IMPORTS

In [ ]:
```
#importing relevant libraries

import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit    #for fitting an exponential
import functools as fnt
from textwrap import wrap
```

CONSTANTS

In [ ]:
```
#relevant constants
pc = 3.086e18                    #cm
pi = np.pi

#galaxy specific constants (taken from SS21_ch11)
R = 10*(10**3)*pc            #radius in cm
omega = 6.481*(10**(-18))    #angular velocity in s^-1
h = 0.5*(10**3)*pc           #height in cm
eta_T = 10**26               #Diffusion coefficient in cm^2 s^-1
to = h*h/eta_T               #diffusion time in s
print('time stepping is normalised to to(s) = ', to)
```

```
time stepping is normalised to to(s) =  2.380849e+16
```

DIFFERENTIAL EQUATIONS

In [ ]:
```
def Br_diff(t, Br, Bp, r_ind, dz, alpha):                                #for b

    dBr_dt = np.array(-1*(0)*(-3*Bp[0] + 4*Bp[1] - Bp[2])/(2*dz) + (2*Br[
    dBr_dt = np.append(dBr_dt, -1*alpha[r_ind]*np.array(Bp[2:] - Bp[:-2])
    dBr_dt = np.append(dBr_dt, -1*alpha[r_ind]*(3*Bp[-1] - 4*Bp[-2] + Bp[
    return dBr_dt

def Bp_diff(t, Br, Bp, r_ind, dz, alpha, S, D, Ralpha, ao = True):

    if ao:          #for alpha-omega dynamo
        dBp_dt = np.array( D*S[r_ind]*Br[0] + (2*Bp[0] - 5*Bp[1] + 4*Bp[2
        dBp_dt = np.append(dBp_dt, D*S[r_ind]*np.array(Br[1:-1]) + np.arr
        dBp_dt = np.append(dBp_dt, D*S[r_ind]*Br[-1] + (2*Bp[-1] - 5*Bp[-
    else:           #for alpha-square omega dynamo
        dBp_dt = np.array( D*S[r_ind]*Br[0] + (Ralpha**2)*alpha[r_ind]*(-
        dBp_dt = np.append(dBp_dt, D*S[r_ind]*np.array(Br[1:-1]) + (Ralph
        dBp_dt = np.append(dBp_dt, D*S[r_ind]*Br[-1] + (Ralpha**2)*alpha[

    return dBp_dt
```

RUNGE KUTTA (4TH ORDER)

For Dirichlet Boundary Conditions

These boundary conditions specify that the time evolution of the boundary values is fixed to some number.

For Neumann Boundary Conditions

These boundary conditions specify that the spatial derivatives of the function values are some number at the boundary.

This Runge-Kutta code solves a general system of differential equations in two variables(since in our case we have Br and Bphi)

In [ ]:
```
def rk4(F, G, X, Y, BX0, BXn, BY0, BYn, t, dt, dz, bc = 'dir'):
    #F is the function for the time evolution of X, in our case Br
    #G is the function for the time evolution of Y, in our case Bp
    #X is the array of X values
    #Y is the array of Y values
    #t is the current time
    #dt is the time step
    #dz is the spatial step
    #bc is the boundary condition type
    #BX0, BY0, BXn, BYn are the boundary values

    #We know the time evolution of the borders
    #So we only need to solve for the interior

    #let len(X) = len(Y) = n
    #Assigning the first boundary value
    if bc == 'dir':
        X_new = np.array([BX0])
        Y_new = np.array([BY0])
    elif bc == 'neu':
        X_new = np.array([4*X[1]/3 - X[2]/3 - 2*dz*BX0/3])
        Y_new = np.array([4*Y[1]/3 - Y[2]/3 - 2*dz*BY0/3])

    #Solving the runge kutta coefficients
    k1 = F(t, X, Y)                                     #len = n
    l1 = G(t, X, Y)                                     #len = n

    k2 = F(t + dt/2, X + k1*dt/2, Y + l1*dt/2)          #len = n
    l2 = G(t + dt/2, X + k1*dt/2, Y + l1*dt/2)          #len = n

    k3 = F(t + dt/2, X + k2*dt/2, Y + l2*dt/2)          #len = n
    l3 = G(t + dt/2, X + k2*dt/2, Y + l2*dt/2)          #len = n

    k4 = F(t + dt, X + k3*dt, Y + l3*dt)                #len = n
    l4 = G(t + dt, X + k3*dt, Y + l3*dt)                #len = n

    #Assigning the interior values
    X_new = np.append(X_new, X[1:-1] + np.array(k1 + 2*k2 + 2*k3 + k4)[1:
    Y_new = np.append(Y_new, Y[1:-1] + np.array(l1 + 2*l2 + 2*l3 + l4)[1:

    #Assigning the last boundary value
    if bc == 'dir':
        X_new = np.append(X_new, BXn)
```

```
        Y_new = np.append(Y_new, BYn)
    elif bc == 'neu':
        X_new = np.append(X_new, 4*X[-2]/3 - X[-3]/3 + 2*dz*BXn/3)
        Y_new = np.append(Y_new, 4*Y[-2]/3 - Y[-3]/3 + 2*dz*BYn/3)

    return X_new, Y_new
```

In [ ]:
```
#defining the grid and timestepping

#grid parameters
z_res = 2*10**2                 #resolution
r_res = 2*10**1                 #resolution
z0 = -1                         #lower z/h limit
zn = 1                          #upper z/h limit
r0 = 0.001                      #lower r/R limit
rn = 1                          #upper r/R limit
dz = (zn-z0)/z_res              #step size
dr = (rn-r0)/r_res              #step size
z = np.linspace(z0, zn, z_res)  #normalised to scale height h
r = np.linspace(r0, rn, r_res)  #normalised to radius R

#time parameters
dt = 0.000049
t0 = 0

print('dt/to =', dt, 'and dz =', dz)
print('The solution is stable when 2*dt/(dz**2) =', 2*dt/(dz**2), '< 1')
```

```
dt/to = 4.9e-05 and dz = 0.01
The solution is stable when 2*dt/(dz**2) = 0.9799999999999999 < 1
```

In [ ]:
```
#constants and other known variables (0 for the diffusion equation)
l0 = 0.1*(10**2)*pc                     #in cm
V0 = 2*10**7                            #in cm/s
omega0 = V0/R                           #in s^-1
alpha0 = (l0**2)*omega0/h               #normalised to l0**2 omega0/h
S0 = -omega0                            #normalised to 1/r

alpha = np.repeat(1, r_res)             #normalised to alpha0 = l0**2 ome
S = 1/r                                 #normalised to S0 = -omega0
D1 = -1
Ralpha = 1                              #Some dummy value, not used in al
r_ind = np.where(r >= 0.2)[0][0]        #Pick r where r/R around 0.2

#Initial seed fields
Br1 = np.cos(pi*z/2)
Bp1 = np.cos(pi*z/2)
Br2 = (np.cos(pi*z/2))**2 + (np.cos(3*pi*z/2))**2
Bp2 = (np.cos(pi*z/2))**2 + (np.cos(3*pi*z/2))**2
Br3 = np.sin(pi*z)
Bp3 = np.sin(pi*z)

#Dirichlet boundary conditions
Br1_0 = 0
Br1_n = 0
Bp1_0 = 0
Bp1_n = 0
```

```
Br2_0 = 0
Br2_n = 0
Bp2_0 = 0
Bp2_n = 0

Br3_0 = 0
Br3_n = 0
Bp3_0 = 0
Bp3_n = 0

#Removing function dependence on known values by specifying them
Br_diff_eq = fnt.partial(Br_diff, r_ind = r_ind, dz = dz, alpha = alpha)
Bp_diff_eq1 = fnt.partial(Bp_diff, r_ind = r_ind, dz = dz, alpha = alpha,
Bp_diff_eq2 = fnt.partial(Bp_diff, r_ind = r_ind, dz = dz, alpha = alpha,
Bp_diff_eq3 = fnt.partial(Bp_diff, r_ind = r_ind, dz = dz, alpha = alpha,
Bp_diff_eq4 = fnt.partial(Bp_diff, r_ind = r_ind, dz = dz, alpha = alpha,
Bp_diff_eq5 = fnt.partial(Bp_diff, r_ind = r_ind, dz = dz, alpha = alpha,
```

In [ ]:
```
#initialising the solutions
Br1_sol1 = np.array([Br1])
Bp1_sol1 = np.array([Bp1])

Br2_sol1 = np.array([Br2])
Bp2_sol1 = np.array([Bp2])

Br3_sol1 = np.array([Br3])
Bp3_sol1 = np.array([Bp3])

t = np.array([t0])

steps= 3*10**4
for i in range(steps):

    Br_temp, Bp_temp = rk4(Br_diff_eq, Bp_diff_eq1, Br1_sol1[-1], Bp1_sol
    Br1_sol1 = np.append(Br1_sol1, [Br_temp], axis = 0)
    Bp1_sol1 = np.append(Bp1_sol1, [Bp_temp], axis = 0)

    Br_temp, Bp_temp = rk4(Br_diff_eq, Bp_diff_eq1, Br2_sol1[-1], Bp2_sol
    Br2_sol1 = np.append(Br2_sol1, [Br_temp], axis = 0)
    Bp2_sol1 = np.append(Bp2_sol1, [Bp_temp], axis = 0)

    Br_temp, Bp_temp = rk4(Br_diff_eq, Bp_diff_eq1, Br3_sol1[-1], Bp3_sol
    Br3_sol1 = np.append(Br3_sol1, [Br_temp], axis = 0)
    Bp3_sol1 = np.append(Bp3_sol1, [Bp_temp], axis = 0)

    t = np.append(t, t[-1] + dt)
```

In [ ]:
```
#initialising the solutions
Br1_sol2 = np.array([Br1])
Bp1_sol2 = np.array([Bp1])

Br2_sol2 = np.array([Br2])
Bp2_sol2 = np.array([Bp2])

Br3_sol2 = np.array([Br3])
Bp3_sol2 = np.array([Bp3])

t = np.array([t0])
```

```
steps= 3*10**4
for i in range(steps):

    Br_temp, Bp_temp = rk4(Br_diff_eq, Bp_diff_eq2, Br1_sol2[-1], Bp1_sol
    Br1_sol2 = np.append(Br1_sol2, [Br_temp], axis = 0)
    Bp1_sol2 = np.append(Bp1_sol2, [Bp_temp], axis = 0)

    Br_temp, Bp_temp = rk4(Br_diff_eq, Bp_diff_eq2, Br2_sol2[-1], Bp2_sol
    Br2_sol2 = np.append(Br2_sol2, [Br_temp], axis = 0)
    Bp2_sol2 = np.append(Bp2_sol2, [Bp_temp], axis = 0)

    Br_temp, Bp_temp = rk4(Br_diff_eq, Bp_diff_eq2, Br3_sol2[-1], Bp3_sol
    Br3_sol2 = np.append(Br3_sol2, [Br_temp], axis = 0)
    Bp3_sol2 = np.append(Bp3_sol2, [Bp_temp], axis = 0)

    t = np.append(t, t[-1] + dt)
```

In [ ]:
```
#initialising the solutions
Br1_sol3 = np.array([Br1])
Bp1_sol3 = np.array([Bp1])

Br2_sol3 = np.array([Br2])
Bp2_sol3 = np.array([Bp2])

Br3_sol3 = np.array([Br3])
Bp3_sol3 = np.array([Bp3])

t = np.array([t0])

steps= 3*10**4
for i in range(steps):

    Br_temp, Bp_temp = rk4(Br_diff_eq, Bp_diff_eq3, Br1_sol3[-1], Bp1_sol
    Br1_sol3 = np.append(Br1_sol3, [Br_temp], axis = 0)
    Bp1_sol3 = np.append(Bp1_sol3, [Bp_temp], axis = 0)

    Br_temp, Bp_temp = rk4(Br_diff_eq, Bp_diff_eq3, Br2_sol3[-1], Bp2_sol
    Br2_sol3 = np.append(Br2_sol3, [Br_temp], axis = 0)
    Bp2_sol3 = np.append(Bp2_sol3, [Bp_temp], axis = 0)

    Br_temp, Bp_temp = rk4(Br_diff_eq, Bp_diff_eq3, Br3_sol3[-1], Bp3_sol
    Br3_sol3 = np.append(Br3_sol3, [Br_temp], axis = 0)
    Bp3_sol3 = np.append(Bp3_sol3, [Bp_temp], axis = 0)

    t = np.append(t, t[-1] + dt)
```

In [ ]:
```
#initialising the solutions
Br1_sol4 = np.array([Br1])
Bp1_sol4 = np.array([Bp1])

Br2_sol4 = np.array([Br2])
Bp2_sol4 = np.array([Bp2])

Br3_sol4 = np.array([Br3])
Bp3_sol4 = np.array([Bp3])

t = np.array([t0])

steps= 3*10**4
```

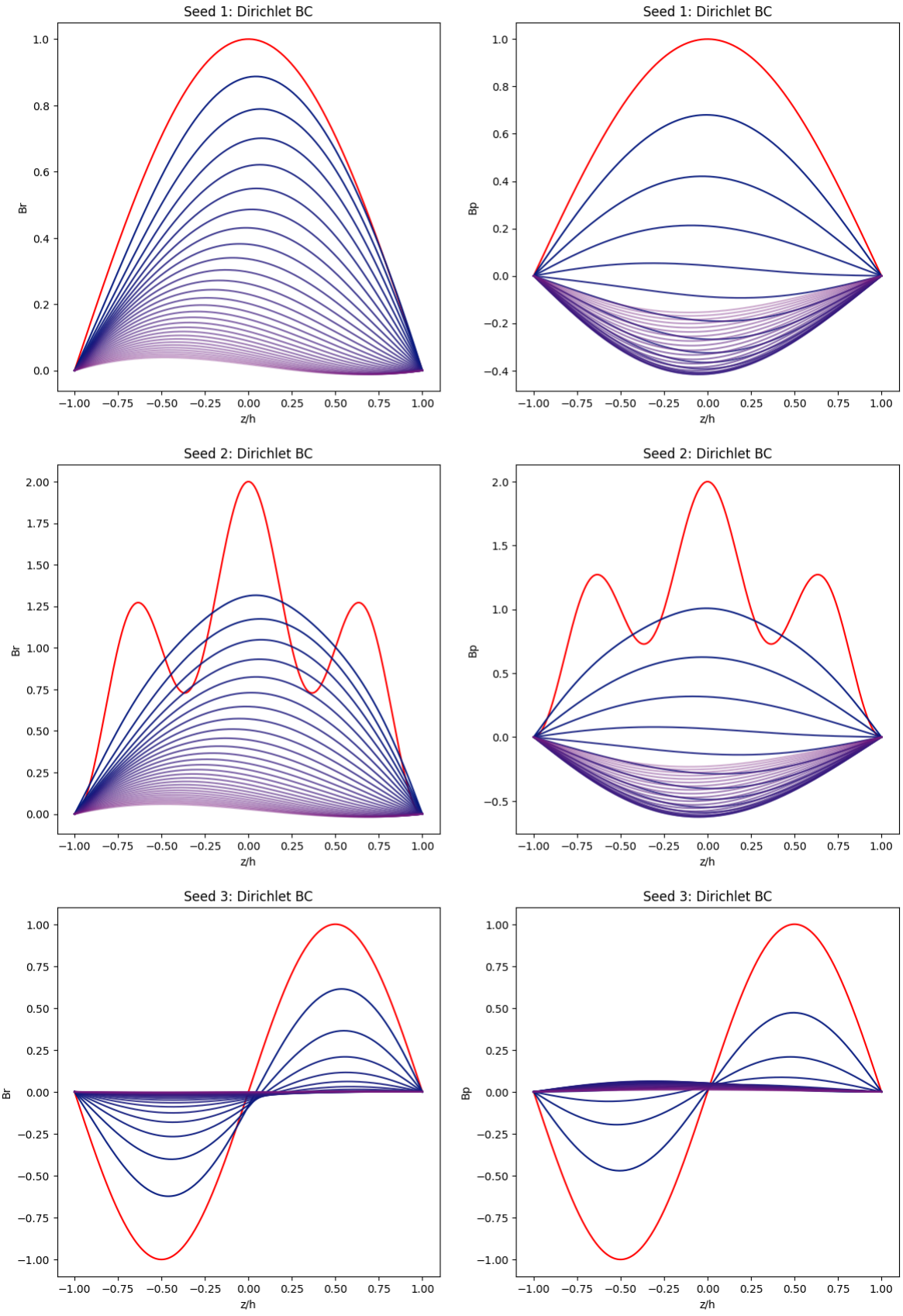
```
for i in range(steps):

    Br_temp, Bp_temp = rk4(Br_diff_eq, Bp_diff_eq4, Br1_sol4[-1], Bp1_sol
    Br1_sol4 = np.append(Br1_sol4, [Br_temp], axis = 0)
    Bp1_sol4 = np.append(Bp1_sol4, [Bp_temp], axis = 0)

    Br_temp, Bp_temp = rk4(Br_diff_eq, Bp_diff_eq4, Br2_sol4[-1], Bp2_sol
    Br2_sol4 = np.append(Br2_sol4, [Br_temp], axis = 0)
    Bp2_sol4 = np.append(Bp2_sol4, [Bp_temp], axis = 0)

    Br_temp, Bp_temp = rk4(Br_diff_eq, Bp_diff_eq4, Br3_sol4[-1], Bp3_sol
    Br3_sol4 = np.append(Br3_sol4, [Br_temp], axis = 0)
    Bp3_sol4 = np.append(Bp3_sol4, [Bp_temp], axis = 0)

    t = np.append(t, t[-1] + dt)
```

In [ ]:
```
#initialising the solutions
Br1_sol5 = np.array([Br1])
Bp1_sol5 = np.array([Bp1])

Br2_sol5 = np.array([Br2])
Bp2_sol5 = np.array([Bp2])

Br3_sol5 = np.array([Br3])
Bp3_sol5 = np.array([Bp3])

t = np.array([t0])

steps= 6*10**4
for i in range(steps):

    Br_temp, Bp_temp = rk4(Br_diff_eq, Bp_diff_eq5, Br1_sol5[-1], Bp1_sol
    Br1_sol5 = np.append(Br1_sol5, [Br_temp], axis = 0)
    Bp1_sol5 = np.append(Bp1_sol5, [Bp_temp], axis = 0)

    Br_temp, Bp_temp = rk4(Br_diff_eq, Bp_diff_eq5, Br2_sol5[-1], Bp2_sol
    Br2_sol5 = np.append(Br2_sol5, [Br_temp], axis = 0)
    Bp2_sol5 = np.append(Bp2_sol5, [Bp_temp], axis = 0)

    Br_temp, Bp_temp = rk4(Br_diff_eq, Bp_diff_eq5, Br3_sol5[-1], Bp3_sol
    Br3_sol5 = np.append(Br3_sol5, [Br_temp], axis = 0)
    Bp3_sol5 = np.append(Bp3_sol5, [Bp_temp], axis = 0)

    t = np.append(t, t[-1] + dt)
```

In [ ]:
```
fig, axs = plt.subplots(3, 2, figsize = (14, 21))
i = 1000
fig.suptitle("\n".join(wrap("Time evolution of Br for D=-1 (blue to red =

axs[0, 0].plot(z, Br1_sol1[0], color = 'red')
axs[0, 0].set_title('Seed 1: Dirichlet BC')
axs[0, 0].set(xlabel='z/h', ylabel='Br')

axs[0, 1].plot(z, Bp1_sol1[0], color = 'red')
axs[0, 1].set_title('Seed 1: Dirichlet BC')
axs[0, 1].set(xlabel='z/h', ylabel='Bp')

axs[1, 0].plot(z, Br2_sol1[0], color = 'red')
axs[1, 0].set_title('Seed 2: Dirichlet BC')
```

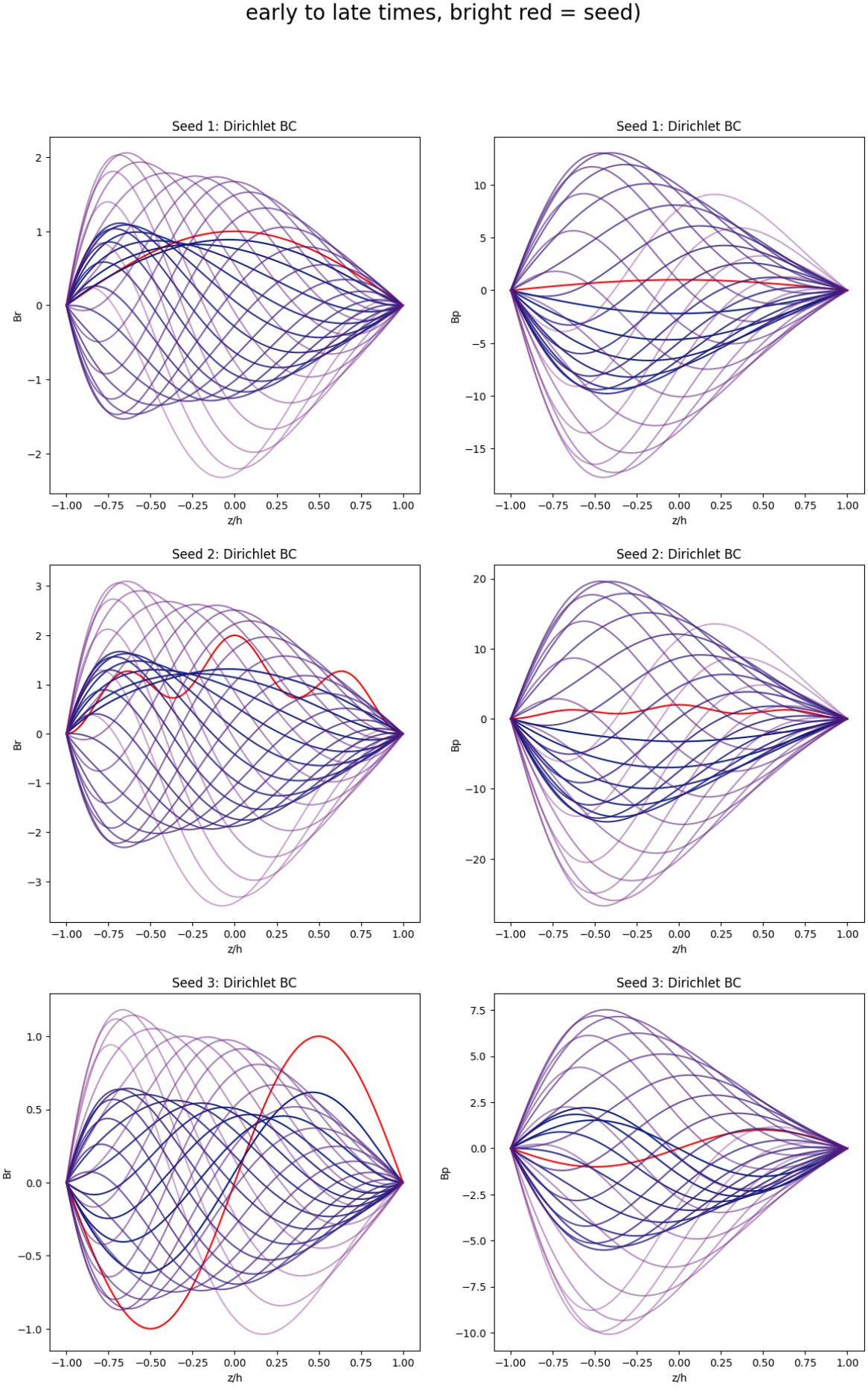
```
axs[1, 0].set(xlabel='z/h', ylabel='Br')

axs[1, 1].plot(z, Bp2_sol1[0], color = 'red')
axs[1, 1].set_title('Seed 2: Dirichlet BC')
axs[1, 1].set(xlabel='z/h', ylabel='Bp')

axs[2, 0].plot(z, Br3_sol1[0], color = 'red')
axs[2, 0].set_title('Seed 3: Dirichlet BC')
axs[2, 0].set(xlabel='z/h', ylabel='Br')

axs[2, 1].plot(z, Bp3_sol1[0], color = 'red')
axs[2, 1].set_title('Seed 3: Dirichlet BC')
axs[2, 1].set(xlabel='z/h', ylabel='Bp')

while i < steps:
    axs[0, 0].plot(z, Br1_sol1[i], color = (round(i/(2*steps), 5), 0.1, 0
    axs[0, 0].set_title('Seed 1: Dirichlet BC')
    axs[0, 0].set(xlabel='z/h', ylabel='Br')

    axs[0, 1].plot(z, Bp1_sol1[i], color = (round(i/(2*steps), 5), 0.1, 0
    axs[0, 1].set_title('Seed 1: Dirichlet BC')
    axs[0, 1].set(xlabel='z/h', ylabel='Bp')

    axs[1, 0].plot(z, Br2_sol1[i], color = (round(i/(2*steps), 5), 0.1, 0
    axs[1, 0].set_title('Seed 2: Dirichlet BC')
    axs[1, 0].set(xlabel='z/h', ylabel='Br')

    axs[1, 1].plot(z, Bp2_sol1[i], color = (round(i/(2*steps), 5), 0.1, 0
    axs[1, 1].set_title('Seed 2: Dirichlet BC')
    axs[1, 1].set(xlabel='z/h', ylabel='Bp')

    axs[2, 0].plot(z, Br3_sol1[i], color = (round(i/(2*steps), 5), 0.1, 0
    axs[2, 0].set_title('Seed 3: Dirichlet BC')
    axs[2, 0].set(xlabel='z/h', ylabel='Br')

    axs[2, 1].plot(z, Bp3_sol1[i], color = (round(i/(2*steps), 5), 0.1, 0
    axs[2, 1].set_title('Seed 3: Dirichlet BC')
    axs[2, 1].set(xlabel='z/h', ylabel='Bp')

    i += 1000

# for ax in axs.flat:
#     ax.set(xlabel='z/h', ylabel='Br')
```

Time evolution of Br for D=-1 (blue to red = early to late times, bright red = seed)



In [ ]:
```
fig2, axs2 = plt.subplots(3, 2, figsize = (14, 21))
i = 1000
fig2.suptitle("\n".join(wrap("Time evolution of Br for D=-15 (blue to red
```

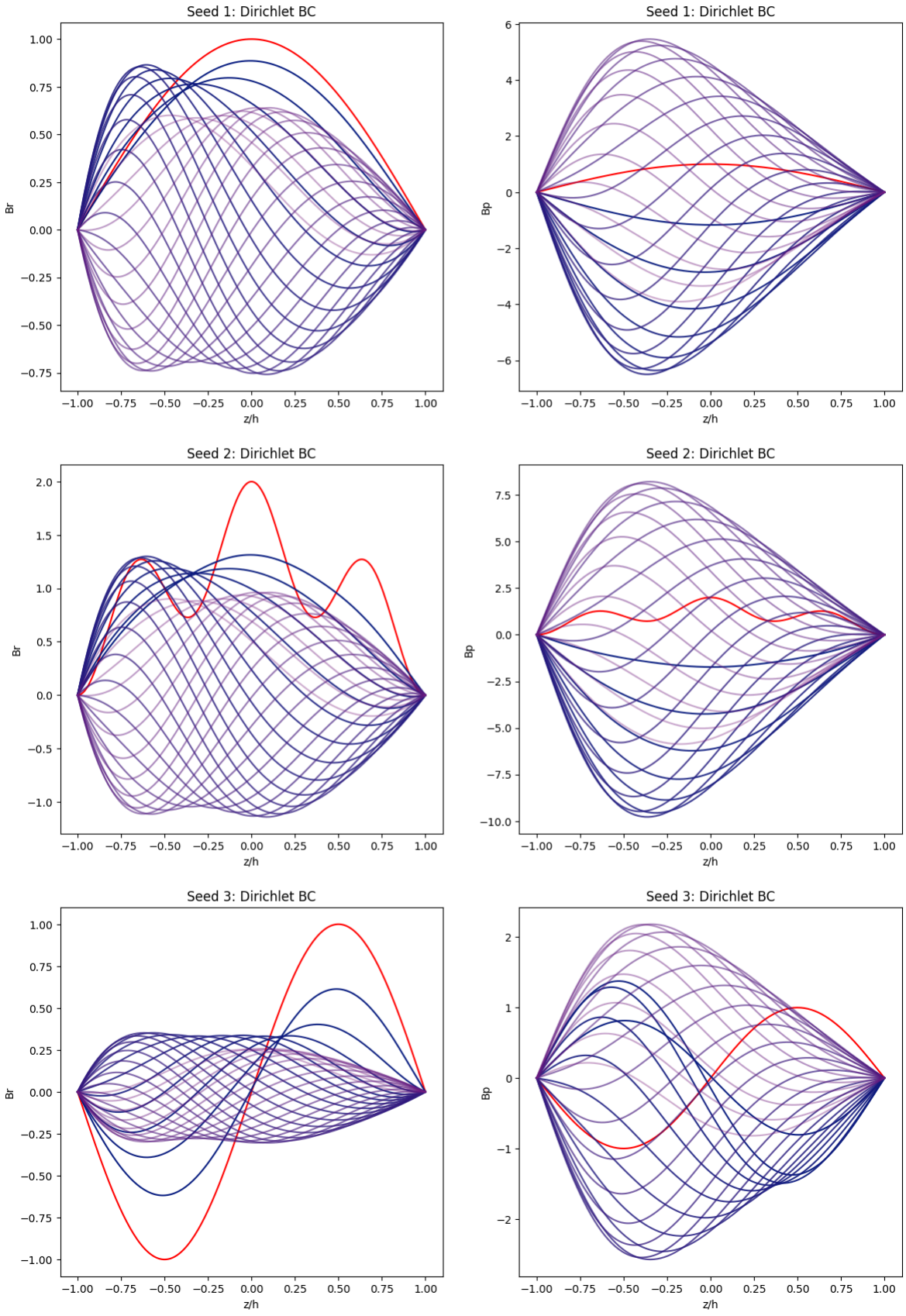
```
axs2[0, 0].plot(z, Br1_sol2[0], color = 'red')
axs2[0, 0].set_title('Seed 1: Dirichlet BC')
axs2[0, 0].set(xlabel='z/h', ylabel='Br')

axs2[0, 1].plot(z, Bp1_sol2[0], color = 'red')
axs2[0, 1].set_title('Seed 1: Dirichlet BC')
axs2[0, 1].set(xlabel='z/h', ylabel='Bp')

axs2[1, 0].plot(z, Br2_sol2[0], color = 'red')
axs2[1, 0].set_title('Seed 2: Dirichlet BC')
axs2[1, 0].set(xlabel='z/h', ylabel='Br')

axs2[1, 1].plot(z, Bp2_sol2[0], color = 'red')
axs2[1, 1].set_title('Seed 2: Dirichlet BC')
axs2[1, 1].set(xlabel='z/h', ylabel='Bp')

axs2[2, 0].plot(z, Br3_sol2[0], color = 'red')
axs2[2, 0].set_title('Seed 3: Dirichlet BC')
axs2[2, 0].set(xlabel='z/h', ylabel='Br')

axs2[2, 1].plot(z, Bp3_sol2[0], color = 'red')
axs2[2, 1].set_title('Seed 3: Dirichlet BC')
axs2[2, 1].set(xlabel='z/h', ylabel='Bp')

while i < steps:
    axs2[0, 0].plot(z, Br1_sol2[i], color = (round(i/(2*steps), 5), 0.1,
    axs2[0, 0].set_title('Seed 1: Dirichlet BC')
    axs2[0, 0].set(xlabel='z/h', ylabel='Br')

    axs2[0, 1].plot(z, Bp1_sol2[i], color = (round(i/(2*steps), 5), 0.1,
    axs2[0, 1].set_title('Seed 1: Dirichlet BC')
    axs2[0, 1].set(xlabel='z/h', ylabel='Bp')

    axs2[1, 0].plot(z, Br2_sol2[i], color = (round(i/(2*steps), 5), 0.1,
    axs2[1, 0].set_title('Seed 2: Dirichlet BC')
    axs2[1, 0].set(xlabel='z/h', ylabel='Br')

    axs2[1, 1].plot(z, Bp2_sol2[i], color = (round(i/(2*steps), 5), 0.1,
    axs2[1, 1].set_title('Seed 2: Dirichlet BC')
    axs2[1, 1].set(xlabel='z/h', ylabel='Bp')

    axs2[2, 0].plot(z, Br3_sol2[i], color = (round(i/(2*steps), 5), 0.1,
    axs2[2, 0].set_title('Seed 3: Dirichlet BC')
    axs2[2, 0].set(xlabel='z/h', ylabel='Br')

    axs2[2, 1].plot(z, Bp3_sol2[i], color = (round(i/(2*steps), 5), 0.1,
    axs2[2, 1].set_title('Seed 3: Dirichlet BC')
    axs2[2, 1].set(xlabel='z/h', ylabel='Bp')

    i += 1000

# for ax in axs2.flat:
#     ax.set(xlabel='z/h', ylabel='Br')
```



In [ ]:
```
fig3, axs3 = plt.subplots(3, 2, figsize = (14, 21))
i = 1000
fig3.suptitle("\n".join(wrap("Time evolution of Br for D=-10 (blue to red
```

```
axs3[0, 0].plot(z, Br1_sol3[0], color = 'red')
axs3[0, 0].set_title('Seed 1: Dirichlet BC')
axs3[0, 0].set(xlabel='z/h', ylabel='Br')

axs3[0, 1].plot(z, Bp1_sol3[0], color = 'red')
axs3[0, 1].set_title('Seed 1: Dirichlet BC')
axs3[0, 1].set(xlabel='z/h', ylabel='Bp')

axs3[1, 0].plot(z, Br2_sol3[0], color = 'red')
axs3[1, 0].set_title('Seed 2: Dirichlet BC')
axs3[1, 0].set(xlabel='z/h', ylabel='Br')

axs3[1, 1].plot(z, Bp2_sol3[0], color = 'red')
axs3[1, 1].set_title('Seed 2: Dirichlet BC')
axs3[1, 1].set(xlabel='z/h', ylabel='Bp')

axs3[2, 0].plot(z, Br3_sol3[0], color = 'red')
axs3[2, 0].set_title('Seed 3: Dirichlet BC')
axs3[2, 0].set(xlabel='z/h', ylabel='Br')

axs3[2, 1].plot(z, Bp3_sol3[0], color = 'red')
axs3[2, 1].set_title('Seed 3: Dirichlet BC')
axs3[2, 1].set(xlabel='z/h', ylabel='Bp')

while i < steps:
    axs3[0, 0].plot(z, Br1_sol3[i], color = (round(i/(2*steps), 5), 0.1,
    axs3[0, 0].set_title('Seed 1: Dirichlet BC')
    axs3[0, 0].set(xlabel='z/h', ylabel='Br')

    axs3[0, 1].plot(z, Bp1_sol3[i], color = (round(i/(2*steps), 5), 0.1,
    axs3[0, 1].set_title('Seed 1: Dirichlet BC')
    axs3[0, 1].set(xlabel='z/h', ylabel='Bp')

    axs3[1, 0].plot(z, Br2_sol3[i], color = (round(i/(2*steps), 5), 0.1,
    axs3[1, 0].set_title('Seed 2: Dirichlet BC')
    axs3[1, 0].set(xlabel='z/h', ylabel='Br')

    axs3[1, 1].plot(z, Bp2_sol3[i], color = (round(i/(2*steps), 5), 0.1,
    axs3[1, 1].set_title('Seed 2: Dirichlet BC')
    axs3[1, 1].set(xlabel='z/h', ylabel='Bp')

    axs3[2, 0].plot(z, Br3_sol3[i], color = (round(i/(2*steps), 5), 0.1,
    axs3[2, 0].set_title('Seed 3: Dirichlet BC')
    axs3[2, 0].set(xlabel='z/h', ylabel='Br')

    axs3[2, 1].plot(z, Bp3_sol3[i], color = (round(i/(2*steps), 5), 0.1,
    axs3[2, 1].set_title('Seed 3: Dirichlet BC')
    axs3[2, 1].set(xlabel='z/h', ylabel='Bp')

    i += 1000

# for ax in axs3.flat:
#     ax.set(xlabel='z/h', ylabel='Br')
```

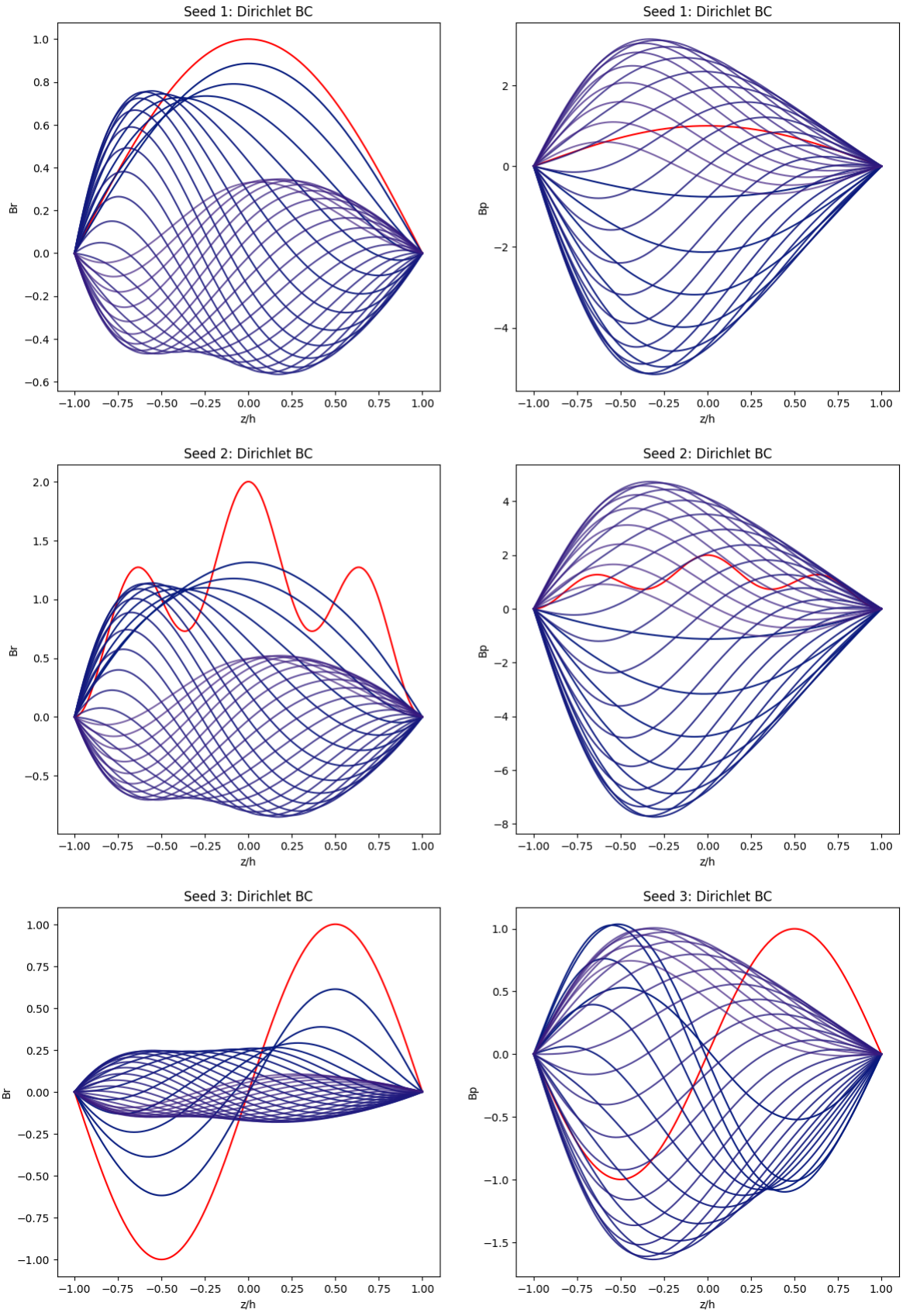
Time evolution of Br for D=-10 (blue to red =
early to late times, bright red = seed)



In [ ]:
```
fig4, axs4 = plt.subplots(3, 2, figsize = (14, 21))
i = 1000
fig4.suptitle("\n".join(wrap("Time evolution of Br for D=-8 (blue to red
```

```
axs4[0, 0].plot(z, Br1_sol4[0], color = 'red')
axs4[0, 0].set_title('Seed 1: Dirichlet BC')
axs4[0, 0].set(xlabel='z/h', ylabel='Br')

axs4[0, 1].plot(z, Bp1_sol4[0], color = 'red')
axs4[0, 1].set_title('Seed 1: Dirichlet BC')
axs4[0, 1].set(xlabel='z/h', ylabel='Bp')

axs4[1, 0].plot(z, Br2_sol4[0], color = 'red')
axs4[1, 0].set_title('Seed 2: Dirichlet BC')
axs4[1, 0].set(xlabel='z/h', ylabel='Br')

axs4[1, 1].plot(z, Bp2_sol4[0], color = 'red')
axs4[1, 1].set_title('Seed 2: Dirichlet BC')
axs4[1, 1].set(xlabel='z/h', ylabel='Bp')

axs4[2, 0].plot(z, Br3_sol4[0], color = 'red')
axs4[2, 0].set_title('Seed 3: Dirichlet BC')
axs4[2, 0].set(xlabel='z/h', ylabel='Br')

axs4[2, 1].plot(z, Bp3_sol4[0], color = 'red')
axs4[2, 1].set_title('Seed 3: Dirichlet BC')
axs4[2, 1].set(xlabel='z/h', ylabel='Bp')

while i < 3*10**4:
    axs4[0, 0].plot(z, Br1_sol4[i], color = (round(i/(2*steps), 5), 0.1,
    axs4[0, 0].set_title('Seed 1: Dirichlet BC')
    axs4[0, 0].set(xlabel='z/h', ylabel='Br')

    axs4[0, 1].plot(z, Bp1_sol4[i], color = (round(i/(2*steps), 5), 0.1,
    axs4[0, 1].set_title('Seed 1: Dirichlet BC')
    axs4[0, 1].set(xlabel='z/h', ylabel='Bp')

    axs4[1, 0].plot(z, Br2_sol4[i], color = (round(i/(2*steps), 5), 0.1,
    axs4[1, 0].set_title('Seed 2: Dirichlet BC')
    axs4[1, 0].set(xlabel='z/h', ylabel='Br')

    axs4[1, 1].plot(z, Bp2_sol4[i], color = (round(i/(2*steps), 5), 0.1,
    axs4[1, 1].set_title('Seed 2: Dirichlet BC')
    axs4[1, 1].set(xlabel='z/h', ylabel='Bp')

    axs4[2, 0].plot(z, Br3_sol4[i], color = (round(i/(2*steps), 5), 0.1,
    axs4[2, 0].set_title('Seed 3: Dirichlet BC')
    axs4[2, 0].set(xlabel='z/h', ylabel='Br')

    axs4[2, 1].plot(z, Bp3_sol4[i], color = (round(i/(2*steps), 5), 0.1,
    axs4[2, 1].set_title('Seed 3: Dirichlet BC')
    axs4[2, 1].set(xlabel='z/h', ylabel='Bp')

    i += 1000

# for ax in axs4.flat:
#     ax.set(xlabel='z/h', ylabel='Br')
```

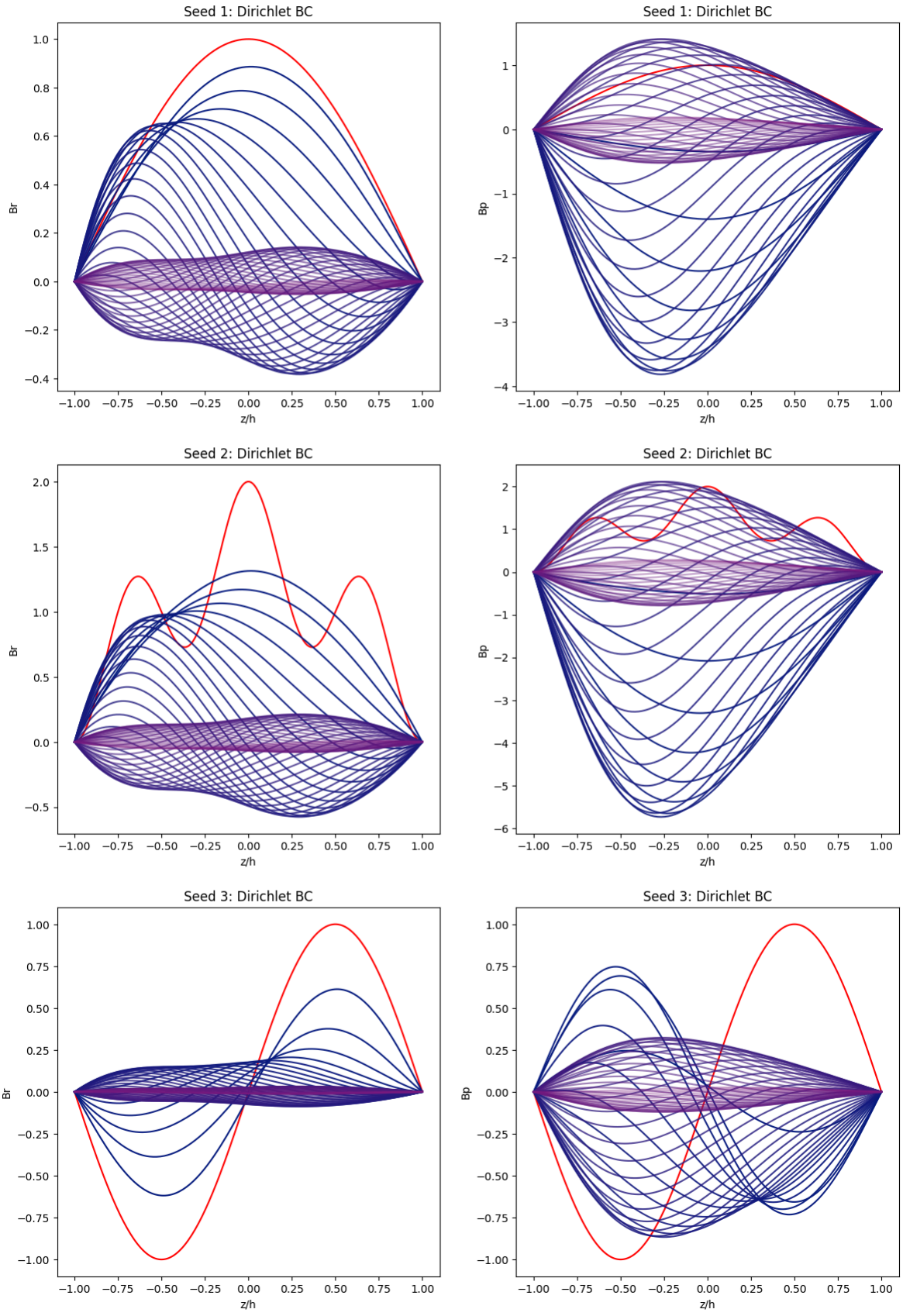
Time evolution of Br for D=-8 (blue to red = early to late times, bright red = seed)



In [ ]:
```
fig5, axs5 = plt.subplots(3, 2, figsize = (14, 21))
i = 1000
fig5.suptitle("\n".join(wrap("Time evolution of Br for D=-1 (blue to red
```

```
axs5[0, 0].plot(z, Br1_sol5[0], color = 'red')
axs5[0, 0].set_title('Seed 1: Dirichlet BC')
axs5[0, 0].set(xlabel='z/h', ylabel='Br')

axs5[0, 1].plot(z, Bp1_sol5[0], color = 'red')
axs5[0, 1].set_title('Seed 1: Dirichlet BC')
axs5[0, 1].set(xlabel='z/h', ylabel='Bp')

axs5[1, 0].plot(z, Br2_sol5[0], color = 'red')
axs5[1, 0].set_title('Seed 2: Dirichlet BC')
axs5[1, 0].set(xlabel='z/h', ylabel='Br')

axs5[1, 1].plot(z, Bp2_sol5[0], color = 'red')
axs5[1, 1].set_title('Seed 2: Dirichlet BC')
axs5[1, 1].set(xlabel='z/h', ylabel='Bp')

axs5[2, 0].plot(z, Br3_sol5[0], color = 'red')
axs5[2, 0].set_title('Seed 3: Dirichlet BC')
axs5[2, 0].set(xlabel='z/h', ylabel='Br')

axs5[2, 1].plot(z, Bp3_sol5[0], color = 'red')
axs5[2, 1].set_title('Seed 3: Dirichlet BC')
axs5[2, 1].set(xlabel='z/h', ylabel='Bp')

while i < steps:
    axs5[0, 0].plot(z, Br1_sol5[i], color = (round(i/(2*steps), 5), 0.1,
    axs5[0, 0].set_title('Seed 1: Dirichlet BC')
    axs5[0, 0].set(xlabel='z/h', ylabel='Br')

    axs5[0, 1].plot(z, Bp1_sol5[i], color = (round(i/(2*steps), 5), 0.1,
    axs5[0, 1].set_title('Seed 1: Dirichlet BC')
    axs5[0, 1].set(xlabel='z/h', ylabel='Bp')

    axs5[1, 0].plot(z, Br2_sol5[i], color = (round(i/(2*steps), 5), 0.1,
    axs5[1, 0].set_title('Seed 2: Dirichlet BC')
    axs5[1, 0].set(xlabel='z/h', ylabel='Br')

    axs5[1, 1].plot(z, Bp2_sol5[i], color = (round(i/(2*steps), 5), 0.1,
    axs5[1, 1].set_title('Seed 2: Dirichlet BC')
    axs5[1, 1].set(xlabel='z/h', ylabel='Bp')

    axs5[2, 0].plot(z, Br3_sol5[i], color = (round(i/(2*steps), 5), 0.1,
    axs5[2, 0].set_title('Seed 3: Dirichlet BC')
    axs5[2, 0].set(xlabel='z/h', ylabel='Br')

    axs5[2, 1].plot(z, Bp3_sol5[i], color = (round(i/(2*steps), 5), 0.1,
    axs5[2, 1].set_title('Seed 3: Dirichlet BC')
    axs5[2, 1].set(xlabel='z/h', ylabel='Bp')

    i += 1000

# for ax in axs5.flat:
#     ax.set(xlabel='z/h', ylabel='Br')
```

Time evolution of Br for D=-1 (blue to red = early to late times, bright red = seed)



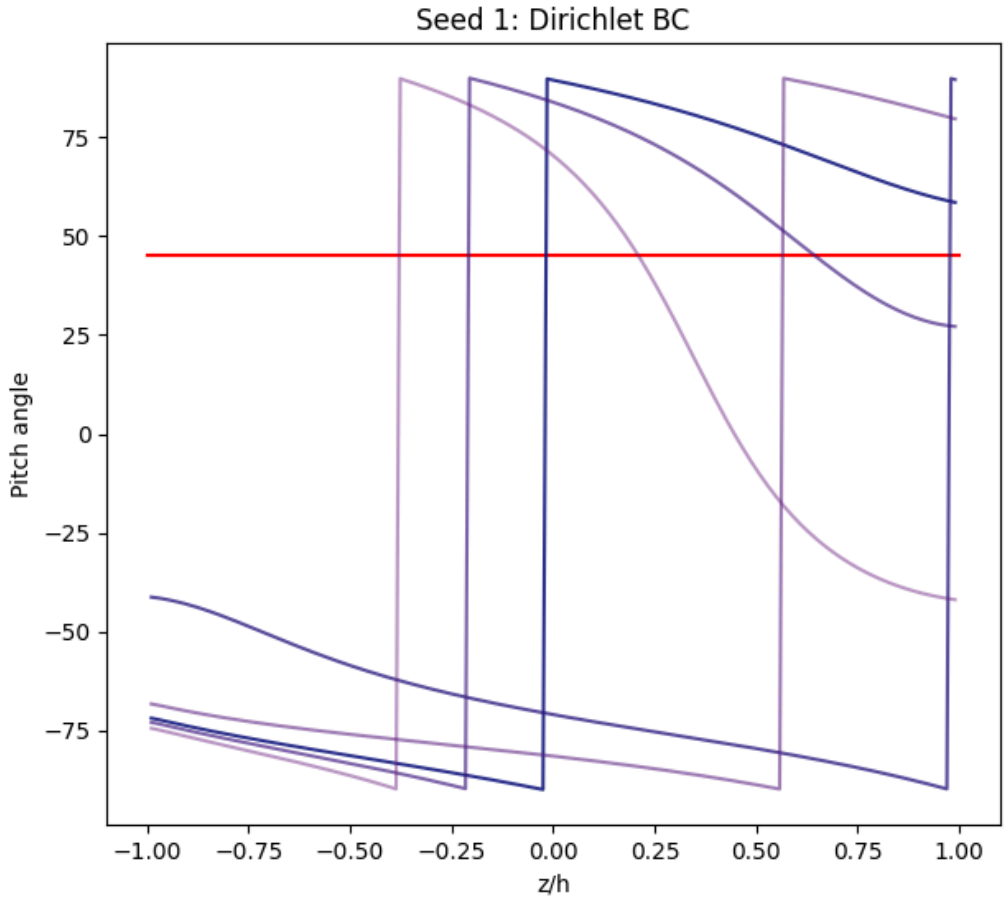
From the above simulations, I think the critical dynamo number lies somewhere between -7 to -9 for alpha = 1. I

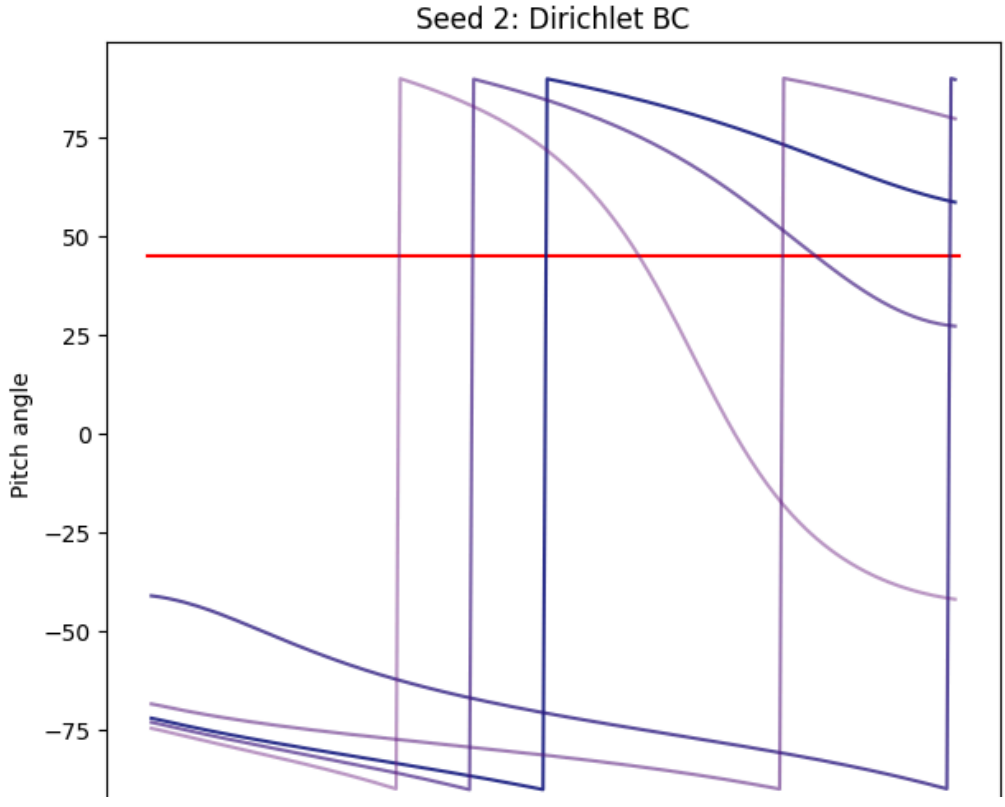
performed the simulation for the -6 for a larger time to show the decay more clearly. We can see that for D < -10, the fields get amplified.

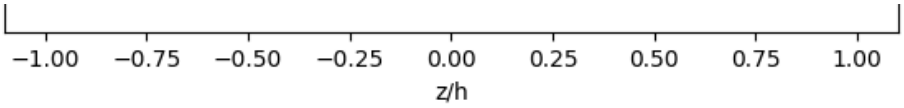
In [ ]:
```
def pitch(Br, Bp):
    return 180*np.arctan(Bp/Br)/np.pi

i = 10000
fig6, axs6 = plt.subplots(3, 1, figsize = (7, 21))
fig6.suptitle("\n".join(wrap("Time evolution of pitch angle (deg) for D=-

axs6[0].plot(z, pitch(Br1_sol5[0], Bp1_sol5[0]), color = 'red')
axs6[0].set_title('Seed 1: Dirichlet BC')
axs6[0].set(xlabel='z/h', ylabel='Pitch angle')

axs6[1].plot(z, pitch(Br2_sol5[0], Bp2_sol5[0]), color = 'red')
axs6[1].set_title('Seed 2: Dirichlet BC')
axs6[1].set(xlabel='z/h', ylabel='Pitch angle')

axs6[2].plot(z, pitch(Br3_sol5[0], Bp3_sol5[0]), color = 'red')
axs6[2].set_title('Seed 3: Dirichlet BC')
axs6[2].set(xlabel='z/h', ylabel='Pitch angle')



while i < 6*10**4:
    axs6[0].plot(z, pitch(Br1_sol5[i], Bp1_sol5[i]), color = (round(i/(2*
    axs6[0].set_title('Seed 1: Dirichlet BC')
    axs6[0].set(xlabel='z/h', ylabel='Pitch angle')

    axs6[1].plot(z, pitch(Br2_sol5[i], Bp2_sol5[i]), color = (round(i/(2*
    axs6[1].set_title('Seed 2: Dirichlet BC')
    axs6[1].set(xlabel='z/h', ylabel='Pitch angle')

    axs6[2].plot(z, pitch(Br3_sol5[i], Bp3_sol5[i]), color = (round(i/(2*
    axs6[2].set_title('Seed 3: Dirichlet BC')
    axs6[2].set(xlabel='z/h', ylabel='Pitch angle')

    i += 10000
```

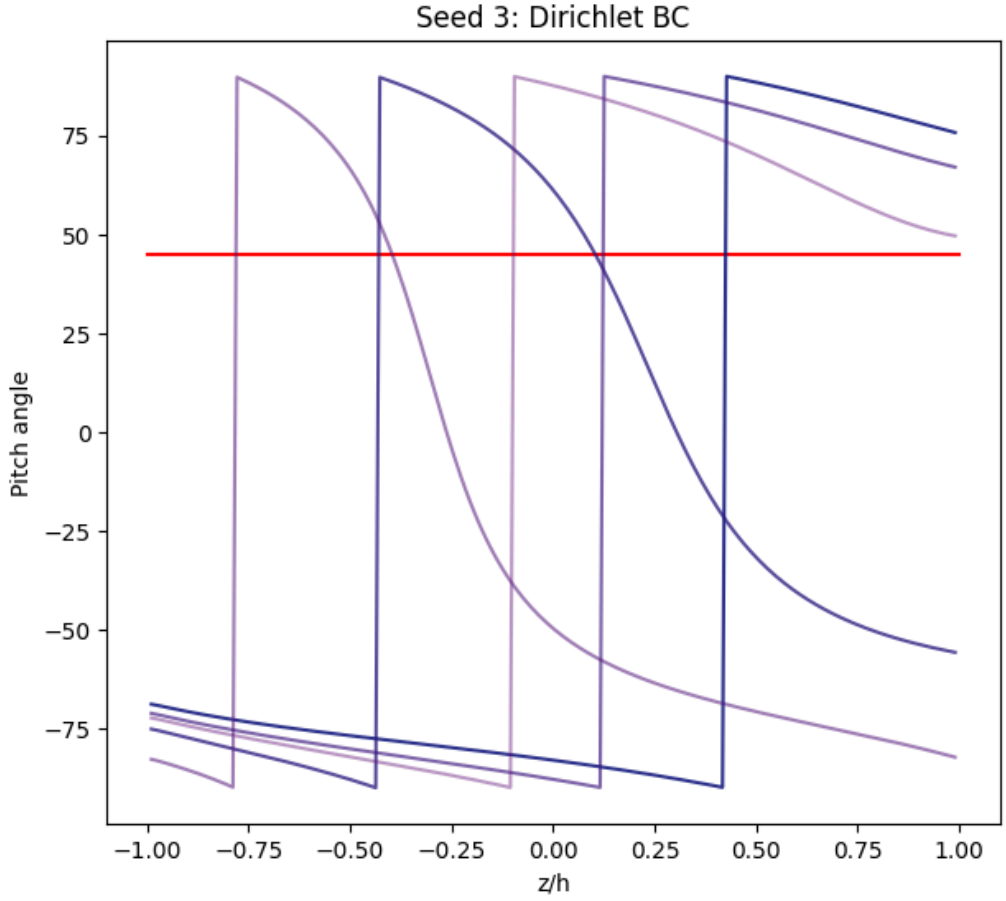
```
/tmp/ipykernel_41349/1302223438.py:2: RuntimeWarning: invalid value encoun
tered in divide
  return 180*np.arctan(Bp/Br)/np.pi
```

Time evolution of pitch angle (deg) for D=-6 (blue to red = early to late times, bright red = seed)







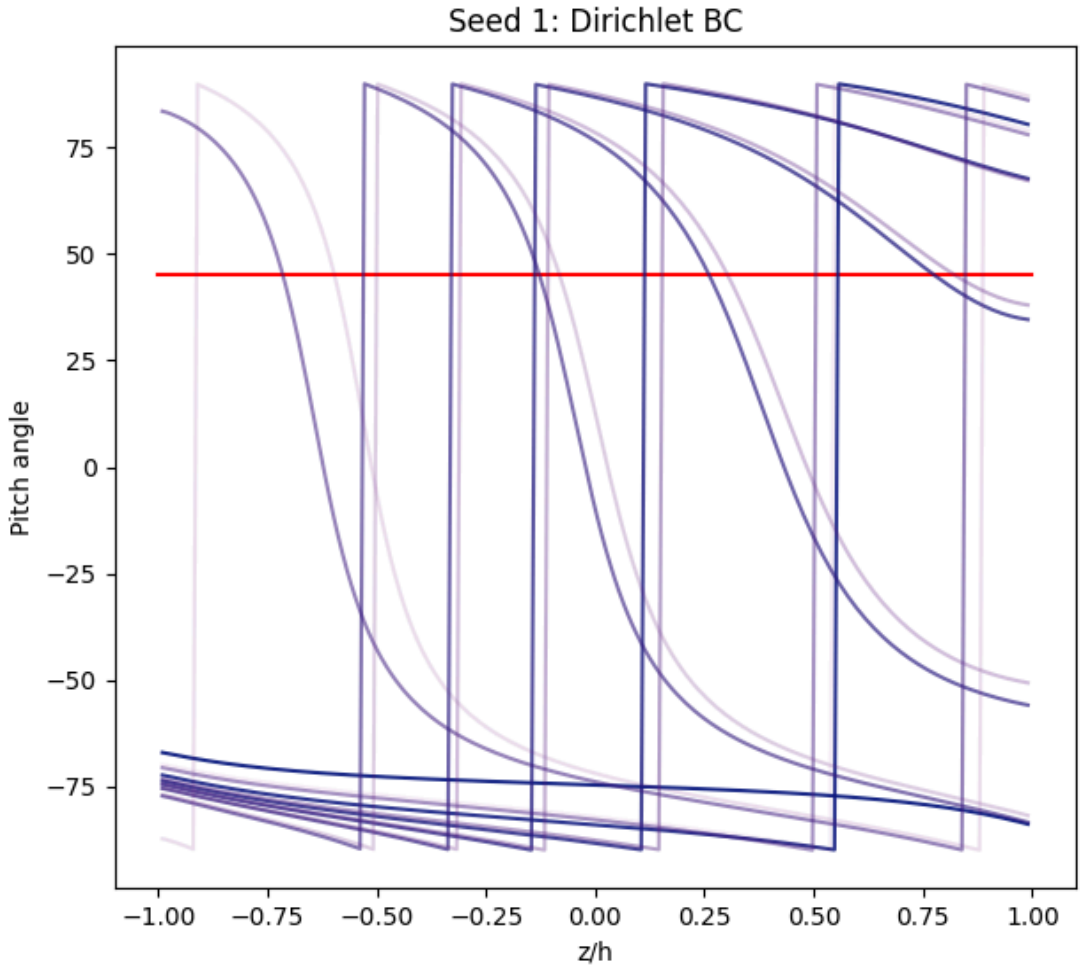


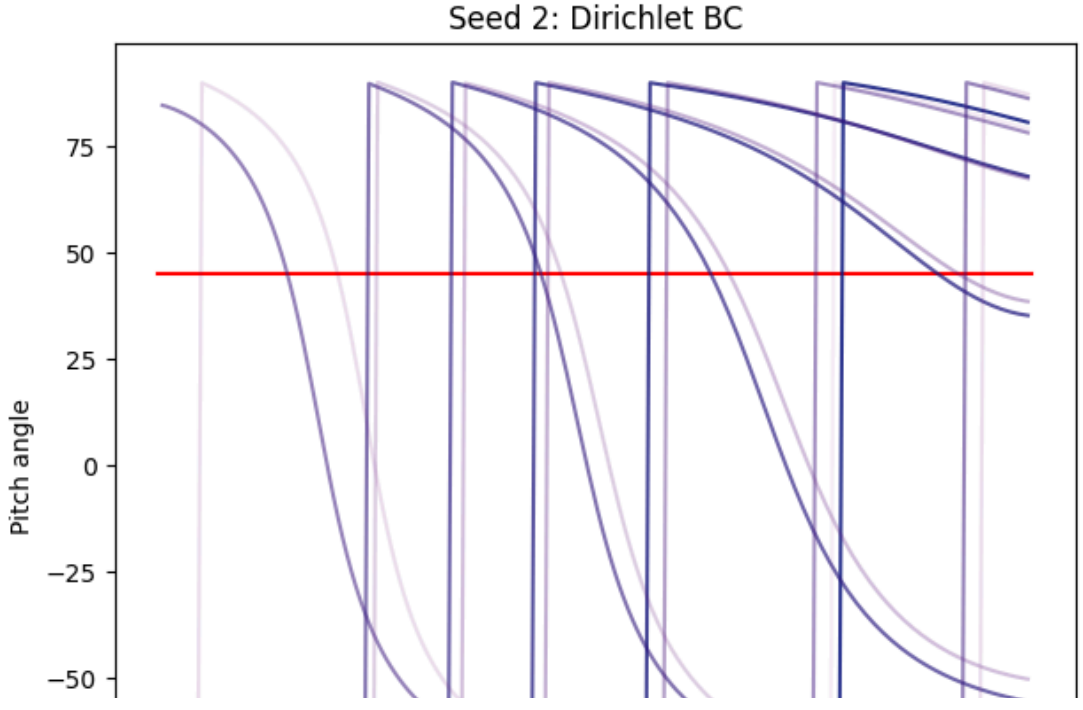
In [ ]:
```
i = 2000
fig6, axs6 = plt.subplots(3, 1, figsize = (7, 21))
fig6.suptitle("\n".join(wrap("Time evolution of pitch angle (deg) for D=-

axs6[0].plot(z, pitch(Br1_sol3[0], Bp1_sol3[0]), color = 'red')
axs6[0].set_title('Seed 1: Dirichlet BC')
axs6[0].set(xlabel='z/h', ylabel='Pitch angle')

axs6[1].plot(z, pitch(Br2_sol3[0], Bp2_sol3[0]), color = 'red')
axs6[1].set_title('Seed 2: Dirichlet BC')
axs6[1].set(xlabel='z/h', ylabel='Pitch angle')

axs6[2].plot(z, pitch(Br3_sol3[0], Bp3_sol3[0]), color = 'red')
axs6[2].set_title('Seed 3: Dirichlet BC')
axs6[2].set(xlabel='z/h', ylabel='Pitch angle')



while i < 3*10**4:
    axs6[0].plot(z, pitch(Br1_sol3[i], Bp1_sol3[i]), color = (round(i/(1*
    axs6[0].set_title('Seed 1: Dirichlet BC')
    axs6[0].set(xlabel='z/h', ylabel='Pitch angle')

    axs6[1].plot(z, pitch(Br2_sol3[i], Bp2_sol3[i]), color = (round(i/(1*
    axs6[1].set_title('Seed 2: Dirichlet BC')
    axs6[1].set(xlabel='z/h', ylabel='Pitch angle')

    axs6[2].plot(z, pitch(Br3_sol3[i], Bp3_sol3[i]), color = (round(i/(1*
```

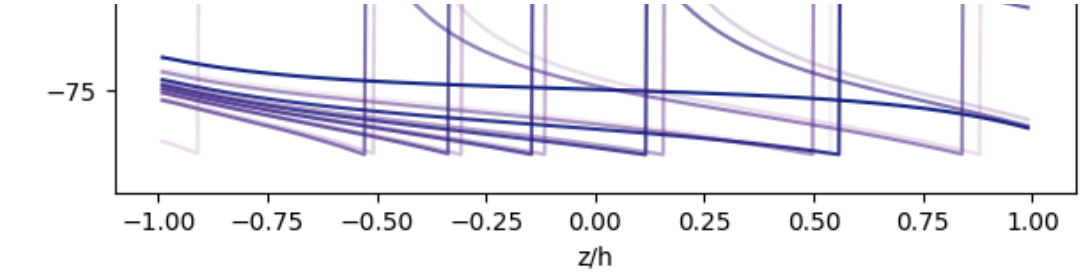
```
    axs6[2].set_title('Seed 3: Dirichlet BC')
    axs6[2].set(xlabel='z/h', ylabel='Pitch angle')

    i += 2000
```

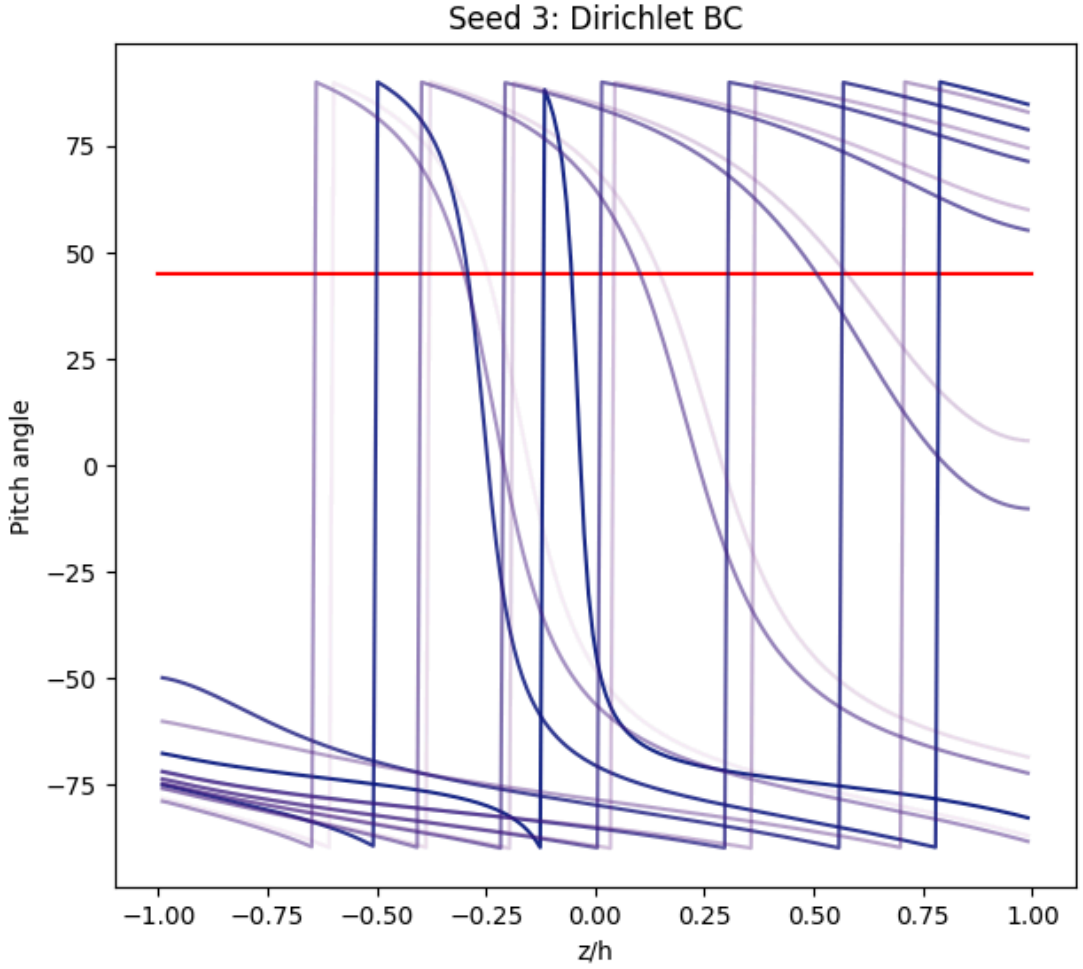
```
/tmp/ipykernel_41349/1302223438.py:2: RuntimeWarning: invalid value encoun
tered in divide
  return 180*np.arctan(Bp/Br)/np.pi
```

Time evolution of pitch angle (deg) for D=-10 (blue to red = early to late times, bright red = seed)









We can see that the pitch angle is not a constant, and that it oscillates with time, and also gets closer to 45 degrees in the case of D = -6. If we continue the simulation for longer, we should see the pitch angle slowly saturate to 45 degrees.

In the case of D = -10, since the field grows, the pitch angle doesn't necessarily have to saturate. We need to perform a longer simulation to see the effect clearly.