets.

2. Acoustic Transducers

(a) Sound Generators : Loudspeakers, earphones, telephone receivers, ringers, buzzers, ultrasonic generators.

(b) Sound Receivers : Dynamic microphones, ultrasound pickups.

(c) Other Audio Frequency Transducers Phonograph pickups.

3. Mechanical Force and Torque Applications

(a) Contact Holding and Lifting : Machine-tool chucks, grippers, load-lifting magnets (electrically switchable), tool holders, door catches, refrigerator seals, advertising signs, toys, and many more.

(b) Traction Devices : Conveyors, separators for ores and other materials, field gradient water purifiers, photocopier rollers.

(c) Couplings and Brakes : Synchronous torque couplings, linear followers, eddy current and hysteresis couplers and brakes, rotary-to linear motion converter.

(d) **Magnetic Bearings and Suspensions** : Passive-watt-hour meters, ultra-centrifuges, record player tone-arm support, textile spinning turbines. Partly active served systems-gyros, satellite momentum and energy wheels, laser beam scanner, turbo-molecular pumps, electro-magnetic tracked vehicle levitation.

(e) **Electro-Balances** : Modern weighing devices from analytical balance to supermarket scales and truck weight stations.

4. Microwave/MM-Wave Devices, Electro Ion Beam Control

(a) **Power Tubes** : Magnetrons (radar, kitchen ovens); PPM focusing for TWTs and klystrons; crossed-field amplifiers, gyrotrons, etc.

(b) **Waveguide Devices** : Biasing ferrite or YIG elements in resonance filters, switches, and isolators.

(c) **Particle Accelerators, Synchrotron Radiation Sources**. Fee Electron Laser Lenses, deflecting magnets, wigglers, undulators.

(d) **Mass Spectrometers** : Deflecting magnets

(e) **Cathode Ray Tubes** : Ion trap, focusing, pin-cushion correction

5. Sensors, Electric Signal Transducers

(a) **Transducers Using Permanent Magnets**: Inductive, Hall effect, magnetoresistive, temperature sensitive elements.

(b) **Quantities Measured** : Position, velocity, acceleration, fluid and heat flow, pressure, vibration, temperature etc.

(c) **Use Areas** : Automotive, industrial, aerospace, computer peripherals (keyboards, read/write head sensors), office equipment.

6. Medical Electronics and Bioengineering

(a) **NMR Imaging Devices** : DC field source for MRI tomographs

(b) **Mechanical Prostheses** : Eyelid muscle assist, dental prostheses, stoma seals, valves, heart-assist pumps, artificial limbs.

(c) **Surgical Clamps** : For incisions and severed blood vessels.

(d) **Diagnostic Aids** : Catheters; sensors/transducers.

(e) **Miniature Hearing Aids** : External devices and implants.

7. Miscellaneous Applications

(a) **Magnetic Locks** : Key and cylinder with encoded magnets

(b) **Magnetic Jewellery** : Necklaces, clasps, earrings

(c) **Electronic Choke** : Steady bias field for core

(d) **Magnetic Bubble Memory** : Bias field for bubble element

(e) **Vacuum Technology** : ton getter pumps, vacuum gauges.

Q. 7. Write the basic principle of Energy Conversion and Energy Balance

Ans. Basic Principle of Energy Conversion and Energy Balance

According to it, energy can neither be created nor destroyed, it can merely be converted from one form into another.

In an energy conversion device, out of the total input energy, some energy is converted into the required form, some energy is stored and the rest is dissipated for a motor, it can be written as

$$\begin{pmatrix} \text{Total electrical} \\ \text{energy input} \end{pmatrix} = \begin{pmatrix} \text{Mechanical} \\ \text{energy output} \end{pmatrix} + \begin{pmatrix} \text{Total energy} \\ \text{stored} \end{pmatrix} + \begin{pmatrix} \text{Total energy} \\ \text{dissipated} \end{pmatrix} \quad \dots (2.44)$$

For generator action, it can be written as

$$\begin{pmatrix} \text{Total mechanical} \\ \text{energy input} \end{pmatrix} = \begin{pmatrix} \text{Electrical energy} \\ \text{output} \end{pmatrix} + \begin{pmatrix} \text{Total energy} \\ \text{stored} \end{pmatrix} + \begin{pmatrix} \text{Total energy} \\ \text{dissipated} \end{pmatrix} \quad \dots (2.45)$$

Now the various forms of energy involved in eq. (1.33) can be represented as

W_e = Electrical energy input from the supply means

W_m = Mechanical energy output

Total energy stored in any device = Energy stored in magnetic field, W_{es} + Energy stored in mechanical system, W_{ms} .

Total energy dissipated = Energy dissipated in electric circuit as ohmic losses + Energy dissipated as magnetic core loss (hysteresis and eddy-current losses) + Energy dissipated in mechanical system (friction and windage losses etc.).

Thus the energy balance equation (2.44) can be written in more simplified terms as

$$W_{ei} = W_{mo} + (W_e + W_m) + (\text{Ohmic energy losses} + \text{Coupling field energy losses}) + (\text{Energy losses in mechanical system}).$$

The subscripts e , m , i , s and o stand for electrical, mechanical, input, stored and output respectively. For example, subscript ei denotes electrical input (energy), subscript ms denotes mechanical stored (energy).

If the appropriate terms are grouped together, then the energy balance equation becomes,

$$(W_{ei} - \text{Ohmic energy losses}) = (W_{mo} + W_{ms} + \text{Mechanical energy losses}) + (W_{es} + \text{Coupling field energy losses}) \quad \dots (2.46)$$

$$W_{elec} = W_{mech} + W_{fd} \quad \dots (2.47)$$

Equation (2.46) leads to the electromechanical energy conversion model.

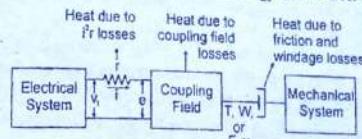


Fig. 2.15 : General Representation of Electromechanical Energy Conversion Device

For the losses conversion system of Fig. 2.15 equation (2.47) can be written in differential form as

Where

- $dW_{elec} = dW_{mech} + dW_{fd}$
- dW_{elec} = Differential electrical energy input to coupling field
- dW_{mech} = Differential mechanical energy output
- dW_{fd} = Differential change in energy stored in the coupling field

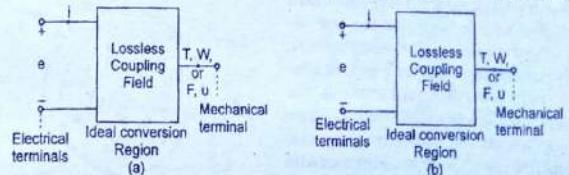


Fig. 2.16 : Representation of Lossless Electromechanical Energy Conversion Device
(a) Motoring Mode and (b) Generating Mode

From Fig. 2.16, the differential electrical energy input in time dt is

$$dW_{ei} = v_i i dt \quad \dots (2.48)$$

Ohmic loss in resistance r in time dt is $i^2 r dt$.

\therefore Differential electrical energy input to the coupling field,

$$\begin{aligned} dW_{elec} &= dW_{ei} - \text{ohmic loss} \\ &= (v_i - ir)i dt = ei dt \end{aligned} \quad \dots (2.49)$$

Equation (2.49) now become

$$ei dt = dW_{mech} + dW_{fd} \quad \dots (2.50)$$

Energy balance equation (2.50) is obtained by applying the principle of conservation of energy to the motoring mode. This equation (2.50) alongwith Faraday's law of induced e.m.f., $\epsilon = -\frac{d\psi}{dt}$, forms the fundamental basis for the analysis of energy-conversion devices.

COUPLING FIELD REACTION

The energy stored in the coupling field must produce action and reaction on the electrical and mechanical system for the conversion of energy.

- (i) In a motor, this reaction is the counter e.m.f. 'e', the coupling field extracts energy proportional to e.i. from the electrical system, converts and delivers energy proportional to T.Wr (or fall) to the mechanical system.
- (ii) In generator, this reaction is the counter torque, opposite to the applied mechanical torque of the prime mover, thus the coupling field extracts mechanical energy proportional to (creation torques V_s speed) from the mechanical system, converts and deliver it as electrical energy proportional to (e.i.) to the electrical system.

Thus it may be seen that coupling field serve as the energy conversion region.

BASIC ASPECTS AND PHYSICAL PHENOMENA INVOLVED IN DOUBLY EXCITED MAGNETIC SYSTEM

Fig. 2.15 illustrates a simple model of a doubly excited magnetic system. This model consists of stator iron, rotor iron and both are of the salient pole type. The stator with N_s turns is energised from source 1 and the rotor with N_r turns is excited from source 2. The m.m.f.s produced by both the stator and rotor windings are in the same direction and magnetic torque T_e is in the anticlockwise direction as shown in Fig. 2.17.

Therefore, the differential electrical energy input dW_{elec} from two energy sources 1 and 2, in Fig. 2.17, is

$$dW_{elec} = i_s d\psi_s + i_r d\psi_r \quad \dots (2.51)$$

Here ψ_s and ψ_r are the instantaneous total flux linkages of stator and rotor windings respectively. Since the magnetic saturation is neglected, ψ_s and ψ_r can be expressed in terms of self and mutual inductances.

$$\therefore \psi_s = L_s i_s + M_{sr} i_r$$

and

$$\psi_r = L_r i_r + M_{rs} i_s$$

where

L_s = Self-inductance of stator winding

L_r = Self-inductance of rotor winding

and

M_{rs} = Mutual inductance between stator & rotor windings

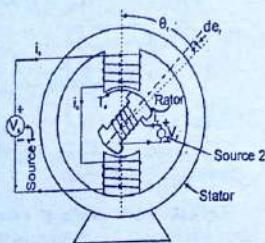


Fig. 2.17 : Doubly-excited Magnetic System

In Fig. 2.17, initially the space angle between rotor and stator axes is θ_r and both the currents i_s and i_r are assumed zeros. Now the stator and rotor coils are switched on to their respective energy sources, so that the currents rise from zero to i_s and i_r respectively. If the rotor is not allowed to move, the dW_{mech} is zero and energy balance equation because

$$dW_{elec} = 0 + dW_{fd}$$

Thus, with the rotor held fixed, all the electric energy supplied by the two supply sources, is stored in the magnetic field.

$$\text{From equation 2.51 } dW_{fd} = dW_{elec} = i_s d\psi_s + i_r d\psi_r$$

$$= i_s d(L_s i_s + M_{sr} i_r) + i_r d(L_r i_r + M_{rs} i_s) \quad \dots (2.53)$$

Since the rotor is not allowed to move, the reluctances and therefore the inductances are constant. In view of this, the differential changes in inductances, i.e. dL_s , dL_r and dM_{sr} in equation 2.53 are all zeros.

Therefore, from equation 2.53

$$dW_{fd} = i_s L_s d i_s + i_s M_{sr} d i_r + i_r L_r d i_r + i_r M_{sr} d i_s$$

$$= i_s L_s d i_s + i_r L_r d i_r + M_{sr} d(i_s i_r)$$

The magnetic field energy stored in establishing the currents from zero to i_s and i_r is given by

$$W_{fd} = L_s \int_0^{i_s} i_s d i_s + L_r \int_0^{i_r} i_r d i_r + M_{sr} \int_0^{i_s i_r} d(i_s i_r)$$

$$= \frac{1}{2} i_s^2 L_s + \frac{1}{2} i_r^2 L_r + M_{sr} i_s i_r \quad \dots (2.54)$$

For obtaining the magnetic torque T_e , assume the rotor to move through virtual displacement $d\theta_r$ in the direction of T_e as shown in Fig. 2.17 now as the rotor moves, the reluctance and inductance varies

So the parameter dW_{elec} and θ dW_{fd} changes and

$$dW_{mech} = T_e d\theta_r$$

Substituting of the values of dW_{elec} , dW_{mech} and dW_{fd} as energy balance equation gives

$$\frac{1}{2} i_s^2 dL_s + \frac{1}{2} i_r^2 dL_r + i_s i_r dM_{sr} = T_e d\theta_r$$

$$\text{So } T_e = \frac{1}{2} I_s^2 \frac{dL_s}{d\theta_r} + \frac{1}{2} I_r^2 \frac{dL_r}{d\theta_r} + i_s i_r \frac{dM_{sr}}{d\theta_r} \quad \dots (2.55)$$

Imp: The torque T_e depends on

- (a) The instantaneous values of current i_s and i_r
- (b) The angular rate of change of inductance.

This equation above reveals some important results.

The north, south poles produced on stator by i_{s1} and south, north poles produced on rotor by i_{r1} , attract each other tending to align their fields, the torque to be developed by the stator and rotor magnetic fields is the electromagnetic or interaction torque it should be noted that the reluctance torque $\left(\frac{1}{2} i_s^2 \frac{dL_s}{d\theta_r} \text{ or } \frac{1}{2} i_r^2 \frac{dL_r}{d\theta_s}\right)$ does not depend on the direction

of current in stator or rotor winding but the inter action torque $\left(i_s i_r \frac{dM_{12}}{d\theta_r}\right)$ does depend on the direction of current i_s and i_r .

Example 2.6 : A dynamometer type moving coil instrument can be represented by two coils as shown in Fig. 2.18. The self and mutual inductances of the two coils are $L_1 = 0.20 \text{ mH}$.

$$L_2 = 0.15 \text{ mH and}$$

$$M_{12} = 0.08 \sin \theta \text{ mH.}$$

The fixed and moving coils are connected in series and the current flowing through them is $I_m \sin \omega t$. The controlling torque produced by the helical restraining spring varies in direct proportion to the deflection angle θ and its value is 0.0144 N-m at $\theta = 90^\circ$.

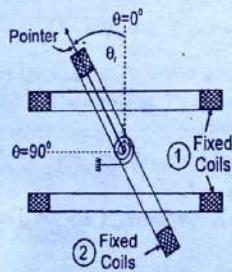


Fig. 2.18 : Dynamometer Type Moving Coil Instrument

- (i) Find an expression for torque in terms of the angular position θ .
- (ii) Find the time average torque in terms of θ .
- (iii) For $I = 5, 10$ and 15 amperes, sketch the variation of average torque with angle θ .

Solution : (i) The magnetic torque is given by

$$T_e = \frac{1}{2} i_1^2 \frac{dL_1}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_2}{d\theta} + i_1 i_2 \frac{dM_{12}}{d\theta}$$

Here $i_1 = i_2 = i_m \sin \omega t$, because the two coils are connected in series.

$$\begin{aligned} T_e &= 0 + (I_m \sin \omega t)^2 \frac{d}{d\theta} (0.08 \sin \theta) 10^{-3} \\ &= 0.8 \times 10^{-4} I_m^2 \sin^2 \omega t \cos \theta \text{ N-m} \end{aligned} \quad \text{Ans.}$$

$$(ii) T_e = 0.8 \times 10^{-4} I_m^2 \left(\frac{1 - \cos 2\omega t}{2} \right) \cos \theta$$

$$\therefore T_{e(\text{av})} = 0.4 \times 10^{-4} I_m^2 \cos \theta \text{ N-m} \quad \text{Ans.}$$

(iii) For $I = 5 \text{ A rms}$,

$$T_{e(\text{av})} = (0.4 \times 10^{-4}) (\sqrt{2} \times 5)^2 \cos \theta = 2 \times 10^{-3} \cos \theta \text{ N-m}$$

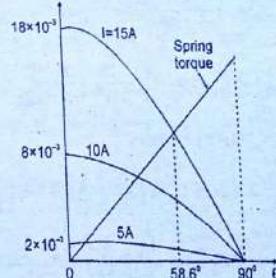


Fig. 2.19 : Variation of torque $T_{e(\text{av})}$ with θ

$$\text{For } I = 10 \text{ A rms}, T_{e(\text{av})} = 8 \times 10^{-3} \cos \theta \text{ N-m}$$

$$\text{For } I = 15 \text{ A rms}, T_{e(\text{av})} = 18 \times 10^{-3} \cos \theta \text{ N-m} \quad \text{Ans.}$$

The variation of $T_{e(\text{av})}$ with different values of θ is shown in Fig. 2.17.

Example 2.7 : Self and mutual-inductances in henrys of two coupled coils are

$$L_1 = 3 + \frac{2}{2x}, L_2 = 2 + \frac{1}{2x}, M_{12} = M_{21} = \frac{1}{2x}$$

over a certain displacement x in metres. The coil resistances are negligible.

For constant currents of $I_1 = 10 \text{ A}$ and $I_2 = -5 \text{ A}$, compute

- The mechanical work done in increasing x from 0.5 to 1m,
- The energy supplied by each electrical source in part (i),
- Change in field energy in part (i).

Hence verify that sum of the energies associated with mechanical work done & field energy is equal to the energy supplied by both sources during the motion from $x = 0.5$ to $x = 1 \text{ m}$.

Solution: For a linear case

$$\begin{aligned} W_{fd}(i_1, i_2, x) &= \frac{1}{2} L_1 i_1^2 + i_1 + \frac{1}{2} L_2 i_2^2 M \\ &= \frac{1}{2} \left(3 + \frac{1}{2x} \right) 100 + \frac{1}{2} \left(2 + \frac{1}{2x} \right) 25 + (-50) \left(\frac{1}{2x} \right) = 175 + \frac{25}{4x} \end{aligned}$$

$$(i) f_e = \frac{\partial W_{fd}(i_1, i_2, x)}{\partial x} = \frac{-25}{4x^2}$$

$$\therefore W_{mech} = \int_{0.5}^{1.0} f_e dx = \int_{0.5}^{1.0} \frac{-25}{4x^2} dx = \frac{25}{4} \left| \frac{1}{x} \right|_{0.5}^1 = -\frac{25}{4} \text{ Watt-sec} \quad \text{Ans.}$$

$$(ii) dW_{elec} = i_1 d\psi_1 + i_2 d\psi_2$$

$$\text{Here } \psi_1 = L_1 i_1 + M i_2 = \left(3 + \frac{1}{2x} \right) 10 + (-5) \left(\frac{1}{2x} \right) = 30 + \frac{5}{2x}$$

$$\psi_2 = L_2 i_2 + M i_1 = \left(2 + \frac{1}{2x} \right) (-5) + \frac{1}{2x} (10) = -10 + \frac{5}{2x}$$

$$\text{Energy supplied by source 1, } W_{elec1} = \int_{\psi_1(x=0.5)}^{\psi_1(x=1.0)} i_1 d\psi_1$$

$$= 10 \left[\left(30 + \frac{5}{2x} \right) \Big|_{x=1.0} - \left(30 + \frac{5}{2x} \right) \Big|_{x=0.5} \right] = -25 \text{ Watt-sec}$$

Similarly, energy supplied by source 2 is

$$W_{elec2} = \int_{\psi_2(x=0.5)}^{\psi_2(x=1.0)} i_2 d\psi_2$$

$$= (-5) \left[\left(-10 + \frac{5}{2x} \right) \Big|_{x=1.0} - \left(-10 + \frac{5}{2x} \right) \Big|_{x=0.5} \right] = 12.5 \text{ Watt-sec}$$



TRANSFORMER

Q. 1. Define a transformer. How is the energy transferred from one circuit to the other?

Ans. A transformer is a static device which transfers electrical energy from one circuit to the other circuit without change in frequency. During this transfer, the voltage level may increase or decrease with a corresponding decrease or increase in current.

It consists of two windings, a primary winding and a secondary winding. The primary winding is connected across an A.C. source from which the input is taken. The energy transfer from one circuit to other circuit takes place with the help of a principle called electromagnetic induction. According to this principle, whenever two coils are mutually coupled to each other and the current in one of the coils is changed uniformly, then an e.m.f. is induced in the second coil. The e.m.f. induced in the second coil can be effectively utilized to deliver the power to any load connected across it.

Q. 2. Distinguish between step-up and step-down transformer.

Ans. Differences between Step-up and Step-down Transformer

Step-up Transformer

- Number of turns in the secondary winding are greater than the number of turns in primary winding i.e., $N_s > N_p$.
- In a step up transformer the voltage produced across the secondary winding is greater than the input voltage present across the primary.
- The secondary current is lesser than the primary current i.e., $I_s < I_p$.
- It finds its application in electrical power transmission systems.

Step-down Transformer

- Number of turns in secondary winding are lesser than the number of turns in primary winding i.e., $N_s < N_p$.
- In a step down transformer the voltage produced across the secondary winding is always lesser than the input voltage present across the primary.
- The secondary current is greater than the primary current i.e., $I_s > I_p$.
- It finds its application in electrical power distribution systems.

Q. 3. Explain the working principle of transformer.

Ans. Working Principle of Transformer :

Machine-1

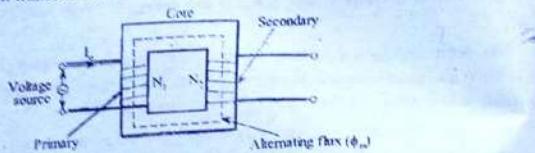
[3-2]

Transformer

The working principle of transformer is based on mutual induction between two coupled coils. According to this principle, a changing flux creates an induced e.m.f. in each turn equal to the derivative of the flux so that the total induced e.m.f. across N turns is,

$$E = \frac{N d\phi}{dt}$$

This can be seen in transformer shown in figure below.



Figure

Transformer core is wound with two windings having N_1 and N_2 number of turns. The winding with N_1 turns is known as primary as it is fed by alternating voltage source. This voltage source also forms a closed path in the primary. So that alternating currents starts flowing through the primary.

Alternating flux set-up by the alternating m.m.f. ($I_0 N_1$), creates an induced e.m.f. E_1 in the primary. During the path completion of the alternating flux through the core, induced e.m.f.s are also created in other than the primary winding. The winding in which e.m.f.s are induced mutually is known as secondary.

Q. 4. Why transformer rating is expressed in terms of kVA?

Ans. In transformers, the copper loss mainly depends on current and that of iron loss depends on voltage and moreover, the losses that occur in transformer are independent of power factor i.e., the losses in transformer does not depend on load power factor. Hence, the reason, why the transformer rating is mention in kVA instead of kW.

Q. 5. Explain the effect of change in load on copper losses of a transformer.

Ans. The copper losses are the heat loss occurring due to resistance of the other conductor of the winding. It is also called the ohmic loss. If I_1 , R_1 , I_2 and R_2 are the primary current, primary winding resistance, secondary current and secondary winding resistance respectively, then,

$$\text{Heat loss due to primary winding} = I_1^2 R_1 \text{ and}$$

$$\text{Heat loss due to secondary winding} = I_2^2 R_2$$

$$\text{Total heat loss in transformer} = \text{Copper loss} = I_1^2 R_1 + I_2^2 R_2$$

The copper loss thus vary to the square of the currents. The currents vary with change in kVA load on transformer. Hence the copper losses are affected by load on the transformer.

Machine-1

[3-3]

Transformer

For a given load the copper losses will be $\left(\frac{1}{n}\right)^2 P_c$.

Where, $\left(\frac{1}{n}\right)$ is the fraction of full load capacity of transformer at which the transformer is operated.

P_c is the copper loss at full load capacity of transformer.

Q. 6. Define efficiency of a transformer.

Ans. It is defined as the ratio of transformer output to its input i.e.

$$\eta = \frac{\text{Output of the transformer}}{\text{Input of the transformer}} \times 100$$

Input of the transformer can also be expressed as the sum of the output and losses. Thus efficiency of a transformer can also be written as,

$$\eta = \frac{\text{Output power}}{\text{Output power} + \text{Losses}} \times 100$$

Where, Output, input and losses are in watts or in kilowatts.

Q. 7. In detail, explain the classification of transformer.

Ans. Transformers are mainly classified into two types, depending upon the type of construction used. They are :

1. Core type transformer

2. Shell type transformer.

1. Core Type Transformer :

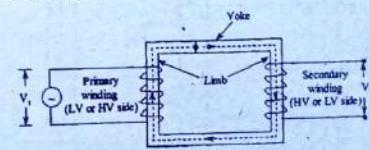


Figure (1)

The schematic representation of core type transformer is shown in figure (1). The core of this transformer is rectangular in shape and consists of two limbs. Cylindrical type winding coils are used in this type of transformer. The primary and secondary windings are both wound on different limbs as shown in figure (1). Here, the winding surrounds a considerable part of the core. There is only one path for the flux to flow which is through the yoke. Hence, core type transformers have only one magnetic circuit.

Machine-I

[3-4]

Transformer

Even though, most of the flux is confined to high permeability core of the transformer there is always some flux leaking through the yoke and non-magnetic materials surrounding the core. This small amount of flux is known as leakage flux. It should always be kept in mind that this leakage flux has to be minimized as soon as possible so that the performance of transformer can improve. Hence in practical condition instead of placing the windings of a core transformer on different limbs, they are interleaved as shown in figure (2), i.e., half of the primary winding and half of the secondary winding has been placed concentrically on each limb due to which the leakage flux reduces.

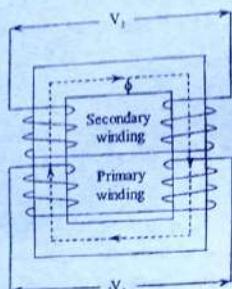


Figure (2)

For remaining answer refer Unit-IV, Q. 11.

2. Shell Type Transformer

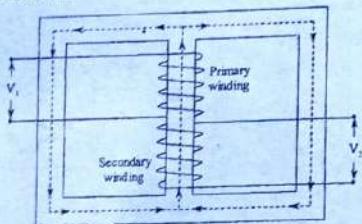


Figure (3)

The schematic representation of shell type of transformer is shown in figure (3). The core of the transformer consists of three limbs. Sandwich type or multilayer type windings are used in this type of transformer. The primary and secondary windings are both wound on the same limb i.e., the central limb. Hence the core surrounds a considerable portion of the windings. The flux in the transformer divides equally in the central limb and flows through the outer two limbs. Therefore,

Machine-I

[3-5]

Transformer

the shell type transformer have two magnetic circuits. In actual construction, in order to reduce the leakage flux, the low voltage and high voltage windings are interleaved on the central limb as shown in figure (4).

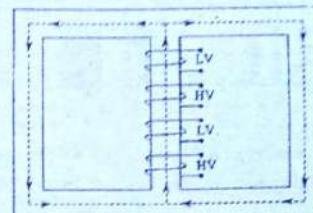


Figure (4)

Q. 8. Discuss the components of a transformer ?

Ans. Construction of a Transformer :

The main components of a transformer are,

- | | |
|---------------------|---------------|
| 1. Core | 2. Windings |
| 3. Conservator tank | 4. Bushings |
| 5. Breather and | 6. Radiators. |

1. Core of a Transformer : The core of a transformer is made up of sheet steel doped with 4% of silicon, which is also called as silicon steel. Thin silicon steel sheets of thickness 0.35 mm to 5 mm depending upon the requirement are cut into particular sizes and shapes are stocked together to form the magnetic core. Since, the core is cut by the alternating flux produced by the flow of alternating currents, this induces an e.m.f. in core, which leads to the flow of eddy currents. This eddy current circulates in the core and gives rise to I²R loss in the core. To reduce the eddy current loss caused by the flow of eddy current, the core is built-up of laminations.

The purpose of the core is to provide low reluctance path between the two windings, so that the flux caused by one winding will fully link the other coil.

2. Windings : A single phase transformer has two windings. The winding connected to the A.C source is called primary winding and the one connected to load is called secondary winding. The alternating voltage whose magnitude is to be changed is applied to primary. The primary and secondary windings consists of a series of turns called coils, which wound round the core. The coils of transformer are of two types. They are :

- (i) Concentric coils and
- (ii) Sandwiched coils.

(i) Concentric Coil : Generally, the concentric windings are used in core type transformers. The whole length of the transformer core limb is concentrically wound with the high voltage

winding and on high voltage winding, low voltage winding is placed concentrically as shown in the figure (1).

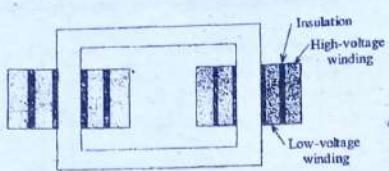


Fig. (1) : Concentric Coil Winding of Two Winding Transformer

The low voltage winding is placed close to the core of a transformer for the reason that the insulation involved is less.

(ii) Sandwiched Coil : Sandwiched coils are used in shell type transformers. These type of transformers consists of series of flat coils with a high voltage winding sandwiched between low voltage windings and the low voltage windings occupy the ends to reduce the insulation as shown in the figure (2).

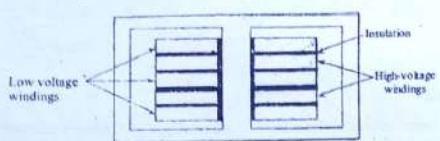


Fig. (2) : Sandwiched Coil Windings of a Two Winding Transformer

3. Conservator Tank : The oil in the transformer main tank is subjected to expand and contract due to the variations in load current. While undergoing expansion and contraction, the oil is subjected to heat. The function of the conservator tank is to help the oil in the tank to settle down by expansion whenever heavy loads appears.

Without a conservator tank, the main tank may burst out, because of the high pressures developed inside the tank.

4. Bushings : Connections from the transformer windings are brought out by means of bushings. The function of the bushings is to give proper insulation for the output leads. Bushings are fixed on the transformer tank. Bushing made up of porcelain are available and can be used up to 33 kV. Capacitor and oil filled type of bushing are used for voltage above 33 kV.

5. Breather : The main transformer tank and some portion of the conservator tank are filled with oil. This oil should not be exposed to the atmosphere directly, because it may absorb the moisture and dust and may loose its electrical properties within a very short time. A breather is provided in order to avoid this.

Breather completely prevents the outside atmospheric moisture and dust from coming into

contact with oil.

6. Radiators : These metal structures are connected around the transformer tank which acts as a heat sink. The function of the radiators is to cool the transformer tank gradually.

Working Principle of Transformer

For answer refer Unit-IV, Q. 3.

Q. 9. Explain the functions of the following in a transformer

- (i) Breather (ii) Conservator
- (iii) Oil.

Ans.

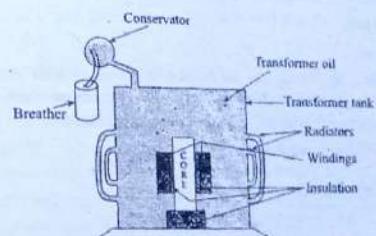
(i) Breather : When the temperature changes, the transformer oil expands or contracts and there is a displacement of air. When the transformer gets cooled, the level of oil goes down and air is drawn in. This is known as breathing. The air coming in is passed through an apparatus known as breather for the purpose of extracting moisture.

Breather consists of calcium chloride or silica gel by which, moisture in the air is absorbed. Calcium chloride or silica gel is in reddish brown colour but after absorbing the moisture changes to bluish colour.

(ii) Conservator : The oil should not come in contact with atmospheric air as it may take up moisture which can spoil its insulating properties. Also air may cause acidity and sludging of oil. To prevent this, transformers are provided with conservators. The main function of conservator is to take-up expansion and contraction of oil without allowing it to come in contact with outside air.

(iii) Oil : It serves the following purposes.

1. Provides additional insulation,



Figure

2. Carries away the heat generated in coils and the core.

3. Protects the paper from dust and moisture.

Oil should be free from dust, small water particles and fibres and also from high viscosity.

Machine-1

[3-8]

Transformer

Transformer oil must possess the property of minimum sludge formation after long periods of heating in the presence of oxygen.

Q. 10. Explain why hysteresis and eddy current losses occur in a transformer. What are the methods to reduce these losses ?

Or,

What are the various losses taking place in transformer ? How these losses can be minimized ?

Ans. Losses in a Transformer :

Transformer, being a stationary electrical equipment, does not have rotational or windage losses. The losses which occur in a transformer are,

(i) Copper losses and

(ii) Iron or core losses.

(i) Copper Losses : These losses occur due to the resistance of the transformer windings and appear as heat resulting in the increase of temperature. If R_1 and R_2 are the primary and secondary resistances and I_1 and I_2 are the primary and secondary currents, then the copper loss in the respective windings is given by,

Copper loss in primary = $I_1^2 R_1$ and

Copper loss in secondary = $I_2^2 R_2$

The total copper loss in a transformer is given by,

Total copper loss = $(I_1^2 R_1 + I_2^2 R_2)$

(ii) Iron or Core Loss : Iron (or) core losses occurs when the core is subjected to alternating flux. These losses are further divided into two types, namely,

(a) Hysteresis loss and

(b) Eddy-current loss.

(a) Hysteresis Loss : Hysteresis loss occurs due to the continuous magnetization and demagnetization of the core. In every cycle of alternating flux, i.e., magnetization, the hysteresis loss appears as heat. Hysteresis loss exists by virtue of property of material.

Hysteresis loss is given by the relation,

$$W_{hyp} = K_{hyp} V f B_{max}^2 \text{ Watts}$$

Where,

K_{hyp} = Constant

f = Frequency

V = Volume of magnetic core

B_{max} = Maximum value of flux density

[3-8]

Machine-1

[3-9]

Transformer

x = Value of x varies from 1.5-2.0.
Usually taken as 1.6

Hysteresis losses can not be eliminated. However, they can be minimized by selecting the core material which possess slow hysteresis coefficient. For example, silicon steel and nickel, iron alloys.

(b) Eddy Current Loss : Alternating flux linking with the core, induces e.m.f. in it and causes circulating currents, known as eddy current to flow within the body of the transformer core. These eddy currents leads to the power loss given by PR and are also called as eddy current losses. These losses depends upon the length of the eddy current path and the effective resistance

Eddy current losses are given by the relation,

$$W_{eddy} = K_{eddy} f^2 B_{max}^2 V \text{ Watts}$$

Where,

K_{eddy} = Constant

f = Frequency

B_{max} = Maximum value of flux density

V = Volume of the core

d = Thickness of the lamination.

Similar to the hysteresis losses, it is impossible to eliminate eddy-current losses. However, they can be reduced by using laminations of thickness suitable to the capacity of the machine. Using laminations, effective resistance to the eddy currents can be increased and length of the circulating current path is decreased, thereby reducing the eddy-current losses.

In transformers, iron or core losses plays a vital role in deciding the equipment rating, temperature rise and efficiency.

Q.11. (a) Develop the exact equivalent circuit of a single-phase transformer and draw phasor diagram of equivalent circuit for lagging power factor.

Ans. : Equivalent Circuit of a Transformer :- The equivalent circuit for any electrical engineering device can be drawn if the equations describing its behaviour are known. If any electrical device is to be analysed and investigated further for suitable modifications, its appropriate equivalent circuit is necessary. The equivalent circuit for electromagnetic devices consists of a combination of resistances, inductances, capacitances, voltages etc. Such an equivalent circuit (or circuit model) can, therefore, be analysed and studied easily by the direct application of electric circuit theory.

In the equivalent circuit, $(r_1 + jx_1)$ and $(r_2 + jx_2)$ are the leakage impedances of the primary and secondary windings respectively. The voltage V_1 is treated as a voltage drop in the direction of I_1 . Recall that the magnitude of V_1 does not change appreciably from no load to full load.

Machine-1 [3-10] Transformer
full load in large transformers. The magnitude of V_i depends on f , N_1 and ϕ_{\max} , since $|V_i| = |E_1|$.

The primary current I_1 consists of two components. One component I_1' is the load component and counteracts the secondary m.m.f. I_2 completely. The other component is exciting current I_e which is composed of I_c and I_m . The current I_e is in phase with V_i in Fig and product $V_i I_e$ gives core loss. The resistance R_e in parallel with V_i represents the core loss P_e such that

$$P_e = I_e^2 R_e = V_i I_e = \frac{(V_i)^2}{R_e} \quad \text{.....(i)}$$

and $R_e = \frac{V_i}{I_e}$

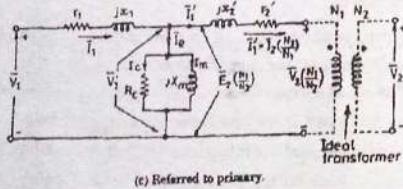
The current I_m lags V_i by 90° and this can, therefore, be represented in the equivalent circuit by a reactance X_m such that

$$X_m = \frac{V_i}{I_m} \quad \text{.....(ii)}$$

R_e and X_m are shown in Fig, which is the exact equivalent circuit of a transformer. The resistance R_e and reactance X_m are called core-loss resistance and magnetizing reactance respectively.

For minor changes in supply voltage and frequency, which is common under normal operation, R_e and X_m are treated constant.

In fig (a) and (b), the ideal transformer has been introduced to show the transformation of voltage and current between primary and secondary windings. Even at this stage, the transformer magnetization curve is assumed linear, since the effect of higher order harmonics can't be represented in the equivalent circuit.



In transformer analysis, it is usual to transfer the secondary quantities to primary side or primary quantities to secondary side. Secondary resistance drop $I_2 r_2$ when transferred to primary side, must be multiplied by the turns ratio N_1/N_2 .

Secondary resistance drop, when transferred to primary =

$$(I_2 r_2) \frac{N_1}{N_2}$$

Machine-1 [3-11] Transformer

$$\begin{aligned} &= \left(I_1 \cdot \frac{N_1}{N_2} \cdot r_2 \right) N_1 \\ &= I_1 \left[\left(\frac{N_1}{N_2} \right)^2 r_2 \right] = I_1 r_2' \end{aligned}$$

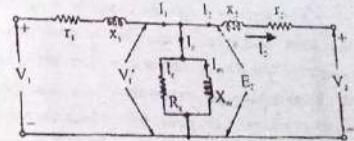
where $r_2' = r_2 \left(\frac{N_1}{N_2} \right)^2$

If resistance r_2 is placed in the primary circuit, then the relation between voltages V_1 and V_2 is unaffected. This resistance r_2 is called the secondary resistance referred to primary. Therefore, the total resistance in the primary circuit is

$$r_{e1} = r_2 + r_1 \left(\frac{N_1}{N_2} \right)^2 = r_2 + r_1' \quad \text{.....(iii)}$$

Hence r_{e1} is called the transformer equivalent (or total) resistance referred to primary winding. Similarly the primary resistance referred to secondary is $r_1 \left(\frac{N_2}{N_1} \right)^2$ and the equivalent (or total) resistance referred to secondary is

$$r_{e2} = r_2 + r_1 \left(\frac{N_2}{N_1} \right)^2 = r_2 + r_1' \quad \text{.....(iv)}$$



Equivalent circuit in a general form.

Secondary leakage reactance drop $I_2 x_2$, when transferred to primary is

$$I_2 x_2 \left(\frac{N_1}{N_2} \right) = I_1 \left(\frac{N_1}{N_2} \right)^2 x_2 = I_1 x_2'$$

The quantity x_2' is called the equivalent or total leakage reactance referred to primary. Likewise, the equivalent or total leakage reactance referred to secondary is

$$x_{e2} = x_2 + x_1 \left(\frac{N_2}{N_1} \right)^2 = x_2 + x_1' \quad \text{.....(v)}$$

The equivalent (or total) leakage impedance referred to primary is

$$z_{e1} = r_{e1} + j x_{e1}$$

The equivalent (or total) leakage impedance referred to secondary is

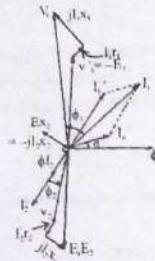
$$z_{e2} = r_{e2} + j x_{e2}$$

Machine-I

[3-12]

Following the above procedure, it can be shown that

$$z_{e1} = \left(\frac{N_1}{N_2} \right)^2 z_{e2} \quad \text{and} \quad z_{e2} = \left(\frac{N_2}{N_1} \right)^2 z_{e1}$$



Q.12. Discuss briefly the essential and desirable conditions to be fulfilled for operating two three-phase transformers in parallel. Draw schematically how a 3-φ transformer can be phased in with another 3-φ transformer.

Ans. : Parallel Operation of Single-phase Transformers :- When electric power is supplied to a locality, city or an area, a single transformer, capable of handling the required power demand, is installed. In some cases, it may be preferable to install two or more transformers in parallel, instead of one large unit. Though two or more transformers may be expensive than one large unit, yet this scheme possesses certain advantages described below.

(i) With two or more transformers, the power system becomes more reliable. For instance if one transformer develops fault, it can be removed and the other transformers can maintain the flow of power, though at a reduced level.

(ii) Transformers can be switched off or on, depending upon the power demand. In this manner, the transformer losses decreases and the system becomes more economical and efficient in operation.

(iii) The cost of a standby (or spare) unit is much less when two or more transformers are installed.

In any case, with the passage of time, electric power demand may become more than the rated kVA capacity of the already existing transformer or transformers. Under such circumstances, the need for extra transformer arises. Since the supply voltage has to remain constant, the extra unit must be connected in parallel.

Note that the parallel operation of transformers requires that their primary windings, as well as their secondary windings, are connected in parallel. In this article, only the parallel operation of single-phase transformers is considered.

Machine-II

[3-13]

Transformer

The various conditions which must be fulfilled for the satisfactory parallel operation of two or more single-phase transformers, are as follows :

(a) The transformers must have the same voltage ratios, i.e., with the primaries connected to the same voltage source, the secondary voltages of all transformers should be equal in magnitude.

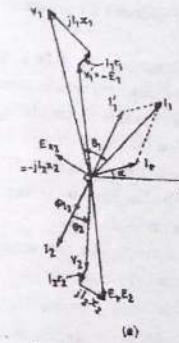
(b) The equivalent leakage impedances in ohms should be inversely proportional to their respective kVA ratings. In other words, the per unit leakage impedances of the transformers based on their own kVA ratings must be equal.

(c) The ratio of equivalent leakage reactance to equivalent resistance, i.e., x_e/r , should be same for all the transformers.

(d) The transformers must be connected properly, so far as their polarities are concerned.

Q.13(a). Develop the phasor diagram of a single-phase transformer under lagging power-factor and leading power-factor loads.

Ans.



Transformer phasor diagram for
(a) lagging p.f. load

(b) Find condition for maximum efficiency of a transformer.

Ans. : Condition for maximum efficiency :- In Eq., P_c is constant and the load voltage V_2 remains practically constant. At a specified value of load $p.f. \cos \theta_2$, the efficiency will be

maximum when $\frac{d\eta}{dI_2} = 0$. Therefore, $\frac{d\eta}{dI_2}$ for Eq. is

$$\frac{d\eta}{dI_2} = \frac{\left[(V_2 I_2 \cos \theta_2 + P_c + I_2^2 r_{e2}) (V_2 \cos \theta_2) \right] - V_2 I_2 \cos \theta_2 (V_2 \cos \theta_2 + 2I_2 r_{e2})}{\left[V_2 I_2 \cos \theta_2 + P_c + I_2^2 r_{e2} \right]^2} = 0$$

Machine-1 [3-14] Transformer

$$\text{or } \left\{ V_1 I_1 \cos \theta_1 + P_c + I_1^2 R_{CL} \right\} V_2 \cos \theta_2 \\ = V_2 I_2 \cos \theta_2 (V_1 \cos \theta_1 + 2I_1^2 \alpha) \quad \text{or} \quad I_2^2 R_{CL} = P_c$$

or Variable ohmic loss, $I_2^2 R_{CL}$ = Constant core loss, P_c

(c) State the necessary conditions for satisfactory operation of two transformers in parallel. State briefly why all transformers cannot be operated in parallel.

Ans. : The various conditions which must be fulfilled for the satisfactory parallel operation of two or more single-phase transformers, are as follows :

(a) The transformers must have the same voltage ratios, i.e., with the primaries connected to the same voltage source, the secondary voltages of all transformers should be equal in magnitude.

(b) The equivalent leakage impedances in ohms should be inversely proportional to their respective kVA ratings. In other words, the per unit leakage impedances of the transformers based on their own kVA ratings must be equal.

(c) The ratio of equivalent leakage reactance to equivalent resistance, i.e., x_e/r_s , should be same for all the transformers.

(d) The transformers must be connected properly, so far as their polarities are concerned.

Out of the conditions listed above, condition (d) must be strictly fulfilled. If the secondary terminals are connected with wrong polarities, large circulating currents will flow and the transformers may get damaged. Condition (a) should be satisfied as accurately as possible since different secondary voltages would give rise to undesired circulating currents. For conditions (b) and (c) some deviation is permissible. Thus the fulfilment of condition (d) is essential, whereas the fulfilment of other conditions is desirable.

Fig shows two single-phase transformers in parallel, connected to the same voltage source on the primary side. Terminals with proper polarity markings have been connected both on the h.v and l.v sides. A further check on the polarities can be applied by connecting a voltmeter V in series with the two secondaries. Zero voltmeter reading indicates proper polarities. If voltmeter reads the sum of the two secondary voltages, the polarities are improper and can be corrected by reversing the secondary terminals of any one transformer.

Q.14.(a) Discuss No Load Test and Short Circuit Test on single phase transformer.

Ans.: No Load Test : The circuit diagram for performing open circuit test on a single phase transformer is given in fig.(a). In this diagram, a voltmeter, wattmeter and an ammeter are shown connected on the low voltage side of the transformer. The high voltage side is left open circuited. The rated frequency voltage applied to the primary, i.e. low voltage side, is varied with the help of a variable ratio auto-transform. When the voltmeter reading is equal to the rated voltage of the l.v. winding, all the three instrument reading are recorded.

Machine-1 [3-15] Transformer

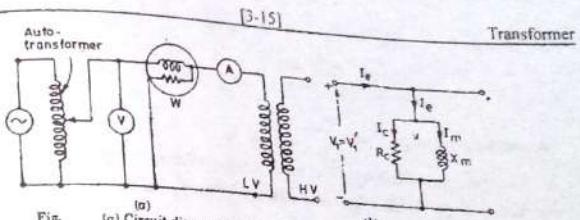


Fig. (a) Circuit diagram for open-circuit test on a transformer and (b) approximate equivalent circuit at no load.

The ammeter records the no-load current or exciting current I_e . Since I_e is quite small (2 to 6% of rated current), the primary leakage impedance drop is almost negligible, and for all practical purposes, the applied voltage V_1 is equal to the induced e.m.f. V_1' . Consequently, the equivalent circuit of fig.(e) gets modified to that shown in fig.(b).

The input power given by the wattmeter reading consists of core loss and ohmic loss. The exciting current being about 2 to 6 per cent of the full load current, the ohmic loss in the primary ($= I_e^2 R_p$) varies from 0.04 per cent ($\frac{2}{100} \times \frac{2}{100} \times 100$) to 0.36 per cent of the full-load primary ohmic loss. In view of this fact, the ohmic loss during open circuit test is negligible in comparison with the normal core loss. Hence the wattmeter reading can be taken as equal to transformer core loss.

A negligible amount of dielectric loss may also exist. Error in the instrument readings may be eliminated if required. Let

$$V_1 = \text{applied rated voltage on l.t. side}, \\ I_e = \text{exciting current (or no-load current)}, \\ \text{and } P_c = \text{core loss}.$$

$$\text{Then } P_c = V_1 I_e \cos \theta_0$$

$$\therefore \text{No load p.f.} \quad = \cos \theta_0 = \frac{P_c}{V_1 I_e}$$

From the phasor, it follows that

$$I_c = I_e \cos \theta_0 \quad \text{and} \quad I_m = I_e \sin \theta_0$$

$$\text{From fig.(b),} \quad I_c = \frac{P_c}{V_1}$$

$$\therefore \text{Core loss resistance } R_{CL} = \frac{V_1}{I_c} = \frac{V_1}{I_e \cos \theta_0} = \frac{V_1^2}{V_1 I_e \cos \theta_0} = \frac{V_1^2}{P_c}$$

$$\text{Also} \quad I_c^2 R_{CL} = P_c$$

$$R_{CT} = \frac{P_c}{I_c^2} = \frac{P_c}{(I_e \cos \theta_0)^2}$$

$$\text{Magnetizing reactance, } X_{mL} = \frac{V_1}{I_m} = \frac{V_1}{I_e \sin \theta_0}$$

Thus the open-circuit test gives the following information: (i) core loss at rated voltage and frequency, (ii) the shunt branch parameters of the equivalent circuit, i.e. R_C and X_m and (iii) turns ratio of the transformer.

Short - circuit test : The low voltage side of the transformer is short-circuited and the instruments are placed on the high voltage side, as illustrated in fig.(a). The applied voltage is adjusted by auto-transformer, to circulate rated current in the high voltage side. In a transformer, the primary m.m.f. is almost equal to the secondary m.m.f., therefore, a rated current in the h.v. winding causes rated current to flow in the l.v. winding.

A primary voltage of 2 to 12% of its rated value is sufficient to circulate rated currents in both primary and secondary windings. From fig.(b), it is clear that the secondary leakage impedance drop appears across the exciting branch (R_e and X_m in parallel).

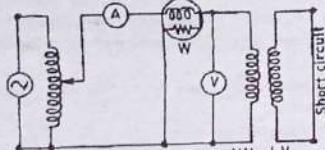


Fig. (a) Connection diagram for short circuit test on a transformer.

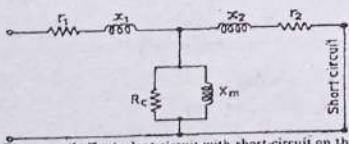


Fig. (b) Equivalent circuit with short-circuit on the secondary side

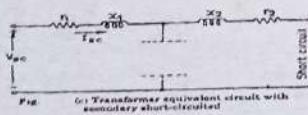


Fig. (c) Transformer equivalent circuit with secondary short-circuited

The instrument readings may be corrected, if required. Let V_{sc} , I_{sc} and P_{sc} be the voltmeter, ammeter and wattmeter readings; then from fig.(c), equivalent leakage impedance referred to h.v. side, $\bar{z}_e H = \frac{V_{sc}}{I_{sc}}$

$$\text{equivalent resistance referred to h.v. side, } r_e H = \frac{P_{sc}}{I_{sc}^2}$$

and equivalent leakage reactance referred to h.v. side,

$$x_e H = \sqrt{z_e^2 H - r_e^2 H}$$

In $r_e H$, $x_e H$ and $z_e H$, the subscript H is used to indicate that these quantities are referred to h.v. side. These parameters can, however, be referred to the values of equivalent resistance and equivalent leakage reactance referred to either side, are used. However, if the leakage impedance parameters for both primary and secondary are required separately, then it is usual to take $\eta = r_2 = \frac{1}{2} r_e$ and $x_1 = x_2 = \frac{1}{2} x_e$, referred to the same side.

Thus, the short-circuit test gives the following information : (i) ohmic loss at rated current and frequency and (ii) the equivalent resistance and equivalent leakage reactance.

Q.14.(b) Briefly explain the concept of an ideal transformer stating the assumptions made.

Ans.: Ideal Two-winding Transformer : In the beginning, a transformer is assumed to be an ideal one, merely for obtaining an easier explanation of what happens in a transformer. For a transformer to be an ideal one, the various assumptions are as follows:

1. Winding resistances are negligible.
2. All the flux set up by the primary links the secondary windings, i.e. all the flux is confined to the magnetic core.
3. The core losses (hysteresis and eddy current losses) are negligible.
4. The core has constant permeability, i.e. the magnetization curve for the core is linear.

As a later stage, the effect of these assumptions will be taken up one by one.

Q.14.(c) Briefly explain the concept of an ideal transformer stating the assumptions made.

Ans.: Ideal Transformer : An ideal transformer is an imaginary transformer which has the following properties:

1. Its primary and secondary winding resistance are negligible.
2. The core has infinite permeability (μ) so that negligible mmf is required to establish the flux in the core.
3. Its leakage flux and leakage inductances are zero. The entire flux is confined to the core and links both windings.
4. There are no losses due to resistance, hysteresis and eddy currents. Thus, the effi-

Machine-I

[3-18]

Transformer

ciency is 100 per cent.

It is to be noted that practical transformer has none of these properties inspite of the fact that its operation is close to ideal.

An ideal iron-core transformer is shown in fig. It consists of two coils wound in the same direction on a common magnetic core. The winding connected to the supply, V_1 , is called the primary. The winding connected to the load, Z_2 , is called the secondary.

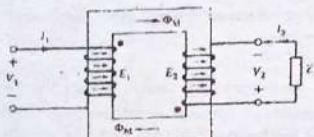


Fig. Ideal iron-core transformer.

Q.15. A 10 kVA, 2500/250 V, single-phase transformer gave the following test results :

Open-circuit test : 250V 0.8 A 50 W

Short-circuit test : 60 V 3 A 45 W

(a) Calculate the efficiency at full-and half-load at 0.8 pf.

(b) Calculate the load (kVA output) at which maximum efficiency occurs and also the value of maximum efficiency at 0.8 pf.

(c) Compute the voltage regulation and the secondary terminal voltage under rated load at power factors of 0.8 lagging.

Ans.: 10 kVA, 2500 / 250 V

OCT : 250 V 0.8 A 50 W

SCT : 60 V 3 A 45 W

(a) at full load, $k = 1$

$$\eta = \frac{kV I \cos \phi}{kV I \cos \phi + P_c + P_{cu}}$$

from above test,

$$P_c = 50W, P_{cu} = \left(45W \times \frac{4^2}{3^2}\right) = 80W; \quad \eta = \frac{10k \times 0.8}{10k \times 0.8 + 80 + 50}$$

$$\eta = 0.984 = 98.4\%$$

at half load,

$$k = 0.5$$

$$P_{cu} = (0.5)^2 \times 80 = 20W$$

$$\eta = \frac{0.5 \times 10k \times 0.8 \times 100}{(0.5 \times 10k \times 0.8) + 20 + 50} \text{ (in \%)} \quad \dots$$

Machine-I

[3-19]

Transformer

$$\eta = 98.28\%$$

(b) For maximum efficiency,

$$\Rightarrow P_c = k_1^2 (P_{cu})$$

$$\Rightarrow 50 = k_1^2 (80)$$

$$\Rightarrow k_1 = \sqrt{\frac{50}{80}} = 0.79, 2P_{cu} = 100W$$

$$\text{kVA output} = k_1 \times 10k = 7.9 \text{ kVA}$$

$$\Rightarrow \eta_{max} = \frac{0.79 \times 10k \times 0.8}{(0.79 \times 10k \times 0.8) + 100}$$

$$\boxed{\eta_{max} = 98.44\%}$$

(c) From SCT, (H.V.S)

$$\Rightarrow Z = \frac{V_{AC}}{I_{AC}} = 20\Omega$$

$$\Rightarrow R = P/I^2 = 5\Omega$$

$$\Rightarrow X = 19.4\Omega$$

$$\Rightarrow Z_{base} = \frac{V_{base}^2}{kVA}$$

for H.V.S, $V_{base} = 2500V$

$$\Rightarrow Z_{base} = \frac{(2500)^2}{(10k)} = 625$$

$$\Rightarrow X(p.u) = \frac{19.4}{625} = 0.031$$

$$\Rightarrow R(p.u) = 0.008, \cos \phi = 0.8$$

$$\Rightarrow VR = X(p.u) \sin \phi + R(p.u) \cos \phi$$

$$\Rightarrow VR = 0.025, \text{ or } 2.5\%, V'_1 = 250V$$

$$\Rightarrow VR = \frac{V'_1 - V_2}{V_2}$$

$$\Rightarrow 0.025 V_2 = 250 - V_2$$

$$\Rightarrow V_2 = 243.9V$$

Q.16. The maximum efficiency of a 100 kVA, 1-phase transformer is 98% and occurs at 80% of full load at 0.8 power factor lagging. If the leakage impedance of the transformer is 5% find the voltage regulation at full load.

Machine-I

[3-20]

Transformer

Ans.: $\eta_{max} = 98\%$, $100kVA$, 80% of full load, at 0.8 p.f.

$$Z_{eo} = 5\%, \quad VR = ?$$

$$VR = e_c \cos \theta + c_s \sin \theta$$

$$\Rightarrow \frac{\text{loss}}{\text{O/P}} + \frac{\text{O/P}}{\text{O/P}} = \frac{1}{\eta}$$

$$\text{loss} = \left(\frac{1}{\eta} - 1 \right) \text{O/P} = \left(\frac{1}{0.98} - 1 \right) \times 100K \times 0.8 \times (8)^2 = 1045 \text{ Watt.}$$

at full load, of maxⁿ efficiency

$$P_{eo} = P_c$$

$$P_c = 522 \text{ Watt}$$

at full load

$$\Rightarrow P_c = \left(\frac{100}{80} \right)^2 \times 522 = 816 \text{ Watt}$$

$$z_s = \frac{816}{100K} = 0.0082$$

$$\Rightarrow e_s = \sqrt{(0.05)^2 - (0.0082)^2}$$

$$\Rightarrow e_s = 0.04933$$

$$\Rightarrow VR = 0.0082 \times \cos \theta + 0.04933 \times \sin \theta$$

$$\Rightarrow VR = 0.0082 \times 0.8 + 0.04933 \times 0.6 = 0.0362$$

$$VR(\%) = 3.62\% \text{ Ans.}$$

Q.17. Open circuit and short circuit tests on a 5 kVA, 200/400V, 50 Hz single phase transformer gave the following test results:

O. C. Test 200V, 2A, 100W (L.v. side)

S. C. Test 40V, 10A, 200W (H.v. side)

Determine the efficiency and approximate regulation of the transformer at full load and 0.8 power factor lagging.

Ans.: 5 KVA, 200 / 400 V, 50 Hz, 1- ϕ X_{mer}

OCT – 200V, 2A, 100W (L.V.S.)

SCT – 40V, 10A, 200W (H.V.S.)

$\eta = ?$, $VR = ?$ at full load and 0.8 P.F. lag.

here, $P_c = 100W$

$$\text{from SCT, } r_{ph} = \frac{200}{(10)^2} = 2\Omega; Z_{eh} = \frac{40}{10} = 4\Omega$$

[3-20]

Transformer

Machine-I

[3-21]

Transformer

$$x_{eh} = \sqrt{16 - 4} = 2\sqrt{3}\Omega; I_{2h} = \frac{5k}{400} = 12.5A$$

$$VR = \frac{I_{2h} e_{eh} \cos \theta_2 + I_{2h} x_{eh} \sin \theta_2}{E_2}$$

$$P_{oh} = 200W; P_0 = 5KVA$$

$$\eta = \frac{P_o \times 0.8}{P_o \times 0.8 + P_{oh} + P_c} = \frac{5000 \times 0.8}{5000 \times 0.8 + 100 + 200} = 93.02\%$$

$$VR = \frac{E_2 - V_2}{E_2}; VR = \frac{12.5 \times 2 \times 0.8 + 12.5 \times 3.464 \times 0.6}{400} = 0.11495$$

$$VR(\%) = 11.495\%.$$

Q.18. A single phase transformer working at unity p.f. has an efficiency of 90% at both half load and at the full load of 500 watt. Determine the efficiency at 75% full load and the maximum efficiency.

Ans.: P.F = 1, $\eta = 90\%$ at half load and full load

$$P = 500W; S = 500VA, \cos \theta = 1$$

$$\Rightarrow \eta_1 = \frac{kS \cos \theta}{kS \cos \theta + \text{loss}}$$

$$\Rightarrow 0.9 = \frac{1 \times 500 \times 1}{1 \times 500 \times 1 + \text{loss}}$$

$$\Rightarrow \text{loss} = P_i + P_c = 55.55W$$

$$\Rightarrow \eta_{1/2} = \frac{1/2 \times 500 \times 1}{1/2 \times 500 \times 1 + P_i + P_c (1/4)} = 0.9$$

$$\Rightarrow P_i + \frac{P_c}{4} = 27.78W$$

$$P_i = 18.52W, P_c = 37.03W$$

$$\eta_{0.75} = \frac{0.75 \times 500 \times 1}{0.75 \times 500 + P_i + P_c (0.75)^2} = 0.905 \times 100 = 90.5\%$$

for maximum efficiency

$$P_i = k^2 P_c, k = \sqrt{\frac{18.52}{37.03}} = 0.707$$

$$\eta_{max} = \frac{(0.707 \times 500 \times 1) \times 100}{0.707 \times 500 \times 1 + 2(18.52)} = 90.52\%$$

Q.19. Two single phase transformer rated 600 kVA and 500 kVA respectively, are connected in parallel to supply a load of 1000 kVA at 0.8 lagging p.f. The resistance and reactance of the first transformer are 3% and 6.5% respectively, and of the second

Machine-1

[3-22]

Transformer

transformer are 1.5% and 8% respectively. Calculate the kVA loading and the p.f. at which each transformer operates.

$$\text{Ans. : } S_1 = 600 \text{ kVA}, \quad S_2 = 500 \text{ kVA}$$

$S_L = 1000 \text{ kVA}$ at 0.8 p.f. lagging

$$Z_1(p.u) = 0.03 + j0.065$$

$$Z_2(p.u) = 0.015 + j0.08$$

$$(Z_1 + Z_2)(p.u) = 0.045 + j0.145$$

$$S_2^*(\text{rated}) = \frac{S_L^* Z_1}{Z_1 + Z_2} = \frac{1000(0.8 + j0.6)(0.03 + j0.065)}{0.045 + j0.145} = 471.53 \angle 29.34^\circ$$

$$\text{p.f.} = \cos \phi_2 = \cos(29.34^\circ) = 0.872 \text{ lagging}$$

$$S_1^*(\text{rated}) = \frac{S_L^* Z_2}{Z_1 + Z_2} = \frac{(800 + j600)(0.015 + j0.08)}{0.045 + j0.145} = 536.1 \angle 43.5^\circ$$

$$\text{p.f.} = \cos \phi_1 = \cos(43.5^\circ) = 0.725 \text{ lagging.}$$

Q.20. The equivalent circuit for a 200/400 V step-up transformer has the following parameters referred to the low-voltage side :

$$\text{Equivalent resistance} = 0.15 \Omega; \quad \text{Equivalent reactance} = 0.37 \Omega$$

$$\text{Core-loss component resistance} = 600 \Omega; \quad \text{Magnetising reactance} = 300 \Omega$$

When the transformer is supplying a load at 10A at a power factor of 0.8 lag, calculate the (i) Primary current and (ii) Secondary terminal voltage.

Ans. : 200/400 referred to primary side,

$$r_{el} = 0.15 \Omega; \quad x_{el} = 0.37 \Omega; \quad R_t = 600 \Omega; \quad X_m = 300 \Omega; \quad \cos \phi_2 = 0.8 \text{ lag}; \quad I_2 = 10 \text{ A}$$

$$(i) I_1 = ?; \quad (ii) V_2 = ?$$

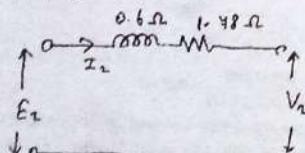
$$\text{Here, } z_{el} = (0.15 + j0.37) \Omega$$

$$z_{el} = z_{el} \left(\frac{1}{a} \right)^2 \quad \therefore \quad a = \frac{200}{400} = \frac{1}{2}$$

$$z_{el} = (0.15 + j0.37) \times 4 = (0.6 + j1.48) \Omega$$

$$\text{Now, } E_2 = 400 \text{ V}$$

$$\text{Here, } \bar{I}_1 = I_1 + \bar{I}_2$$

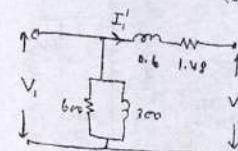


Machine-1

[3-23]

Transformer

$$I_e = \frac{200}{600} + \frac{200}{300} = \left(\frac{1}{3} + \frac{2}{3} \right) = \frac{1}{a}; \quad I_1' = I_2 \left(\frac{1}{a} \right)$$



$$\Rightarrow I_1' = 4I_2 = (10 \angle -2.16) \times 4 = (9.993 - j0.377) \times 4 = (39.97 - j1.51) \text{ A}$$

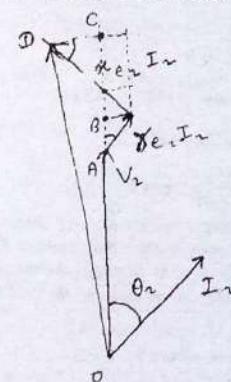
$$\Rightarrow I_1 = 39.97 - j1.51 + 0.333 + j0.667 = (40.3 - j0.843) \text{ A}$$

$$\text{Here, } OD = OC = OA + AB + BC$$

$$\Rightarrow E_2 = [V_2 + r_{el} I_2 \cos \phi_2 + x_{el} I_2 \sin \phi_2]$$

$$\Rightarrow \cos \phi_2 = 0.8, \quad \sin \phi_2 = 0.6$$

$$\Rightarrow r_{el} I_2 \cos \phi_2 + x_{el} I_2 \sin \phi_2 = 4.8 + j8.88 = 13.68 \text{ V}$$



$$\Rightarrow V_2 = E_2 - 13.68 = 386.32 \text{ V} \quad \text{Ans.}$$

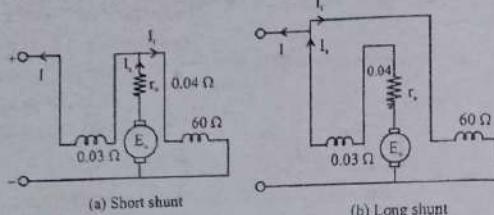
$$\Rightarrow I_1 = (40.3 - j0.843) = 40.3 \angle -1.2^\circ \quad \text{Ans.}$$

Q.21. A short shunt d.c. compound generator supplies 200 A at 100 V. The resistance of armature, series field and shunt field windings are 0.04, 0.03 and 60 Ω respectively. Find the emf generated. Also find the emf generated if the same machine is connected as a long-shunt machine.

Machine-1

[3-24]

Ans. : Short-shunt dc compound generator



Here, $r_d = 0.04 \Omega$

$$r_{f_n} = 0.03 \Omega, V_f = 100V,$$

$$r_{shf} = 60 \Omega, I = 200 A,$$

$$I_f = \frac{100}{60} = \frac{5}{3} A; I_a = 200 + \frac{5}{3} = \frac{605}{3} A$$

for short shunt,

$$E_{ath} = V_f + I r_{shf} + I_a r_d$$

$$= 100 + (200 \times 0.03) + \frac{605}{3} \times 0.04 = 114.07 V.$$

for long shunt, $E_{ath} = V_f + I_a(r_{shf} + r_d)$

$$= 100 + (0.07) \left(\frac{605}{3} \right) = 114.12 V.$$

Q.22. A d.c. series motor having a resistance of 1Ω drives a fan for which the torque varies as the square of the speed. At $220 V$, the set runs at 350 r.p.m and takes $25 A$. The speed is to be raised to 500 r.p.m. by increasing the voltage. Determine the necessary voltage and the corresponding current assuming the field to be unsaturated.

Ans. : $r = 1 \Omega$

$$T_e \propto n^2; V_1 = 220 V; N_1 = 350 \text{ r.p.m}; I_1 = 25 A$$

$$N_2 = 500 \text{ r.p.m by increasing voltage}$$

$$V_2 = ?; I_2 = ?; I \propto \phi; I_1 \propto \phi_1$$

$$\frac{E_{a_1}}{E_{a_2}} = \frac{N_1 I_1}{N_2 I_2}$$

..... (i)

$$E_{a_1} = V_{f_1} - r I_1$$

$$= 220 - 1 \times 25 = 195 V$$

Transformer

[3-25]

Transformer

Machine-1

$$E_{a_2} = V_{f_2} - r I_2$$

$$T_e \propto n^2, T_e \propto I_2$$

$$T_e \propto I^2$$

$$n^2 \propto I^2; \frac{n_1}{n_2} = \frac{I_1}{I_2}; \frac{350}{500} = \frac{25}{I_2}$$

$$I_2 = \frac{25 \times 500}{350} = 35.71 A$$

$$\text{from eq. (i)} \quad \frac{195}{E_{a_2}} = \frac{350 \times 25}{500 \times 35.71}$$

$$E_{a_2} = 398 V$$

$$\text{from eq. (ii)} \quad V_{f_2} = 398 + 1 \times 35.71 = 433.67 V.$$

Q.23. Write short notes any two:

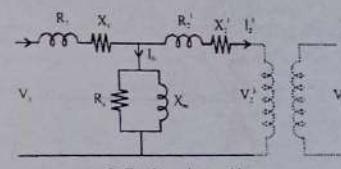
(a) Equivalent circuit of transformer referred to primary (only draw) and also secondary

(b) Vector diagram of transform at leading p.f. at full load.

(c) Autotransformer

Ans.: (a) Equivalent circuit of transformer referred to primary (only draw) and also secondary:

Circuit diagram is referred to primary side



$$R_2^1 = a^2 R_1; X_2^1 = a^2 X_1; V_2^1 = a V_1;$$

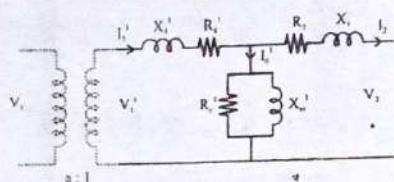
$$I_2^1 = \frac{I_2}{a}$$

$$I_1 = I_2^1 + I_0$$

Circuit diagram referred to secondary side

Machine-1

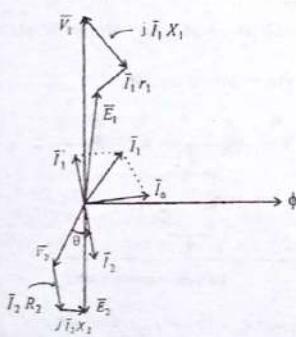
[3-26]



$$\begin{aligned} \Rightarrow V_1^1 &= \frac{V_1}{a}; & I_1^1 &= a I_1 \\ \Rightarrow X_1^1 &= \frac{X_1}{a^2}; & R_1^1 &= \frac{R_1}{a^2} \\ \Rightarrow R_c^1 &= \frac{R_c}{a^2}; & R_m^1 &= \frac{X_m}{a^2} \\ R_0^1 &= a R_0. \end{aligned}$$

(b) Vector diagram of Transform at leading p.f. at full load:

A vector diagram for leading p.f.



θ is leading power factor angle.

(c) Autotransformer : So far two-winding transformers have been discussed wherein the windings are electrically isolated. When the primary and secondary windings are electrically connected so that a part of the winding is common to the two as shown in fig., the transformer is known as an autotransformer. Such a transformer is particularly economical where the volt-

Transformer

[3-27]

Transformer

Machine-1

age ratio is less than 2 in which case electrical isolation of the two windings is not essential. The major applications are induction motor starters, interconnection of HV systems at voltage power supplies. The autotransformer has lower reactance, lower losses, smaller exciting current and better voltage regulation compared to its two-winding counterpart. All this is on account of the fact that in an autotransformer a part of the energy transfer is through the conduction process.

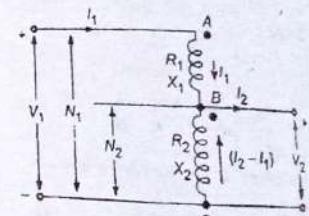


Fig. Autotransformer

Figure shows a single-phase autotransformer having N_1 turns primary with N_2 turns tapped for a lower voltage secondary. The winding section BC of N_2 turns is common to both primary and secondary circuits. In fact it is nothing but a conventional two-winding transformer connected in a special way. The winding section AB must be provided with extra insulation being at higher voltage.

It will be assumed here that the magnetizing current is negligible, but it can easily be determined by a no-load test and accounted for.

With reference to fig., the two winding voltage and turn-ratio is

$$a = \frac{V_1 - V_2}{V_2} = \frac{N_1 - N_2}{N_2}; N_1 > N_2$$

As an autotransformer its voltage and turn-ratio is

$$a' = \frac{V_1}{V_2} = \frac{N_1}{N_2} > 1$$

It is easy to see that eqn., are related as

$$a' = 1 + a.$$

Q.24. The following data were obtained on a 20kVA, 50Hz, 2000/200 V distribution transformer:

	Voltage (V)	Current (A)	Power (W)
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Machine-1

[3-28]

Transformer

OC test with HV

open circuited 200 4 120

SC test with LV

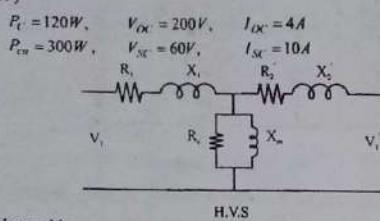
short circuited 60 10 300

Draw the approximate equivalent circuit of the transformer referred to the HV and LV sides respectively.

Ans.: 20kVA, 50Hz, 2000/200V

OCT (HV)

SCT (LV)



Low voltage side,

$$Z_{SC} = 6 \Omega$$

$$\Rightarrow R_{SC} = \frac{P_{SC}}{I_{SC}^2}$$

$$\Rightarrow R_{SC} = 3 \Omega, \quad R'_c = 3.33 \Omega, \quad X'_m = 0.506 \Omega$$

$$\Rightarrow X_{SC} = 3\sqrt{3} \Omega = 5.2 \Omega$$

H.V.S.

$$Z = \frac{V_{OC}}{I_{OC}} = 50 \Omega$$

$$R_C = \frac{V_{OC}^2}{P_C} = \frac{(200)^2}{120} = 333.33 \Omega$$

$$X_m = 50.6 \Omega$$

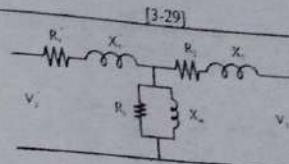
$$\Rightarrow R_n = R'_{SC} = \left(\frac{2000}{200} \right)^2 \times 3 = 300 \Omega$$

$$\Rightarrow X_n = X'_{SC} = 520 \Omega$$

Machine-1

[3-29]

Transformer



For H.V.S

$$R_H = R_{sc} = R_1 + R_2 = 300 \Omega$$

$$R_h = R_{SC} = 520 \Omega$$

$$R_c = 333.33 \Omega$$

$$X_m = 50.6 \Omega$$

$$X_h = X_1 + X_2$$

$$R_L = R_n = 3 \Omega$$

$$R_L = R_2 + R_1$$

$$X_L = X_2 + X_1$$

$$X_L = X_m = 5.2 \Omega$$

$$R'_c = 3.33 \Omega$$

$$X'_m = 0.506 \Omega$$

Q.25. A 100 kVA, 1100/230 V, 50Hz transformer has an HV winding resistance of 0.1Ω and a leakage reactance of 0.4Ω. The LV winding has a resistance of 0.006Ω and a leakage reactance of 0.01Ω. Find the equivalent winding resistance, reactance and impedance referred to the HV and LV sides. Convert these to pu values.

Ans.: 100 kVA, 1100/230V, 50Hz

$$R_1 = 0.1 \Omega \quad R_2 = 0.006 \Omega$$

$$X_1 = 0.4 \Omega \quad X_2 = 0.01 \Omega$$

for H.V. side,

$$\therefore R_{eq} = R_1 + \left(\frac{1100}{230} \right)^2 R_2$$

$$\Rightarrow R_{eq} = 0.24 \Omega \quad \Rightarrow X_{eq} = X_1 + \left(\frac{1100}{230} \right)^2 X_2$$

$$\Rightarrow X_{eq} = 0.4 + 0.23 = 0.63 \Omega$$

$$Z_{eq} = R_{eq} + j X_{eq} = (0.24 + j 0.63) \Omega$$

for L.V. side,

$$\Rightarrow R_{eq} = R_2 + \left(\frac{230}{1100} \right)^2 R_1$$

$$\Rightarrow R_{eq} = 0.0104 \Omega$$

$$\Rightarrow X_{eq} = X_2 + \left(\frac{230}{1100} \right)^2 X_1$$

$$\Rightarrow X_{eq} = 0.0275 \Omega$$

[3-30]

Machine-1

$$\Rightarrow Z_{eq} = R_{eq} + j X_{eq} = (0.0104 + j 0.0275) \Omega$$

in Pu system,

$$Z_S = \frac{[\text{base voltage}]^2}{\text{kVA base}} = 100 \text{ kVA}, \quad Z_S = \frac{Z_{eq}}{\text{base kVA}}$$

base voltage = 1100 V, $Z(\text{pu}) = \frac{Z_{eq}}{Z_S}$

for H.V. side
 $Z(\text{pu}) = (0.02 + j 0.052)$

for L.V. side
 $Z(\text{pu}) = (0.02 + j 0.052)$

Q.26. In a transformer, if the load current is kept constant, find the power factor at which the maximum efficiency occurs.

Ans.: Condition for Maximum Efficiency :

The pre-unit (pu) efficiency at load current I_2 is

$$\eta = \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + I_2^2 R_{eq} + P_i} = \frac{V_2 \cos \phi_2}{V_2 \cos \phi_2 + I_2 R_{eq} + (P_i / I_2)}$$

Equation shows that the efficiency varies with the load. The plot of efficiency η versus load is shown in fig.

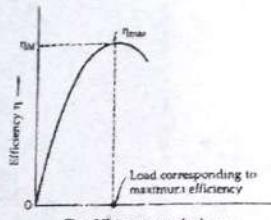


Fig. Efficiency versus load curve

It is seen that the efficiency is low at small loads and reaches a maximum value for a certain load. The efficiency then decreases as the load is increased. Sometimes it is desired to know under what conditions and at which value of the load, the transformer will operate at maximum efficiency.

At maximum efficiency

$$\frac{d\eta}{dI_2} = 0 \text{ and } \frac{d^2\eta}{dI_2^2} < 0$$

Since V_2 and $\cos \phi_2$ are constants for a given load, the efficiency will be a maximum when the denominator $Dr = V_2 \cos \phi_2 + I_2 R_{eq} + \frac{P_i}{I_2}$ is a minimum.

Transformer

[3-31]

Machine-1

For a minimum value of the denominator Dr

$$\frac{d}{dI_2} Dr = 0 \text{ and } \frac{d^2 Dr}{dI_2^2} > 0$$

$$\frac{d}{dI_2} Dr = \frac{d}{dI_2} (V_2 \cos \phi_2 + I_2 R_{eq} + \frac{P_i}{I_2}) = 0 + R_{eq} - \frac{P_i}{I_2^2}$$

For a minimum Dr,

$$R_{eq} - \frac{P_i}{I_2^2} = 0$$

$$I_2^2 R_{eq} = P_i$$

$$\text{Also, } \frac{d^2 Dr}{dI_2^2} = \frac{d}{dI_2} \left(R_{eq} - \frac{P_i}{I_2^2} \right) = 0 + \frac{2P_i}{I_2^3} > 0$$

Since $\frac{d^2 Dr}{dI_2^2}$ is positive, the expression is a condition for the minimum value of Dr, and therefore the condition for maximum value of efficiency.

The efficiency of a transformer for a given power factor is a maximum when the variable copper loss is equal to the constant iron loss.

Q.27. The maximum efficiency of a 100 kVA, single-phase transformer is 98% and occurs at 80% of full-load at 0.8 p.f. If the leakage impedance of the transformer is 5%, find the voltage regulation at rated load of 0.8 p.f. lagging.

Ans.: 100 kVA

$$\eta_{max} = 0.98; \quad I = 0.8I_{f1}$$

0.8 p.f.

$$Z(\text{pu}) = 0.05; \quad X = ? \quad R = ?$$

$$\Rightarrow \eta_{max} = \frac{kVI \cos \phi}{kVI \cos \phi + 2P_{cu}} \Rightarrow 0.98 = \frac{1 \times 100 \times 0.8}{1 \times 100 \times 0.8 + Z P_{cu}}$$

$$\Rightarrow P_{cu} = 816.33W$$

$$R(\text{pu}) = \frac{P_{cu} \times k^2}{kVI \cos \phi} = 0.0082 \Omega$$

$$X(\text{pu}) = 0.0493 \Omega$$

VR at 0.8 p.f. lagging i.e., $\cos \phi = 0.8$ and $\sin \phi = 0.6$

$$\Rightarrow VR = R(Pu) \cos \phi + X(Pu) \sin \phi \Rightarrow VR = 3.616\%$$

Machine-I	[3-32]	Transformer
<p>Q.28. List few applications of transformers.</p> <p>Ans.: Application of Transformers :</p> <p>Transformers are used in a number of applications:</p> <ul style="list-style-type: none"> (a) To change the level of voltage and current in electric power systems. (b) As impedance-matching device for maximum power transfer in low-power electronic and control circuits. (c) As a coupling device. (d) To isolate one circuit from another, since primary and secondary are not interconnected. (e) To measure voltage and currents, these are known as instrument transformers. (f) Converting hvac to hvdc in combined ac/dc power systems. <p>Transformers are extensively used in ac power system because of the following reasons:</p> <ul style="list-style-type: none"> (1) Electric energy can be generated at the most economic level (11 kV – 33 kV). (2) Stepping of the generated voltage to high voltage, extra high voltage EHV, or to even ultra high voltage UHV to suit the power transmission requirement to minimise losses and increase transmission capacity of lines. (3) The transmission voltage is stepped down in many stages for distribution and utilisation of domestic commercial and industrial consumers. <p>Q.29. The efficiency of a 400-kVA, single-phase transformer is 98.77% when delivering full-load at 0.8 p.f. lagging and 99.13% at half full-load at unity p.f. Calculate (i) iron loss and (ii) full-load copper loss.</p> <p>Ans.: 400 kVA</p> <p>$\eta = 98.77\%$ at full load; p.f. = 0.8 lagging $\eta = 99.13\%$ at 1/2 load; p.f. = 1</p> $\Rightarrow \eta = \frac{P_0 \times 1}{P_0 \times 1 + P_L} = \frac{VI \cos \phi}{VI \cos \phi + P_L} \quad \Rightarrow \quad 0.9877 = \frac{400 \times 0.8}{400 \times 0.8 + P_L}$ $\Rightarrow P_L = 3.985 \text{ kW}$ <p>Secondly,</p> $\Rightarrow 0.9913 = \frac{400 \times 1/2}{400 \times 1/2 + P_i + (1/2)^2 P_{cv}} \quad \dots \quad (i)$ $\Rightarrow P_i + \frac{P_{cv}}{4} = 1.7553 \text{ kW} \quad \dots \quad (i)$ $\Rightarrow P_i + P_{cv} = 3.985 \text{ kW} \quad \dots \quad (ii)$	[3-32]	Transformer
Machine-I	from eqn.(i) and (ii)	[3-33] Transformer
	$P_i = 1.012 \text{ kW}; \quad P_{cv} = 2.973 \text{ kW}$	
		<p>Q.30. What are the advantages and disadvantages of a 3-phase transformer over three single-phase bank of transformers?</p> <p>Ans.: Advantages :</p> <p>It is found that generation, transmission and distribution of electrical power are more economical in three phase system, than single phase system. For three phase system three using single three phase transformers or by using a bank of three single phase transformers. Both are having some advantages over other. Single three transformers costs around 15% less than bank of 3-single phase transformers. Again former occupies less space than later. For very big transformer, it is impossible large 3-phase transformers to the site and it is easier to transport 3-single phase transformers which is erected separately to form a 3-phase unit. Another advantage using bank of 3-single phase transformers is that if one unit of bank becomes out of order than the bank can be run as open delta.</p> <p>Disadvantages :</p> <ol style="list-style-type: none"> 1) The standard 10 transformer in the US has 120 / 240 V secondary. The secondary winding can be connected to allow the cluster mounted bank to provide 120 / 240 V 30, 4-wire or 120 / 208 V 30, 4-wire. Most 30 transformer only offer 1 secondary voltage. 2) If a 10 transformer fails, you can replace just that one, not scrap the whole bank. 3) Each of the 10 transformer weight substantially less than the corresponding 30 transformers. This allow the a line truck to hang transformers where a 30 transformer might need a crane. 4) For smaller loads, or in rarer locations, only 20 primary is needed to furnish 30, 4-wire service using two 10 transformer connected with open - wye primary and open delta secondary. <p>Q.31. Why is transformer rating in kVA?</p> <p>Ans.: Rating of the Transformer : It is to be noted that, since the copper loss depends on current and core loss depends on voltage, the total loss in the transformer depends on the volt-ampere product, and not on the phase angle between voltage and current, that is independent of the load power factor. The rating of the transformer is, therefore, in volt-ampères (VA) and not in watts (W). In actual practice, the rating of the transformer is specified in VA, kVA or MVA depending upon its size.</p> <p>Q.32. A 100-kVA, 2200/440 V transformer has $R_1 = 0.3 \Omega$, $X_1 = 1.1 \Omega$, $R_2 = 0.01 \Omega$, and</p>

Machine-1

[3-34]

Transformer

$X_2 = 0.035 \Omega$. Calculate (i) the equivalent impedance of the transformer referred to the primary and (ii) total copper losses.

Ans.: 100 kVA

220 / 440 V

$$R_1 = 0.3 \Omega$$

$$X_1 = 0.1 \Omega$$

$$R_2 = 0.01 \Omega$$

$$X_2 = 0.035 \Omega$$

$$I_1 = \frac{100 \text{ kVA}}{V_1} = 45.45$$

$$(i) R_{eq1} = R_1 + \left(\frac{2200}{440} \right)^2 R_2$$

$$\Rightarrow R_{eq1} = 0.3 + 0.25 = 0.55 \Omega$$

$$\Rightarrow X_{eq1} = X_1 + \left(\frac{2200}{440} \right)^2 X_2$$

$$= 1.1 + 0.875$$

$$= 1.975 \Omega$$

$$\Rightarrow Z_{eq1} = R_{eq1} + j X_{eq1}$$

$$= (0.55 + j 1.975) \Omega$$

$$(ii) P_{cu} = I_1^2 R_{eq1}$$

$$P_{cu} = (45.45)^2 (0.55) = 1136.36 W$$

Q.33. What is voltage regulation? Develop an expression for calculating the voltage regulation of a two winding transformer.

Ans. Voltage regulation :- The voltage regulation of a transformer is defined as the arithmetical difference in the secondary terminal voltage between no load ($I_2 = 0$) and full rated load ($I = I_{2n}$) at a given power factor with same value of primary voltage for both rated load and no load.

$$\text{Voltage regulation} = \frac{|V_{2n}| - |V_{2f}|}{|V_{2f}|} \times 100$$

where, V_{2f} = rated secondary terminal voltage at rated load.

V_{2f} = no load secondary terminal voltage with the same value of primary voltage for both rated load and no load.

Expression for regulation :- Let us consider the equivalent circuit of the transformer referred to the secondary as shown in figure.

Machine-1

[3-35]

Transformer

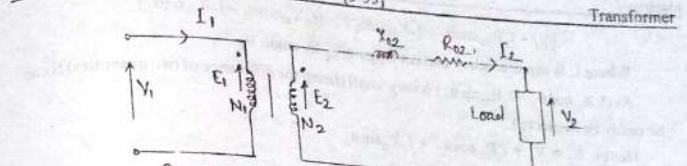


Fig. Approximate equivalent circuit of transformer referred to as secondary

When the load is connected to the secondary side, current I_2 will start flowing. Depending upon the nature of the load, current I_2 may be lagging the voltage V_2 for resistive load and leading the voltage V_2 for capacitive load.

Hence on the basis of the above current voltage phasor relation, the load has a lagging power factor, a unity power factor and a leading power factor respectively.

(i) Lagging power factor:- The phasor diagram of the transformer referred to as the secondary, when supplying a load of lagging power factor load, as shown in figure.

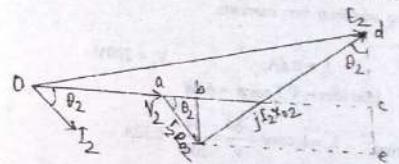


Fig. Phasor diagram of transformer on lagging load, referred to as secondary

E_2 = the no load voltage

V_2 = the load voltage

$I_2 R_{2f}$ = the resistive drop referred to secondary

$I_2 X_{2f}$ = the reactive drop referred to secondary

θ_2 = the angle between V_2 and I_2 , i.e., $\cos \theta_2$ is p.f. of the load.

$oa = V_2$; $ab = I_2 R_{2f} \cos \theta_2$; $bc = I_2 X_{2f} \sin \theta_2$;

$cd = (I_2 X_{2f}) \cos \theta_2 - I_2 R_{2f} \sin \theta_2$

From figure,

$$E_2^2 = (oc)^2 + (cd)^2 = (oa + ab + bc)^2 + (cd)^2$$

Machine-1

[3-36]

Transformer

$$= (V_2 + I_2 R_{22} \cos \theta_2 + I_2 X_{22} \sin \theta_2)^2 + (I_2 X_{22} \cos \theta_2 - I_2 R_{22} \sin \theta_2)^2$$

Where I_2 is the secondary current lagging V_2 by angle θ_2 .

As $(I_2 X_{22} \cos \theta_2 - I_2 R_{22} \sin \theta_2)$ is very small (being the difference of two quantities) it can be easily be neglected.

$$\text{Hence } E_2 = V_2 + I_2 R_{22} \cos \theta_2 + I_2 X_{22} \sin \theta_2$$

Percentage voltage regulation

$$\Rightarrow \frac{E_2 - V_2}{E_2} \times 100\% = \left(\frac{I_2 R_{22} \cos \theta_2}{E_2} + \frac{I_2 X_{22} \sin \theta_2}{E_2} \right) \times 100\%$$

$$= (R_{pu} \cos \theta_2 + X_{pu} \sin \theta_2) \times 100\%$$

Where (R_{pu}) and (X_{pu}) are the total p.u resistance and reactance respectively.

$$\therefore \left[R_{pu} = \frac{I_2 R_{22}}{E_2}; X_{pu} = \frac{I_2 X_{22}}{E_2} \right] \quad \text{Ans.}$$

Q.34. A transformer takes a current of 0.6A and absorb 64W when primary is connected to its normal supply of 200V, 50Hz, the secondary being on open circuit. Find the magnetizing and iron loss currents.

Ans. Given :-

$$I_0 = 0.6A; \quad V_1 = 200V$$

$$\text{Iron loss} = V_1 I_0 \cos \phi = 64W$$

$$\text{Iron current, } I_e = I_0 \cos \phi = \frac{64}{200} = 0.32A$$

Magnetising current,

$$I_m = \sqrt{I_0^2 - I_e^2} = \sqrt{(0.6)^2 - (0.32)^2} = 0.507 = 0.51A \quad \text{Ans.}$$

Q.35. Discuss the advantages, disadvantages and application of autotransformer.

Ans. Advantage of Auto transformer :-

1. Its efficiency is more when compared with the conventional one.
2. Its size is relatively very smaller.
3. Voltage regulation of auto transformer is much better.
4. Lower cost.
5. Low requirements of excitation current.
6. Less copper is used in its designed and construction.

Disadvantage of Auto transformer :-

1. Because of electrical conductivity of the primary and secondary windings the lower

Machine-1

[3-37]

Transformer

voltage circuits is liable to be impressed upon by higher voltage. To avoid breakdown in the lower voltage circuits it becomes necessary to design the low voltage circuit to withstand higher voltage.

2. The leakage flux between the primary and secondary windings is small and hence the impedance is low. This results into severe short circuit current under fault conditions.

3. The connections on primary and secondary sides have necessarily to be same, except when using interconnected starting connections. This introduces complications due to changing primary and secondary phase angle particularly in case of delta/delta connection.

Application :-

1. Used in both synchronous motors and induction motors.
2. Used in electrical apparatus testing labs since the voltage can be smoothly and continuously varied.
3. They find application as boosters in AC feeders to increase the voltage levels.

Q.36. Explain Scott connection of a 3-phase transformer.

Ans.

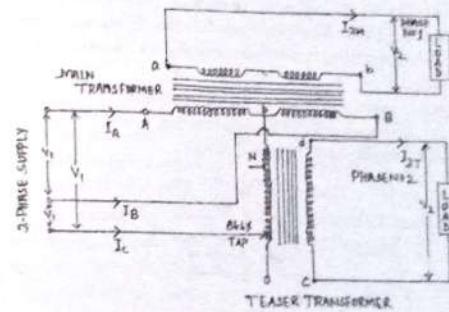


Fig: Scott-Connected Transformer

In some cases such as for electric furnaces, it is desired to work with two phase current. Since now a days $3-\phi$ ac supply is available it is necessary to convert from 3-phase to 2-phase. This is achieved by means of scott-connection. It employs two identical transformers one with primary centre tap, known as main transformer and the other with primary 0.866 tap, known as teaser transformer. The main transformer primary centre tap point D is con-

Machine-1

[3-38]

Transformer

nected with one end of the teaser primary as shown in above figure. Now, ends A, B and C are connected to 3-phase ac supply and true two phase four wire supply is obtained as shown in figure. The two phase side makes use of all the turns on teaser winding as well as all those on the main winding. This connection will cause the voltage per turn of the two transformer to be equal.

Since point D is located midway on A, B so V_{CD} leads V_{AB} by 90° i.e., voltage across the primaries of the transformer are 90° apart. It follows that the secondary voltage are 90° apart. With equal fluxes in the core, the secondary winding require an equal number of turns (Say N_2) to give equal secondary voltages. Hence the two transformer have unequal ratios of transformations.

$$\text{Transformation ratio of main transformer } K = \frac{N_2}{N_1}$$

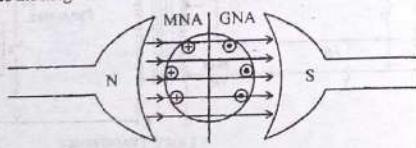
$$\text{Transformation ratio of teaser transformer} = \frac{N_2}{0.866 N_1} = 1.15$$

Q.37. Explain the armature reaction phenomena in a DC machine. What methods can be used to reduce the effect of armature reaction?

Ans. The armature reaction simply shows the effect of armature field on the main field. In other words the armature reaction represents the impact of the armature flux on the main field flux. The armature field produced by the main field flux. The armature field is produced by the armature conductor when current flows through them. And the main field is produced by magnetic poles.

The armature flux causes two effects on the main field flux

- (i) The armature reaction distorted the main field flux
- (ii) It reduces the magnitude of the main field flux.



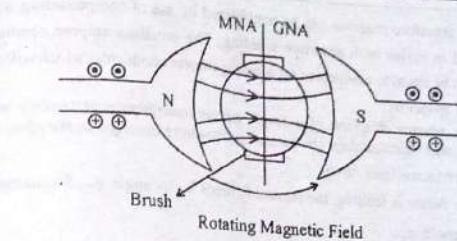
Magnetic Neutral Axis

When no load is connected to the generator the armature current becomes zero. In this condition only the MMF of the main poles exists in the generator. The MMF flux is uniformly distributed along the magnetic axis. The diagram below shows the two poles dc generator. The arrow shows the direction of the magnetic flux ϕ_M . The magnetic neutral axis or plane is perpendicular to the axis of the magnetic flux.

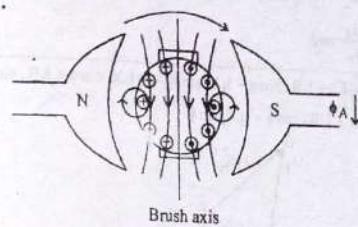
Machine-1

[3-39]

Transformer



The MNA coincides with the geometrical neutral axis (GNA). The brushes of the DC machines are always placed in this axis, and hence this axis is called the axis of commutation.



Consider the condition in which only the armature conductors carrying current and no current flows through the main poles. The direction of current remains same in all the conductors which lying under one pole. The direction of current induces in the conductor is given by the Fleming right hand rule. And the direction of flux generates in the conductors is given by the cork screw rule. The direction of current on the left sides of the armature conductor goes into paper. The armature conductors combine their MMF for generating the fluxes through the armature in the downward direction.

The following methods are used in order to reduce the effect of armature reaction :

- (i) The armature reaction causes the distortion in main field flux. This can be reduced if the reluctance of the path of the cross-magnetising field is increased. The armature teeth and air gap at pole tips offer reluctance to armature flux. Thus by increasing length of air gap, the armature reaction effect is reduced.
- (ii) If reluctance at pole tips is increased it will reduce distorting effect of armature reaction. By using special construction in which leading and trailing pole tip portions of laminations are alternatively omitted.

- (ii) The effect of armature reaction can be neutralized by use of compensating winding. It is always placed in series with armature winding. The armature ampere conductors under pole shoe must be equal to compensating winding ampere conductors which will compensate armature mmf perfectly.

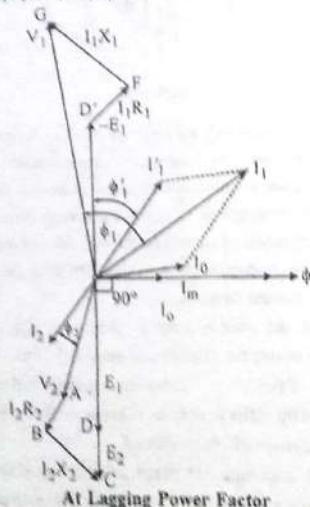
Q.38. Draw the phasor diagram of a single phase transformer at leading and lagging power factor load. Also explain the various parameters involved in the phasor diagram.

Ans. Leading Power factor load $\cos\phi_2$

As load power factor is leading, the current I_2 leads V_2 by angle ϕ_2 . So change is to draw I_2 leading by angle ϕ_2 .

For resistive capacitive load

$$\begin{aligned} \Rightarrow E_2 &= \sqrt{(V_2 + I_2 R_2 \cos\phi - I_2 X_2 \sin\phi)^2 + (I_2 X_2 \cos\phi + I_2 R_2 \sin\phi)^2} \\ \Rightarrow E_2 &= V_2 + I_2 R_2 \cos\phi - I_2 X_2 \sin\phi \\ \Rightarrow E_1 &= \frac{E_2}{K} \text{ and} \\ \Rightarrow V_1 &= \sqrt{(E_1 + I_1 R_1 \cos\phi - I_1 X_1 \sin\phi)^2 + (I_1 X_1 \cos\phi + I_1 R_1 \sin\phi)^2} \\ \Rightarrow V_1 &= E_1 + I_1 R_1 \cos\phi - I_1 X_1 \sin\phi \end{aligned}$$



Leading Power factor load $\cos\phi_2$

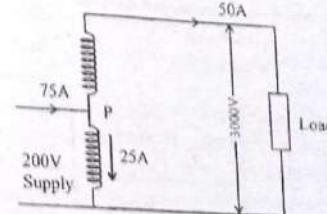
As load power factor is lagging $\cos\phi_2$, the current I_2 lags V_2 by angle ϕ_2 . So only changes in drawing the phasor diagram is to draw I_2 lagging V_2 by ϕ_2 . $I_2 X_2$ leads I_2 by 90° and $I_1 X_1$ leads I_1 by 90° .

For resistive-inductive load

$$\begin{aligned} \Rightarrow E_2 &= \sqrt{(V_2 + I_2 R_2 \cos\phi + I_2 X_2 \sin\phi)^2 + (I_2 X_2 \cos\phi - I_2 R_2 \sin\phi)^2} \\ \Rightarrow E_2 &\approx V_2 + I_2 R_2 \cos\phi - I_2 X_2 \sin\phi \\ \Rightarrow E_1 &= \frac{E_2}{K} \text{ and} \\ \Rightarrow V_1 &= \sqrt{(E_1 + I_1 R_1 \cos\phi + I_1 X_1 \sin\phi)^2 + (I_1 X_1 \cos\phi - I_1 R_1 \sin\phi)^2} \\ \Rightarrow V_1 &\approx E_1 + I_1 R_1 \cos\phi + I_1 X_1 \sin\phi \end{aligned}$$

Q.39. A 5 kVA, 200 V/100 V, 50 Hz, single phase ideal two winding transformer is used to step up a voltage of 200 V to 300 V by connecting it like an auto transformer. Show by the autotransformer (without over loading any of the HV and LV coil). How much conduction of this kVA is transferred magnetically and how much is transferred by electrical conduction.

Ans.



Two winding transformer as an autotransformer

Rated voltage of HV coil is = 220V

Rated voltage of LV coil is = 100 V

Phase turn ratio is $a = 200/100 = 2$

Rated current of each HV coil is $= 5000/200 = 25A$

Rated current of each LV coil is $= 5000/100 = 50A$

Since the load is in series with LV coil, so load current flowing through the LV coil. Thus a maximum of 50A can be drawn by the load otherwise overloading of the coils take place.

Output KVA = $300 \times 50 / 1000 = 15 \text{ KVA}$

Input KVA = Output KVA = 15 KVA

Machine-1

[3-42]

Transformer

A transformer is ideal. Now, the question is how much current is flowing in the HV and in which direction. However this is quite easy since supply and load currents are already their direction as shown in fig. Applying KCL at the junction P, we get

$$\text{Current through HV coil } I_{HV} = 75 - 50 = 25A$$

It is important to note that as a two winding transformer, kVA handling capacity is 5 kVA, the rating of the transformer. However, the same transformer when connected as auto transformer kVA handling capacity becomes 15 kVA without overloading any of the coils.

$$\text{kVA transferred magnetically} = \text{kVA of either HV or LV coil}$$

$$= 200 \times 25\text{VA} = 100 \times 50\text{VA} = 5\text{kVA}$$

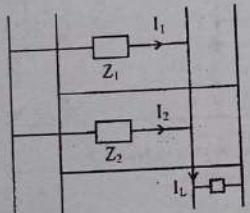
$$\text{kVA transferred magnetically} = 5 \text{ kVA}$$

$$\text{kVA transferred electrically} = \text{total kVA transferred kVA}$$

$$= 15 - 5 = 10 \text{ kVA}$$

Q.40. Two single phase transformers rated 600 kVA and 500 kVA respectively, are connected in parallel to supply a load of 1000 kVA at 0.8 lagging power factor. The resistance and reactance of the first transformer are 3% and 6.5% respectively and of the second transformer 1.5% and 8% respectively. Calculate the kVA loading and the power factor at which each transformer operates.

Ans. $S_1 = 600 \text{ kVA}$ $S_2 = 500 \text{ kVA}$
 $S_L = 1000 \text{ kVA}$ 0.8 pf lagging



$$S_1 = 600 \text{ kVA} \quad S_2 = 500 \text{ kVA}$$

choose kVA base of 1000 kVA

$$\Rightarrow \bar{Z}_1(\text{PU}) = (0.03 + j0.065) \times \frac{1000}{600}$$

$$\Rightarrow \bar{Z}_1(\text{PU}) = 0.05 + j0.018$$

Machine-1

[3-43]

Transformer

$$\Rightarrow \bar{Z}_2(\text{PU}) = (0.015 + j0.08) \times \frac{1000}{500}$$

$$\Rightarrow \bar{Z}_2(\text{PU}) = (0.03 + j0.16)$$

$$\Rightarrow \bar{S}_1 = \frac{\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} \bar{S}_L$$

$$\Rightarrow \bar{S}_1 = \frac{0.03 + j0.16}{0.08 + j0.268} \times 1000(0.8 - j0.6)$$

$$\Rightarrow \bar{S}_1 = 584.2 \angle -30.9^\circ \text{kVA}$$

$$\Rightarrow \bar{S}_1 = 584.2 \text{ kVA at } 0.858 \text{ pf lagging}$$

$$\Rightarrow \bar{S}_2 = \frac{\bar{Z}_1}{\bar{Z}_1 + \bar{Z}_2} \bar{S}_L$$

$$\Rightarrow \bar{S}_2 = \frac{0.05 + j0.108}{0.08 + j0.268} \times 1000(0.8 - j0.6)$$

$$\Rightarrow \bar{S}_2 = 426.5 \angle -45.15^\circ \text{kVA}$$

$$\Rightarrow \bar{S}_2 = 426.5 \text{ kVA at } 0.705 \text{ lagging pf}$$

Q.41. Explain the short-circuit test and open-circuit test on transformer. Why are these tests to be performed?

Ans. Open Circuit Test on Transformer : Open circuit test or no load test on a transformer is performed to determine 'no load loss (core loss) and no load current I_0 '. The ammeter reading gives the no load current I_0 . As I_0 itself is very small, the voltage drops due to this current can be neglected. The input power is indicated by the Wattmeter (W). And as the other side of transformer is open circuited there is no output power. Hence this input power only consists of core losses and copper losses. As we know, no load current is so small that these copper losses can be neglected. Hence the input power is almost equal to the core losses. Thus the wattmeter reading gives the core losses of the transformer.

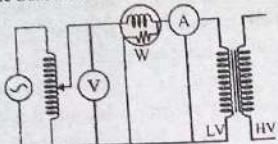
Short Circuit Test on Transformer : The ammeter reading gives primary equivalent of full load current (ISC). The voltage applied for full load current is very small as compared to rated voltage. Hence core loss due to small applied voltage can be neglected. Thus the wattmeter reading can be taken as copper loss in the transformer. Hence we can see that the short circuit test gives copper losses of transformer and approximate equivalent resistance and reactance of the transformer.

Machine-1

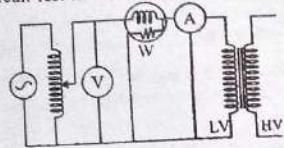
[3-44]

Transformer

These tests are performed to find the parameters of equivalent circuit of transformer and losses of the transformer.



Open Circuit Test on Transformer



Short Circuit Test on Transformer

Q.42. A 100 kVA, 2200/220V transformer has leakage reactance drop of 8% and resistance drop of 2%. find its voltage regulation at full load and 0.8 p.f. lagging. Also find the p.f. at which the regulation will be zero.

$$\text{Ans. PU resistance} = \frac{2}{100} = 0.02; \quad \text{PU reactance} = \frac{8}{100} = 0.08$$

Full load Secondary Current

$$I_2 = \frac{KVA \times 1000}{V_2}; \quad I_2 = \frac{100 \times 1000}{220} = 454 \text{ amp}$$

Voltage Regulation at 0.8 lagging power factor

$$\begin{aligned} &\Rightarrow V.R.\% = \frac{I_2 R_2 \cos\phi + I_2 X_2 \sin\phi}{V_2} \times 100 \\ &\Rightarrow V.R.\% = \frac{454 \times 0.02 \times 0.8 + 454 \times 0.08 \times 0.6 \times 100}{220} \\ &\Rightarrow V.R.\% = \frac{7.264 + 21.792}{220} \times 100 \quad \therefore \sin\phi = 0.6 \\ &\Rightarrow \boxed{V.R.\% = 13.20\%} \quad \text{As } \cos\phi = 0.8 \\ &\qquad \qquad \qquad \phi = 36.86 \\ &\qquad \qquad \qquad \sin\phi = 0.6 \end{aligned}$$

For zero voltage regulation

$$E_2 = V_2 \quad \text{i.e.} \quad E_2 - V_2 = 0$$

Machine-1

[3-45]

Transformer

$$VR \cos\phi - VX \sin\phi = 0$$

$$VR \cos\phi = VX \sin\phi$$

$$\tan\phi = VR / VX$$

$$\cos\phi = \cos \left\{ \tan^{-1} \left(\frac{VR}{VX} \right) \right\}$$

This is the leading p.f. of which voltage regulation becomes zero while supplying the load.

13V. Calculate :

- (i) the number of primary and secondary turns;
- (ii) the net cross-section area of the core for a maximum flux density of 1.4 T.

Ans. (i) Number of primary turns

$$N_1 = \frac{E_1}{\text{EMF per turn}} \quad N_1 = \frac{2310}{13} = 177 \text{ Ans.}$$

Number of secondary turns

$$N_2 = \frac{E_2}{\text{EMF per turn}} \quad N_2 = \frac{220}{13} = 16 \text{ Ans.}$$

Maximum value of flux

$$\phi_{\max} = \frac{\text{EMF per turn}}{4.44f}; \quad \phi_{\max} = \frac{13}{4.44 \times 50} = 0.05 \text{ Wb}$$

(ii) Net cross-sectional area of core

$$a = \frac{\phi_{\max}}{B_{\max}}; \quad a = \frac{0.05}{1.4}; \quad \boxed{a = 0.035 \text{ m}^2} \quad \text{Ans.}$$

□□□



DC MACHINE

Q. 1. Explain the basis principle of operation of a D.C. generator.

Ans. Principle of Working of a D.C. Generator : Production of dynamically induced e.m.f. is the operating principle of D.C. generator. Dynamically induced e.m.f. is the e.m.f. induced in the conductor after the magnetic flux cuts it.

According to Faraday's second law, its magnitude is proportional to the rate of change of flux linkages. Hence, the D.C. generator must possess the magnetic field, a conductor and motion of the conductor relative to the field for the production of dynamically induced e.m.f. Figure shows that the flux lines of magnet (either permanent or electromagnet type) are perpendicular to the plane of the rectangular coil. Rectangular coil is made of copper and whose sides are 12, 22, 21, 11.

Say, the coil rotates in clockwise direction from its initial position 0° . At initial position (i.e., 0°), the induced e.m.f. of coil is zero even though the flux linkage is maximum. This is because of minimum rate of change of flux linkage which results from the sliding of the coil along the flux lines. As soon as the coil starts its rotation, the rate of change of flux linkage also starts increasing until it reaches its maximum value at position $\frac{\pi}{2}$.

Therefore, at this instant, induced e.m.f. is maximum though the coil has minimum flux linkage.

Similarly, at further position like π and $\frac{3\pi}{2}$, the induced e.m.f. of coil has minimum and maximum values respectively. Here, induced e.m.f.s of coil sides 12 and 12 are considered, as they aid each other but not the coil sides 11 and 22. Because coil sides 11 and 22 move in same direction under the same flux. Therefore, its magnitude of induced e.m.f.s are equal but opposite in direction which results zero resultant e.m.f. The induced e.m.f. of coil side 12 is in one direction (say positive) upto position π . After that it is reversed for a period of another π (i.e. negative). The same effect takes place in reverse order for coil side 12. Hence, induced e.m.f.s, thereby currents of coil reverses its direction after every π rotation. Such a periodic reversal of coil current known as alternating current. But the D.C. generator output is not in alternating nature.

In order to obtain the D.C. output, the two coil ends are connected to two split rings, *a* and *b* on which brushes (carbon or copper type) *A* and *B* are arranged as shown in figure. The two split rings are insulated from each other and also from armature shaft on which they are mounted. The two split rings are collectively known as commutator.

As we know for the first π rotation of the coil, the induced e.m.f. of coil side 12, thereby its current is positive, at this instant the current following path is 12aABb2'1'. Thus, here brush, *A* acts

as positive output terminal and *B* as negative one as shown in the following figure. For further π rotation of the coil also, the brushes have same polarity though the coil and split rings changes its polarity.

Thus, the induced e.m.f.s are in alternating nature, but the D.C. generators output is in unidirectional because of rectifying action of split rings.

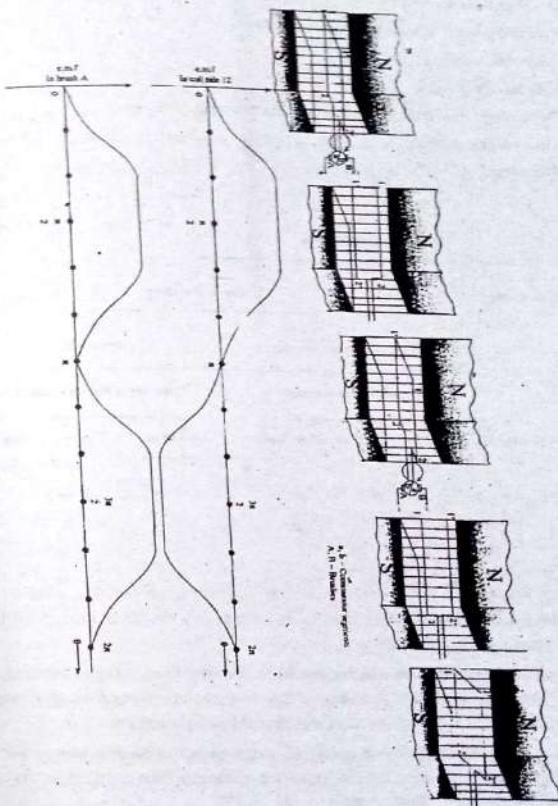


Figure: Simple D.C. Generator with Two Segments Commutator

Q. 2. Write a brief account on different types of D.C armature windings.

Or,

Mention various types of windings used in D.C machines and briefly discuss their relative merits.

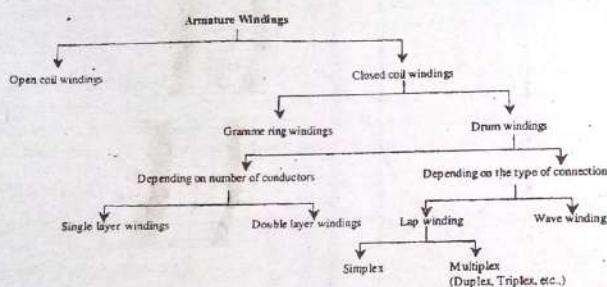
Ans. Classification :

Armature windings are classified into two types :

1. Open coil winding
2. Closed coil winding.

Further closed coil winding are classified into two types. They are

- (i) Gramme ring winding
- (ii) Drum winding.



Figure

1. **Open Coil Winding :** It is that winding which does not close on itself i.e., a closed circuit will be formed until some external connection is made to a source or load. Such type of winding is employed in A.C. machines.

2. **Closed Coil Winding :** It is that winding which closes on itself i.e., in such a winding, if one starts tracing through it, one will come back to the starting point without passing through any external connection. This type of winding is preferred for D.C machine.

(i) **Gramme Ring Winding :** This is an early form of winding that the armature core consists of a ring made of iron laminations with tapes taken from the wire at regular intervals and connected to the commutator segments.

Depending on the number of conductors they are classified as,

(a) **Single Layer Winding :** Single layer winding is one in which one armature slot consist of one conductor or one coil side and this winding is rarely used.

(b) **Two Layer Winding :** Two layer winding is one in which two conductors or two coil sides are placed in each slot. Usually one side of every coil lies in the top of one slot and other side lies in the bottom of some other slot at a distance of approximately one pole apart. This type of winding is mainly used for medium size machine.

In dynamo of larger ratings, it is often necessary to place several coil sides in a single slot usually 4, 6, 8 or more than 8 conductors in a slot are rarely used. With 2 coil sides in each slot, all the coil sides lying in upper half are numbered odd whereas, those at lower half are numbered as even.

The use of several sides in a single slot has following advantages,

- (i) Size of machine reduces for higher rating and hence it is economical.
- (ii) Improved commutation.
- (iii) Resultant output E.M.F is high.

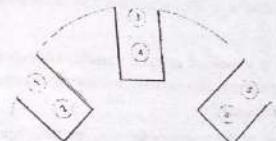


Figure (1)

Advantages :

1. There is no crossing on conductors in the winding which makes it simple.
2. Same winding can be employed for 2, 4, 6 or 8 poles theoretically.

Disadvantages :

1. Wastage of copper as the half portion of the coil lying at the back is not linked with pole flux and hence that position merely act as connections.
2. Maintenance and repairs are more costly.
3. Insulating of winding is also difficult.
4. The E.M.F induced in ring winding is equal to half of the E.M.F induced in drum winding for same pole flux and same number of coils.

(ii) **Drum Winding :** In this type of winding, the conductors are housed in slots over armature surface and connected to one another by front and back connectors.

Advantages :

1. All the armature copper except the end connections is active.

2. Coils can be preformed and insulated before placing on the armature. Hence, the cost is reduced.

Depending on the type of connection there are classified as,

1. **Lap Winding :** In this type of winding the completing end of one coil is connected to a commutator segment and to the start end of adjacent coil located under the same pole and similarly all coils are connected. This type of winding is known as lap because the sides of successive coils overlap each other.

Lap winding may be simplex (single) or multiplex (doublex or triplex) winding. In simplex lap winding the connection of the winding is that there are as many parallel paths as there are number of poles.

Whereas for duplex, the number of parallel paths are equal to twice that of the number of poles and for triplex it is thrice. For this reason, the lap winding is called multiple or parallel winding. The sole purposes of such type of windings are :

(a) To increase the number of parallel paths enabling the armature current to increase i.e., for high current output.

(b) To improve commutation as the current per conductor decreases.

2. Wave winding :

In wave winding, the coils which are carrying current in one direction are connected in series circuit and the coils carrying current in opposite direction are connected in another series circuit. A wave winding is shown in figure (2).

If after passing once around the armature the winding falls in a slot to the left of its starting point (i.e., A', B') then the winding is said to be retrogressive. If it falls one slot to the right it is progressive.

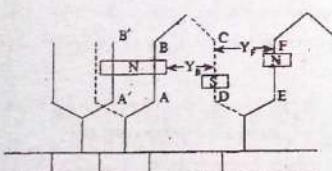


Figure (2)

Notes :

The following are the important points to be remembered pertaining to wave winding.

- Both pitches Y_e and Y_s are odd and of same sign.
- Back and front pitches may be equal or differ by 2 and are merely equal to pole pitch.

$$(iii) \text{ Resultant pitch, } Y_r = Y_p + Y_s = \frac{Z \pm 2}{P}$$

P = Number of poles

Z = Total number of conductors.

$$(iv) \text{ Commutator pitch, } Y_c = Y_A \text{ (Average pitch)}$$

$$= \frac{\text{Number of commutator bars} \pm 1}{\text{Number of pair of poles}}$$

(v) Number of parallel paths are equal to $2m$, where m is the multiplicity.

(vi) The number of brushes required are two irrespective of the number of poles.

(vii) If I_a is the total armature current then current carried by each path or conductor is $I_a/2$.

Q. 3. State the purpose of magnetic yoke in a D.C. machine.

Ans. 1. The magnetic yoke or the frame is the outermost metal structure, which provides mechanical strength to the whole machine and acts as a protective cover.

2. The field poles are held firmly on the inner side of yoke.

3. The yoke offers low reluctance path for the magnetic flux produced by the main poles.

4. The internal parts such as rotor, armature winding etc., are protected from external damage by this magnetic yoke.

Q. 4. Write the principle of working of a D.C. generator.

Or,

Explain the generating action of a D.C. machine.

Ans. D.C. generator is a D.C. machine which takes mechanical energy as input and converts it into electrical energy as output. D.C. generator works on the principle of Faraday's law of electromagnetic induction i.e., whenever a moving conductor is placed in a magnetic field, dynamically induced e.m.f. is produced in the conductor or whenever a conductor cuts the magnetic flux dynamically, e.m.f. is produced. As a result, this e.m.f. causes a current to flow in the conductor if the conductor circuit is closed. Therefore, for a machine operating as a generator, external driving force is given as input and a D.C output is obtained.

Q. 5. List any four comparisons between lap and wave windings.

Or,

Differentiate lap and wave windings.

Ans. Lap Winding

1. In this winding all the pole groups of the coils generating e.m.f. in the same direction at an instant of time are connected in parallel by the brushes.
2. Lap winding is also known as parallel winding.
3. The number of parallel paths is equal to the number of poles i.e., $A = p$.

4. The number of brushes required by this winding is always equal to the number of poles.

Wave Winding

1. In this winding all the coils carrying current in the same direction are connected in series i.e., coils carrying current in one direction are connected in one series circuit and coils carrying current in opposite direction are connected in other series circuit.
2. Wave winding is also known as series winding.
3. The number of parallel paths is always equal to 2, i.e., $A = 2$.
4. The number of brushes required by this winding is always equal to 2.

Q. 6. What purpose is served by the pole shoe in a D.C machine ?

Ans. The field magnets in a D.C machine consists of pole cores and pole shoes. The cross-section of pole core is small than that of pole shoe. The purpose served by the pole shoe in a D.C machine are as follows :

- (i) It provides mechanical strength and support to the field winding.
- (ii) The pole shoes spread out the flux in the air gap i.e., flux per pole entering the armature increases and hence reduces the reluctance of the magnetic path in air gap.
- (iii) Due to reduced cross-section of the pole core, less copper is required for the field winding.

Q. 7. What is armature reaction ? List the different effects of it.

Ans. Armature Reaction : The effect of magnetic field set up by the armature current on the distribution of flux under the main poles of a D.C generator or a D.C motor is known as armature reaction.

Effects : The armature m.m.f produces two undesirable effects on the main field. They are,

1. Distortion of the main field flux wave along the air gap periphery i.e., cross magnetization.
2. Net reduction in the main field flux per pole i.e., demagnetization.

Q. 8. Discuss the role of interpoles in improving commutation.

Ans. 1. Interpoles are small in size compared to main poles of D.C machine and placed in between them.

2. Cross magnetizing effect in interpolar region is minimized by interpoles.

3. Interpoles are connected in series with the armature winding placed on the yoke.

The magnitude of m.m.f produced by interpoles is such that it is sufficient to neutralize the armature core flux and also reactance e.m.f. of the coil to be commutated.

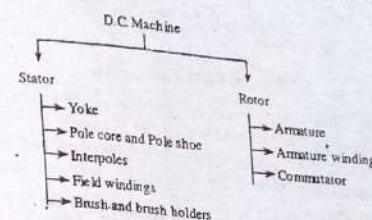
Hence, no current flows in the coil to be commutated which results in sparkless commutation.

Thus, interpoles are helpful in improving the commutation when its developed flux is sufficient to neutralize the cross magnetizing effect as well as reactance e.m.f. of the coil to be commutated.

Q. 9. With neat diagram give the constructional features of D.C machine.

Ans. A Direct Current (D.C) machine is an energy conversion device. It converts electrical

energy to mechanical energy while working as a motor and mechanical energy to electrical energy while working as generator. Hence, it is known as electromechanical energy conversion device. It consists of stator and rotor. Stator is the assembly of main parts like yoke, main poles, pole shoe, inter pole windings etc. and the rotor is the assembly of armature, winding, etc.



Yoke : Yoke is the outer cover of the machine supporting and protecting the internal parts. It is made of low reluctance material like silicon steel or cast iron, since, it has to carry the magnetic flux i.e., to provide the closed for the flux produced through the poles.

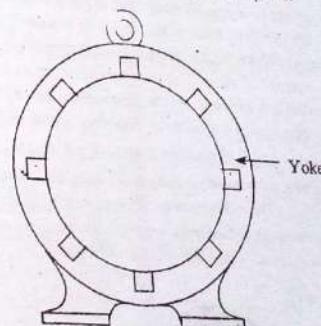


Figure (1): Yoke

Pole Core and Pole Shoe : Pole core is generally a solid material and pole shoe is a laminated one in small machines, but the pole shoe and pole core both are laminated made of annealed steel in modern days. The purpose of the pole core is to provide flux and to support the field windings, whereas the pole shoe is stretched so as to provide uniform air gap along the armature core and also to provide uniform flux distribution in the air gap.

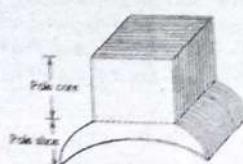


Figure (2): Pole Core and Pole Shoe

Brush and Brush Holders : Brushes are the structures placed on the rotating commutator through which the unidirectional current is to be collected. Generally it is made of carbon, which can give smooth surface at the contacts so as to reduce the spark and wear and tear of the commutator bars. These are fixed to the stator core (yoke) by means of brush holders.



Figure (3): Brush



Figure (4): Wound Field Coil

Field Windings : The field windings are wound initially on a wooden former and then installed into the pole core. These are generally made of low resistivity materials like copper or aluminium. There are two ways of connecting the field winding to the armature in case of self-excited machine. They can be connected in series or shunt. If it is connected in series less numbered turns with larger cross-sectional conductors are used. If it is connected in shunt, the winding would be of large turns, whose cross-section is less, so as to withstand for whole supply voltage.

Interpoles : These are the pole structures generally smaller than main poles and is placed in between the main poles. These windings of the interpoles are of less turns since it is connected in series with armature windings. The main purpose of these inter poles is to reduce the armature reaction, thereby reducing the sparks at the brush contacts. The polarity of the inter poles is made same as that of the main pole ahead of it in the direction of rotation.

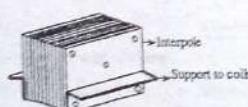


Figure (5): Interpole and Support to Coil

Rotor : The rotor is generally rotating part which carries the armature, armature windings and the commutator on the same shaft. The armature core is made of laminated silicon steel. The main purpose is to hold the armature windings and to provide the low reluctance path for the flux.

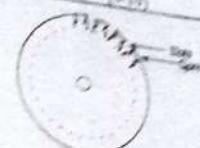


Figure (6): Armature Core (End View)

Armature Windings : The windings placed in the armature are called the armature windings and are generally classified into two ways,

- (i) Lap windings
- (ii) Wave windings

Lap windings are preferred for higher currents and lower voltages and wave windings are preferred for higher voltages and lower currents.

Q. 10. Derive the e.m.f. equation of a D.C generator.

Or,

Derive the expression of e.m.f. generated in case of generator from the first principles.

Or,

Derive the expression of e.m.f. generated in case of D.C machine.

Ans. E.M.F Equation

Let, ϕ = Flux per pole in webers

Z = Total number of armature conductors

= Number of slots + Number of conductors per slot

p = Number of poles

A = Number of parallel paths in armature

N = Armature rotation in r.p.m

E = E.M.F induced in any parallel path of armature.

As per Faraday's law of electromagnetic induction,

Average e.m.f. generated per conductor

$$= \frac{d\phi}{dt} \text{ Volts } (\because N = 1) \quad \dots (1)$$

During one revolution of armature in a ' P ' pole generator, each armature conductors cuts the magnetic flux ' P ' times so that flux cut per one conductor in one revolution is,

$$d\phi = \phi p \text{ Webers} \quad \dots (2)$$

Armature revolves $\frac{60}{N}$ times in one second. Therefore the time required by it for one revolution is dt .

$$\text{i.e., } dt = \frac{60}{N} \text{ seconds} \quad \dots (3)$$

Substituting equations (2) and (3) in equation (1), we get,

$$\text{i.e., e.m.f. generated/conductor} = \frac{d\phi}{dt} = \frac{\phi p N}{60} \text{ Volts.}$$

The total number of armature conductors per parallel path = $\frac{Z}{4}$

$$\therefore \text{Total e.m.f. generated/path} = \frac{\phi p N}{60} \times \frac{Z}{A}$$

$$E = \frac{\phi p N Z}{60 A}$$

Where, A = for simplex wave winding

$A = p$ = Number of poles for simplex lap windings.

Q. 11. Explain the concept of armature reaction and how it overcomes.

Or,

What is armature reaction ? What are the effects of armature reaction ? How the armature reaction is minimized ?

Or,

Illustrate the effect of armature reaction with neat diagrams.

Or,

What do you understand by demagnetizing and cross magnetizing effects of armature reaction in D.C. machine ?

Or,

What is armature reaction ? Describe the effects of armature reaction on the operation of D.C. machine.

How armature reaction is minimized ?

Or,

With the help of neat sketches, explain the effect of armature reaction on the air gap flux in a D.C. generator.

Ans. Reduction in main field flux per pole reduces the generated voltage and torque, whereas distortion of main field flux gives three harmful effects. They are increase in iron losses, poor commutation and sparking.

Consider a two-pole machine as shown in figure (1) at no-load i.e., having no armature currents. The main field flux is shown on a horizontal phasor OA which is produced by field m.m.f. (I/N_s).

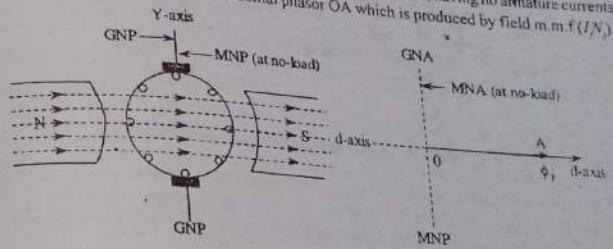


Figure (1)

The geometrical neutral plane and the magnetic neutral plane are coincident at no-load, i.e., magnetic lines of force intersect the MNP at right angles.

If D.C. machine is loaded then the armature winding receives the current. These currents are shown in figure (2) by dots under south pole and by crosses under north pole. These currents setup armature flux. Armature flux ϕ_a is shown by a vertical phasor OB. ϕ_a is produced by armature m.m.f. I/N_a . If the D.C. machine is working as generator, then its armature must be driven clockwise by prime mover and anti-clockwise for motoring operation.

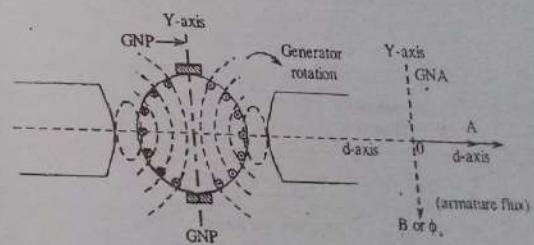


Figure (2)

From figure (2) it is seen that ϕ_a is perpendicular to ϕ_m , i.e., the path of armature flux crosses the path of main field flux. This effect is known as cross magnetizing effect.

If the current is flowing in both the winding, the resultant flux distribution is obtained by superimposing the two fluxes as shown in figure (3).

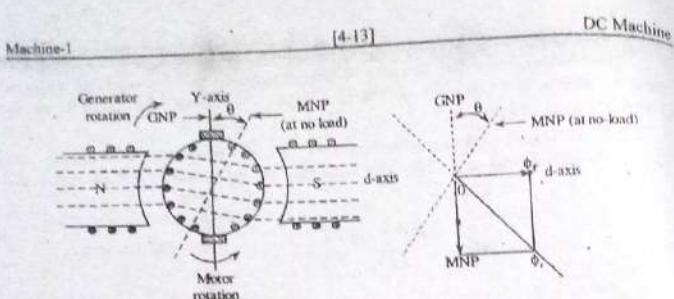


Figure 3: Resultant of Main Field and Armature Fluxes

From above, it is clear that armature flux aids the main field flux at upper end of north pole and at lower end of south pole. The D.C machine practically gets saturated and the strengthening effect is very low as compared with weakening effect and the resultant flux get decreased from its no-load value.

This effect on armature flux is called demagnetizing effect. This effect reduces the total flux/pole and found to be 1 to 5% from no-load to full-load.

Methods to Minimize Cross Magnetization : The cross magnetizing effect of armature m.m.f can be reduced at the design and construction stage of a D.C machine. They are as follows:

(a) **High Reluctance Pole Tips :** By flattening the pole faces slightly so that the air gap is longer at the pole tips rather than at the centre of the pole results in increase in reluctance of the pole tips and the magnitude of armature cross flux is reduced and the distortion of the resultant flux density wave is minimized. This can be achieved by using chamfered or eccentric pole face.

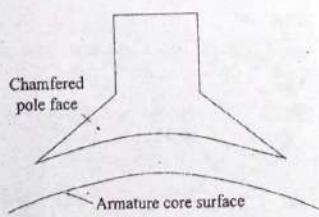


Figure 4

(b) **Reduction in Armature Flux :** To reduce armature cross flux without reducing the main field flux, it is required to create more reluctance in the path of armature flux. This is done by using field pole laminations as shown in figure (5) having four rectangular poles punches.

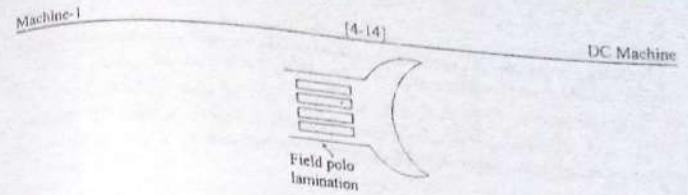


Figure 5

The reluctance offered to armature flux is more pronounced due to four air gap openings introduced in the path of cross flux.

(c) **Strong Main Field Flux :** During the design of D.C machine, it should be ensured that main field m.m.f should be strong when compared to full load armature m.m.f. The distortion produced by armature cross flux can be minimized by increasing the ratio of main field m.m.f to the full load m.m.f.

(d) **Interpoles :** Interpoles are small poles placed in between the main poles. These are connected in series with armature, so that they carry armature current. The e.m.f. induced by the interpoles neutralizes the effect of armature m.m.f in the interpolar region, thus making commutation sparkless.

(e) **By Using Compensating Winding :** A compensating winding is an auxiliary winding embedded in slots located in the faces of main poles. This winding is connected in series with armature in such a manner so that the direction of current flowing in this winding should be quite opposite to the direction of current flowing in the armature conductors. The m.m.f produced by the compensating winding should be equal to the m.m.f produced by the armature conductors. To maintain a uniform distribution of flux in the main poles and to neutralize the effect of armature reaction, compensating windings are provided. This winding adds cost of the machine and doubles the armature copper loss, but it makes the machine to withstand the most violent fluctuations of load that is applied to it.

Q. 12. Explain D.C motor principle and its working.

Ans. Principle of Operation of a D.C Motor : D.C motor is a D.C machine for which electrical energy is given as input and mechanical energy is obtained as output. D.C motor works on the principle of Faraday's Laws of electromagnetic induction i.e., whenever a current carrying conductor is placed in a magnetic field, it experiences a mechanical force.

The magnitude of the mechanical force experienced is given as,

$$F = Bl/I \text{ newton}$$

Where, B is the flux density (Wb/m^2 or Tesla)

I is the current flowing through the conductor (Ampere)

l is the length of the conductor (Meters).

Also the direction of force experienced is given by Flemings left hand rule.

Machine-1

[4-15]

DC Machine

Working of a D.C Motor: The force on a current carrying conductor of length l meters and carrying current I amperes in a magnetic field of flux density B Wb/m^2 is.

$$F = BIl \text{ Newtons}$$

When a D.C machine is connected to a voltage supply, the action of an armature with current carrying conductors under magnetic field flux is as shown in the figure (a).

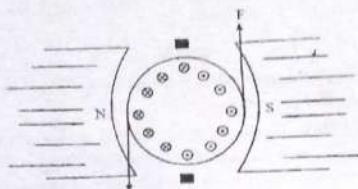


Figure (a): Armature Current with Stator Field of D.C Motor

Inward direction ← and → Outward direction

Figure (b): Directional Arrow of the Current

Under the north pole, the direction of current carrying conductors is into the plane of this paper and is denoted by the crosses which are the ends of the directional arrows of the current. The current in the conductors under the south pole have a direction out of the plane of the paper and are denoted by dots which are the heads of the directional arrows of the current. The magnetic flux of the armature core is at right angles to the main field flux. By applying Fleming's left hand rule, the armature rotates in anti-clockwise direction.

The physical phenomenon is explained below,

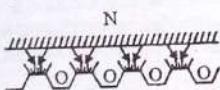


Figure (c): Field Flux without Armature Current

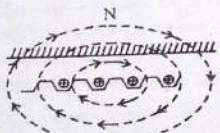


Figure (d): Armature Flux without Field Flux

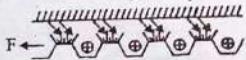


Figure (e): Resultant Field Flux with Armature Flux

Machine-1

[4-16]

DC Machine

- When there is no armature current, the field flux converges symmetrically and enters the teeth of the armature core.
- When there is no field flux, the armature flux is clockwise around the conductors in the armature core.
- When the armature flux and field flux act together, the resultant flux is inclined to the teeth of the armature core.
- The resultant flux tries to straighten itself by pulling the teeth of the armature core and causing the armature to move with a force F . The force F multiplied by the radius gives the torque and sum of all such torque is the total torque.

Q. 13. Classify the D.C motors based on field excitation with neat figures.

Ans. Based on field excitation the D.C motors are mainly classified into two types. They are,

1. Separately excited D.C motor.
2. Self excited D.C motor.

1. Separately Excited D.C Motor

The field coil and armature of these motors are supplied from different supply sources.

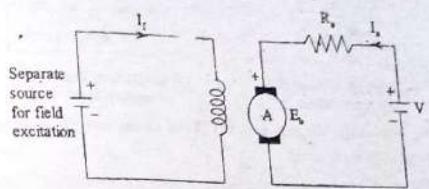
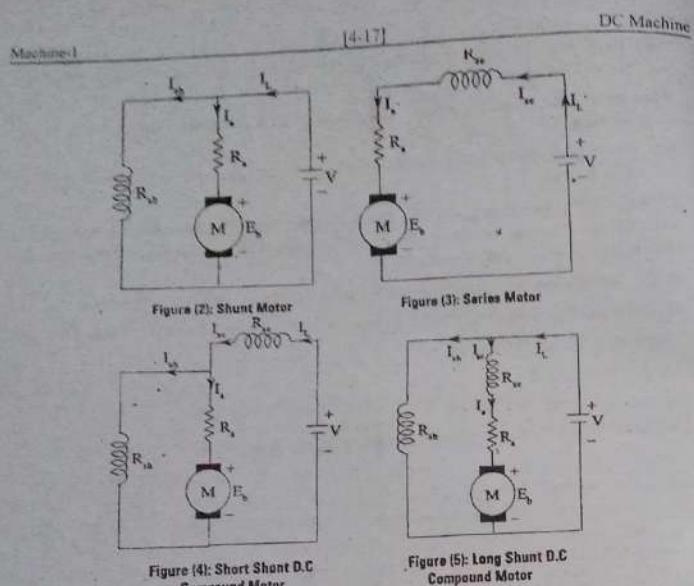


Figure (f)

2. Self Excited D.C Motor

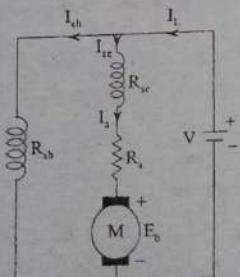
The field coils of these motors are supplied power from the armature. The different types of self excited D.C motors are,

- (a) Shunt motor
- (b) Series motor
- (c) Compound motor
 - (i) Short shunt D.C compound motor
 - (ii) Long shunt D.C compound motor.



Q. 14. Draw the schematic diagram of D.C long shunt motor. Also write the back e.m.f., current and voltage equations.

Ans. D.C long shunt motor comes under the category of D.C compound motors. In long shunt D.C compound motors, the series field winding is connected in series with the armature and this combination is connected in parallel with the shunt field winding. The schematic diagram of D.C long shunt compound motor is shown in figure below,



Machine-1 [4-18] DC Machine

$$\text{Armature current, } I_a = \text{Series field current, } I_s = I_t - I_{sh}$$

$$\text{Shunt field current, } I_{sh} = \frac{V}{R_{sh}}$$

$$\begin{aligned} \text{Terminal voltage, } V &= E_b + I_a R_a + I_{sh} R_{sh} \\ &= E_b + I_a R_a + I_a R_{sh} \\ &= E_b + I_a (R_a + R_{sh}) \end{aligned}$$

$$\Rightarrow \text{Back e.m.f. } E_b = V - I_a (R_a + R_{sh})$$

If brush drop is considered then we get,

$$\text{Back e.m.f. } E_b = V - I_a (R_a + R_{sh}) - \text{Brush drop.}$$

Q. 15. Derive the torque equation of D.C. Motor.

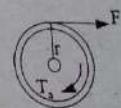
Or,

Derive an expression for the electromagnetic torque produced by D.C motor.

Or,

Derive the expression for torque developed in the armature of a D.C motor. State the factors on which the torque depends.

Ans. Torque Developed in a D.C. Motor :



Let,

F – Force in Newton

r – Radius of armature in meter

T_a – Armature torque in N-m

S – Circumferential distance

Since, torque is the twisting movement produced across the armature.

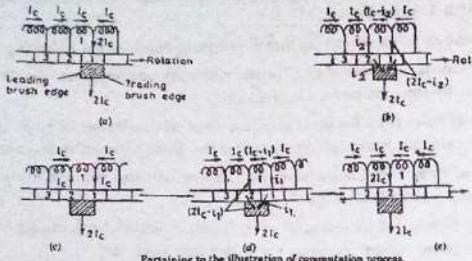
Let,

N – Speed of armature in r.p.m.

$$\frac{N}{60} \text{ – Speed in r.p.m.}$$

$$\frac{60}{N} \text{ – Time taken for one revolution.}$$

Mechanical work done per second,



Pertaining to the illustration of commutation process.

In Fig (a) the brush is fully on bar 1 and coil 1 carries current $I_c = \frac{I_a}{a}$ from L (left) to R (right). The brush delivers a current $2I_c$. The direction of armature rotation is taken from left to right.

As soon as the brush makes contact with bar 2, coil 1 gets short-circuited and current in it starts decreasing from I_c . The current from bar 2 to brush is, say i_2 , and, therefore, the current in coil is $I_c - i_2$ from L to R. Bar 1 delivers $2I_c - i_2$ to the brush so that the output current is again $2I_c$, Fig (b). If the area of copper-carbon contacts decide the distribution of current, then i_2 would increase and $2I_c - i_2$ would decrease linearly. When brush makes equal areas of contacts with bars 1 and 2, each bar delivers I_c to the brush and coil 1 carries no current, Fig. (c). With further rotation of the armature and bars, area of contact between bar 1 and brush, starts decreasing and therefore the current delivered by bar 1 decreases from I_c to say i_2 . Now the coil 1 carries current $I_c - i_2$ from R to L, Fig (d) and the current carried by bar 2 is $(2I_c - i_2)$ so that output current is again $2I_c$ as before. When brush breaks contact with bar 1 and is fully on bar 2, the short circuit of coil 1 is over and it carries current I_c from R to L as shown in Fig (e). The time required by the coil current to change from $+I_c$ to $-I_c$ is called the commutation period T_c . In other words, the commutation period may be defined as the time measured from the instant the brush is fully on bar 1, to the instant the brush is fully on bar 2. It can be computed from the relation,

$$T_c = \frac{\text{Brush width}}{\text{Commutator peripheralspeed}}$$

The nature of current flowing in the local circuit of the coil being commutated, depends on the following factors :

- (a) Resistance of the copper-carbon contacts.
- (b) Resistance of the coil being commutated.
- (c) e.m.f.s. induced in the commutated or short circuited coil, due to its :-

(i) self-inductance and
(ii) mutual inductance with other coils undergoing commutation simultaneously. Note that the e.m.f due to mutual inductance effect is present only when the brush width is more than one bar width.

(d) e.m.f induced in the coil due to its rotation in the armature cross flux.

Q.17. On what factors does the current in commutated coil depends ?

Ans. : Resistance Commutation :- In the present section, the effect of e.m.fs induced in the commutated coil is ignored—however their effect is discussed qualitatively at a later stage. For the time being, the effect of armature coil resistance and brush contact resistance is only taken into account. For studying the effect of these resistance on the commutation process, and let

$$R_c = \text{coil resistance}$$

$$r_1 = \text{resistance between bar 1 and brush}$$

$$\text{and } r_2 = \text{resistance between bar 2 and brush}$$

With no e.m.fs. induced in the commutated coil as assumed before, the Kirchhoff's voltage law, for the local circuit consisting of brush, bar 1, coil 1 undergoing commutation and bar 2, from Fig. (b), is

$$(2I_c - i_2)r_1 + (I_c - i_2)R_c - i_2r_2 = 0 \quad \text{or} \quad i_2 = \frac{R_c + 2r_1}{R_c + r_1 + r_2}$$

In coil 1 the current is given by

$$i_c = I_c - i_2 = I_c \left[1 - \frac{R_c + 2r_1}{R_c + r_1 + r_2} \right]$$

$$\text{or } i_c = I_c \left[\frac{r_2 - r_1}{R_c + r_1 + r_2} \right] = I_c \left[\frac{r_1 + r_2 - 2r_1}{r_1 + r_2 + R_c} \right]$$

$$\text{or } i_c = I_c \left[\frac{1 - \frac{2r_1}{r_1 + r_2}}{1 + \frac{R_c}{r_1 + r_2}} \right] \quad \dots \dots \dots (i)$$

If coil resistance R_c is small as compared with the copper-carbon resistance r_1 and r_2 , then $\frac{R_c}{r_1 + r_2}$ may be neglected. With this, the coil current, from Eq (i) is given by

$$i_c = I_c \left[1 - \frac{2r_1}{r_1 + r_2} \right]$$

If A_1 and A_2 are the areas, between bar 1 and brush, and between bar 2 and brush respectively, then since resistance is inversely proportional to area, we get coil current i_c as

Machine-I [4-23] DC Machine

$$i_c = I_c \left[1 - \frac{2 \frac{1}{A_1}}{\frac{1}{A_1} + \frac{1}{A_2}} \right] = I_c \left[1 - \frac{2A_2}{A_1 + A_2} \right] \quad \dots \dots \dots \text{(ii)}$$

With the rotation of the commutator to the right, area A_1 decreases and area A_2 increases linearly.

Thus, at time $t = 0$, the brush is fully on bar 1,

$$\therefore A_2 = 0 \text{ and } i_c = I_c$$

$$\text{At time } t = \frac{T_c}{4}, A_1 = 3A_2, \therefore i_c = \frac{1}{2} I_c$$

$$\text{At time } t = \frac{T_c}{2}, A_1 = A_2; \therefore i_c = 0$$

$$\text{At time } t = \frac{3T_c}{4}, 3A_1 = A_2, \therefore i_c = -\frac{1}{2} I_c$$

At time $t = T_c$, brush is fully on bar 2,

$$A_1 = 0 \text{ and } i_c = -I_c$$

It is also obvious from fig. (e) that at $t = T_c$ i.e., just after the commutation of coil 1 is over $i_c = -I_c$.

The plot of coil current variation with $R_c = 0$ is shown in Fig. Under such a condition, there will be no sparking at the brush and the commutation is referred to as the straight line or linear commutation, because the current varies linearly with T_c .

If coil resistance R_c is not neglected, then from Eqs. (i) and (ii),

At time $t = 0$, brush is fully on bar 1, $\therefore i_c = I_c$ as before.

$$\text{At time } t = \frac{T_c}{4}, A_1 = 3A_2; i_c = \frac{I_c/2}{1 + \frac{R_c}{R_i}}, \text{i.e., } i_c < \frac{I_c}{2}$$

$$\text{At time } t = \frac{T_c}{2}; A_1 = A_2, i_c = \frac{\frac{R_c}{2} + r_2}{1 + \frac{R_c}{R_i} + r_1 + r_2}, \text{i.e., } i_c = 0 \text{ as before}$$

$$\text{At time } t = \frac{3T_c}{4}, 3A_1 = A_2, i_c = \frac{-\frac{1}{2} I_c}{1 + \frac{R_c}{R_i} + r_1 + r_2}, \text{i.e., } i_c < -\frac{I_c}{2}$$

and at time $t = T_c$, brush is fully on bar 2, $\therefore i_c = -I_c$ as before.

The variation of coil current i_c with R_c included, is shown in Fig. Such type of commutation is referred to as the resistance commutation.

In fractional kW d.c. machines, resistance commutation provides good commutation and this is achieved by using carbon brushes so that

Machine-I [4-24] DC Machine

- (i) copper-carbon resistance is larger as compared with the coil resistance and
- (ii) The brush contact drop is larger as compared with the e.m.f.s induced in the commutated coil.

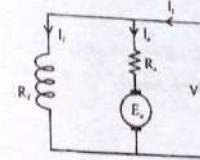
Q.18. Derive expression for torque-speed characteristics of a d.c. shunt motor.

Ans.: Torque speed

$$E_a = V_t - I_a R_a = k'_a \phi n$$

$$T = ka \phi I_a$$

$$I_a = \frac{T}{k_a \phi}$$



From eqn. (i)

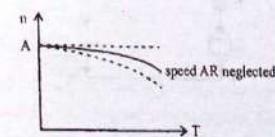
$$n = \frac{V_t}{k'_a \phi} - \frac{T}{k_a \phi} \left(\frac{1}{k'_a \phi} \right)$$

if V_t, ϕ are constant, then we get

$$A = \frac{V_t}{k'_a \phi}, B = \frac{1}{k_a \phi^2 k'_a}$$

$$n = A - TB$$

from eqn.(ii)

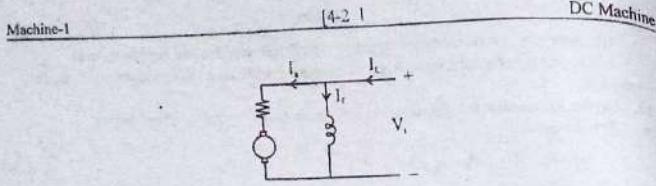


Q.19. A 4-pole shunt generator with lapconnected armature having field and armatures of 50Ω and 0.1Ω respectively supplies sixty 100 V, 40 watt lamps. Calculate the total armature current per path and generated e.m.f. Allow a contact drop of 1 V per brush.

Ans.: $P = 4, a = p, R_f = 50\Omega, R_a = 0.1\Omega$

$$\text{load power} = 40 \times 60 = 2.4 \text{ kW}$$

$$I_a = ?, V_t = 100 \text{ V}, E_a = ?, V_b = 1 \text{ V}$$



$$I_f = \frac{V_t}{R_f} = \frac{100}{50} = 2A ; \quad I_L = \frac{P_L}{V_t} = 24A$$

$$\Rightarrow I_a = I_L + I_f = 26A$$

$$\Rightarrow E_a = V_t + I_a R_a$$

$$\Rightarrow E_a = 100 + 26 \times 0.1 = 102.6V$$

I_{f1} = per path armature current,

$$\Rightarrow I_{f1} = \frac{I_a}{4} = \frac{26}{4} = 6.5A$$

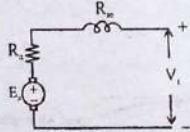
if we consider voltage drop of 1V,

$$E_a = V_t + I_a R_a + V_h = 102.6 + 1 = 103.6V$$

Q.20. A 240 V d.c. series motor takes 40 A when giving its rated output at 1500 r.p.m. Its resistance is 0.3Ω. Calculate the value of resistance that must be added to obtain the rated torque

(i) at starting and (ii) at 1000 r.p.m.

Ans.: $V_t = 240V$, $I_2 = 40A$, $n_1 = 1500$ r.p.m., $R_a + R_{se} = 0.3\Omega$, $R = ?$



(i) at starting

$$E_a = 0$$

$$\Rightarrow V_t = I_a(R + R_a + R_{se}) \Rightarrow 240 = 40(R + R_a + R_{se})$$

$$\Rightarrow R = 6 - 0.3 = 5.7\Omega$$

$$(ii) n_2 = 1000 r.p.m.$$

$$\frac{\phi_2}{\phi_1} = \frac{I_{f2}}{I_{f1}} = \frac{I_{L2}}{I_{L1}} = \frac{I_{a2}}{I_{a1}}$$

Machine-1 [4-2-1] DC Machine

$$\Rightarrow \frac{E_{a2}}{E_{a1}} = \frac{n_2 \phi_2}{n_1 \phi_1} \Rightarrow \frac{E_{a2}}{E_{a1}} = \frac{V_t - I_{a2}(R + R_a + R_{se})}{V_t - I_{a1}(R_a + R_{se})}$$

for rated torque, $I_{a1} = I_{a2}$

$$\Rightarrow \frac{E_{a2}}{E_{a1}} = \frac{1000}{1500} = \frac{2}{3} \Rightarrow \frac{2}{3} = \frac{240 - 40(R + 0.3)}{240 - 40(0.3)}$$

$$\Rightarrow R = 1.9\Omega$$

Q.21. The magnetization characteristic for a 4-pole, 110 V, 1000 r.p.m. shunt generator is as follows:

Field current :	0	0.5	1	1.5	2	2.5	3
OC voltage :	5	50	85	102	112	116	120

Armature is lap-connected with 144 conductors. Field resistance is 45Ω. Determine

(i) voltage the machine will build up at no-load, (ii) the critical resistance, (iii) the speed at which the machine just fails to excite and (iv) the residual flux per pole.

Ans.: $P = 4$; $V = 110V$; $\eta = 1000$ r.p.m.; $a = P$
 $z = 144$; $R_f = 45\Omega$

(i) $I_{L1} = 0$

$$V_0 = 110V, E_{a0} = 120V$$

(ii) $R_{cr} = \frac{50}{0.5} = 100\Omega$

(iii) $\eta_2 = \eta_c = \text{critical speed}$
 $\eta_c = ?$

$$V_{fA} = R_f I_{f1} = 45 \times 1 = 45V$$

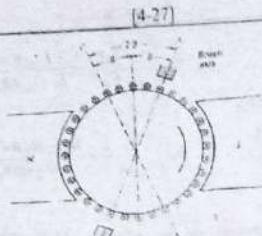
$$V_{acc} = 85V$$

$$\eta_c = 1000 \times \frac{45}{85} = 529.4 \text{ r.p.m.}$$

Q.22. Describe cross magnetising and demagnetising ampere turns in detail.

Ans.: Demagnetizing and Cross Magnetizing Ampere Turns: We have seen that the generating voltage in the commutating coils should be made to oppose the commutating coils cut a flux in the direction as that in the post-commutation period for a dc generator and that in the precommutation period for dc motor. For this purpose, the brushes may be shifted from the GNP through an angle β° electrical in the direction of rotation for a generator. For a motor the brushes are given a backward shift.

The nature of demagnetizing and cross-magnetizing ampere turns can be calculated by considering.



The defining eqn, for the separately excited dc generator are as follows:

$$V_f = R_f I_f$$

$$E_a = V + I_f R_a$$

$$E_a = K_a \Phi \omega_n$$

$$V = I_L R_L$$

$$I_a = I_L$$

If there were no armature reaction, the generated voltage V_0 would be constant as shown by a straight line in fig. Because of the demagnetizing effect of armature reaction there is a voltage drop ΔV_{AR} .

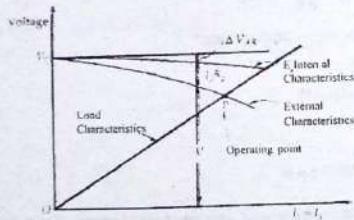


Fig. Terminal characteristics of a separately excited dc generator.

There is a voltage drop $I_a R_a$ across R_a . The generator external characteristic ($V - I_f$) defined by the relation

$$V = E_a - I_a R_a$$

is shown in fig. The point of intersection between the generator external characteristic and the load characteristic given by the relation $V = I_f R_L$ determines the operating point P. The operating point gives the operating values of terminal voltage V and terminal current I_L .

Q.23. Write short notes any two.

(a) Present day uses of DC

Ans.: (a) **Present day uses of DC Machines:** At present time bulk of electrical energy is generated in the form alternating current. Hence the use of d.c. generators is very limited. They are mainly used in supplying excitation of small and medium range alternators. For motor drive, the present trend is to generate a.c. and then to convert a.c. into d.c. by rectifiers. Thus, dc generators have generally been superseded by rectified ac supplies for many applications.

Direct current motor are very commonly used as variable-speed drives and in applications where severe torque variations occur.

The main applications of the three types of d.c. motors are given below.

Series motors. These motors are used where high starting torque is required and speed can vary, for example, traction cranes, etc.

Shunt motors. These motors are used where constant speed is required and starting conditions are not severe, for example, lathes, centrifugal pumps, fans, blowers, conveyors, lifts etc.

Compound motors. These motors are used where high starting torque and fairly constant speed is required, for example presses, shears, conveyors, elevators, rolling mills, heavy planers etc.

Small d.c. machines are used primarily as control devices such as tachogenerators for speed sensing and servomotors for positioning and tracking.

Q.24. A 250 V d.c. shunt motor runs at 700 r.p.m. on no-load with no extra-resistance in the field and armature circuit. Determine-

(i) the resistance to be placed in series with the armature for a speed of 400 r.p.m. when taking a total current of 16 a.m.p.;

(ii) the resistance to be placed in series with the field to produce a speed of 1000 r.p.m. when taking an armature current of 18 a.m.p.

Armature resistance = 0.5Ω , Field resistance = 150Ω . Assume that the useful flux per pole is proportional to the field current.

Ans.: 250 V, $R_a = 0.5\Omega$, $R_f = 150\Omega$

$$n_1 = 700 \text{ r.p.m.}$$

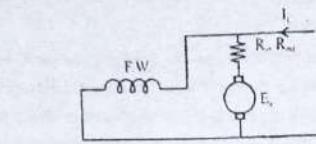
$$(i) n_2 = 400 \text{ r.p.m.}$$

$$R_{ext}, I_a = 16 \text{ amp}$$

Machine-1

[4-29]

DC Machine



$$I_f = \frac{250}{150} = 1.67 \text{ A}$$

$$I_a = 14.33 \text{ A}$$

$$E_{a1} = V_t = 250 \text{ V}$$

$$\Rightarrow E_{a2} = V_t - I_a (R_a + R_{sh}) \quad \Rightarrow \quad E_{a2} = 250 - 14.33(0.5 + R_{sh})$$

$$\frac{E_{a1}}{E_{a2}} = \frac{n_1}{n_2} = \frac{700}{400} = 1.75 \quad \Rightarrow \quad \frac{E_{a1}}{E_{a2}} = \frac{V_t}{V_t - I_a (R_a + R_{sh})}$$

$$\Rightarrow R_a + R_{sh} = 7.5 \Omega \quad \Rightarrow \quad R_{sh} = 7 \Omega$$

$$(ii) \quad n_2 = 1000 \text{ r.p.m.}$$

$$I_a = 18 \text{ A}$$

$$\Rightarrow I_{f1} = \frac{V_t}{R_f}, \quad I_{f2} = \frac{V_t}{R_f + R}$$

$$(I_{f1}/I_{f2}) = (\phi_1/\phi_2)$$

$$\Rightarrow \frac{\phi_1}{\phi_2} = \frac{R_f + R}{R_f} \quad \Rightarrow \quad \frac{E_{a1}}{E_{a2}} = \frac{n_1 \phi_1}{n_2 \phi_2}$$

$$\Rightarrow \frac{E_{a1}}{E_{a2}} = \frac{V_t}{V_t - I_a R_a} \quad \Rightarrow \quad \frac{250}{250 - 9} = \frac{7}{10} \times \frac{(150 + R)}{150}$$

$$\Rightarrow R = 72.3 \Omega.$$

Q.25. Explain the use of interpoles and compensating windings in a d.c. machine.

Ans.: Commutating Poles or Interpoles: Interpoles are narrow poles attached to the stator yoke, and placed exactly midway between the main poles. Interpoles are also called commutating poles or compoles. The interpole windings are connected in series with the armature, because the interpoles must produce fluxes that are directly proportional to the armature

Machine-1

[4-30]

DC Machine

current. The armature and interpole mmfs are affected simultaneously by the same armature current. Consequently, the armature flux in the commutating zone which tends to shift the magnetic neutral axis, is neutralized by an appropriate component of interpole flux. The neutral plane is, therefore, fixed in position regardless of the load.

Compensating Windings : Compensating windings are the most effective means for eliminating the problems of armature reaction reaction and flashover by balancing the armature mmf. Compensating windings are placed in slots provided in pole faces parallel to the direction of currents in the compensating winding must armature windings. The currents in the compensating winding must be opposite to that in the armature winding just below the pole faces. Thus, compensating winding produces an mmf that is equal and opposite to the armature mmf. In effect the compensating winding demagnetizes or neutralizes the armature flux produced by the armature conductors lying just under the pole faces. The flux per pole is then undisturbed by the armature flux regardless of the load conditions.

The major drawback with the compensating windings is that they are very costly. Their use can only be justified in the following special cases:

1. In large machines subject to heavy overloads or plugging.
2. In small motors subject to sudden reversal and high acceleration.

Q.26. What are the advantages and disadvantages of carbon brushes in a d.c. machine?

Ans.: Advantages brush D.C. motor :

The D.C. brush motor is one of the earliest of all electrical motor designs. It is usually the motor of choice for the majority of torque control and variable speed application. The following discusses the advantages and dis-advantages of using a brush d.c. motor in machinery and automated processes.

* The brush d.c. motor has a simple construction, Therefore may not require a controller. When a controller is chosen, it is typically a simple and inexpensive drive design.

* If the field created by permanent magnets, a brush d.c. motor is said to be a permanent magnet d.c. motor. If created by electromagnetic windings, the d.c. brush motor is often said to be a shunt wound brush d.c. motor. Today because of cost effectiveness and reliability, the PMDC motor is the motor of choice for the application involving of fractional horse power brush d.c. motor usual as most application upto about 2.0 horse power.

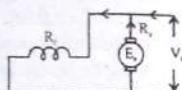
* Opposing the stator field is the armature field, which is generated by a changing electromagnetic flux coming from windings located on the rotor of the d.c. brush motor. The magnetic poles of the armature field will attempt to line up with the opposite magnetic poles generated by the stator field. Next, section of the rotor where the electricity enters the rotor winding is called the commutator. The electricity is carried between the brush motor rotor and the stator by conductive graphite - copper brushes which contact rings on stator.

Dis-advantages - brush d.c. motor :

- * A brush d.c. motor is less reliable in control at lowest speeds.
- * A brush d.c. motor is physically larger than other motors producing equivalent torque.
- * A brush d.c. motor is considered high maintenance, which is not true of brushless d.c. motors.
- * A brush d.c. motor is vulnerable to dust which decreases performance.

Q.27 A d.c. shunt machine when runs as a motor on no-load takes 400 W and runs at 1000 r.p.m. The field current and armature resistance are 1 A and 0.5Ω respectively. Calculate the efficiency of the machine, when running (i) as a generator delivering 40 A at 220 V and (ii) as a motor taking 40 A from a 220-V supply.

Ans.: $V_t, I_{f(n)} = 400W, n_t = 1000 \text{ r.p.m.}, I_f = 1A, R_a = 0.5\Omega$



(i) as a motor

$$I_L = 40A$$

$$V_t = 220V$$

$$I_{L(n)} = 1.82A$$

$$I_{a(n)} = 0.82A$$

$$P_t = (V_t, I_{a(n)} - R_a I_{a(n)}^2) + I_f, V_t$$

$$= 220 \times 0.82 - (0.82)^2 \times 0.5 + 1 \times 220 \\ = 400W$$

$$P_V = I_a^2 R_a = 760.5W$$

$$\eta_m = \frac{P_0 - P_{LS}}{P_0} = \frac{V_t I_{L(n)} - P_{LS}}{V_t I_{L(n)}} \times 100 = 86.82\%$$

(ii) as a generator

$$I_{L(n)} = 1.82A, I_{a(n)} = 1 + 1.82 = 2.82A$$

$$\Rightarrow P_k = V_t, I_{a(n)} - I_{a(n)}^2 R_a + I_f^2 R_f$$

$$\therefore P_k = 836.4$$

$$P_V = I_a^2 R_a = (40 + 1)^2 \times 0.5 = 840.5W$$

$$\eta_G = \frac{P_0}{P_0 + P_L} = \frac{V_t I_L}{V_t I_L + P_{LS}} = \frac{220 \times 40}{220 \times 40 + 840.5 + 836.4} = 0.8399 \times 100 = 83.99\%$$

Q.28. A dc series motor connected to a 440V supply, runs at 600 rpm, when taking a current of 50 A. Calculate the value of resistance which when inserted in series with the motor, will reduce the speed to 400 rpm, the gross torque being then half its previous value. The resistance of the motor is 0.2Ω

Ans. Before an extra resistance is introduced

$$E = V - I_a R_a$$

$$E = 440 - 50 \times 0.2$$

$$E = 430V$$

$$E = K_f \phi N = K_f T_a N \quad [\text{For a series motor } \phi \text{ at } I_a]$$

$$K_f = \frac{E}{T_a N} = \frac{430}{50 \times 600} = 0.143; \quad T = K_f \phi I_a = K_f I_a^2$$

When torque is half

$$\frac{T}{T_a} = \frac{K_f I_a^2}{K_f I_a^2} = \frac{50 \times 50}{(I_a)^2}$$

$$T_a = \sqrt{\frac{50 \times 50}{2}} = 35.35A$$

At this armature current I_a and with a resistance R introduced in the circuit, the induced emf E_1 is given by

$$E_1 = V - I_a (R_a + R)$$

$$E_1 = K_f I_a N_1 = 0.0143 \times 35.35 \times 400$$

$$E_1 = 202$$

$$E_1 = V - I_a (R_a + R)$$

$$220 = 440 - 35.35 (0.2 + R)$$

$$R = 6.53\Omega \quad \text{Ans.}$$

Q.29. Explain the process of voltage build up in a dc shunt generator. State the various conditions under which a dc shunt generator fails to excite.

Ans. Process of voltage build up in dc shunt Generator:- When the armature is rotated, the residual flux in field winding will induce small voltage in armature. The induced voltage in armature generates a flux and it will add with field flux and the net flux will increase further. This process will be repeating until the actual terminal voltage is reached. Once the terminal voltage is reached then the winding will get saturated and hence there won't be any further increase in flux, also the voltage gets constant.

Following are the various conditions under which a dc shunt generator fails to excite.

(i) **No residual Magnetism of field** : Rotating armature will not induce any voltage when there is no field present in the field poles so no voltage will not induce any field current and so no voltage will build up.

- (ii) **Open Field connection** : Field will not increase with time, as there is no current in the field. So armature voltage will not build up.
- (iii) **Field connection reversed** : In this the slight induced voltage at the armature will always try to oppose the direction of field and hence voltage will not increase.
- (iv) **Field circuit resistance too high** : Too high resistance will prevent the flow of required field current to raise the armature field voltage further.

Q.30. A dc generator is connected to a 220V dc mains. The current delivered by the generator to the mains is 100A. The armature resistance is 0.1Ω . The generator is driven at a speed of 400 rpm. Calculate (a) the induced emf (b) the electromagnetic torque (c) the mechanical power input to the armature neglecting iron, windage and friction losses (d) the power input and output of the armature when the speed drop to 350 rpm. State whether the machine is generating or motoring. Assume constant flux.

Ans. (a) $E = V + I_a R_a$
 $E = 220 + 100 \times 0.1$

[E = 230] Ans.

(b) $T_e = \frac{EI_a}{\omega} = \frac{230 \times 100}{\frac{2\pi \times 400}{60}}$
 $T_e = 549.09 \text{ N-m}$

(c) Mechanical power input =
 Electromagnetic power developed + Iron loss in the armature + windage and friction losses.

Neglecting iron, windage and friction losses

Input to armature = ωT or $E_g I_a = \frac{2\pi NT}{60} = \frac{2\pi \times 400 \times 549.09}{60}$

Mechanical power input = 22988.5 Watts

□□□

UNIT 5

DC machine – motoring and generation

Q. 1. What is good and bad commutation ? Give the causes of poor commutation.

Ans. Good Commutation : The commutation in which there is no sparking at the brushes is called good commutation. In a good commutation the surface of the commutator remains unaffected even for continuous operation of D.C machine. A good and satisfactory commutation is one in which the current in the coil undergoing commutation must reverse completely.

Bad Commutation : The commutation in which sparking occurs at the brushes is called a bad commutation or poor commutation. In a bad commutation the surface of the commutator gets damaged during the continuous operation of D.C machine. As a result of sparking, the temperature at the contact of brush and commutator increases sharply which may cause pitting of commutator segments.

Causes of Poor Commutation : Poor commutation is caused due to :

1. Poor mechanical conditions like uneven surface of commutators, vibrations of brushes, etc.
2. Poor electrical conditions such as increased voltage between the bars of commutators, increase in the current density of brushes, etc.

Q. 2. Draw the schematic diagram of separately excited D.C generator. Also write the current and voltage equations.

Ans. Electromagnet type D.C generators are mainly classified into two types. They are, separately excited D.C generator and self excited D.C generator. In separately excited D.C generator, the field winding is excited by a separate source of D.C supply and the armature supplies the load as shown in figure below.

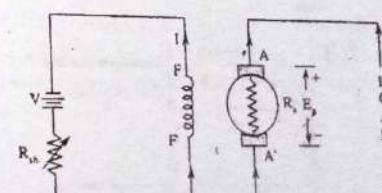


Figure: Separately Excited D.C Generator

Machine-I

[5-2] DC machine : motoring and generation

The e.m.f. generated in case of a separately excited D.C. generator will be the sum of the supply voltage the armature resistance drop. And armature current will be equal to the load current.

The current and voltage equations are as follows :

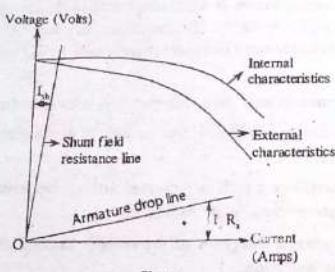
$$\text{Armature current } (I_a) = \text{Load current } (I_L)$$

$$\therefore I_a = I_L$$

$$\text{Generated E.M.F. } E_g = V + I_a R_a$$

Q. 3. Draw the external and internal characteristics of a separately excited D.C. generator.

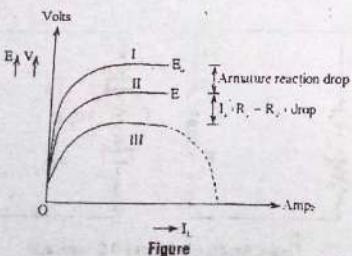
Ans. The variation of induced e.m.f. (E) of the armature with armature current (I_a) when the generator is operated at rated speed is called the internal characteristics. Whereas the variation of terminal voltage (V) with that of load current (I_L) when the generator is operated at rated speed is called the external characteristics of generator. The internal and external characteristics of a separately excited D.C. generator are shown in figure below.



Figure

Q. 4. Draw the OCC, internal and external characteristics of D.C. series generator.

Ans. The O.C.C, internal and external characteristics of a D.C. series generator are shown in figure below,



Figure

Machine-I

[5-3] DC machine : motoring and generation

In figure

Curve I-Represents the O.C.C.

Curve-II Represents the internal characteristics and

Curve-III Represents the external characteristics.

Q. 5. What is critical speed ? Explain the significance of critical speed.

Ans. Critical Speed : It is defined as the speed at which the given field resistance is equal to the critical resistance. It is the speed at which the generator just fails to build up its voltage without any external resistance in the field circuit. It is denoted by "N".

Significance : If the speed of the D.C. machine is reduced to a value such that the field resistance line does not pass through the O.C.C then the machine fails to induce i.e., it may not generate any appreciable voltage. The speed at which the generator fails to build up the voltage is known as the critical speed. Below this speed the field resistance line does not cut the O.C.C. Hence fails to build up the voltage. Therefore the machine should not be operated below the critical speed as it fails to induce. Hence, it should always be operated above the critical speed.

Q. 6. Draw the sketches for different methods of excitation of D.C. generators and write the respective generated emf equations.

Or,

What are the different types of D.C. generators according to the ways in which fields are excited ? Show the connection diagram of each type and obtain voltage and current relationship in each case.

Or,

What are the different types of D.C. generators according to the ways in which fields are excited ? Show the connection diagram of each type.

Ans. Types of D.C. Generators : A.D.C machine is an electromechanical energy conversion device. It requires magnetic flux conductors and the relative motion for the energy conversion. Based on the production of magnetic flux (i.e. exciting the field windings) the D.C. machines are classified as follows.

Machine-1

[5-4] DC machine : motoring and generation

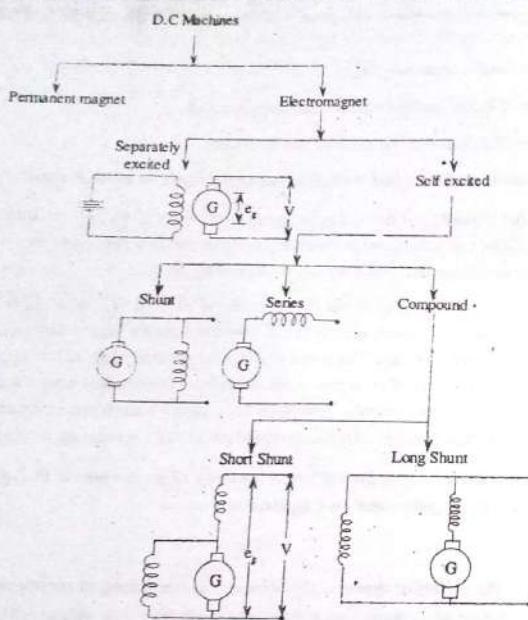


Figure (1)

1. Permanent Magnet Type Machines

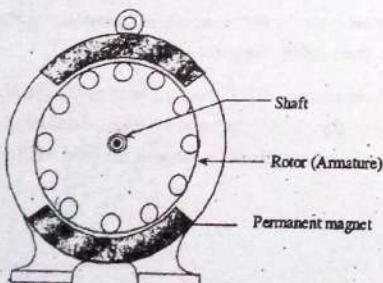


Figure (2): Permanent Magnet Type Machine

Machine-1

[5-5] DC machine : motoring and generation

This type of machines are of low rating and consists of the magnetic poles fixed in the inner periphery and the armature coils feed or being fed by the supply in case of generator and motor respectively.

2. Electromagnetic Type Machines

(a) Separately Excited

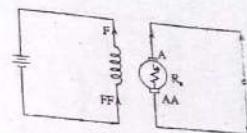


Figure (3)

In the separately excited machine the field windings are being fed by a separate D.C source (battery) and the e.m.f. generated in case of generator would be the sum of the supply voltage and the armature resistance drop.

(b) Self Excited : In case of self excited machines, the field windings are connected to the armature, so that the armature could feed the field coils. Let us consider, these conditions of connecting field winding in case of generators.

(i) Shunt Generator

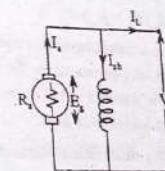


Figure (4)

If the field winding is connected across the armature winding then the machine is called the shunt machine. In the shunt generator, as the shunt winding has to overcome the generated voltage it has to be made with higher turns of lower cross-sectional conductors. They have higher resistance as compared to the series coils, but the current is less.

\therefore Line current = Armature current - Shunt field current

$$I_L = I_a - I_{sh} \text{ and}$$

$$E_g = V + I_a R_a$$

Generated e.m.f. = Terminal voltage + Armature resistance drop

Machine-1

[5-6] DC machine : motoring and generation

Where, I_a is the armature current = (Line current) and R_a is the armature resistance.

(ii) Series Generator : In this type of generator the field coils are connected in series with the armature terminals and the conductors would be of higher cross-section and with lesser number of turns.

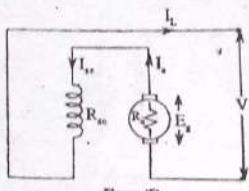


Figure (5)

The current flowing through the coil would be the same as that of the armature current. So, it is made with higher cross-sections.

Line current = Series field current

= Armature current

$$I_L = I_{sh} = I_a$$

Generated emf, E_g = Terminal voltage + Armature resistance drop + Series drop

$$\begin{aligned} E_g &= V + I_a R_a + I_a R_{sh} \\ &= V + I_a (R_a + R_{sh}) \end{aligned}$$

(iii) Compound Generator : The combination of two windings i.e., series winding and shunt field winding is considered as a compound generator. In the compound generator, normally the field of the shunt will be more than the series field and will be less than the individual shunt machine. The same is the case with the series field also.

Based on the type of connection of the shunt field to the armature and series field it is classified as,

(a) Long shunt

(b) Short shunt.

(a) Long Shunt Machine

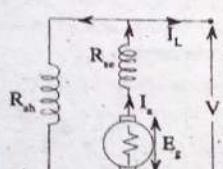


Figure (6)

Machine-1

[5-7] DC machine : motoring and generation

In this type of compound machine, the series current and the armature current is made same the shunt connection is made after the series connection is done.

Series field current = Armature current

$$I_a = I_{sh} = I_L + I_{sh}$$

I_L = Load current

I_{sh} = Shunt field current

and

$$\begin{aligned} V_L &= V_{sh} = E_g - I_a R_a - I_{sh} R_{sh} \\ &= E_g - I_a (R_a + R_{sh}) \end{aligned}$$

(b) Short Shunt Machine

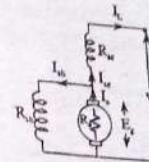


Figure (7)

In this type of machine, the series field current is made same to that of the line current. The connection of shunt field is done first and then the series field.

$$V = E_g - I_a R_a - I_{sh} R_{sh}$$

$$V_{sh} = E_g - I_a R_{sh}$$

and

$$I_a = I_{sh} + (I_a \text{ or } I_L)$$

Whatever the connection of machine either long or short shunt, if the flux produced by the series field aids with the shunt field then the machine is called cumulative compound machine or if the series field opposes the shunt field then the machine is called the differential compound machine.

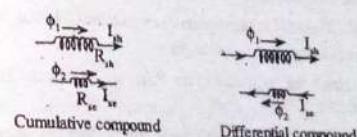


Figure (8)

Q. 7. Explain the method of build-up of e.m.f. in a shunt generator.

Or,

Explain the process of building up of voltage in a D.C shunt generator and give conditions to be satisfied for voltage build up.

Ans. Voltage Build-up of E.M.F. : During the manufacturing of a generator the pole are excited by a separate source, after which some magnetism is left over in it. This is known as residual magnetism. Therefore there is always some residual magnetism present in the poles which helps in the building up of the e.m.f. voltage.

To understand the voltage build-up, let us consider a shunt generator to be loaded (i_p). Before loading, it is allowed to build-up its voltage. As we know that there is some residual magnetism (ϕ_r) present in the poles, so rotate the armature in its presence. Thus, residual magnetism (ϕ_r) present in the poles, so rotate the armature in its presence. Thus, residual magnetism (ϕ_r) cuts the conductors and induces small e.m.f. (e_1) in it, which further produces flux ϕ_1 (i.e. greater than ϕ_r). Now, this flux ϕ_1 cuts again the conductors to induce e.m.f. of greater value e_2 (i.e., $e_2 > e_1$), which leads to the production of still greater flux ϕ_2 (i.e., $\phi_2 > \phi_1$). Now, similarly this process continues until it generates the maximum e.m.f. value. Thus, in this manner, generator builds-up its e.m.f. which can be further explained in the open circuit characteristics graphically.

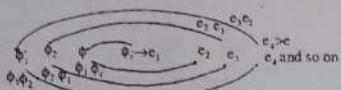


Figure 1: Building-up of E.M.F.

Conditions for Build-up of E.M.F.

The following are the important conditions to be fulfilled in order to build-up the e.m.f. in a generator.

1. The poles used in construction of D.C. shunt generator should possess residual magnetism.
2. The field winding must be properly connected to the armature so as to aid residual flux.
3. When shunt field winding of D.C. shunt generator is excited on NO-load or open circuit. Its resistance should be less than the critical resistance.
4. When shunt field winding of D.C. shunt generator is excited ON-load. Its resistance should be more than critical resistance.
5. In case a series generator the resistance of the external circuit must be less than the critical resistance.

Q. 8. Explain experimental determination of critical field resistance for a self excited generator.

Or,

What is critical field resistance? How do you calculate the critical field resistance practically?

Ans. Critical Field Resistance : It is defined as the amount of field resistance required to generate e.m.f. in the armature winding.

Or,

Critical field resistance is defined as the field resistance which holds the rated voltage of generator.

Steps to Determine Critical Field Resistance

1. Open circuit characteristic of given D.C. generator is plotted by using the experimental set up shown in figure (1).

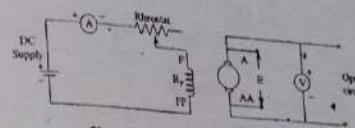


Figure 1: Experimental Setup for O.C.C.

Steps of Plot O.C.C Curve : O.C.C. is a graph plotted between e.m.f. generated and field current.

- (a) Given D.C. generator is operated at rated speed by decreasing the field resistance in steps from '0' to a definite value.
- (b) The readings of ammeter and voltmeter are noted at each step.
- (c) The field resistance is increased until the voltmeter shows the voltage reading as 25% more than the rated voltage.

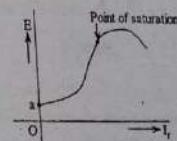


Figure 2

Here is noted that even with the increase in field current by decreasing field resistance the e.m.f. generated after point b is reduced, it is because of saturation. The O.C.C. curve is starting from point a instead of origin (O) because of the presence of residual magnetism in the generator poles which generates small e.m.f. E , when the D.C. generator runs at rated speed.

2. The field resistance line at No Load is noted. For example, if field resistance of D.C. generator is 100Ω at No load, then by considering the voltage and current axis as $(100\text{ V}, 1\text{ A})$ field resistance line is drawn as shown in figure (3).

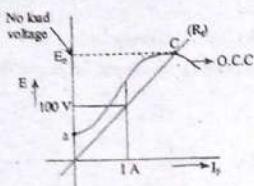


Figure (3)

3. The field resistance line cuts the O.C.C. curve at point 'C' with respect to point 'C' on O.C.C. the voltage on Y-axis is No load voltage (E_0).
4. The line OL is drawn tangent to O.C.C curve

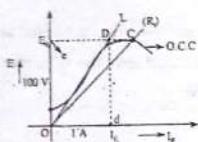


Figure (4)

Taking the value of current on current axis when tangent line OL passing through dotted line of No load e.m.f. critical resistance is calculated.

Critical resistance is the slope of tangent line OL.

$$\text{Therefore, } R_c = \frac{O_d}{O_e} \text{ ohms}$$

5. Critical resistance can also be calculated at different rated speeds by following the above procedure.

Q. 9. Draw a neat graph to show open circuit characteristics of a separately excited DC generator. Why is a field regulator necessary for this machine ?

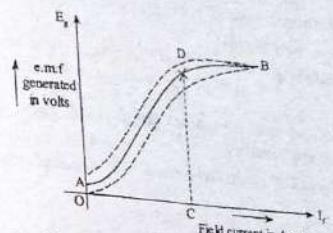


Figure: Open Circuit Characteristics of Separately Excited D.C Generator

From the figure,

AB = No-load curve

OA = Flux voltage due to residual magnetism

OC = Variation in field current with respect to origin.

Point D represents the saturation point where the load characteristics of a separately excited D.C generator saturates and follow the decrease in the generated e.m.f. on further increase of field current.

Importance of Field Regulator :

Field regulator is nothing but the resistance offered by the field circuit in order to maintain the speed of the generator as desired i.e., to increase (or) decrease and to set constant along the operation.

In a separately excited D.C generator, the e.m.f generated mainly depends on two factors.

1. Rotating speed of the machine.
2. Strength of field employed.

Practically, the rotating speed of the armature in a D.C generator can not be easily controlled except by varying the speed of the prime-mover. These variations can be implemented within the close limits. In addition to this, some sufficient speed is present at which the engine must run most economically, which is undesirable to vary from this sufficient speed in order to adjust the output voltage of the generator. Thus, to overcome the variations in speed, it is compulsory to vary the magnetic flux, which can be accomplished by inserting a variable resistance i.e., a rheostat in the field circuit. Due to this, a wide range of control of current across field circuit can be done, which in turn vary the generated e.m.f. and the speed of the generator engine can be varied as desired. As a result, for the above operations the field regulator i.e., field resistance employed is important for better efficiency of generators.

Q. 10. Discuss in detail, the load characteristics of D.C series and shunt generators.

Or,

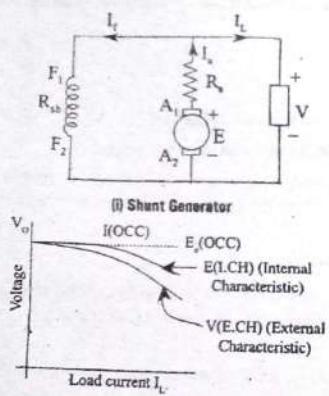
Explain the different characteristics of

- D.C series generator (both)
- D.C shunt generator.

Or,

Sketch the internal and external characteristics of D.C shunt and series generators. What are their fields of application?

Ans. Characteristics of D.C Shunt Generators : The circuit of a shunt generator is shown in figure 1 (i).



(ii) Characteristics of Shunt Generator

Figure (1)

The field current will be $I_f = \frac{V}{R_{sh}}$. Thus, the resistance of the field is much higher than the armature resistance.

Terminal voltage, $V = E - I_a R_a$

The load current, $I_L = I_a - I_f$

The characteristics of the generator is shown in figure 1 (ii), the curves of voltage plotted against load current I_L .

E_o is the ideal voltage, when there is no armature reaction and no drop in field flux. E is the induced e.m.f. when the generator is connected to the load. The curve of E drops compared to the curve of E_o due to armature reaction and drop in field flux. The variation of terminal voltage V is like the curve of E . As the load current increases the curve of V falls below the curve of E by a difference of armature resistance drop $I_a R_a$.

The power generated, $P_g = E_o I_a$

The power delivered, $P_d = VI_L$

Characteristics of D.C. Series Generator

A series generator with the armature $A_1 A_2$ and series field winding $S_1 S_2$ is shown in figure (2). The current flowing through series field winding is same as the current flowing through the connected load.

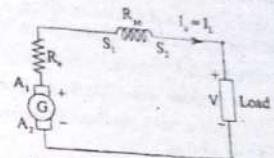


Figure (2): Series Generator Connection

Where,

R_s = Series resistance

R_a = Armature resistance

E = Induced e.m.f.

V = Terminal voltage

I_L = Load current

I_a = Armature current

E_o = O.C.C. voltage

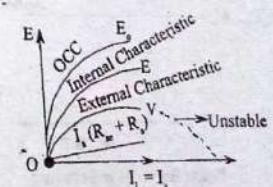


Figure (3): Characteristics of a Series Generator

The induced e.m.f (E) in the armature is lesser than (E_a) the O.C.C voltage, because of armature reaction there is a drop in the field flux on load and internal voltage drops. The curve for E shows the internal characteristic.

In a series generator, load current ' I_L ' is same as the armature current ' I_a '. If R_w and R_a are the resistance of series field and armature respectively, then the terminal voltage $V = E - I_a(R_w + R_a)$. The external characteristics fall below the internal characteristics and is illustrated by the curve drawn for V .

The power generated by the generator, $P_g = EI_a$

The power delivered to the load, $P_L = VI_a$

Applications of Shunt Generators

1. Generally, series generators find applications in acting as boosters to compensate the voltage resistance drop in the main feeders of the D.C distribution systems.

2. They are not used for power supply because of their rising characteristics.

Applications of Series Generators

1. Generally, series generators find applications in acting as boosters to compensate the voltage resistance drop in the main feeders of the D.C distribution systems.

2. They are not used for power supply because of their rising characteristics.

Q. 11. Explain constant and variable losses. Draw the graph of losses v/s load.

Ans. Constant Losses : The constant losses are fixed losses and does not vary with respect to load. These losses are further classified as,

(i) **Core Losses :** These losses include Hysteresis loss (W_h) as well as Eddy current loss (W_e).

$$W_h = \eta B_m^{1.6} f \text{ watts}$$

$$W_e = KB_m^2 f^2 \text{ watts}$$

(ii) **Mechanical Losses :** These losses include friction and windage losses.

Variable Losses : Variable losses vary with load and include copper losses.

Graph of Losses Vs Load

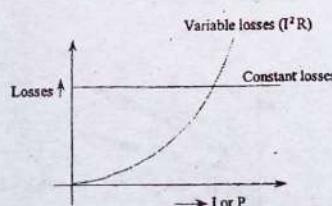


Figure: Graph of Losses Vs Load

Q. 12. List out various types of D.C motors. Mention their applications.

Or,

What are the different types of D.C. motors and give their applications ?

Ans Different Types of D.C Motors : The kind of connection of the field to the armature divides D.C motors into different types as follows,

1. **Separately Excited Motors :** The field is supplied power from a separate source and not from the armature.

2. **Self-excited Motors :** The field is supplied power from the armature. There are three types of self excited motors as follows,

(a) **Series Motor :** The field is in series with the armature.

(b) **Shunt Motor :** The field is in parallel with the armature.

(c) **Compound Motor :** The field has both series and shunt fields. There are two types of compound motors as follows.

(i) **Long Shunt Compound Motor :** The series field is connected in series with armature.

(ii) **Short Shunt Compound Motor :** The series field is connected in parallel with shunt field.

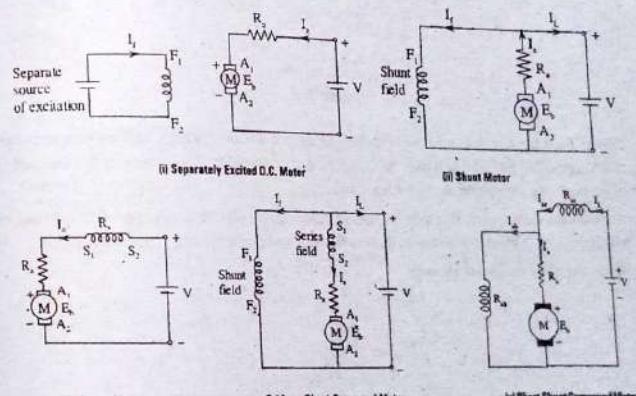


Figure (i): Types of D.C Motors

Machine-I

[5-16] DC machine : motoring and generation

The usage of D.C motor for particular application depends upon comparison, not only on advantages and disadvantages but also on economic and technical features with other competing (A.C) motors.

Applications of Separately Excited D.C Motors

The operating characteristics of separately excited D.C motors are similar to those of D.C shunt motors. From separately excited D.C motor very accurate speeds can be obtained. So these motors are used where wide range of speed control is required i.e., variation of speed from very low speed to very high speeds. The applications of separately excited D.C motors are in steel rolling mills, for paper machines, diesel electric propulsion of ships etc.

Applications of D.C Series Motors

From speed-torque characteristics graph of a D.C series motor as shown in figure (2), we can conclude that,

$$\text{Speed, } N \propto \frac{1}{\sqrt{T}} \quad (\because V \text{ is constant})$$

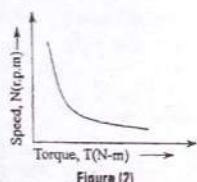


Figure (2)

Hence from figure (2), it is known that D.C series motor provides high starting torque so that these are applicable for driving hoists, trains, excavators, cranes etc. Series motors can be used to drive permanently connected loads, such as fan load.

Series motor is also applicable to battery driven automobiles with the chopper control mechanism by this application pollution is avoided. For traction purpose, series motor is the only choice.

Applications of D.C Shunt Motors

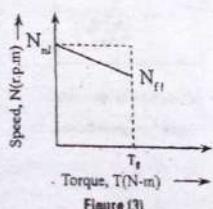


Figure (3)

Machine-II

[5-17] DC machine : motoring and generation

It is known from figure (3), that the speed drops from no-load to full-load is very small hence shunt motors are suitable for constant-speed with medium starting torque applications. Shunt motors are applicable in industry where wide range of speed control is required. Like lathes, centrifugal pumps, reciprocating pumps, fans, blowers, conveyors, wood working machines, spinning and weaving machines etc. Shunt motors are also applicable to steel and aluminum rolling mills and in Ward-Leonard system of speed control when its field winding is separately-excited.

Applications of D.C Compound Motors

A compound motor with a strong series field has its characteristic approach that of a series shunt motor. Weak series field causes more dropping speed-torque characteristic than with an ordinary shunt motor. Such compound motors with more steeper characteristics are used where load fluctuates between wide limits intermittently.

A differential compound motor is almost never used. In this motor, shunt and series fields oppose each other and is possible that at some state of operation, there may be zero flux in the air gap. When this occurs, motor speed becomes dangerously high and armature current increases to a very high value, this shows a differential compound motor is associated with increased armature current at high operating speed. This motor may also draw increased armature current during its starting.

This increased armature current during starting or during high-speed operation becomes dangerously so high that it may damage the commutator and armature windings. So, a differential compound motor is rarely used in practice.

Q. 13. Explain various losses and their equations, that takes place in D.C long shunt compound generator and short shunt compound motor.

Ans.

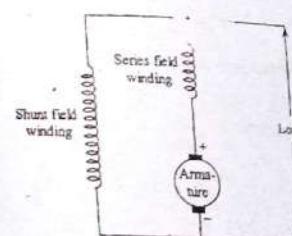


Figure (1)

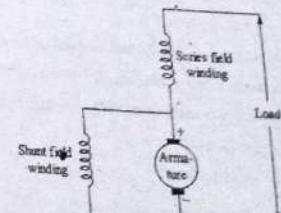


Figure (2)

The circuit diagrams for long shunt compound generator and short shunt compound motor are shown in figure (1) and figure (2) respectively. Both of them consists of a combination of few series and few shunt windings connected to the armature.

Machine-1

[5-18] DC machine : motoring and generation

The losses occurring in both the D.C. machines are,

- (i) Constant losses
- (ii) Variable losses.

Constant losses include core or iron losses and mechanical losses. These losses are fixed for a given D.C. machine and does not vary with load whereas variable losses are the copper losses which vary with load for a given D.C. machine.

Following are the losses that occur in a D.C. machine (Generator/Motor).

- (i) Core or iron losses
- (ii) Mechanical losses
- (iii) Copper losses.

(i) Core or Iron Losses

They are also called as magnetic losses or no-load losses. They include hysteresis loss and eddy current loss.

Hysteresis loss,

$$W_h = \eta B_m^{1.6} f \text{ Watts}$$

Eddy current loss,

$$W_e = K B_m^{1.6} f \text{ Watts}$$

Where,

B_m = Maximum flux density

f = Frequency

(ii) Mechanical Losses

They are of two types,

- (a) Windage loss
- (b) Friction loss.

The sum of iron losses and mechanical losses collectively is called as stray loss.

∴ Stray losses = Iron losses + Mechanical losses

They are constant and depend on design data of machine.

(iii) Copper Losses

They are as follows :

- (a) Armature copper loss, $P_a = I_a^2 R_a$
- (b) Series field copper loss, $P_{sf} = I_{sf}^2 R_{sf}$
- (c) Shunt field copper loss, $P_{sh} = I_{sh}^2 R_{sh}$
- (d) Compensating winding copper loss
- (e) Interpole winding copper loss

[5-18] DC machine : motoring and generation

Machine-1

(f) Brush contact drop copper loss.

The copper loss of machine depends on electrical loading and is proportional to load hence called as variable loss of machine. The first three i.e., a to c are called conventional copper losses which affect the efficiency of machine.

Q. 14. Draw and explain the electrical characteristics of D.C. series and shunt motor.

Or,

Draw the speed characteristics of different types of D.C. motors.

Ans. Characteristics of Motor : The characteristics of a motor can be listed as below :

1. Torque and armature current : T_a/I_a characteristics (electrical characteristics).
2. Speed and armature current : N/I_a characteristics.
3. Speed and torque : N/T_a characteristics (mechanical characteristics).

We know that $T_a \propto \phi I_a$

$$N \propto \frac{E_b}{\phi} \quad \dots (1)$$

$$\text{and} \quad N \propto \frac{E_b}{\phi} \quad \dots (2)$$

Series Motor Characteristics

1. Torque and Armature Current Characteristics

In this case, field windings also carry the armature current hence upto the saturation point $I_a \propto \phi$. On light loads, the armature current is small hence the flux is small. But, when I_a increases flux also increases. Hence the torque increases as the square of the current i.e., $T_a \propto I_a^2$. So, the curve drawn between T_a and I_a is a parabola up to saturation point, flux ϕ is almost independent of excitation current and hence $T_a \propto I_a$. Thus, the characteristic becomes a straight line.

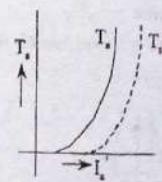


Figure (1)

From the torque current characteristic it is evident that as long as the field T_a of motor is not saturated, the series motor exerts a torque proportional to the square of current i.e., starting torque is very high. Hence, series motors are used where large starting torque is required, such as in hoists, electric railways, trolleys and electric vehicles.

2. Speed and Armature Current Characteristics

We know that, $N \propto \frac{E_b}{\phi} \propto \frac{E_b}{I_a}$

\therefore On no-load the speed is dangerously high, the series motors are never started on no-load the reason being.

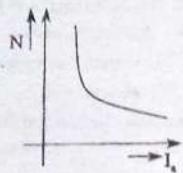


Figure (2)

When the motor is connected across supply mains without load, the current draw is small and hence ϕ is small and hence speed tends to increase. With increase in speed, E_b increases $\left[\because N \propto \frac{E_b}{\phi} \right]$

and thus the field current decreases $\left[\because I_a = \frac{V - E_b}{R_a} \right]$ which in turn leads to decrease in flux and hence speed increases seriously. This process continues until the armature gets damaged.

Hence, series motors are not suitable for the services where the load may be entirely removed.

3. Speed and Torque Characteristics

These characteristics can be obtained or derived from the above two characteristics mentioned i.e., from figures (1) and (2).

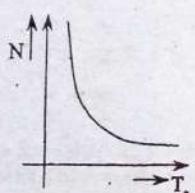


Figure (3)

Therefore, armature torque and speed has inverse relationship as shown in figure (3).

1. T_a/I_a Characteristics

If the applied voltage 'V' is kept constant, the field current will remain constant and hence flux is also constant and is maximum. But as the load increases, the flux decreases lightly due to armature reaction.

Hence, with the increase in I_a , T_a also increases linearly passing through origin.

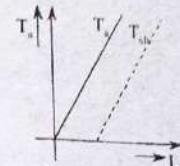


Figure (4)

2. N/I_a Characteristics

We know that $N \propto \frac{E_b}{\phi}$

Since ' ϕ ' is practically constant in shunt motor.

$$\therefore N \propto E_b \propto (V - I_a R_a)$$

With the increase in load current I_a , the I_a also increases $\left[\because I_a = I_a - I_{sh} \text{, as } I_{sh} \text{ is constant} \right]$. With the increase in load current, the speed slightly falls due to increase in voltage drop in armature.

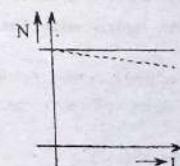


Figure (5)

Since voltage drop in armature at full-load is very small as compared to applied voltage, drop in speed from no-load to full load is very small and for all practical purposes the shunt motor is taken as a constant speed motor.

Because of the constancy of speed they are best suited for driving line shafts, milling machine's conveyors, fans etc.

3. N/T_a Characteristics

The speed and torque (N/T_a) characteristics are referred as mechanical characteristics. The mechanical characteristics of D.C. shunt motor are shown in figure (6).

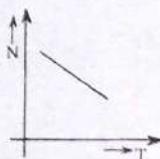


Figure (6)

Q. 15. What is the necessity of speed control of DC motor ?

Ans. The term speed control refers to the variation of speed of a motor as per our requirement.

Motors are said to be beneficial when the motors operate as per the load requirements. For loads that require variable speed operations, it is necessary to use the motors with varying speeds.

Therefore, the necessity of speed control of DC motor arises. The other reasons for preferring speed control of DC motors are,

1. In DC motors, the speed can be very easily controlled.
2. The speed control achieved in DC motor is very smooth and such smooth speed control is not possible in A.C. motors.
3. In DC motors, speed can be controlled over a wide range.

Q. 16. Explain the speed-current, torque-current and speed-torque characteristics of a dc shunt motor.

Ans. For d.c. motors, the supply voltage is usually constant and the quantities of common interest are speed, torque etc. The following are the three important operating characteristics of d.c. motors.

- (i) Speed-armature current characteristic.
- (ii) Torque-armature current characteristic and
- (iii) Speed-torque characteristic.

The object of this article is to describe these operating characteristics for different types of d.c. motors.

D.C. Shunt Motor :- For constant supply voltage, the field current is constant. At small values of armature current the demagnetizing effect of armature reaction is almost negligible and

therefore the air gap flux is unaffected. For larger values of armature (or load) currents, the demagnetizing effect of armature reaction decreases the air gap flux slightly.

The speed of a d.c. motor, is given by

$$\omega_m = \frac{E_a}{K_a \varphi} \quad \text{But} \quad E_a = V_i - I_a r_s$$

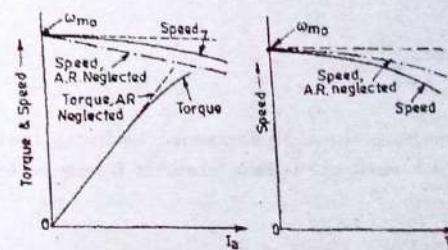
$$\therefore \omega_m = \frac{V_i - I_a r_s}{K_a \varphi} \quad (1)$$

(i) Speed current characteristic :- For constant supply voltage V_i and constant field current I_f , the motor speed is affected by $I_a r_s$ drop and demagnetizing effect of armature reaction. With the increase of I_a , the demagnetizing effect of armature reaction increases which reduces drop $I_a r_s$ increases and the numerator $(V_i - I_a r_s)$ decreases therefore the motor speed tends to decrease. With the increase of I_a , the numerator decrement is more than the denominator decrement in view of this, the speed of d.c. shunt motor with increase of I_a drops only slightly from its no-load speed ω_{m0} . Since I_f at no-load is negligibly small the shunt motor no-load speed ω_{m0} is given by

$$\omega_{m0} = \frac{V_i}{K_a \varphi}$$

In case the effect of armature reaction (AR) is neglected, then the denominator of Eq. is constant. As a consequence, speed drops faster with I_a . Fig (a) illustrate speed current characteristics of a shunt motor with and without AR. The curve marked speed is with AR included.

(ii) Torque-current characteristic :- The expression $T_e = K_a \varphi I_a$ reveals that if the flux φ is constant as in a shunt motor, the torque would increase linearly with armature current I_a . However, for larger I_a , the net flux decreases due to the demagnetizing effect of armature reaction. In view of this, the torque current characteristic deviates from the straight line, as illustrated in Fig. (a). In case the effect of AR is neglected, T_e versus I_a characteristic would be a straight line as shown.



D.C. shunt motor, (a) speed current and torque-current characteristics and
(b) speed torque characteristics.

(iii) Speed-torque characteristic :- The speed-torque characteristic is also called the mechanical characteristic and under steady state conditions, it can be obtained as follows.

From Eq. (i)

$$\omega_m = \frac{V_1 - I_a r_s}{K_a \varphi}$$

But

$$T_e = K_a \varphi I_a \text{ or } I_a = \frac{T_e}{K_a \varphi}$$

Substituting this value of I_a in Eq. (i)

$$\begin{aligned} \omega_m &= \frac{1}{K_a \varphi} \left[V_1 - \frac{T_e r_s}{K_a \varphi} \right] = \frac{V_1}{K_a \varphi} - r_s \frac{T_e}{K_a^2 \varphi^2} \\ &= \omega_{m0} - r_s \frac{T_e}{K_a^2 \varphi^2} \end{aligned} \quad \dots \dots \dots \text{(ii)}$$

It is seen from Eq. (ii) that with increase of T_e , the speed drops. Note that for larger T_e , larger I_a is required and this has the effect of reducing the air gap flux φ , due to saturation and armature reaction. Since with increase of T_e , φ is reduced, T_e/φ^2 increases at a faster rate and the speed drops more rapidly with the increase of torque in a shunt motor as shown in Fig. (b).

In effect of AR is neglected, then $(K_a \varphi)^2$ in Eq. (ii) remains constant. As a result, the speed drop with T_e is slow as shown in Fig. (b).

Q. 17. Explain the speed-current torque-current and speed-torque characteristics of a d.c. series motor.

Ans. Characteristics of A D.C. Series Motor :

The motor speed N is given by

$$N \propto \frac{V - I_a (R_a + R_{ic})}{\Phi}$$

At low values of I_a , the voltage drop $I_a (R_a + R_{ic})$ is negligibly small in comparison with V . Therefore

$$N \propto \frac{V}{\Phi}$$

Since V is constant

$$N \propto \frac{1}{\Phi}$$

In a series motor, the flux Φ is produced by the armature current flowing in the field winding so that $\Phi \propto I_a$. Hence the series motor is a variable flux machine. Equation now becomes

$$N \propto \frac{1}{I_a}$$

Thus, for the series motor, the speed is inversely proportional to the armature current.

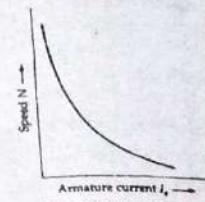


Fig. Speed-armature current characteristic of a d.c. series motor.

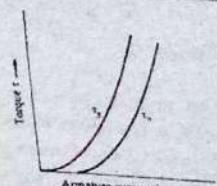


Fig. Torque/armature current characteristic of a d.c. series motor.

Speed / torque characteristic : The speed / torque characteristic of a series motor can be derived from its speed-armature current (N/I_a) and torque-armature current (T/I_a) characteristics as follows:

For a given value of I_a find r from T/I_a curve and N from N/I_a curve. This gives one point (r, N) on speed-torque (N/r) curve. Repeat this procedure for a number of values of I_a and find the corresponding values of speed and torque $(r_1, N_1), (r_2, N_2)$ etc. These points are plotted to get the speed-torque characteristic of a d.c. series motor.

Q. 18. Define and explain the critical field circuit resistance and the critical speed of a DC generator.

Ans. We have assumed that the generator is on no load during the buildup process. The following equations describe the steady-state operation:

$$I_a = I_f$$

$$V = E_a - I_a R_a = E_a - I_f R_a$$

Since the field current I_f in a shunt generator is very small, the voltage drop $I_f R_a$ can be neglected,

and

$$V = E_a$$

The E_a versus I_f curve is the magnetization curve shown in fig.

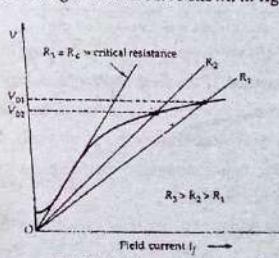


Fig. Effect of field resistance on no-load voltage

For the field circuit

$$V = I_f R_f$$

The straight line given by

$$V = I_f R_f$$

is called the field - resistance line.

The solution of the no-load terminal voltage V_0 of the generator. Thus, the intersection point P of the magnetization curve and the field-resistance line gives the no-load terminal voltage $V_0 (= bP)$ and the corresponding field current (Ob). Normally, in the shunt generator the voltage builds up to the value given by the point P. At this point $E_a = I_f R_f = V_0$.

If the field current corresponding to point P is increased further, there is no further increase in the terminal voltage.

Figure shows the voltage buildup in the dc shunt generator for various field circuit resistances.

A decrease in the resistance of the field circuit reduces the slope of the field-resistance line resulting in a higher voltage. If the speed remains constant. An increase in the resistance of the field circuit increases the slope of the field resistance line, resulting in a lower voltage. If the field circuit resistance is increased to R_c , which is termed as the critical resistance of the field, the field resistance line becomes tangent to the initial part of the magnetization curve. When the field resistance is higher than this value the generator fails to excite.

Figure shows the variations of no-load voltage with fixed R_f and variable speed of the armature.

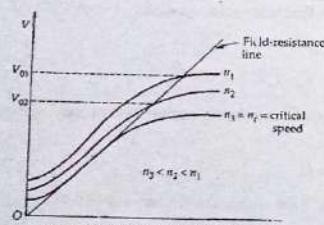


Fig. Variation of no-load voltage with speed.

The magnetization curve varies with the speed and its ordinate for any field current is proportional to the speed of the generator. If the field resistance is kept constant and the speed is reduced, all the points on the magnetization curve are lowered, and the point of intersection of the magnetization curve and the field resistance line moves downwards. At a particular speed, called the critical speed, the field-resistance line becomes tangential to the magnetization curve. Below the critical speed the voltage will not build up.

In brief, the following conditions must be satisfied for voltage buildup in a self-excited dc generator.

- 1. There must be sufficient residual flux in the field poles.
- 2. The field terminals should be connected in such a way that the field current increases flux in the direction of residual flux.
- 3. The field circuit resistance should be less than the critical field circuit resistance.
- If there is no residual flux in the field poles, disconnect the field from the armature circuit and apply a dc voltage to the field winding. This process is called flashing the field. It will induce some residual flux in the field poles.

Q. 19. Write down the names of losses which occur in a d.c. machine.

Ans. Losses in DC Machines : The losses that occur in dc machines can be divided into five basic categories:

- | | |
|--|-------------------------------|
| 1. Electrical or copper losses ($I^2 R$ losses) | 2. Core losses or iron losses |
| 3. Brush losses | 4. Mechanical losses |
| 5. Stray-load losses | |

Electrical or Copper Losses or Winding Losses : These losses are the winding losses. The copper losses are present because of the resistance of the windings. Currents flowing through these windings produce ohmic losses. The windings that may be present in addition to the armature winding are the field windings, interpole and compensating windings.

Armature copper losses $I_a^2 R_a$ where I_a is armature current and R_a is armature resistance. These losses are about 30 per cent of total full-load losses.

Copper loss in the shunt field of a shunt machine $= I_{sh}^2 R_{sh}$ where I_{sh} is the current in the shunt field and R_{sh} is the resistance of the shunt field winding. The shunt regulating resistance is included in R_{sh} .

Copper loss in the series field of a series machine $= I_{se}^2 R_{se}$ where I_{se} is the current through the series field winding and R_{se} is the resistance of the series field winding.

In a compound machine, both shunt and series field losses occur. These losses are about 20% of full load losses.

Copper loss in interpole windings $= I_i^2 R_i$ where R_i is the resistance of interpole windings.

Copper loss in compensating winding if any $= I_c^2 R_c$ where R_c is the resistance of compensating winding.

Magnetic Losses or Core Losses or Iron Losses :

The core losses are the hysteresis losses and eddy-current losses. Since machines are usually operated at constant flux density and constant speed, these losses are almost constant. These losses are about 20 per cent of full-load losses.

Brush Losses :

There is a power loss at the brush contacts between the copper commutator and the carbon brushes. In practice, this loss depends upon the brush contact voltage drop and the armature current I_a . It is given by

$$P_{BD} = V_{BD} I_a$$

Machine-1

[5-28] DC machine : motoring and generation

The voltage drop across a set of brushes is approximately constant over a large range of armature currents. Unless stated otherwise, the brush voltage drop is usually assumed to be about 2 V. The brush drop loss is, therefore, taken as 2 I_a .

Mechanical Losses :

The losses associated with mechanical effects are called mechanical losses. They consist of bearing friction loss and windage loss. Windage losses are those associated with overcoming air friction between the moving parts of the machine and the air inside the machine for cooling purposes. These losses are usually very small.

Stray-Load Losses :

Stray-load loss consists of all losses, not covered above. These are the miscellaneous losses that result from such factors as (i) the distortion of flux because of armature reaction, (ii) short circuit currents in the coil, undergoing commutation, etc. These losses are very difficult to determine. The indeterminate nature of the stray-load loss makes it necessary to assign it a reasonable value. For most machines stray losses are taken by convention to be one percent of the full-load output power. The term stray powerless should not be confused with stray load loss.

Q. 20. Write short notes on the following :

(a) Advantage of d.c. series motor

Ans. (a) Advantage of d.c. series motor :

Advantage - A DC series motor has its field winding in series with Armature winding. Because of this the armature current is the field current too. The following equations are valid for a series Motor:

Torque = $K \times I^2$ where K is motor constant and I_a is armature current.

Because of this the torque changes quadratically with the armature current. Hence a series DC motor is used in applications requiring High Starting torque. Locomotives requiring high starting torque

(i) The series motor has the highest starting torque for a given power rating. When the motor is overloaded, the speed drops and the torque increases. While delivering high torque during an overload, the series motor draws less current and power from the source compared to a shunt or compound motor.

(ii) The main application where series motors are best suited are for traction applications.

Torque of the series motor is proportional to square of the field current. Therefore Starting torque for the series motor is quite high. Because of this property series motors are widely used in electric traction and in crane application

Q. 21. A 230V, 10 HP d.c series motor draws a line current of 36A when delivering rated power at its rated speed of 1200 r.p.m. The armature circuit resistance is 0.2Ω and the series field winding resistance is 0.1Ω . Assume the magnetization curve to be linear.

Machine-1

[5-29] DC machine : motoring and generation

- Find the speed of motor when it draws a line current of 20A.
- What is the developed torque at new condition?
- How does this torque compare with the original value? Why?

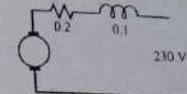
Ans. 230V, 10 HP, $I_1 = 36A$

$n = 1200 \text{ rpm}$, $r_s = 0.2\Omega$, $r_i = 0.1\Omega$

$\phi \propto I$

$$(i) E_{a1} = 230 - 36 \times (0.1 + 0.2) = 219.2V$$

$$= 219.2V$$



$$I_2 = 20A$$

$$E_{a2} = 230 - 20(0.1 + 0.2) = 224V$$

$$n_2 = ?$$

$$\frac{\phi_2}{\phi_1} = \frac{I_1}{I_2}, \quad \frac{\phi_1}{\phi_2} = \frac{36}{20}$$

$$\Rightarrow \frac{E_{a1}}{E_{a2}} = \frac{n_1 \phi_1}{n_2 \phi_2} \Rightarrow \frac{219.2}{224} = \frac{n_1}{n_2} \times \frac{36}{20}$$

$$\Rightarrow n_2 = \frac{1200 \times 36 \times 224}{219.2 \times 20} = 2207.3 \text{ rpm}$$

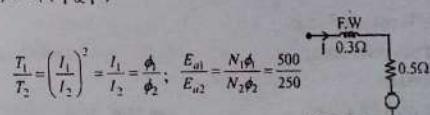
$$(ii) T_2 = \frac{E_{a2} I_2}{w_2} = \frac{224 \times 20 \times 60}{2\pi \times 2207.3} = 19.38 \text{ Nm} \quad (iii) T_1 = \frac{E_{a1} I_1}{w_1} = \frac{219.2 \times 36 \times 60}{2\pi \times 1200} = 62.796 \text{ Nm}$$

We can say that when current decreases the flux also decreases So, speed increases and hence torque decreases.

$$\% \text{ decrease in torque} = \frac{T_2 - T_1}{T_1} \times 100 = -69.13\%$$

Q.22. A 200V d.c series motor runs at 500 rpm when taking a current of 25A. The resistance of armature is 0.5Ω and that of field is 0.3Ω . If the current remains constant, calculate the resistance necessary to reduce the speed to 250 rpm.

Ans. 200V, $N_1 = 500 \text{ r.p.m}$, $I_1 = 25A$, $r_s = 0.5\Omega$, $r_i = 0.3\Omega$, $R_F = ?$, $N_2 = 250 \text{ r.p.m}$, $I_2 = I_1$, $T \propto \psi I$, $1 \propto \psi$, $T \propto I^2$,



$$E_{a2} = \frac{180}{2} = 90; \quad E_{a1} = 200 - 25(0.3 + 0.5)$$

Machine-1

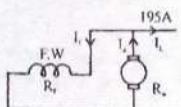
[5-30] DC machine : motoring and generation

$$\begin{aligned} E_{o1} &= 180, \quad E_{o2} = 200 - 25(R_g + 0.8) \\ 90 &= 200 - 25(R_g + 0.8), \quad 200 - 90 = 25(R_g + 0.8) \\ R_g &= \frac{110}{25} - 0.8 = 3.6\Omega \quad \text{Ans.} \end{aligned}$$

Q. 23. A shunt generator delivers 195 A at a terminal p.d. of 250 V. The armature resistance and shunt field resistance are 0.02Ω and 50Ω respectively. The iron and friction losses equal 950 W. Find –

- (i) e.m.f. generated;
- (ii) Cu losses;
- (iii) output of the prime mover;
- (iv) mechanical, electrical and commercial efficiencies

Ans. $V_t = 250\text{V}$, $R_a = 0.02\Omega$, $R_f = 50\Omega$, $P_L = 950\text{W}$



$$\begin{aligned} (i) \quad I_f &= \frac{V_t}{R_f} \\ &\Rightarrow I_f = \frac{250}{50} = 5\text{A} \\ &\Rightarrow I_a = I_f + I_L = 200\text{A} \\ &\Rightarrow E_o = V_t + I_a R_a = 250 + (200 \times 0.02) \\ &\Rightarrow E_o = 254\text{V} \end{aligned}$$

$$\begin{aligned} (ii) \quad P_{cu} &= I_f^2 R_f + I_a^2 R_a \\ &\Rightarrow P_{cu} = (5)^2 \times 50 + (200)^2 \times 0.02 \\ &\Rightarrow P_{cu} = 2.05\text{ kW} \end{aligned}$$

$$\begin{aligned} (iii) \quad P_i &= E_o I_a \\ &\Rightarrow P_i = (254)(200) \\ &\Rightarrow P_i = 50800 \end{aligned}$$

$$\begin{aligned} (iv) \quad \text{Mechanical output } (P_{mif}) &= V_t I_L - P_{cu} &= 46.7\text{kW} \\ \text{electrical output } (P_{co}) &= V_t I_L &= 48.75\text{kW} \\ \text{commercial output } (P_{co}) &= P_0 = 45.75\text{kW} \end{aligned}$$

Machine-1

[5-31] DC machine : motoring and generation

$$P_i = E_o I_a = 56.8\text{kW}$$

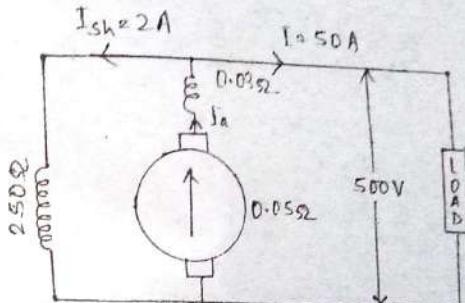
$$\eta_m = \frac{P_{mif}}{P_i} = 91.93\%$$

$$\eta_e = \frac{P_{eo}}{P_i} = 95.96\%$$

$$\eta_c = \frac{P_{co}}{P_i} = 90.06\%$$

Q. 24. A long-shunt compound generator delivers a load current of 50A at 500V and has armature, series field and shunt field resistances of 0.05Ω , 0.03Ω and 250Ω respectively. Calculate the armature current and generated emf. Allow 1V per brush for contact drop.

Ans.



$$I_{sh} = \frac{500}{200} = 2\text{A}$$

Current through armature and series winding is
 $= 50 + 2 = 52\text{A}$

Voltage drop on series field winding
 $= 52 \times 0.03 = 1.56\text{V}$

Armature voltage drop
 $I_a R_a = 52 \times 0.05 = 2.6\text{V}$

Drop at brushes $= 2 \times 1 = 2\text{V}$

Now, $E_g = V + I_a R_a + \text{series drop} + \text{brush drop}$
 $= 500 + 2.6 + 1.56 + 2 = 506.16 \text{ V Ans.}$

Q. 25 A 220V d.c. shunt motor having an armature resistance of 0.25Ω carries an armature current of 50A and runs at 600 rpm. If the flux is reduced by 10% by field regulator, find the speed assuming load torque remains the same.

Ans. Given :

$$V = 220 \text{ V}, \quad I_{a1} = 50 \text{ A}, \quad R_a = 0.25 \text{ A}$$

$$N_1 = 600 \text{ rpm}$$

$$E_1 = V - I_a R_a = 250 - 50 \times 0.25 = 237.5 \text{ V}$$

Conditions after reducing flux

$$\phi_2 = 0.9 \phi_1$$

$$\text{load torque} \quad \tau \propto \phi I_a$$

Since the load torque remains the same

$$\tau_2 = \tau_1; \quad \phi_2 I_{a2} = \phi_1 I_{a1}$$

$$I_{a2} = \frac{\phi_1}{\phi_2} I_{a1} = \frac{50}{0.9} = 55.56 \text{ A}$$

$$E_2 = V - I_a R_a = 250 - 55.56 \times 0.25 \\ = 236.1 \text{ V}$$

$$\frac{N_2}{N_1} = \frac{E_2 \phi_1}{E_1 \phi_2} \quad N_2 = \frac{E_2 \phi_1}{E_1 \phi_2} N_1$$

$$= \frac{236.1 \times 600}{237.5 \times 0.9} = 662.73 \text{ rpm Ans.}$$

