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Assignment - 8

Q1) For the following MA(3) process $y_t = \mu + \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} + \theta_3\epsilon_{t-3}$, where ϵ_t is a zero mean white noice process with variance σ^2. At which lag auto correlation will be zero.

Ans)

Given MA(3) process: $y_t = \mu + \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} + \theta_3\epsilon_{t-3}$ → (1)

where $\{\epsilon_t \sim WN(0, \sigma^2)\}$

$\therefore E[\epsilon_t] = 0$ and $V[\epsilon_t] = \sigma^2$.

Then,

- $E[y_t] = E[\mu + \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} + \theta_3\epsilon_{t-3}]$

$= \mu + 0 + 0 + 0 + 0 = \mu$

- $Var(y_t) = Var\{\mu + \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} + \theta_3\epsilon_{t-3}\}$

$= 0 + \sigma^2 + \theta_1^2\sigma^2 + \theta_2^2\sigma^2 + \theta_3^2\sigma^2$

$= \sigma^2(1 + \theta_1^2 + \theta_2^2 + \theta_3^2)$

- Now, Auto Covariance function ACVF at lag $\tau = 0$

$\gamma(0) = E[\{y_t - E(y_t)\}\{y_t - E(y_t)\}]$

$= E[(y_t - \mu)^2] = E[y_t^2 - 2\mu y_t + \mu^2]$

$= E(y_t^2) - 2\mu E(y_t) + \mu^2$

$$= Var[y_t] - \{E(y_t)\}^2 - 2\mu E[y_t] + \mu^2$$

$$= \sigma^2(1+\theta_1^2+\theta_2^2+\theta_3^2) - \mu^2 - 2\mu^2 + \mu^2$$

$$\gamma(0) = \sigma^2(1+\theta_1^2+\theta_2^2+\theta_3^2) - 2\mu^2$$

- At lag $\tau = 1$,

$$\gamma(1) = E[(y_t - \mu)(y_{t-1} - \mu)]$$

$$= E\Big[(\epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} + \theta_3\epsilon_{t-3})(\epsilon_{t-1} + \theta_1\epsilon_{t-2} + \theta_2\epsilon_{t-3} + \theta_3\epsilon_{t-4})\Big]$$

$$= \theta_1 E[\epsilon_{t-1}^2] + \theta_1\theta_2 E(\epsilon_{t-2}^2) + \theta_2\theta_3 E(\epsilon_{t-3}^2)$$

$$\because E[\epsilon_i\epsilon_j] = \begin{cases} 0, & i \neq j \\ \sigma^2, & i = j \end{cases}$$

$\because \epsilon_i$ iid $WN(0, \sigma^2)$

$$\therefore \gamma(1) = \sigma^2(\theta_1 + \theta_1\theta_2 + \theta_2\theta_3)$$

- Similarly at lag $\tau = 2$

$$\gamma(2) = E[(y_t - \mu)(y_{t-2} - \mu)]$$

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$$= E\left[\left(\epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \theta_3 \epsilon_{t-3}\right)\cdot\left(\epsilon_{t-2} + \theta_1 \epsilon_{t-3} + \theta_2 \epsilon_{t-4} + \theta_3 \epsilon_{t-5}\right)\right]$$

$$= \theta_2 E[\epsilon_{t-2}^2] + \theta_1\theta_3 E[\epsilon_{t-3}^2]$$

$$= \sigma^2 (\theta_2 + \theta_1 \theta_3)$$

At lag $\tau = 3$,

$$\gamma(3) = E[(y_t - \mu)(y_{t-3} - \mu)]$$

$$= E\left[\left(y_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \theta_3 \epsilon_{t-3}\right)\left(\epsilon_{t-3} + \theta_1 \epsilon_{t-4} + \theta_2 \epsilon_{t-5} + \theta_3 \epsilon_{t-6}\right)\right]$$

$$= \theta_3 E[\epsilon_{t-3}^2] = \theta_3 \sigma^2$$

Then clearly $\gamma(q) = 0$ if $|q| > 3 \rightarrow$ (2)

Now, Autocorelation function P at lag K is defined as

$$P(K) = \frac{\gamma(K)}{\gamma(0)}$$

Hence from (2) $P(K) = 0 \Rightarrow \gamma(K) = 0 \Rightarrow \underline{|K| > 3}$

Ans

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Q2) A time series with linear time period:
$y_t = \alpha + \delta \times t$
(a) How one can eliminate this trend?

Ans)
Given: $y_t = \alpha + \delta t \rightarrow (1)$

then $y_{t-1} = \alpha + \delta(t-1) \rightarrow (2)$

So, eq (1) - eq (2), we get

$$y_t - y_{t-1} = \alpha + \delta t - \alpha - \delta(t-1)$$

$$\boxed{y_t = \delta + y_{t-1}}$$

Q3) An AR(1) process: $y_t = 1.5 y_{t-1} + u_t$ is a non stationary process (by transforming in MA process).

Ans)
Given: $y_t = 1.5 y + u_t \rightarrow (1)$

$\{\phi_1 = 1.5\}$

Then eq (1) can we written as $y_t = \phi_1 y_{t-1} + u_t \rightarrow (2)$

From eq (2), $y_{t-1} = \phi_1 y_{t-2} + u_{t-1}$

$y_{t-2} = \phi_1 y_{t-3} + u_{t-2}$

Putting above eq in eq (2)

$$y_t = \phi_1 (\phi_1 y_{t-2} + u_{t-1}) + u_t$$

$$= \phi_1^2 (\phi_1 y_{t-3} + u_{t-2}) + \phi_1 u_{t-1} + u_t$$

$$= \phi_1^3 (\phi_1 y_{t-4} + u_{t-3}) + \phi_1^2 u_{t-2} + \phi_1 u_{t-1} + u_t$$

$$\Rightarrow y_t = \phi_1^{p+1} y_{t-(p+1)} + \sum_{j=0}^{p} \phi_1^j u_{t-j} \rightarrow (3)$$

Now if $p \to \infty$, eqⁿ (3) becomes

$$y_t = \sum_{j=0}^{\infty} \phi_1^j u_{t-j}$$, which represents MA(∞)

Process and is non-stationary if $|\phi_1| > 1$ which is the exact case here since $|\phi_1| = |1.5| > 1$

Hence, the given process is non-stationary.

Q5) Whath are the different steps in the Box-Jenkins approach for time series modeling?

Ans)

The Box-Jenkins approach mainly has 4 steps:

i) Transform data to achine stationarity

ii) Identify the model i.e. the parameters ARMA (p,d,q)

iii) Estimation of the parameters

iv) Diagnostic analysis: testing residuals.

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Q6) Define the auto correlation function (ACF) and PACF.

Ans)

- Given a stochastic process, the auto covariance function is a function that gives the covariance of the process with itself at pairs of time points.

The ACF $\rho_{t,s}$ is given by

$$\rho_{t,s} = Corr(y_t, y_s) \qquad t, s = 0, \pm 1, \pm 2, \ldots$$

$$\text{where } Corr(y_t, y_s) = \frac{Cov(y_t, y_s)}{\sqrt{Var(y_t) Var(y_s)}} = \frac{\gamma_{t,s}}{\sqrt{\gamma_{t,t} \cdot \gamma_{s,s}}}$$

- PACF can be defined as the strength of the relationship between two variables after excluding the effect of one or more variables in between.

PACF of order h say $\alpha(h)$ is the partial correlation coefficient between Y_t and Y_{t+h} conditional on intermediate values of the process.

For example, PACF can be represented as

$$\frac{Cov[(Y_t, X_{t-3}) | X_{t-1}, X_{t-2}]}{\sqrt{Var(Y_t | X_{t-1}, X_{t-2}) \cdot Var(X_{t-3} | X_{t-1}, X_{t-2})}}$$

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Q7) What is meant by weak stationarity and covariance stationarity?

Sol)

- Weak Stationarity: A stochastic process $\{Y_t\}$ is said to be weakly stationary under the following conditions:-

i) The mean funⁿ is constant over time.

ii) $Cov[Y_t, Y_{t-K}] = \gamma_{t,t-K} = \gamma_{0,K}$ $\forall$ time t and lag K

- Covariance Stationarity:- A stochastic process is called covariance stationary if its first and second moments are time invariants.

i) $E[Y_t] = E[Y_{t-1}] = \mu \quad \forall t$

ii) $Var[Y_t] = \gamma_0 < \infty \quad \forall t$

iii) $Cov(Y_t, Y_{t-K}) = \gamma_K \quad \forall t, \forall K$

Q8) How will you determine the order of an auto regressive process? Explain.

Ans) There is no straight forward method to determine the correct model order. As one increases the order of the model, the root mean square error generally decreases quickly up to some order and then more slowly. An order just after the point at which the RMSE flattens out is usually the best choice of order.

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Q9) Describe a method of estimation of parameters of an AR(1) model.

Ans)

Considering a simple case of AR(1) model

$$y_t = \phi y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim WN(0, \sigma^2) \rightarrow (1)$$

The true lag-1 auto correlation $\rho_1 = \phi$.
A method of moments estimator would simply estimate and equate the true lag-1 auto correlation to the sample lag-1 correlation, say u_1

$$\therefore \hat{\phi} = u_1$$

Q10) Derive the stationarity conditions for an AR(p) process.

Ans)

Since, we know that if the parameters of a stochastic process does not depends on time then the process is stationary, therefore an AR process can be transformed in a MA process (Convergent process)

$$\text{Let, } y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + u_t \rightarrow (1)$$

the AR(p) process to be considered with $U_t \sim WN(0, \sigma^2)$ without loss of generality, we may assume $\delta = 0$

Then eqⁿ (1) can be written as

$$y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2} - \dots - \phi_p y_{t-p} = u_t$$

$$\Rightarrow (1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p) y_t = u_t$$

B being the backward shift operator such that $B^n X_t = X_{t-n}$

$$\Rightarrow \phi(B) y_t = u_t \rightarrow (2)$$

$\left\{ \text{where } \phi B \text{ is the characteristic polynomial s.t. } \phi B = (1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p) \right\}$

Let $\theta_1, \theta_2, \dots \theta_p$ be the ~~pro~~ p roots of polynomial ϕB equated to zero.

$$\therefore \phi B = (B - \theta_1)(B - \theta_2) \dots (B - \theta_p)$$

$$= (1 - \theta_1^{-1} B)(1 - \theta_2^{-1} B) \dots (1 - \theta_p^{-1} B) \rightarrow (3)$$

From (2) $y_t = [\phi B]^{-1} u_t \rightarrow (4)$

$\therefore$ We get y_t in the form of linear combinations of u_t's with finite coefficients, then AR(p) process can be written as MA process which is possible if $[\phi B]^{-1}$ is a convergent function.

Using eqⁿ (3) and (4)

$$y_t = \frac{1}{\prod_{i=1}^{p} (1 - \theta_i^{-1} B)} \cdot u_t$$

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$$\Rightarrow y_t = \left[c_1 (1-\theta_1^{-1} B)^{-1} + c_2 (1-\theta_2^{-1} B)^{-1} + \dots + c_p (1-\theta_p^{-1} B)^{-1} \right] u_t$$

{ By using Partial function }

The R.H.S. be convergent if $B = \left| \frac{1}{\theta^{-1}} \right| < 1$

$$\Rightarrow |\theta^{-1}| > 1$$

$\Rightarrow$ The AR(P) process is stationary if the roots of the characteristic polynomial lie outside the unit circle.

Q11) Derive the Yule-Walker equations satisfied by the a.c.f. of an AR(p) process.

Ans)

Model: $y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + u_t$

$\{ u_t \sim WN(0, \sigma^2) \}$

Now, Auto Covariance at lag K, $\gamma(K) = Cov(y_t, y_{t-K})$

$$\Rightarrow \gamma(K) = Cov(\delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + u_t, y_{t-K})$$

$$= \sum_{i=1}^{p} \phi_i Cov(y_{t-i}, y_{t-K}) + Cov(u_t, y_{t-K})$$

$$= \sum_{i=1}^{p} \phi_i \gamma(K-i) + \begin{cases} \sigma^2, & K=0 \\ 0, & K \neq 0 \end{cases}$$

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$$\left\{\begin{aligned} \therefore \gamma(0) &= \phi_1 \gamma(1) + \phi_2 \gamma(2) + \dots + \phi_p \gamma(p) + \sigma^2 \\ \gamma(k) &= \phi_1 \gamma(1) + \phi_2 \gamma(2) + \dots + \phi_p \gamma(p) \end{aligned}\right\}$$

Above are the Yule-Walkers Equations

Q12) Describe least squares estimations of parameters of the AR(2) model.

Ans) Model:

$$y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t \rightarrow (1) \quad \{\varepsilon_t \sim WN(0, \sigma^2)$$

Parameters of eq^n (1) : ϕ_1, ϕ_2

We introduce a possibly non-zero mean μ into our model and treat it as another parameters to be estimated by least squares.
Then model (1) becomes mean centered

$$y_t - \mu = \delta + \phi_1 (y_{t-1} - \mu) + \phi_2 (y_{t-2} - \mu) + \varepsilon_t \rightarrow (2)$$

Studying model (2) as a regression model withs predictors variables y_{t-1}, y_{t-2} and response variable y_t, the least

square estimation proceds by mimimizing sum of the squares of differences

$$y_t - \mu - \phi_1 (y_{t-1} - \mu) - \phi_2 (y_{t-2} - \mu)$$

We now propose the conditional sum of squares (CSS) functions

$$S_c(\phi_1, \phi_2, \mu) = \sum_{t=3}^{n} \left[(y_t - \mu) - \phi_1 (y_{t-1} - \mu) - \phi_2 (y_{t-2} - \mu) \right]^2$$

and evaluate the values of ϕ_1, ϕ_2, μ that would minimize $S_c(\phi_1, \phi_2, \mu)$.

We equate $\frac{\partial S_c}{\partial \phi_1}$, $\frac{\partial S_c}{\partial \phi_2}$, $\frac{\partial S_c}{\partial \mu}$ to zero.

The conditional least square estimates of ϕ_i's $(i=1,2,\ldots)$ are approximately obtained by solving the samples Yule-Walker equations.

Q13) Explain unit root hypothesis for AR(1) model:

$$x_t = \mu + \rho x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim WN(0, \sigma^2)$$

Ans)

$$\Rightarrow x_t = \mu + \rho x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim WN(0, \sigma^2)$$

$$\Rightarrow \phi(B) x_t = \mu + \varepsilon_t \rightarrow (1)$$, where $\phi(B) = (1 - \phi_1 B)$ and B is Backward shift operator.

Here $\phi_1 = p$.

If any root of $\phi(B) = 0$ is 1 then $\phi(1) = 0$

If x_t has unit root then x_t is non-stationary.

The hypothesis of interest are:

eigodic

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H_0 (x_t is non-stationary) : $\phi_1 = 1$
H_1 (x_t is stationary) : $|\phi_1| < 1$

A t-test sums natural to test H_0 but the ergodic theorem and MD CLT do not apply:
The t-statistic does not have the usual distributions

Explanation:

The test statistic $t_{\phi_1=1} = \dfrac{\hat{\phi}_1 - 1}{SE(\hat{\phi}_1)}$

$\hat{\phi}_1 \rightarrow$ least squares estimate
$SE(\hat{\phi}_1) \rightarrow$ usual standard error estimate.

The test is one sided left tail test and if $\{x_t\}$ is stationary, it can be shown that

$$\sqrt{T}\,(\hat{\phi}_1 - \phi_1) \xrightarrow{d} N(0, 1 - \phi_1^2)$$

$$\Rightarrow \hat{\phi}_1 \overset{A}{\sim} N(\phi_1, \tfrac{1}{T}(1 - \phi_1^2))$$

$\Rightarrow t_{\phi_1=1} \sim N(0,1)$. However, under null hypothesis of non-stationarity the above result gives.

$\hat{\phi} \overset{A}{\sim} N(1, 0)$ which makes no sense.
Under the unit root null, $\{x_t\}$ is not stationary and eigodic and the usual sample moments do not converge to fixed constants.

Modified approach: We reparameterize the AR (1) process

$$\Delta x_t = x_t - x_{t-1} = \mu + (p-1) x_{t-1} + \varepsilon_t$$

$$\Rightarrow \Delta x_t = \mu + \alpha_0 x_{t-1} + \varepsilon_t \rightarrow ② \quad , \alpha_0 = p-1 = \phi_1 - 1$$

$$\therefore \phi(B) = (1 - \alpha_0 B)$$

We want to know if process ① has unit root, i.e. we have to choose between two models.

$H_0 : \phi(B) x_t = \mu + \varepsilon_t$
$H_1 : \phi'(B) x_t = \mu + \varepsilon_t$

The null hypothesis establishes that the system has unity as root and is non-stationary.
The alternate hypothesis states that it is less than 1 and the process is stationary.

If the model has unit root, then $\phi'(B) = 0$

$\Rightarrow \phi'(1) = 0$
$\Rightarrow \alpha_0 = 1$

Hence, $H_0 : \alpha_0 = 1$
$H_1 : |\alpha_0| < 1$

$$t = \frac{\hat{\alpha}_0 - 1}{S.t.\ (\hat{\alpha}_0)}$$

Q14) Write down the ACF of order K for an AR(1) model $x_t = 0.7x_{t-1} + \varepsilon_t$, where ε_t is a white noise process. Show that this AR(1) model can be expressed as a MA process of infinite order.

Ans)

Model: $x_t = 0.7x_{t-1} + \varepsilon_t \rightarrow ①$ $\{\varepsilon_t \sim WN(0,\sigma^2)$

- Recursing back one period

$$x_{t-1} = 0.7x_{t-2} + \varepsilon_{t-1}$$

Putting in ①

$$x_t = 0.7(0.7x_{t-2} + \varepsilon_{t-1}) + \varepsilon_t$$

$$= (0.7)^2 x_{t-2} + 0.7\varepsilon_{t-1} + \varepsilon_t \rightarrow ②$$

- Recursing back second period on eq ①

$$x_{t-2} = 0.7x_{t-3} + \varepsilon_{t-2}$$

Putting in eq ②

$$x_t = (0.7)^2[0.7x_{t-3} + \varepsilon_{t-2}] + 0.7\varepsilon_{t-1} + \varepsilon_t$$

$$= (0.7)^3 x_{t-3} + (0.7)^2\varepsilon_{t-2} + 0.7\varepsilon_{t-1} + \varepsilon_t$$

Similarly, recursing for p^{th} period:

$$x_t = (0.7)^{p+1} x_{t-(p+1)} + \sum_{i=0}^{p} (0.7)^i \varepsilon_{t-i} \rightarrow ③$$

Equation ③ is invertible since $0.7<1$

$\therefore$ As $p \to \infty$

$$x_t = 0 + \sum_{i=0}^{\infty} (0.7)^i \varepsilon_{t-i}$$

$$\Rightarrow x_t = \sum_{i=0}^{\infty} (0.7)^i \varepsilon_{t-i} \to ④ \text{, a } MA(\infty) \text{ process.}$$

Hence we can conclude that the gives AR(1) process can be expressed as MA process of order ~~MA~~ ∞.

Now consider a $MA(\infty)$ model:

$$x_t = \alpha + \sum_{K=0}^{\infty} \theta_K \varepsilon_{t-K} \to ⑤$$

Comparing eqⁿ ④ and ⑤

$$\theta_K = (0.7)^K, \ \varepsilon_t = \varepsilon_t$$

$$\text{Mean}(x_t) = \alpha = 0$$

$$\text{Var}(x_t) = \sigma^2 \sum_{K=0}^{\infty} \theta_K^2 = \sigma^2 \left(1 + (0.7)^2 + (0.7)^2 + \dots\right)$$

$$\Rightarrow \text{Var}(x_t) = \frac{\sigma^2}{1-(0.7)^2} = \frac{\sigma^2}{0.51}$$

ACVF at lag K, $\gamma(K) = \sigma^2 \sum_{i=0}^{\infty} \theta_i \theta_{i+K}$

$$= (0.7)^K \sigma^2 \sum_{j=0}^{\infty} (0.7)^{2j}$$

$$\because \gamma(0) = \text{Var}(x_t) = \frac{\sigma^2}{0.51}$$

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Then clearly $\gamma(k) = (0.7)^k \gamma(0)$

$$\Rightarrow \frac{\gamma(k)}{\gamma(0)} = \rho(k) = (0.7)^k \Big\} \text{ ACF of order } k$$

Q15) List the process are given below are stationary/ invertible process.

Soln)

(i) $y_t = \mu + u_t + 0.5\, u_{t-1} + y_{t-1} + 1.2\, y_{t-2}$

$$\Rightarrow (1 - B - 1.2B^2)\, y_t = \mu + (1 + 0.5B)\, u_t$$

$$\Rightarrow \Phi(B)\, y_t = \mu + \Theta(B)\, u_t$$

In order to check if the ARMA (2,1) process is stationary or not, we evaluate roots of $\Phi(B) = 0$ and $\Theta(B) = 0$ and verify $|B| > 1$.

For stationarity,

$$\Phi(B) = 0 \Rightarrow 1 - B - 1.2B^2 = 0$$

$$\Rightarrow |B| = \left| \frac{1 \pm \sqrt{1 + 4.8}}{2 \times 1.2} \right| = 1.4,\ 0.58$$

$\therefore |B| \not> 1$

For invertibility, $\Theta(B) = 0 \Rightarrow 1 + 0.5B = 0$

$\Rightarrow B = -2 \Rightarrow |B| = 2 > 1$

$\therefore$ The process is invertible.

ii) $y_t = 1.5 y_{t-1} + u_t - 0.5 u_{t-1}$

$\Rightarrow (1 - 1.5B) y_t = (1 - 0.5B) u_t$

$\Rightarrow \Phi(B) y_t = \Theta(B) u_t$

For stationarity,

$\Theta(B) = 0$

$\Rightarrow 1 - 0.5B = 0$

$\Rightarrow B = 2 > 1$

$\therefore$ The process is invertible

iii) $y_t = 2 y_{t-1} + 0.8 y_{t-2} + u_t$

$\Rightarrow (1 - 2B - 0.8B^2) y_t = u_t$

$\Rightarrow \Phi(B) y_t = u_t$

Checking for stationarity:

$\Phi(B) = 0 \Rightarrow 1 - 2B - 0.8B^2 = 0$

$\Rightarrow |B| = \left| \dfrac{2 \pm \sqrt{4 + 3.2}}{-2 \times 0.8} \right| = \left| \dfrac{1 \pm 1.34}{0.8} \right| = 2.925, 0.425$

$\therefore$ Not all $|B| > 1$

$\therefore$ The process is not stationary.

v) $x_t = 0.5x_{t-1} + \varepsilon_t - 0.3e_{t-1}$

$\Rightarrow (1-0.5B)x_t = (1-0.3B)e_t$

$\Rightarrow \phi(B)x_t = \Theta(B)e_t$

For stationarity,

$\Phi(B) = 0 \Rightarrow 1-0.5B = 0$

$\Rightarrow B = 2 > 1$

For invertibility,

$\Theta(B) = 0 \Rightarrow 1-0.3B = 0$

$\Rightarrow B = 3.33 > 1$

$\therefore$ The process is stationary and invertible

iv) $y_t = 6y_{t-1} - 4y_{t-2} + u_t - 2u_t$

$\Rightarrow (1-6B+4B^2)y_t = (1-2)u_t$

$\Rightarrow \Phi(B)y_t = \Theta(B)u_t$

For stationarity, $\Phi(B) = 0 \Rightarrow 1-6B+4B^2 = 0$

$4B^2 - 6B + 1 = 0$

$|B| = \left|\frac{+36 \pm \sqrt{36-16}}{2\times4}\right| = \left|\frac{36 \pm 4.47}{8}\right| = 5.058,\ 3.941$

$\Rightarrow |B| > 1$

For invertible,

$$\theta(B) = 0$$

$$\Rightarrow -1 \neq 0$$

$\therefore$ The process is stationary.